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**DIRECT PRICE DISCRIMINATION AND PRODUCT
DIFFERENTIATION IN THE HOTELLING
FRAMEWORK**

Tesi di Dottorato di: STEFANO COLOMBO

Matricola: 3406127

Anno Accademico 2007 – 2008



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Coordinatore: Ch.mo Prof. Massimo Bordignon

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Introduction

Price discrimination is a business practice which has received a lot of attention by economists. However, the analysis has been addressed for a long time toward monopoly price discrimination. Only from the second half of the Eighties, the implications of price discrimination have been investigated in oligopolistic settings. Notwithstanding the progress in this field, many interactions of price discrimination in oligopolistic markets are still to be understood. The aim of this thesis is to contribute from a theoretical point of view to the economic understanding of price discrimination in spatial oligopolies.

The thesis is composed by three chapters. In Chapter 1, we provide a selective survey of the main contributions regarding price discrimination and product differentiation in the Hotelling framework. The contributions surveyed in Chapter 1 can be classified into two broad categories: a group of papers studies the implications of price discrimination in spatial oligopoly, while another group of papers studies the relationship between product differentiation and sustainability of collusion. The first group of contributions emphasizes that: i) price discrimination tends to decrease equilibrium prices with respect to uniform price regime; ii) firms are usually trapped into a Prisoner Dilemma, since the dominant strategy for each firm is to price discriminate, but the equilibrium profits are lower than under uniform pricing. The second chapter of this thesis investigates further on these issues. The second group of contributions shows that collusion is easier to sustain the more the firms are differentiated. In general, these papers do not allow for price discrimination: in the third chapter of this thesis we consider the case of price discrimination.

In Chapter 2, by using the Hotelling duopoly, we study the firms' incentive to price discriminate when the product differentiation degree is endogenous. Two different versions of a three-stage game are considered. In the first version, firms first simultaneously choose which variety to produce, then they choose whether to price discriminate or not, then they set the price schedules. The Prisoner Dilemma arises: firms price discriminate and profits are lower than under uniform pricing. In the second version of the game, firms first choose the pricing policy and then they choose the variety. Interestingly, in this case the equilibrium is characterized by uniform pricing and no Prisoner Dilemma exists. This is due to the emerging of a *product differentiation*

effect: the possibility to price discriminate induces a lower product differentiation degree, which in turn increases the competition between firms and makes price discrimination less attractive for each firm.

In Chapter 3, we extend the traditional analysis of the relationship between product differentiation and sustainability of collusion within the Hotelling framework to the case in which firms may price discriminate. Three different collusive schemes are studied: optimal collusion on discriminatory prices, optimal collusion on a uniform price, and collusion not to discriminate. The analysis yields the following results. The sustainability of the first and the third collusive scheme does not depend on the product differentiation degree. Instead, contrary to the traditional findings, the sustainability of the second collusive scheme depends *negatively* on the product differentiation degree. We consider also the possibility that firms collude on a suboptimal discriminatory price schedule and on a suboptimal uniform price. In both cases if optimal collusion is not sustainable, suboptimal collusion is not sustainable too¹.

¹ Chapter 2 of this thesis has been presented in preliminary versions at the Universities of Montpellier, Milano-Bicocca, Milano Cattolica, Siena, Torino (Collegio Carlo Alberto) and Pavia. I'm indebted with seminar participants for helpful comments and suggestions. All remaining errors are my own. All chapters have been written under the supervision of Michele Grillo, which helped me with suggestions, criticisms, and encouragement. Again, all remaining errors are my own.

A selective survey on direct price discrimination and product differentiation in the Hotelling framework

Stefano Colombo

Abstract

A selective survey of the most important contributions regarding direct price discrimination and product differentiation in the Hotelling framework is presented. The survey focuses in particular in key contributions in the areas of i) direct price discrimination and variety choice, and ii) product differentiation and sustainability of collusion.

JEL codes: D40; L11; L41

Keywords: Price discrimination; Product differentiation; Collusion

1.1. Definitions and focus of the research

Price discrimination is a widespread business practice. It is frequently used by firms, and consumers meet everyday practices which involve price discrimination. Therefore, it is worth to start with a definition of price discrimination. We adopt the most popular one, which is due to Stigler (1987): a firm is said to price discriminate when two similar or identical products are sold at prices that are in different ratios to their marginal costs². Moreover, it is well known that price discrimination is possible only when certain conditions are satisfied: firms must have some market power and must have the ability to sort consumers, and consumers who purchase at a discount price must be prevented from reselling to other consumers (Varian, 1989).

Economists distinguish price discrimination between *direct* and *indirect* price discrimination (Stole, 2007). Direct price discrimination occurs when the price offers (uniform prices, two-part tariffs, or other price menus) of a firm vary across consumers or groups of consumers according to verifiable characteristics. For this to be possible the firm has to be able to distinguish between consumers or groups of consumers on the basis of some verifiable characteristics. Conversely, indirect price discrimination arises when the consumers are still differentiated, but the firm cannot distinguish between them. In this case, the firm may induce self-selection providing a menu of choices: since each consumer selects accordingly to his unverifiable characteristics, his choice reveals “indirectly” his characteristics. Indirect price discrimination is also called “second-degree” price discrimination.

Inside the category of direct price discrimination one can distinguish between first-degree and third-degree price discrimination. If the price offers of a firm vary across *each consumer* and across *each unit* purchased of the good, we refer to first-degree (or perfect) price discrimination. Instead, if the price offers of a firm vary across *groups of consumers* (but not within each group of consumers), we refer to third-degree price discrimination³. There are many examples in the real world of direct price

² However, “a general-equilibrium theorist might rightly point out that goods delivered at different dates, at different locations, in different states of nature, or in different quality are distinct economic goods and thus that the scope of “pure” price discrimination is very limited” (Tirole, 1988).

³ The first author who refers to the existence of three degrees of price discrimination is Pigou (1920). His taxonomy has been extensively used by generations of economists, although his definition of the second-degree price discrimination is quite different with respect to the textbook definition of the second degree

discrimination practices. Young citizens receiving special discounts to visit museum, or teachers paying more than students for purchasing books, are third-degree price discrimination practices, based on age (the former example) or occupation (the latter example). Moreover, the recent and rapid development of the internet as a medium of communication and commerce, allows firms to identify individual consumers with great accuracy, and this in turn may allow firms to engage in personalized prices (perfect price discrimination). For example, the books retailer Books.com has adopted in 1998 a perfect price discriminating strategy where different consumers paid different prices for the same item depending on their shopping behaviour (Bailey, 1998). Similarly, many firms send personalized coupons via e-mails, the face values of whom depend on each consumer's willingness to pay as implied by the personal characteristics of the consumers (Allenby and Rossi, 1999). Examples of firms adopting this practice are provided by the largest competitors in the North America long-distance telephone market (like AT&T, MCI and Sprint) and direct marketing companies (Land's End and L.L. Bean). Also financial services firms and banks like Citigroup engage in perfect price discrimination through personalized discounts on card fees (Chen and Iyer, 2002)⁴.

The focus of this research is on direct price discrimination in spatial oligopolies. Inside this category, we investigate around two main questions: using a spatial duopoly model with horizontally differentiated firms, we study the firms' incentive to price discriminate when the product differentiation degree is endogenous (second chapter), and we analyse how the possibility to price discriminate affects the sustainability of a collusive agreement between the competing firms (third chapter).

This chapter is a selective survey: we concentrate on those papers which are in some way linked to the analysis developed in chapters 2 and 3 of this thesis. Therefore, in no way this survey can be considered exhaustive with regard to price discrimination literature, which is "*a collection of several related theories, whose relevance depends*

price discrimination. For an interesting analysis of Pigou's second-degree price discrimination see Philips (1983, p. 12-13) and Stole (2007, p. 4).

⁴ The importance of e-commerce and personalized prices has increased in Italy as well. For example, a top manager of a firm operating in the industry of tourism and travels states: "*E-mails are effective, cheap and fundamental to understand the willingness to pay of the consumer and to update the database. Using the information that customers provide about the satisfaction of their past experience, it is possible to provide by e-mail a series of proposals (for example, the hotels where to stay) on the basis of the consumer's profile*" (quoted in "Italia Oggi", 23-1-2008).

upon the specific economic contest" (Stole, 2007). The structure of this chapter is the following. Section 1.2. is dedicated to the traditional analysis of price discrimination in monopoly. In Section 1.3. price discrimination in spatial oligopolistic models is analysed through the main findings of the literature. Section 1.4. concerns collusion in spatial models.

1.2. Price discrimination in monopoly

In the following, we consider the effects of first-degree and third-degree price discrimination on equilibrium prices, profits, consumer surplus and total welfare in a monopolistic environment.

As already pointed out, first-degree price discrimination occurs when the firm's price offers vary between each consumer and each unit purchased of the good. Consider the following setting. Suppose to have n -consumers with different downward sloping demand curves, and a monopolistic firm with constant marginal cost, c , and zero fixed costs. Suppose first that the monopolist cannot distinguish between each consumer⁵, but it knows the aggregate demand function, which is given by $Q = Q(p)$. In this case, the best the firm can do is fixing the monopolistic price, p^m , which results from the maximization of $(p-c)Q(p)$. Now suppose that the firm perfectly knows each individual demand function, $q^i = q(p^i)$, where superscript i indicates the i -consumer. The monopolist can maximize the profits in two ways. A first strategy consists in contracting with each consumer and offering him a different price for each unit he buys: the monopolist sets for each unit a price equal to the maximum willingness to pay of the consumer for that unit, and in this way the firm is able to extract the whole consumer surplus. An identical result can be obtained by the firm through an appropriate system of personalized two-part tariffs: the optimal pricing scheme consists in applying a price equal to the marginal cost c to the marginal unit and demanding a personalized fixed

⁵ Suppose also that the monopolist cannot distinguish between groups of consumers, otherwise it could engage in third-degree price discrimination.

premium from each consumer equal to the net surplus of that consumer at price c^6 . With regard to the effect of monopolistic perfect price discrimination on welfare and profits, it is immediate to note that under perfect price discrimination total welfare is maximized, since the price of the marginal unit is equal to the marginal cost. Moreover, total welfare coincides with the profits, and the consumer surplus is zero. On the contrary, when the monopolist does not price discriminate, total welfare is lower, since $p^m > c$, and the consumer surplus is positive. It follows that profits are necessarily higher under perfect price discrimination than under the non-discriminatory monopolistic price.

If we consider third-degree price discrimination, the implications of price discrimination are more ambiguous. Suppose to have two groups of consumers, 1 and 2. The aggregate demand function of each group is given by $q^i = q(p^i)$, where $i = 1, 2$. The monopolist can discriminate between the groups, but not within each group. The equilibrium price resulting in each group is determined by the well-known inverse elasticity rule: $(p^i - c)/p^i = 1/\varepsilon^i$, where ε^i indicates the demand elasticity of group i . Therefore, the monopolist charges more in the market where the elasticity is lower. Clearly, if the monopolist were forced to set the same price in the two markets, it would be damaged by such prohibition: in fact, under the discriminatory price regime, the monopolist “at worst” can always set a uniform price if it is profitable to do so. From the point of view of the consumers, if all consumers are served under the uniform price regime and if the uniform equilibrium price arising when price discrimination is not possible is between p^1 and p^2 , the consumers in the low-elasticity market are better off when the monopolist sets a uniform price, while the consumers in the high-elasticity market are damaged by the uniform price regime⁷. When uniform pricing implies the exclusion of some consumers from the market, price discrimination benefits those

⁶ Referring to two different profit-maximization practices becomes redundant when consumers have unit demand functions. Unit demand functions mean that consumers wish to consume one or zero unit of the good: in this case, the optimal strategy for the firm consists simply in setting a system of personalized prices where each consumer pays a sum equal to his willingness to pay.

⁷ The idea that when there are two or more groups of consumers monopolistic third-degree discrimination raises the price for some consumers and lowers it for the other consumers with respect to the uniform price regime has remained unquestioned for a long time in the literature of monopolistic price discrimination. However, Nahata *et al.* (1990) show that this is not a general result: if the profit functions on the markets represented by each group of consumers are not concave, it is possible that the monopolistic third-degree price discrimination raises (lowers) prices for *all* consumers with respect to the uniform price regime. In this case, banning price discrimination benefits (damages) all consumers.

consumers and does not harm the other consumers⁸. An even more subtle issue is the effect of third-degree price discrimination on total welfare. It can be shown (Schmalensee, 1981; Varian, 1985) that if the output does not increase passing from the uniform price regime to price discrimination, total welfare is lower under price discrimination than under uniform price⁹. However, when the output under price discrimination is greater than under uniform price, we cannot know the direction of the total welfare change: total welfare might be higher or lower than under the uniform price regime.

1.3. Competitive price discrimination in spatial models

The aim of this section is to understand the implications of direct price discrimination when spatially differentiated firms compete, by reviewing the most relevant contributions in the area.

The spatial competition literature begins with the seminal work of Hotelling (1929). The well-known Hotelling linear market consists of a bounded segment over which consumers characterized by unit demand functions are uniformly distributed. Two firms are located along the segment. Two alternative interpretations of the Hotelling model are possible. We define these two interpretations respectively as the “geographical” interpretation and the “product differentiation” interpretation. Consider first the

⁸ Consider the following example. There are two types of consumers: both types of consumers have unit demand functions, but the willingness to pay of type-1 consumers is 1, while the willingness to pay of type-2 consumers is 2. Moreover, suppose that there are two type-1 consumers and four type-2 consumers. If the monopolist cannot price discriminate, it sets a uniform price equal to 2 and only type-2 consumers are served; if the monopolist can price discriminate, it sets a price equal to 1 on type-1 consumers and equal to 2 on type-2 consumers: all consumers are served and they are all (weakly) better off under price discrimination. Layson (1994) derives general conditions that determine when price discrimination induces a monopolist to serve a market that would not be served under the uniform price regime.

⁹ The intuition is the following. Third-degree price discrimination causes marginal rate of substitution to differ among consumers, so the total output is sub-optimally distributed from a total welfare point of view (“unequal marginal utilities effect”: Armstrong, 2008). Therefore, it is necessary that the total output increases in order to compensate the misallocation of output induced by third-degree price discrimination: if output does not increase under price discrimination, total welfare must be higher under uniform pricing. There are many papers that generalize the Schmalensee (1981) and Varian (1985) result: see, among the others, Hausman and Mackie-Mason (1986) for increasing-returns-to-scale production technologies, Schwartz (1990) for non-linear cost functions, and, more recently, Cowan (2007) for non-linear demand functions.

“geographical” interpretation. In this case, the distance between a consumer and a firm is a physical distance, which can be measured in kilometres, miles, and so on. To say that a firm is located at point a means that the mill of the firm is located at point a in a physical space, and that real transportation costs must be sustained by the firm when it carries the product from the mill to the consumer, or, vice versa, by the consumer when he goes and takes up the product at the firm’s mill. When firms and consumers are physically located in different points of the space, there are two different pricing methods that a firm may adopt: it can use *delivered prices*, in which the transportation costs are sustained by the firm which carries the product to the consumers at their locations, or it can use *free on board (f.o.b.) prices*, in which the transportation costs are sustained by the consumer which goes to the mill and takes the good or pays independent couriers to transport the good from the mill to him. Consider now the “product differentiation” interpretation. In this case, the segment does not represent a real space along which the consumers are distributed, but it represents the set of product varieties from which the firm chooses the variety to produce. Therefore, to say that a firm is located at point a means that the firm is producing the variety a , instead of the varieties $b, c...$ The location of a consumer represents the preferred variety of that consumer within the whole set of the possible varieties of the good. It follows that the more the variety produced is different from the preferred variety of a consumer, the lower is the utility that such consumer obtains from consuming the good: such “disutility” costs are commonly defined as “transportation” costs, even if there is no a real distance between the consumer and the firm (in other words, transportation costs are only a metaphor of the distance between the different varieties). Given that under the “product differentiation” interpretation the “transportation” costs are necessarily paid by the consumers, there is no possibility for *delivered prices*: the pricing practices are necessarily of the *f.o.b.* type^{10, 11}.

¹⁰ One has to be careful when he considers price discrimination in the contest of delivered prices. Consider the following example. Suppose that there is a cement producer serving two costumers, located at 5 km and 10 km from the mill. Suppose further that the costs of production are zero and the transportation costs (sustained by the producer) are linear in the distance, and assume that the cost of transporting one unit of cement for 1 km is equal to 1. The producer sells the cement at a price of 10 to the nearer costumer, while applies a price of 20 to the more distant costumer. Is the producer price discriminating? The answer is negative, since the ratio between prices and marginal costs is the same, and therefore there is not price discrimination in the sense of Stigler (see section 1.1). In general, when delivered prices are considered, it is not sufficient that prices are different between consumers in order to observe price discrimination, since the cost of serving each consumer varies with the consumer’s location.

In both interpretations, firms which do not locate in the same point of the market are differentiated. More specifically, they are *horizontally* differentiated. In fact, for equal prices no firm is preferred by all consumers, since some consumers prefer one firm and some others prefer the rival: in particular, when both firms set the same price each consumer prefers the nearer firm¹². Hotelling (1929) considers the case of uniform pricing. He erroneously suggests that an equilibrium in locations exists and yields back-to-back locations at the centre of the market: that is, minimal differentiation between the firms emerges as the unique equilibrium. The intuition is that, in order to obtain a larger demand, each firm has the incentive to locate nearer to the centre than the rival: these back-to-back locations at the centre of the market finish when firms are located at the same point (the middle of the segment).

As noted by D'Aspremont *et al.* (1979), the Hotelling's result is invalid: no location-price equilibrium exists under the Hotelling's hypothesis. Instead, for a slightly modified version of the Hotelling's framework (quadratic transportation costs instead of linear transportation costs), firms are shown to locate at the two extremities of the market at the first stage of a two-stage game in which firms first choose where to locate and then set prices. The so-called Maximum Differentiation Principle can be explained as the result of two different forces that work in opposite directions. On one hand, if a firm locates near to the rival, it serves more consumers: *for given prices*, this causes the profits to increase (*demand effect*). On the other hand, when a firm increases the differentiation from the rival, it reduces the cross-price elasticity, allowing for higher equilibrium prices, and, *ceteris paribus*, for higher profits (*strategic effect*). In this model, the *strategic effect* prevails over the *demand effect*: this determines the maximum differentiation result.

For about fifty years, spatial models have not been used to consider the implications of price discrimination. The first paper that explicitly addresses the issue of price

Even before Stigler (1987), Philips (1983) clarifies this point: "*If the price difference fully reflects the difference in the cost of carrying the good from the seller's location to the buyers' location, then nobody would argue, I'm sure, that a discriminatory practice is involved*" (p. 5-6). Instead, when *f.o.b.* prices are considered, price discrimination arises every time in which consumers pay different prices, because the cost of serving the consumers does not change with the position of the consumers.

¹¹ Our research is developed by using the "product differentiation" interpretation of the Hotelling framework. Therefore, in chapter 2 and 3 of this thesis, *f.o.b.* prices are used. Instead, the literature we survey in chapter 1 has been partially developed by using the "geographical" interpretation of the Hotelling framework, and *delivered* prices are frequently used.

¹² A situation in which, for equal price, all consumers prefer one firm to the other indicates that the two firms are *vertically* differentiated.

discrimination inside spatial models is Lederer and Hurter (1986). The authors assume that firms simultaneously decide where to locate, and then simultaneously decide which perfect discriminatory price schedule to apply. Firms carry the product directly to the consumers, sustaining the transportation costs: that is, *delivered* prices are assumed. The original framework of Hotelling (1929) is extended to allow firms to locate on a compact subset of the plane instead than on a linear segment. Moreover, uniform distribution of the consumers is not assumed, while the hypothesis of unit demand functions is maintained. The authors show that in such model a location – (discriminatory) price equilibrium always exists. In particular, the equilibrium locations minimize the transportation costs. Given that the total output is fixed, in terms of total welfare only transportation costs matter (prices determine how much of the total surplus goes to the consumers and how much goes to the firms): therefore, equilibrium locations maximize total welfare.

Following Lederer and Hurter (1986), other researchers have extended the analysis of the implications of price discrimination in competitive spatial models. Hamilton *et al.* (1989) study the characteristics of the location-price equilibrium arising in the Hotelling model when price discrimination is assumed. In their paper, delivered prices are assumed and transportation costs are linear. The main innovation with respect to Lederer and Hurter (1986) consists in supposing that each consumer has a downward sloping linear demand function: such demand functions are identical for all consumers. Two different two-stage games are studied. In both games, the firms first simultaneously choose where to locate. Then, in the first game, the firms choose simultaneously which delivered price to apply to each consumer (Bertrand competition), while in the second game, the firms choose simultaneously which quantity to deliver to each consumer, letting the market-clearing condition determine the price at each location (Cournot competition)¹³. For sufficiently low transportation costs, a unique location-price equilibrium is showed to exist in both cases. Some relevant differences between the Bertrand equilibrium and the Cournot equilibrium are pointed out. Each consumer pays a lower price under Bertrand than under Cournot. In Cournot, firms locate at the centre of the segment (no differentiation arises), while in Bertrand firms are moderately differentiated (they locate between the first and the third quartile). Since the

¹³ Note that Hamilton *et al.* (1989) consider a model of third-degree price discrimination: the price set by each firm varies across each consumer but not across each unit of the good bought by each consumer.

quantity sold at each location is higher and the transportation costs are lower in Bertrand than in Cournot, it follows that the aggregate welfare is higher in Bertrand than in Cournot. Finally, profits are higher in Cournot than in Bertrand.

Along the same line of investigation, Hamilton and Thisse (1992) consider the case of non-linear pricing. As in Hamilton *et al.* (1989), delivered prices and downward-sloping linear demand functions are assumed. A two-stage game is supposed. In the first stage, the firms simultaneously choose where to locate; in the second stage each firm offers to each consumer a two-part tariff depending on his location in the market. The price varies across each consumer *and*, given the two-part structure of the tariff, across each unit of the good purchased by each consumer: therefore, firms perfectly price discriminate. The authors show that when non-linear discriminatory pricing can be used in the second stage of the game, the two firms locate at $1/4$ and $3/4$ respectively in the first stage of the game, and total welfare is maximized. Together, the works by Hamilton *et al.* (1989) and Hamilton and Thisse (1992) show that the welfare maximizing implication of price discrimination obtained by Lederer and Hurter (1986) depends heavily on the possibility for firms to perfectly price discriminate. When third-degree price discrimination is assumed (Hamilton *et al.*, 1989), optimal locations do not arise; on the contrary, when perfect price discrimination is assumed (Hamilton and Thisse, 1992) optimal locations arise in equilibrium even with downward-sloping demand functions. These results seem to suggest that for firms to locate efficiently along the market maximal flexibility in pricing is needed.

Thisse and Vives (1988), by adopting the “geographical” interpretation of the Hotelling model, compare the f.o.b. uniform price regime with the delivered discriminatory price regime, assuming that firms are exogenously located at the endpoints of the Hotelling segment (maximal differentiation). Uniform distribution of consumers along the segment, unit demand functions and linear transportation costs are assumed. Thisse and Vives (1988) show that the equilibrium discriminatory prices are all lower than the equilibrium uniform prices. It follows that consumers are better off under the discriminatory price regime, while the reverse is true for the firms. Moreover, when firms are free to choose whether to price discriminate or not, the authors show that only discriminatory prices arise in equilibrium even if the uniform price equilibrium would yield higher profits to both firms. Thisse and Vives (1988) consider also the case

in which each firm may credibly commit not to price discriminate before to set the price schedule. A two-stage game is supposed. In the first stage of the game each firm commits to uniform pricing or does not commit. In the second stage the firms compete on prices contingently on the price policy chosen in the first stage. When firms have chosen not to commit in the first stage, they compete with unrestricted price schedules, and the resulting equilibrium is characterized by both firms setting discriminatory prices. The interesting question therefore is the following: does an equilibrium characterized by both firms committing in the first stage emerge? Thisse and Vives (1988) show that the answer is negative, since in the first stage both firms choose not to commit. Therefore, price discrimination in the second stage emerges as the unique equilibrium outcome. This happens because being unconstrained during the second stage of the game gives to each firm more flexibility to respond to its rival's action. Hence the individual incentives lead both firms to choose not to commit in the first stage, and this situation yields to a typical Prisoner Dilemma.

In the Thisse and Vives (1988) model, price discrimination is beneficial for consumers, since price discrimination determines a reduction in all prices with respect to the case of uniform price. Such consequence of price discrimination derives from the fact that price discrimination forces firms to compete more fiercely for each consumer, because lowering the price for a given consumer does not imply a reduction of the price applied to the other consumers as would be in the case of uniform pricing. The following quotation by Hoover (1948) clarifies the point: *“the difference between market competition under [uniform] f.o.b. pricing...and under discriminatory delivered prices is something like the difference between trench warfare and guerrilla warfare. In the former case all the fighting takes place along a definite battle line; in the second case the opposing forces are intermingled over a broad area”* (p. 57). It is interesting to note that competition policy has opposed for a long time discriminatory pricing practices. For example, in the United States, the Robinson-Patman Act states that it is *“unlawful...to discriminate in price between different purchasers of commodities of like grade and quality...where the effect of such discrimination may be substantially to lessen competition or tend to create a monopoly in any line of commerce, or to injure, destroy or prevent competition with any person who either grants or knowingly receives the benefit of such discrimination, or with consumers of either of them”*. However, the

Thisse and Vives (1988) analysis provides an example in which a legislation like the Robinson-Patman Act, instead of promoting competition between firms, weakens it. In fact, given the Prisoner Dilemma mechanism, one should not expect to observe competing firms to adopt a uniform price policy. Or, if one observes it, he should infer the existence of some collusive agreement which purpose is precisely to avoid the lower profits equilibrium induced by price discrimination¹⁴. On the contrary, the existence of a legislation that impedes the firms to price discriminate allows them to obtain the uniform price equilibrium without colluding.

In the model of Thisse and Vives (1988), the effect of price discrimination on the equilibrium prices is univocal: equilibrium discriminatory prices are all below the uniform equilibrium prices. However, the general necessary condition for such a phenomenon to occur has been provided more recently by Corts (1998). His analysis is not developed within a spatial framework, but it can be easily extended to locational models¹⁵: given the importance of its implications we devote some words on it.

Suppose to have two differentiated firms, A and B . Firms can divide the total market into two submarkets, 1 and 2. Define with:

$$b_j^i(p_{-j}), \quad j = A, B \quad \text{and} \quad i = 1, 2,$$

the best-response function of firm j in market i . We say that market i is the *strong* market for firm j if firm j prefers to set a higher price in market i than in market $-i$ for any given price set by the rival. Formally, market i is the *strong* market for firm j if the following condition holds:

$$b_j^i(p_{-j}) > b_j^{-i}(p_{-j}), \quad \forall p_{-j}, \quad j = A, B \quad \text{and} \quad \forall i, \quad i = 1, 2,$$

Conversely, market $-i$ is the *weak* market for firm j .

Corts (1998) provides the following definition:

¹⁴ Thisse and Vives (1988) sum up this concept stating that: “*uniform f.o.b. pricing is not evidence of a more competitive environment*” (p. 124). In fact, the opposite is true: uniform pricing is evidence of a less competitive environment.

¹⁵ See the appendix in this chapter.

Def: An oligopoly exhibits *best-reply symmetry* when the following condition holds:

$$b_j^i(p_{-j}) > b_j^{-i}(p_{-j}) \leftrightarrow b_{-j}^i(p_j) > b_{-j}^{-i}(p_j), \forall p_j, p_{-j}, j = A, B \text{ and } \forall i, i = 1, 2 \quad (1)$$

In words, an oligopoly exhibits best-reply symmetry when the two firms have the same *strong* market and the same *weak* market. If condition (1) is not respected, an oligopoly is said to exhibit *best-reply asymmetry*. In words, an oligopoly exhibits best-reply asymmetry when the *strong* market of one firm is the *weak* market of the other firm, and vice-versa.

Finally, define with:

$$b_j^u(p_{-j}), \quad j = A, B$$

the uniform-price best-reply function of firm j , that is, the best-reply function of firm j when both firms set a uniform price in the two markets. The importance of the notion of *best-reply symmetry* for analysing the impact of price discrimination can be easily understood looking at figures 1.a and 1.b.

Figure 1.a

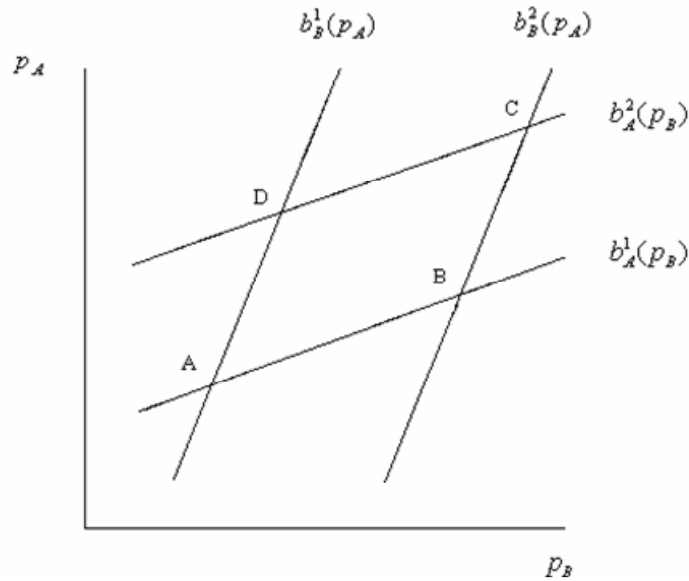


Figure 1.b

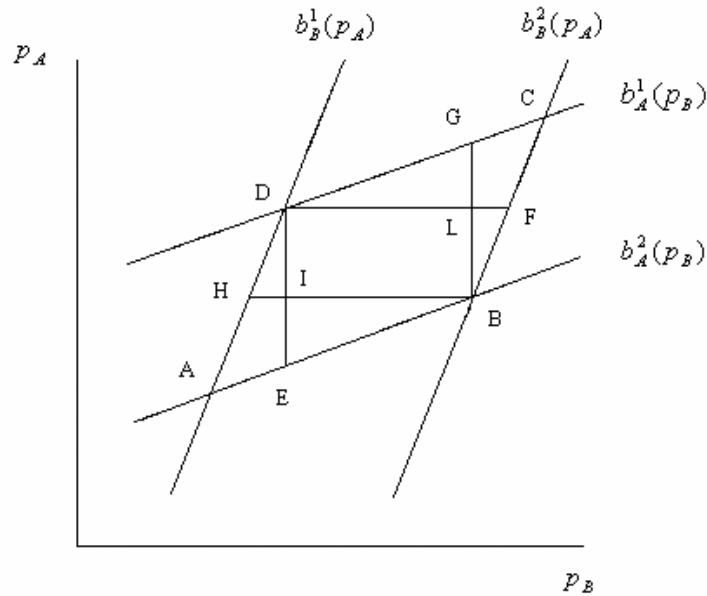


Figure 1.a illustrates a situation of best-reply symmetry. Given that $b_A^2(p_B) > b_A^1(p_B)$ and $b_B^2(p_A) > b_B^1(p_A)$, market 2 is the *strong* market for both firms (and conversely market 1 is the *weak* market for both firms). Equilibrium discriminatory prices in market 1 are therefore defined by point A, while the equilibrium discriminatory prices in market 2 are defined by point C. Under general conditions¹⁶, the graph of $b_j^u(p_{-j})$, with $j = A, B$, must lie in the region between the submarket best-reply functions: $b_A^2(p_B) > b_A^u(p_B) > b_A^1(p_B)$ and $b_B^2(p_A) > b_B^u(p_A) > b_B^1(p_A)$. Therefore, the equilibrium prices when both firms do not discriminate must lie in the area ABCD. It follows that uniform equilibrium prices must be lower than the discriminatory equilibrium prices in the *strong* market, while they must be higher than the discriminatory equilibrium prices in the *weak* market. That is, when there is best-reply symmetry, moving from uniform pricing to price discrimination causes some prices to increase and some others to decrease. However, such ambiguity does not necessarily arise when best-reply asymmetry occurs. Consider picture 1.b, where the case of best-reply asymmetry is

¹⁶ It is sufficient to assume that the profit functions in each submarket are continuous and concave (Corts, 1998).

depicted. Since $b_A^2(p_B) < b_A^1(p_B)$ and $b_B^2(p_A) > b_B^1(p_A)$, market 1 is the *strong* market for firm *A* and the *weak* market for firm *B*, while the reverse is true for market 2. When firms price discriminate, equilibrium prices in market 1 are represented by point D, while equilibrium prices in market 2 are represented by point B. As before, under uniform price regime, the equilibrium uniform prices must lie in the area ABCD. Therefore, it is possible that when both firms set a uniform price in the two markets, the equilibrium prices are higher than the equilibrium discriminatory prices. This means that moving from the uniform price regime to the discriminatory price regime causes all prices to fall: Corts (1998) calls this phenomenon *all-out competition*. This happens when the uniform price equilibrium is in the area LFCG. It may also be that the equilibrium discriminatory prices are higher than the equilibrium uniform prices: this situation occurs when the uniform price equilibrium is in the area AEIH. However, it may also be that the uniform equilibrium prices are between the equilibrium discriminatory prices: in this case the equilibrium prices under the uniform price regime are in some point of the area DIBL. It is also possible that passing from the discriminatory price regime to the uniform price regime one firm sets a uniform price higher or lower than both its equilibrium discriminatory prices, while the other firm sets a uniform price which is between its equilibrium discriminatory prices. These cases occur when the uniform best-reply functions of the firms intersect in the areas HID, EBI, BFL and DLG. Summing up, *best-reply asymmetry* is the necessary condition to avoid the ambiguity that typically characterizes the price effects of price discrimination, but it is not also a sufficient condition.

The analysis of Corts (1998) becomes particularly relevant for spatial frameworks: in the appendix of this chapter we show that the Hotelling model in the D'Aspremont *et al.* (1979) version is characterized by the asymmetry of the best-reply functions and by *all-out competition*¹⁷. The existence of best-reply asymmetry should not be surprising. In spatial markets, firms rank in opposite way the consumers. For any given price set by the rival, each firm finds it more profitable to set a higher price to the consumers which are located nearer to it than to the rival: the nearer consumers represent its strong market, while the farther consumers represent its weak market. Since the consumers

¹⁷ We also show that a standard locational model with vertically differentiated firms is characterized by best-reply asymmetry and *all-out competition*.

who are nearer to one firm are at the same time the farther consumers for the other firm, the strong market for one firm is the weak market for the other firm, and vice versa¹⁸.

However, best-reply asymmetry is a necessary, but not a sufficient, condition for *all-out competition*. A recent paper by Ulph and Vulkan (2000) provides some insights on the role of transportation costs in determining the occurrence of *all-out competition* in a model characterized by best-reply asymmetry. The authors extend the analysis of the price effects of (perfect) price discrimination in the Hotelling framework to the case of transportation costs which are not constrained to be linear or quadratic. Indeed, firms are assumed to be located at the endpoints of the segment and the transportation costs for a consumer buying from the firm located at the left endpoint are defined by t^β , where $\beta \geq 1$ and β takes integer values. Interestingly, Ulph and Vulkan (2000) show that whether or not prices and profits are lower under perfect price discrimination than under uniform price regime depends crucially on the nature of the transportation cost function, that is, on the value of the parameter β . In particular, when β is low ($\beta = 1, 2$) the prices are everywhere lower under perfect price discrimination than under uniform pricing (i.e. *all-out competition* occurs), and obviously profits are lower too. When $\beta = 3, 4$ the equilibrium discriminatory prices are higher than the equilibrium uniform price for the consumers nearer to the firms, while they are lower for the consumers nearer to the middle of the segment: the overall effect is that profits are lower under price discrimination, but *all-out competition* is absent. Finally, when β is sufficiently high ($\beta \geq 5$), the higher profits that the firms are able to extract from the nearer consumers due to price discrimination outweigh the losses from the consumers located nearer to the middle of the segment: the result is that when both firms price discriminate profits are higher than when both firms set a uniform price. The authors identify two effects of perfect price discrimination: first, a *surplus extraction effect*, and, second, an *intensified competition effect*. The *surplus extraction effect* refers directly to the well-known property of first-degree price discrimination in monopoly: since the price can be targeted on the basis of the willingness to pay of each consumer, price discrimination allows extracting the whole surplus and therefore it increases profits. However, price discrimination is likely to increase competition between firms which

¹⁸ Stole (2007) states that “*locational models seem especially susceptible to all-out competition*”, which in turn requires the oligopoly to exhibit best-reply asymmetry.

make use of it¹⁹: this is the *intensified competition effect*, the impact of whom on profits is negative. The magnitude of the transportation costs affects the relative strength of these two effects, and thus determines the overall impact of perfect price discrimination over the profits of the firms. When the transportation costs are low (linear or quadratic), the competition between the firms is intense (consumers do not particularly care about the differentiation between the firms), and the *intensified competition effect* prevails over the *surplus extraction effect*. All prices and profits are lower when both firms price discriminate than under uniform pricing. When transportation costs increase, some consumers (those nearer to the firms) are locked in. The *surplus extraction effect* starts to work on these consumers, and it contrasts the *intensified competition effect* which works on the consumers located far from the firms: for the former consumers discriminatory prices are higher than the uniform price, while for the latter the reverse is true. The impact on profits depends on which effect is prevailing: when transportation costs are sufficiently low, the number of consumers locked in is quite low, and the *intensified competition effect* outweighs the *surplus extraction effect*; when transportation costs increase, the number of consumers locked in increases too, and, after a given threshold ($\beta = 5$), the *surplus extraction effect* outweighs the *intensified competition effect*: in this case profits under price discrimination are higher than under the uniform price regime.

More recently, Liu and Serfes (2004) show that the Prisoner Dilemma result found by Thisse and Vives (1988) is valid also under the hypothesis that firms are not able to perfectly price discriminate between the consumers distributed along the Hotelling segment. The authors develop an interesting framework where firms may discriminate between an exogenous number of groups of consumers: at the limit, their framework converges to perfect price discrimination. Firms are exogenously located at the endpoints of the segment and transportation costs are linear. There exists an information technology that allows firms to partition the consumers into distinct groups²⁰. When a firm owns such technology, it is able to divide the market into a given number of sub-segments (assumed to be of equal length) and to recognize the sub-segment where each consumer is located, while it cannot distinguish between the consumers belonging to the

¹⁹ “Competitive price discrimination may intensify competition by giving firms more weapons with which to wage their war” (Corts, 1998).

²⁰ For example, think to a database of past consuming behaviours.

same sub-segment. The firms can price discriminate between the sub-segments, but not within the same sub-segment. The number of the sub-segments indicates the quality of the information technology: a higher number indicates a better quality of the technology. At the limit, when the information technology allows a firm to individuate the position of each consumer along the market, perfect price discrimination occurs. Liu and Serfes (2004) suppose a two-stage game. In the first stage, the firms simultaneously choose whether or not to buy (with no costs) the information technology, the quality of whom is exogenously given. At the second stage, each firm sets the price schedule, which must be non-discriminatory if the firm has not bought the information technology at the first stage, while it can be discriminatory if the firm has bought the information technology. The authors show that when a firm has bought the information technology, it uses it in setting prices: that is, every firm price discriminates when it can do it. Moreover, when both firms price discriminate, the equilibrium profits of each firm are a U-shape function of the information quality: initially better information reduces equilibrium profits, but eventually firms' profits increase with better information²¹. In any case, profits when both firms price discriminate are lower than when both firms do not price discriminate. Finally, the dominant strategy of each firm in the first stage of the game consists in buying the information technology that allows to price discriminate in the successive stage of the game, unless the information quality is very low. The standard Prisoner Dilemma problem arises: the individual incentive leads each firm to price discriminate and this induces equilibrium profits that are lower for both firms with respect to the uniform pricing equilibrium.

The Prisoner Dilemma problem is shown to arise also in a vertical differentiation model with perfect price discrimination and endogenous choice of the quality level by the firms. Choudary *et al.* (2005) consider a model where two firms compete in both the quality and the price of the products they offer. The utility of a consumer is given by: $u(\theta) = \theta q - p$, where $\theta \in [0,1]$ indicates the consumer valuation for quality, p is the

²¹ This is due to the fact that when each sub-segment is shared between firms, an information refinement intensifies the competition and prices and profits fall. However, when information quality is sufficiently high, some sub-segments (those nearer to the endpoints of the segment) start to be monopolized by the nearer firm. Further information improvements increase the number of sub-segments monopolized by the nearer firm, and therefore profits increase.

price and q is the quality²². Unit demand functions are assumed. The firms have marginal costs that depend on the quality level: a quadratic quality cost function, $c(q) = q^2$, is supposed²³. Perfect price discrimination implies that a firm can apply a different price to each consumer depending on his valuation of quality, θ . A two-stage game is assumed: in the first stage of the game both firms simultaneously choose the quality level, and in the second stage both firms choose simultaneously the price schedule. The authors show that when only the low-quality firm price discriminates, the quality equilibrium is characterized by both firms offering a lower quality product than under the uniform price regime, while when only the high-quality firm price discriminates both firms offer a higher quality product than under the uniform price regime. Finally, when both firms price discriminate, the high-quality firm offers a lower quality good than under the uniform price regime, while the low-quality firm offers a higher quality good than under the uniform price regime (i.e. a lower differentiation in quality between firms emerges). Moreover, in the second stage of the game both firms have the incentive to adopt first-degree price discrimination, regardless whether the other firm price discriminates or not. But this induces a “bad” equilibrium, because when both firms perfectly price discriminate their profits are lower than under the uniform price regime. The intuition is similar to the case of horizontal price discrimination. Price discrimination is profitable since it allows each firm to design the best price to be set to each consumer given the price set to that consumer by the rival, without taking into account the prices applied to the other consumers (*surplus extraction effect*). However, price discrimination also increases competition (*intensified competition effect*). In this model, the *surplus extraction effect* prevails and both firms price discriminate: the final result is a generalized reduction of the profits.

²² The vertical nature of the differentiation between the firms emerges from the utility function: if firms set the same price all consumers buy from the firm which offers the product characterized by higher quality.

²³ The analysis of Choudary *et al.* (2005) extends also to generic cost functions: $c(q) = q^\alpha$, with $\alpha \geq 1$. Here we consider only the simplest case with quadratic quality costs.

1.4. Collusion in spatial models

In this section, the relationship between product differentiation and sustainability of collusion in the Hotelling framework is considered by surveying the main contributions on this issue. As it turns out, all papers (with one notable exception) assume that firms price uniformly²⁴.

The first paper that explicitly studies the effects of the differentiation degree between the firms on the collusive pricing within a spatial model is Chang (1991)²⁵. The Hotelling linear market is assumed. The differentiation degree between the two firms is represented by the distance between their locations on the market: the more the firms are distant, the higher is the differentiation degree²⁶. The differentiation degree is exogenously given and the firms are assumed to be symmetric. Price discrimination is totally absent from the analysis. Clearly, if firms cannot collude, the impact of the differentiation degree over the equilibrium price is immediate. The firm's ability to set a high price for its product depends on the substitutability between its product and the rival's product. The more the firms are similar (low differentiation degree), the higher is the competition and the lower is the equilibrium price. On the contrary, the relationship between the differentiation degree and the sustainability of collusion is not so obvious, since it depends also on the effect of the product differentiation degree on the collusive profits and on the incentives of a firm participating to the agreement to deviate²⁷. In order to analyze this issue, Chang (1991) supposes an infinitely repeated game. Moreover, the *grim trigger* strategy *a la* Friedman (1971) for supporting collusion is assumed²⁸. A collusive agreement is said to be sustainable if the sub-game perfect equilibrium entails both firms to set the collusive price. In other words, collusion is

²⁴ Indeed, the main contribution of the third chapter of this thesis consists in analysing the sustainability of a collusive agreement between horizontally differentiated firms when firms are assumed to be able to price discriminate.

²⁵ Other papers that investigate the relationship between product differentiation and sustainability of collusion adopting non-spatial frameworks are Deneckere (1983), Majerus (1988) and Ross (1992). Hackner (1994) instead considers the relationship between sustainability of collusion and product differentiation in a model with vertically differentiated firms.

²⁶ An alternative interpretation of the differentiation degree between the firms within the Hotelling model considers the transportation costs parameter. See, for example, Schultz (2005).

²⁷ See for example Tirole (1988, p. 242) and Motta (2004, p. 146-147).

²⁸ The *grim trigger* strategy means that in the first period of the game each firm sets the collusive price. In any subsequent period firms continue to set the collusive price as long as both firms have set the collusive price in the past. If either firm deviates from the collusive price at time t , from time $t+1$ onward both firms revert to the Nash equilibrium price.

sustainable if every firm has never the incentive to deviate from the agreement. To clarify the point, define with Π^C , Π^D and Π^N respectively the one-shot collusive profits, the one-shot deviation profits and the one-shot punishment (or Nash) profits of each firm. Further, define with $\delta \in (0,1)$ the market discount factor, which is assumed to be exogenous and identical for each firm. The market discount factor provides a measure of the importance that the firms attribute to the future: if the future is important, δ is high, while the reverse is true when little importance is attributed to the future. When a firm is considering the possibility to deviate from a collusive agreement, it has to compare the short-term advantage that it obtains from the deviation with the long-term disadvantage which derives from the disruption of the collusive agreement. If the future is important enough, the long-term losses outweigh the short-term gains, no deviation occurs and collusion is sustainable. The opposite is true if the future is not important enough. In other words, collusion is sustainable if and only if the discounted value of the profits that each firm obtains under collusion is higher than the discounted value of the profits that a firm obtains deviating from the collusive agreement. Formally, the following incentive-compatibility constraint must be satisfied:

$$\sum_{t=0}^{\infty} \delta^t \Pi^C \geq \Pi^D + \sum_{t=1}^{\infty} \delta^t \Pi^N ,$$

After some manipulations, the constraint can be rewritten as:

$$\delta \geq \delta^* = \frac{\Pi^D - \Pi^C}{\Pi^D - \Pi^N} \quad (2)$$

The parameter δ^* is called in literature the “critical discount factor”²⁹. Equation (2) says that if the market discount factor is higher than the critical discount factor (that is, if the future is important enough) the collusive agreement is sustainable, otherwise it is not sustainable, since every firm has the incentive to deviate from the collusive price. It

²⁹ Since $\delta = \frac{1}{1+r}$, where r is the interest rate, the incentive-compatibility constraint is sometimes

expressed as: $r \leq r^* = \frac{\Pi^C - \Pi^N}{\Pi^D - \Pi^C}$.

follows that the critical discount factor provides a measure of the sustainability of the agreement: the higher is δ^* the smaller is the set of the market discount factors that support collusion (that is, collusion is less easy to sustain).

The profit functions (Π^C , Π^D and Π^N) depend on the differentiation degree between the firms. In particular, the Nash profits are a monotonically increasing function of the differentiation degree, due to the fact that the lower is the differentiation, the lower is the non-cooperative price, and, consequently, the lower are the punishment profits. Chang (1991) shows that the collusive profits are non-monotonic with respect to the differentiation degree, and achieve a maximum when firms are located at the first and the third quartile of the market. Finally, the deviation profits are a monotonically decreasing function of the differentiation degree. The shape of the deviation profits is the result of two opposite effects. First, for a given collusive price, lower product differentiation degree allows for a higher deviation price, which in turn induces greater deviation profits; second, the collusive price is lower the more the firms are far from the optimal locations, and this reduces the deviation profits. When firms are strongly differentiated (that is, they are located outside the first and the third quartile) and product substitutability increases, the two effects go in the same direction, because the collusive price approaches the maximum. Therefore, deviation profits necessarily increase moving from the endpoints to the first and the third quartile of the segment. Instead, when firms are weakly differentiated (that is, they are located inside the first and the third quartile) and product substitutability increases, the two effects go in opposite directions, because the collusive price moves away from the maximum. The overall impact of lower differentiation on the deviation profits depends upon the relative strength of these two effects. Chang (1991) shows that the first effect dominates, and therefore deviation profits increase also when the firms move from the first and the third quartile toward the centre of the segment.

More importantly, Chang (1991) shows that the overall impact of the differentiation degree over the critical discount factor is negative: that is, collusion is easier to sustain when the firms are more differentiated. Therefore, the relationship between the differentiation degree and the sustainability of a collusive agreement on a uniform price within the Hotelling framework is not ambiguous: similar firms are more in troubles in sustaining collusion. Furthermore, Chang (1991) demonstrates that, for any product

differentiation degree, sustaining collusion becomes more difficult as the transportation costs become smaller. The intuition behind this result is that the incentive to cheat increases when the transportation costs become smaller: this occurs because when the transportation costs are low a slight undercut allows the deviating firm to obtain a large market share.

Chang (1992) extends the analysis considering a different situation: firms may relocate, paying a fixed cost, during the punishment phase. As in Chang (1991), the purpose is to analyse the relationship between the degree of product differentiation and the sustainability of the collusive agreement. First, Chang (1992) shows that even when relocation is possible collusion is more difficult to sustain when the initial (exogenous) differentiation degree is low: as in Chang (1991), the gains from the defection become greater relative to the benefit of maintaining the collusion as the products become stronger substitutes. Second, Chang (1992) shows that, for any product differentiation degree, collusion is more difficult to sustain with a lower fixed cost of relocation. This occurs because when the relocation costs are low the firms optimally relocate during the punishment phase obtaining higher non-cooperative equilibrium profits with respect to the case of fixed locations: since punishment is less severe, the implication is that collusion is more difficult to sustain.

Friedman and Thisse (1993) focus on the location equilibrium when a “partial” collusion is supposed. Within the Hotelling framework, firms interact for an infinite number of periods. At the first period the firms choose the location. From the second period onward, the firms choose the price. Collusion is “partial” because the firms choose non-collusively the locations, while they collude on price³⁰. The authors assume that the share of the total collusive profits pertaining to each firm is proportional to the individual market power, which is represented by the share of the one-shot Nash profits of each firm and depends on the firms’ locational choices. Friedman and Thisse (1993) show that minimum differentiation between firms emerges as the unique equilibrium outcome. This is due to the fact that firms’ ability to punish one another for cheating is maximized by locating at the centre of the segment. An interesting implication of this result is that minimum differentiation degree, which D’Aspremont *et al.* (1979) proved not to arise within the Hotelling framework, arises when partial collusion is supposed.

³⁰ For a survey of partial-collusion theories, see Grillo (1999).

Hackner (1995) turns back to the analysis of the relationship between the differentiation degree and the sustainability of collusion, following the approach used by Chang (1991) and Chang (1992). The main difference with respect to those works is that Hackner (1995) supposes that firms collude not only on the uniform price but also on the differentiation degree. Therefore, his work can be seen as an extension of the work by Chang (1991), where locations are exogenous, and Chang (1992), where the locational choice is endogenized during the punishment phase but not during the collusive phase³¹. First, as noted also by Chang (1991), the profit maximizing collusive price is easier to sustain the higher the collusive differentiation degree is. Second, collusive profits are maximized when firms are located at the first and the third quartile of the segment. It follows that when the market discount factor is high enough to sustain optimal location-price collusion, the firms agree to locate at the first and the third quartile. Things become more interesting when optimal location-price collusion cannot be sustained as a sub-game perfect equilibrium because the market discount factor is too low. In this case, Hackner (1995) demonstrates that firms collude on a higher differentiation degree, because this allows them to set the profit maximizing collusive price. Apart from the case of a very low market discount factor, the profit maximizing collusive price can be sustained by increasing differentiation; when differentiation cannot increase further because the firms are located at the endpoints of the segment, firms collude on a price lower than the profit maximizing collusive price. The main conclusion of Hackner (1995) is that within the Hotelling model there is tendency, also highlighted by the works by Chang (1991) and Chang (1992), to increase the differentiation degree in order to facilitate the collusive agreement³².

A different approach is followed by Liu and Serfes (2007), which building on Liu and Serfes (2004) introduce the hypothesis of price discrimination in evaluating the sustainability of a collusive agreement. Here the focus is not on the relationship between product differentiation and sustainability of collusion (firms are assumed to be maximally differentiated), but on the relationship between the precision of information about consumers and the sustainability of collusion. Firms can collude on two aspects:

³¹ However, Hackner (1995) assumes that the relocation costs during the punishment phase are negligible, while in Chang (1992) the relocation costs are kept general.

³² The U.S. Department of Justice Merger Guidelines (1992) instead claims that differentiation is pro-competitive, since homogeneity increases “*both the ability to reach terms of coordination and to detect or punish deviations from those terms*”.

the information acquisition and the price schedules. Three collusive schemes are supposed:

- 1) firms agree to buy the information technology and agree on the discriminatory price schedule to apply;
- 2) firms agree not to buy the information technology and agree on the uniform price schedule to apply;
- 3) firms agree not to buy the information technology but do not agree on the uniform price schedule to apply.

Clearly, the second and the third collusive scheme are motivated by the fact that the first collusive scheme may be very difficult to implement since it involves a huge number of different prices. As usual, the sustainability of each collusive scheme is measured by the critical discount factor. The more interesting innovation of this analysis consists in the possibility for the firms to price discriminate. In particular, since price discrimination is the most profitable strategy for each firm, deviation entails price discrimination. The punishment phase is characterised by both firms price discriminating as well. Liu and Serfes (2007) show that the first collusive scheme yields higher collusive profits than the second, which in turn yields higher profits than the third collusive scheme. This is not surprising: the first collusive scheme allows firms to make use of the advantages of price discrimination (extraction of the consumer surplus) without bearing the costs of the intensified competition induced by price discrimination when the price schedules are set non-cooperatively. Both in the second and in the third collusive scheme the firms do not price discriminate; in the second scheme they collude on price, while in the third scheme the price is set non-cooperatively: it follows that the second collusive scheme yields higher collusive profits than the third. Furthermore, Liu and Serfes (2007) show that the first collusive scheme is easier to sustain than the second: this implies that the second scheme is dominated by the first scheme and therefore it never occurs in equilibrium. Instead, the third collusive scheme is easier to sustain than the first when the quality of the information technology is low. On the contrary, when the quality of the information technology is high, the first scheme is easier to sustain than the third and therefore dominates it. Finally, Liu and Serfes (2007) demonstrate that the critical discount factor for any collusive scheme increases with the quality of the information: that is, collusion becomes more difficult to sustain when firms have better information.

The intuition behind this result is the following. Consider the second and the third collusive scheme. When the quality of information increases, the collusive profits clearly do not change, since these collusive schemes imply the refusal to use the information technology; the deviation profits increase, because the deviating firm can now better target the deviation price schedule; the punishment profits decrease, since the competition becomes fiercer. Liu and Serfes (2007) show that the effect of better information on the deviation profits outweighs the effect on the punishment profits: it follows that the critical discount factor of the second and the third collusive scheme must increase with the precision of the information. In the first collusive scheme better information increases both the collusive profits and the deviation profits, and lowers the Nash profits. Even in this case the effect of larger deviation profits outweighs the effect of larger collusive profits and smaller punishment profits, and therefore the sustainability of the first collusive scheme decreases with higher quality of information. These results have a straightforward and relevant implication for antitrust analysis: the collection and the use of consumer information should not be discouraged, because they make collusion less sustainable. However, when collusion is sustainable, less information is beneficial for the consumers³³.

1.5. Appendix

As noted in section 1.3., the effect of price discrimination on the equilibrium prices in oligopolies exhibiting best-reply symmetry is quite similar to its effect in monopoly: some equilibrium discriminatory prices are higher than the equilibrium uniform price while other equilibrium discriminatory prices are lower than the equilibrium uniform price. Instead, if best-reply asymmetry occurs, the equilibrium discriminatory prices may be all lower than the equilibrium uniform prices (*all-out competition*): best-reply asymmetry is the necessary (but not sufficient) condition for *all-out competition* to

³³ Suppose that the market discount factor is so high that equation (2) is satisfied for any level of the information quality: since collusion is always sustainable, colluding firms that can price discriminate are able to extract more consumer surplus than colluding firms that cannot price discriminate. It follows that the prohibition of price discrimination would not prevent firms to collude, but would limit the amount of consumer surplus that the firms would be able to extract.

occur. In this appendix we show that the Hotelling model and a typical vertical differentiation model are characterized by best-reply asymmetry and *all-out competition*.

Horizontally differentiated firms

Assume a linear market of length 1. Consumers are uniformly distributed along the segment. Define with $x \in [0,1]$ the location of each consumer. Each point in the linear market represents a certain variety of a given good. For a consumer positioned at a given point, the preferred variety is represented by the point in which the consumer is located: the more the variety is far from the point in which the consumer is located, the less it is appreciated by the consumer. Each consumer buys no more than 1 unit of the good. Define with v the maximum price that a consumer is willing to pay for buying his preferred variety. Suppose that v is equal for all consumers and that it is large enough to guarantee that each consumer always buys the good. There are two firms, A and B , competing in the market. Marginal costs are zero. Firm A is located at a , while firm B is located at b . Without loss of generality, assume $0 \leq a < b \leq 1$. Define with p_A the price set by firm A and with p_B the price set by firm B . The utility of each consumer depends on v , on the price set by the firm from which he buys, and on the distance between its preferred variety and the variety produced by the firm. The parameter t , equal for all consumers, measures the importance attributed by the consumer to the distance between his preferred variety and the variety offered by the firm. Quadratic transportation costs are assumed. The utility of a consumer located at x when he buys from firm A is given by: $u = v - p_A - t(x - a)^2$, while his utility when he buys from firm B is given by: $u = v - p_B - t(x - b)^2$.

Suppose that firms cannot price discriminate. Define with x^\wedge the consumer which is indifferent between buying from firm A or from firm B . Equating the utility in the two cases and solving for x it follows:

$$x^\wedge = \frac{a+b}{2} + \frac{p_B^u - p_A^u}{2t(b-a)}$$

where the superscript indicates that we are in the uniform price regime case. The demand functions of firm A and firm B are given respectively by:

$$q_A^u = x^\wedge = \frac{a+b}{2} + \frac{p_B^u - p_A^u}{2t(b-a)}; \quad q_B^u = 1 - x^\wedge = \frac{2-a-b}{2} - \frac{p_B^u - p_A^u}{2t(b-a)}$$

The profit function of each firm is given by: $\Pi_j = p_j^u q_j^u$, with $j = A, B$. Maximizing the profit functions with respect to the price we obtain the following best-reply functions:

$$b_A^u(p_B^u) = \frac{t(b-a)(a+b)}{2} + \frac{p_B^u}{2}; \quad b_B^u(p_A^u) = \frac{t(b-a)(2-a-b)}{2} + \frac{p_A^u}{2}$$

The equilibrium uniform prices are the following:

$$p_A^{u*} = \frac{t(2+a+b)(b-a)}{3}; \quad p_B^{u*} = \frac{t(4-a-b)(b-a)}{3}$$

Suppose now that both firms are able to recognize whether a consumer is $x \leq x^*$ or $x \geq x^*$, where $x^* \in (0,1)$. Therefore, there are two markets: a market composed by consumers $x \leq x^*$ (market 1), and a market composed by consumers $x \geq x^*$ (market 2). Firms can price discriminate between the two markets. Denote by p_j^i , with $j = A, B$ and $i = 1, 2$, the price applied by firm j in market i . The indifferent consumer in market 1 is given by:

$$x^{\wedge 1} = \frac{a+b}{2} + \frac{p_B^1 - p_A^1}{2t(b-a)}$$

while the indifferent consumer in market 2 is given by:

$$x^{\wedge 2} = \frac{a+b}{2} + \frac{p_B^2 - p_A^2}{2t(b-a)}.$$

The demand of the two firms in each market is given by:

$$q_A^1 = x^{\wedge 1} = \frac{a+b}{2} + \frac{p_B^1 - p_A^1}{2t(b-a)}; \quad q_B^1 = x^* - x^{\wedge 1} = \frac{2x^* - a - b}{2} - \frac{p_B^1 - p_A^1}{2t(b-a)}$$

$$q_A^2 = x^{\wedge 2} - x^* = \frac{a+b-2x^*}{2} + \frac{p_B^2 - p_A^2}{2t(b-a)}; \quad q_B^2 = 1 - x^{\wedge 2} = \frac{2-a-b}{2} - \frac{p_B^2 - p_A^2}{2t(b-a)}$$

The profit functions are given by: $\Pi_j^i = p_j^i q_j^i$, with $j = A, B$ and $i = 1, 2$. The best-reply functions in the two markets are the following:

$$b_A^1(p_B^1) = \frac{t(b-a)(a+b)}{2} + \frac{p_B^1}{2}, \quad b_B^1(p_A^1) = \frac{t(b-a)(2x^* - a - b)}{2} + \frac{p_A^1}{2}$$

$$b_A^2(p_B^2) = \frac{t(b-a)(a+b-2x^*)}{2} + \frac{p_B^2}{2}, \quad b_B^2(p_A^2) = \frac{t(b-a)(2-a-b)}{2} + \frac{p_A^2}{2}$$

Since $b_A^1(p_B) > b_A^2(p_B)$ and $b_B^2(p_A) > b_B^1(p_A)$ for any p_B and p_A , the oligopoly exhibits best-reply asymmetry. Moreover, market 1 (2) and market 2 (1) are respectively the *strong* (*weak*) market and the *weak* (*strong*) market for firm A (B). Solving the system of the best-reply functions, the following equilibrium discriminatory prices arise³⁴:

³⁴ In calculating the equilibrium discriminatory prices, we are implicitly assuming that both prices are strictly positive, which in turn requires that: $4x^* - 2 < a + b < 4x^*$. If this condition does not hold, one of the two firms is a constrained monopolist in one of the markets. Suppose for example that: $4x^* < a + b$. In this case firm B sets a zero price in market 1. Firm A acts as a constrained monopolist in market 1, and sets the highest price which allows it to serve the whole market given that firm B is setting a price equal to zero. Therefore, the optimal discriminatory price by firm A in market 1 is the solution of: $q_A^1(p_B^1 = 0) = x^*$ with respect to p_A^1 . It follows that: $p_A^1 = t(b-a)(a+b-2x^*)$. Comparing the equilibrium discriminatory price in the left segment with the equilibrium uniform price, it is easy to note that the *all-out competition* may not occur. In particular, the equilibrium discriminatory price is higher than the equilibrium uniform price when: $3x^* + 1 < a + b$. The intuition is the following. When consumers in market 1 are far from both firms, the nearer firm (i.e. firm A) is more likely to act as a monopolist: this is what the condition $4x^* < a + b$ says. When consumers in market 1 are *very* distant from both firms (that is, when $4x^* < 3x^* + 1 < a + b$ holds), not only firm A becomes a monopolist, but it

$$\begin{cases} p_A^{1*} = \frac{t(b-a)(a+b+2x^*)}{3}; & p_B^{1*} = \frac{t(b-a)(4x^*-a-b)}{3} \\ p_A^{2*} = \frac{t(b-a)(2+a+b-4x^*)}{3}; & p_B^{2*} = \frac{t(b-a)(4-a-b-2x^*)}{3} \end{cases}$$

A simple comparison between the equilibrium discriminatory prices and the equilibrium uniform prices, p_A^u and p_B^u , shows that the equilibrium discriminatory prices are all lower than the equilibrium uniform prices. Therefore, *all-out competition* occurs³⁵.

Vertically differentiated firms

Suppose to have two firms, A and B ³⁶. Firm A produces a good of quality s_A , while firm B produces a good of quality s_B . Assume that $\Delta = s_A - s_B > 0$: that is, firm A is the high-quality firm, while firm B is the low-quality firm. Marginal costs are zero. Define with p_A the price set by firm A and with p_B the price set by firm B . Assume that consumers are uniformly distributed over $[\underline{g}, \bar{g}]$, where the parameter g indicates the preference for quality. The utility of a consumer g when he buys from firm A is given by: $u = gs_A - p_A$, while his utility when he buys from the firm B is given by: $u = gs_B - p_B$.

Assume for the moment that: $\bar{g} > 2\underline{g}$ (that is, there is high heterogeneity of the consumers). Suppose that firms cannot price discriminate. Define with g^{\wedge} the

is also able to set in market 1 a price which is higher than the equilibrium price that would result in absence of discrimination. This is due to the fact that, given the assumption of quadratic transportation costs, the more the consumers are distant from the firms, the more the nearer firm is preferred by consumers with respect to the farther firm. Of course, the analysis is similar when $a+b < 4x^*-2$. In this case firm B is the constrained monopolist in market 2 and it sets: $p_B^{2*} = t(b-a)(2x^*-a-b)$, which is higher than the equilibrium uniform price when $a+b < 3x^*-2$.

³⁵ Our finding is consistent with Bester and Petrakis (1996) result. Using a symmetric model with third-degree price discrimination, they show that all equilibrium discriminatory prices are lower than the equilibrium uniform price. The same result occurs in our exercise when symmetry is assumed. In fact, symmetry implies $a+b=1$ and $x^*=1/2$, from which it follows that: $4x^*-2 < a+b < 4x^*$, which is a sufficient condition for *all-out competition* to occur.

³⁶ The vertical model proposed here follows Tirole (1988, p. 296).

consumer which is indifferent between buying from firm A or from firm B . Equating the utility functions in the two cases, we get:

$$\mathcal{G}^\wedge = \frac{P_A^u - P_B^u}{\Delta}$$

where the superscript indicates that we are in the uniform price regime case. The demand functions of firm A and firm B are given respectively by:

$$q_A^u = \bar{\mathcal{G}} - \mathcal{G}^\wedge = \bar{\mathcal{G}} - \frac{P_A^u - P_B^u}{\Delta} ; \quad q_B^u = \mathcal{G}^\wedge - \underline{\mathcal{G}} = \frac{P_A^u - P_B^u}{\Delta} - \underline{\mathcal{G}}$$

The profit function of each firm is given by: $\Pi_j = p_j^u q_j^u$, with $j = A, B$. After standard calculations, the best-reply functions are:

$$b_A^u(p_B^u) = \frac{\Delta \bar{\mathcal{G}} + P_B^u}{2} ; \quad b_B^u(p_A^u) = \max\left[\frac{P_A^u - \Delta \underline{\mathcal{G}}}{2}, 0\right]$$

The equilibrium uniform prices are the following:

$$p_A^{u*} = \frac{\Delta(2\bar{\mathcal{G}} - \underline{\mathcal{G}})}{3} ; \quad p_B^{u*} = \frac{\Delta(\bar{\mathcal{G}} - 2\underline{\mathcal{G}})}{3}$$

Suppose now that both firms are able to recognize whether a consumer is $\mathcal{G} \leq \mathcal{G}^*$ or $\mathcal{G} \geq \mathcal{G}^*$, where $\mathcal{G}^* \in (\underline{\mathcal{G}}, \bar{\mathcal{G}})$. Therefore, there are two markets: market 1, which is composed by low-quality consumers ($\mathcal{G} \leq \mathcal{G}^*$), and market 2, which is composed by high-quality consumers ($\mathcal{G} \geq \mathcal{G}^*$). Firms can price discriminate between the two markets. Define with p_j^i , with $j = A, B$ and $i = 1, 2$, the price applied by firm j in market i . The indifferent consumer in market 1 is given by:

$$\mathcal{G}^{\wedge 1} = \frac{P_A^1 - P_B^1}{\Delta}$$

while the indifferent consumer in market 2 is given by:

$$g^{\wedge 2} = \frac{p_A^2 - p_B^2}{\Delta}$$

The demand of the two firms in each market is given by:

$$q_A^1 = g^* - g^{\wedge 1} = g^* - \frac{p_A^1 - p_B^1}{\Delta} ; \quad q_B^1 = g^{\wedge 1} - \underline{g} = \frac{p_A^1 - p_B^1}{\Delta} - \underline{g}$$

$$q_A^2 = \bar{g} - g^{\wedge 2} = \bar{g} - \frac{p_A^2 - p_B^2}{\Delta} ; \quad q_B^2 = g^{\wedge 2} - g^* = \frac{p_A^2 - p_B^2}{\Delta} - g^*$$

The profit functions are given by: $\Pi_j^i = p_j^i q_j^i$, with $j = A, B$ and $i = 1, 2$. Straightforward calculations show that the best-reply functions of each firm in the two markets are the following:

$$b_A^1(p_B^1) = \frac{\Delta g^* + p_B^1}{2}, \quad b_B^1(p_A^1) = \max\left[\frac{p_A^1 - \Delta \underline{g}}{2}, 0\right]$$

$$b_A^2(p_B^2) = \frac{\Delta \bar{g} + p_B^2}{2}, \quad b_B^2(p_A^2) = \max\left[\frac{p_A^2 - \Delta g^*}{2}, 0\right].$$

The condition for the existence of best-reply asymmetry is always satisfied. In fact, $b_A^2(p_B) > b_A^1(p_B)$ and $b_B^1(p_A) \geq b_B^2(p_A)$ are always contemporaneously satisfied, since it is: $\underline{g} < g^* < \bar{g}$. Therefore, market 1 (2) and market 2 (1) are respectively the *strong (weak)* and the *weak (strong)* market for firm B (A). Why does the low-quality firm set a higher price in the low-quality market than in the high-quality market for any given price set by the high-quality firm? Both consumers in market 1 and market 2 prefer, at equal prices, firm A to firm B (this is precisely the nature of vertical differentiation). However, the preference for the high-quality firm is less strong in market 1 than in market 2. Therefore, firm B is less disadvantaged in market 1 than in market 2: for this

reason firm B sets a higher price in market 1 (the low-quality consumers market), that therefore represents its *strong* market.

Solving the system defined by the best-reply functions, we obtain the following equilibrium discriminatory prices:

$$\begin{aligned}
 1) \quad & \begin{cases} p_A^{1*} = \frac{\Delta(2\underline{g}^* - \underline{g})}{3}; & p_B^{1*} = \frac{\Delta(\overline{g}^* - 2\underline{g})}{3} \\ p_A^{2*} = \frac{\Delta(2\overline{g} - \overline{g}^*)}{3}; & p_B^{2*} = \frac{\Delta(\overline{g} - 2\overline{g}^*)}{3} \end{cases} & \text{if } 2\underline{g} < \overline{g}^* < \frac{\overline{g}}{2} \\
 2) \quad & \begin{cases} p_A^{1*} = \frac{\Delta \overline{g}^*}{2}; & p_B^{1*} = 0 \\ p_A^{2*} = \frac{\Delta(2\overline{g} - \overline{g}^*)}{3}; & p_B^{2*} = \frac{\Delta(\overline{g} - 2\overline{g}^*)}{3} \end{cases} & \text{if } \overline{g}^* < \min[2\underline{g}; \frac{\overline{g}}{2}] \\
 3) \quad & \begin{cases} p_A^{1*} = \frac{\Delta(2\underline{g}^* - \underline{g})}{3}; & p_B^{1*} = \frac{\Delta(\overline{g}^* - 2\underline{g})}{3} \\ p_A^{2*} = \frac{\Delta \overline{g}}{2}; & p_B^{2*} = 0 \end{cases} & \text{if } \overline{g}^* > \max[2\underline{g}; \frac{\overline{g}}{2}] \\
 4) \quad & \begin{cases} p_A^{1*} = \frac{\Delta \overline{g}^*}{2}; & p_B^{1*} = 0 \\ p_A^{2*} = \frac{\Delta \underline{g}}{2}; & p_B^{2*} = 0 \end{cases} & \text{if } \frac{\overline{g}}{2} < \overline{g}^* < 2\underline{g}
 \end{aligned}$$

It is easy to show that the equilibrium discriminatory prices are in any case all lower than the equilibrium uniform prices, p_A^u and p_B^u ³⁷. Therefore, *all-out competition* occurs.

³⁷ For case 1) it is sufficient to recall that: $\underline{g} < \overline{g}^* < \overline{g}$. For case 2) we have to compare p_A^{1*} with p_A^u . For *all-out competition* not to occur it must be: $p_A^{1*} = \Delta \overline{g}^* / 2 > \Delta(2\overline{g} - \underline{g}) / 3 = p_A^u$, which implies: $\overline{g}^* > (4\overline{g} - 2\underline{g}) / 3$. However, $p_A^{1*} = \Delta \overline{g}^* / 2$ only if: $2\underline{g} > \overline{g}^*$. We show now that $\overline{g}^* > (4\overline{g} - 2\underline{g}) / 3$ and $2\underline{g} > \overline{g}^*$ cannot be contemporaneously verified. In fact, it should be: $2\underline{g} > (4\overline{g} - 2\underline{g}) / 3$, which implies $\overline{g} < 2\underline{g}$, which is impossible by assumption. For case 3) we have to compare p_A^{2*} with p_A^u . For *all-out*

Finally, suppose that: $\bar{g} < 2\underline{g}$ (that is, there is low heterogeneity of the consumers). Under uniform price regime, the whole market is served by the high-quality firm at a price equal to³⁸: $p_A^{u*} = \frac{\Delta\bar{g}}{2}$. Under price discrimination, we are in the case 4) defined above³⁹. All consumers in both markets are served only by the high-quality firm at prices: $p_A^{1*} = \frac{\Delta\underline{g}}{2}$ and $p_A^{2*} = \frac{\Delta\bar{g}}{2}$. Hence, the equilibrium price under price discrimination is strictly lower than the equilibrium uniform price for the low-quality consumers, while the equilibrium discriminatory price is equal to the equilibrium uniform price for the high-quality consumers.

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competition not to occur it must be $p_A^{2*} = \Delta\bar{g}/2 > \Delta(2\bar{g} - \underline{g})/3 = p_A^{u*}$, which implies $\bar{g} < 2\underline{g}$, which is impossible by assumption. Finally, the proof for case 4) follows from the proofs of case 2) and case 3).

³⁸ See Tirole (1988, p. 297).

³⁹ Indeed, $\bar{g} < 2\underline{g}$ implies both $\underline{g} < 2\underline{g}$ and $\bar{g} < 2\underline{g}$, from which it follows: $\bar{g}/2 < \underline{g} < 2\underline{g}$.

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Discriminatory prices, endogenous product differentiation and the Prisoner Dilemma problem

Stefano Colombo

Abstract

In the Hotelling framework, the equilibrium first-degree discriminatory prices are all lower than the equilibrium uniform price. When firms' varieties are fixed, price discrimination emerges as the unique equilibrium in a game in which every firm may commit not to discriminate before setting the price schedule. This chapter assumes endogenous product differentiation degree and shows that uniform pricing emerges as the unique equilibrium in a game in which every firm may commit not to discriminate before choosing the variety. Price discrimination still is the unique equilibrium outcome when firms may commit only after the variety choice. When third-degree price discrimination is introduced the main results do not change.

JEL codes: D43; L11

Keywords: Price discrimination; Commitment; Variety.

2.1. Introduction

Price discrimination is frequently used by firms. However, in oligopoly it may be possible that the equilibrium discriminatory prices are all lower than the equilibrium uniform price. This phenomenon is called *all-out competition*⁴⁰. When *all-out competition* occurs, equilibrium profits under the discriminatory price regime are lower than the equilibrium profits under the uniform price regime.

All-out competition typically emerges in the Hotelling (1929) framework with linear or quadratic transportation costs. Thisse and Vives (1988) study the case of perfect price discrimination within the Hotelling (1929) model when firms are maximally differentiated. First, they show that when firms can perfectly price discriminate and simultaneously choose the price schedule, uniform pricing is never an equilibrium. Then, Thisse and Vives (1988) assume a two-stage game, where in the first stage each firm chooses the pricing policy, while in the second stage the price schedules are set. They show that even when every firm may credibly commit to uniform pricing before setting the price schedule, the discriminatory prices still arise in equilibrium, since no-commitment is the dominant strategy for each firm in the first stage of the game conditioned on the equilibrium path in the second stage of the game. This situation gives rise to a typical prisoner dilemma: both firms would be better off setting uniform prices, but the dominant strategy of each firm induces the discriminatory equilibrium.

However, assuming that the product differentiation degree is exogenous doesn't seem to describe well those markets where firms compete both on price and variety. In contrast, this chapter investigates the firms' incentives to price discriminate when the product differentiation degree is *endogenous*. Two different versions of a three-stage game are considered. In the first version, firms first simultaneously choose which variety to produce, then they choose whether to price discriminate or not, then they set the price schedules. A prisoner dilemma arises: firms price discriminate, all discriminatory prices are lower than the uniform prices, and profits are lower than under uniform pricing. In the second version of the game, the first two stages are reversed: firms first choose the pricing policy and then they choose the variety. Interestingly, in this case the (unique) sub-game perfect equilibrium is characterized by uniform pricing:

⁴⁰ Corts (1998).

both firms commit to uniform pricing in the first stage, and no prisoner dilemma is present. This is due to the emerging of a *product differentiation effect*: the possibility to price discriminate induces a lower product differentiation degree, which in turn increases the competition between firms and makes the no-commitment strategy less profitable for each firm. The formalization of the product differentiation effect, which was missing (as far as we know) from the price discrimination literature, is the main contribution of this chapter, and we believe it may enhance the understanding about firms' incentives to price discriminate.

This chapter is organized as follows. In Section 2.2. we describe the model and we briefly recall the well-known variety-price equilibrium under the hypothesis of uniform price regime. In section 2.3. we analyse the variety-price equilibrium when the firms can perfectly price discriminate. In section 2.4. we analyze the two versions of the three-stage game. Section 2.5. concludes. In the appendix we extend the model to consider third-degree price discrimination, and we show that the main results do not change.

2.2. Uniform price

Assume a linear market of length 1. Consumers are uniformly distributed along the segment. Define with $x \in [0,1]$ the location of each consumer. Each point in the linear market represents a certain variety of a given good. For a consumer positioned at a given point, the preferred variety is represented by the point in which the consumer is located: the more the variety is far from the point in which the consumer is located, the less it is appreciated by the consumer. Each consumer consumes no more than 1 unit of the good. Define with v the maximum price that a consumer is willing to pay for buying his preferred variety. Suppose that v is equal for all consumers. Suppose further that v is large enough to guarantee that each consumer always buys the good.

There are two firms, A and B , competing in the market. Both firms have zero marginal costs. Define with a the variety chosen by firm A and with b the variety chosen

by firm B . Without loss of generality, assume: $0 \leq a \leq b \leq 1$. Define with \bar{p}^A the uniform price set by firm A and with \bar{p}^B the uniform price set by firm B ⁴¹.

The utility of a consumer depends on v , on the price set by the firm from which he buys, and on the distance between his preferred variety and the variety produced by the firm. We assume quadratic disutility costs. Define with t , equal for all consumers, the importance attributed by the consumer to the distance between his preferred variety and the variety offered by the firm. The utility of a consumer located at x when he buys from firm A is given by: $u_x = v - \bar{p}^A - t(x-a)^2$, while the utility of a consumer located at x when he buys from firm B is given by: $u_x = v - \bar{p}^B - t(x-b)^2$. Define with x^* the consumer which is indifferent between buying from firm A or from firm B for a given couple of varieties, a and b , and for a given couple of uniform prices, \bar{p}^A and \bar{p}^B . Equating the utility in the two cases and solving for x it follows:

$$x^* = \frac{a+b}{2} + \frac{\bar{p}^B - \bar{p}^A}{2t(b-a)}$$

Given the uniform distribution of the consumers, x^* is the demand function of firm A and $1-x^*$ is the demand function of firm B . It is well known that in a two-stage game in which firms first choose varieties and then choose the uniform price, the unique sub-game perfect equilibrium implies maximal differentiation, as the following proposition indicates:

Proposition 1 (D'Aspremont *et al.* 1979): *in a two-stage game in which the firms first simultaneously choose the variety and then simultaneously decide the [uniform] price,*

⁴¹ Given the interpretation of the linear market that we are adopting, the “transportation costs” are necessarily sustained by the consumers: therefore, prices are *f.o.b.*. However, the linear market can also have a “spatial” interpretation: in this case each point of the segment represents a point in the physical space. Since the distance between a consumer and the firm implies now effective transportation costs, two pricing methods are possible: *f.o.b.* prices, when the transportation costs are sustained by the consumer which goes and takes up the product at the firm’s mill, and *delivered* prices, when the transportation costs are sustained by the firm that carries the product from the mill to the consumer. Thisse and Vives (1988) adopt a spatial interpretation of the market and assume *delivered* prices.

there is a unique sub-game perfect equilibrium, defined by $a^* = 0$ and $b^* = 1$, and $\bar{p}^A = \bar{p}^B = t$.

Given the equilibrium varieties and the equilibrium prices, the equilibrium profits for each firm are: $\Pi^A = \Pi^B = t/2$.

2.3. Perfect discriminatory prices

We study now the variety-price equilibrium when both firms can perfectly price discriminate between consumers. We suppose a two-stage game, in which the firms first decide which variety to produce and then compete on prices. Before to start, note that the fact that the firms have the possibility to price discriminate does not imply that the firms effectively price discriminate: a firm may decide to price uniformly even if it can price discriminate. In the following we show that when firms can price discriminate, they do it. Consider a consumer located in x . Define with p_x^J the price charged by firm $J = A, B$ to the consumer x . The utility of that consumer when he buys from firm A is given by: $u_x = v - p_x^A - t(x - a)^2$, while his utility when he buys from firm B is given by: $u_x = v - p_x^B - t(x - b)^2$. The consumer buys from the firm which gives him the higher utility. If the utility of the consumer is the same when he buys from firm A and when he buys from firm B , we suppose that he buys from the nearer firm⁴². Suppose that consumer x is nearer to firm A than to firm B . For a given couple of firms' varieties and for a given price set by firm B , the best thing firm A can do is setting a price that gives the consumer the same utility he receives from firm B : this is the highest possible price that guarantees that consumer x buys from A . Suppose instead that the consumer x is nearer to firm B . For a given couple of firms' varieties and for a given price set by firm B , in order to serve consumer x the best thing firm A can do is giving him a slightly

⁴² This assumption is common in spatial models, and it is necessary to avoid the technicality of ε -equilibria when both firms price discriminate. For more details about this assumption, see among the others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

higher utility than the utility provided to him by firm B . Of course, an analogous reasoning holds for firm B .

The following proposition defines the equilibrium price schedule for any couple of varieties.

Proposition 2: *when the firms can perfectly price discriminate between the consumers, the equilibrium prices in the second stage of the game are the following:*

$$p_x^A * (a, b) = \begin{cases} t(x-b)^2 - t(x-a)^2 & \text{if } x \leq (a+b)/2 \\ 0 & \text{if } x \geq (a+b)/2 \end{cases}$$

$$p_x^B * (a, b) = \begin{cases} 0 & \text{if } x \leq (a+b)/2 \\ t(x-a)^2 - t(x-b)^2 & \text{if } x \geq (a+b)/2 \end{cases}$$

Proof. Suppose that x is near to firm A , that is, $x < (a+b)/2$. Consider firm B . First, we show that $p_x^B > 0$ cannot be an equilibrium. When $p_x^B > 0$, the best-reply of firm A consists in setting: $p_x^A = p_x^B + t(x-b)^2 - t(x-a)^2$: the consumer x obtains the same utility and buys from firm A . But firm B has now the incentive to undercut firm A by setting a price equal to: $p_x^{B'} = p_x^B - \varepsilon$, where ε is a positive and infinitely small number. Since p_x^B is higher than 0 by hypothesis and ε is a positive and infinitely small number by definition, $p_x^{B'}$ is higher than 0. Therefore, $p_x^B > 0$ cannot be an equilibrium, because firm B would obtain higher profits by setting $p_x^{B'}$. We show instead that $p_x^B = 0$ is an equilibrium. The best-reply of firm A is: $p_x^A = t(x-b)^2 - t(x-a)^2$. With such a price firm B obtains zero profits from consumer x , which buys from firm A , but it has no incentive to change the price, because increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss. It follows that $p_x^A = t(x-b)^2 - t(x-a)^2$ and $p_x^B = 0$ represents the (unique) price equilibrium. The proof for $x > (a+b)/2$ is symmetric to the proof for $x < (a+b)/2$. Finally, when the consumer is at the same distance from the two firms, that is $x = (a+b)/2$, the standard Bertrand's result holds: the unique price

equilibrium when two undifferentiated firms compete on price is represented by both firms setting a price equal to the marginal cost (which in this case is zero). ■

Using Proposition 2, the firms' profits can be written directly as functions of a and b :

$$\Pi^A(a, b) = t(b - a)(a + b)^2 / 4 \quad (1)$$

$$\Pi^B(a, b) = t(b - a)(2 - a - b)^2 / 4 \quad (2)$$

The equilibrium varieties in the first stage of the game are defined in the next proposition:

Proposition 3: *in the first stage of the game the unique Nash equilibrium is given by $a^* = 1/4$ and $b^* = 3/4$.*

Proof. Maximizing (1) and (2) with respect to a and b we get:

$$\partial \Pi^A / \partial a = t(b^2 - 3a^2 - 2ab) / 4 = 0 \quad (3)$$

$$\partial \Pi^B / \partial b = t(3b^2 - a^2 + 2ab - 8b + 4) / 4 = 0 \quad (4)$$

Consider equation (3) as a function of b . This equation has two solutions: $b = 3a$ and $b = -a$. The second solution is impossible, since neither a or b can be negative, and $a = b = 0$ does not solve equation (4). Therefore it must be: $b = 3a$. Substituting it in equation (4) and solving with respect to a we obtain two solutions: $a = 1/4$ and $a = 1/2$. The second solution is impossible, since we have $b = 3a = 3/2 > 1$, which is impossible. Therefore, the only admissible values which solve the system defined by equations (3) and (4) are $a^* = 1/4$ and $b^* = 3/4$. ■

The following proposition compares the variety-price equilibrium under perfect price discrimination with the variety-price equilibrium under the uniform price regime:

Proposition 4:

- a) *All prices are lower under perfect price discrimination than under uniform price. Therefore, profits are lower under perfect price discrimination than under uniform price.*
- b) *The surplus of each consumer is higher under perfect price discrimination than under uniform price, and the more the consumer is located near to the middle of the market the higher is the difference.*
- c) *Total welfare is maximized under perfect price discrimination.*

Proof.

a) Substituting $a = 1/4$ and $b = 3/4$ into the equilibrium discriminatory price schedules, it follows: $p_x^A * = t(1/2 - x)$, $\forall x \in [0, 1/2]$ and $p_x^B * = t(x - 1/2)$, $\forall x \in [1/2, 1]$. It follows that $\forall x \in [0, 1/2]$ we get $\bar{p}^A * = \bar{p}^B * = t > t(1/2 - x) = p_x^A *$, and $\forall x \in [1/2, 1]$ we get $\bar{p}^A * = \bar{p}^B * = t > t(x - 1/2) = p_x^B *$. Under price discrimination total profits are: $\Pi^D = t/4$, while under the uniform price regime they are: $\Pi^U = t$. Then: $\Delta\Pi = \Pi^D - \Pi^U = -3t/4 < 0$.

b) Under price discrimination, the surplus of a consumer located at $x \in [0, 1/2]$ is given by: $CS_x^D = v - p_x^A * - t(a^D * - x)^2 = v - t(1/2 - x) - t(1/4 - x)^2$, while the surplus of a consumer located at $x \in [1/2, 1]$ is given by: $CS_x^D = v - p_x^B * - t(b^D * - x)^2 = v - t(x - 1/2) - t(3/4 - x)^2$. Under uniform price, the surplus of a consumer located at $x \in [0, 1/2]$ is: $CS_x^U = v - \bar{p}^A * - t(a^U * - x)^2 = v - t - tx^2$, while the surplus of a consumer located at $x \in [1/2, 1]$ is: $CS_x^U = v - \bar{p}^B * - t(b^U * - x)^2 = v - t - t(1 - x)^2$. Define: $\Delta CS \equiv CS_x^D - CS_x^U$. It follows that: $\Delta CS = t(3x/2 + 7/16) > 0$, $\forall x \in [0, 1/2]$, and $\Delta CS = t(-3x/2 + 31/16) > 0$, $\forall x \in [1/2, 1]$. Moreover, $\partial\Delta CS/\partial x > 0 \forall x \in [0, 1/2]$ and $\partial\Delta CS/\partial x < 0 \forall x \in [1/2, 1]$.

c) Since the output is the same under the uniform price regime and the discriminatory price regime and the prices have only a redistributive effect, total welfare depends only on the disutility costs, which in turn are determined by the equilibrium varieties. Define

with \hat{a} and \hat{b} the optimal varieties from the total welfare point of view. They are: $(\hat{a}, \hat{b}) = \arg \min DC = \arg \min \left\{ \int_0^x t(z-a)^2 dz + \int_x^1 t(z-b)^2 dz \right\}$, where DC indicates the total disutility costs. The proof has two steps: first we calculate the optimal sharing of consumers, and then we calculate the optimal values of a and b .

$$1) \frac{\partial DC}{\partial x} = \frac{\partial \left\{ \int_0^x t(z-a)^2 dz + \int_x^1 t(z-b)^2 dz \right\}}{\partial x} = a^2 - 2ax + 2bx - b^2 = 0 \rightarrow x^* = \frac{a+b}{2}$$

$$2) DC(a, b) = \int_0^{\frac{a+b}{2}} t(x-a)^2 dx + \int_{\frac{a+b}{2}}^1 t(x-b)^2 dx = \frac{t(a^3 - ab^2 - b^3 + a^2b + 4/3 + 4b^2 - 4b)}{4}$$

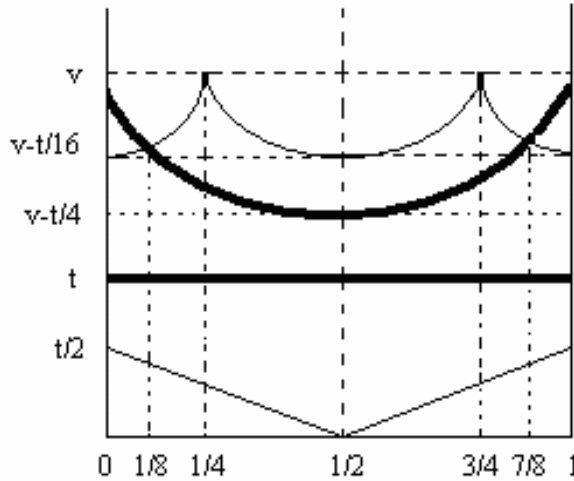
$$\partial DC / \partial a = t(3a^2 - b^2 + 2ab) / 4 = 0 \tag{5}$$

$$\partial DC / \partial b = t(-2ab - 3b^2 + a^2 + 8b - 4) / 4 = 0 \tag{6}$$

Since equations (5) and (6) coincide respectively with equations (3) and (4), the optimal varieties \hat{a} and \hat{b} coincide with the equilibrium varieties $a^* = 1/4$ and $b^* = 3/4$. ■

The characteristics of the variety-price equilibrium under the two pricing regimes are summarized in the following figure:

Figure 1: Illustration of Proposition 4



The thin and sloped lines in the bottom part of the graph represent the equilibrium prices set by the firms to each consumer under perfect price discrimination, while the

bold and flat line represents the equilibrium prices under uniform price. It is immediate to see that all equilibrium discriminatory prices are below the price line under the uniform price regime (*all-out competition*), and that the discriminatory prices decrease moving from consumers located at the endpoints to consumers located at the middle.

From the consumers' point of view, the surplus depends on the price paid and on the disutility costs sustained. The curves in the upper part of the graph describe the surplus, gross of the price, of each consumer: the bold curve refers to the uniform price regime while the thin curve refers to the discriminatory price regime. Under the discriminatory price regime, the gross consumer surplus is maximum for consumers located at $1/4$ and $3/4$, and decreases the more the consumers are distant from these points. The minimum gross consumer surplus is at points 0 , $1/2$ and 1 . Under the uniform price regime the gross consumer surplus is maximum at points 0 and 1 and it is minimum at point $1/2$. The net consumer surplus is given by the difference between the upper curves and the price lines. In Proposition 4 we state that the surplus of each consumer is higher under price discrimination than under the uniform price regime. For consumers located between $1/8$ and $7/8$ this is immediate, since both the disutility costs and the prices decrease passing from the uniform price regime to the discriminatory price regime. For the other consumers we observe two opposite effects: the disutility costs increase under price discrimination (since the firms now are farther from these consumers) but the equilibrium prices decrease. In order to prove that even for these consumers the surplus is higher under the discriminatory price regime than under the uniform price regime it is sufficient to compare the surplus of the most external consumers in the two cases, since the consumers located at point 0 and 1 are the best-positioned consumers under the uniform price regime and the worst-positioned consumers under the discriminatory price regime. Under uniform pricing, the surplus of the consumers located at points 0 and 1 is equal to $v - t$; under perfect price discrimination, the same consumers obtain a surplus which is equal to $v - 9t/16$. Since the surplus of these consumers increases passing from the uniform price regime to the discriminatory price regime, the same must be true for all other consumers.

Finally, in Proposition 4 we state that total welfare is maximized under price discrimination. Since the total output is the same under the uniform price regime and

under the discriminatory price regime and since prices have only a redistributive effect, total welfare depends only on the equilibrium varieties which determine the total disutility costs sustained by the consumers: the equilibrium varieties under price discrimination, $1/4$ and $3/4$, minimize the total disutility costs and therefore maximize total welfare.

2.4. A three-stage model

In section 2.3 we have shown that perfect price discrimination yields lower profits than uniform pricing. Now, suppose that each firm can choose the pricing policy before setting the price schedule. When product differentiation is exogenous and maximum, Thisse and Vives (1988) show that even if there is the possibility to commit to uniform pricing, price discrimination emerges in equilibrium⁴³. In this section we ask whether this result is still valid when varieties are endogenously chosen by the firms. Two different versions of a three stage game are considered: in the first version, firms first choose variety, then they choose whether to price discriminate or not, and finally they set the prices; in the second version, firms first choose the pricing policy, then choose the variety, and finally set the price schedules.

Game 1

Timing: at time 1, both firms simultaneously choose the variety; at time 2 both firms simultaneously decide whether to commit to uniform pricing (U) or not (D); at time 3 both firms simultaneously choose the price schedule.

⁴³ There are many ways in which a firm may commit to uniform pricing. One may imagine an explicit contract between the firm and the consumers. An example of such contract is the *most-favoured nation clause*, which engages a firm to offer a consumer the same price as its other consumers: if the clause is not respected, the firm must pay back the consumer the difference between the price he effectively paid and the lowest price fixed by the firm (Corts, 1998). Moreover, one may consider a more subtle (and perhaps more common) type of commitment to uniform pricing. Since a firm can price discriminate only if it is able to identify consumers (or groups of them), when a firm has the possibility to obtain specific consumers' information but it abstains from doing it, it is committing not to price discriminate (Liu and Serfes, 2004).

We solve the game by backward induction. Consider the third stage of the game. Suppose that one firm has committed at stage 2 while the other has not committed. If the utility of the consumer is the same when he buys from the discriminating firm and when he buys from the non discriminating firm, we assume that he buys from the discriminating firm⁴⁴. We state the following proposition:

Proposition 5: *if firm A has committed and firm B has not committed, the equilibrium prices in the third stage of the game are the following:*

$$\begin{aligned} \bar{p}^A * (a, b) &= t(b-a)(a+b)/2 \\ p_x^B * (a, b) &= \begin{cases} -t(b-a)(a+b)/2 + 2tx(b-a) & \text{if } x \geq (a+b)/4 \\ 0 & \text{if } x \leq (a+b)/4 \end{cases} \end{aligned}$$

If firm A has not committed and firm B has committed, the equilibrium prices in the third stage of the game are the following:

$$\begin{aligned} p_x^A * (a, b) &= \begin{cases} t(b-a)(2+a+b)/2 - 2tx(b-a) & \text{if } x \leq (2+a+b)/4 \\ 0 & \text{if } x \geq (2+a+b)/4 \end{cases} \\ \bar{p}^B * (a, b) &= t(b-a)(2-a-b)/2 \end{aligned}$$

Proof. Suppose that firm A has committed while firm B has not committed. Consider a generic consumer x . The best-reply of firm B consists in setting: $p_x^B = \bar{p}^A + t(x-a)^2 - t(x-b)^2$, where \bar{p}^A is the uniform price set by firm A. If firm A sets $\bar{p}^A > t(x-b)^2 - t(x-a)^2$, firm B can always serve the consumer x by undercutting the uniform price set by firm A without pricing below zero: therefore consumer x will always buy from firm B and firm A will obtain zero profits. In order to have a positive demand, firm A must set a uniform price such that: $\bar{p}^A \leq t(x-b)^2 - t(x-a)^2$, which

⁴⁴ This assumption is necessary to avoid ε -equilibria in the sub-games when only one firm price discriminates, and it can be easily rationalized noting that the discriminating firm can always offer to the consumer a utility which is strictly larger than the utility he receives from the non-discriminating firm simply by setting a price equal to $\hat{p}_x - \varepsilon$, where \hat{p}_x is the discriminatory price which makes the consumer x indifferent between the two firms and ε is a positive small number.

cannot be undercut by firm B . Therefore, the highest uniform price that firm B cannot undercut is given by: $\bar{p}^A = t(x-b)^2 - t(x-a)^2$. Solving for x , we obtain the most at the right consumer served by firm A : $x^{*'} = [t(b^2 - a^2) - \bar{p}^A] / 2t(b-a)$. Given that consumers are uniformly distributed, the demand of firm A is $x^{*'}$, while the demand of firm B is $1 - x^{*'}$. The profits of firm A are: $\Pi^A = \bar{p}^A x^{*'}$, while the demand of firm B is $1 - x^{*'}$. The profits of firm A are: $\Pi^A = \bar{p}^A x^{*'}$. Maximizing the profits with respect to \bar{p}^A it follows: $\bar{p}^{A*} = t(b^2 - a^2)/2$. Substituting \bar{p}^A into the best-reply function of firm B we get the equilibrium price schedule of the discriminating firm. The proof of the second part of the proposition proceeds in the same way. ■

We can write the firms' profits directly as functions of a and b in the four possible cases: (U,U), (U,D), (D,U) and (D,D)⁴⁵. We do it in the following table:

Table 1

$\Pi^A \backslash \Pi^B$	U	D
U	$t(b-a)(2+a+b)^2/18$; $t(b-a)(4-a-b)^2/18$	$t(b-a)(a+b)^2/8$; $t(b-a)(4-a-b)^2/16$
D	$t(b-a)(2+a+b)^2/16$; $t(b-a)(2-a-b)^2/8$	$t(b-a)(a+b)^2/4$; $t(b-a)(2-a-b)^2/4$

It is immediate to see that, for any couple of varieties, the dominant strategy of each firm is D. Given that at the second stage both firms do not commit, in the third stage they price discriminate and the equilibrium varieties are given by Proposition 3.

The following proposition summarizes and defines the unique sub-game perfect equilibrium:

⁴⁵ The profit functions in (D,D) are simply the functions (1) and (2); the profit functions in (U,D) and (D,U) come from Proposition 5; the profit functions in (U,U) can be obtained by standard calculations (see, for example, Tirole, 1988, p. 281).

Proposition 6: *in game 1, the unique sub-game perfect equilibrium is given by $a^* = 1/4$ and $b^* = 3/4$, (D,D), $p_x^{A*} = t(1/2 - x)$ and $p_x^{B*} = 0$ for $x \leq 1/2$, and $p_x^{A*} = 0$ and $p_x^{B*} = t(x - 1/2)$ for $x \geq 1/2$.*

Proof. Consider Table 1. If firm A chooses U, then firm B chooses D for any a and b , since $1/16 > 1/18$. When firm A chooses D, firm B chooses D for any a and b , since $1/4 > 1/8$. Then, D is the dominant strategy for firm B . The same is true for firm A . It follows that in the second stage of the game the equilibrium is given by both firms choosing D. The rest of the Proposition follows from Propositions 2 and 3. ■

Proposition 6 shows that the Prisoner Dilemma is present in game 1, since both firms do not commit even if this strategy is conducive to lower equilibrium profits. That is, assuming endogenous choice of the varieties before the commitment decision does not alter the Thisse and Vives (1988) result: in equilibrium, firms price discriminate.

Game 2

Timing: at time 1 both firms simultaneously decide whether to commit or not; at time 2 both firms simultaneously choose the variety; at time 3 both firms simultaneously choose the price schedule.

As usual, in order to solve the game we start from the last stage. We already have the equilibrium prices and the equilibrium varieties when both firms set a uniform price (Proposition 1) and when both price discriminate (Propositions 2 and 3). Moreover, we already know the equilibrium prices when one firm has committed and the other has not committed (Proposition 5). Therefore, it remains to calculate the equilibrium varieties in the sub-games arising when only one firm has committed in the first stage. Equilibrium varieties in these sub-games are defined by the following proposition:

Proposition 7: *if at the first stage of the game firm A has chosen U and firm B has chosen D, the equilibrium varieties at the second stage are given by $a^* = 1/3$ and*

$b^* = 1$; if at the first stage of the game firm A has chosen D and firm B has chosen U , the equilibrium varieties at the second stage are given by $a^* = 0$ and $b^* = 2/3$.

Proof. Maximize the profit functions in (U,D) of Table 1. It follows: $\partial\Pi^A/\partial a = t(b^2 - 3a^2 - 2ab)/8$ and $\partial\Pi^B/\partial b = t(4 - a - b)(4 + a - 3b)/16$. Consider the latter equation. Since it is always positive, firm B locates at the right extremity of the segment: that is, $b = 1$. Put it into the first equation and solve. There are two solutions: $a = 1/3$ and $a = -1$. Since the latter solution is impossible, the equilibrium varieties are $a^* = 1/3$ and $b^* = 1$. The second part of Proposition 7 is demonstrated in the same way. Maximize the profit functions in (D,U) of Table 1. It follows: $\partial\Pi^A/\partial a = t(2 + a + b)(-2 - 3a + b)/16$ and $\partial\Pi^B/\partial b = t(2 - a - b)(2 + a - 3b)/8$. The first equation is always negative: therefore, firm A has always the incentive to move to the left, that is, $a = 0$. Substitute into the second equation and solve. There are two solutions: $b = 2/3$ and $b = 2$. Since the second solution is impossible (b cannot be higher than 1) the unique equilibrium varieties are $a^* = 0$ and $b^* = 2/3$. ■

Since we have the equilibrium prices (third stage) and the equilibrium varieties (second stage) in all possible cases, we can write the equilibrium profits of each firm directly as functions of the pricing policy decision at the first stage of the game. The equilibrium profits are summarised in the following table:

Table 2

$\Pi^A \backslash \Pi^B$	U	D
U	$t/2; t/2$	$4t/27; 8t/27$
D	$8t/27; 4t/27$	$t/8; t/8$

The next proposition follows directly from Table 2:

Proposition 8: in game 2, the unique sub-game perfect equilibrium is given by (U,U) , $a^* = 0$ and $b^* = 1$, and $\bar{p}^A = \bar{p}^B = t$.

Perhaps surprisingly, if the variety decision is taken when the decisions regarding the pricing policy have already been taken, there exists a (unique) sub-game perfect equilibrium in which both firms commit to uniform pricing. On the contrary, when the decision whether to commit or not is taken after the variety decisions, the equilibrium is characterized by price discrimination. The intuition is the following. When the product differentiation degree is fixed at the pricing policy stage (*Game 1*), the possibility to price discriminate creates two effects: a *surplus extraction effect* and an *intensified competition effect* (Ulph and Vulkan, 2000). The first effect refers to the ability for each firm to extract more consumer surplus when it can customize the prices, while the second effect refers to the fact that price flexibility induces firms to compete more fiercely. In *Game 1*, the surplus extraction effect dominates the intensified competition effect, and each firm decides to price discriminate, even when each firm has the opportunity to commit to uniform pricing. Instead, when the varieties are chosen after the choice of the pricing policy (*Game 2*), a third effect, which indirectly influences the intensified competition effect, arises: we call it the *product differentiation effect*. As it turns out, product differentiation is lower (and then competition is fiercer) when firms do not commit to uniform pricing⁴⁶. Both the product differentiation effect and the intensified competition effect work in favour of the uniform pricing policy, and as a result these two effects dominate the surplus extraction effect and each firm commits to uniform pricing at the first stage of the game⁴⁷.

To sum up, the occurrence of the *product differentiation effect* seems to be the cause of the emerging of the uniform pricing equilibrium, because it makes price discrimination less attractive for firms. However, one may ask why the product differentiation effect arises, that is, why is product differentiation lower when firms price discriminate. In what follows we try to answer to this question by analysing the incentives that drive the variety choice of firm *A* (for firm *B* the reasoning is analogous).

Consider first the case in which both firms set a uniform price. When firm *A* chooses its variety, it faces two opposite effects: the *demand effect* and the *strategic effect*

⁴⁶ When both firms commit, the equilibrium distance between the firms is 1 (Proposition 1); if one firm commits not to price discriminate while the other does not commit, the equilibrium distance is $2/3$ (Proposition 7); when both firms do not commit, the equilibrium distance is $1/2$ (Proposition 3).

⁴⁷ We thank an anonymous referee for suggesting this analysis.

(Tirole, 1988). The former refers to the fact that firm A wants to move toward the centre to increase its market share given the price structure; the latter refers to the fact that firm A wants to move toward the endpoint of the segment in order to increase the product differentiation and reduce the price competition. In the Hotelling framework, the *strategic effect* dominates the *demand effect* and firm A locates at the extremity of the segment.

Consider now the case in which both firms price discriminate. We observe three different effects. First, firm A wants to move toward the centre because in this way some consumers, which otherwise would be nearer to firm B , become nearer to firm A . This incentive is analogous to the *demand effect*. Consider instead the effect of a movement of firm A toward the centre on the equilibrium prices applied on those consumers which are nearer to firm A even before the movement. Rewriting the equilibrium price schedule defined in Proposition 2, we get: $p_x^{A*} = t(b-a)(a+b-2x)$. The price applied on consumer x depends on two factors. First, it depends on the distance between the two firms (the first term): the more the firms are differentiated, the higher is the price applied on consumer x , because the difference between the two firms perceived by the consumer is higher (and therefore the advantage of firm A over firm B is higher). Clearly, this effect (let call it the *increasing differentiation effect*⁴⁸) stimulates firm A to move toward the endpoint of the segment. Second, the price applied on consumer x depends on the distance between consumer x and firm A (second term): for a given product differentiation degree, the more the consumer is near to the firm the higher is the price. This effect (let call it the *minimizing disutility costs effect*) stimulates firm A to minimize its distance from the consumers buying from it. Note that this last incentive is absent under the uniform price regime, because in that case the (uniform) price depends only on the product differentiation degree, but not on the distance between the firm and its consumers. To sum up, when both firms price discriminate the *demand effect* pushes the firms towards the centre of the segment, the *increasing differentiation effect* pushes the firms towards the endpoints of the segment, and the

⁴⁸ Note that the *increasing differentiation effect* is logically different from the *strategic effect*, even if both push firm A to locate at the beginning of the segment. Indeed, the *increasing differentiation effect* considers the effect of a change in location of firm A on the prices it applies on its own consumers, while the *strategic effect* considers the effect of a change in location of firm A on the price set by firm B . It is clear that when both firms price discriminate the *strategic effect* does not occur, since firm B always sets a price equal to the marginal cost on firm A ' consumers.

minimizing disutility costs effect pushes the firms towards the middle of their respective markets. In our model, the equilibrium between these three forces arises when firms choose respectively varieties at 1/4 and 3/4.

Consider now the case in which firm A price discriminates while firm B sets a uniform price to all consumers. Recall that firm A locates at point 0 (Proposition 7): why does the discriminating firm locate as far as possible from the rival as under the uniform price regime? The reason can be found by looking at the incentives which are at work when only firm A discriminates. As when both firms price discriminate, we observe the *demand effect*, the *increasing differentiation effect* and the *minimizing disutility costs effect*. However, the *strategic effect* is also at work when only firm A discriminates. This can be seen by looking at the equilibrium discriminatory price schedule: $p_x^A = \bar{p}^B * + t(x - b)^2 - t(x - a)^2 = t(b - a)(2 - a - b)/2 + t(b - a)(a + b - 2x)$.

The second term is identical to the case where both firms price discriminate, and sums up the *increasing differentiation effect* and the *minimizing disutility costs effect*. The first term instead shows that by increasing the product differentiation degree firm A is able to lower the price competition by firm B (*strategic effect*). As it turns out, the *strategic effect* together with the *increasing differentiation effect* dominates the *demand effect* and the *minimizing disutility costs effect*, and firm A locates at the beginning of the segment.

Finally, consider the case in which firm A sets a uniform price, while firm B price discriminates. Which are the incentives working now? As usual, the *demand effect* works, because the more firm A is near to the centre of the segment the more numerous are the consumers which firm A is able to steal from firm B . Instead, the *strategic effect* is not present, because the product differentiation degree does not alter the price competition by firm B which always sets a zero price on firm A 's consumers. However, a different reason prevents firm A from locating very far from the extremity of the market: indeed, when firm A moves toward the centre, the consumers located at the beginning of the segment become more and more distant, and they can be served only by lowering the (uniform) price. Therefore, given the variety chosen by the rival, firm A moves toward the centre until the increase of profits due to the higher demand equates the decrease of profits due to the reduction of the price on all consumers. In our model, this

occurs when firm A locates at $1/3$ (given the position of firm B at the right endpoint of the market).

2.5. Conclusion

We used the Hotelling's framework to investigate on the effects of the possibility to perfectly price discriminate when product differentiation is endogenous. We obtain the following results. If firms cannot commit to uniform pricing before competing on price, price discrimination emerges as the unique sub-game perfect equilibrium and firms locate respectively at $1/4$ and $3/4$ (Propositions 2 and 3). Equilibrium first-degree discriminatory prices are all lower than the equilibrium uniform price of a two-stage variety-price game where price discrimination is impossible (Proposition 4). If firms can commit to uniform pricing before competing on price but after choosing the variety, the unique equilibrium is characterized by price discrimination (Proposition 6). On the contrary, if firms can commit to uniform pricing before competing on price and before choosing the variety, the unique equilibrium is characterized by uniform pricing (Proposition 8).

2.6. Appendix

In this appendix we extend the model in order to analyse third-degree price discrimination. Using the framework developed by Liu and Serfes (2004), we show that uniform pricing is the unique sub-game perfect equilibrium when the pricing policy is decided before the variety decision. Other results regard equilibrium varieties, firms' profits, consumer surplus and total welfare: when firms can imperfectly price discriminate, the equilibrium varieties are very close to the varieties that maximize total welfare; firms' profits are lower under imperfect price discrimination than under uniform pricing; consumer surplus and total welfare are higher under imperfect price discrimination than under uniform pricing.

2.6.1. The model

Following Liu and Serfes (2004), we suppose that there is an information technology which allows firms to partition the consumers into different groups. We assume that the information technology partitions the linear market into n sub-segments indexed by m , with $m = 1, \dots, n$. Each sub-segment is of equal length, $1/n$. It follows that sub-segment m can be expressed as the interval $\left[\frac{m-1}{n}; \frac{m}{n} \right]$. A firm can price discriminate between consumers belonging to different sub-segments, but not between the consumers belonging to the same sub-segment. The cost of using the information technology is zero. Define with p_m^J the price set by firm $J=A,B$ on consumers belonging to sub-segment m . Clearly, when firm J cannot price discriminate, it must be $p_m^J = p_{m'}^J$, $\forall m, m'$. Finally, assume that $n = 2^k$, with $k = 1, 2, 3, 4, \dots$. Therefore, n measures the precision of consumer information: the higher is n , the higher is the information precision.

Differently from Liu and Serfes (2004), firms are not exogenously located at the endpoints of the market: as in the previous part of the chapter we are interested in the variety-price equilibria, so we allow for endogenous choice of the variety by the firms.

2.6.2. Imperfect discriminatory prices

This section extends the analysis developed in section 2.3. of this chapter to the case of third degree price discrimination. A two-stage game is supposed: the firms first decide which variety to produce and then compete on prices. Here we assume that both firms can (imperfectly) price discriminate. The utility of the consumer x belonging to sub-segment m when he buys from firm A is therefore: $u_x = v - p_m^A - t(x-a)^2$, while his utility when he buys from firm B is given by: $u_x = v - p_m^B - t(x-b)^2$. Consider segment m . Define x_m^* as the consumer on segment m which is indifferent between buying from firm A or from firm B for a given couple of varieties, a and b , and for a given

couple of discriminatory prices, p_m^A and p_m^B . Equating the utility in the two cases and solving for x it follows:

$$x_m^* = \frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)}$$

Therefore, the demand of firm A and firm B on sub-segment m is respectively:

$$d_m^A = \frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)} - \frac{m-1}{n} \quad (7)$$

$$d_m^B = \frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p_m^A}{2t(b-a)} \quad (8)$$

It follows that the profits of firm A and firm B on sub-segment m are respectively:

$$\Pi_m^A = p_m^A d_m^A = p_m^A \left[\frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)} - \frac{m-1}{n} \right]$$

$$\Pi_m^B = p_m^B d_m^B = p_m^B \left[\frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p_m^A}{2t(b-a)} \right]$$

Define $m_A \equiv \frac{n(a+b)}{2} - 1$ and $m_B \equiv \frac{n(a+b)}{2} + 2$. Clearly, $m_B > m_A$. The following proposition defines the equilibrium price schedules for any couple of varieties:

Proposition 9: *the equilibrium prices and the equilibrium demand of each firm in the second stage of the game are the following:*

- $m_A < m < m_B$

$$p_m^{A*} = \frac{t(b-a)}{3} \left(\frac{4-2m}{n} + a+b \right), \quad d_m^{A*} = \frac{2-m}{3n} + \frac{a+b}{6}$$

$$p_m^{B*} = \frac{t(b-a)}{3} \left(\frac{2+2m}{n} - a-b \right), \quad d_m^{B*} = \frac{m+1}{3n} - \frac{a+b}{6}$$

- $m \leq m_A$

$$p_m^{A*} = t(b-a)(a+b-2\frac{m}{n}), \quad d_m^{A*} = \frac{1}{n}$$

$$p_m^{B*} = 0, \quad d_m^{B*} = 0$$

- $m \geq m_B$

$$p_m^{A*} = 0, \quad d_m^{A*} = 0$$

$$p_m^{B*} = t(b-a)(\frac{2m-2}{n} - a - b), \quad d_m^{B*} = \frac{1}{n}$$

Proof. We refer directly to the proof provided by Liu and Serfes (2004) for their Proposition 1. The only difference is that firm A chooses a instead of 0, and firm B chooses b instead of 1. ■

Using Proposition 9, the firms' profits can be written directly as functions of a and b (the subscript indicates that both firms are price discriminating). They are:

$$\begin{aligned} \Pi_{DD}^{A*} &= \sum_{m=1}^{m_A} \frac{t(b-a)}{n} (a+b-2\frac{m}{n}) + \sum_{m=m_A+1}^{m_B-1} \frac{t(b-a)}{3} (\frac{4-2m}{n} + a+b) (\frac{2-m}{3n} + \frac{a+b}{6}) = \\ &= \frac{t(b-a)[9n^2(a+b)^2 - 18n(a+b) + 40]}{36n^2} \end{aligned} \quad (9)$$

$$\begin{aligned} \Pi_{DD}^{B*} &= \sum_{m=m_A+1}^{m_B-1} \frac{t(b-a)}{3} (\frac{2+2m}{n} - a - b) (\frac{m+1}{3n} - \frac{a+b}{6}) + \sum_{m=m_B}^n \frac{t(b-a)}{n} (\frac{2m-2}{n} - a - b) = \\ &= \frac{t(b-a)[9n^2(2-a-b)^2 - 18n(2-a-b) + 40]}{36n^2} \end{aligned} \quad (10)$$

We state the following proposition:

Proposition 10: *the equilibrium varieties in the first stage of the game are the following:*

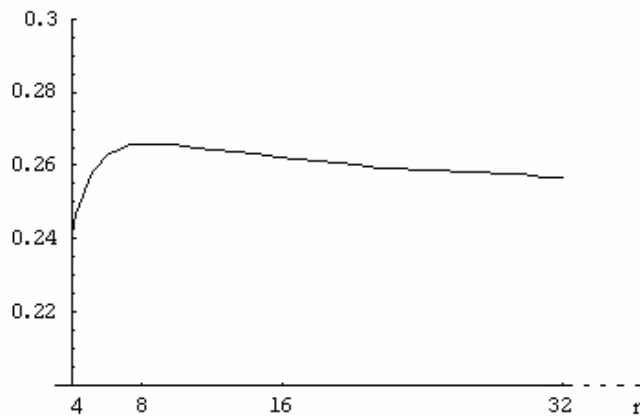
- $a^* = 0, b^* = 1, \text{ if } n = 2$
- $a^* = \frac{9n^2 - 40}{36n^2 - 36n}, b^* = 1 - a^*, \text{ if } n \geq 4$

Proof. The equilibrium varieties come from the solution of: $\begin{cases} \partial \Pi_{DD}^A / \partial a = 0 \\ \partial \Pi_{DD}^B / \partial b = 0 \end{cases}$. Note that

when $n = 2$, $\partial \Pi^A / \partial a$ is always negative and $\partial \Pi^B / \partial b$ is always positive: the corner solution follows. For $n \geq 4$ an interior solution exists. ■

The following figure illustrates the equilibrium variety of firm A (firm B is symmetric) for $n \geq 4$ ⁴⁹. Firm A locates just below $1/4$ when the market is partitioned in 4 sub-segments; it locates just above $1/4$ when the market is partitioned in 8 sub-segments and afterwards the equilibrium variety decreases monotonically with n and converges to $1/4$ at the limit. Maximal differentiation emerges in equilibrium only for a very low-quality information technology ($n = 2$): in this case the firms locate as in the absence of price discrimination.

Figure 2: Illustration of Proposition 10



The next propositions compare the variety-price equilibrium when price discrimination is possible with the variety-price equilibrium under the uniform price regime.

⁴⁹ Clearly, n does not take all values, but only 4,8,16,32,64... In order to better illustrate the pattern of the equilibrium locations as n increases we draw a continuous line.

Proposition 11. *All equilibrium prices are lower under imperfect price discrimination than under uniform price. Therefore, profits are lower under imperfect price discrimination than under uniform price.*

Proof. Consider first the case with $n \geq 4$. Look at firm A . By Proposition 10, we have that: $a^* + b^* = 1$. Therefore, the equilibrium prices (Proposition 9) can be written as:

$$p_m^{A*} = \begin{cases} \frac{t(1-2a^*)}{3} \left(\frac{4-2m}{n} + 1 \right) & \text{if } m_A < m < m_B \\ t(1-2a^*) \left(1 - 2\frac{m}{n} \right) & \text{if } m \leq m_A \end{cases}$$

First, note that the equilibrium price for $m \leq m_A$ is always higher than the equilibrium price for $m_A < m < m_B$. Indeed, since p_m^{A*} is decreasing in m , the lowest equilibrium price for $m \leq m_A$ occurs when $m = m_A$, while the highest equilibrium price for $m_A < m < m_B$ occurs when $m = m_A + 1$. Substituting a^* and b^* into m_A , and then substituting m_A into p_m^{A*} , we have: $p_{m_A(a^*,b^*)}^{A*} = \frac{2t(1-2a^*)}{n} > \frac{4t(1-2a^*)}{3n} = p_{m_A(a^*,b^*)+1}^{A*}$.

Therefore, the comparison between the uniform equilibrium price, $\bar{p}^{A*} = t$, and the discriminatory prices can be limited to the comparison between \bar{p}^{A*} and the highest discriminatory price for $m \leq m_A$. The highest discriminatory price occurs when $m = 1$. Substituting $m = 1$ into p_m^{A*} , we get $t(1-2a^*)(1-2/n)$, which is always lower than t since both terms in the round brackets are positive and lower than 1. Consider now the case with $n = 2$. In this case, both firms have a positive demand in both sub-segments. Consider firm A . Its equilibrium prices in sub-segment 1 and sub-segment 2 are respectively: $p_1^{A*} = 2t/3$ and $p_2^{A*} = t/3$. Clearly: $\bar{p}^{A*} > p_1^{A*} > p_2^{A*}$. Moreover, since the output is constant, equilibrium profits are necessarily lower under price discrimination than under uniform price for any n . ■

Proposition 12: *Consumer surplus and total welfare are higher under imperfect price discrimination than under uniform price.*

Proof. The consumer surplus and the total welfare can be written respectively as:

$$CS = v - \Pi^T - DC$$

$$W = \Pi^T + CS = \Pi^T + v - \Pi^T - DC = v - DC$$

where $\Pi^T \equiv \Pi_{DD}^A + \Pi_{DD}^B$ is the sum of the profits of each firm, and DC are the total disutility costs. From Proposition 11 we know that profits are lower under price discrimination than under uniform price. Moreover, disutility costs are lower under price discrimination than under uniform price (apart from the case of $n = 2$, where disutility costs are the same than under the uniform price regime), since firms locate near to the socially optimal varieties, $1/4$ and $3/4$. It follows that consumer surplus and total welfare increase passing from the uniform price regime to the discriminatory price regime. ■

2.6.3. A three-stage model

Suppose now the following three-stage model (see *Game 2* in section 2.4.)⁵⁰, in which firms choose the pricing policy before choosing the variety. The timing of the game is the following: at the first stage of the game the firms simultaneously choose the pricing policy; at the second stage of the game the firms simultaneously choose the variety; at the third stage of the game the firms simultaneously set the price schedules.

We solve the game by backward induction. At the third stage firms compete on prices, given the varieties and the commitment decision. We need to calculate the equilibrium prices when one firm has committed and the other has not committed. The following proposition defines the equilibrium prices in such case:

Proposition 13: *if firm A has committed and firm B has not committed, the equilibrium prices at the third stage of the game are the following:*

⁵⁰ The correspondent third-degree version of *Game 1* is briefly discussed at the end of the chapter.

$$p_m^{B*} = \frac{t(b-a)(a+b)}{2} + \frac{t(b-a)}{2n}$$

$$p_m^{B*} = \begin{cases} \frac{t(b-a)}{n} & \text{if } m = m^{\wedge} - 1 \\ \frac{t(b-a)}{2} \left(\frac{4m-3}{n} - a - b \right) & \text{if } m \geq m^{\wedge} \end{cases}$$

where $m^{\wedge} = \frac{n(a+b)+7}{4}$

If firm A has not committed and firm B has committed, the equilibrium prices in the third stage of the game are the following:

$$p_m^{A*} = \begin{cases} \frac{t(b-a)}{n} & \text{if } m = m^{\circ} + 1 \\ \frac{t(b-a)}{2} \left(2 + a + b + \frac{1-4m}{n} \right) & \text{if } m \leq m^{\circ} \end{cases}$$

$$p_m^{B*} = \frac{t(b-a)(2-a-b)}{2} + \frac{t(b-a)}{2n}$$

where $m^{\circ} = \frac{n(2+a+b)-3}{4}$

Proof. Suppose that firm A has committed while firm B has not committed. Consider the sub-segment m . The demand of firm B is:

$$d_m^B = \frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p^{\wedge A}}{2t(b-a)} \quad (11)$$

The profits obtained by firm B from the sub-segment m are therefore:

$$\Pi_m^B = p_m^B \left[\frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p^{\wedge A}}{2t(b-a)} \right] \quad (12)$$

Maximizing equation (12) with respect to p_m^B , we obtain the optimal discriminatory price in sub-segment m given the price set by the non-discriminating firm and given the varieties chosen at the first stage of the game. We get:

$$p_m^B = t(b-a)\left(\frac{m}{n} - \frac{a+b}{2}\right) + \frac{\bar{p}^A}{2} \quad (13)$$

Inserting equation (13) into equation (7), we obtain the demand of firm A in each sub-segment:

$$d_A^m = \frac{a+b}{4} - \frac{m}{2n} - \frac{\bar{p}^A}{4t(b-a)} + \frac{1}{n} \quad (14)$$

The demand of firm A is zero in the most at the right sub-segments. More precisely:

$$d_A^m \leq 0 \leftrightarrow m \geq m^{\wedge} \equiv n\left[\frac{a+b}{2} - \frac{\bar{p}^A}{2t(b-a)} + \frac{2}{n}\right]$$

The demand of firm A is $1/n$ in the most at the left sub-segments. More precisely:

$$d_A^m \geq 1/n \leftrightarrow m \leq m^{\wedge\wedge} \equiv n\left[\frac{a+b}{2} - \frac{\bar{p}^A}{2t(b-a)}\right]$$

Note that $m^{\wedge} - m^{\wedge\wedge} = 2$. It follows that only in sub-segment $m^{\wedge} - 1$ ⁵¹ both firms have a positive demand. The profits of firm A are therefore defined by the following equation (the subscript indicates that firm A sets a uniform price while firm B price discriminates):

⁵¹ Note that $m^{\wedge} - 1 = m^{\wedge\wedge} + 1$. Therefore, one may indifferently refer to sub-segment $m^{\wedge} - 1$ or to sub-segment $m^{\wedge\wedge} + 1$.

$$\Pi_{UD}^A = \bar{p}^{-A} \sum_{m=1}^{m^{\wedge}} \frac{1}{n} + \bar{p}^{-A} \left[\frac{a+b}{4} - \frac{m^{\wedge}-1}{2n} - \frac{\bar{p}^{-A}}{4t(b-a)} + \frac{1}{n} \right] = \bar{p}^{-A} \left[\frac{a+b}{2} - \frac{\bar{p}^{-A}}{2t(b-a)} + \frac{1}{2n} \right] \quad (15)$$

Maximizing equation (15) with respect to \bar{p}^{-A} we obtain the optimal uniform price set by the non-discriminating firm:

$$\bar{p}^{-A*} = \frac{t(b-a)(a+b)}{2} + \frac{t(b-a)}{2n} \quad (16)$$

Inserting equation (16) in equation (13), and substituting m with $m^{\wedge}-1$ (in which we insert equation (16) again), we obtain the optimal discriminatory price in the only sub-segment in which both firms sell a positive amount. That is:

$$p_{m^{\wedge}-1}^B = \frac{t(b-a)}{n} \quad (17)$$

The demand of firm B in sub-segment $m^{\wedge}-1$ is obtained inserting equation (17) in equation (11). It follows:

$$d_{m^{\wedge}-1}^B = \frac{1}{2n}$$

The optimal discriminatory prices in sub-segments $m \geq m^{\wedge}$ are obtained by solving:

$d_m^B(\bar{p}^{-A*}) = 1/n$. It follows:

$$p_m^{B*} = \frac{t(b-a)}{2} \left(\frac{4m-3}{n} - a - b \right)$$

The proof of the second part of the proposition proceeds in the same way, and therefore it is omitted. ■

By Proposition 13, the profits of the two firms when one sets a uniform price while the other discriminates follow immediately:

Corollary of Proposition 13: *if firm A has committed and firm B has not committed the equilibrium profits are:*

$$\Pi_{UD}^A * = \frac{t(b-a)}{8n^2} [n^2(a+b)^2 + 2n(a+b) + 1]$$

$$\Pi_{UD}^B * = \frac{t(b-a)}{16n^2} [n^2(4-a-b)^2 - 2n(4-a-b) + 5]$$

If firm A has not committed and firm B has committed the equilibrium profits are:

$$\Pi_{DU}^A * = \frac{t(b-a)}{16n^2} [n^2(2+a+b)^2 - 2n(2+a+b) + 5]$$

$$\Pi_{DU}^B * = \frac{t(b-a)}{8n^2} [n^2(2-a-b)^2 + 2n(2-a-b) + 1]$$

Now we move to the second stage of the game. We already calculated the equilibrium varieties when both firms set a uniform price (Proposition 1) and when both firms set discriminatory prices (Proposition 10). It remains to calculate the equilibrium varieties when one firm has committed and the other has not committed. The following Proposition defines the equilibrium varieties in this case:

Proposition 14: *if firm A has chosen U and firm B has chosen D, the equilibrium varieties at the second stage of the game are given by $a^* = \frac{1}{3} - \frac{1}{3n}$ and $b^* = 1$.*

If firm A has chosen D and firm B has chosen U, the equilibrium varieties at the second stage of the game are given by $a^ = 0$ and $b^* = \frac{2}{3} + \frac{1}{3n}$.*

Proof. Suppose that firm A has chosen U and firm B has chosen D in the first stage of the game. Then, taking the derivative of Π_{UD}^B with respect to b , it results: $\partial \Pi_{UD}^B / \partial b = t[5 + 4n(b - 2) + n^2(16 - 16b - a^2 + 2ab + 3b^2)] / 16n^2$, which is always positive. Therefore the discriminating firm, B , locates at the right endpoint of the market. Substituting $b^* = 1$ into Π_{UD}^A and maximizing it with respect to a , we get: $a^* = 1/3 - 1/3n$. The second part of the proposition can be proved in the same way. ■

Since we have the equilibrium prices (third stage) and the equilibrium varieties (second stage) in all possible cases, we can write the equilibrium profits of each firm directly as functions of the pricing policy at the first stage of the game, by substituting the equilibrium prices and the equilibrium varieties in the appropriate profit functions. The equilibrium profits are summarised in the following table⁵²:

Table 3

$\Pi^B \backslash \Pi^A$	U	D
U	$\frac{t}{2}, \frac{t}{2}$	$\frac{t(1+2n)^3}{54n^3}, \frac{t(1+2n)(10-9n+16n^2)}{108n^3}$
D	$\frac{t(1+2n)(10-9n+16n^2)}{108n^3}, \frac{t(1+2n)^3}{54n^3}$	$\frac{t(9n^2-18n+40)^2}{648n^3(n-1)}, \frac{t(9n^2-18n+40)^2}{648n^3(n-1)}$

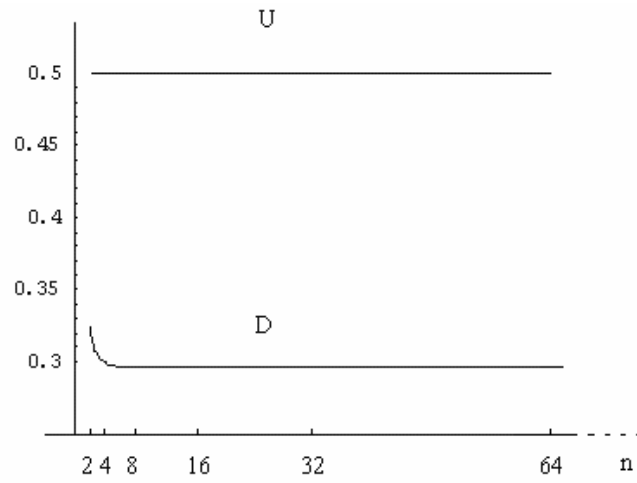
We state the following proposition:

Proposition 15: *the (unique) sub-game perfect Nash equilibrium entails uniform pricing by both firms.*

Proof. Suppose that firm A chooses U. The pattern of the profits of firm B as function of n when it chooses U and when it chooses D is depicted in the following picture:

⁵² In Table 3 we consider only $n \geq 4$. If $n=2$, firms maximally differentiate even when both firms price discriminate (Proposition 10), and the equilibrium profits in (D,D) are $0.27t$.

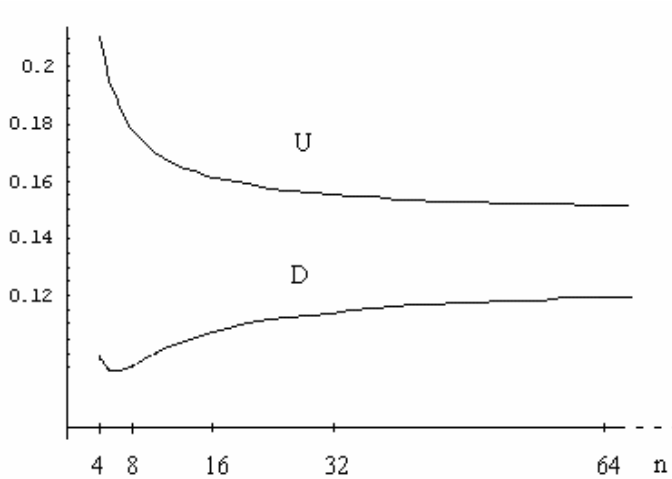
Figure 3



Then, firm *B* always chooses U when firm *A* chooses U.

Suppose now that firm *A* chooses D. The pattern of the profits of firm *B* as function of *n* (with $n \geq 4$) when it chooses U and when it chooses D is depicted in the following picture:

Figure 4



Hence, firm *B* always prefers to commit when firm *A* chooses D. When $n=2$, direct calculations show that firm *B* obtains profits equal to $0.29t$ by choosing U and profits

equal to $0.27t$ by choosing D (see footnote 52). Therefore the dominant strategy of firm A is U. The same reasoning is valid also for firm B , and this completes the proof. ■

Then, when the pricing policy is decided before the variety, discriminatory prices do not arise in equilibrium, independently on the number of partitions of the consumers.

Remark. In what follows we briefly discuss the third-degree version of *Game 1*. Using Table 1, equations (9) and (10), and Corollary of Proposition 13, we can write the payoffs function at the second stage of the game (the pricing policy stage):

Table 4

$\Pi^B \backslash \Pi^A$	U	D
U	$\frac{t(b-a)(2+a+b)^2}{18}$; $\frac{t(b-a)(4-a-b)^2}{18}$	$\frac{t(b-a)[n^2(a+b)^2 + 2n(a+b)+1]}{8n^2}$; $\frac{t(b-a)[5-2n(4-a-b)+n^2(4-a-b)^2]}{16n^2}$
D	$\frac{t(b-a)[5-2n(2+a+b)+n^2(2+a+b)^2]}{16n^2}$; $\frac{t(b-a)[n^2(2-a-b)^2 + 2n(2-a-b)+1]}{8n^2}$	$\frac{t(b-a)[9n^2(a+b)^2 - 18n(a+b)+40]}{36n^2}$; $\frac{t(b-a)[9n^2(2-a-b)^2 - 18n(2-a-b)+40]}{36n^2}$

It can be shown (for $n \geq 16$) that there exists a unique variety equilibrium in the first stage of the game coinciding with the varieties indicated in Proposition 10, and that the unique equilibrium in the second stage is DD. The proof follows.

Let call the two firms generically “firm I ” and “firm J ” and their respective locations i and j , where $I, J \in \{A, B\}$ and $i, j \in \{a, b\}$: if in equilibrium $i < j$, then $I = A$ and $J = B$, while if in equilibrium $i > j$, then $I = B$ and $J = A$. We show that in the first stage of the game no variety equilibrium exists that induces an asymmetric pricing policy equilibrium in the second stage. We start with the following observation coming directly from Table 4: if firms locate symmetrically, in the second stage of the game the equilibrium is DD. Consider now the necessary conditions on i and j for DU to arise in the second stage of the game. The necessary conditions can be obtained by solving

$\Pi_{DU}^B > \Pi_{DD}^B$. Without loss of generality, assume that the non discriminating firm is J and define the R^2 -set inducing DU with S , that is, $S \equiv \{(i, j) \in R^2 : \Pi_{DU}^B > \Pi_{DD}^B, j > i\}$. Now, consider firm J : given i , is the best reply of firm J to set j such that $(i, j) \in S$? If at least one \tilde{j} such that $(i, \tilde{j}) \notin S$ exists that guarantees higher profits to firm J when the rival plays i , then j such that $(i, j) \in S$ cannot be the best reply of firm J , and $(i, j) \in S$ cannot be an equilibrium. We consider $\tilde{j} = 1 - i$: since symmetric locations induce DD equilibrium in the second stage, we are sure that $(i, \tilde{j}) \notin S$. Then, we compare $\Pi_{DU}^J((i, j) \in S)$ with $\Pi_{DD}^J((i, \tilde{j}))$. It can be shown that $\Pi_{DU}^J((i, j) \in S)$ is always lower than $\Pi_{DD}^J((i, \tilde{j}))$ ⁵³: therefore, choosing j such that $(i, j) \in S$ cannot be the best reply of firm J when firm I plays i . No variety equilibrium inducing DU can exist in the first stage of the game. The proof for the non-existence of variety equilibria inducing UD in the second stage equilibrium is analogous.

Moreover, it can be easily verified by looking at Table 4 that for any possible couple of varieties, UU is never an equilibrium in the second stage of the game. Therefore, only variety equilibrium inducing DD can exist in the first stage of the game. Consider now the variety equilibrium defined in Proposition 10. When firm I plays i^* , the equilibrium emerging in the second stage of the game is DD for any possible j (this can be easily verified by substituting i^* into Table 4 and noting that neither $\Pi_{DU}^B > \Pi_{DD}^B$ nor $\Pi_{UD}^A > \Pi_{DD}^A$ are possible). Therefore, firm J plays j^* as shown in Proposition 10, and (i^*, j^*) is the unique equilibrium. It follows that in the second stage of the game both firms choose not to commit.

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⁵³ The file with the calculations for $n=16, 32, 64, 128, 256$ is available upon request. When $n = 2$ direct computations show instead that the unique equilibrium in the second stage of the game is UU for any a and b . Unfortunately, we have not been able to obtain the variety equilibrium for $n=4$ and $n=8$.

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Product differentiation, price discrimination and collusion

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Abstract

The existing literature which analyses the relationship between the product differentiation degree and the sustainability of a collusive agreement on price assumes that firms cannot price discriminate, and concludes that there is a negative relationship between the product differentiation degree and the critical discount factor. This chapter, in contrast, assumes that firms are able to price discriminate. Within the Hotelling framework, three different collusive schemes are studied: optimal collusion on discriminatory prices; optimal collusion on a uniform price; collusion not to discriminate. We obtain that the critical discount factor of the first and the third collusive scheme does not depend on the product differentiation degree, while the critical discount factor of the second collusive scheme depends positively on the product differentiation degree. Moreover, we show that suboptimal collusion is more difficult to sustain than optimal collusion.

JEL codes: D43; L11; L41

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3.1. Introduction

Product differentiation affects both the way in which the firms compete and the way in which they collude. When the firms produce differentiated goods, their pricing decisions depend on the substitutability between the products: if the products are good substitutes no firm can command a high price for its product. Therefore, the lower is the product differentiation degree the lower is the non-cooperative equilibrium price. For this reason, the firms may try to coordinate their pricing decisions in order to jointly raise the price above the competitive level. However, a low product differentiation degree not only increases the opportunity for collusion, but it also increases the incentive to cheat from the collusive agreement. Indeed, with highly substitutable products, a cheating firm can capture a large fraction of the market and obtain large short-term profits by slightly lowering the price unilaterally. Therefore, the impact of the product differentiation degree on the sustainability of a collusive agreement is not *a priori* an obvious issue.

The relationship between the product differentiation degree and the ability of the firms to collude has been studied by, among others, Chang (1991), Chang (1992) and Hackner (1995). Chang (1991) employs the spatial competition framework of Hotelling (1929) with quadratic transportation costs. He assumes fixed and symmetric locations of the firms. The sustainability of the cartel agreement is measured by the minimum discount factor supporting the joint maximum profits as a sub-game perfect equilibrium of an infinitely repeated game. Chang (1991) shows that collusion is easier to sustain the more differentiated are the firms. Indeed, the critical discount factor monotonically increases as the product differentiation decreases. A similar result is found in Chang (1992), where the initial degree of differentiation is exogenous, but firms can relocate once the collusive agreement has been broken. Chang (1992) concludes that a higher initial product differentiation degree makes collusion easier to sustain. Hackner (1995) instead considers the possibility that firms collude not only with respect to the price but also with respect to the location. When the market discount factor is high enough, firms collude to locate at $1/4$ and $3/4$. The lower is the market discount factor the more the firms collude on a higher product differentiation degree in order to keep collusion from

breaking down. Hackner (1995) concludes that there is “*a fairly general tendency within the Hotelling framework for differentiation to facilitate collusion*” (p. 293).

All these articles are characterized by the assumption that firms cannot price discriminate. In this chapter we remove this hypothesis, and we study how the product differentiation degree affects the sustainability of a collusive agreement *when firms can price discriminate*. At our knowledge, the only article that studies the sustainability of collusion taking into account price discrimination is Liu and Serfes (2007). They assume that firms are maximally differentiated on the Hotelling segment, while they allow for different customer-specific information quality. Firms have access to information of a given quality which allows them to partition consumers into different groups and charge each group with a different price. Higher information quality is modelled as a refinement of the partition. At the limit, firms know the position of each consumer in the market and can charge each consumer with a different price (perfect price discrimination). Liu and Serfes (2007) show that collusion becomes more difficult to sustain as the quality of consumer-specific information improves. Better information allows for higher collusive profits and harsher punishment, but at the same time makes deviation more profitable: this last effect dominates, and the critical discount factor is a positive function of the quality of information.

On the one hand, our analysis is less general than Liu and Serfes (2007), since we consider only the limit case of perfect price discrimination⁵⁴. On the other hand, our analysis is more general, since we do not limit the analysis to the case of maximally differentiated firms, but we allow for different product differentiation degrees. As in Liu and Serfes (2007), we study three different collusive schemes: 1) collusion on discriminatory prices; 2) collusion on a uniform price; 3) collusion not to discriminate. In the first collusive scheme firms coordinate on the price to be applied to each consumer, without the constraint that the price must be equal for all consumers. Clearly, this collusive scheme yields the highest collusive profits, since it allows the colluding firms to perfectly target the price on the willingness to pay of each consumer. However, such collusive scheme may be very difficult to implement, since it requires colluding on a huge number of prices. A less “extreme” collusion is represented by the second collusive scheme: here firms try to coordinate on a uniform price. This scheme is less

⁵⁴ In the appendix, however, we extend the analysis of two of the collusive schemes we analyzed to the case of third-degree price discrimination.

profitable, but it is easier to implement, because it requires firms to agree only on one price. Finally, in the third collusive scheme firms do not agree directly on the price(s), but simply agree not to price discriminate. Since in the spatial competition framework price discrimination causes lower equilibrium profits⁵⁵, firms have the incentive to coordinate in order to compete less fiercely: an agreement not to discriminate has precisely this purpose⁵⁶.

For each collusive scheme we search the minimum discount factor which is needed to sustain the joint maximum profits. We are mainly interested in the following question: how does the easiness of collusion change with the product differentiation degree? A linked question is the following: which collusive scheme is easier to sustain in equilibrium for any given product differentiation degree? We obtain the following results. The sustainability of the first and the third collusive scheme does not depend on the product differentiation degree. The sustainability of the second collusive scheme instead depends *negatively* on the product differentiation degree. This result contrasts with the findings by Chang (1991), Chang (1992) and Hackner (1995): the hypothesis of price discrimination reverses the relationship between the sustainability of collusion and the product differentiation degree. Moreover, in contrast with Chang (1991), the sustainability of collusion depends *negatively* on the transportation costs. We obtain also that, independently on the product differentiation degree, the first collusive scheme is easier to sustain than the second collusive scheme, which in turn is easier to sustain than the third collusive scheme. In addition, we consider the possibility that firms collude on a suboptimal discriminatory price schedule and on a suboptimal uniform price. In both cases we obtain that if optimal collusion is not sustainable, suboptimal collusion is not sustainable too. Finally, in the appendix we extend the analysis of the second and the third collusive scheme to a third-price discrimination framework *a la* Liu and Serfes (2004, 2007), and we show that the results do not change.

⁵⁵ See for example Thisse and Vives (1988).

⁵⁶ Each of the collusive schemes we study in this chapter is well documented in European antitrust cases. Examples of the first collusive scheme are: *Cast Iron and Steel* (D. Comm., Oct. 17, 1983) and *Pre-insulated Pipes* (D. Comm., Oct. 21, 1998); examples of the second collusive scheme are: *Austrian Banks* (D. Comm., June 12, 2002) and *Specialty Graphite* (D. Comm., Dec. 17, 2002); examples of the third collusive scheme are: *IFTRA Glass* (D. Comm., May 15, 1974), *IFTRA Aluminium* (D. Comm., July 15, 1975) and *Far East Trade Tariff Charges and Surcharges Agreement (FETTCSA)* (D. Comm., May 16, 2000).

This chapter is structured as follows. In section 3.2. the model is introduced. In section 3.3. we describe the infinitely repeated game. In section 3.4. the sustainability of each collusive scheme is studied, while in section 3.5. the model is extended to include the possibility of suboptimal collusion. Section 3.6. concludes. The appendix generalizes the second and the third collusive scheme results to the case of third-degree price discrimination.

3.2. The model of differentiated firms

Assume a linear market of length 1. Consumers are uniformly distributed along the market. Define with $x \in [0,1]$ the location of each consumer. Each point in the linear market represents a certain variety of a given good. For a consumer positioned at a certain point, the preferred variety is represented by the point in which the consumer is located: the more the variety is far from the point in which the consumer is located, the less it is appreciated by the consumer. Each consumer consumes no more than 1 unit of the good. Define with v the maximum price that a consumer is willing to pay for buying his preferred variety.

There are two firms, A and B , competing in the market. Both firms have identical constant marginal costs, which are assumed to be zero. Following Chang (1991), we consider symmetric firms. Firm A produces the variety $a \in [0,1/2]$ and firm B , given the symmetry assumption, produces the variety $1-a$. The parameter a measures the product differentiation: when $a = 0$, firms are maximally differentiated; when $a = 1/2$ firms are identical. Finally, define with p_x^J the price charged by firm $J = A, B$ to the consumer x : clearly, when firm J sets a uniform price, it must be $p_x^J = p_{x'}^J$ for every $x, x' \in [0,1]$.

The utility of a consumer depends on v , on the price set by the firm from which he buys, and on the distance between his preferred variety and the variety produced by the firm. Following D'Aspremont *et al.* (1979), we assume quadratic transportation costs. Define with t , equal for all consumers, the importance attributed by the consumer to the distance between his preferred variety and the variety offered by the firm. The utility of

a consumer located at x when he buys from firm A is given by: $u_x^A = v - p_x^A - t(x - a)^2$, while the utility of a consumer located at x when he buys from firm B is given by: $u_x^B = v - p_x^B - t(x - 1 + a)^2$. As in Chang (1991), Chang (1992) and Hackner (1995) we assume $w \equiv v/t \geq 5/4$: this assumption is sufficient to guarantee that under any optimal collusive agreement the entire market is served.

3. The infinitely repeated game

Suppose that firms interact repeatedly in an infinite horizon setting. As in Chang (1991), Chang (1992) and Hackner (1995), a *grim strategy* is assumed (Friedman, 1971)⁵⁷. Moreover, there is perfect monitoring. Define Π^C , Π^D and Π^N respectively as the one-shot collusive profits, the one-shot deviation profits and the one-shot punishment (or Nash) profits for each firm: obviously, $\Pi^D > \Pi^C > \Pi^N$. Define $\delta \in (0,1)$ as the market discount factor, which is assumed to be exogenous and common for each firm. It is well known that collusion is sustainable as a sub-game perfect Nash equilibrium if and only if the discounted value of the profits that each firm obtains under collusion exceeds the discounted value of the profits that each firm obtains deviating from the tacit agreement. Formally, the following incentive-compatibility constraint must be satisfied:

$$\sum_{t=0}^{\infty} \delta^t \Pi^C \geq \Pi^D + \sum_{t=1}^{\infty} \delta^t \Pi^N,$$

The incentive-compatibility constraint can be rewritten as follows:

⁵⁷ The *grim strategy* implies that firms start by charging the collusive price schedule, p^C . The firms continue to set p^C until one firm has played p^D in the previous period, where p^D is the price schedule set by a firm which deviates from the collusive agreement. If a firm sets p^D at time t , from $t+1$ onward both firms play p^N , where p^N is the equilibrium price schedule emerging in the non-cooperative constituent game (Nash price).

$$\delta \geq \delta^* = \frac{\Pi^D - \Pi^C}{\Pi^D - \Pi^N} \quad (1)$$

Define δ^* as the critical discount factor. Equation (1) says that if the market discount factor is greater than the critical discount factor collusion is sustainable, otherwise it is not sustainable. Then, the critical discount factor measures the sustainability of the agreement: the greater is δ^* the smaller is the set of market discount factors which support collusion.

3.4. Sustainability of the collusive schemes

The stage game

Given the varieties produced by the firms and given the price set by each of them, each consumer buys from the firm which gives him the higher utility. If the utility of a consumer is the same when he buys from firm A and when he buys from firm B , we assume that he buys from the nearer firm⁵⁸. The following proposition defines the Nash prices for any a :

Proposition 1: *when the firms can perfectly price discriminate between the consumers, the equilibrium prices during the punishment stage are the following:*

$$p^{A,N} = \begin{cases} t(1 - 2x - 2a + 4ax) & \text{if } x \leq 1/2 \\ 0 & \text{if } x \geq 1/2 \end{cases} \quad (2)$$

$$p^{B,N} = \begin{cases} 0 & \text{if } x \leq 1/2 \\ -t(1 - 2x - 2a + 4ax) & \text{if } x \geq 1/2 \end{cases} \quad (3)$$

⁵⁸ This assumption is largely used in spatial models, since it allows avoiding the technicality of ε -equilibria. See also Chapter 2.

Proof⁵⁹. Suppose $x < 1/2$. Consider firm B . First, we show that $p_x^B > 0$ cannot be an equilibrium price. When $p_x^B > 0$, the highest price firm A can set in order to serve consumer x is: $p_x^A = p_x^B + t(1 - 2x - 2a + 4ax)$. But now firm B has convenience to undercut p_x^A in order to serve consumer x . Therefore $p_x^B > 0$ cannot be an equilibrium price. Second, we show that $p_x^B = 0$ is an equilibrium price. When $p_x^B = 0$, the highest price firm A can set in order to serve consumer x is: $p_x^A = t(1 - 2x - 2a + 4ax)$. With such a price firm B obtains zero profits from consumer x , which buys from firm A , but it has no incentive to change its price, because increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss. On the other hand, firm A is setting the highest possible price which guarantees it to serve consumer x : therefore, firm A has no incentive to change its price. It follows that $p_x^A = t(1 - 2x - 2a + 4ax)$ and $p_x^B = 0$ represents the (unique) price equilibrium. The proof for $x > 1/2$ is symmetric. Finally, when $x = 1/2$, the standard Bertrand's result holds: the unique price equilibrium when two undifferentiated firms compete on price is represented by both firms setting a price equal to the marginal cost, which in this case is zero. ■

The punishment profits of each firm follow directly from Proposition 1:

$$\Pi^N = \frac{t}{4}(1 - 2a) \quad (4)$$

Optimal collusion on discriminatory prices

Consider the first collusive scheme. Define p^* as the optimal collusive discriminatory price schedule. Since the individual price can be perfectly targeted to each consumer, the optimal collusive price is the highest price which satisfies the participation constraint of the consumer. Therefore:

⁵⁹ Clearly, Proposition 1 in this chapter coincides with Proposition 2 in Chapter 2 when the hypothesis of firms' symmetry is introduced. For the sake of clarity we provide the proof again.

$$p^* = \begin{cases} v - t(x - a)^2 & \text{if } x \leq 1/2 \\ v - t(x - 1 + a)^2 & \text{if } x \geq 1/2 \end{cases} \quad (5)$$

Given the assumption on w , p^* is strictly positive for any a and x and the whole market is served. It is immediate to note that consumers located at $x \in [0, 1/2]$ buy from firm A , while consumers located at $x \in [1/2, 1]$ buy from firm B . Consumer surplus is totally transferred to the firms, and the collusive profits of each firm are equal to:

$$\Pi^C = \frac{v}{2} - \frac{t}{2} \left(a^2 - \frac{a}{2} + \frac{1}{12} \right) \quad (6)$$

Suppose that firm A deviates from the collusive agreement. Define with \hat{p}_1 the price that makes the consumer located in x indifferent between buying from firm A and from firm B , which is setting the collusive price p^* . Solving $u_x^A(\hat{p}_1) = u_x^B(p^*)$ with respect to \hat{p}_1 we get: $\hat{p}_1 = p^* + t(1 - 2x - 2a + 4ax)$. We assume that when the consumer is indifferent between the deviating firm and the colluding firm, he buys from the deviating firm⁶⁰. Therefore, \hat{p}_1 is the highest price which allows firm A to steal a consumer from firm B . However, firm A may be impeded from setting \hat{p}_1 : this occurs when \hat{p}_1 is too high for the participation constraint of the consumer or when it is too low for the participation constraint of the firm. Since p^* extracts the whole consumer surplus, it represents a natural upper bound for the deviation price, while the marginal cost (equal to zero) is the lower bound for the deviation price. Then:

$$p^D = \max\{0; \min[p^*; \hat{p}_1]\}, \quad (7)$$

where p^D is the deviation price schedule. The following lemma fully characterizes the deviation price schedule:

⁶⁰ This assumption can be rationalized noting that the deviating firm can always offer to the consumer a utility which is strictly larger than the utility he receives from the colluding firm by setting a price equal to $\hat{p}_1 - \varepsilon$, where ε is a positive small number.

Lemma 1. *When firms collude on p^* the deviation price schedule is:*

$$p^D = v - t(x - a)^2 \quad (8)$$

Proof. First, consider the consumers located at $x \leq 1/2$. Since $t(1 - 2x - 2a + 4ax) \geq 0 \quad \forall x \leq 1/2$, it follows that $\hat{p}_1 \geq p^*$ (i.e. the participation constraint of the consumer binds). Moreover, given the assumption on w , $p^* \geq 0$ (i.e. the participation constraint of the deviating firm does not bind). Then, by equation (7), it must be: $p^D = p^*$. Finally, using equation (5) it follows that: $p^D = v - t(x - a)^2, \forall x \leq 1/2$. Next, consider the consumers located at $x \geq 1/2$. Since $t(1 - 2x - 2a + 4ax) \leq 0 \quad \forall x \geq 1/2$, it follows that $\hat{p}_1 \leq p^*$ (i.e. the participation constraint of the consumer does not bind). Moreover, given the assumption on w , $\hat{p}_1 \geq 0$ for $x \geq 1/2$ (i.e. the participation constraint of the firm does not bind). By equation (7) it must be: $p^D = \hat{p}_1$. Substituting equation (5) into \hat{p}_1 we get: $p^D = v - t(x - a)^2, \forall x \geq 1/2$. ■

The deviation profits are the following:

$$\Pi^D(a) = \int_0^1 [v - t(x - a)^2] dx = v - t(a^2 - a + \frac{1}{3}) \quad (9)$$

By substituting equations (4), (6) and (9) into equation (1) we obtain the critical discount factor:

$$\delta_1^* = \frac{1}{2} \quad (10)$$

Then, the sustainability of the optimal collusive discriminatory price schedule *does not depend* on the product differentiation degree. The fact that the substitutability of the products of the two firms is high or low does not have any impact on the likelihood that the tacit agreement will be disrupted by the defection of one member of the cartel. When firms can perfectly price discriminate during the deviation and during the

punishment phase, collusion can be sustained if and only the market discount factor exceeds $1/2$, irrespectively of the position of the firms in the market.

Optimal collusion on uniform price

Suppose now that the firms, instead of colluding on the optimal discriminatory price schedule, collude on the optimal uniform price, \bar{p}^C . Chang (1991) and Hackner (1995) show that, under the hypothesis that $w \geq 5/4$, joint profit maximization implies full market coverage. Therefore, profits are maximized by raising the price until the farthest consumer is indifferent between buying and not buying. It follows that when $a \leq 1/4$ the consumer located in the middle of the segment ($x = 1/2$) receives zero utility at the profit maximizing collusive uniform price. Similarly, when $a \geq 1/4$, the consumers at the endpoints of the segment ($x = 0$ and $x = 1$) receive zero utility at the profit maximizing collusive uniform price. Hence, the optimal collusive uniform price is:

$$\bar{p}^C = \begin{cases} v - t(1/2 - a)^2 & \text{if } a \leq 1/4 \\ v - ta^2 & \text{if } a \geq 1/4 \end{cases} \quad (11)$$

At this point it is convenient to handle separately the case of $a \leq 1/4$ and the case of $a \geq 1/4$. The relevant equations will be identified by the appropriate subscript. We start from the case in which the firms are highly differentiated, $a \leq 1/4$. The collusive profits of each firm are the following:

$$\Pi_{a \leq 1/4}^C = \frac{v}{2} - \frac{t}{2} \left(\frac{1}{2} - a \right)^2 \quad (12)$$

Suppose that firm A deviates. Define $\hat{p}_{2;a \leq 1/4}$ as the price that makes the consumer located in x indifferent between buying from firm A and from firm B , which is setting the collusive price \bar{p}^C . Solving $u_x^A(\hat{p}_{2;a \leq 1/4}) = u_x^B(\bar{p}^C)$ with respect to $\hat{p}_{2;a \leq 1/4}$ we get: $\hat{p}_{2;a \leq 1/4} = \bar{p}^C + t(1 - 2x - 2a + 4ax)$. Following the reasoning introduced in the previous

subsection, the deviation price is equal to $\hat{p}_{2;a \leq 1/4}$, provided that $\hat{p}_{2;a \leq 1/4}$ is lower than p^* and higher than 0. That is:

$$p_{a \leq 1/4}^D = \max\{0; \min[p^*; \hat{p}_{2;a \leq 1/4}]\} \quad (13)$$

The following lemma describes the deviation price schedule:

Lemma 2. *Suppose $a \leq 1/4$. When firms collude on \bar{p}^C the deviation price schedule is:*

$$p_{a \leq 1/4}^D = v - t(-4ax - \frac{3}{4} + a^2 + a + 2x) \quad (14)$$

Proof. First, we show that: $\hat{p}_{2;a \leq 1/4} \leq p^*$. The utility of each consumer (except the farthest one) paying \bar{p}^C has to be positive, since \bar{p}^C is obtained by setting the utility of the farthest consumer equal to zero. Therefore, $u_x^A(\bar{p}^C) \geq 0$. Given the indifference condition, it follows that $u_x^A(\hat{p}_{2;a \leq 1/4}) \geq 0$. Recall that the optimal discriminatory price schedule yields zero consumer surplus, that is: $u_x^A(p^*) = 0$. Therefore: $u_x^A(\hat{p}_{2;a \leq 1/4}) \geq u_x^A(p^*)$, which in turn implies $\hat{p}_{2;a \leq 1/4} \leq p^*$. Next, we prove that $\hat{p}_{2;a \leq 1/4} \geq 0$. Using equation (11) in $\hat{p}_{2;a \leq 1/4}$ we obtain the following equation: $\hat{p}_{2;a \leq 1/4} = v - t(-4ax - 3/4 + a^2 + a + 2x)$. The condition $\hat{p}_{2;a \leq 1/4} \geq 0$ can be rewritten as: $w > -4ax - 3/4 + a^2 + a + 2x$. Since $w \geq 5/4$, the condition is always verified when: $2(1-x) > a(1+a-4x)$. Both the l.h.s. and the r.h.s. of the last inequality are linearly decreasing in x . Therefore, it is sufficient to consider the extreme values of x : when $x=0$ we get $2 > a(1+a)$, and when $x=1$ we get $0 > a(a-3)$. Hence, the l.h.s. is always larger than the r.h.s.. It follows that $\hat{p}_{2;a \leq 1/4} \geq 0, \forall x$. Therefore, by equation (13), we get: $p_{a \leq 1/4}^D = \hat{p}_{2;a \leq 1/4} = v - t(-4ax - 3/4 + a^2 + a + 2x)$. ■

Given the deviation price schedule, all consumers are served by the deviating firm, and the deviation profits are:

$$\Pi_{a \leq 1/4}^D(a) = \int_0^1 p_{a \leq 1/4}^D dx = v - t\left(\frac{1}{2} - a\right)^2 \quad (15)$$

By inserting equations (4), (12) and (15) into equation (1) we obtain the critical discount factor:

$$\delta_{a \leq 1/4}^*(a, w) = \frac{\frac{w}{2} - \frac{1}{8} - \frac{1}{2}a^2 + \frac{1}{2}a}{w - \frac{1}{2} - a^2 + \frac{3}{2}a} \quad (16)$$

Consider now the case of lower product differentiation degree, $a \geq 1/4$. The collusive profits are:

$$\Pi_{a \geq 1/4}^C(a) = \frac{v}{2} - \frac{t}{2}a^2 \quad (17)$$

Suppose firm A cheats. As usual, define $\hat{p}_{2; a \geq 1/4}$ as the price which solves $u_x^A(\hat{p}_{2; a \geq 1/4}) = u_x^B(\bar{p}^C)$. Therefore: $\hat{p}_{2; a \geq 1/4} = \bar{p}^C + t(1 - 2x - 2a + 4ax)$, which is the deviation price schedule provided that it is lower than p^* and higher than 0. Then:

$$p_{a \geq 1/4}^D = \max\{0; \min[p^*; \hat{p}_{2; a \geq 1/4}]\} \quad (18)$$

The deviation price schedule is fully characterized by the following lemma.

Lemma 3. *Suppose $a \geq 1/4$. When firms collude on \bar{p}^C the deviation price schedule is:*

$$p_{a \geq 1/4}^D = v - t(a^2 - 4ax - 1 + 2x + 2a) \quad (19)$$

Proof. The proof for $\hat{p}_{2;a \geq 1/4} \leq p^*$ is identical to the case described in Lemma 2. We prove now that: $\hat{p}_{2;a \geq 1/4} \geq 0$. By substituting equation (11) into $\hat{p}_{2;a \geq 1/4}$, we obtain: $\hat{p}_{2;a \geq 1/4} = v - t(a^2 - 4ax - 1 + 2x + 2a)$. Rearranging, the condition $\hat{p}_{2;a \geq 1/4} \geq 0$ can be rewritten as: $w > a^2 - 4ax - 1 + 2x + 2a$. Since $w \geq 5/4$, the condition is always verified when: $9/4 - 2x > a(a - 4x + 2)$. Both the l.h.s. and the r.h.s. of the last inequality are linearly decreasing in x . Therefore, it is sufficient to consider the extreme values of x : when $x = 0$ we get $9/4 > a(a + 2)$, and when $x = 1$ we get $1/4 > a(a - 2)$. Hence, the l.h.s. is always larger than the r.h.s.. It follows that $\hat{p}_{2;a \geq 1/4} \geq 0$ for any x and $1/4 \leq a \leq 1/2$. Then, by equation (18), $p_{a \geq 1/4}^D = \hat{p}_{2;a \geq 1/4} = v - t(a^2 - 4ax - 1 + 2x + 2a)$. ■

Therefore, the deviating firm serves the whole market and the deviation profits are:

$$\Pi_{a \geq 1/4}^D(a) = \int_0^1 p_{a \geq 1/4}^D dx = v - ta^2 \quad (20)$$

By inserting equations (4), (17) and (20) into (1), we get the critical discount factor:

$$\delta_{a \geq 1/4}^*(a, w) = \frac{\frac{w}{2} - \frac{1}{2}a^2}{w - \frac{1}{4} + \frac{1}{2}a - a^2} \quad (21)$$

Define:

$$\delta_2^*(a, w) = \begin{cases} \delta_{a \leq 1/4}^*(a, w) & \text{if } a \leq 1/4 \\ \delta_{a \geq 1/4}^*(a, w) & \text{if } a \geq 1/4 \end{cases} \quad (22)$$

We state the following proposition:

Proposition 2: $\delta_2^*(a, w)$

a) is a continuous function;

b) is monotonically decreasing both in a and w ;

c) takes values between $1/2$ and $g(w)$, where $g(w) \in (1/2; 2/3]$ and $\partial g(\cdot)/\partial w < 0$.

Proof.

a) In order to prove that $\delta_2^*(a, w)$ is a function we need to verify that it takes a unique value for all points of its domain. This is certainly true when $a \in [0, 1/4)$ and $a \in (1/4, 1/2]$. It remains to verify it when $a = 1/4$. Hence, we need to prove that $\delta_{a \leq 1/4}^*(a = 1/4; w) = \delta_{a \geq 1/4}^*(a = 1/4; w)$. By substituting $a = 1/4$ in equations (16) and

(21), we get: $\delta_{a \leq 1/4}^*(a = 1/4; w) = \frac{w-1/16}{2w-3/8} = \delta_{a \geq 1/4}^*(a = 1/4; w)$. Moreover, since

$\delta_{a \leq 1/4}^*(a, w)$ and $\delta_{a \geq 1/4}^*(a, w)$ are continuous, $\delta_{a \leq 1/4}^*(a = 1/4; w) = \delta_{a \geq 1/4}^*(a = 1/4; w)$ implies that $\delta_2^*(a, w)$ is continuous in $a \in [0, 1/2]$ and $w \in [5/4, \infty)$.

b) Consider the derivative of δ_2^* with respect to w . When $a \leq 1/4$ we obtain: $\partial \delta_{a \leq 1/4}^* / \partial w = (2a-1) / [2(1-3a+2a^2-2w)^2] \leq 0$, while when $a \geq 1/4$ we obtain: $\partial \delta_{a \geq 1/4}^* / \partial w = (4a-2) / (1-2a+4a^2-4w)^2 \leq 0$. Therefore δ_2^* decreases as w increases.

Consider now the derivative of δ_2^* with respect to a . When $a \leq 1/4$ we get: $\partial \delta_{a \leq 1/4}^* / \partial a = -[4w + (1-2a)^2] / 4(1-3a+2a^2-2w)^2 < 0$. When $a \geq 1/4$ we obtain: $\partial \delta_{a \geq 1/4}^* / \partial a = -4(w-a+a^2) / (1-2a+4a^2-4w)^2$. The derivative is negative if and only if $w > a - a^2$, which is always true since the maximum value of the r.h.s. is $1/4$.

c) Since $\delta_2^*(a, w)$ is decreasing both in a and in w , its maximum is in $(a = 0; w = 5/4)$. It results: $\delta_2^*(a = 0; w = 5/4) = 2/3$. On the contrary, the critical discount factor takes the minimum value when $a = 1/2$ and $w \rightarrow \infty$. Note that $\delta_2^*(a = 1/2, w) = 1/2$ for any w . Conversely, since $\lim_{w \rightarrow \infty} \delta_{a \leq 1/4}^* = \lim_{w \rightarrow \infty} \delta_{a \geq 1/4}^* = 1/2$ for any a , it follows that $\lim_{w \rightarrow \infty} \delta_2^* = 1/2$ for any a . ■

Proposition 2 shows that there is a *negative* relationship between the product differentiation degree and the sustainability of the collusive agreement, as well as a *negative* relationship between the transportation costs and the sustainability of the

collusive agreement. These findings contrast with Chang (1991) paper, where a *positive* relationship between the product differentiation degree and the sustainability of the collusive agreement as well as a *positive* relationship between the transportation costs and the sustainability of the collusive agreement are shown to exist. In the following, we try to describe the mechanism behind such reversion. First, note that the sign of the derivative of the critical discount factor depends on the value taken by the following function, which is simply the numerator of the derivative of the critical discount factor (equation (1)) with respect to the variable i :

$$\Gamma = \frac{\partial \Pi^D}{\partial i}(\Pi^C - \Pi^N) - \frac{\partial \Pi^C}{\partial i}(\Pi^D - \Pi^N) + \frac{\partial \Pi^N}{\partial i}(\Pi^D - \Pi^C),$$

where $i = t, a$. That is, when $\Gamma > 0$ the derivative of the critical discount factor is positive, and vice-versa.

From equations (4), (15) and (20) of this chapter and equations (6), (9) and (10) in Hackner (1995)⁶¹ paper it is immediate to note that: $\Pi_u^N > \Pi_d^N$ and $\Pi_u^D < \Pi_d^D$, where the subscript indicates the uniform price model (Chang, 1991, Hackner, 1995) and the discriminatory price model (this chapter) respectively, while the superscript indicates the Nash profits and the deviation profits respectively. The explanation is the following. When a firm can use discriminatory deviation prices, it can better target the prices it uses to steal consumers from the rival, and therefore deviation profits are larger. On the contrary, when *both* firms compete with discriminatory prices, competition is fiercer, and consequently Nash profits are lower.

Consider now $i = t$. By comparing the derivatives of equations (4), (15) and (20) of this chapter with the derivatives of equations (6), (9) and (10) in Hackner (1995) we get:

$$\frac{\partial \Pi_u^D}{\partial t} < \frac{\partial \Pi_d^D}{\partial t} < 0 \quad \text{and} \quad \frac{\partial \Pi_u^N}{\partial t} > \frac{\partial \Pi_d^N}{\partial t} > 0.$$

The intuition behind the first inequality is the following. When transportation costs increase, each consumer is more loyal to the nearer firm. Therefore, it becomes more difficult for the cheating firm to steal

⁶¹ Hackner (1995) defines the relevant equations for the model with uniform price, while in Chang (1991) they are left implicit. So we refer directly to Hackner (1995) paper. Moreover, in order to simplify the exposition, we refer only to the case where the deviating firm serves the whole market: a sufficient condition for this to occur in the uniform price model is $w \geq 13/4$ (see Hackner, 1995, p. 296).

consumers from the rival. When the cheating firm uses a uniform price, the deviation price reduces for *all consumers* as a consequence of a larger t . Instead, when the deviating firm uses discriminatory prices, a larger t allows *increasing* prices on those consumers which are nearer to the cheating firm⁶². This effect partially counterbalances the reduction of the prices applied on the more distant consumers, and therefore the deviation profits are less sensitive (in absolute value) to variations in t in the discriminatory price model. The intuition behind the second inequality is the following. The equilibrium Nash price in the uniform price model is given by⁶³: $t(1-2a)$, while the equilibrium Nash prices in the discriminatory price model are given by⁶⁴: $t(1-2a-2x+4ax)$. With discriminatory prices, the individual price does not depend only on the transportation costs and on the distance between the firms as in the uniform price model, but also on the location of the consumer, x . In particular, the more the consumer is indifferent between the firms, the less the price depends on t : at the limit, when the consumer is completely indifferent between the firms ($x=1/2$), the equilibrium price is 0 for every t (i.e. transportation costs do not matter for the equilibrium price on this consumer). In general, the dependency of the equilibrium discriminatory prices on x reduces the dependency of equilibrium discriminatory prices on t : therefore the Nash profits in the discriminatory price model are less sensitive to t with respect to the uniform price model.

Finally, from equations (12) and (17)⁶⁵ of this chapter, we get: $\frac{\partial \Pi^C}{\partial t} < 0$. In fact, the greater is t the smaller is the collusive price needed to serve the furthest consumer.

Now, it is possible to identify the impact of the discriminatory price assumption over the Γ -function. *Ceteris paribus*, the fact that the derivative of the deviation profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller in the discriminatory price model decreases Γ with respect to the uniform price model; the

⁶² Consider the derivative of equations (14) and (19) of this chapter with respect to t and observe that they are positive for low values of x .

⁶³ See, for example, D'Aspremont *et al.*, (1979).

⁶⁴ See Proposition 1. Here we consider only consumers located in the first half of the segment. The analysis proceeds in the same way for the other consumers.

⁶⁵ Equations (12) and (17) of this chapter coincide with equations (4) and (5) in Hackner (1995) paper respectively.

fact that the deviation profits are greater in the discriminatory price model increases Γ with respect to the uniform price model; and, finally, the fact that the Nash profits are smaller in the discriminatory price model decreases Γ with respect to the uniform price model⁶⁶. In our framework, the lower sensitivity of the deviation profits and the higher level of the deviation profits in the discriminatory price model outweigh the impact of the lower sensitivity and the lower level of the Nash profits, and change the sign of Γ , which now is positive: thus, the relationship between the transportation costs and the critical discount factor reverses with respect to the uniform price model.

Consider now $i = a$, with $a \leq 1/4$. Again, from equations (4) and (15) of this chapter

and equations (6) and (9) in Hackner (1995) paper we get: $\frac{\partial \Pi_u^N}{\partial a} < \frac{\partial \Pi_d^N}{\partial a} < 0$ and

$\frac{\partial \Pi_u^D}{\partial a} > \frac{\partial \Pi_d^D}{\partial a} > 0$ ⁶⁷. Moreover, from equation (12), we get: $\frac{\partial \Pi^C}{\partial a} > 0$ ⁶⁸. Therefore,

ceteris paribus, the fact that the derivative of the deviation profits is smaller in the discriminatory price model decreases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the deviation profits are greater in the discriminatory price model decreases Γ with respect to the uniform price model; and, finally, the fact that the Nash profits are smaller in the discriminatory price model increases Γ with respect to the uniform price model⁶⁹. Again, the lower sensitivity of the deviation profits together with the higher level of the deviation profits in the discriminatory price model outweigh the impact of the lower sensitivity and the lower level of the Nash profits, and change the sign of Γ : the higher is the product differentiation degree, the higher is the critical discount factor, while the opposite is true in the uniform price model.

⁶⁶ Equations (12) and (15) of this chapter yield: $\partial \Gamma / \partial \Pi_a^N = -\partial \Pi_a^D / \partial t + \partial \Pi_a^C / \partial t = 1/2(1/2 - a)^2 > 0$ for $a \leq 1/4$; from equations (17) and (20), we get: $\partial \Gamma / \partial \Pi_a^N = -\partial \Pi_a^D / \partial t + \partial \Pi_a^C / \partial t = a^2/2 > 0$ for $a \geq 1/4$.

⁶⁷ The sign of the derivatives with respect to a is the opposite of the sign of the derivatives with respect to t . The intuition behind the lower (absolute) value of the derivatives in the discriminatory price model is analogous to the explanation developed for the transportation costs, once one takes into account the reversion of the sign of the derivatives.

⁶⁸ Indeed, when firms move from the endpoints of the segment to $1/4$ and $3/4$ the furthest consumers become nearer.

⁶⁹ Note from equations (12) and (15) that: $\partial \Gamma / \partial \Pi_a^N = -\partial \Pi_a^D / \partial a + \partial \Pi_a^C / \partial a = -t(1/2 - a) < 0$.

Finally, consider $i = a$, with $a \geq 1/4$. Now, from equations (4), (17) and (20) of this chapter and equations (6) and (10) in Hackner (1995) paper we obtain the following inequalities: $\frac{\partial \Pi_u^N}{\partial a} < \frac{\partial \Pi_d^N}{\partial a} < 0$, $\frac{\partial \Pi_u^D}{\partial a} > 0$, $\frac{\partial \Pi_d^D}{\partial a} < 0$, and $\frac{\partial \Pi^C}{\partial a} < 0$ ⁷⁰. Then, *ceteris paribus*, the fact that the derivative of the deviation profits in the discriminatory price model is negative instead of positive decreases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the deviation profits are greater in the discriminatory price model decreases Γ with respect to the uniform price model⁷¹; and, finally, the fact that the Nash profits are smaller in the discriminatory price model decreases Γ with respect to the uniform price model⁷². The reversion of the sign of the derivative of the deviation profits together with the higher level of the deviation profits and the lower level of the Nash profits in the discriminatory price model outweigh the impact of the lower sensitivity of the Nash profits, and change the sign of Γ : the relationship between the product differentiation degree and the critical discount factor is therefore reverted with respect to the uniform price model.

Collusion not to discriminate

In the third collusive scheme the firms do not jointly fix the price schedules. Instead, they agree not to price discriminate. Once firms have established to set a uniform price to all consumers, competition determines which price is effectively set by the firms.

⁷⁰ The intuition behind $\frac{\partial \Pi_u^N}{\partial a} < \frac{\partial \Pi_d^N}{\partial a} < 0$ is analogous to the intuition developed for t , when one takes into account the reversion of the sign of the derivatives. $\frac{\partial \Pi^C}{\partial a} < 0$ is due to the fact that when firms move from $1/4$ and $3/4$ to the middle of the segment, the furthest consumers (located at the endpoints of the segment) become more distant. With regard to $\frac{\partial \Pi_u^D}{\partial a} > 0$ and $\frac{\partial \Pi_d^D}{\partial a} < 0$, Chang (1991, p. 464) notices that when $a \geq 1/4$ a lower product differentiation degree has two opposite effects on the deviation profits. First, for a given collusive price, a lower product differentiation degree allows for a higher deviation price, which in turn induces greater deviation profits; second, the collusive price is lower when firms are nearer, and this reduces the deviation profits. In the uniform price model the first effect prevails, and the deviation profits increase with a ; in the discriminatory price model the second effect dominates, and the deviation profits decrease with a .

⁷¹ Indeed, from equations (4) and (17), we get: $\frac{\partial \Gamma}{\partial \Pi_d^D} = -\frac{\partial \Pi^C}{\partial a} + \frac{\partial \Pi_u^N}{\partial a} / \frac{\partial a}{\partial a} = ta - t/2 < 0$.

⁷² Note from equations (17) and (20) that: $\frac{\partial \Gamma}{\partial \Pi_d^N} = -\frac{\partial \Pi_d^D}{\partial a} + \frac{\partial \Pi_u^C}{\partial a} / \frac{\partial a}{\partial a} = ta > 0$.

Define with \tilde{p}^C the uniform price which results from the competition between the firms, when firms have collusively decided not to price discriminate. It is well known that the equilibrium uniform price and the equilibrium profits are respectively (D'Aspremont *et al.*, 1979):

$$\tilde{p}^C = t(1 - 2a) \quad (23)$$

$$\tilde{\Pi}^C = \frac{t}{2}(1 - 2a) \quad (24)$$

A straightforward implication of equation (23) is that when $a = 1/2$ collusive profits are nil and equal to the punishment profits. We simplify the analysis making the reasonable assumption that in this case firms have no incentive to collude. Therefore, the rest of the analysis is limited to the case of $a < 1/2$.

Suppose that firm A deviates. Define \hat{p}_3 as the price which solves $u_x^A(\hat{p}_3) = u_x^B(\tilde{p}^C)$. Therefore: $\hat{p}_3 = \tilde{p}^C + t(1 - 2x - 2a + 4ax)$. Then:

$$\tilde{p}^D = \max\{0; \min[p^*; \hat{p}_3]\} \quad (25)$$

The deviation price schedule is fully characterized by the following lemma.

Lemma 4. *When firms collude not to discriminate, the deviation price schedule is:*

$$\tilde{p}^D = t(2 - 4a - 2x + 4ax) \quad (26)$$

Proof. The proof for $\hat{p}_3 \leq p^*$ is identical to the case described in Lemma 2. We prove that: $\hat{p}_3 \geq 0$. Substituting equation (23) into \hat{p}_3 , we obtain: $\hat{p}_3 = t(2 - 2x - 4a + 4ax)$. Note that \hat{p}_3 is continuous and strictly decreasing in x . Therefore, \hat{p}_3 is positive for

every x if and only if it is non-negative when $x = 1$. Since $\hat{p}_3(x=1) = 0$, \hat{p}_3 is strictly positive for every $x < 1$. Hence, $\tilde{p}^D = \hat{p}_3 = t(2 - 2x - 4a + 4ax)$ ■

Therefore, the cheating firm serves the whole market and the deviation profits are:

$$\tilde{\Pi}^D(a) = \int_0^1 \tilde{p}^D = t(1 - 2a) \quad (27)$$

During the punishment phase the firms compete fiercely. The equilibrium price schedules are defined by equations (2) and (3), while the punishment profits are defined by equation (4). Therefore, the critical discount factor is obtained inserting equations (4), (24) and (27) into equation (1). It follows:

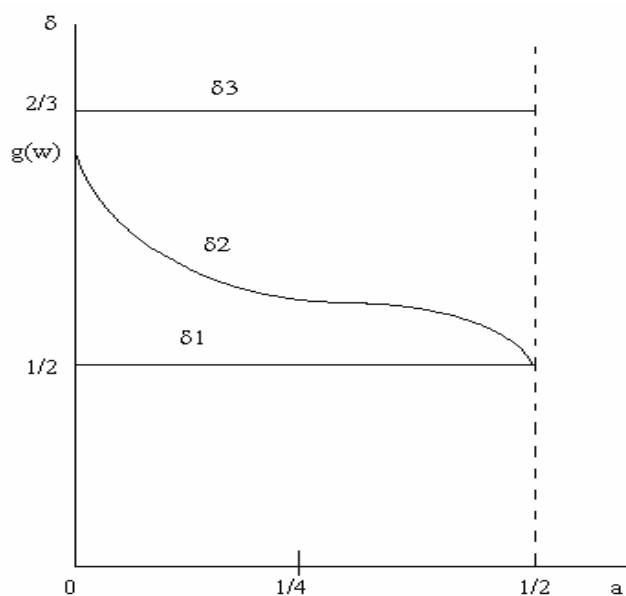
$$\delta_3^* = \frac{2}{3} \quad (28)$$

As for the first collusive scheme, the product differentiation degree does not influence the sustainability of the collusive agreement, since the critical discount factor is equal to $2/3$ for every value of a .

Figure 1 summarizes the results. The critical discount factor of the first and the third collusive scheme does not depend on the position of the firms in the market. Instead, the critical discount factor of the second collusive scheme decreases when the product differentiation degree decreases. Moreover, the critical discount factor of the second scheme is always between the critical discount factor of the first scheme ($1/2$) and the critical discount factor of the third scheme ($2/3$). This implies that the first collusive agreement is always easier to sustain than the second collusive agreement, which in turn is always easier to sustain than the third collusive agreement. Compare now the collusive profits in the three collusive schemes (equations (6), (12), (17) and (24)). Obviously, the first collusive scheme yields the largest collusive profits, while the third collusive scheme yields the smallest collusive profits. The collusive profits under the second collusive scheme are in an intermediate position. Therefore, the first collusive

scheme dominates the other two collusive schemes: it yields greater profits and it is easier to sustain. However, this does not imply that collusion on a uniform price or collusion not to discriminate will never arise. Even if these schemes are less profitable and more difficult to sustain, they may be less “costly” to implement than the collusion on discriminatory prices, since an agreement regarding a huge number of prices (as the first collusive scheme) may be very time-demanding and difficult to reach. Unfortunately, our model does not allow taking into consideration this aspect.

Figure 1



3.5. Suboptimal collusion

In the previous section we considered the sustainability of optimal collusion. Now we ask whether suboptimal collusion is sustainable when optimal collusion is not sustainable. In the following we show that suboptimal collusion is never sustainable when optimal collusion is not sustainable⁷³.

⁷³ Notice that the answer cannot be known *a priori* if the suboptimal collusive price is lower than the optimal collusive price: both the collusive profits and the deviation profits are lower and the net effect on the critical discount factor is *a priori* ambiguous. On the contrary, the answer is *a priori* negative if the suboptimal collusive price is higher than the optimal collusive price, since collusive profits are lower and the deviation profits are higher. Therefore, in the remaining part of this section we refer only to suboptimal collusive prices which are lower than the optimal collusive prices.

Suboptimal collusion on discriminatory prices

Consider the following setting. The two firms collude on the prices to set to the consumers located at points x' and $1-x'$, with $x' < 1/2$. Suppose first that the firms collude in the optimal way, that is, they set the collusive prices $p^C(x') = p^*(x')$ and $p^C(1-x') = p^*(1-x')$, respectively on consumer x' and consumer $1-x'$. Of course, $p^*(x') = p^*(1-x')$. The collusive profits each firm obtains from these consumers are:

$$\Pi^C(x'; 1-x') = p^*(x') \quad (29)$$

Define: $T(x') \equiv t(1-2x'-2a+4ax')$. Note that: $T(x') \geq 0$ if $x' \leq 1/2$ and $T(x') \leq 0$ if $x' \geq 1/2$ ⁷⁴. Moreover, note that: $T(x') = -T(1-x')$.

Consider the punishment profits. From equations (2) and (3) we can write:

$$\Pi^N(x'; 1-x') = T(x') \quad (30)$$

Suppose that firm A deviates. Using Lemma 1 we get:

$$p^D \equiv [p_{x'}^D; p_{1-x'}^D] = [p^*(x'); p^*(x') - T(x')] \quad (31)$$

It follows that the deviation profits firm A obtains from consumers x' and $1-x'$ are:

$$\Pi^D(x'; 1-x') = 2p^*(x') - T(x') \quad (32)$$

Now, observe that:

$$\Pi^D(x'; 1-x') - \Pi^N(x'; 1-x') = 2[p^*(x') - T(x')] = 2[\Pi^D(x'; 1-x') - \Pi^C(x'; 1-x')] \quad (33)$$

⁷⁴ Indeed, $T(x') \equiv t(1-2x'-2a+4ax') = t(1-x'-a)^2 - t(x'-a)^2$ measures the advantage (disadvantage) of firm A over firm B in serving consumer $x' \leq 1/2$ ($\geq 1/2$), because it says how much firm A may increase (decrease) its price above (below) the price set by the rival without losing (serving) consumer x' .

Obviously, if optimal collusion regards not only consumers x' and $1-x'$ but also, say, consumers x'' and $1-x''$, x''' and $1-x'''$, and so on, the previous result does not change: the difference between the deviation profits and the Nash profits is always equal to the double of the difference between the deviation profits and the collusive profits. Define W as the set of consumers served with optimal collusive discriminatory prices. From (33) it follows that:

$$Z = \sum_{x \in W} [\Pi^D(x; 1-x) - \Pi^N(x; 1-x)] = 2 \sum_{x \in W} [\Pi^D(x; 1-x) - \Pi^C(x; 1-x)] = 2\Omega \quad (34)$$

Suppose now that the two firms set suboptimal collusive prices on the consumers located at \tilde{x} and $1-\tilde{x}$, with $\tilde{x} \leq 1/2$. Suboptimal collusive prices are defined as the optimal collusive prices minus a strictly positive amount. That is: $p^C(\tilde{x}) = p^*(\tilde{x}) - k$ and $p^C(1-\tilde{x}) = p^*(1-\tilde{x}) - k$, respectively for consumer \tilde{x} and consumer $1-\tilde{x}$, with $k > 0$. The collusive profits each firm obtains from consumers \tilde{x} and $1-\tilde{x}$ are:

$$\Pi^C(\tilde{x}; 1-\tilde{x}) = p^*(\tilde{x}) - k \quad (35)$$

The punishment profits are:

$$\Pi^N(\tilde{x}; 1-\tilde{x}) = T(\tilde{x}) \quad (36)$$

Suppose that firm A deviates from the collusive agreement. Define \hat{p}_s as the price which solves: $u_x^A(\hat{p}_s) = u_x^B(p^* - k)$, with $x = \tilde{x}, 1-\tilde{x}$. It is: $\hat{p}_s = p^*(x) - k + T(x)$, with $x = \tilde{x}, 1-\tilde{x}$. Then:

$$p_s^D = \max\{0; \min[p^*(x); p^*(x) - k + T(x)]\}, \quad x = \tilde{x}, 1-\tilde{x} \quad (37)$$

We look for: $p_S^D \equiv [p_{\tilde{x}}^D; p_{1-\tilde{x}}^D]$, that is, the deviation prices applied by firm A on consumer \tilde{x} and consumer $1-\tilde{x}$ respectively. First, note that $p_{\tilde{x}}^D = 0$ is impossible for every k . In fact, for it to be possible, it must be: $0 > \min[p^*(\tilde{x}); p^*(\tilde{x}) - k + T(\tilde{x})]$. But $0 > p^*(\tilde{x})$ is clearly impossible, and $0 > p^*(\tilde{x}) - k + T(\tilde{x})$ is impossible as well, since it contradicts the assumption that the collusive profits must be higher than the punishment profits⁷⁵. Second, note that $p_{1-\tilde{x}}^D = p^*(\tilde{x})$ is impossible for every k , since it would imply: $p^*(\tilde{x}) = \min[p^*(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$.

Assume for the moment $a < 1/2$. Depending on the value of k , four cases are possible:

Case 1) $p_S^D = [p^*(\tilde{x}); 0]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (38) \\ 0 > p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (39) \end{cases}$$

Case 2) $p_S^D = [p^*(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (40) \\ 0 < p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (41) \end{cases}$$

Case 3) $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) - k + T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (42) \\ 0 < p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (43) \end{cases}$$

⁷⁵ Indeed, $\Pi^C > \Pi^N$ implies: $p^*(\tilde{x}) - k > T(\tilde{x})$, which is impossible if $p^*(\tilde{x}) - k + T(\tilde{x}) < 0$.

Case 4) $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x}), 0]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) - k + T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (44) \\ 0 > p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (45) \end{cases}$$

Clearly, the deviation profits firm A obtains from consumers \tilde{x} and $1 - \tilde{x}$ depend on the deviation prices. Therefore:

$$\text{Case 1) } \Pi^D(\tilde{x}; 1 - \tilde{x}) = p^*(\tilde{x}) \quad (46)$$

$$\text{Case 2) } \Pi^D(\tilde{x}; 1 - \tilde{x}) = 2p^*(\tilde{x}) - k - T(\tilde{x}) \quad (47)$$

$$\text{Case 3) } \Pi^D(\tilde{x}; 1 - \tilde{x}) = 2p^*(\tilde{x}) - 2k \quad (48)$$

$$\text{Case 4) } \Pi^D(\tilde{x}; 1 - \tilde{x}) = p^*(\tilde{x}) - k + T(\tilde{x}) \quad (49)$$

We state the following Lemma:

Lemma 5: the following inequalities hold:

$$\text{Case 1) } \Pi^D(.) - \Pi^N(.) = p^*(\tilde{x}) - T(\tilde{x}) < 2k = 2[\Pi^D(.) - \Pi^C(.)] \quad (50)$$

$$\text{Case 2) } \Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - k - 2T(\tilde{x}) < 2(p^*(\tilde{x}) - T(\tilde{x})) = 2[\Pi^D(.) - \Pi^C(.)] \quad (51)$$

$$\text{Case 3) } \Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - 2k - T(\tilde{x}) < 2(p^*(\tilde{x}) - k) = 2[\Pi^D(.) - \Pi^C(.)] \quad (52)$$

$$\text{Case 4) } \Pi^D(.) - \Pi^N(.) = p^*(\tilde{x}) - k < 2T(\tilde{x}) = 2[\Pi^D(.) - \Pi^C(.)] \quad (53)$$

Proof. Inequalities (51) and (52) are immediately verified. Consider inequality (50). Recall that $p_S^D = [p^*(\tilde{x}), 0]$ occurs only if: $p^*(\tilde{x}) - k - T(\tilde{x}) < 0$ (condition (39)). For inequality (50) not to hold it must be: $p^*(\tilde{x}) - T(\tilde{x}) - k > k$, but this contradicts condition (39). Therefore, condition (39) always implies inequality (50). Consider inequality (53). Recall that $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x}), 0]$ occurs only if: $p^*(\tilde{x}) - k - T(\tilde{x}) < 0$ (condition (45)). For inequality (53) not to hold it must be:

$p^*(\tilde{x}) - k - T(\tilde{x}) > T(\tilde{x})$, but this contradicts condition (45). Therefore, condition (45) always implies condition (53). ■

Obviously, if suboptimal collusion regards not only consumers \tilde{x} and $1 - \tilde{x}$ but also, say, consumers \tilde{x}' and $1 - \tilde{x}'$, \tilde{x}'' and $1 - \tilde{x}''$, and so on, the previous result does not change: the difference between the deviation profits and the Nash profits is always less than the double of the difference between the deviation profits and the collusive profits. Define \tilde{W} as the set of consumers served with suboptimal collusive discriminatory prices. From lemma 5) it follows:

$$\Delta \equiv \sum_{x \in \tilde{W}} [\Pi^D(x; 1-x) - \Pi^N(x; 1-x)] < 2 \sum_{x \in \tilde{W}} [\Pi^D(x; 1-x) - \Pi^C(x; 1-x)] \equiv 2\Psi \quad (54)$$

Since it must be $W \cup \tilde{W} = X$, where X is the set of all consumers, and $W \cap \tilde{W} = \emptyset$, putting together equation (34) and inequality (54) it follows:

$$\begin{aligned} \delta^* &= \frac{\sum_{x \in X} \Pi^D(x; 1-x) - \sum_{x \in X} \Pi^C(x; 1-x)}{\sum_{x \in X} \Pi^D(x; 1-x) - \sum_{x \in X} \Pi^N(x; 1-x)} = \\ &= \frac{\sum_{x \in W} \Pi^D(x; 1-x) + \sum_{x \in \tilde{W}} \Pi^D(x; 1-x) - \sum_{x \in W} \Pi^C(x; 1-x) - \sum_{x \in \tilde{W}} \Pi^C(x; 1-x)}{\sum_{x \in W} \Pi^D(x; 1-x) + \sum_{x \in \tilde{W}} \Pi^D(x; 1-x) - \sum_{x \in W} \Pi^N(x; 1-x) - \sum_{x \in \tilde{W}} \Pi^N(x; 1-x)} = \frac{\Omega + \Psi}{Z + \Delta} \geq \frac{1}{2} \end{aligned}$$

Since $Z = 2\Omega$ and $\Delta < 2\Psi$, the critical discount factor is equal to 1/2 only when Ψ and Δ are equal to 0, that is, when $\tilde{W} = \emptyset$, or, in other words, when all consumers are served with optimal collusive discriminatory prices. Conversely, when Ψ and Δ are different from 0 (that is, when $\tilde{W} \neq \emptyset$), the critical discount factor is strictly greater than 1/2. The straightforward implication is that if optimal collusion is not sustainable, sub-optimal collusion is not sustainable too.

Finally, consider the case of $a = 1/2$. It implies $T(\tilde{x}) = 0$. Consider again conditions (38) – (45). It is immediate to see that conditions (38) and (40) are never verified.

Moreover, conditions (44) and (45) cannot be contemporaneously verified. It follows that cases 1), 2) and 4) never occur. Conversely, conditions (42) and (43) are always verified. Therefore, case 3) always occurs, and the deviation price schedule is given by: $p_s^D = [p^*(\tilde{x}) - k; p^*(\tilde{x}) - k]$. The deviation profits are: $\Pi^D = 2p^*(\tilde{x}) - 2k$, while the punishment profits are: $\Pi^N = 0$. It follows that:

$$\Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - 2k = 2[\Pi^D(.) - \Pi^C(.)] \quad (55)$$

When $a = 1/2$ and firms collude in a suboptimal way, the difference between the deviation profits and the Nash profits is equal to the double of the difference between the deviation profits and the collusive profits. It follows that the critical discount factor is equal to 1/2. Note that these results are consistent with Proposition 2. Optimal collusion on a uniform price can be seen as a form of suboptimal collusion with respect to optimal collusion on discriminatory prices: Proposition 2 says that the critical discount factor is strictly larger than 1/2 when firms are differentiated and it is equal to 1/2 when firms are undifferentiated. In this section we have shown that *any possible suboptimal collusive agreement* induces a critical discount factor larger than 1/2 if firms are differentiated and equal to 1/2 if firms are undifferentiated.

Suboptimal collusion on uniform price

We consider now the second collusive scheme: firms collude on a uniform price to be applied to all consumers. What happens to the critical discount factor if firms collude on a suboptimal uniform collusive price?

Proposition 3: *the critical discount factor is a decreasing function of the collusive price.*

Proposition 3 implies that when optimal collusion is not sustainable, suboptimal collusion cannot be sustainable too, because it increases the critical discount factor. In what follows we prove Proposition 3.

Let $\tilde{p}^C \leq \bar{p}^C$ denote the collusive uniform price⁷⁶. When $\tilde{p}^C = \bar{p}^C$ we are clearly in the optimal collusion case described in section 3.4.. Therefore, we concentrate on $\tilde{p}^C < \bar{p}^C$. First, since when $\tilde{p}^C = \bar{p}^C$ all consumers are served, the same must be true when firms sub-optimally collude, that is when $\tilde{p}^C < \bar{p}^C$. The sub-optimal collusive profits of each firm are therefore the following:

$$\tilde{\Pi}^C = \frac{\tilde{p}^C}{2} \quad (56)$$

Now consider the deviation price. Suppose that firm A cheats. Define \hat{p}_U as the price which solves: $u_x^A(\hat{p}_U) = u_x^B(\tilde{p}^C)$. We get: $\hat{p}_U = \tilde{p}^C + t(1 - 2x - 2a + 4ax)$. The deviation price is therefore:

$$\tilde{p}_U^D = \max\{0; \min[p^*; \tilde{p}^C + t(1 - 2x - 2a + 4ax)]\} \quad (57)$$

Before proceeding, note that the following equation is always true: $\tilde{p}^C + t(1 - 2x - 2a + 4ax) = \min[p^*; \tilde{p}^C + t(1 - 2x - 2a + 4ax)]$. The intuition is simple. Lemma 2 and Lemma 3 show that: $\bar{p}^C + t(1 - 2x - 2a + 4ax) < p^*$. Since $\tilde{p}^C < \bar{p}^C$, it must be: $\tilde{p}^C + t(1 - 2x - 2a + 4ax) < p^*$. Therefore, the deviation price is simply:

$$\tilde{p}_U^D = \max\{0; \tilde{p}^C + t(1 - 2x - 2a + 4ax)\} \quad (58)$$

Intuitively, the smaller is the suboptimal collusive uniform price the more difficult for the cheating firm is to steal a consumer without setting a price lower than zero (the marginal cost). This allows us to derive a condition for the whole market to be served by the cheating firm. First, note that the second term in (58) is decreasing in x . Therefore, if the consumer located in x is served by the deviating firm, all consumers

⁷⁶ Note that we do not consider $\tilde{p}^C > \bar{p}^C$, because in this case the critical discount factor is unambiguously higher than under optimal collusion (see also footnote 73).

located at the left of x must be served as well. The consumer located in x is served by the cheating firm when: $\tilde{p}^C + t(1 - 2x - 2a + 4ax) \geq 0$, from which it follows:

$$x \leq \frac{\tilde{p}^C}{2t(1-2a)} + \frac{1}{2} \equiv \zeta(\tilde{p}^C) \quad (59)$$

On the contrary, if $x > \zeta(\tilde{p}^C)$ the consumer located in x cannot be stolen by the deviating firm. It follows that if $\zeta(\tilde{p}^C)$ is higher than 1, all consumers are served by firm A . By solving $\zeta(\tilde{p}^C) \geq 1$ with respect to the price we obtain the necessary and sufficient condition for the whole market to be served by the deviating firm. This condition reduces to:

$$\tilde{p}^C \geq t(1-2a) \quad (60)$$

Hence, if the suboptimal collusive uniform price is high enough (i.e. if condition (60) is satisfied) the entire market is served by the cheating market, otherwise a subset of consumers continues to be served by firm B . Incidentally, note that condition (60) is always satisfied when $a = 1/2$.

Suppose first that inequality (60) is satisfied. The deviation profits are the following (the subscript w indicates that the whole market is served by the deviating firm):

$$\tilde{\Pi}_w^D = \int_0^1 [\tilde{p}^C + t(1 - 2x - 2a + 4ax)] dx = \tilde{p}^C \quad (61)$$

By substituting equations (4), (56) and (61) into equation (1) we get the critical discount factor:

$$\tilde{\delta}_w^*(\tilde{p}^C) = \frac{2\tilde{p}^C}{4\tilde{p}^C - t(1-2a)} \quad (62)$$

The derivative of the critical discount factor with respect to the collusive price is the following:

$$\frac{\partial \tilde{\delta}_w^*(\tilde{p}^C)}{\partial \tilde{p}^C} = -\frac{2t(1-2a)}{[4\tilde{p}^C - t(1-2a)]^2} \leq 0 \quad (63)$$

Therefore, colluding on a suboptimal collusive uniform price increases the critical discount factor (or leaves it unchanged when firms are not differentiated), and this makes the collusive agreement less (or equally) sustainable.

Suppose now that condition (60) does not hold. The deviating firm does not serve all consumers, but only the consumers located at the left of x^* , where $x^* = \zeta(\tilde{p}^C) < 1$. Hence, the deviation profits are the following (the subscript f indicates that only a fraction of the market is served by the deviating firm):

$$\tilde{\Pi}_f^D = \int_0^{\zeta(\tilde{p}^C)} [\tilde{p}^C + t(1-2x-2a+4ax)] dx = \frac{\tilde{p}^{C2}}{4t(1-2a)} - \frac{t}{2}(a - \frac{1}{2}) + \frac{\tilde{p}^C}{2} \quad (64)$$

By substituting equations (4), (56) and (64) into equation (1) we obtain the critical discount factor:

$$\tilde{\delta}_f^*(\tilde{p}^C) = \frac{K\tilde{p}^{C2} + Z}{K\tilde{p}^{C2} + \tilde{p}^C} \quad (65)$$

with $K \equiv \frac{1}{2t(1-2a)}$ and $Z \equiv t(\frac{1}{2} - a)$. After some manipulations, the derivative of the critical discount factor can be written as follows:

$$\frac{\partial \tilde{\delta}_f^*(\tilde{p}^C)}{\partial \tilde{p}^C} = \frac{K\tilde{p}^C(\tilde{p}^C - 2Z) - Z}{(K\tilde{p}^{C2} + \tilde{p}^C)^2} \quad (66)$$

Since $K > 0$ and $Z > 0$, a sufficient condition for the derivative to be negative is $\tilde{p}^c - 2Z < 0$. Note that this condition coincides with: $\tilde{p}^c < t(1-2a)$, which is always satisfied when the deviating firm cannot serve the whole market (see condition (60)). Therefore, the derivative of the critical discount factor with respect to price is always negative. This means that lowering the collusive price below the optimal uniform collusive price makes the collusion less sustainable, even in the case in which the deviating firm cannot serve the entire market. This completes the proof of Proposition 3.

3.6. Conclusion

In this chapter we analyzed the relationship between the product differentiation degree and the sustainability of three different collusive schemes. The main innovation of our analysis is represented by the possibility for firms to perfectly price discriminate. We obtain the following results. The critical discount factor for the first collusive scheme (optimal collusion on discriminatory prices) is equal to $1/2$ for any product differentiation degree. The second collusive scheme (optimal collusion on uniform price) is more difficult to sustain than the first, since the critical discount factor is between $1/2$ and $2/3$. Moreover, the greater is the product differentiation degree and the greater are the transportation costs, the greater is the critical discount factor: these findings contrast with previous results obtained under the hypothesis of uniform price (Chang, 1991, Chang, 1992 and Hackner, 1995). Finally, the sustainability of the third collusive scheme (collusion not to discriminate) does not depend on the product differentiation degree, and it is always equal to $2/3$. In the last section we extend the model to consider the possibility for the firms to sub-optimally collude. Both when firms collude on a suboptimal discriminatory price schedule and when they collude on a suboptimal uniform price, the critical discount factor is always greater or equal to the critical discount factor obtained under optimal collusion (collusion is less sustainable).

3.7. Appendix

In this appendix we study the sustainability of the second and the third collusive scheme when imperfect direct price discrimination *a la* Liu and Serfes (2004) is assumed⁷⁷. The Nash profits are the following⁷⁸:

$$\Pi^N = \frac{t(1-2a)(9n^2-18n+40)}{36n^2} \quad (67)$$

The second collusive scheme. Suppose that firm *A* deviates. Assume that v is sufficiently high, so that it is always optimal for the deviating firm to serve the whole market. Therefore, the deviation price schedule is defined in such a way to make the consumers located at the endpoint of each sub-segment indifferent between buying from the deviating firm and the colluding firm⁷⁹. Such consumers are located at m/n , with

$m = 1, \dots, n$, and the indifference condition is: $v - \ddot{p}^D - t\left(\frac{m}{n} - a\right)^2 = v - \bar{p}^C - t\left(\frac{m}{n} - 1 + a\right)^2$,

from which it follows:

$$\ddot{p}^D = \begin{cases} v - t\left(\frac{1}{2} - a\right)^2 + t\left(\frac{m}{n} - 1 + a\right)^2 - t\left(\frac{m}{n} - a\right)^2 & \forall a \leq \frac{1}{4} \\ v - ta^2 + t\left(\frac{m}{n} - 1 + a\right)^2 - t\left(\frac{m}{n} - a\right)^2 & \forall a \geq \frac{1}{4} \end{cases}$$

The deviation profits are:

$$\Pi^D = \sum_{m=1}^n \frac{\ddot{p}^D}{n} = \begin{cases} \frac{4vn - t(2a-1)[n(2a-1) - 4]}{4n} & \forall a \leq \frac{1}{4} \\ \frac{vn - t(1-2a+na^2)}{n} & \forall a \geq \frac{1}{4} \end{cases} \quad (68)$$

⁷⁷ See chapter 2 for the description of the Liu and Serfes (2004) framework.

⁷⁸ Consider equations (9) and (10) in chapter 2 when firms are assumed to be symmetric.

⁷⁹ If the consumer located at the endpoint of a sub-segment receives the same utility from firm *A* and from firm *B*, it follows that all consumers in the same sub-segment at the left of such consumer receives a (strictly) higher utility from firm *A* than from firm *B*, because the transportation costs are lower.

Inserting equations (12), (17), (67) and (68) into equation 1, we get the critical discount factor:

$$\delta^* = \begin{cases} \frac{9n[(2a-1)(n(2a-1)-8)-4wn]}{4[(2a-1)(-20-9n+9n^2(a-1))-18wn^2]} & \forall a \leq \frac{1}{4} \\ \frac{18n(2-4a+na^2-wn)}{40+18+9n^2+36a^2n^2-2a(40+18n+9n^2)-36wn^2} & \forall a \geq \frac{1}{4} \end{cases}$$

Differentiating δ^* with respect to a and w we get respectively:

$$\frac{\partial \delta^*}{\partial a} = \begin{cases} -\frac{9n^2(40-54n+9n^2)(1-4a+4a^2+4w)}{4[20+9n+18a^2n^2-a(40+18n+27n^2)+n^2(9-18w)]^2} & \forall a \leq \frac{1}{4} \\ -\frac{36n^2(40-54n+9n^2)(w-a+a^2)}{[40+18+36a^2n^2-2a(40+18n+9n^2)+n^2(9-36w)]^2} & \forall a \geq \frac{1}{4} \end{cases}$$

$$\frac{\partial \delta^*}{\partial w} = \begin{cases} -\frac{9n^2(1-2a)(40-54n+9n^2)}{2[20+9n+18a^2n^2-a(40+18n+27n^2)+n^2(9-18w)]^2} & \forall a \leq \frac{1}{4} \\ -\frac{18n^2(1-2a)(40-54n+9n^2)}{[40+18+36a^2n^2-2a(40+18n+9n^2)+n^2(9-36w)]^2} & \forall a \geq \frac{1}{4} \end{cases}$$

It is easy to see that for $n \geq 8$ the critical discount factor is decreasing both in a and w , while for $n = 2$ and $n = 4$ it is increasing both in a and w . Therefore, for a sufficiently high information quality, the more the firms are differentiated and the higher are the transportation costs, the *less* collusion is sustainable.

The third collusive scheme. As before, suppose that firm A deviates and assume that v is sufficiently high, so that it is always optimal for the cheating firm to serve the whole market. Therefore, the deviation price schedule is obtained by solving the following

indifference condition: $v - \hat{p}^D - t\left(\frac{m}{n} - a\right)^2 = v - \check{p}^C - t\left(\frac{m}{n} - 1 + a\right)^2$. We get:

$$\hat{p}^D = t(1-2a) + t\left(\frac{m}{n} - 1 + a\right)^2 - t\left(\frac{m}{n} - a\right)^2$$

The deviation profits are:

$$\Pi^D = \sum_{m=1}^n \frac{\hat{p}^D}{n} = \frac{t(1-2a)(n-1)}{n} \quad (69)$$

Inserting equations (24), (67) and (69) we obtain the critical discount factor:

$$\delta^* = \frac{18n^2 - 36n}{27n^2 - 18n - 40}$$

As for perfect price discrimination, the sustainability of collusion does depend neither on the product differentiation degree nor on transportation costs.

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