

# Communities in criminal networks: A case study

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## Abstract

Criminal organizations tend to be clustered to reduce risks of detection and information leaks. Yet, the literature has so far neglected to explore the relevance of subgroups for their internal structure. The paper applies methods of community analysis to explore the structure of a criminal network representing the individuals' co-participation in meetings. It draws from a case study on a large law enforcement operation (“Operazione Infinito”) tackling the 'Ndrangheta, a mafia organization from Calabria, a southern Italian region. The results show that the network is significantly clustered and that communities are partially associated with the internal organization of the 'Ndrangheta into different “locali” (similar to mafia families). Furthermore, community analysis methods can effectively predict the leadership roles (above 90% precision in classifying nodes as either bosses or non-bosses) and the locale membership of the criminals (up to two thirds of any random sample of nodes). The implications of these findings on the interpretation of the structure and functioning of the criminal network are discussed.

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## 1. Introduction

Academics and law enforcement agencies are increasingly applying network analysis to organized crime networks. While the current applications mainly focus on the identification of the key criminals through centrality measures (Varese, 2006b; Morselli, 2009; Calderoni, 2014) and other individual attributes (Carley et al., 2002; Morselli and Roy, 2008; Malm and Bichler, 2011; Bright et al., 2015), the analysis of the subgroups and their influence on the criminal activities received very limited attention so far.

Subgroups are a natural occurrence in criminal networks. Criminal organizations may structure themselves in functional, ethnic, or hierarchical units. Furthermore, the constraints of illegality limit information sharing to prevent leaks and detection, as criminal groups face a specific efficiency vs. security trade-off (Morselli et al., 2007). This makes criminal organizations globally sparse but locally clustered networks, often showing both scale-free and small-world properties (Malm and Bichler, 2011). Also, the larger the criminal organization, the most likely and relevant is the presence of subgroups. These considerations suggest that the analysis of subgroups in criminal networks may provide insight on both the internal structure of large organized crime groups and on the best preventing and repressive strategies against them.

The mafias are a clear example of large organized crime groups, often comprising several families or clans with a specific hierarchy and a strong cohesion. These units may show different interactions among them, ranging from open conflict to pacific cooperation. Each mafia family is a subgroup within a larger criminal network, and inter-family dynamics are determinant for the activities of the mafias. Nevertheless, possibly due to the difficulties in gathering reliable data, the literature has so far neglected the role of the family in the structure and the activities of the mafias.

In the literature of network analysis (e.g., Boccaletti et al., 2006; Barrat et al., 2008; Newman, 2010), one of the most challenging areas of investigation in recent years is *community analysis*, which is aimed at revealing possible subnetworks (i.e., groups of nodes called communities, or clusters, or modules) characterized by comparatively large internal connectivity, namely whose nodes tend to connect much more with the other nodes of the group than with the rest of the network. A large number of contributions have explored the theoretical aspects of community analysis and proposed a broad set of algorithms for community detection (Fortunato, 2010). Most notably, community analysis has revealed to be a powerful tool for deeply understanding the properties of a number of real-world complex systems in virtually any field of science, including biology (Jonsson et al., 2006), ecology (Krause et al., 2003), economics (Piccardi et al., 2010), information (Flake et al., 2002; Fortuna et al., 2011) and social sciences (Girvan and Newman, 2002; Arenas et al., 2004).

This paper aims to apply the methods of community analysis to criminal networks analyzing the co-participation in the meetings of a large mafia organization. The exercise aims to explore the relevance of subgroups in criminal networks, with a specific focus on the characterization of mafia clans and families and the identification of bosses. The case study draws data from a large law enforcement operation in Italy (“Operazione Infinito”), which arrested more than 150 people and concerned the establishment of several ’Ndrangheta (a mafia from Calabria, a southern Italian region) groups in the area around Milan, the capital city of the Lombardy region and Italy’s “economic capital” and second largest city. The exploration has a double relevance. First, it improves the understanding of the internal functioning of criminal organizations, demonstrating that the Infinito network is clustered in subgroups, and showing that the subgroups identified by community analysis overlap with the internal organization of the ’Ndrangheta. Second, it may contribute in the development of law enforcement intelligence capacities, providing tools for early identification of the internal structure of a criminal group.

The internal organization of the ’Ndrangheta provides an interesting oppor-

tunity to explore the relevance of subgroups in criminal networks. Indeed, this  
60 mafia revolves around the blood family (Paoli, 2003; Varese, 2006a). One or  
several 'Ndrangheta families, frequently connected by marriages, godfathering  
and similar social ties, form a "ndrina". The "ndrine" from the same area may  
form a "locale", which controls a specific territory (Paoli, 2007). The "locale"  
is the main structural unit of the 'Ndrangheta. Each "locale" has a number of  
65 formal charges, tasked with specific functions: the boss of the "locale" is the  
"capobastone" or "capolocale", the "contabile" (accountant) is responsible for  
the common fund of the locale, the "crimine" (crime) oversees violent actions,  
and the "mastro di giornata" (literally "master of the day") takes care of the  
communication flows within the "locale".

70 Since the organization in "locali" plays such an important role in the struc-  
ture of the 'Ndrangheta, our investigation is specifically oriented to assess their  
significance in the sense of community analysis. Therefore, after illustrating  
some details on the network data (Sec. 2), we first quantify the cohesiveness  
of each "locale" in the Infinito network, discovering a quite diversified picture  
75 where very cohesive "locali" coexist with others apparently not so significant.  
The results of community analysis (Sec. 3) show that the Infinito network is  
significantly clustered, suggesting that subgroups play an important role in its  
internal organization. If we try and match the clusters obtained by community  
analysis with the "locali" composition, we interestingly discover that in most  
80 cases clusters correspond either to "locali" or to unions of them. In the last  
two sections, we use the results from community analysis to identify the bosses  
and the locale membership of each network participant. Section 4 shows that  
community analysis can effectively identify the bosses in the Infinito network,  
yielding up to approximately 93% of correct predictions (nearly 60% for bosses  
85 only). Section 5 demonstrates the utility of community analysis in identifying  
the "locale" membership of the nodes. The best method correctly attributes  
the "locale" of up to 65% of a random sample of nodes.

## 2. The Infinito network

“Operazione Infinito” was aimed at disentangling the organizational structure of the 'Ndrangheta in Lombardy, with a special care in charting the hierarchical structure and the different “locali” existing in the region. The documentation<sup>2</sup> provides information on a large number of meetings among members. Indeed, most of the investigation focused on meetings occurring in private (e.g., houses, cars) or public places (e.g. bars, restaurants or parks). The two sets, namely meetings and participants, define a standard bipartite (two-mode) network. The projection of the bipartite network onto the set of 256 participants leads to a (one-mode) weighted, undirected network, whose largest connected component – which we will denote hereafter as the *Infinito network* – has  $N = 254$  nodes and  $L = 2132$  links (the density is  $\rho = 2L/(N(N - 1)) = 0.066$ ). The weight  $w_{ij}$  is the number of meetings co-participation between nodes  $i$  and  $j$ , and it ranges from 1 to 115. However, the mean value of the (nonzero) weights is  $\langle w_{ij} \rangle = 1.88$  and about 70% of them is 1, denoting that only very few pairs of individuals co-attended a large number of meetings. Similarly, the distributions of the nodes degree  $k_i$  and strength  $s_i = \sum_j w_{ij}$  display a quite strong heterogeneity: indeed, their average values are, respectively,  $\langle k_i \rangle = 16.8$  and  $\langle s_i \rangle = 31.5$ , but the most represented individual in the sample has both degree and strength equal to 1.

The affiliation of an individual to the “locale”, namely the group controlling the criminal activities in a specific territory, is formal and follows strict traditional rules. Each “locale” has a boss who is responsible of all the activities in front of the higher hierarchical levels (see Calderoni (2014) for further details). The investigation activity of “Operazione Infinito” was able to associate 177 individuals (out of 254) to one of the 17 “locali” identified in Milan area, the region under investigation. Of the remaining ones, 35 were known to belong to

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<sup>2</sup>Pretrial detention order issued by the preliminary investigation judge upon request by the prosecution (Tribunale di Milano, 2011).

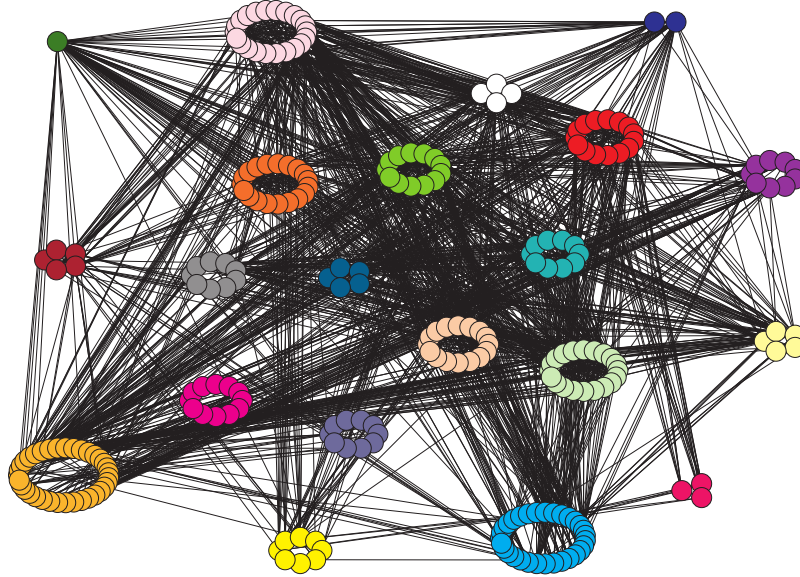


Figure 1: The Infinito network: nodes are grouped and colored according to the “locali” partition (Table 1).

115 “locali” based in Calabria (the region of Southern Italy where the ’Ndrangheta had origin and still has its headquarters), 3 came from a Lombardy “locale” not in the area of investigation (Brescia), and 8 were known to be non affiliated to ’Ndrangheta, whereas the correct classification of the remaining 31 individuals remained undefined. The Infinito network is displayed in Fig. 1. The figure<sup>3</sup>  
120 puts in evidence the 17 “locali” and the other groups above described.

As a first analysis, we assess whether the partition defined by the “locale” membership is significant in the sense of community analysis, namely whether the intensity of intra-“locale” meetings is significantly larger than that of the contacts among members of different “locali”. If so, this would confirm, on one  
125 hand, the actual modular structure of the crime organization; on the other hand, it would provide a tool for investigations, as the composition of the “locali” could

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<sup>3</sup>All network figures in the paper were produced with Pajek (Batagelj and Mrvar, 2004).

endogenously be derived by mining meetings data.

We denote by  $C_k$  the subgraph induced by the nodes belonging to “locale”  $k$ . We quantify the cohesiveness of  $C_k$  by the *persistence probability*  $\alpha_k$ , namely the probability that a random walker, which is in one of the nodes of  $C_k$ , remains in  $C_k$  at the next step. This quantity, which proved to be an effective tool for mesoscale network analysis (Piccardi, 2011; Della Rossa et al., 2013), reduces in an undirected network to:

$$\alpha_k = \frac{\sum_{i \in C_k} \sum_{j \in C_k} w_{ij}}{\sum_{i \in C_k} \sum_{j \in \{1, 2, \dots, N\}} w_{ij}}, \quad (1)$$

namely to the fraction of the strength of the nodes of  $C_k$  that remains within  $C_k$  (the same quantity is referred to as *embeddedness* by some authors (e.g., Hric et al., 2014)). Radicchi et al. (2004) defined *community* a subnetwork which has  $\alpha_k > 0.5$ . Obviously, the larger  $\alpha_k$ , the larger is the internal cohesiveness of  $C_k$ . Notice that, since  $\alpha_k$  tends to grow with the size  $N_k$  of  $C_k$  (trivially,  $\alpha_k = 1$  for the entire network), large  $\alpha_k$  values must be checked for their statistical significance. We derive the empirical distribution of the persistence probabilities  $\bar{\alpha}_k$  of the connected subgraphs of size  $N_k$  (we do that by randomly extracting 1000 samples), and we quantify the significance of  $\alpha_k$  by the *z-score*:

$$z_k = \frac{\alpha_k - \mu(\bar{\alpha}_k)}{\sigma(\bar{\alpha}_k)}. \quad (2)$$

A large value of  $\alpha_k$  (i.e.,  $\alpha_k > 0.5$ ) reveals the strong cohesiveness of the subgraph  $C_k$ , while a large value of  $z_k$  (i.e.,  $z_k > 3$ ) denotes that such a cohesiveness is not trivially due to the size of the subgraph, but it is anomalously large with respect to the subgraphs of the same size.

Table 1 summarizes the values of  $\alpha_k$  and  $z_k$  computed on the subgraphs corresponding to the “locali” (see Fig. 1). Notice that L2 to L18 actually refer to the 17 “locali” under investigation, all based in Milan area (Milan itself plus 16 small-medium towns); L19 collects the individuals, participating in some of the meetings, belonging to any of the Calabria “locali”, and L20 contains those affiliated to Brescia, not subject to investigation and whose members participated in the meetings only occasionally; L0 are the individuals with non

| “locale”   |                        | $N_k$     | $\alpha_k$  | $z_k$       |
|------------|------------------------|-----------|-------------|-------------|
| <i>L0</i>  | <i>not specified</i>   | 31        | 0.08        | -3.15       |
| <i>L1</i>  | <i>not affiliated</i>  | 8         | 0.03        | -0.84       |
| L2         | Bollate                | 13        | 0.25        | 1.31        |
| L3         | Bresso                 | 15        | 0.39        | 2.72        |
| L4         | Canzo                  | 2         | 0.10        | 0.47        |
| L5         | Cormano                | 22        | 0.41        | 3.96        |
| L6         | Corsico                | 4         | 0.12        | 0.21        |
| <b>L7</b>  | <b>Desio</b>           | <b>19</b> | <b>0.63</b> | <b>6.40</b> |
| L8         | Erba                   | 9         | 0.37        | 2.44        |
| <b>L9</b>  | <b>Giussano</b>        | <b>10</b> | <b>0.63</b> | <b>5.26</b> |
| L10        | Legnano                | 10        | 0.20        | 0.77        |
| L11        | Limbate                | 1         | 0           | -           |
| L12        | Mariano Comense        | 9         | 0.27        | 1.40        |
| <b>L13</b> | <b>Milano</b>          | <b>16</b> | <b>0.62</b> | <b>5.78</b> |
| L14        | Pavia                  | 5         | 0.13        | 0.25        |
| L15        | Pioltello              | 20        | 0.43        | 3.83        |
| L16        | Rho                    | 5         | 0.18        | 0.78        |
| <b>L17</b> | <b>Seregno</b>         | <b>12</b> | <b>0.93</b> | <b>8.73</b> |
| L18        | Solaro                 | 5         | 0.06        | -0.42       |
| <i>L19</i> | <i>Calabria locali</i> | 35        | 0.19        | -0.97       |
| <i>L20</i> | <i>Brescia</i>         | 3         | 0.17        | 0.98        |

Table 1: Testing the “locale” partition. In bold, the four “locale” with significant cohesiveness ( $\alpha_k > 0.5$ ).

specified affiliation, L1 those who are not affiliated. Overall, only 4 “locale”  
155 out of 17 reveal strong – and statistically significant – cohesiveness, proving  
to actually behave as communities in the sense of network analysis. Most of  
the other ones, however, display very mild cohesiveness. It cannot be claimed,  
therefore, that the “locale” partition as a whole is significant in functional terms.  
In the next section, we analyze whether the network is actually organized around  
160 a different clusterization.

### 3. Community analysis

Given a partition  $C_1, C_2, \dots, C_K$  of the nodes of a weighted, undirected network into  $K$  subgraphs, the *modularity*  $Q$  (Newman, 2006; Arenas et al., 2007) is given by

$$Q = \frac{1}{2s} \sum_{k=1,2,\dots,K} \sum_{i,j \in C_k} \left( w_{ij} - \frac{s_i s_j}{2s} \right), \quad (3)$$



|    | $N_k$ | $\alpha_k$ | $z_k$ |
|----|-------|------------|-------|
| C1 | 12    | 0.93       | 9.07  |
| C2 | 18    | 0.72       | 7.79  |
| C3 | 25    | 0.66       | 9.85  |
| C4 | 25    | 0.63       | 9.11  |
| C5 | 45    | 0.68       | 8.20  |
| C6 | 62    | 0.78       | 8.30  |
| C7 | 67    | 0.67       | 5.72  |

Table 2: Results of max-modularity community analysis

165 where  $s = \sum_i s_i/2$  is the total link weight of the network. Modularity  $Q$  is the (normalized) difference between the total weight of links internal to the subgraphs  $C_k$ , and the expected value of such a total weight in a randomized “null network model” suitably defined (Newman, 2006). Community analysis seeks the partition with the largest  $Q$ : large values ( $Q \rightarrow 1$ ) typically reveal  
170 a high network clusterization. Although the exact max- $Q$  solution cannot be obtained because computationally unfeasible even for small-size networks (Fortunato, 2010), many reliable sub-optimal algorithms are available: here we use the so-called “Louvain method” (Blondel et al., 2008).

The result is a partition with 7 clusters ( $Q = 0.48$ ), whose data are reported  
175 in Table 2. All clusters are strongly cohesive ( $\alpha_k$  much larger than 0.5, with large  $z_k$ ). Overall, the Infinito network is therefore strongly clusterized, with community size from small (12) to medium-large (67, about 26% of the network size).

The max-modularity partition of the Infinito network is displayed in Fig.  
180 2. The patterns of node colors – which refer to the “locali”, see Fig. 1 – denote a non trivial relationship between the “locali” partition and the max-modularity partition. To disentangle this aspect, we pairwise compare the “locali”  $L_0, L_1, \dots, L_{20}$  (Table 1) and the communities  $C_1, C_2, \dots, C_7$  obtained by max-modularity (Table 2), quantifying similarities by *precision* and *recall* (e.g.,  
185 Baeza-Yates and Ribeiro-Neto, 1999). Let  $m_{hk}$  be the number of nodes classified both in  $L_h$  and in  $C_k$ . Then the precision  $p_{hk} = m_{hk}/|C_k|$  is the fraction of the nodes of  $C_k$  that belongs to  $L_h$  whereas, dually, the recall  $r_{hk} = m_{hk}/|L_h|$  is the fraction of the nodes of  $L_h$  that belongs to  $C_k$ . If we interpret  $L_h$  as the

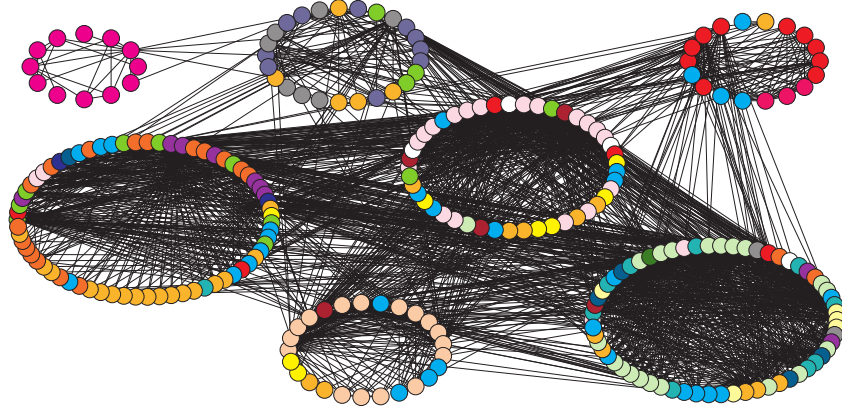


Figure 2: The Infinito network: nodes are grouped according to the max-modularity partition (Table 2) and colored according to the “locali” partition (Table 1).

“true” set and  $C_k$  as its “prediction”, then the precision quantifies how many  
 190 of the predicted nodes are true, and the recall how many of the true nodes are  
 predicted. Then  $p_{hk} = r_{hk} = 1$  if and only if the sets  $L_h$  and  $C_k$  coincide, while  
 $p_{hk} \rightarrow 1$  if most of the nodes of  $C_k$  belong to  $L_h$ , and  $r_{hk} \rightarrow 1$  if most of the  
 nodes of  $L_h$  are included in  $C_k$ .

Figure 3 (upper panels) summarizes the results of this analysis by a graphical  
 195 representation of the precision and recall matrices. We firstly note that “locale”  
 L17 perfectly matches community C1 (it is the community in the upper-left  
 corner of Fig. 2). Moreover, “locale” L13 can be approximately identified with  
 C3, whereas C2 corresponds to a large extent to the union of L3 and L20, and  
 C4 to the union of L9 and L12. But also the last three columns of the recall  
 200 matrix clearly put in evidence that C5, C6 and C7 actually behave, to a large  
 extent, as unions of “locali”. This clearly emerges from the lower panels of  
 Fig. 3, where the precision/recall analysis is performed again but after “locali”  
 have been partially aggregated in 7 supersets: the diagonal dominance of the  
 matrices  $p_{hk}, r_{hk}$  highlights that, overall, the Infinito network is quite strongly  
 205 compartmentalized (see again Table 2), and the compartments coincide to a

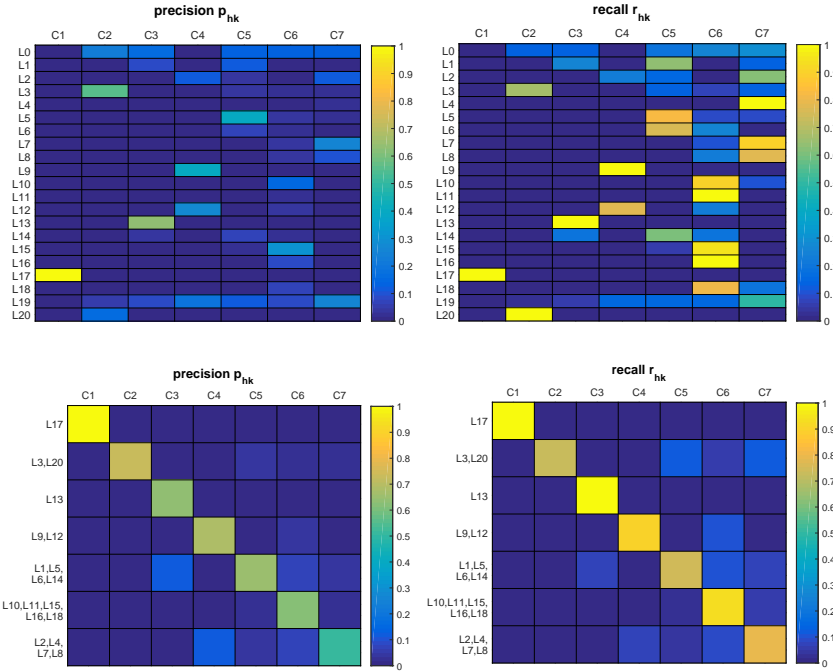


Figure 3: Precision/recall matrices of the comparison between the “locali” and the max-modularity communities. Above: the “locali”  $L_0, L_1, \dots, L_{20}$  are compared with the communities  $C_1, C_2, \dots, C_7$ . Below: after “locali” have been partially aggregated, the diagonal dominance of the precision/recall matrices evidences that communities coincide to a large extent with unions of “locali”.

large extent with single “locali” or unions of them.

These findings support the intuition that subgroups are important elements in the internal organizations of the mafias. The clusterization of unions of “locali” may suggest that clans or families may have closer connections with a few others. Several investigations showed that “locali” may raise and decline, compete or collaborate, merge or separate. Based on meeting co-participation patterns, community analysis methods can effectively reveal a clusterization closely connected with the formal structure of the mafia. The next two sections will explore whether community analysis techniques can further contribute to identifying the bosses and the “locale” membership.

#### 4. Identifying bosses

In this section we focus on the relation between the hierarchical role of individuals within the 'Ndrangheta organization, and the pattern of their meeting attendance, as modeled by the Infinito network. The aim is to explore whether  
220 the results from community analysis can provide tools to identify individuals with leading roles, who will be referred to as *bosses* from now on. As already pointed out in Sec. 1, the 'Ndrangheta relies on a formal hierarchy with multiple ranks and offices. In particular, each locale normally appoints a few major officers: the capobastone or capolocale is the head of the locale; the contabile  
225 is the accountant who manages the common fund of the group; the crimine (crime) oversees violent actions; the mastro di giornata (master of the day) ensures the flow of information within the locale (Calderoni, 2014). Information on the actual number and roles of the offices in the 'Ndrangheta is incomplete. Yet, in some investigations the suspects discuss about the different offices: these  
230 conversations are sometimes tapped by the police, as in the Infinito case.

The judicial documentation classifies 34 of the 254 nodes of the Infinito network as bosses. Calderoni (2014), working on the unweighed network, investigated the correlation between a set of node centrality measures (including degree, strength, betweenness, closeness, and eigenvector centrality) and the  
235 boss role of the node, finding that betweenness is by far the most effective predictor. Indeed, the average betweenness of bosses turns out to be about 15 times larger than that of non-bosses, testifying a brokering role of bosses within the criminal network.

Here we want to further improve the predictive performance by exploiting the  
240 information provided by community analysis. As a matter of fact, the partition induced by max-modularity has the effect of placing each node in a specific position in terms of intra-/inter-community connectivity, an information that can potentially be useful in assessing its functional role.

#### 4.1. $z$ - $P$ analysis

245 We follow the  $z$ - $P$  analysis approach proposed by Guimera and Amaral (2005) (see also Guimera et al. (2005)) where, after community analysis has identified a partition into  $K$  modules, the intra- vs inter-community role of each node  $i$  is quantified by a pair of indexes  $(z_i, P_i)$ . We denote by  $c(i) \in \{1, 2, \dots, K\}$  the community node  $i$  belongs to, and by  $s_i^{c(i)} = \sum_{j \in c(i)} w_{ij}$  the  
250 *internal strength* of  $i$ , i.e., the strength directed towards nodes of  $c(i)$ . By straightforwardly extending the definitions of Guimera and Amaral (2005) to the case of weighted networks, we define the *within-community strength* as

$$z_i = \frac{s_i^{c(i)} - \mu(s_i^{c(i)})}{\sigma(s_i^{c(i)})}, \quad (4)$$

where  $\mu(s_i^{c(i)})$  and  $\sigma(s_i^{c(i)})$  are the mean and standard deviation of  $s_i^{c(i)}$  over all nodes  $i \in c(i)$ , and the *participation coefficient* as

$$P_i = 1 - \sum_{c=1}^K \left( \frac{s_i^c}{s_i} \right)^2, \quad (5)$$

255 where  $s_i^c = \sum_{j \in c} w_{ij}$  is the strength of node  $i$  directed towards nodes of community  $c$ . The normalized internal strength  $z_i$  measures how strongly a node is connected within its own community. On the other hand,  $P_i$  quantifies to what extent a node tends to be uniformly connected to all communities ( $P_i \rightarrow 1$ ) rather than only to its own community ( $P_i \rightarrow 0$ ).

260 Figure 4 shows the results of the  $z$ - $P$  analysis of the Infinito network (notice that we normalize  $z_i$  to take values in the  $[0, 1]$  interval, i.e.,  $z_i \rightarrow (z_i - \min z_i) / (\max z_i - \min z_i)$ ). The figure highlights that bosses tend to concentrate on the upper-right part of the plot, namely they have both within-module strength  $z_i$  and participation coefficient  $P_i$  larger than average. As a matter of  
265 fact, the ratio between the values of the two indicators for bosses and non-bosses is 2.51 for  $z_i$ , and 2.30 for  $P_i$ . It seems, therefore, that leading individuals have a twofold characterization, namely a connectivity larger than average within their own community, and at the same time the capability of connecting to a large number of the other communities. In order to get the most effective prediction,

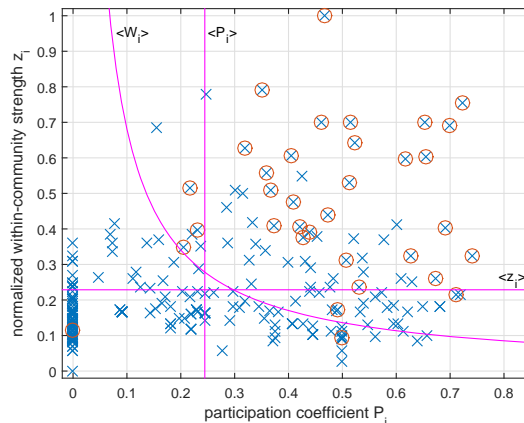


Figure 4: z-P analysis of the Infinito network. Each node is identified by a cross corresponding to the  $(z_i, P_i)$  coordinates: bosses are highlighted by red circles. The magenta lines correspond to the average value of  $z_i$ ,  $P_i$ , and  $W_i$ , over all nodes.

270 we can combine the role of  $z_i$  and  $P_i$  in a unique indicator defined as the product  $W_i = z_i P_i$ . The ratio between the  $W_i$  value for bosses and non-bosses is 5.46: as evidenced in Fig. 4, only 2 bosses out of 34 have  $W_i$  lower than average.

We now want to explicitly quantify the predictive ability of the z-P analysis in identifying the leading roles within the criminal network, and compare it  
 275 with a non community-based indicator such as the betweenness  $b_i$ . For that, first notice that all the indicators  $b_i$ ,  $z_i$ ,  $P_i$ , and  $W_i$  induce a ranking in the set of 254 nodes. Table 3 summarizes the performance of the above indicators in terms of their predictive precision, assuming to know the exact number of bosses to be guessed (i.e., 34). In other words, we count how many of the top-34 nodes in the  
 280 relevant indicator's ranking are actually bosses. While the P-score alone seems unable to effectively capture the leading nodes, the z-score and the betweenness both identify 23 bosses (although the two sets are slightly different), but the zP-score outperforms all the methods identifying 25 bosses over 34.

One may wonder to what extent the above performances are influenced by  
 285 the assumption of knowing exactly the number of bosses, an information not available in reality. For these reasons, we refine our analysis and compute the

| method            | precision |
|-------------------|-----------|
| zP-score $W_i$    | 0.735     |
| z-score $z_i$     | 0.677     |
| betweenness $b_i$ | 0.677     |
| P-score $P_i$     | 0.294     |

Table 3: Identifying bosses: for each method, the precision is computed as the fraction of true bosses among the top 34 nodes ranked by the related indicator.

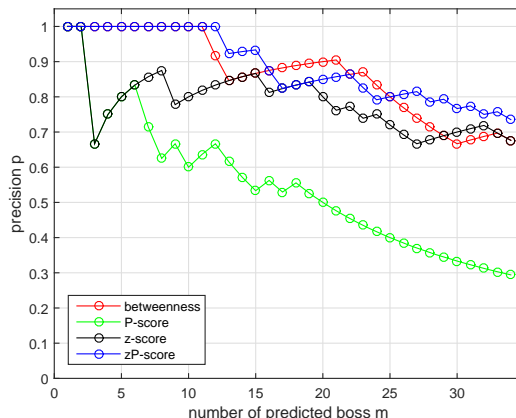


Figure 5: Identifying bosses: for a given number of predicted boss  $m$ , the precision is computed as the fraction of true bosses among the top  $m$  nodes ranked by the related indicator.

precision  $p$  for all methods as a function of the number  $m = 1, 2, \dots, 34$  of guessed bosses, i.e., we take the top- $m$  nodes for each index and we compute how many of them actually correspond to bosses:

$$p = \frac{\# \text{ of nodes correctly guessed among } m \text{ nodes}}{m}. \quad (6)$$

290 The precision  $p$  as a function of  $m$  is depicted, for all methods, in Fig. 5. Overall, the zP-score has the best performance, with 100% precision up to  $m = 12$  and a good performance even for the largest  $m$  values. Betweenness is a valid alternative, displaying comparable performances except for large  $m$ .

#### 4.2. Integrating network-based measures

295 We complement the previous analysis through a set of multiple logistic regressions estimating the influence of different factors on the probability of being

| variable         | min | max | mean  | st.dev. |   | 2 | 3   | 4   | 5   | 6   |
|------------------|-----|-----|-------|---------|---|---|-----|-----|-----|-----|
| 1 boss           | 0   | 1   | 0.134 | 0.341   |   |   |     |     |     |     |
| 2 betweenness    | 0   | 100 | 4.23  | 11.7    | 2 | - | .77 | .73 | .76 | .83 |
| 3 strength       | 1   | 361 | 31.6  | 48.2    | 3 |   | -   | .81 | .84 | .88 |
| 4 z-score        | 0   | 1   | 0.228 | 0.158   | 4 |   |     | -   | .82 | .67 |
| 5 zP-score       | 0   | 100 | 12.5  | 17.2    | 5 |   |     |     | -   | .70 |
| 6 n. of meetings | 1   | 179 | 7.29  | 16.7    | 6 |   |     |     |     | -   |
| 7 mafia charge   | 0   | 1   | 0.5   | 0.5     |   |   |     |     |     |     |

Table 4: Descriptive statistics (left) and Pearson’s correlation coefficients (right) of the variables used in the regression (all correlations are statistically significant at  $p < 0.001$  level). To improve the readability of the results, betweenness and zP-score have been normalized to the  $[0, 100]$  range.

a boss. This integrates and expands the analyses of Calderoni (2014, 2015), which were restricted to the individual centrality measures on the subset of meetings with more than 3 participants (215 nodes).

300 The dependent dichotomous variable is derived from the judicial documents (1 for bosses, and 0 for non-bosses). Independent variables include two of the network centrality measures retained in Calderoni (2014), namely the betweenness and the strength, and the z-score and zP-score from the previous subsection. The models also include two control variables: the first is the number of  
305 meetings attended by each individual, the second (mafia charge) is a dummy one describing whether an individual was charged with the offence of mafia-type association in the court order, a possible bias in the network (Table 4).

Given the low number of bosses in the sample (34 out of 254), in the logistic regressions we adopt the penalized maximum likelihood estimation proposed by  
310 Firth (1993). This method compensates for low numbers in one of the categories of the dependent variable, making it a good approach for the Infinito network. As for the standard logistic regressions, it models a dichotomous dependent variable  $y$  (in this case, the boss attribute) as a linear combination of independent variables  $x_i$  ( $y = a + b_1x_1 + b_2x_2 + \dots$ ). The outcomes can be  
315 expressed as odds ratio (OR), where  $OR = \exp(b_i)$ . In the present application, OR expresses the change in the probability that a node is a boss per unitary increase in any independent variable, all other variables equal. For  $OR = 1$  the



probability is the same, for  $OR > 1$  it increases, and for  $OR < 1$  it decreases. For example,  $OR = 1.1$  means that a unitary increase in the independent variable implies a +10% increase in the probability of a node being a boss. Since  
320 the logistic regression predicts the value of the dependent variable based on the values of the independent variables, comparison between predicted and observed values enables to assess its predictive power (percentage of correct predictions) (Hosmer et al., 2013).

325 The results are summarized in Table 5. Model I replicates the best model from Calderoni (2014) on a wider sample, yielding very similar results. A unit increase in betweenness centrality provides +11% increase in the probability of being a boss, all other variables equal. The strength contributes with a +3.5% increase in probability. The model correctly classifies 94.1% of the population  
330 and 61.8% of bosses (compare with a random probability of 13.3%). Model II relies only on the control variables, mafia charge and number of meetings. Both are significant and positive. Yet the overall capacity of the model is lower than the first one (90.9%), with a remarkable decrease in the identification of bosses (41.2%). Model III includes both individual centrality measures and the  
335 controls. Both strength and betweenness maintain their significant and positive effect, whereas the controls are non-significant. The prediction success are similar to model I, especially for bosses. Models IV to VI test the community measures identified in the previous section. Model IV shows that z-score has no significant impact on the probability of being a boss, once tested along with  
340 the control variables. Conversely, in Model V the zP-score has a statistically significant and positive influence (+9.8% per unitary increase of zP-score) despite the presence of the controls. The last model (VI) includes the controls and both betweenness and zP-score. The latter results as the only significant variable with an impact of +8.6% on the probability of being a boss, all other  
345 variables equal. Overall, the share of correct predictions is slightly lower than models I and III, with the best results in model VI (92.9% and 58.8% for total correct predictions and correct boss predictions, respectively).

The regressions corroborate the results of the previous section. Network

|                        | I        | II       | III     | IV      | V        | VI      |
|------------------------|----------|----------|---------|---------|----------|---------|
| strength               | 1.035*** | -        | 1.032** | -       | -        | -       |
| betweenness            | 1.111**  | -        | 1.108*  | -       | -        | 1.035   |
| mafia charge           | -        | 12.87*   | 4.501   | 8.400*  | 5.444    | 5.172   |
| n. of meetings         | -        | 1.167*** | 0.982   | 1.116** | 1.057    | 1.046   |
| z-score                | -        | -        | -       | 33.34   | -        | -       |
| zP-score               | -        | -        | -       | -       | 1.098*** | 1.086** |
| true non-bosses        | 218      | 217      | 217     | 217     | 216      | 216     |
| false non-bosses       | 13       | 20       | 13      | 16      | 15       | 14      |
| true bosses            | 21       | 14       | 21      | 18      | 19       | 20      |
| false bosses           | 2        | 3        | 3       | 3       | 4        | 4       |
| precision (total) [%]  | 94.1     | 90.9     | 93.7    | 92.5    | 92.5     | 92.9    |
| precision (bosses) [%] | 61.8     | 41.2     | 61.8    | 52.9    | 55.9     | 58.8    |

Table 5: Results of Firth’s logistic regressions on bosses. The upper part of the table reports the odds ratio with the statistical significance ( $*p < 0.05$ ,  $**p < 0.01$ ,  $***p < 0.001$ ), the bottom part summarizes the predictive capabilities (percentage of correct predictions) of Models I-VI described in the text.

analysis measures can effectively predict the leadership roles of individuals in a  
350 criminal network. All network measures perform better than naturally observ-  
able variables such as the two controls. Centrality measures are effective and  
yield the highest share of correct predictions. Among community measures,  
zP-score has a significant capacity to predict bosses. In a model with central-  
ity measures and controls, zP-score is the only statistically significant variable,  
355 indicating a strong capacity to capture the behavior of leaders in criminal net-  
works.

These findings expand the literature on leadership in criminal networks, as  
previous studies mainly relied on centrality measures only, often finding that  
betweenness centrality identified leadership roles within crime groups (Morselli,  
360 2009; Calderoni, 2014). Whereas the previous studies pointed out the role of  
brokering positions, they neglected the analysis of subgroups and its implications  
for leadership. The application of community analysis measures shows that  
criminal leaders not only have a notable brokering capacity, but also manage to  
balance the connection within and outside their group. These results advocate  
365 for expanding the concept of brokerage beyond individuals measures. In fact,  
bosses not only meet unconnected individuals, but also have a crucial function  
in bridging their group with other groups.

## 5. Identifying the “locale” membership

In this section we consider the problem of identifying the “locale” membership of those individuals for which such an information is unknown. In the Infinito network (254 nodes), this problem arises for 31 nodes (see Table 1, row L0).

The problem can be set in the general framework of *label prediction* (Zhang et al., 2010): we are given a set of network nodes  $X = \{x_1, x_2, \dots, x_{254}\}$  and a set of labels  $L = \{L_1, L_2, \dots, L_{20}\}$  which, in our case, code the “locali” of the criminal organization (Table 1). The majority of the nodes have a label:  $L_h$  is assigned to node  $x_i$  (and we write  $L(x_i) = L_h$ ) if  $x_i$  is affiliated to “locale”  $L_h$ . The correspondence nodes/labels is, however, partially unknown, since there are 31 nodes of  $X$  whose labeling is unknown and must be predicted based on the network structure and on the known labels.

A very general approach to the above problem relies on the notion of *node similarity*, based on the assumption that the more two nodes are similar (in a sense to be defined – see below), the more likely their label is the same. Therefore, once defined a similarity score  $s_{ij}$  between nodes  $(x_i, x_j)$ , the probability that the unlabeled node  $x_i$  has label  $L_h$  is assumed equal to

$$p(L(x_i) = L_h) = \frac{\sum_{\{x_j | j \neq i, L(x_j) = L_h\}} s_{ij}}{\sum_{\{x_j | j \neq i, L(x_j) \in L\}} s_{ij}}, \quad h = 1, 2, \dots, 20. \quad (7)$$

In words,  $p(L(x_i) = L_h)$  counts the relative abundance of nodes labeled  $L_h$  in the network, and weights each of these nodes by its similarity to  $x_i$ . The label predicted for node  $x_i$  is the one attaining the largest  $p(L(x_i) = L_h)$ .

### 5.1. Node similarities

We consider and test four definitions of the similarity score  $s_{ij}$ : (i) and (ii) are very popular and find many applications in social network analysis (e.g., Lü and Zhou (2011)), (iii) and (iv) exploit the partition found by max-modularity community analysis (Sec. 3).

395 (i) *Common Neighbors (CN)*: denoting by  $\Gamma(x_i)$  the set of nodes neighbors to  $x_i$ , we let

$$s_{ij} = |\Gamma(x_i) \cap \Gamma(x_j)|, \quad (8)$$

where  $|Q|$  denotes the number of elements of the set  $Q$ .

(ii) *Weighted Common Neighbors (wCN)*: it generalizes the above definition by exploiting the information on link weights (Lü and Zhou, 2010):

$$s_{ij} = \sum_{k \in \{\Gamma(x_i) \cap \Gamma(x_j)\}} \frac{w_{ik} + w_{kj}}{2}. \quad (9)$$

400 (iii) *Common Community (CC)*: a binary indicator, stating that similarity is equivalent to the membership to the same community:

$$s_{ij} = \begin{cases} 1, & \text{if } c(i) = c(j), \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where  $c(i)$  denotes the community node  $i$  belongs to.

(iv) *Weighted Common Neighbors - Common Community (wCN-CC)*: it combines (ii) and (iii). It is equal to the Weighted Common Neighbors similarity, but it is nonzero only when  $(x_i, x_j)$  are in the same community:

$$s_{ij} = \begin{cases} \sum_{k \in \{\Gamma(x_i) \cap \Gamma(x_j)\}} \frac{w_{ik} + w_{kj}}{2}, & \text{if } c(i) = c(j), \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

## 5.2. Results

The label identification procedure, with the different node similarities above defined, has been tested on the Infinito network. Unfortunately, the specificity  
410 of the case does not allow one to validate the method on the 31 nodes which are actually unlabeled – their “locale” is unknown by definition. Thus the procedure has been applied to the 177 nodes with known label  $L2, L3, \dots, L18$  (the “locali” in Milan area, the region under investigation – see Table 1), assuming their label is unknown and trying to recover it.

415 In order to mimic the real situation, in which an entire pool of labels have to  
be simultaneously identified, in our experiments we assume that the labels of  $m$   
nodes have to be reconstructed at the same time, and we test the effectiveness  
of the procedure by letting  $m$  increasing from 1 to 30. For each  $m$ , we randomly  
extract  $5 \times 10^3$  samples of  $m$  nodes in “locali”  $L2, L3, \dots, L18$ , and predict  
420 simultaneously their labels via equation (7). For each sample, we compute the  
precision as the fraction of correct guesses. More in detail, for each node under  
test we increment a success counter  $s$  by 1 if the label which maximizes the  
probability (7) is the correct node label, while if the probability of  $r > 1$  labels  
is equally maximal in (7) we increment the counter by  $1/r$  if the correct node  
425 label is one of them. For the  $m$ -node sample, the precision of the reconstruction  
is eventually given by  $s/m$ .

Figure 6 summarizes the results, in terms of mean and standard deviation  
of the precision over the samples, for all  $m = 1, 2, \dots, 30$  and for the four  
similarity measures above defined. In principle, we expect that the larger  $m$ ,  
430 the more difficult the prediction task, since the latter is based on a smaller  
set of known labels. In this respect, the results are rather counterintuitive.  
Firstly, the average precision is largely insensitive to  $m$ , and ranges from about  
45% to 65% according to the similarity measure adopted. Notably, the best  
performing method (wCN-CC) exploits the analysis of the community structure  
435 of the network. Secondly, the variability of the precision rate displays a clear  
decreasing trend as  $m$  increases. This behaviour is due to a sort of “large  
numbers” effect: when very few labels are to be guessed, the success depends  
very much on the specific nodes under scrutiny. When a large pool of nodes  
are instead investigated, successes and failures tend to balance in a proportion  
440 which mildly depends on the specific set of nodes. Overall, this latter analysis  
confirms that, on the Infinito network, the precision of the label reconstruction  
procedure can reach a proportion of about two thirds, even for sets of the same  
order of magnitude of the real unlabeled set  $L0$ .

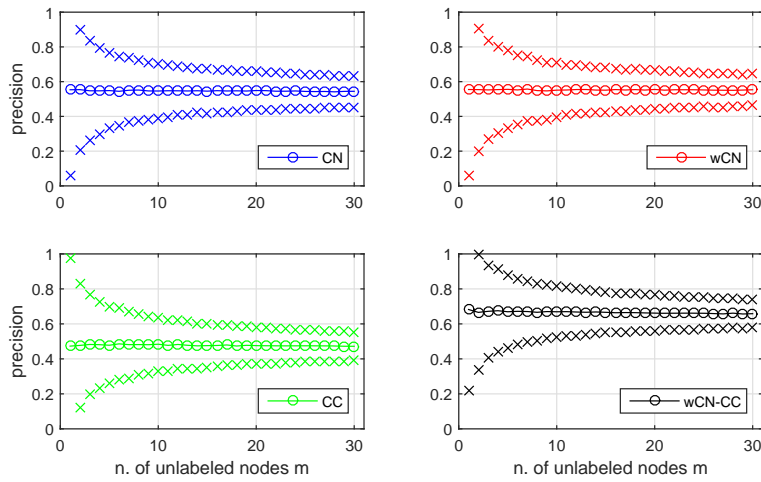


Figure 6: Precision of the label identification methods with respect to the number  $m$  of unlabeled nodes. The curves represent the average precision (circles) plus/minus standard deviation (crosses) over  $5 \times 10^3$  random samples of  $m$  nodes (CN: Common Neighbors; wCN: Weighted Common Neighbors; CC: Common Community; wCN-CC: Weighted Common Neighbors - Common Community).

## 6. Concluding remarks

445 This paper applied community analysis methods to investigate the structure of a mafia organization. Focusing on meeting participation as a proxy for the relationships among criminals, community analysis assessed the clusterized structure of the mafia and showed that it often mirrors the internal subdivision of the mafia among several clans or “locali”, or unions of them. This supports  
450 the intuition that subgroups matter in this type of organizations.

In the light of these findings, the study tested the capacity of community analysis techniques to identify relevant characteristics of the criminal organization, namely leadership roles and “locali” membership. The results show that the zP-score, which captures the interplay between a node connectivity  
455 within its community and to the other communities, can effectively single out the bosses of the mafia. Furthermore, the most effective method for identifying the “locale” membership of the nodes focuses again on the connectivity within the same community.

Overall, these findings reinforce the idea that the tools of network analysis  
460 can be fruitfully adopted to enhance the understanding of the structure and function of organized crime, albeit their use as a support for law enforcement intelligence still needs further exploration.

The research can be extended in many directions. First of all, a deeper structural analysis on a pool of criminal networks would be needed, aimed at  
465 assessing whether peculiar structural attributes turn out to be recurrent in such networks. Then, coming back to the problem of community detection, other methods might prove to be more effective – including those specifically devoted to bipartite networks, as it is our data structure before projection (see Sec. 2). Finally, once the structure has been thoroughly understood, the challenge  
470 is clearly that of linking it with the function of the network, namely to fully understand how structural properties relate to criminal activities.

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