Risk aversion heterogeneity and the investment-uncertainty relationship

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Abstract A simple dynamic general-equilibrium model of savings and investment is populated by agents with Epstein-Zin preferences. Households are heterogeneous in their risk aversion, and trade riskless assets to share the aggregate risk. In equilibrium a higher volatility increases the certainty-equivalent future return for low-risk-averse individuals, who hold a long position in risky assets. The certainty-equivalent return may also increase for high-risk-averse agents, who hold safe assets. In response to a rise in certainty-equivalent expected returns, savings and investment decrease due to a limited willingness to substitute consumption over time. This generates a negative response of aggregate investment to an increase in systematic volatility.

Keywords: Aggregate investment; volatility; risk aversion; heterogeneity.
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1 Introduction

Agents populating an economy differ along many dimensions, and risk aversion is a trait for which heterogeneity is of paramount importance.

Barsky et al. (1997) document that the risk tolerance of agents responding to the Health and Retirement Study (HRS) varies depending on their age, gender, and educational attainments. Ethnic and religious factors also play a role, although a less prominent one. Kimball et al. (2009) find a significant dispersion in risk aversion for the survey responses in the Panel Study of Income Dynamics (PSID), uncovering also a covariance in risk attitudes among family members. Heterogeneity in preferences is also detected by studies based on experiments, such as von Gaudecker et al. (2011), who target a large set of Dutch households, and by analysis of financial market data, such as Chiappori and Paiella (2011), who focus on Italy.

Risk aversion heterogeneity provides a basis for studies focusing on income and wealth distributions. Coen-Pirani (2004) populates an endowment economy with two types of Epstein-Zin agents who differ only in the risk aversion parameter; the more-risk-averse agents choose a portfolio with a lower average return, but also with a lower variance, which can lead them to dominate the long-run distribution of wealth. More recently, Cozzi (2014) develops a model with heterogenous expected-utility maximizers who face uninsurable income shocks *à la* Krusell and Smith (1998); he finds that preference heterogeneity can account for the degree of wealth inequality observed in the U.S.[1]

Epstein-Zin preferences allow to disentangle risk aversion from intertemporal elasticity of substitution, and hence allow to better understand their respective roles. Therefore, we adopt such a class of non-recursive preference, presenting a model in which Epstein-Zin agents are heterogeneous in their attitudes toward risk.

We highlight the conditions on preferences such that an increase in volatility has negative effects on investment, as suggested by most empirical contributions, and we argue that these conditions are consistent with the available empirical estimates. This is interesting since the theoretical liter-

[1] Roche (2011) shows how time discount rate, risk aversion, and intertemporal substitution contribute to shaping capital accumulation decisions. In his model, the more effectively patient agents dominate the wealth distribution, and they may well be the ones characterized by the lowest risk aversion.

ature, with the notable exception of Angeletos (2007), can hardly explain a negative 'investment-uncertainty' relation.

The tension between the theoretical literature on competitive firms' investment and the empirical reaction of investment to a volatility shock originates in the flexibility effect. As highlighted by Hartman (1972) and Abel (1983), a firm which incurs an installation cost when deciding upon its capital level, but which is able to freely adjust labour after the realization of the shock, can exploit the asymmetry in the timing of decisions. In fact, when the marginal cost is low, the firm can increase production by adjusting the flexible inputs, while output is reduced when the cost is high. Therefore, the firm's marginal revenue of capital is convex in the size of the productivity shock, so that a mean-preserving spread in productivity increases the expected marginal revenue of capital, and therefore the (risk-neutral) firm's investment activity.

Here, we develop a general equilibrium framework populated by individuals whose labour supply is elastic, and we focus on the case of systematic risk, which is appropriate when financial markets are fully operational. The former choice is dictated by the desire to keep the flexibility effect completely operational. In fact such effect is at play in full in our economy, since firms can modify the usage of labour without inducing changes in the real wage. Accordingly, the expected return to capital is increasing in the volatility of the shock. In our economy, as in Coen-Pirani (2004), the presence of mature financial markets allows investors characterized by different degrees of relative risk aversion to trade safe bonds. The low risk-averse households take a long position in the risky asset, while the high-risk averse families own riskless bonds. These portfolio effects are of crucial importance for our results: they imply that an higher aggregate volatility increases not only the expected return, but also the certainty-equivalent return for most investors. When the elasticity of intertemporal substitution is below unity, investors' incentive to smooth utility over time implies that a higher certainty-equivalent capital yield in the future induces an increase in current consumption, and hence a reduction in investment.

In the homogeneous agent version of this framework, which is close to the one that has been studied by Saltari and Ticchi (2007), an higher aggregate volatility can easily *reduce* the certainty-equivalent capital income, which is what Saltari and Ticchi label the risk aversion effect. Accordingly, for values of risk aversion close to the currently available estimates, and for an intertemporal substitution below unity, the reduction in perceived future returns creates an incentive to shrink current consumption to smooth utility over time, which, in their model, leads to a larger investment.

Notice that our framework offers elements of interest that go beyond the specific point we make.

From the methodological perspective, we provide a neat illustration of the fact that the aggregate behavior of an heterogeneous agents economy can be different from the behavior originating from a representative agent characterized by preference parameters that corresponds to those of the 'average' heterogeneous agent. Moreover, to stylize aging or the arrival of new family members, the risk aversion of each household changes stochastically over time. This effect is exploited to guarantee the existence of a non degenerate ergodic wealth distribution, and may help keeping manageable the dynamics of heterogeneous agents macro-models dealing with financial issues.

Indeed, Epstein-Zin preferences have been extensively used in recent research in macroeconomics and finance.

Bansal and Yaron (2004) are able to explain both the equity premium and the risk-free rate puzzles, adding to a standard endowment model some persistency in the growth rate for dividends, and considering a time-varying volatility. In fact, with non-expected utility, agents care about the variance of future utility, and hence about the variance of future wealth, so that future uncertainty plays a remarkable role in raising the current equity premium. Bansal and Yaron's (2004) results have been extended to a production economy by Croce (2014). He considers persistence in productivity growth, which involves an element of 'long-run risk', finding that some convex capital adjustment costs are important for the explanation of the puzzles. The framework we propose poses a challenge to this literature. In fact, in our model the aggregate risk is born by the individuals who can stand it better, which may weaken the long-run risk channel. Notice that the effects of the heterogeneity in risk aversion are limited by portfolio constraints, an element that should be taken into account in analyzing this issue.[2]

[2] A number of paper recognizes the importance of portfolio constraints in heterogeneous agents models. In an early paper Coen-Pirani (2005) studies margin requirements in an economy with Epstein–Zin investors with unit elasticity of intertemporal substitution. He finds that portfolio constraints do not affect stock returns because income and

The existence of an highly correlated long-run component in output allows Colacito and Croce (2013) to simultaneously account for the tendency of high interest rate countries to appreciate, and for the disconnect between exchange rates and consumption growth differentials. In fact, a positive long-run news, while increasing the domestic interest rate, also calls for a stronger demand for foreign assets, which induces depreciation. According to Kollman (2016) this framework generates realistic exchange rate volatility when all agents trade in complete markets, but at the price of large movements in countries' net foreign asset positions. Instead, when only a fraction of households has access to markets, the model generates a realistic volatility both of real exchange rate and of net foreign wealth. Populating the model with heterogeneous agents may constitute an alternative to the presence of 'hand-to-mouth' consumers, which is a distinctive feature of Kollman's model. The potential importance of risk aversion heterogeneity has recently be recognized by Stathopoulos (2017). He shows how a time-varying heterogeneity in risk aversion allows to obtain an high international correlation of asset prices with a modest cross-country correlation of consumption growth, thereby providing an explanation complementing the one proposed for this puzzle by Colacito and Croce (2011). Our framework might prove useful in analyzing this issue without invoking the high degree of risk aversion that can be induced by habit formation preferences.

Our model is solved by means of numerical techniques; following the pertinent literature, we compute how aggregate investment reacts to an increase in the variance characterizing the productivity process. We proxy 'uncertainty' matching the standard deviation of the return on capital with the stock market volatility, and assuming that this standard deviation follows a Markov process. We parameterize the average risk aversion and its degree of heterogeneity across agents consistently with the empirical literature, and we find that an elasticity of intertemporal substitution below unity implies a negative investment-uncertainty relation. Moreover, we consider the effects of portfolio constraints framed in line with those prescribed by U.S. regulations.

The remainder of the paper is organized as follows. After a brief discussion of the literature on investment under uncertainty, Section 2 describes the model and characterizes the recursive

substitution effects cancel each other when households have log-intertemporal preferences. More recently, Fostel and Geanakoplos (2008), and Chabakauri (2013) study the impact of margin requirements in economies characterized by various types of preference heterogeneity.

equilibrium, and Section 3 considers the household maximization problem. Section 4 solves the model numerically and presents the simulation results, while Section 5 draws the conclusions. The Appendixes contain the more technical material.

1.1 Related literature

While in the early 1990s, as reported by Carruth *et al.* (2000), the contributions based on aggregate data did not find a statistically significant negative investment-uncertainty relation, more recent macro- and microeconometric analysis has tended to be conclusive. In particular, most of the contributions based on micro data point to a negative investment-uncertainty relationship. Leahy and Whited (1996) find a negative correlation between investment and the firm's returns variance. Robust evidence is provided by Guiso and Parigi (1999), who find that an increase in volatility has negative effects that are stronger for firms with a relevant market power or for those that are characterized by high irreversibilities. When both the market power and the degree of irreversibility are low, the investment-volatility relationship is still negative, although less significant. However, according to Ghosal and Loungani (2000), the negative impact of profit volatility on investment is substantially greater for U.S. industries composed of small firms. Bloom *et al.* (2007) quantify uncertainty following Leahy and Whited (1996) and find that a significant negative effect on investment is obtained when considering the interaction between volatility in stock returns and sales. An increase in volatility negatively affects investment as well, while the level of past volatility is less significant. Panousi and Papanikolaou (2012) find that idiosyncratic volatility at the firm level induces a reduction in investment in U.S., which is due to managerial risk aversion. However, they also find an effect, although barely significant, of systematic volatility on investment.

The empirical evidence can hardly be interpreted by the literature building upon the competitive models by Hartman (1972) and Abel (1983). When the elasticity of demand is finite – as in Caballero (1991) – a firm increasing output in response to a positive productivity shock brings down its goods price, which weakens the flexibility effect. Caballero uncovers a negative investment-uncertainty relation when couples such a weakened effect with asymmetries in the adjustment costs. In fact, to reduce the risk of suffering high resale costs, firms shrink their investment when uncertainty increases.

Many contributions incorporating various types of irreversibilities have found a negative effect of risk on investment. In a volatile environment, if conditions worsen, the irreversibility constraint may become binding, which generates a "value-of-waiting". When business conditions become more risky, this effect becomes stronger. Chirinko and Schaller (2009) identify an 'irreversibility premium' in the discount rate used by a panel of U.S. firms. However, Abel and Eberly (1999) identify the 'hangover effect', according to which investment irreversibility prevents the firm from selling capital when the marginal revenue product of capital is low, so that the average long-run capital stock is higher than in the case with full reversibility. Higher uncertainty reinforces both the value-of-waiting effect and the hangover effect, so that no unambiguous result can be obtained. When investment, in addition to being irreversible, is also indivisible, the-value-of waiting is higher. In fact, not only the irreversibility constraint may become binding if conditions worsen, but also a firm – sinking an investment project – loses the possibility of investing in the future in a more favorable situation. Nevertheless, the effects of returns volatility on investment are not clear cut: a riskier project may attain larger values sooner, which prompts for an earlier investment, as suggested by Sarkar (2000).

To explain the negative relationship between investment and risk in competitive markets, several contributions have considered the role of risk aversion in general-equilibrium frameworks. Craine (1989) and Zeira (1990) concentrate on the effect of risk on the reallocation of savings and investment across sectors, but in their frameworks aggregate savings (and investment) are a constant fraction of output, and therefore their papers are not focused on the effects of aggregate volatility on aggregate investment.[3] In Saltari and Ticchi (2007) a risk aversion effect is at play: individuals take their decisions considering the certainty-equivalent return on capital and not the return itself, and hence the increase in future returns may actually involve a reduction in the certainty-equivalent return, which creates – when the elasticity of intertemporal substitution is lower than 1 – an incentive to shrink current consumption to smooth the utility levels over time.

Saltari and Ticchi's (2007) general equilibrium approach is shared by Angeletos (2007), who

[3] In Craine's setup this is an outcome of the log-preference specification, while in Zeira's overlapping-generations model, it is assumed that each member of the young generation saves entirely the real wage she gets.

presents a model of idiosyncratic capital income risk. He finds that, in absence of risk-sharing, capital accumulation suffers from an increase in volatility. This is due to the fact that risk averse agents value more their (riskless) human wealth as the capital income risk increases, and this 'wealth effect' reduces savings. In Angeletos' model, labour supply is rigid, which dampens the Hartman-Abel effect, because, at the aggregate level, the increase in labour demand fostered by an increase in the marginal revenue of capital partly resolves into an increase in wage.[4] Hence, he consider a case which is polar to the one we analyze, so that our contribution can be seen as a complement to his own.

2 The model

This section describes a discrete-time dynamic general-equilibrium model in which investors – who are heterogeneous in their degree of risk aversion – trade stocks and riskless assets in order to share the aggregate risk.

2.1 The technology

Competitive firms operate a constant-returns-to-scale Cobb Douglas production function, $Y_t = A_t K_t^{1-\alpha} L_t^{\alpha}$, in which Y_t is output at time t, K_t is the stock of capital, L_t is the labour employed, and A_t denotes productivity. The evolution of productivity, for a given standard deviation, is modeled as a first-order Markov process; we let A_k^v indicate that productivity is at level k while the standard deviation of the process governing productivity is v, and $\pi_{kz} = \Pr(A_{t+1} = A_z^v | A_t = A_k^v)$ be the probability that productivity takes state z at time $t+1$, conditional on being at level k at time t. Following e.g. Chen (2010), we assume that also the standard deviation for productivity follows a Markov process, with transition probabilities denoted by $\nu_{lh} = \Pr(v_{t+1} = \sigma_h | v_t = \sigma_l)$.[5]

[4] In response to a spread in productivity, labour moves from less to more efficient firms, which increases the average revenues.

[5] Many papers in the pertinent literature – such as Craine (1989), Zeira (1990), and Saltari and Ticchi (2007) – assume i.i.d. productivity shocks. In Sub-section 4.4 we shall discuss what happens in our model when the persistency of the productivity shock is reduced.

2.2 Preferences

The economy is populated by a continuum of infinitely-lived households, who derive utility from consumption and supply labour, thus incurring a disutility cost. Every household is characterized by a Kreps-Porteus non-expected utility, modeled as in Epstein and Zin (1989). This specification allows for the independent parameterization of attitudes toward risk and intertemporal substitutability.

The households populating our model share the same subjective discount factor under certainty, β, and the same elasticity of intertemporal substitution, $1/R$, but differ in the coefficient of risk aversion for timeless lotteries. In fact there exist J types of families characterized by different relative risk aversion coefficients, γ_j. With no loss of generality, let $\gamma_j < \gamma_{j+1}$, for $j = 1, 2, ..., J-1$. The risk aversion of each family may change over time, due to aging or to the arrival of new family members. This effect is modeled with the assumption that, in every period, there is a constant probability of transition from group j to group s, denoted by λ_{js}. Obviously, $\sum_{s=1}^{J} \lambda_{js} = 1$, for $j = 1, 2, ..., J$. The share of agents that in period t belongs to group j is denoted by $z_{j,t}$, with $\sum_{s=1}^{J} z_{s,t} = 1$. The possibility of a change in risk aversion for our infinitely lived families is considered not only because it is realistic, but also because the wealth distribution, when risk aversion is heterogeneous, may well be dominated in the long run by one group of agents.

We denote by $U_{j,t}$ the utility from time t onward of an household characterized by risk aversion γ_j. This is defined recursively as:

$$U_{j,t} = \left\{ (1-\beta)(C_{j,t} - \phi L_{j,t})^{1-R} + \beta \left(\sum_{s=1}^{J} \lambda_{js} \left(E_t \left[U_{s,t+1} \right]^{1-\gamma_s} \right)^{\frac{1}{1-\gamma_s}} \right)^{1-R} \right\}^{\frac{1}{1-R}} \tag{1}$$

in which $C_{j,t}$ is consumption, $L_{j,t}$ is labour supplied by an household of type j, ϕ is the constant opportunity cost of supplying labour, and $E_t[.]$ denotes the conditional expectation operator. Notice that the homotheticity of the Kreps-Porteus preferences – together with the linearity of the budget constraint – allows for the use of a representative household for each type; notice also that the amount of labour directly affects the utility the representative agent receive from consumption, in the spirit of Greenwood *et al.* (1988). This specification – which is the same as that adopted by

Saltari and Ticchi (2007) – is analytically convenient because it implies that, in equilibrium, the real wage is constant, being equal to the marginal disutility of labour. The elasticity in the labour supply guarantees that our results are obtained in a settings in which the flexibility effect is fully operational.

2.3 Markets and assets

In each period, there exist spot markets for output, labour, equities, and for riskless bonds. Output is taken as the numeraire, and it is traded on a competitive market; the (nominal and real) wage w_t is determined on the labour market, which is competitive as well. Risky assets entitle holders to the ownership of capital, so that the price of one equity is identical to the price of one unit of capital, and equal to 1; each equity hold at the beginning of period t carries the right to a dividend equal to D_t. The variables that affects dividends, the riskless rate r_t, and the wage w_t , will be specified at a later stage. The differences in the degree of risk aversion enrich the possibility of financial trading among agents. In fact, the more risk averse families can purchase riskless bonds issued by the less risk averse agents. The issuing of riskless bonds by the less-risk-averse families, who want to increase their equity holdings, stylizes the obtaining of credit from banks or from other financial institutions, and also the purchase of equities issued by levered firms. Riskless assets trade for one period: a bond issued in period t, which is paid $x_t \equiv 1/(1+r_t)$, entitles to one units of output in period $t+1$. Notice that riskless bonds are in zero net supply.

2.4 Budget constraints

The one-period budget constraint facing an household of type j is simply

$$K_{j,t+1} + x_t B_{j,t+1} + C_{j,t} = (1+D_t) K_{j,t} + B_{j,t} + w_t L_{j,t}. \tag{2}$$

The homotheticity of the Kreps-Porteus utility implies that stock and bond holdings, for any given interest rate and any given capital stock, are proportional to individual wealth, so that, defining $W_{j,t} \equiv (1+D_t) K_{j,t} + B_{j,t}$, and $q_{j,t}$ as the fraction of $W_{j,t}$ that is invested in risky assets and yields the return D_t, we have that:

$$K_{j,t+1} = q_{j,t} \left(W_{j,t} + w_t L_{j,t} - C_{j,t} \right),$$

and

$$x_t B_{j,t+1} = \left(1 - q_{j,t}\right) \left(W_{j,t} + w_t L_{j,t} - C_{j,t} \right).$$

Combining the two expressions above with (2), and exploiting the definition for $W_{j,t}$ we obtain

$$W_{j,t+1} = \left(\left(1 + D_{t+1}\right) q_{j,t} + \frac{1 - q_{j,t}}{x_t} \right) \left(W_{j,t} + w_t L_{j,t} - C_{j,t} \right), \tag{3}$$

while the aggregate budget constraint is:

$$\sum_{j=1}^{J} z_{j,t} W_{j,t+1} = \left(1 + D_{t+1}\right) \sum_{j=1}^{J} z_{j,t} q_{j,t} \left(W_{j,t} + w L_{j,t} - C_{j,t} \right) + \sum_{j=1}^{J} z_{j,t} \frac{1 - q_{j,t}}{x_t} \left(W_{j,t} + w_t L_{j,t} - C_{j,t} \right).$$

2.5 The timing of decisions

At the beginning of period t firms decide how much capital to use. Then, the productivity shock materializes, and agents perceive the change in volatility should this take place. Having observed the realization of the shock, firms adjust the amount of labour employed, and households decide upon their period t consumption level; finally changes in agents' risk aversion occurs, and households, choosing $q_{j,t}$, allocate their portfolio for period $t + 1$.

[Figure 1 about here]

Figure 1 shows the time line. The assumption that each firm can adjust the amount of labour employed within the period is standard in the literature. Many contributions, such as Hartman (1972), Abel (1983), Caballero (1991), and Lee and Shin (2000), assume that the firm chooses its capital level after the realization of the shock, incurring an installation cost that induces the convexity of the firm's marginal revenue. Instead, we assume that capital must be decided upon in advance without paying an adjustment cost. Our assumption can be interpreted as depicting a situation in which installation costs rapidly increase with the compression of the time span during which the investment is implemented: when investment is planned in advance, it involves negligible adjustment costs; on the contrary, a late implementation brings about prohibitively high expenses. While a short installation period actually increases the adjustment costs, we underscore that our assumption, besides being analytically convenient, preserves the convexity of the firm's marginal revenue, which is the crucial ingredient for the Hartman-Abel flexibility effect. [6]

The hypothesis according to which the period $t+1$ variance for productivity is perceived during period t is introduced for two reasons. First, we believe reasonable that a regime switch – which may be associated to the introduction of new technologies, to shocks in the availability of natural resources, or possibly to major political events – takes some time to deploy its effect on productivity. Second, such an assumption makes our model comparable with those presented in the relevant literature, which analyzes the reactions to an increase in future volatility. Notice, finally, that the change in risk aversion occurs at the end of each period, so that each household's portfolio decisions is based on her own risk aversion index.[7]

2.6 Recursive equilibrium

Each household decides upon the share of risky assets at the end of the period, when all the information, including the next-period risk aversion have been disclosed. Consumption levels are chosen upon, and labour supply decisions are taken, when only productivity and next-period volatility are known. Accordingly, the utility-maximization problem must be solved in two stages.

At the end of period t agents solve their portfolio problem, which amounts to obtain

$$\hat{q}_{s,t}(A_t, \Gamma_t, \sigma_{t+1}) = \arg\max_{q_{s,t}} E\left[V_s(W_{s,t+1}, \Gamma_{t+1}, A_{t+1}, \sigma_{t+1}))^{1-\gamma_s}\,\middle|\, A_t, \Gamma_t, \sigma_{t+1}, \gamma_{t+1} = \gamma_s\right]^{\frac{1}{1-\gamma_s}}, \quad (4)$$

[6] The absence of adjustment cost, coupled with the preferences specified in (1), implies that the aggregate intertemporal budget constraint is linear-in-capital (refer to Section 3), so that the adjustment following a shock lasts for only one period. This non-realistic implication seems inconsequential for the issue which is analyzed in this paper.

[7] **The fact that each portfolio decision is based on one risk aversion index speeds up the convergence of the routine. This choice does not alter in a significant way our results, given that the probabilities of a change in risk aversion are very low.**

in which $V_s(W_{s,t}, \Gamma_t, A_t, \sigma_t)$ denotes the period t maximum value for an households of type s, and Γ_t is the $\{\Gamma_{j,t}\}$ vector, $\Gamma_{j,t} \in [0,1]$ being the share of aggregate wealth held by a type j households (i.e., $\Gamma_{j,t} \equiv W_{j,t}/W_t$).

For an household belonging to group j at time t and to group s at time $t+1$, the relevant constraint is

$$W_{s,t+1} = \rho(D_{t+1}, r_t, q_{s,t}) \left(W_{j,t} + w_t L_{j,t} - C_{j,t}\right), \tag{5}$$

in which we have defined for compactness the portfolio return as

$$\rho(D_{t+1}, r_t, q_{s,t}) \equiv (1 + D_{t+1})\, q_{s,t} + (1 + r_t)\,(1 - q_{s,t}).$$

Households take into account the evolution for Γ_t, which is given by

$$\Gamma_{m,t+1} \left(\equiv \frac{W_{m,t+1}}{W_{t+1}}\right) = \rho(D_{t+1}, r_t, q_{m,t}) \frac{W_t}{W_{t+1}} \frac{(W_{i,t} + w_t L_{i,t} - C_{i,t})}{W_t}, \text{ for } i, m = 1, 2, ..., J. \tag{6}$$

When the labour and consumption decision needs to be taken, the intertemporal utility problem is:

$$V_j(W_{j,t}, \Gamma_t, A_t, \sigma_t) = \max_{\{L_{j,t}, C_{j,t}\}} \left\{(1-\beta)(C_{j,t} - \phi L_{j,t})^{1-R}\right.,$$
$$\left. + \beta \left(\sum_{s=1}^{J} \lambda_{js} \left[E\left[V_s(W_{s,t+1}, \Gamma_{t+1}, A_{t+1}, \sigma_{t+1}))^{1-\gamma_s} \right| A_t, \Gamma_t, \sigma_{t+1}, \gamma_{t+1} = \gamma_s \right]^{\frac{1}{1-\gamma_s}} \right]^{1-R} \right) \right\}^{\frac{1}{1-R}}. \tag{7}$$

Firms finance their capital stock by issuing equities, each of which therefore represents one unit of capital, and carries the right to obtain the payment of a share of the firms' cash-flow. The representative firm's maximizes the discounted stream of dividends it pays out; the absence of adjustment costs leads to a sequence of static problems, so that the firm actually targets its cash flow, which, for each period i, resolves into $K_{t+i}D_{t+i}$. Bearing in mind that capital must be chosen at the beginning of each period, while labour is decided upon after the realization of the productivity shock, the period t firm's problem is therefore formulated as:

$$\max_{K_t} E\left[\max_{L_t} K_t D_t \middle| A_{t-1}, \sigma_t\right] = \max_{K_t} \; E\left[\max_{L_t} A_t K_t^{1-\alpha} L_t^{\alpha} - w_t L_t - \delta K_t \middle| A_{t-1}, \sigma_t\right], \tag{8}$$

where δ is the constant depreciation parameter, and w_t is wage.

We now define the recursive equilibrium for this economy.

Definition 1 *The recursive equilibrium is represented by:*

- *the households' value functions* $\{V_j(W_{j,t}, \Gamma_t, A_t, \sigma_t)\}$, *for* $j = 1, 2, ...J$,
- *the households' decision rules* $\{C_{j,t}, L_{j,t}, q_{j,t}\}$, *for* $j = 1, 2, ...J$,
- *the representative firm's decision rules* $\{K_t, L_t\}$,
- *the law of motion (6) for* $j, s = 1, 2, ...J$,
- *the interest rate* r_t, *and the wage rate* w_t,

such that

- *the value functions* $\{V_j(W_{j,t}, \Gamma_t, A_t, \sigma_t)\}$, *and decision rules* $\{C_{j,t}, L_{j,t}, q_{j,t}\}$, *for* $j = 1, 2, ...J$, *solve problems (7) and (4) under constraint (5), given the interest rate* r_t, *the wage rate* w_t, *the law of motion (6), the transition probabilities* $[\lambda_{js}]$, *and the period* t *distribution for types* $[z_{j,t}]$, *for* $j, s = 1, 2, ...J$;
- *the decision rules* $\{K_t, L_t\}$ *solve problem (8);*

- *the markets for goods, labour, riskless assets, and equities clear:*

$$Y_t = \sum_{j=1}^{J} z_{j,t}[C_{j,t} + I_{j,t}],$$
$$L_t = \sum_{j=1}^{J} z_{j,t} L_{j,t},$$
$$0 = \sum_{j=1}^{J} z_{j,t}(1 - q_{j,t}) W_{j,t},$$
$$K_t = \sum_{j=1}^{J} z_{j,t} q_{j,t} W_{j,t}.$$

- *The law of motion (6) is consistent with the equilibrium for $j, s = 1, 2, ...J$.*

The following two sections are devoted to the characterization of the equilibrium.

3 Individual optimization problems

This section describes the firms' and the households' optimization problems for given pricing functions, and for a given evolution of the wealth distribution.

3.1 Firms

Once the firm has decided upon the capital stock, and it has observed the realization of the productivity shock, its problem is:

$$\max_{L_t} A_t K_t^{1-\alpha} L_t^{\alpha} - w_t L_t - \delta K_t,$$

The optimal amount of labour is given by $\hat{L}_t = ((\alpha/w_t)A_t)^{\eta} K_t$, where $\eta \equiv 1/(1-\alpha)$. Notice that $\eta \in (1, \infty)$ since $\alpha \in (0, 1)$. Having chosen $L_t = \hat{L}_t$, the firm's cash flow becomes: $\xi_t A_t^{\eta} K_t - \delta K_t$, where $\xi_t = (1-\alpha)\left(\frac{\alpha}{w_t}\right)^{\alpha\eta}$. Defining the marginal revenue product of capital as

$$MR(w_t, A_t) \equiv \xi_t A_t^{\eta}, \tag{9}$$

the beginning-of-period firm's problem can be formulated as:

$$\max_{K_t} E\left[\left(MR(w_t, A_t) - \delta\right) K_t \middle| A_{t-1}, \sigma_t\right].$$

Accordingly, the amount of capital operated by firms is determined by the supply provided by households. Given that $\eta > 1$, the profit function is convex in the random variable as in Abel (1983), Hartman (1972), and many others. Notice that the dividend per unit of capital is given by the marginal revenue product of the latter, net of depreciation, so that, letting $MR(w_t, A_t) \equiv MR_t$, we have that $D(MR_t) = MR_t - \delta$. Notice also that the higher is the labour income share α, the larger is also η, and hence the stronger is the Hartman-Abel flexibility effect.

3.2 Households

We consider the individual agent's maximization problem for a given interest rate function and for a given evolution of the wealth distribution.

The independence of the return on equities from the capital stock implies that the riskless rate is independent from aggregate capital as well, which simplifies the solution of the household intertemporal problem. Denoting by $r_t(\Gamma_t)$ the riskless rate, the portfolio return can be written as

$$\rho(MR_{t+1}, \Gamma_t, q_{s,t}) \equiv (1 + D(MR_{t+1}))q_{s,t} + (1 + r_t(\Gamma_t))(1 - q_{s,t}), \text{ for } s = 1, 2, ..., J,$$

so that the budget constraint (5) becomes

$$W_{s,t+1} = \rho(MR_{t+1}, \Gamma_t, q_{s,t}) \left(W_{j,t} + w_t L_{j,t} - C_{j,t}\right), \text{ for } j, s = 1, 2, ..., J. \tag{10}$$

Accordingly, households solve problems (7) under constraint (10). In Appendix A we provide the details of the solution, showing that consumption and labour supply are proportional to wealth, and that the real wage is constant, being equal to the marginal disutility of labour. Notice that, when w_t is constant, so it is ξ_t. Accordingly, Equation (9) becomes $MR(A_t) \equiv \xi A_t^{\eta}$, and the only source of variability for the marginal revenue of capital is the stochastic productivity A_t. Accordingly, also the dividend depend only on A_t.

In the Appendix, we verify that the value function $V_j(W_{j,t}, \Gamma_t, A_t, \sigma_t)$ is, for each type j,

$$V_j(W_{j,t}, \Gamma_t, A_t, \sigma_t) = \psi_j(\Gamma_t, A_t, \sigma_t) W_{j,t}. \tag{11}$$

To complete the description of the value function $V_j(W_{j,t}, \Gamma_{t,}, A_t, \sigma_t)$ it is necessary to characterize the individual portfolio choices, which – as detailed in Appendix A – affect the maximum value through wealth accumulation. Hence, we prove

Proposition 2 *1*

Denote by $\underline{MR}$ the marginal revenue of capital corresponding to the lowest realization for productivity. If $r(\Gamma_t) < E\left[MR_t | A_{t-1}, \sigma_t\right] - \delta$, the optimal portfolio choice $\hat{q}_{j,t}(A_t, \Gamma_t, \sigma_{t+1})$ is unique and $\hat{q}_{j,t}(A_t, \Gamma_t, \sigma_{t+1}) \in \left[0, \frac{1+r(\Gamma_t)}{\delta + r(\Gamma_t) - \underline{MR}}\right)$ for $j = 1, 2, ...J$.

Proof. Please refer to Appendix B.

The assumption $r(\Gamma_t) < E\left[MR_t | A_{t-1}, \sigma_t\right] - \delta$ simply requires that the endogenous riskless rate be lower than the expected returns on risky assets; in an economy populated by risk averse agents, this assumption is always satisfied in equilibrium. Taking a short position in riskless assets to finance her investment in equities, an agent may expose herself to the risk of obtaining a negative wealth level, which happens when $\rho(MR_{t+1}, \Gamma_t, q_{j,t}) < 0$. As wealth approaches zero, its marginal utility tends to infinity even for the least risk-averse agents populating the economy. The upper bound for the optimal portfolio shares $\hat{q}_{j,t}$ in Proposition (1) – $(1 + r(\Gamma_t)) / (\delta + r(\Gamma_t) - \underline{MR})$ – is the maximum value guaranteeing a finite expected marginal utility of wealth.

4 Simulation results

To compute the effect of an increase in volatility on investment, it is necessary to solve the households' and firms' problems guaranteeing the consistency of the individual law of motions with the equilibrium, which can be done only numerically. Hence, we first illustrate the parameters values that we use, we then underscore the effects characterizing a homogeneous-agent version of our economy to help developing some intuition for the heterogeneous-agent framework. Finally, the model is solved using a policy function iteration procedure[8] and the results of several simulation exercises are offered.

4.1 Benchmark parameterization

The model parameters are chosen to stylize the U.S. economy. Table 1 presents our baseline parameterization, in Column (A), and the various alternative that we consider.

[Table 1 around here]

Since our model is calibrated using a time period equal to one year, we let $\beta = 0.96$. Reasonable variations in the value of the time discount factor do not significantly affect our results.

As recently restated by Havránek (2015) the elasticity of intertemporal substitution is likely to be below unity;[9] hence we fix its reciprocal, R, at 2. It is important to underscore that $R = 1$ is a threshold discriminating the sign of the reaction of investment to a change in uncertainty. As long as $R > 1$, its value does not qualitatively affect our results, while some quantitative implications will be discussed later.[10] We test in Sub-Section 4.4.1 the robustness of our results for the case of an elasticity of intertemporal substitution higher than 1.

The coefficient of relative risk aversion and its dispersion – affecting portfolio choices – play a key role. Contributions such as Barsky *et al.*'s (1997), Halek and Eisenhauer (2001), Kimball *et al.* (2008, 2009) provide guidance in choosing these parameters.[11] Barsky *et al.* (1997) analyze the households from the University of Michigan HRS of 1992, estimating a mean of risk aversion close to 12. Taking into account the status quo bias that may affect Barsky *et al.*'s results, Kimball

[8] In all the exercises, we compute the aggregate investment for each productivity level, each volatility level, and for all the possible wealth distributions (i.e., we consider $\Gamma_{t,1} \in [0,1]$, $\Gamma_{t,2} \in [0,1]$, with $\Gamma_{t,1} + \Gamma_{t,2} \leq 1$). The numerical algorithm is briefly described in Appendix D.

[9] Using metadata, Havránek (2015) concludes that the empirical studies at the micro level point to an average elasticity of intertemporal substitution as low as 0.2, while, for asset holders, the mean estimate reaches 0.4. His results also suggest that the empirical literature does not support calibrations greater than 0.8, an upper bound obtained for the asset holders' elasticity of intertemporal substitution.

[10] Guvenen (2009), considers the case of heterogeneity in intertemporal substitution in a macroeconomic model that is consistent with the key features of asset prices behavior. With heterogeneous elasticity of intertemporal substitution, our result would still be valid, as long as it is lower than unity for all agents.

[11] Some recent studies exploit financial market data to assess the RRA index, as well as to highlight that it varies across investors. Chiappori and Paiella (2011) focus on Italian families holding risky financial assets (which means about one half of the total). According to their estimates, the mean risk aversion is 4.2, with the median being 1.7, which suggests the presence of a significant heterogeneity in risk-aversion. An issue with this literature is that it is reasonable to think that the households that do not hold risky assets are more risk averse than those participating in the stock market.

et al. (2008) estimated an average risk aversion of 8.2, with a standard deviation of 6.8. Because risk aversion tends to increase with age – and the individuals in the HRS sample were aged 51 to 61 years old – Kimball *et al.* (2009) analyze the participants in the PSID 1996, which comprised ages 20 to 69. Assuming that the relative risk aversion is lognormally distributed, they obtained an average relative risk aversion of 4.2, a value significantly lower than the ones obtained using the HRS sample. Our benchmark parameterization is based on this results. In particular, to limit the number of state variables in the system, our numerical analysis considers three types of agents. Using three Gauss-Hermite quadrature points, we approximate the Kimball et al. (2009) log-normal distribution for relative risk aversion with the vector $\gamma = [0.64,\ 2.86,\ 12.94]$. The standard deviation is 4, which is in line with previous studies. Notice that distribution of types associated with such a vector of risk aversion parameters are: $[0.1\bar{6},\ 0.6\bar{6},\ 0.1\bar{6}]$. When considering the possibility of type changes, we calibrate the probabilities of change so that the long-run stationary distribution of types matches the distribution above. In Sub-section 4.4 we present some sensitivity analysis focused on the elasticity of intertemporal substitution and on the risk aversion coefficient.

The risk attitudes of each household may change over time due to aging, improvement in the educational attainments, and so on. Moreover, the composition of the family may change, as well. While e.g. Kimball et al. (2009) uncover a covariance in risk preferences among family members, the size of such effect is weak. This is confirmed for example by Necker and Voskort, (2014) who use the German Socio-Economic Panel, finding that fathers' earnings risk is positively but weakly related to their sons' earnings risk. To encompass these issues, each household is assumed to face constant probabilities of transition to the groups characterized by different degrees of risk aversion. Our simulations consider three alternative parameterizations for the transition matrix. In the first, $\Lambda = [\lambda_{js}]$ is the identity matrix. While this choice – assuming away changes in relative risk aversion – implies a degenerate long-run wealth distribution, it also implies the smallest set of wealth distributions involving a negative effect on investment of an increase in volatility. Then, the alternative parameterizations based on the transition matrixes

$$\Lambda' = \begin{bmatrix} 0.96 & 0.04 & 0 \\ 0.01 & 0.98 & 0.01 \\ 0 & 0.04 & 0.96 \end{bmatrix} \text{ and } \Lambda'' = \begin{bmatrix} 0.94 & 0.06 & 0 \\ 0.015 & 0.97 & 0.015 \\ 0 & 0.06 & 0.94 \end{bmatrix}$$

are considered.[12] The data presented in Kimball et al. (2009) suggests that, from their 20s to their 60s, 38% of agents moves into the high-risk aversion group from the other categories (that initially amount to the 77% of the population). Matrix Λ' stylizes this chance, taking into account that this shift to high-risk aversion is likely to take place by progressive steps. Matrix Λ'' encompasses the fact that the agents entering into stage are likely to be characterized by a risk tolerance different from the one of their ancestors. In fact, correlation of risk aversion among family members is weak, as found in the literature. As shown by the simulation exercises, a lower persistence in risk aversion makes it (marginally) easier to obtain a negative investment-uncertainty relation. Accordingly, our fairly conservative choices do not operate in favour of the attainment of our result.

It is assumed that – for a given volatility level – the technology shock can take two states; the transition probabilities characterizing this first-order Markov process are chosen in such a way that the business cycles are symmetric and last on average six years. This requires $\pi_{11} = \pi_{22} = 0.66$. We also assume that the standard deviation for A_t can take only two values ($v_t = \{\sigma_l, \sigma_h\}$). The transition probabilities for the standard deviation are chosen so that a change in regime takes place on average once in twenty periods. Hence, we assume $\nu_{lh} = \nu_{hl} = \nu = 0.05$; the choice for v has quantitative implications that prove to be modest for the net investment to output ratio. The normalization adopted for productivity guarantees that the unconditional mean for the return to capital (net of depreciation) is 7% when the standard deviation for returns takes its low value. Notice that, when instead $v_t = \sigma_h$, the expected return increases. This is due to the flexibility effect described in Sub-section 3.1. The low value for the standard deviation, σ_l, is set to 15%.

[12] **Notice that decomposing matrixes Λ' and Λ'' using eigenvalues and eigenvectors, one can easily show that** $\lim_{T\to\infty} (\Lambda')^T = \lim_{T\to\infty} (\Lambda'')^T = \begin{bmatrix} 0.1\bar{6} & 0.6\bar{6} & 0.1\bar{6} \\ 0.1\bar{6} & 0.6\bar{6} & 0.1\bar{6} \\ 0.1\bar{6} & 0.6\bar{6} & 0.1\bar{6} \end{bmatrix}$.

The 'supporting material' presents the histogram for the long-run wealth distribution.

This represents the unique departure from the parameterization adopted in models similar to our one. Such a departure is motivated by the fact that the stock market volatility – being far higher than the output variance – takes value similar to the one we choose: because our argument relies on portfolio choices, we believe sensible to adopt a value for the standard deviation of returns which is consistent with the stock market volatility. σ_h is 20% higher than σ_l. In Sub-section 4.4 we offer the result obtained departing from the baseline scenario changing either the persistence parameter π_{11} or the possibility of a volatility shock v.

The baseline labour income share and capital depreciation have been set, respectively, to 0.65 and to 0.10. Sub-section 4.4 contains some sensitivity analysis concerning these parameters as well.

4.2 A representative agent economy

We now devote some attention to a version of our economy populated by homogeneous agents who live only for two periods, t and $t+1$. In this environment, the utility function (1) specializes to:

$$U_t = \left\{ (1-\beta)(C_t - \phi L_t)^{1-R} + \beta E\left[(1+\xi A_{t+1}^{\eta} - \delta)^{1-\gamma} \middle| A_t, \sigma_{t+1} \right]^{\frac{1-R}{1-\gamma}} K_{t+1}^{1-R} \right\}^{\frac{1}{1-R}}.$$

Adapting the logic of Colacito and Croce (2013), in Appendix C we show that a quadratic approximation of $E\left[(1+\xi A_{t+1}^{\eta} - \delta)^{1-\gamma} \middle| A_t, \sigma_{t+1} \right]^{\frac{1-R}{1-\gamma}}$ implies

$$sign\left[\frac{\partial I_t}{\partial Var\left[A_{t+1} \middle| A_t, \sigma_{t+1} \right]} \right] = sign\left[(1-R)\left(\alpha\left(1-\delta\right) + \left(\alpha - \gamma\right) \xi \bar{A}_t^{\eta} \right) \right], \tag{12}$$

in which $\bar{A}_t \equiv E\left[A_{t+1} \middle| A_t, \sigma_{t+1} \right]$.

To interpret this result, consider first the special case of complete capital depreciation ($\delta = 1$), which is considered by Saltari and Ticchi, (2007). The flexibility effect is captured by α: the larger is the labour share, the more convex is the firm's marginal revenue of capital, and hence the stronger is the increase in expected returns induced by a mean-preserving spread in productivity. Were the representative agent risk neutral ($\gamma = 0$), a low elasticity of intertemporal substitution ($R > 1$) would induce the household to shrink investment, due to the strong desire to smooth utility over time. But when agents are risk averse, they take their decision considering the certainty-equivalent of future utility, and not utility itself. Such a risk aversion effect – reducing the evaluation of future utility – calls for more capital accumulation. In fact, an higher investment increases the certainty-equivalent consumption in the second period, when it is lower. When $\gamma > \alpha$, the risk aversion effect prevails, and investment is increased by a rise in volatility.

Saltari and Ticchi (2007) provide numerical illustrations for the case $\delta < 1$. Indeed, they show that capital depreciation makes larger the threshold for risk aversion allowing for a negative investment-uncertainty relation with a low elasticity of intertemporal substitution. In their own words (p. 634), capital depreciation 'allows the flexibility effect to operate for more periods, which in turn implies that the agents' risk aversion has to be relatively higher for the risk aversion effect to prevail'.[13]

To investigate this point, we considered an economy characterized by our benchmark parameterization but for risk aversion, which is of course assumed identical for all agents. In such an homogeneous-agent version of our infinite-horizon economy, the investment-volatility relationship is negative for $\gamma < 2.68$. Accordingly, when all the agents are characterized by $\gamma = 4.2$, which is the average value reported by Kimball *et al.* (2009), the response of investment to a change in volatility is positive. The second row in Table 2 shows, for sensible values of the depreciation parameter, the maximum value for γ allowing for a negative reaction of investment to future volatility (which is labeled $\bar{\gamma}^{Hom}$). The third row offers instead the corresponding threshold value for the heterogeneous-agent economy, $\bar{\gamma}^{Het}$. This is the highest value for the average of the risk-aversion parameters which allows a negative investment-uncertainty relation; such an average is computed using the hergodic distribution of agent types.[14]

[Table 2 around here]

The results above are suggestive of the relevance of heterogeneity in the attitudes toward risk to generate a negative response of investment to an increase in volatility.

[13] In a model populated by expected-utility maximizers, Femminis (2008) finds the same result, and he ascribes it to the fact that, when $\delta < 1$, a share of the period t real resources can be (safely) transferred to the following period, which makes the risk aversion effect less important.

[14] $\bar{\gamma}^{Het}$ it is obtained by changing the average risk aversion until a virtual independence of investment from changes in future volatility is reached. In these exercises the average risk aversion is moved in steps by 0.01 while keeping unchanged its standard deviation.

4.3 Simulation results for the benchmark economy

To illustrate the mechanisms at work in our model, we first describe how the consumption-to-wealth ratio reacts to an increase in perceived volatility, considering, in turn, only two of the three types of agents populating our model. Then, we present the results for the general case.

4.3.1 From the representative agent economy to the general case

Figure 2 helps understanding how the intuition developed for the representative agent case extends. More specifically, each panel in Figure 2 displays the changes in the consumption-to-wealth ratios for a specific class of agent, when the economy is populated only by that class and by one of the other two. In these exercises, the dynamics in the relative risk aversion index is governed by the identity matrix (a case which is specific but representative).[15]

[Insert Figure 2 about here]

Panels (a_1) and (a_2) focus on the behavior of low risk-aversion households. Panel (a_1) describes an economy populated by low- and by high- risk aversion agents (and hence with no households characterized by $\gamma = 2.86$); this panel shows that the low risk-aversion agents increase their consumption in response to a rise in volatility. This happens because the risk aversion effect is weak for this group (refer to Sub-section 4.2): aiming to smooth utility over time, these household increase their current consumption when the expected return on capital increases due to the flexibility effect. The rise in the consumption to wealth ratio is stronger the smaller is their wealth share (and hence the higher is the wealth share for the $\gamma = 12.94$ households). In fact, the smaller is the fraction of wealth they own, the higher is the optimal share of equity in their portfolio (which reaches 625% for the case in which they own almost no wealth), and hence the higher is the increase in their certainty-equivalent yield. Panel (a_2), represents the case in which there are no high risk-averse households, and hence the wealth is split between low and intermediate risk-aversion agents. The response of low risk-aversion agents' consumption is positive, although quantitatively less important than that presented in Panel (a_1): risk is now shared between agents who only moderately differ in

[15] The choice to consider $\Lambda = I$ is convenient because, when two types of agents own no wealth, the model reduces to a homogeneous agent framework (since there is no possibility of a change in γ_j), which allows for comparisons.

their degrees of risk aversion.

The behavior of the households with an the intermediate degree of risk-aversion is considered in Panels $(b_1) - (b_2)$. Panel (b_1) is drawn for the case in which there are no agents characterized by the low degree of risk-aversion. When the $\gamma = 2.86$ agents hold the entire wealth, their response is (mildly) negative, which is consistent with the fact, stated in Sub-section (4.2), that in a representative agent economy the increase in future volatility raises investment for $\gamma > 2.68$.[16] More interestingly, when the wealth is shared between intermediate and high risk-aversion agents, the response of consumption becomes positive for the $\gamma = 2.86$ agents. In fact, they bear most of the risk, so that for them the flexibility effect, which implies an increase in the return on risky capital, outweights the risk-aversion effect. When there are no agents characterized by a high degree of risk-aversion (Panel (b_2)), and the $\gamma = 2.86$ households own a negligible share of wealth, the response of their consumption to an increase in uncertainty is positive because they own almost no risky capital, and the increase in volatility causes an increase in the riskless rate.

Panels (c_1) and (c_2) describe the behavior of highly risk-averse households. Panel (c_1) depicts what happens when the wealth is split between high- and low- risk aversion agents. When the high risk-aversion agents own a large share of wealth, they reduce their consumption, which instead (very mildly) increases when their wealth share is limited. In fact, when the high risk-aversion households own a large fraction of wealth, their equity portfolio share gets close to 1, and an increase in volatility decreases their consumption ratio because the risk-aversion effect dominates the flexibility effect. On the contrary, when they own a small wealth share, the risk aversion effect loses its clout, since they do not need to bear virtually any risk, which is instead borne by the low-risk aversion agents. Accordingly, their consumption can (marginally) increases. Panel (c_2), is drawn for the case in which there are no low risk-averse households: given that risk is shared among agents who significantly suffer it (although up to different degrees), the response of consumption to an increase in volatility is always negative.

[16] When $\Gamma_{t,1} = \Gamma_{t,2} = 0$, the model reduces to a homogeneous agent framework, in which risk-aversion is 2.86. We verified that the result obtained by our routine are actually the same that would characterize a representative agent model with $\gamma = 2.86$.

4.3.2 Consumption and investment in the heterogeneous agents economy

Figure 3 shows how the effects described above put together in the general case: the three panels of Figure 3 consider any possible wealth distribution (i.e., $\Gamma_{t,1} \in [0,1]$, $\Gamma_{t,2} \in [0,1]$, with $\Gamma_{t,1}+\Gamma_{t,2} \leq 1$) for the transition matrixes described in Sub-section 4.1. The shaded areas indicate where the response to an increase in volatility involves an increase in investment.[17] The border between the shaded and the white areas represents the cases in which investment is independent from future volatility.

Small probabilities of a change in the degree of risk aversion imply an enlargement of the area in which the volatility-investment relation is negative. In fact, every agent is aware of the probability of changing her type, which is taken into full account. Because a high-risk averse individual faces the probability of becoming less risk averse, she takes into account the possible increase in the certainty-equivalent of future wealth, which decreases her current savings. This effect dominates, from a quantitative perspective, the opposite force acting on the intermediate risk-averse agents.

[Insert Figure 3 about here]

To provide a further account of our results, we generated the time series for consumption, investment and output simulating our economic system for one million periods. Table 3 presents the data obtained in this exercise. The first two columns considers the case of the transition matrix Λ', and shows that, in response to a perceived change in volatility, the increase in consumption of the low risk-aversion households more than compensates the increase in savings chosen by the middle and high risk-averse agents. Accordingly, the ratio between investment (net of depreciation) and output declines in response to an increase in volatility. Notice that the percentage change in consumption (aggregated across states using their ergodic probabilities), $\Delta\%$, is modest, but notice also that the elasticity of the change with respect to the increase in return, $\epsilon\%$, is relevant. This is due to the fact that the change in returns which is induced by an increase in volatility is limited (from 7% to 7.0047%, refer to Table 1). From Table 3 we also see that the normalized standard deviation of net investment is around seven times higher than that for aggregate consumption both in the case

[17] These results have been obtained integrating over the productivity nodes using their ergodic probabilities.

of low volatility and in case of high volatility, which matches the stylized facts. Finally, we highlight that the normalized standard deviation for consumption of the high-risk aversion households is close to that for the middle-risk aversion families. To understand this counterintuitive point, consider first that output stochastically grows over time,[18] so that the data on which we elaborate are detrended. Consider, moreover, that the high-risk aversion agents virtually refuse to bear any risk; hence, when the outcome of the productivity process is favorable, their consumption growth is below average, while the converse happens when productivity is low. This is the source for the variability of consumption of the high-risk aversion households. Table 3 allows to appreciate the stark contrast between our heterogeneous agent economy and the homogeneous agent one. In particular, notice that the elasticity of the net investment-to-output ratio is -26.5% with heterogeneous households, while is 49.6% in the representative agent case.[19]

[Insert Table 3 about here]

4.3.3 Portfolio shares and equity returns

Table 3 also presents some financial data, including the average portfolio choice for the three groups. Notice that the agents belonging to the low risk-aversion group bear a large fraction of the aggregate risk (on average, with $q_l = 3.86$ and a wealth share very close to 16.7%, they own around 64.4% of the risky capital). When volatility increases, they do not change significantly their portfolio share (actually, in most cases it marginally decreases), and the portfolio yield increases. Since they are very mildly risk averse, the flexibility effect prevails on the risk aversion effect, so that the (certainty-equivalence) of their future return increases. This, when households aim at smoothing consumption over time, involves a reduction in investment and savings. The middle risk-aversion households bear a relatively small fraction of the aggregate risk (on average, with $q_m = 0.52$ and a wealth share of about 66.6, they own around 34.8% of the risky capital). When volatility increases, these

[18] As in basic endogenous growth models, growth is induced by the linearity in capital characterizing output (refer to Sub-section 2.1).

[19] Appendix E makes available a Figure displaying the histogram of the stationary distribution of wealth when the transition matrixes is Λ' (the results for Λ'' are almost identical). We find that the average value for the wealth shares is essentially determined by the long-run distribution of types (which is $[0.1\bar{6}, 0.6\bar{6}, 0.1\bar{6}]$); we also obtain that not only the average value for the wealth shares imply a negative investment-uncertainty relation, but the relation is negative in virtually the 100% of the realized wealth distributions.

agents (marginally) increase their share of equities. The expected return on their wealth increases, but the certainty-equivalence of future return decreases. Accordingly, given the low elasticity of intertemporal substitution, the middle risk-aversion families decide to save more to compensate for the future reduction in the certainty-equivalent consumption. A similar reasoning applies for the high risk-aversion households.[20]

Table 3 highlights the main limitation of the model: from the last two rows we see that the riskless rate is close to the expected returns on capital, so that the equity premium is far too small. Actually, it is smaller that the premium that can be computed for the homogeneous agent version of our model. This does not come as a surprise: risk is traded among agents, and a large fraction of it is allocated to the agents who bear it more easily, accordingly its price gets lower.

4.4 Sensitivity analysis

The robustness of our results is tested considering various alternative scenarios. Since the results for the transition matrixes described in Sub-section 4.1 are quantitatively similar, we focus on the case with Λ'.

As highlighted in Sub-section 3.1 the flexibility effect is stronger the higher is α, since a larger labour income share increases the convexity of the marginal revenue product function. The first two columns in Table 4 report the results for Parameterizations (B) and (C) in Table 1. Comparing the two columns, it is clear that – when α is raised from 0.60 to 0.70, the percentage change in the consumption-to-output ratio (marginally) increases for all the three classes of households, so that the negative effect of an increase in future volatility on net investment becomes stronger. Not surprisingly, the changes in investment for the benchmark parameterization (Table 3, first column) are in between those reported here.

[Insert Table 4 about here]

The potential importance of the depreciation parameter has already emerged in Sub-section 4.2. Here, we modify the baseline parameterization by raising δ to 0.15, and then lowering it to 0.05.

[20] The variability over time of the portfolio decisions is limited: the standard deviation of the portfolio shares (normalized using the corresponding average) is, at most, close to 5%.

The pertinent results are shown in the third and in the fourth columns of Table 4. As expected, the lowering of the depreciation parameter makes quantitatively stronger the negative investment-uncertainty relationship we have obtained. In fact, the lower δ, the higher the share of real resources that can be (safely) transferred to the following period, which makes the risk aversion effect less important.

When we focus our attention on the transition probabilities characterizing the first-order Markov process for productivity, we preserve the symmetry of the business cycles. We first consider the case of very high persistency, assuming that $\pi_{11} = \pi_{22} = 0.80$; we then assume persistency away, by letting $\pi_{11} = \pi_{22} = 0.50$. The result for these two cases are offered in Columns (F) and (G), which show that, with high persistency, our model fails to obtain a negative reaction of investment to a volatility shock. This result is ascribed to the fact that the persistency in productivity gives more weight to the risk aversion effect. In fact, a change in productivity affects more heavily the utility-evaluation of capital due to its long-lasting effects on output. Hence, also the effects of a mean-preserving spread in productivity on the state-contingent evaluation of capital are more relevant. When instead persistency is assumed away, our preferred result becomes quantitatively stronger than in the baseline case.

Finally, we raised the frequency of the changes of regime by letting, $\nu(= \nu_{lh} = \nu_{hl}) = 0.10$. In this case, the higher v induces weaker effects on consumption and on investment. When a change is more frequent, its effects last for a shorter time span, and therefore are perceived as less important.

4.4.1 Elasticity of intertemporal substitution and relative risk aversion

Our previous analysis implies that the intertemporal elasticity of substitution governs the sign and the size of the impact of the other effects (Equation (12)). Some recent contributions such as Bansal and Yaron (2004) and Barro (2009) suggests that – to match important qualitative features of the data – the elasticity of intertemporal substitution needs to be larger than unity. Others, as Guvenen (2006), prompt a value close to unity (at least for stockholders). Accordingly, it is interesting to verify what happens in our economy when R is lower than (but close to) 1. Table 4, Column (I) presents the data obtained simulating an economy characterized by our benchmark values for all the parameters but for R, which is now set to 0.9. The results are in line with our previous intuition. An elasticity of substitution above 1 reverses the sign of the effects of an increase in volatility for each class of agents, for aggregate consumption and for investment. The size of the effects on investment is lower than those presented in Table 2, which is consistent with the fact that the value for R is much closer to unity. We have also verified that the area in the wealth distribution space such that the response to an increase in volatility involves an increase in investment is complementary to the one displayed in Panel (b) of Figure 3.

Finally, we test the robustness of our conclusion by simulating an economy in which, keeping unchanged the mean risk aversion, its dispersion is reduced first by 25% and then by 50%. Column (L) shows that in the first case the negative response of investment to a perceived increase in volatility is preserved, although it is quantitatively weaker. In this case, the average portfolio share for the low-risk aversion agents is 3.30. Instead, when the dispersion in risk aversion in reduced by 50% the risk-shifting effect is limited enough (since, on average, $q_l = 2.58$) that the investment-uncertainty relation becomes positive.

4.4.2 Portfolio constraints

In the U.S., the Federal Reserve Bank Regulation T[21] allows an investor to borrow up to the 50% of the total purchase value of equities when opening a new position; moreover, a position can be maintained as long as the amount of the initial loan is worth less than the 75% of the purchased portfolio. These requirements imply that $q_{j,t} \leq 2$ and $q_{j,t} \leq 4$, respectively.[22] Because our result depends on the fact that aggregate risk is transferred to the less risk averse families, one might be induced to think that binding portfolio constraints may severely weaken our result. As it is shown in Table 4, Columns (N) shows that, when we add the constraint $q_{j,t} \leq 4$ in a variant of our model otherwise characterized by the benchmark parameterization, we obtain that the results are virtually unchanged, which is not surprising if we take into account the fact that the long-run-equilibrium wealth share in risky assets is around 3.86. When instead the more severe constraint $q_{j,t} \leq 2$ is

[21] Available at: http://www.federalreserve.gov/bankinforeg/reglisting.htm

[22] In most papers dealing with credit constraints, the amount an agent can borrow is constant over time. Instead, as, for example, Coen-Pirani (2005), we assume that this amount depends positively on her wealth. As a household's wealth evolves over time, so does its maximum borrowing capability.

considered, the size of the investment-uncertainty relation is reduced, as pointed out in Column (O).

5 Concluding remarks

With a realistic distribution for the relative risk aversion index and a low elasticity of intertemporal substitution, our model generates a negative impact of an increase in future volatility on aggregate investment.

The channel we identify is new: in our heterogeneous agents model, the aggregate risk is born by the individuals who can stand it better. Accordingly, the risk aversion effect loses much of its clout, and the flexibility effect induces a reduction in savings and investment.

Our economy is competitive, but we conjecture that our result can be extended to a monopolistic competition general equilibrium framework. When the elasticity of demand is finite, the incentive to adjust the production level in response to a demand shock is less important for any individual firm, since – increasing output in response to a positive shock – it brings down its good price. Although weakened, however, the flexibility effect does not disappear. Also, in a general equilibrium framework, the flexibility effect, affecting all the firms, increases the expected aggregate output, which, in its turn, raises each firm's estimate for its future demand, and hence for its future returns.

Our results are important in several respects.

Our model helps to explain why an increase in volatility, such as the one that followed September 11, 2001, or the large oil shocks of the 1970s, can slow down aggregate investment. In general, a better understanding of the investment-uncertainty relationship may help providing relevant policy implications; in fact, the design of an appropriate stabilization policy should take into account how investment responds to an increase in aggregate volatility.

From a methodological perspective, we provide a neat example of the fact that the aggregate behavior which is obtained in heterogeneous agent economies can be very different from the one that is generated in a representative agent model.

Our result are grounded on portfolio choices by heterogeneous agents. In a fully dynamic environment, these choices may easily lead to a 'degenerate' solution, implying the dominance of a single class of agents. To avoid these solutions, we assumed that the risk aversion of each household changes stochastically over time. This approach seems useful is devising manageable models allowing to push further ahead the analyis of some classic puzzles, such as the equity premium and the risk-free rate ones. For these issue, the interplay between risk-aversion heterogeneity and portfolio constraints can in fact be relevant. Also, the heterogeneity in portfolio positions is interesting in models explaining international finance conundrums, such as the forward premium puzzle or the disconnect between exchange rates and consumption growth differentials; in particular, heterogeneity could help explaining the dimension of the movements in foreign wealth. The analysis of asset pricing behavior in the presence of risk aversion heterogeneity is an intriguing issue, the analysis of which can be eased by a framework similar to our one.

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Parameter		(A)	(B)	(C)	(D)	(E)	(F)	(G)
Time discount factor	β	0.96						
Inverse EIS	R	2						
		0.64						
Risk aversion vector	γ	2.86						
		12.94						
Labour income share	α	0.65	0.60	0.70				
Capital depreciation parameter	δ	0.10			0.15	0.05		
Return on capital	$\xi A_t^\eta - \delta$							
- high productivity, low variance		7.7425						
- low productivity, low variance		6.2575						
- high productivity, high variance		7.8962	7.8958	7.8966				
- low productivity, high variance		6.1131	6.1128	6.1134				
Prob. (good state | good state)	π_{11}	0.66					0.80	0.50
Prob. (bad state | bad state)	π_{22}	0.66					0.80	0.50
Standard dev. of returns – low	σ_l	15 %						
Standard dev. of returns – high	σ_h	18 %						
Prob. of volatility change	ν	0.05						
Portfolio constraint	q	∞						

Parameter		(A)	(H)	(I)	(L)	(M)	(N)	(O)
Time discount factor	β	0.96						
Inverse EIS	R	20.92		0.9				
		0.64			0.96	1.55		
Risk aversion vector	γ	2.86			3.27	3.70		
		12.94			11.12	8.84		
Labour income share	α	0.65						
Capital depreciation parameter	δ	0.10						
Return on capital	$\xi A_t^\eta - \delta$							
- high productivity, low variance		7.7425						
- low productivity, low variance		6.2575						
- high productivity, high variance		7.8958						
- low productivity, high variance		6.1128						
Prob. (good state | good state)	π_{11}	0.66						
Prob. (bad state | bad state)	π_{22}	0.66						
Standard dev. of returns – low	σ_l	15 %						
Standard dev. of returns – high	σ_h	18 %						
Prob. of volatility change	ν	0.05	0.10					
Portfolio constraint	q	∞					4	2

Table 1: Alternative parameterizations - annual model.

An empty cell implies that the corresponding parameter takes a value identical to the one in the baseline parameterization (A).

δ	0.05	0.10	0.15	0.20
$\bar{\gamma}^{Hom}$	3.60	2.68	2.17	1.85
$\bar{\gamma}^{Het}$	5.34	4.60	4.28	4.09

Table 2: Upper bounds for risk aversion: homogeneous versus heterogeneous agents economies.

		Parameterization (A)		Parameterization (A)		Homogeneous Agent	
		Transition matrix: Λ'		Transition matrix: Λ''		$(\gamma = 4.20)$	
		$\Delta\%$	$\epsilon\%$	$\Delta\%$	$\epsilon\%$	$\Delta\%$	$\epsilon\%$
	$\gamma_l = 0.64$	1.89722	28.512	1.68604	25.339		
C_j/Y	$\gamma_m = 2.86$	-0.16136	-2.425	-0.13426	-2.018	n.a.	n.a.
	$\gamma_h = 12.94$	-0.91339	-13.727	-0.83535	-12.554		
$\sum_j C_j/Y$		0 06051	0.909	0.05801	0.871	-0.11385	-1.7109
$(I - \delta K)/Y$		-1.76129	-26.469	1.68829	-25.372	3.30450	49.6616
		$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_l}$	$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_h}$	$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_l}$	$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_h}$	$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_l}$	$\frac{\sigma}{\mu}\Big\vert_{v_t=\sigma_h}$
	$\gamma_l = 0.64$	0.09710	0.11581	0.09714	0.11550		
C_j	$\gamma_m = 2.86$	0.07647	0.09176	0.07638	0.09145	n.a.	n.a.
	$\gamma_h = 12.94$	0.07504	0.08976	0.07495	0.08947		
$\sum_j C_j$		0.07849	0.09419	0.07842	0.09389	0.07855	0.09390
$I - \delta K$		0.54076	0.64177	0.53957	0.64019	0.53950	0.64131
Y		0.08328	0.09997	0.08321	0.09966	0.08339	0.09969
		σ_l	σ_h	σ_l	σ_h	σ_l	σ_h
	$\gamma_l = 0.64$	3.86298	3.84528	3.84896	3.83136		
q_j	$\gamma_m = 2.86$	0.52221	0.52678	0.51906	0.52365	n.a.	n.a.
	$\gamma_h = 12.94$	0.00000	0.00000	0.00000	0.00000		
	r	6.98850	6.98926	6.98886	6.99001	6.97184	6.96727
	$E\left[\xi A_t^{\eta} - \delta\right] - r$	0.01150	0.01544	0.01114	0.01469	0.02816	0.03743

Table 3: Consumption, investment, average portfolio shares and risk-free rate.

$\Delta\%$ is the percentage change in a detrended variable, $\epsilon\%$ is the elasticity of the change in the detrended variable with respect to the increase in return; $\sigma/\mu|_{v_t=\sigma_i}$ is the standard deviation of a detrended variable normalized using its mean, when the standard deviation for productivity is σ_i; q_j is the wealth share invested in risky assets by agents with risk aversion γ_j, r is the risk-free rate, and $E\left[\xi A_t^{\eta} - \delta\right] - r$ is the risk premium. All the values have been obtained by integrating across states using the pertinent long-run relative frequencies.

		Alternative parameterizations – transition matrix Λ'					
		(B)	(C)	(D)	(E)	(F)	(G)
		$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$
	γ_l	1.87970	1.88674	1.83529	2.02007	**3.13030**	1.17860
C_j/Y	γ_m	-0.16938	-0.16339	-0.21438	-0.33016	**-0.17276**	-0.04653
	γ_h	-0.85245	-0.84681	-0.89761	-0.71732	**-2.62302**	-0.38808
$\sum_j C_j/Y$		0.06272	0.68824	0.01780	0.19953	**-0.03012**	0.10499
$(I-\delta K)/Y$		-1.53378	-2.43657	-0.64126	-4.44091	**0.87529**	-3.06279
		(H)	(I)	(L)	(M)	(N)	(O)
		$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$	$\Delta\%$
	γ_l	1.21433	**-0.49108**	1.94080	**1.69942**	1.91298	2.19148
C_j/Y	γ_m	-0.11112	**0.03491**	-0.21390	**-0.21691**	-0.16399	-0.15962
	γ_h	-0.51543	**0.32317**	-1.06027	**-1.19404**	-0.92191	-1.41596
$\sum_j C_j/Y$		0.04531	**-0.00516**	-0.01011	**-0.05991**	0.06008	0.02556
$(I-\delta K)/Y$		-1.31916	**0.06425**	-0.29391	**1.74054**	-1.74883	-0.74354

Table 4: Consumption and investment for alternative parameterizations.

$\Delta\%$ is the percentage change in a detrended variable obtained by integrating across states using its long-run relative frequencies. The coloums in bold concern parameterizations implying a *positive* investment-uncertainty relationship.

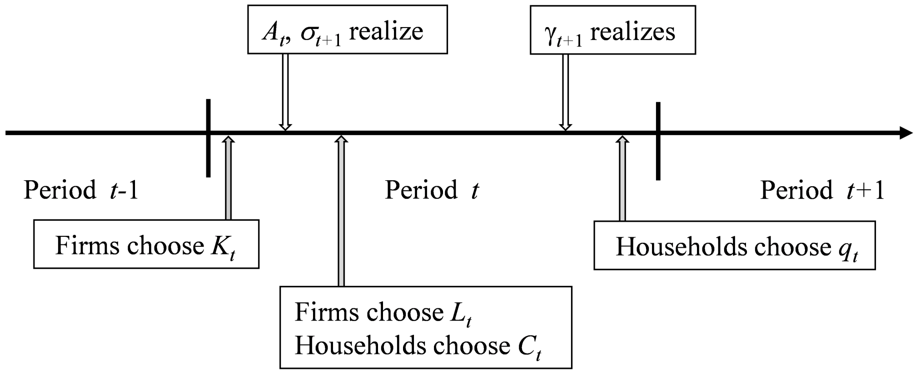

Figure 1: The time line.

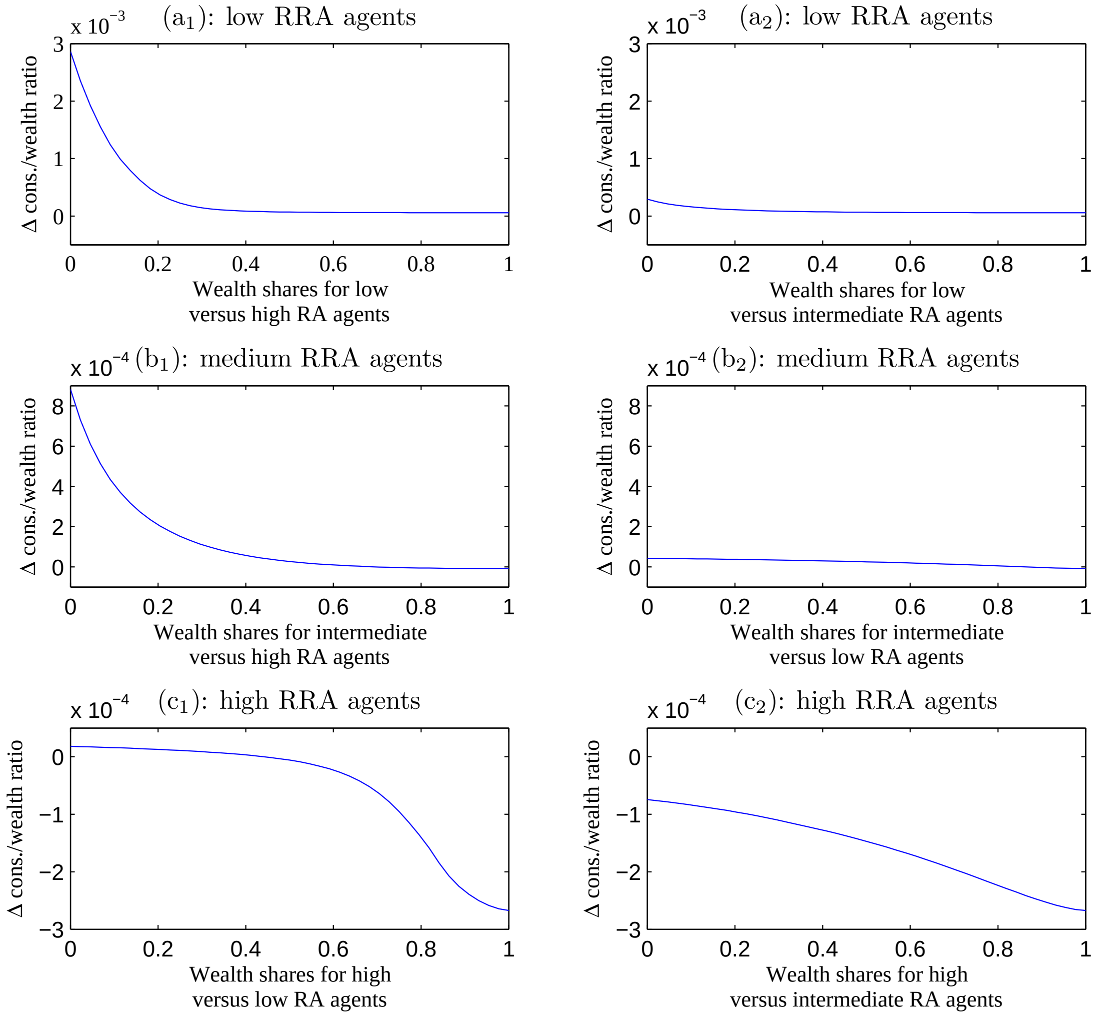

Figure 2: Response to an increase in volatility of the consumption-to-wealth ratios for the three classes of agents, as a function of their wealth shares.

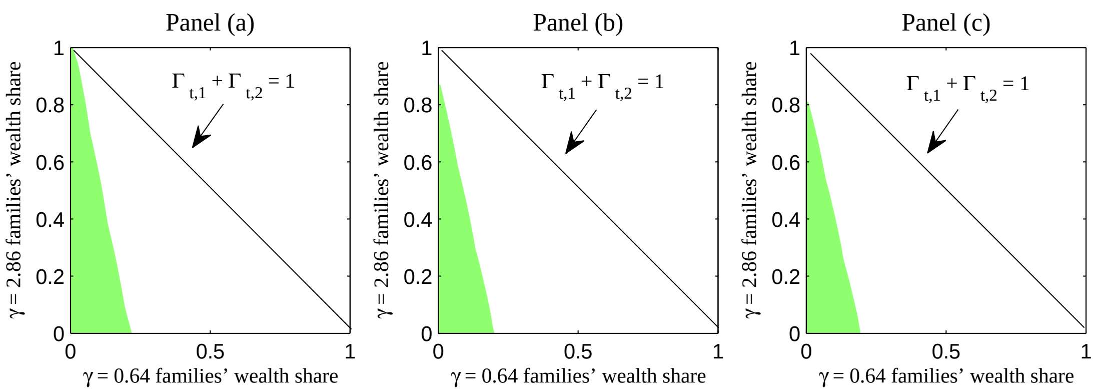

Figure 3: When $\gamma = \{0.64,\ 2.86,\ 12.94\}$ and $R = 2$, the investment-volatility relation is positive in the shaded areas in Panels (a), (b), and (c), computed for Λ, Λ', and Λ'', respectively.