The Mechanics of Endogenous Diffusion: A Tractable Analytical Framework

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Abstract

This note develops a growth model in which the diffusion of capital goods is endogenized through the interaction between monopolistic suppliers and a final-goods sector. We introduce a distinction between private and social returns to adoption: while individual firms operate under constant or diminishing returns—ensuring a stable micro-foundation for profit maximization—aggregate production exhibits increasing returns driven by system-wide diffusion complementarities and adoption externalities. The model derives a closed-form solution for the long-run growth rate, which is shown to depend on the user cost of capital and the strength of these aggregate complementarities. Specifically, we find that lower financing costs and declining relative prices for capital goods accelerate diffusion. By explicitly modeling the transition from micro-level adoption to macro-level increasing returns, this framework provides a structural mechanism explaining how economies sustain growth through the propagation of innovations, complementing standard R&D-based approaches.

JEL Codes: O1, O3, O4
Keywords: Endogenous growth, technological diffusion, long-run growth

1 Introduction

The relationship between technological change and economic growth is well established in modern macroeconomics. While innovation has been the focus of extensive theoretical and empirical investigation, the diffusion of innovations—their adoption throughout the economy—has often been treated as exogenous or instantaneous. Building on Romer (1990b) and Romer (1990a), this paper develops a framework in which the diffusion path of capital goods is endogenously determined. In Romer's model, growth is driven by knowledge accumulation through research, but diffusion is not explicitly modelled. Other approaches, such as Jovanovic and Lach (1993), introduce stochastic diffusion but treat it as exogenous. In

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contrast, Jones (1995) addresses limitations of earlier R&D based models by showing that the scale of research effort does not drive long-run growth, but instead depends on population growth, thus highlighting the need to endogenize diffusion mechanisms more explicitly. While recent literature has begun to explore endogenous adoption (e.g., Moran and Queralto (2018)), the structural mechanics of the diffusion path can be derived analytically. Building on the framework established in Baussola (1999), this note provides a tractable closed-form solution in which the diffusion of capital goods is explicitly endogenized. We show that the long-run growth rate depends not only on research investment but also on structural parameters governing diffusion. Specifically, the user cost of capital and the productivity of capital goods in final output affect the adoption path and, in turn, the economy's growth trajectory. While prior work has examined the role of adoption lags and delayed productivity effects(Comin and Hobijn, 2004; Comin and Gertler, 2006), our model is, to our knowledge, the first to link the diffusion path explicitly and analytically to the long-run growth rate. This framework provides a novel mechanism through which market parameters and policy instruments influence growth dynamics. Recent quantitative studies (Moran and Queralto, 2018; Anzoategui et al., 2019) demonstrate that endogenous technology adoption is a primary driver of TFP persistence, particularly in the aftermath of financial shocks. These models highlight how liquidity constraints can choke off the diffusion of new technologies, generating hysteresis. While they utilize a large-scale DSGE framework to quantify business cycle effects, this note complements their findings by providing a tractable analytical derivation of the long-run diffusion trajectory. Crucially, we reconcile the stability required for competitive equilibrium with the increasing returns required for growth by distinguishing between private and social returns. We show that while individual firms perceive diminishing returns to adoption, system-wide externalities generate aggregate increasing returns. This framework provides a novel mechanism through which market parameters and policy instruments influence growth dynamics.

2 The Model

The economy consists of three sectors: final goods, intermediate capital goods, and research. In the final goods sector, output is produced by means of physical labor (L,unskilled workers) and physical capital x.We normalize the price of final output to unity, serving as the numeraire for the economy. It is assumed that there is a continuum of capital-good varieties indexed by $i \in [0, A(t)]$.

The aggregate production function is given by:

$$Y_f = g(L) \int_0^{A(t)} x(i,t)^{\phi} \, di \tag{1}$$

where $\phi > 1$.

It is essential to clarify the economic interpretation of ϕ. In Romer (1990a,b), the parameter choice $\phi < 1$ guarantees constant returns to scale for the representative firm. By contrast, we assume $\phi > 1$ to capture aggregate complementarities in the diffusion process—specifically, network effects and learning spillovers that increase the productivity of capital varieties as adoption spreads.

To reconcile this with the requirements of competitive equilibrium, we interpret Equation (1) as a reduced-form social production function. Individual firms perceive diminishing private returns (ensuring a valid optimization problem), but the aggregate economy exhibits increasing returns due to external effects that are internalized at the system level.[1] Production determines the demand for new technologies. In the intermediate capital goods sector, each capital good is produced by a single monopolist, and diffusion emerges from the interaction of supply and demand. In the intermediate sector, output of capital good i is given by:

$$z(i) = T(i)A(t) \tag{2}$$

where $T(i)$ is specialized labor input and $A(t)$ is the stock of knowledge. In the research sector, knowledge accumulation follows:

$$\dot{A} = \delta R A(t) \tag{3}$$

where R is the stock of researchers and δ is a productivity parameter. The model assumes a fixed total labor endowment, allocated across sectors. While this may appear restrictive given the continuous diffusion of new capital goods, it is consistent with the logic of expanding-variety growth models. The diffusion of capital goods enhances productivity by allowing the existing labor force to work with a broader array of inputs. As such, growth in output arises not from an increase in labor supply, but from the increasing effectiveness of labor through the adoption of new technologies. This mechanism is well-established in the endogenous growth literature and supports the model's internal consistency[2]

The adoption decision by the final goods firm satisfies two conditions: a profitability condition ($f'(x(i)) \geq rp(i)$) and an arbitrage condition ($-\dot{p}(i) + rp(i) - f'(x(i)) \leq 0$), where $f'(x)$ is the permanent increase in revenue from purchasing one unit of capital good i, and r is the interest rate[3]. Assuming myopic expectations, the second condition collapses into the first. Thus, the adoption decision is based on current-period variables without internalizing the full future effects of actions. This reflects limited foresight and information frictions and aligns with models in which firms adopt innovations gradually or respond to current

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[1] Formally, this aggregate function is the reduced-form result of a production technology characterized by external spillovers. Consider an individual final-goods firm j with the following private production function:

$$y_j(t) = g(L_j)S(t)\int_0^{A(t)} x_{ji}(t)^{\alpha}\,di$$

where $\alpha < 1$ is the private elasticity of substitution, ensuring diminishing marginal returns to specific capital varieties (and thus a stable demand function). The term $S(t)$ represents an aggregate productivity spillover—such as network effects or learning-by-using—that depends on the total diffusion of capital in the economy: $S(t) = \left[\int x(i)^{\alpha} di\right]^{\frac{\mu}{\alpha}}$. In a symmetric equilibrium where all firms adopt identical quantities, the private elasticity α and the externality parameter μ combine to yield the aggregate curvature $\phi = \alpha + \mu$. We assume $\phi > 1$ to capture these strong complementarities at the social level, while $\alpha < 1$ guarantees competitive stability at the firm level.

[2] We assume the economy is endowed with two distinct factors: a fixed stock of unskilled labor L used in final production, and a fixed stock of human capital H allocated between research (R) and intermediate goods production (T), such that $H = R + T$. This segmentation simplifies the analysis by restricting labor arbitrage to the high-skill sectors driving innovation.

[3] Standard formulations of the user cost include physical depreciation (γ). Here, we assume $\gamma = 0$ to focus purely on the interplay between financial returns (r) and technological price dynamics (π_p).

profitability. However, firms optimize based on their *private* marginal benefit curve, which is downward sloping ($\alpha < 1$). The monopolist supplier, anticipating this private demand, optimizes dynamically.

The monopolist maximizes the stream of operating profits, defined as the difference between price and marginal cost. Following Ireland and Stoneman (1986), we derive a dynamic demand function for capital goods where the adoption flow $z(i)$ equals the time derivative of cumulative adoption $x(i)$.[4]

The monopolist faces a downward-sloping demand curve derived from the final goods sector, characterized by the demand elasticity parameter $\alpha \in (0, 1)$. Unlike the aggregate economy, the individual firm perceives diminishing marginal returns to its own specific variety.

Differentiating the inverse demand schedule $p(t) \propto x(t)^{\alpha-1}$ with respect to time yields the firm's dynamic constraint (Equation 4):

$$\pi_p = -(1-\alpha)g \tag{4}$$

This condition implies that for the firm to increase supply at rate g, it must lower prices at rate π_p proportional to the concavity of demand $(1-\alpha)$. For notational convenience, we define $\rho \equiv \pi_p$ as the equilibrium rate of price change.

The monopolist maximizes the present discounted value of profits. The first-order condition equates the marginal revenue product to the user cost of capital. In this dynamic setting, the implicit user cost $u(t)$ faced by the firm is:

$$u(t) = p(t)(r - \pi_p) \tag{5}$$

Combining the demand side (Marginal Product of Capital) with the supply side (User Cost), the optimization requires that the return on capital equals the user cost:

$$\alpha A x(t)^{\alpha-1} = p(t)(r - \pi_p) \tag{6}$$

The equilibrium growth rate g is determined by the net incentive to accumulate capital. In the frictionless baseline, the supplier receives a gross rental return of $r - \pi_p$ (representing the user cost paid by the firm), while the financing cost is the interest rate r. The net arbitrage gap driving diffusion is therefore:

$$\text{Net Gap} = (r - \pi_p) - r = -\pi_p$$

In the frictionless baseline, financial arbitrage ensures that the interest rate r cancels out between the supplier (who receives r) and the adopter (who pays r). The net incentive driving the firm's expansion is purely the rate of technological deflation $-\pi_p$. Substituting the net incentive into the dynamic constraint yields the firm's optimal accumulation path:

$$g = \frac{-\pi_p}{1-\alpha} \tag{7}$$

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[4] See the Appendix for the Hamiltonian derivation maximizing positive operating profits.

Finally, to ensure the stability of this path, the transversality condition requires that the discount rate exceeds the growth of the firm's value. Given the deflationary mechanism, this condition is:

$$r > g + \pi_p \tag{8}$$

This set of equations (4)-(8) describes the rational behavior of the monopolist. As shown in Appendix A, when aggregated across all sectors with knowledge spillovers, this micro-behavior translates into the general equilibrium growth rate determined by the social parameter ϕ.

It is then possible to show that from this, and combining the knowledge accumulation and capital production dynamics, the growth rate of final output evolves according to:[5]

$$\frac{\dot{Y}_f}{Y_f} = \delta R \left[\frac{e^{\delta R t} - \phi e^{\phi \delta R t}}{e^{\delta R t} - e^{\phi \delta R t}} \right] \tag{9}$$

As $t \to \infty$, since $\phi > 1$ (reflecting aggregate increasing returns), the term $e^{\phi \delta R t}$ dominates. Consequently, the output growth rate converges to:

$$g_Y = \lim_{t \to \infty} \frac{\dot{Y}_f}{Y_f} = \phi \delta R \tag{10}$$

This result implies that the long-run growth rate of the economy exceeds the rate of variety expansion (δR). The presence of diffusion externalities ($\phi > 1$) acts as a growth multiplier, magnifying the impact of innovation on aggregate output. Substituting the supplier's arbitrage condition ($\delta R = g = \frac{-\rho}{1-\alpha}$), the aggregate growth rate becomes[6]:

Substituting the supplier's equilibrium condition derived in the frictionless baseline ($\delta R = g = \frac{-\rho}{1-\alpha}$), the aggregate long-run growth rate becomes:

$$g_Y = \phi \left[\frac{-\rho}{1-\alpha} \right] \tag{11}$$

With the social parameter $\phi > 1$, the denominator is positive. Consequently, positive long-run growth is sustained by the rate of technological deflation (negative ρ), which continuously expands the effective real demand for new capital varieties. The term ϕ acts as a "social multiplier," converting the rate of variety adoption into a higher rate of aggregate output growth.

To incorporate demand-side financing constraints, we introduce an adopter-side financing wedge $\eta \geq 1$. We interpret $\eta = 1$ as the frictionless benchmark where the financing cost equals the risk-free rate, and $\eta > 1$ as a regime characterized by credit frictions, collateral constraints, or risk premia. Furthermore, we maintain the assumption that while capital suppliers optimize dynamically, adopters form myopic expectations: they maximize static profits period-by-period, ignoring future capital gains or losses. Consequently, the adopter's perceived user cost is simply the interest rate ($u_A = r$). The effective growth rate becomes:

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[5] See Appendix B for the full derivation.

[6] We note that while micro-level adoption of specific technologies often follows an S-shaped trajectory due to learning lags or adjustment costs, as modeled in (Anzoategui et al., 2019), our framework characterizes the aggregate balanced growth path. The diffusion rate g derived here represents the optimal steady-state speed of accumulation, effectively capturing the intensive margin of adoption that sustains long-run expansion.

$$g = \frac{u_S - \eta\, u_A}{1-\alpha}, \qquad u_S = r - \pi_p, \quad u_A = r, \quad \eta \geq 1. \tag{12}$$

3 Policy Implications

Our model highlights a policy channel that operates through the user cost of capital. The interest rate r plays a dual role[7]. From the capital-goods producer's viewpoint, a higher r increases the user cost u and therefore the real return on capital production, accelerating diffusion. From the adopter's side, however, a higher r raises financing costs.

Accordingly, the sensitivity of growth to the interest rate depends on the financial wedge:

$$\frac{\partial g}{\partial r} = \frac{1-\eta}{1-\alpha}. \tag{13}$$

The condition $\eta > 1$ is necessary for the existence of a credit channel. In a frictionless setting ($\eta = 1$), the adopter's cost and supplier's return move symmetrically with r, rendering the growth rate insensitive to interest rate changes. However, when financial frictions are present ($\eta > 1$), the marginal financing cost for adopters outweighs the intertemporal arbitrage benefit for suppliers. Consequently, monetary easing (lowering r) reduces the financing burden more than it dampens supply incentives, resulting in a net acceleration of diffusion. Moran and Queralto (2018) find that expansionary monetary shocks (lower r) lead to significant increases in adoption expenditure and TFP. In our model, this confirms that the adopter-side financing channel (characterized by $\eta > 1$) dominates the supplier's intertemporal arbitrage incentive. Our analytical result that monetary and credit conditions (η) drive the diffusion rate provides a transparent structural micro-foundation for the quantitative findings of Anzoategui et al. (2019), who attribute the post-2008 productivity slowdown to a freeze in adoption liquidity. Furthermore, capital goods price dynamics play a critical role. Since $\rho = \pi_p$, a decrease in capital goods prices (deflation, $\rho < 0$) increases the numerator ($r - \rho$), thereby boosting the growth rate. This suggests that structural factors that foster technological efficiency (i.e., lower the relative price of equipment) are potent drivers of long-run diffusion and growth.

The parameter ϕ governs the curvature of the aggregate productivity function. Economically, it captures the degree of complementarity and the strength of diffusion externalities across capital goods. For a given user cost, a higher ϕ (stronger social returns) reduces the denominator ($\phi - 1$), ostensibly amplifying the growth rate, though also moderating the responsiveness to policy changes.

4 Conclusion

This note has explored the mechanics of endogenous diffusion as a distinct engine of long-run growth. By explicitly modeling the interaction between a representative final-goods demander and monopolistic capital-goods suppliers, we have derived a growth trajectory

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[7] Since final output is the numeraire, r represents the real interest rate. We assume monetary policy influences this real rate through standard transmission channels.

that depends not only on the creation of new ideas but on the economic incentives governing their propagation.

A central feature of our framework is the interpretation of the curvature parameter ϕ. To reconcile self-sustaining growth with the requirements of competitive equilibrium, we interpret $\phi > 1$ as capturing aggregate complementarities—such as network effects and learning spillovers—rather than internal economies of scale. In this setup, individual firms perceive constant returns to scale, ensuring the validity of the profit-maximization problem. However, at the social level, the accumulation of capital varieties generates an external effect that sustains increasing returns ($\phi > 1$), thereby preventing the growth rate from diminishing to zero.

Our analysis yields two main policy implications. First, the diffusion speed is highly sensitive to the user cost of capital, defined by the interplay between financing conditions (r) and capital goods price dynamics (ρ). Policies that reduce financial frictions or foster technological deflation (lowering ρ) directly widen the arbitrage gap for suppliers, accelerating the equilibrium rate of adoption. Second, the parameter ϕ acts as a structural transmission mechanism: high aggregate complementarities imply that policy interventions can have magnified effects on long-run growth by leveraging the external returns of adoption.

Finally, the derivation highlights the consistency between monopolistic pricing and free entry. While suppliers optimize adoption paths to maximize operating profits via markups, the free-entry condition in the research sector ensures that, in equilibrium, the net present value of a new variety is driven to zero. This logic suggests that diffusion-based growth models can be seamlessly integrated with R&D-based frameworks, offering a more comprehensive view of how financing, capital-goods inflation, and structural externalities jointly shape an economy's long-run growth rate. Further extensions could consider delayed productivity gains from adoption. The model assumes immediate productivity gains, though in practice, technologies such as ICT and AI often exhibit delayed effects due to adjustment costs, learning-by-using, or institutional frictions. Incorporating these frictions would yield smoother diffusion trajectories that better align with empirical observations. Furthermore, the current framework assumes the infinite durability of capital goods. Future work could introduce physical depreciation (γ), which would modify the user cost to $u = r + \gamma - \pi_p$. While this would raise the effective cost of adoption, it could also generate replacement cycles that interact with the diffusion of new vintages, potentially altering the pace of aggregate modernization. Finally, extensions could consider heterogeneity in firms' adoption behavior or non-monopolistic capital goods markets. The framework also offers insights into cross-country heterogeneity, illustrating how differences in user-cost components and diffusion curvature can generate persistent gaps in adoption intensity and productivity growth, even among economies with similar innovation inputs or demographics.

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Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the author used Gemini (Google) to assist with LaTeX coding and to improve the readability and fluency of the manuscript. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication

References

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Baussola, M. (1999). *Technological change, diffusion and output growth.* PhD thesis, University of Warwick. Available at http://webcat.warwick.ac.uk/record=b1367925 S1.

Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.

Comin, D. and Hobijn, B. (2004). Cross-country technology adoption: making the theories face the facts. *Journal of monetary Economics*, 51(1):39–83.

Ireland, N. and Stoneman, P. (1986). Technological diffusion, expectations and welfare. *Oxford Economic Papers*, 38(2):283–304.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy*, 103(4):759–784.

Jovanovic, B. and Lach, S. (1993). Diffusion lags and aggregate fluctuations. new name: Product innovation and the business cycle.

Moran, P. and Queralto, A. (2018). Innovation, productivity, and monetary policy. *Journal of Monetary Economics*, 93:24–41.

Romer, P. M. (1990a). Capital, labor, and productivity. *Brookings papers on economic activity. Microeconomics*, 1990:337–367.

Romer, P. M. (1990b). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.

Appendix A: Derivation of the Optimal Diffusion Path

We adopt a two-step approach. First, we solve the monopolistic producer's optimization problem based on the private demand curve (ensuring micro-stability). Second, we impose the free-entry condition and aggregate the results to derive the social growth path.

Step 1: Private Demand and Monopoly Markup

The representative final goods producer maximizes profits (expressed in units of the numeraire final good) given a private production elasticity $\alpha < 1$. The Inverse Demand Curve faced by the monopolist for variety i is:

$$p(i) = \frac{\alpha\Gamma}{r} x(i)^{\alpha-1} \tag{A.1}$$

Since $\alpha < 1$, this demand curve is downward sloping.

Step 2: The Monopolist's Hamiltonian

The monopolist chooses the investment rate $z(i) = \dot{x}(i)$ to maximize the present value of *operating profits*. We assume the monopolist entering the market at time v charges a price $p(i)$ and faces a marginal production cost $c(t)$. The objective is:

$$\max_{z} \int_{\nu}^{\infty} [p(i) - c(t)] z(i) e^{-r(t-\nu)} \, dt \tag{A.2}$$

Using the Current Value Hamiltonian H_c with costate λ:

$$H_c = [p(i) - c(t)] z(i) + \lambda z(i) \tag{A.3}$$

Substituting the demand function $p(i)$:

$$H_c = \left[\frac{\alpha\Gamma}{r} x(i)^{\alpha-1} - c(t) + \lambda \right] z(i) \tag{A.4}$$

Because the Hamiltonian is linear in z, the singular control solution requires the marginal benefit of supply to equal the marginal cost. Optimizing with respect to price yields the standard monopoly markup rule:

$$p(i) = \frac{c(t)}{\alpha} \tag{A.5}$$

Since $\alpha < 1$, the markup $1/\alpha > 1$, ensuring positive operating profits $[p(i) - c(t)] > 0$.

The optimization is subject to the standard transversality condition:

$$\lim_{t\to\infty} e^{-r(t-v)} \lambda(t) x(i,t) = 0 \tag{A.6}$$

The transversality condition requires that the discount rate exceeds the growth rate of value. This implies $r > g + \pi_p$ (or equivalently, $r - \rho > g$), ensuring the convergence of the objective integral. Also, the transversality condition implies a limit on the social multiplier ϕ. While externalities ($\phi > 1$) are necessary to drive endogenous growth, they cannot be so strong that they cause the growth rate g to violate the condition $r > g + \pi_p$, which would render the economy unstable.

Robustness check

To demonstrate that our results hold under perfect foresight regarding price dynamics, we apply the integration by parts technique (Ireland and Stoneman, 1986). The monopolist maximizes the present value of profits from sales flow $z(t) = \dot{x}(t)$:

$$\Pi = \int_v^\infty [p(t) - c(t)]\dot{x}(t)e^{-r(t-v)}\, dt \tag{A.7}$$

Applying integration by parts to the revenue term $\int_v^\infty p(t)\dot{x}(t)e^{-r(t-v)}\, dt$:

$$\begin{aligned}\int_v^\infty p(t)\dot{x}(t)e^{-r(t-v)}\, dt &= \left[p(t)x(t)e^{-r(t-v)}\right]_v^\infty - \int_v^\infty x(t)[\dot{p}(t) - rp(t)]e^{-r(t-v)}\, dt && \text{(A.8)}\\ &= \int_v^\infty x(t)[rp(t) - \dot{p}(t)]e^{-r(t-v)}\, dt && \text{(A.9)}\end{aligned}$$

(assuming standard transversality conditions where $e^{-r(t-v)}p(t)x(t) \to 0$ as $t \to \infty$).

The term in brackets, $u(t) = rp(t) - \dot{p}(t)$, represents the **user cost of capital** (or rental price). The objective function can thus be rewritten as maximizing the stream of rental revenues from the installed stock $x(t)$ minus investment costs:

$$\Pi = \int_v^\infty \left[u(t)x(t) - c(t)z(t)\right] e^{-r(t-v)}\, dt \tag{A.10}$$

This transformation reveals that the demand constraint faced by the monopolist is governed by the user cost $u(t)$. Consequently, the static inverse demand derived under myopic expectations ($rp(i)$) is equivalent to the dynamic condition under perfect foresight, provided the price is interpreted as the user cost. This validates the use of $\rho = \pi_p$ in the growth equation. Conclusion: the monopolist is effectively maximizing the stream of rental income ($x \cdot u$) minus production costs. This confirms that the demand driver is indeed the user cost $r - \pi_p$, validating our focus on ρ.

Step 3: Micro-Founded Price Dynamics The dynamics of the model are rooted in the optimization problem of the individual monopolist. The firm faces a downward-sloping demand curve derived from the final goods production function, $y(i) = Ax(i)^\alpha$. Differentiating the inverse demand schedule $p(t) \propto x(t)^{\alpha-1}$ with respect to time yields the perceived price dynamics:

$$\pi_p = -(1-\alpha)g \tag{A.11}$$

Since $\alpha \in (0, 1)$, this confirms that for an individual firm, capital accumulation ($g > 0$) is always associated with a decline in the relative price ($\pi_p < 0$). This micro-foundation justifies the deflationary mechanism in Equation (4) of the main text.

Aggregate Output Dynamics While the equilibrium rate of capital accumulation g is determined by the interaction of firm-level incentives and price dynamics, the growth rate of final output g_Y reflects the social value of that accumulation.

Given the aggregate production function characterized by the elasticity ϕ (where $\phi > 1$ captures knowledge spillovers and variety effects), the growth rate of final output is a multiple of the rate of physical accumulation:

$$g_Y = \phi \cdot g \tag{A.12}$$

Substituting the equilibrium capital growth rate from Equation (7), we obtain the reduced-form solution for the economy's growth path:

$$g_Y = \phi \left(\frac{-\pi_p}{1-\alpha} \right) \tag{A.13}$$

This equation encapsulates the central mechanism of the model: technological deflation ($-\pi_p$) drives capital accumulation, which is then amplified by aggregate increasing returns (ϕ) to generate robust economic growth.

Step 5: Free Entry and Resource Allocation

The derivation above maximizes the operating profit of the monopolist. To close the model, we invoke the free entry condition, which governs the flow of resources between the intermediate capital goods sector and the research sector. Potential monopolists enter the intermediate sector by purchasing a new design from the research sector at price P_A. Equilibrium requires that this sunk entry cost must equal the discounted present value of the future stream of operating profits:

$$P_A = \int_v^\infty [p(i) - c(t)] z(i) e^{-r(t-v)} \, dt \tag{A.14}$$

This condition is symmetric: it determines the allocation of resources to R&D (equating the marginal cost of research to the price P_A) and simultaneously ensures zero long-run profits in the intermediate goods sector. It thus closes the general equilibrium system but does not alter the optimal diffusion path g derived from the Hamiltonian.

Appendix B: Derivation of Aggregate Output Growth

Using the transformation adopted in Jovanovic and Lach (1993), we express aggregate output in terms of capital vintage (ν). Given that the index is $i = e^{\delta R \nu}$, we have:

$$Y_f = g(L) \int_0^t \delta R e^{\delta R \nu} x(t,\nu)^\phi \, d\nu \tag{B.1}$$

Substituting the optimal supply trajectory $x(t,\nu) = e^{g(t-\nu)}$ where $g = \delta R$ (assuming balanced growth where diffusion speed matches innovation speed):

$$Y_f = g(L) \delta R \int_0^t e^{\delta R \nu} \left(e^{\delta R (t-\nu)} \right)^\phi d\nu \tag{B.2}$$

Rearranging terms to group by vintage ν:

$$Y_f = g(L)\delta R e^{\phi\delta Rt} \int_0^t e^{\delta R(1-\phi)\nu}\, d\nu \tag{B.3}$$

Solving the integral:

$$\int_0^t e^{\delta R(1-\phi)\nu}\, d\nu = \left[\frac{e^{\delta R(1-\phi)\nu}}{\delta R(1-\phi)}\right]_0^t = \frac{e^{\delta R(1-\phi)t} - 1}{\delta R(1-\phi)} \tag{B.4}$$

Substituting back into the expression for Y_f:

$$Y_f = \frac{g(L)}{\delta R(1-\phi)} \delta R e^{\phi\delta Rt} \left(e^{\delta R(1-\phi)t} - 1\right) \tag{B.5}$$

Simplifying:

$$Y_f = \frac{g(L)}{1-\phi} \left(e^{\phi\delta Rt} \cdot e^{\delta Rt} e^{-\phi\delta Rt} - e^{\phi\delta Rt}\right) \tag{B.6}$$

$$Y_f = \frac{g(L)}{1-\phi} \left(e^{\delta Rt} - e^{\phi\delta Rt}\right) = \frac{g(L)}{\phi - 1} \left(e^{\phi\delta Rt} - e^{\delta Rt}\right) \tag{B.7}$$

To find the growth rate, we differentiate Y_f with respect to t:

$$\dot{Y}_f = \frac{g(L)}{\phi - 1} \left(\phi\delta R e^{\phi\delta Rt} - \delta R e^{\delta Rt}\right) \tag{B.8}$$

Dividing $\dot{Y}_f$ by Y_f:

$$\frac{\dot{Y}_f}{Y_f} = \frac{\phi\delta R e^{\phi\delta Rt} - \delta R e^{\delta Rt}}{e^{\phi\delta Rt} - e^{\delta Rt}} = \delta R \left[\frac{\phi e^{\phi\delta Rt} - e^{\delta Rt}}{e^{\phi\delta Rt} - e^{\delta Rt}}\right] \tag{B.9}$$

Taking the limit as $t \to \infty$, and noting that with $\phi > 1$, the term $e^{\phi\delta Rt}$ dominates $e^{\delta Rt}$:

$$\lim_{t\to\infty} \frac{\dot{Y}_f}{Y_f} = \delta R \left(\frac{\phi e^{\phi\delta Rt}}{e^{\phi\delta Rt}}\right) = \phi\delta R \tag{B.10}$$

Thus, the economy converges to a Balanced Growth Path where aggregate output grows at rate $\phi\delta R$.