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# The Value of Transparency in Multidivisional Firms*

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**Abstract**

We study internal incentives, transparency and firm performance in multidivisional organizations. Two independent divisions of the same firm design internal incentives, and decide whether to publicly disclose their performances. In each division a risk-neutral principal deals with a risk-averse (exclusive) agent under moral hazard. Each agent exerts an unverifiable effort that creates a spillover on the effort cost of the other agent. We first study the determinants of the optimal principal-agent contract with and without performance transparency. Then, we show how effort spillovers affect the equilibrium communication behavior of each division. Both principals commit to disclose the performance of their agents in equilibrium when efforts are complements, while no communication is the only equilibrium outcome when efforts are substitutes.

**JEL codes:** D43, D82, L14.
**Keywords:** Multidivisional Firms, Transparency, Moral Hazard.



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# 1 Introduction

Many large firms are heavily decentralized and delegate important tasks to independent divisions, whose objectives are not always aligned — see, e.g., Groves and Loeb (1979) and Wettstein (1994) among others. Lining up divisions' incentives often requires accurate information, whose costs may depend on the strategic interaction between the divisions' staff. For example, using information about one division to improve the performance of another may be difficult when divisions compete for the same budget, operate in the same or related markets, or one supplies the other with goods or services.

This paper examines the link between incentives, transparency and performance in multidivisional firms. Knowledge sharing mechanisms are traditionally believed to enhance efficiency: because they stimulate learning and imitation — see, e.g., Griffin and Hauser (1992) — or because, by improving firms' internal transparency, they enhance stakeholders' ability to access capital markets — see, e.g., Milgrom (1981) and Roberts (1992). However, these mechanisms may also affect firms' organization and contractual choices. What is the effect of transparency on the trade-off between risk and incentives in multidivisional firms? What are the costs and benefits of information sharing when divisions impose externalities one on the other?

To address these issues we study a firm composed of two independent divisions, each ruled by a principal (top manager) dealing with an exclusive agent (mid-level manager or worker). Top managers simultaneously and independently design incentive contracts for their workers and decide whether or not to publicly disclose their division's performance — i.e., they choose whether to be transparent or not.[1] Agents' effort is unverifiable and produces cross-division spillovers (externalities): an agent's marginal cost of effort depends on the effort exerted by the other agent, either negatively (complementary efforts) or positively (substitute efforts). Divisions are heterogeneous with respect to the volatility of their profits, and contracts can only be based on observed performances, which are an imperfect measure of agents' efforts and are correlated across divisions.

We start by analyzing how each principal conditions his own agent's wage to the performance of the other division (when he can do so), and study the determinants of equilibrium contracts with and without communication (transparency). This allows us to highlight the specificity of our contracting game relative to the benchmark in which there is only one principal-agent pair.

As expected, under secret contracts, incentives equal those of the benchmark when principals do not communicate. Instead, principals enforce steeper own-performance bonuses when both choose to be transparent. The wedge between incentives with and without multiple divisions widens as divisions become more connected: the more correlated are their performances, the steeper the incentives that principals offer in equilibrium.

The cross-performance bonus is determined by the need to diversify risk and its sign depends on

[1] A transparent division commits to publicly disclose its performance. This allows the principal of the other division to condition the payment pledged to his own agent not only on his performance (own-performance bonus), but also on the performance of the other agent (cross-performance bonus). The latter is, in fact, a form of relative performance evaluation.

the correlation between divisions' performances. When the divisions' performances are positively correlated, each principal rewards his agent if the other division under-performs, because a bad performance by the other division is a signal that his own agent may under-perform too. In this case the cross-performance bonus is negative. By contrast, when the correlation between divisions' performances is negative, each principal rewards his agent if the other division performs well to optimally diversify risk. Hence, the cross-performance bonus is positive. As intuition suggests, this bonus decreases in the volatility of the other division's performance: if performance becomes noisier, the value of information received by that division decreases.

Equilibrium efforts are unambiguously higher than in the case of a single principal-agent pair if effort externalities are positive, while the comparison yields ambiguous predictions when efforts are substitutes. In this case, two contrasting effects are at play. On the one hand, both principals offer steeper incentives than in the benchmark, thus boosting the agents' efforts. However, since efforts are strategic substitutes, the increase of one agent's effort implies a reduction of the other agent's effort. It turns out that the first effect dominates when the performance of the division from which a principal receives information is sufficiently noisy — i.e., receiving information from a transparent division mitigates moral hazard and enhances effort even if agents impose negative externalities one on the other.

Next, we characterize the equilibria of the communication game. Principals first decide whether to communicate or not, and then offer contracts to their agents. It turns out that, in equilibrium, the communication behavior of each principal depends on the nature of effort spillovers. When efforts are complements there is a unique equilibrium in which both divisions choose to be transparent, while no communication is the unique equilibrium when efforts are substitutes. Even though divisions are heterogeneous as to profits' volatility, asymmetric equilibria in which only one principal shares information do not exist.

Two main effects shape the incentives to share information in our setting. First, a principal's decision to share information generates an indirect strategic effect on his agent's effort. Indeed, the information disclosed to the other division is used to increase the effort of that division's agent, which indirectly affects the effort of the agent working for the principal who discloses his division's performance. Second, when a principal commits to disclose the performance of his division, he is also directly affecting his own agent's effort. This is because a principal's disclosure decision affects the effort of the agent working in the other division, which in turn determines the agency costs that the principal has to pay to control his own agent. Hence, any change in the effort of the other division modifies the fixed component of the wage that each principal pays to his own agent.

Taken together, these results offer novel predictions both on the type of vertical contracts that shape the internal organization of multidivisional firms, and on the process of communication among their independent profit centers. The model provides testable implications for: the determinants of divisions' incentives to share information; the link between the power of incentives and cross-division externalities; the impact of monitoring and contractual power on their internal structure; the limits to decentralization.

The rest of the paper is organized as follows. Section 2 relates our work to the received literature. Section 3 outlines the model. Section 4 characterizes the agents' effort choices and discusses some important features of the equilibrium contracts. Section 5 presents the equilibrium contracts in the subgames following the first-stage transparency decisions. In Section 6, we characterize the equilibrium disclosure choices. Section 7 concludes. Proofs are in the Appendix.

## 2 Related literature

Our findings contribute to the literature on multidivisional firms. Stemming from Hirshleifer (1957), this literature has examined the resource allocation problem of the headquarter of a divisionalized firm, whose objective is to harmonize incentives among different, and possibly competing, divisions. An efficient allocation of resources is achieved either directly, through a centralized design of incentives — see, e.g., Faulí-Oller and Giralt (1995) and Groves and Loeb (1979) among others — or, indirectly, through transfer prices between units — see, e.g., Baldenius and Reichelstein (2006) and Harris, Kriebel, and Raviv (1982). In our model, units' contracting decisions are fully decentralized — i.e., they cannot be set by the headquarter. This seems reasonable when contracts can be secretly renegotiated.

Another strand of related literature examines firms' optimal organization form as a response to information asymmetries within firms — see, e.g., Aghion and Tirole (1995), Berkovitch, Israel, and Spiegel (2010), Besanko, Régibeau, and Rockett (2005), Maskin, Qian, and Xu (2000), Qian, Roland, and Xu (2006) and Rotemberg (1999) among many others. All these papers develop theories in which firms' organizational structure is endogenous, and is determined by the trade-offs between the costs and the benefits of divisionalization. In our model the firm structure is exogenous: our objective is to analyze the potential conflicts between units and determine how asymmetries between them determine their decisions on contracts and transparency.

In this respect, our model is closer in spirit to the growing literature linking the issue of optimal contract design to that of communication between principals. Calzolari and Pavan (2006) devise a sequential game in which principals dealing with the same (privately informed) agent learn through costly contracting and then share with the rival the elicited information. More recently, Piccolo and Pagnozzi (2013) have extended this idea to the case of vertical hierarchies. In these models, players acquire private information by contracting with common parties, and create new private information by taking decisions that affect both rivals and contractual counterparts. The approach taken in this literature is different from the one employed in oligopoly models — e.g., Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985) — where the information shared by competitors is exogenous. In these models there are no incentive issues within firms and communication may simply help to overcome coordination problems, thereby facilitating implicit collusion.[2]

[2]The standard industrial organization approach to information sharing has been applied in the management literature studying information sharing within supply chains. In these models a manufacturer deals with competing retailers and the information sharing decision depends on the contract type and the form of competition (see, e.g., Ha

On the moral hazard side, Maier and Ottaviani (2009) study the costs and benefits of transparency in a common agency game in which principals commit to share information about the common agent's performance. We study the case of exclusive deals. In contrast to Maier and Ottaviani (2009), we find that with exclusive deals the type of externalities that divisions impose one on the other is crucial to determine the equilibrium degree of transparency. This difference arises because in our model the agents' efforts are imperfect complements and can even be substitutes, while in their framework the effort of the common agent affects the profits of both principals. In a nutshell, while in Maier and Ottaviani (2009) the main issue is free-riding — i.e., each principal would like the other to pay for the agent's effort — in our model this problem disappears when efforts are substitutes.

Finally, our paper also contributes to the literature on relative performance evaluation. The possibility of linking an agent's compensation to the performance of another agent *de facto* allows principals to enforce relative performance evaluations. While the existing literature has mainly focused on the problem of a single principal dealing with several agents that free-ride one on the other — see, e.g., (Bolton and Dewatripont, 2005, Ch. 8) — in our context information sharing provides a natural tool to enforce the relative performance evaluation of agents that serve principals with (possibly) conflicting objectives. To the best of our knowledge, only Bertoletti and Poletti (1996) analyze a similar idea in a context with adverse selection and risk neutral agents. Lottery contracts in their setting allow to implement the first-best outcome.

## 3 The model

*Players and environment.* Consider a firm consisting of two independent divisions (departments or profit centers). Division $i$ ($i = 1, 2$) is modeled as a principal-agent pair, composed by a principal (manager) $P_i$, and an exclusive agent (mid-level manager, or employee) $A_i$. Each division carries over a project that yields a gross profit $y_i$, which is linear in $A_i$'s unverifiable effort, $a_i$, and in an additive random component, $\varepsilon_i$. That is,

$$y_i = a_i + \varepsilon_i, \tag{1}$$

where $\varepsilon_i \sim N(0, \sigma_i^2)$ for every $i$. We allow $\varepsilon_1$ and $\varepsilon_2$ to be correlated, with $\mathrm{Cov}(\varepsilon_1, \varepsilon_2) = \rho\sigma_1\sigma_2$. The parameter $\rho \in [-1, 1]$ denotes the correlation index between the divisions' (gross) profits $y_1$ and $y_2$.

*Preferences.* Principals are risk neutral and offer contracts (wages) to their exclusive agents. $P_i$ maximizes his expected (gross) profit net of the wage paid to $A_i$. Thus, $P_i$'s objective function is

$$\mathbb{E}[y_i - w_i(\cdot)], \tag{2}$$

and Tong (2008); Li and Zhang (2008), among many others). We contribute to this literature by studying the impact of divisions internal incentives on cross-divisions communication strategies. In a sense, our approach innovates upon this literature by opening the black-box of divisions.

where $w_i(\cdot)$ denotes $A_i$'s wage, whose structure depends on the transparency choice of each division and will be introduced shortly.

Agents are risk-averse with CARA preferences and additively separable effort cost — i.e., $A_i$'s certainty utility is

$$u_i(w_i, a_i, a_j) = 1 - e^{-r(w_i - \psi_i(a_i, a_j))}.$$

The function $\psi_i(a_i, a_j)$ measures $A_i$'s cost of exerting effort $a_i$, which depends also on $A_j$'s effort. The parameter $r > 0$ indicates the absolute risk-aversion index, which (for simplicity) is common to both agents. Following Faulí-Oller and Giralt (1995) and Berkovitch, Israel, and Spiegel (2010), we assume that

$$\psi_i(a_i, a_j) = \frac{a_i^2}{2} - \delta a_i a_j.$$

This specification for the effort cost includes a standard quadratic component $(a_i^2/2)$ and an interaction term $(\delta a_i a_j)$, which reflects potential externalities that agents may exert on each other. The parameter $\delta \in (-1, 1)$ is key to our analysis: it measures the type of (strategic) interaction between agents' efforts.[3] Agents' efforts are strategic substitute when $\delta < 0$, as they create negative spillovers across business units — e.g., when these units compete for the same budget, or when they operate on the same or related markets. When $\delta > 0$, instead, agents' efforts are strategic complements. This case captures, for example, the positive externalities generated by investments in basic R&D and informative advertising campaigns that benefit not only the unit enacting them, but also the other units of an organization. Alternatively, positive spillovers may emerge when business units jointly invest in essential facilities, such as distributional networks, that allow them to market their products more effectively and reduce operating costs.

*Communication, contracts and timing.* A transparent division discloses its performance to the other division. Principals choose their disclosure rule (all or nothing) independently. Further, we assume that this choice is publicly observable and there is full commitment to it. This gives rise to three scenarios: one in which both principals choose to be transparent *(full communication)*, one in which none does *(no communication)*, and one in which only one does *(partial or one-sided communication)*.

The timing of the game is as follows:

- $(T = 0)$ Principals decide whether to disclose their agents' performance.
- $(T = 1)$ The chosen transparency regime is observed by all players. Principals offer contracts.
- $(T = 2)$ Agents choose efforts, the projects' returns realize and principals disclose information (if they committed to do so). Payments are made.

[3] Note that this interaction is on the intensive margin only. We rule out externalities on the extensive margin which play no role under secret contracts.

Contracts are secret and, hence, have no strategic value.[4] We restrict the analysis to the class of linear contracts — i.e., each principal offers a wage

$$w_i(y_i, y_j) = \alpha_i + \beta_i y_i + \mathbb{I}_j \gamma_i y_j \quad \forall i, j = 1, 2, \tag{3}$$

where the indicator function $\mathbb{I}_j \in \{0, 1\}$ takes value 1 if $P_j$ adopts a transparent regime and shares with $P_i$ the information regarding $A_j$'s performance, and equals 0 otherwise. Hence, $\alpha_i$ is the fixed wage component, $\beta_i$ measures the responsiveness of $A_i$'s pay to his own performance (the standard bonus in this literature), while $\gamma_i$ measures how $A_i$'s wage reacts to $A_j$'s performance (the cross-performance component of the wage).

The reason why we focus on linear contracts is twofold. First, this restriction is standard in the CARA-normal setting — see, e.g., (Bolton and Dewatripont, 2005, Ch. 5). This approach hinges on Hölmstrom and Milgrom (1987), which shows that linear contracts are optimal in a dynamic CARA-normal model if effort is chosen in continuous time by the agent and (at each stage) the principal rewards him based on the overall history of past performances.[5] The same type of argument justifies the use of linear contracts in our setting, because secret contracts imply that managerial compensations have no strategic value. Second, we consider linear contracts to compare our results with those of Maier and Ottaviani (2009), who impose the same restriction in a setting with common agency.

The possibility of conditioning the wage of one agent on the performance of the other *de facto* introduces a simple form of relative performance evaluation. To avoid the possibility that agents (collectively) undo the effect of these contracts we rule out side transfers across divisions. Each player's outside option is normalized to zero.

*Equilibrium concept.* The equilibrium concept is *Perfect Bayesian Equilibrium* (PBE). Since contracts are private, we have to make an assumption on each agent's off-equilibrium beliefs about behavior in the other division. Following most of the literature on private contracts (e.g., Caillaud, Jullien, and Picard (1995) and Martimort (1996)), we assume that agents have *passive beliefs*: regardless of the contract offered by his principal, an agent always believes that the other principal offers the equilibrium contract and the other agent exerts the equilibrium effort. This assumption captures the idea that, since principals are independent and act simultaneously, a principal cannot signal to his agent information that he does not possess about the other principal's contract — i.e., the *no signal what you do not know* requirement introduced by Fudenberg and Tirole (1991).

We solve the model under two natural assumptions that must hold in every possible scenario. The first prescribes that an agent is willing to exert positive effort at equilibrium — i.e., $a_i > 0$. The

[4]The commitment value of observable contracts has been extensively analyzed in the traditional industrial organization literature. We will abstract from this issue by assuming secrecy, which is natural when public contracts can be secretly renegotiated, or when division principals can overturn the contractual rules chosen by the firm's headquarter — see, e.g., Katz (1991).

[5]Although standard in the applied contract theory literature, it must be noted that the restriction to linear contracts is not without loss of generality. Mirrlees (1999) argues that (already in a single principal-agent model) discontinuous contracts are nearly first-best.

second is that an agent's effort cost and its derivative cannot be negative — i.e., $\psi_i(a_i, a_j) \geq 0$ and $\frac{\partial}{\partial a_i}\psi_i(a_i, a_j) \geq 0$ for every admissible pair $(a_i, a_j)$. The latter, in turn, implies that the marginal cost of effort is positive. In the Appendix we provide a formal statement of these assumptions.

*The single principal-agent pair benchmark.* Before turning to the equilibrium analysis, recall that without cross-division spillovers ($\delta = 0$), the agent's effort choice satisfies the first-order condition $a_i = \beta_i$, and the optimal contract entails

$$\beta_i^* = a_i^* = \frac{1}{1 + r\sigma_i^2}.$$

Hence, both a higher volatility ($\sigma_i^2$) and a higher risk aversion index ($r$) induce $P_i$ to offer a low-powered incentive scheme. This is because more uncertainty makes the realized profit $y_i$ a worse indicator of $A_i$'s effort and greater risk-aversion commands a larger risk-premium for the agent — see, e.g., (Bolton and Dewatripont, 2005, Ch. 4) and (Laffont and Martimort, 2002, Ch. 4). Throughout the paper we study how the introduction of the correlation term $\rho$ and the effort interaction parameter $\delta$ shapes equilibrium contracts and transparency.

## 4 Contract design: preliminary insights

In this section we characterize how the contract offered by one principal is affected by the choice on transparency made by the other principal. To this purpose, we first study the effort game between agents and then turn to analyze the principals' contract choices.

*Optimal effort.* Suppose that $A_j$ chooses effort $a_j$ in equilibrium. Using the wage function in (3) together with the performance structure in (1) yields

$$w_i(y_i, y_j) = \alpha_i + \beta_i \underbrace{(a_i + \varepsilon_i)}_{y_i} + \mathbb{I}_j \gamma_i \underbrace{(a_j + \varepsilon_j)}_{y_j}. \tag{4}$$

Hence, agent $A_i$'s certainty equivalent is[6]

$$CE_i(a_i, a_j) \equiv \alpha_i + \beta_i a_i + \mathbb{I}_j \gamma_i a_j - \frac{r}{2}[\sigma_i^2 \beta_i^2 + \mathbb{I}_j(\sigma_j^2 \gamma_i^2 + 2\beta_i \gamma_i \sigma_i \sigma_j \rho)] - \frac{a_i^2}{2} + \delta a_i a_j. \tag{5}$$

[6]Due to the noise in the performance measures, agents receive an uncertain wage for any pair of efforts they choose. When this noise is normally distributed and the gents' utility has the CARA form, it is convenient to carry out the analysis in terms of the certainty equivalent each agent obtains upon choosing a given level of effort, holding fixed the effort of the other agent. By definition, the certainty equivalent is the certain payment that gives an agent the same expected utility obtained with the original gamble:

$$1 - e^{-rCE_i(\cdot)} = 1 - \mathbb{E}\left[e^{-r\left[\alpha_i + \beta_i a_i + \mathbb{I}_j \gamma_i a_j + \beta_i \varepsilon_i + \mathbb{I}_j \gamma_i \varepsilon_j - \frac{a_i^2}{2} + \delta a_i a_j\right]}\right].$$

It follows that the effort level that maximizes $A_i$'s expected utility is

$$a_i(a_j) \equiv \beta_i + \delta a_j, \tag{6}$$

which defines $A_i$'s best reaction to $A_j$'s effort. This function depends in a direct way only on the sensitivity of $A_i$'s wage to own performance $y_i$ — i.e., $\beta_i$ — but not on the cross-performance bonus $\gamma_i$: in fact, $P_j$'s decision on the transparency regime has no direct impact on $A_i$'s optimal effort. This is because the performance of each agent depends exclusively on his own effort and not on that of the other agent.[7] Nevertheless, $A_i$'s reaction function depends on $A_j$'s effort, so the choice of $P_j$ on the transparency regime indirectly affects $A_i$'s effort insofar as it affects $A_j$'s effort, as will be explained shortly.

Using the expressions in (6), it is then easy to prove the following lemma.

**Lemma 1** *Regardless of the transparency regime, if principals are expected to offer $\beta_i$ and $\beta_j$ in equilibrium, the agents' equilibrium effort choices are*

$$a_i(\beta_i, \beta_j) \equiv \frac{\beta_i + \delta\beta_j}{1-\delta^2} \quad \forall i, j = 1, 2, \tag{7}$$

*with $\frac{\partial a_i(\beta_i,\beta_j)}{\partial \beta_i} > 0$, and $\frac{\partial a_i(\beta_i,\beta_j)}{\partial \beta_j} \geq 0$ if, and only if, $\delta \geq 0$.*

The positive sign of the impact of $\beta_i$ on $a_i$ is intuitive. Moreover, note that the impact of $\beta_j$ on $a_i$ depends on the sign of $\delta$. A higher value of $\beta_j$ induces $A_i$ to work more if efforts generate positive spillovers ($\delta > 0$): with complementarity, a higher effort by $A_j$ reduces $A_i$'s marginal cost of effort. Hence, an increase in $\beta_j$ raises $a_j$ and translates into a higher effort's value by agent $A_i$. The opposite holds true when efforts are substitutes and generate negative spillovers ($\delta < 0$).

*Optimal (linear) contracts.* Suppose that $A_j$ is expected to exert effort $a_j$ in equilibrium. Regardless of the first-stage $P_i$'s decision on transparency, $A_i$'s (ex ante) participation constraint binds in equilibrium — i.e., (5) is equal to zero — so that

$$\alpha_i + \beta_i a_i(a_j) + \mathbb{I}_j \gamma_i a_j \equiv \frac{r}{2}[\sigma_i^2\beta_i^2 + \mathbb{I}_j(\sigma_j^2\gamma_i^2 + 2\beta_i\gamma_i\sigma_i\sigma_j\rho)] + \frac{a_i(a_j)^2}{2} - \delta a_i(a_j)a_j. \tag{8}$$

Using the linear performance in (1) together with the wage structure in (4), $P_i$'s objective function is

$$\mathbb{E}[a_i(a_j) + \varepsilon_i - (\alpha_i + \beta_i a_i(a_j) + \mathbb{I}_j\gamma_i a_j + \beta_i\varepsilon_i + \mathbb{I}_j\gamma_i\varepsilon_j)] = a_i(a_j) - [\alpha_i + \beta_i a_i(a_j) + \mathbb{I}_j\gamma_i a_j].$$

[7] See a previous version of the paper for a model in which each agent's effort affects not only his own performance, but also the other agent's performance.

Hence, (8) implies that $P_i$'s maximization problem writes as

$$\max_{(\beta_i,\gamma_i)} \left\{ a_i\left(a_j\right) - \frac{r}{2}\left[\sigma_i^2\beta_i^2 + \mathbb{I}_j(\sigma_j^2\gamma_i^2 + 2\beta_i\gamma_i\sigma_i\sigma_j\rho)\right] - \frac{a_i\left(a_j\right)^2}{2} + \delta a_i\left(a_j\right)a_j \right\}, \tag{9}$$

where $a_i(a_j)$ is given by (6).

The first-order conditions with respect to $\beta_i$ and $\gamma_i$ are, respectively

$$\frac{\partial a_i\left(a_j\right)}{\partial \beta_i} = r\left[\sigma_i^2\beta_i + \mathbb{I}_j\gamma_i\sigma_i\sigma_j\rho\right] + \frac{\partial a_i\left(a_j\right)}{\partial \beta_i}\left[a_i\left(a_j\right) - \delta a_j\right], \tag{10}$$

$$-r\mathbb{I}_j(\sigma_j^2\gamma_i + \beta_i\sigma_i\sigma_j\rho) = 0, \tag{11}$$

where $\frac{\partial a_i(a_j)}{\partial \beta_i} = 1$ by equation (6). Indeed, because contracts are secret, any (out of equilibrium) change in $\beta_i$ affects $A_i$'s effort only through its direct effect on (6) — i.e., holding $A_j$'s effort fixed at the (conjectured) equilibrium level.

These first-order conditions highlight how the basic trade-off between risk and incentives changes in a multidivisional firm in which, due to transparency, the principal of each division may tailor his agent's compensation to the performance of the agents working for other divisions. The first-order condition with respect to $\beta_i$ in equation (10) has a simple interpretation. Its left-hand side is standard and represents the marginal benefit associated with an increase in the own performance bonus. The two terms on the right-hand side capture the impact of a higher bonus on $A_i$'s risk premium and effort cost. First, a higher bonus $\beta_i$ makes $A_i$ more responsive to his own performance. Hence, it increases the agent's risk exposure and calls for higher insurance: a standard cost of providing high-powered incentives. Other things being equal, this extra cost depends not only on $A_i$'s risk aversion ($r$) and the volatility of division-$i$'s profit ($\sigma_i$), but also on whether principal $P_j$ commits to disclose information. If he does, agent $A_i$'s insurance will also depend on the sensitivity of his wage to $A_j$'s performance ($\gamma_i$), the correlation index ($\rho$) and the volatility of division-$j$'s profit ($\sigma_j$). The second term on the right-hand side of (10) shows that, by increasing agent $A_i$'s effort, a higher own-performance bonus makes it more costly for the agent to exert effort. Also, it illustrates the effect of the interaction term $\delta a_j$ on $P_i$'s optimal contract. When $\delta$ is negative, so that efforts are strategic substitutes, principal $P_i$ is less willing to offer a high-powered incentive if agent $A_j$ is expected to exert high effort in equilibrium. The opposite holds when efforts are complements ($\delta > 0$).

The first-order condition with respect to $\gamma_i$ in (11) represents the main novelty of introducing the choice on transparency in a setting with multiple divisions. Clearly, this condition matters only if $P_j$ discloses $A_j$'s performance to $P_i$ — i.e., if $\mathbb{I}_j = 1$. Two effects determine the cross-performance bonus $\gamma_i$. First, by making $A_i$'s compensation more responsive to $A_j$'s performance, $P_i$ induces $A_i$ to take a higher risk, for which he needs to be compensated. Second, there is a risk-diversification effect: when $A_i$'s wage is tailored to his opponent's performance, increasing $\gamma_i$ spurs $P_i$'s expected profits as long as the agents' performances are negatively correlated ($\rho < 0$), whereas it decreases $P_i$'s expected profits if the agents' performances are positively correlated ($\rho > 0$). The strength of this risk-diversification effect clearly depends on the magnitude of the own-performance bonus $\beta_i$.[8] If $\beta_i$ is equal to zero then there is no need for risk-diversification because $A_i$'s wage does not depend on $A_j$'s performance. By contrast, when $\beta_i$ is positive, $A_i$'s wage positively depends on his own performance. Hence, to diversify $A_i$'s risk exposure, $P_i$ chooses a positive cross-performance bonus $\gamma_i$ when $\varepsilon_1$ and $\varepsilon_2$ are negatively correlated, and a negative one when they are positively correlated.

## 5 Equilibrium analysis

We now characterize the equilibrium efforts and contracts chosen in every subgame following the principals' (first-stage) communication decisions.

### 5.1 No communication

Consider the subgame in which none of the divisions commits to disclose its performance — i.e., $\mathbb{I}_i = \mathbb{I}_j = 0$ — and let $\alpha_i^n + \beta_i^n y_i$ be the wage offered by $P_i$.

**Proposition 1** *In the regime without communication*

$$\beta_i^n \equiv \frac{1}{1 + r\sigma_i^2} = \beta_i^* \quad \forall i = 1, 2.$$

*At equilibrium, agent $A_i$ exerts effort*

$$a_i^n \equiv \frac{1 + r\sigma_j^2 + \delta(1 + r\sigma_i^2)}{\left(1 - \delta^2\right)\left(1 + r\sigma_i^2\right)\left(1 + r\sigma_j^2\right)} \geq 0 \quad \forall i = 1, 2.$$

*Moreover, $a_i^n \geq a_j^n$ if, and only if, $\sigma_i^2 \leq \sigma_j^2$, $a_i^n$ decreases with $\sigma_i^2$, and it increases with $\sigma_j^2$ if, and only if, $\delta < 0$.*

Because we assumed unobservable contracts and passive beliefs, principals offer the same bonus as in the standard principal-agent problem (which obtains when $\delta = 0$) — i.e. $\beta_i^n = \beta_i^*$. This is because the effort of agent $A_i$ reacts to a change in its own bonus $\beta_i$ only via its direct impact on $a_i$ — see equation (6).

Finally, using the equilibrium condition (7) and the expression for the equilibrium bonus $\beta_i^n$, we compare the agents' efforts to that exerted in the single principal-agent model.

**Corollary 1** *If there are positive effort spillovers, $\delta \geq 0$, then $a_i^n \geq a_i^*$. By contrast, if there are negative spillovers, $\delta < 0$, then $a_i^n \geq a_i^*$ if $\sigma_j^2$ is large enough.*

Both agents exert more effort than in the single principal-agent model when spillovers are positive. That is, $a_i^n > a_i^*$ when $\delta > 0$, with $a_i^n = a_i^*$ if $\delta = 0$. The reason is that principals

[8] For the sake of exposition, we assume here that $\beta_i$ is positive. We later verify that in equilibrium $\beta_i > 0$.

can exploit the synergies between their agents to implement higher efforts at lower costs. When, instead, efforts are substitutes the result is non-obvious because of the strategic effect linking the agents' effort choices. The result shows that, in the regime without communication, $a_i^n$ exceeds $a_i^*$ when $\sigma_j^2$ is large enough. This is because, since $A_j$'s effort is decreasing in $\sigma_j^2$, as $\sigma_j^2$ increases the division-$j$'s monitoring power weakens and the externality that $A_j$ imposes on $P_i$ becomes negligible.

### 5.2 Full transparency

Consider the subgame in which both divisions disclose their performance — i.e., $\mathbb{I}_i = \mathbb{I}_j = 1$ — and let $\alpha_i^t + \beta_i^t y_i + \gamma_i^t y_j$ be $P_i$'s wage offer.

**Proposition 2** *Assume that both principals choose to be transparent, then*

$$\beta_i^t \equiv \frac{1}{1 + r\sigma_i^2 (1-\rho^2)} \quad \forall i = 1, 2, \tag{12}$$

$$\gamma_i^t \equiv -\frac{\sigma_i}{\sigma_j} \frac{\rho}{1 + r\sigma_i^2 (1-\rho^2)} \quad \forall i, j = 1, 2. \tag{13}$$

*Hence,* $\beta_i^t \geq \beta_i^n = \beta_i^*$ *and* $\gamma_i^t \geq 0$ *if, and only if,* $\rho \leq 0$. *Moreover,*

$$a_i^t \equiv \frac{1 + \delta + r(1-\rho^2)(\sigma_j^2 + \delta\sigma_i^2)}{(1-\delta^2)(1 + r\sigma_i^2 (1-\rho^2))(1 + r\sigma_j^2 (1-\rho^2))} \geq 0 \quad \forall i, j = 1, 2,$$

*with* $a_i^t \geq a_j^t$ *if, and only if,* $\sigma_j \geq \sigma_i$. *Effort* $a_i^t$ *decreases with* $\sigma_i^2$, *while it increases with* $\sigma_j^2$ *if, and only if,* $\delta < 0$.

As for the case without communication (Proposition 1), and for the same reasons discussed above, the equilibrium contracts offered by $P_i$ and $P_j$ do not depend on the interaction parameter $\delta$. However, the equilibrium contracts under full transparency differ from those obtained in the no communication regime along two fundamental dimensions. First, since the divisions' performances are correlated, with full transparency principals can optimally diversify the risk taken by their agents, thereby reducing agency costs and implementing steeper incentive schemes — i.e., $\beta_i^t \geq \beta_i^n = \beta_i^*$. Moreover, one can easily verify that

$$\frac{\partial \beta_i^t}{\partial \rho} = \frac{2r\rho\sigma_i^2}{\left(1 + r\sigma_i^2 (1-\rho^2)\right)^2} \geq 0 \quad \Leftrightarrow \quad \rho \geq 0.$$

A U-shaped relationship arises between the own-performance coefficient $\beta_i^t$ and the correlation index $\rho$: when the divisions' performances are strongly correlated (either positively or negatively) principals exploit more heavily risk-diversification to reduce agency costs. This, in turn, allows them to provide steeper incentives.

Equation (13) establishes the link between $A_i$'s equilibrium wage and $A_j$'s performance. The sign of this coefficient depends on the correlation index $\rho$ and again hinges on the risk-diversification logic discussed above. When performances are positively correlated ($\rho > 0$) a bad performance by $A_j$ likely causes $A_i$ to underperform. Hence, principal $P_i$ rewards his agent when division $j$ underperforms ($y_j < 0$). Thus, $\gamma_i^t < 0$ in this case. By the same token, when performances are negatively correlated ($\rho < 0$) risk-diversification induces $P_i$ to reward $A_i$ when the other division performs well — i.e., $\gamma_i^t > 0$. Finally, notice that $\gamma_i^t$ is decreasing in $\sigma_j^2$: receiving information from a division whose performance is noisy has little value. As a consequence, the incentive scheme is optimally less responsive to such information.

Differentiating with respect to $\rho$ and $\sigma_i$, one can also verify that

$$\begin{aligned} \frac{\partial \gamma_i^t}{\partial \rho} &= -\frac{\sigma_i}{\sigma_j}\frac{1+\sigma_i^2 r\left(1+\rho^2\right)}{\left(1+r\sigma_i^2\left(1-\rho^2\right)\right)^2} < 0, \\ \frac{\partial \gamma_i^t}{\partial \sigma_i} &= -\frac{\rho\left(1-r\sigma_i^2\left(1-\rho^2\right)\right)}{\sigma_j\left(1+r\sigma_i^2\left(1-\rho^2\right)\right)^2} \geq 0 \quad \Leftrightarrow \quad -\rho\left[2-\frac{1}{\beta_i^t}\right] \geq 0. \end{aligned} \tag{14}$$

The derivative in (14) measures the impact of the correlation index $\rho$ on the cross-performance bonus $\gamma_i^t$. As $\rho$ grows larger in absolute value, the divisions' profits become more correlated. Thus, to diversify risk, principal $P_i$ responds with a reduction of the (absolute) value of $\gamma_i^t$ if performances are positively correlated, and with an increase in $\gamma_i^t$ otherwise.

The impact of $\sigma_i$ on $\gamma_i^t$ is shaped by two contrasting effects. First, holding $\beta_i$ fixed, a larger $\sigma_i$ implies more need for risk-diversification because agent $A_i$ takes more risk. Hence, $\gamma_i$ must increase in absolute value: a direct risk-diversification effect. Second, a larger $\sigma_i$ implies a lower $\beta_i$ because division-$i$'s performance is noisier. This induces a lower $\gamma_i$ because agent $A_i$ takes less risk: an indirect risk-shifting effect. The tension between these two effects depends on the sign of $\rho$ and the magnitude of the own-performance bonus $\beta_i^t$. Assume first that $\rho > 0$, so that $\gamma_i^t < 0$. Then, a larger $\sigma_i$ tends to increase $\gamma_i^t$ in absolute value when $\beta_i^t < \frac{1}{2}$, because the direct risk-diversification effect dominates the indirect risk-shifting effect. Next, assume $\rho < 0$, so that $\gamma_i^t > 0$. In this case, a larger $\sigma_i$ tends to increase $\gamma_i^t$ when $\beta_i^t < \frac{1}{2}$, because the direct risk-diversification effect dominates the indirect risk-shifting effect.

Finally, combining condition (7) with the equilibrium bonus $\beta_i^t$, Corollary 2 below compares agents' efforts in the full transparency regime to the equilibrium effort in the benchmark model.

**Corollary 2** *If there are positive effort spillovers, $\delta \geq 0$, then $a_i^t \geq a_i^*$. By contrast, if there are negative spillovers, $\delta < 0$, then $a_i^t \geq a_i^*$ if, and only if, $\sigma_j^2$ is large enough.*

The economic intuition behind these results is the same as that offered for Corollary 1.

### 5.3 Partial (one-sided) communication

Finally, consider the subgame in which only one principal ($P_1$, say) chooses to be transparent. Principal $P_2$ has a competitive advantage: he can use the additional information provided by $P_1$ to control $A_2$'s effort, whereas $P_1$ can only condition $A_1$'s wage on his own performance. Let $\alpha_1^{t,n} + \beta_1^{t,n} y_1$ be $P_1$'s wage offer and $\alpha_2^{n,t} + \beta_2^{n,t} y_2 + \gamma_2^{n,t} y_1$ be $P_2$'s wage offer in equilibrium.

**Proposition 3** *When only $P_1$ is transparent $\beta_1^{t,n} = \beta_1^n$, $\beta_2^{n,t} = \beta_2^t$ and $\gamma_2^{n,t} = \gamma_2^t$. Agent $A_i$'s effort decreases with $\sigma_i^2$. Moreover, it increases with $\sigma_j^2$ if, and only if, $\delta < 0$.*

If only one principal commits to be transparent, the equilibrium contracts are the same as those in Propositions 1 and 2 above. However, the principal that receives information copes more effectively with the moral hazard problem he has with his own agent, as compared to the other principal who draws inference about his agent's effort based on his own performance only.

Finally, using the equilibrium condition (7), it can be easily shown that

$$a_1^{t,n} \equiv \frac{\beta_1^n + \delta\beta_2^t}{1-\delta^2} = \frac{1+\delta+r\delta\sigma_1^2 + r\sigma_2^2(1-\rho^2)}{\left(1-\delta^2\right)\left(1+r\sigma_1^2\right)\left(1+r\sigma_2^2\left(1-\rho^2\right)\right)},$$

$$a_2^{n,t} \equiv \frac{\beta_2^t + \delta\beta_1^n}{1-\delta^2} = \frac{1+\delta+r\sigma_1^2 + r\sigma_2^2\delta(1-\rho^2)}{\left(1-\delta^2\right)\left(1+r\sigma_1^2\right)\left(1+r\sigma_2^2\left(1-\rho^2\right)\right)},$$

implying that

$$a_i^{t,n} - a_i^t = \frac{1}{1-\delta^2}\left[\beta_i^n - \beta_i^t\right] \leq 0,$$

and

$$a_i^{n,t} - a_i^n = \frac{1}{1-\delta^2}\left[\beta_i^t - \beta_i^n\right] \geq 0.$$

Hence, agent $A_i$'s effort is higher when principal $P_j$ commits to be transparent, regardless of principal $P_i$'s disclosure decision.

## 6 Communication at equilibrium

Solving the model backward, in this section we characterize the equilibrium of the whole game. We first study how the transparency choice of one principal affects his agent's effort, holding fixed the transparency choice of the other principal. The insights offered by this simple exercise are useful to understand the forces that drive the equilibrium outcome of the game.

**Proposition 4** *Effort choices satisfy the following properties:*

*(i)* $a_i^t \geq a_i^{n,t}$ *for every $i = 1, 2$ if, and only if, $\delta \geq 0$;*

*(ii)* $a_i^n \geq a_i^{t,n}$ *for every $i = 1, 2$ if, and only if, $\delta \leq 0$.*

Holding fixed $P_j$'s (first-stage) behavior, $P_i$'s decision to be transparent increases $A_i$'s effort if and only if agents exert positive externalities one on the other ($\delta \geq 0$). In fact, the information that $P_i$ discloses to $P_j$ is used to increase $A_j$'s effort, which in turn boosts $A_i$'s effort due to complementarity. The opposite is true when efforts are strategic substitutes ($\delta \leq 0$).

Building on this result, we can now characterize the equilibrium of the game. Principals' expected profits when they choose to disclose information ($t$) or not ($n$) are illustrated in Figure 1.

| | | $P_2$ | |
|---|---|---|---|
| | | $t$ | $n$ |
| $P_1$ | $t$ | $\pi_1^t, \pi_2^t$ | $\pi_1^{t,n}, \pi_2^{n,t}$ |
| | $n$ | $\pi_1^{n,t}, \pi_2^{t,n}$ | $\pi_1^n, \pi_2^n$ |

Figure 1: Communication Game.

Observe first that transparency does not affect principals' expected profits when there are no effort externalities ($\delta = 0$) or when divisions' performances are uncorrelated ($\rho = 0$). If there is no strategic interaction between the agents, communication only allows principals to enforce welfare enhancing relative performance evaluations. Hence, it is easy to verify that principals coordinate on the equilibrium with full transparency, which clearly maximizes their joint profits. Similarly, if division performances are uncorrelated, disclosing information about own performance to the other division has no impact on the effort choice of that division's agent. Hence, for $\rho = 0$ principals are indifferent between being transparent or not. By contrast, when $\delta$ and $\rho$ are different from zero strategic considerations shape the divisions' equilibrium behavior. We now discuss them and find the conditions under which communication occurs at equilibrium.

Consider first an outcome in which both principals choose to be transparent. This is an equilibrium if, and only if, the following holds

$$\pi_i^t \geq \pi_i^{n,t} \quad \forall i = 1, 2.$$

Using the objective function (9), the above condition can be split in two parts

$$\pi_i^t - \pi_i^{n,t} \equiv \underbrace{a_i^t - \psi(a_i^t, a_j^t) - \left[a_i^{n,t} - \psi(a_i^{n,t}, a_j^t)\right]}_{\text{Strategic Effect}} + \underbrace{\delta a_i^{n,t}\left[a_j^t - a_j^{t,n}\right]}_{\text{Fixed-fee Effect}} \geq 0. \tag{15}$$

First, principal $P_i$'s decision to be transparent has an indirect effect on agent $A_i$'s effort: the information disclosed to $P_j$ is used to increase $A_j$'s effort, which indirectly affects $A_i$'s effort. *De facto*, the disclosure of a division's performance can be interpreted as a commitment device to rise the rival's effort. This gives rise to a strategic effect that is captured by the first term in equation (15). Specifically, the difference in the first term measures $P_i$'s gain from disclosing information holding $A_j$'s effort equal to its (candidate) equilibrium level $a_j^t$. Moreover, when $P_i$ commits to be transparent he is also directly affecting $A_i$'s expected utility. This is because the information he discloses impacts on $A_j$'s effort, which in turn determines $A_i$'s effort cost. Hence, holding $A_i$'s effort equal to its deviation level $a_i^{n,t}$, any change in $a_j$ induced by $P_i$'s first-stage decision modifies the fixed component of $A_i$'s wage, which impacts on division $i$'s expected profit.

Next, consider an outcome of the game in which both principals refrain from being transparent. This is an equilibrium if, and only if, the following holds

$$\pi_i^n \geq \pi_i^{t,n} \quad \forall i = 1, 2.$$

Using the objective function stated in equation (9) the above condition can be rewritten as

$$\pi_i^n - \pi_i^{t,n} \equiv \underbrace{a_i^n - \psi(a_i^n, a_j^n) - \left[a_i^{t,n} - \psi(a_i^{t,n}, a_j^n)\right]}_{\text{Strategic Effect}} \underbrace{- \delta a_i^{t,n}\left[a_j^{n,t} - a_j^n\right]}_{\text{Fixed-fee Effect}} \geq 0. \tag{16}$$

Again, principal $P_i$'s incentive not to disclose information can be split into two components, reflecting the role of the strategic and the fixed-fee effects discussed above. First, when principal $P_i$ refuses to be transparent, he gives up the possibility of influencing agent $A_j$'s effort choice through $P_j$'s contract. Thus, depending on whether efforts are strategic complements or substitutes, by refusing to communicate $P_i$ decreases or increases $A_i$'s performance. Second, holding fixed $A_i$'s effort, when principal $P_i$ decides not to disclose information he shifts agent $A_j$'s effort downward, which has an impact on $A_i$'s effort cost thereby affecting the fixed component of the wage that $P_i$ must pay to $A_i$.

It turns out that the direction of the effects just described and the signs of (15) and (16) uniquely depend on the sign of $\delta$.

**Proposition 5** *For any admissible value of $\delta$, the equilibrium exists and is unique. It features full communication if $\delta \geq 0$, and no communication if $\delta < 0$.*

To understand the result, consider first a fully transparent equilibrium — i.e., equation (15). The proposition shows that both effects discussed above are positive when efforts are strategic complements. In fact, $P_i$'s decision to share information increases $A_j$'s equilibrium effort, which, because $\delta > 0$, leads $A_i$ to choose a higher effort, and, at the same time, allows $P_i$ to reduce the fixed component of $A_i$'s wage. Thus, when efforts are strategic complements, principals will communicate at equilibrium.

What happens if efforts are strategic substitutes? Note that, even in this case, $P_i$'s decision to disclose information spurs $A_j$'s effort because performances are correlated and the risk-diversification effect discussed above allows $P_j$ to lower agency costs and increase his agent's effort. This means that the sign of the fixed-fee effect is negative when $\delta < 0$, because $a_j^t > a_j^{t,n}$ from Proposition 4. Instead, the sign of the strategic effect is ambiguous: even if the choice of not disclosing information allows $P_i$ to increase $A_i$'s effort — i.e., $a_i^t < a_i^{n,t}$ for $\delta < 0$ — this does not increase $P_i$'s profits. In fact, when efforts are strategic substitutes and $A_j$ exerts high effort, it is in $P_i$'s best interest to keep $A_i$'s effort low and take advantage of the performances' correlation. Indeed, Proposition 5 establishes that the negative effects of transparency prevail on the positive ones for both principals when efforts are substitutes, thereby excluding the full transparency regime from the possible equilibrium outcomes when $\delta < 0$.

Consider now an equilibrium without communication — i.e., equation (16). If efforts are strategic substitutes, the strategic effect and the fixed-fee effect are both positive. By disclosing information principal $P_i$ indirectly spurs $A_j$'s effort, which increases $A_i$'s (marginal) cost of effort because $\delta < 0$. This reduces $A_i$'s effort, meaning that the strategic effect is positive. At the same time, it increases the fixed component of the wage paid to $A_i$, implying that also the fixed-fee effect is positive. As a consequence, if efforts are substitutes equation (16) holds true and principals do not communicate at equilibrium.

What happens when efforts are strategic complements? Again, although the sign of the fixed-fee effect is negative,[9] that of the strategic effect is ambiguous. In fact, when efforts are strategic complements, other things being equal, principal $P_i$ would like to disclose information in order to increase agent $A_i$'s effort and exploit complementarity. Proposition 5 establishes that the net effect is negative for both principals, so that no communication is not an equilibrium when efforts are complements ($\delta > 0$).

Finally, it is clear from the above discussion that asymmetric equilibria with unilateral information sharing cannot exist.

## 7 Concluding remarks

The analysis developed in this paper has offered novel insights about: the determinants of divisions' incentives to share information about their performances; the link between the power of incentives, efforts and cross-division externalities; the impact of monitoring and contractual power on their internal structure. The results have been derived under a few simplifying assumptions that are worth discussing. First, we assumed that contracts are linear. It is well known that discontinuous contracts (wages) might perform better. Given the hypothesis of secret contracts, we believe that this property is likely to remain valid also in our framework. Second, we have assumed that principals commit to disclose information at the outset of the game. More generally, it would be interesting to know how these incentives change if principals lack this commitment power. Our conjecture is that, without commitment, each principal may strategically select the states of nature to be disclosed ex-post, so as to influence the effort of the other agent to his own advantage. Third, in our model agents are ex-ante identical and divisions do not compete to attract them. Clearly, when divisions compete to attract efficient types, information disclosure about past performances may act as a signal device that makes rivals aware of own agents' productivity. Hence, divisions with lower cash flows may be unable to retain efficient agents; this may create an assortative matching that could be worth investigating. Finally, a somewhat natural extension of the model worth exploring is its infinitely repeated version in which contract and information sharing decisions may allow principals to achieve more cooperative outcomes. One question that could be analyzed in that extended framework is whether information disclosure about agents' performances is substitute or complement to information disclosure about contracts. We leave these questions to future research.

[9] That is, the sign of this effect is the opposite of the one we obtain with strategic complementarities, because $a_j^{n,t} > a_j^n$ when $\delta > 0$.

## References


Aghion, P., and J. Tirole (1995): "Some implications of growth for organizational form and ownership structure," *European Economic Review*, 39, 440–455.

Baldenius, T., and S. Reichelstein (2006): "External and Internal Pricing in Multidivisional Firms," *Journal of Accounting Research*, 44(1), 1–28.

Berkovitch, E., R. Israel, and Y. Spiegel (2010): "A Double Moral Hazard Model of Organization Design," *Journal of Economics & Management Strategy*, 19(1), 55–85.

Bertoletti, P., and C. Poletti (1996): "A Note on Endogenous Firm Efficiency in Cournot Models of Incomplete Information," *Journal of Economic Theory*, 71, 303–310.

Besanko, D., P. Régibeau, and K. E. Rockett (2005): "A multi-task principal-agent approach to organizational form," *Journal of Economic Theory*, 53(4), 437–467.

Bolton, P., and M. Dewatripont (2005): *Contract Theory*. MIT Press.

Caillaud, B., B. Jullien, and P. Picard (1995): "Competing Vertical Structures: Precommitment and Renegotiation," *Econometrica*, 63(3), 621–646.

Calzolari, G., and A. Pavan (2006): "On the optimality of privacy in sequential contracting," *Journal of Economic Theory*, 130, 168–204.

Clarke, R. N. (1983): "Collusion and the Incentives for Information Sharing," *The Bell Journal of Economics*, 14(2), 383–394.

Faulí-Oller, R., and M. Giralt (1995): "Competition and Cooperation Within a Multidivisional Firm," *Journal of Industrial Economics*, 43(1), 77–99.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. MIT Press, Cambridge, MA.

Gal-Or, E. (1985): "Information Sharing in Oligopoly," *Econometrica*, 53(2), 329–343.

Griffin, A., and J. R. Hauser (1992): "Patterns of Communication Amond Marketing, Engineering and Manifacturing — A Comparison Between Two New Product Teams," *Management Science*, 38(3), 360–373.

Groves, T., and M. Loeb (1979): "Incentives in a Divisionalized Firm," *Management Science*, 25(3), 221–230.

Ha, A. Y., and S. Tong (2008): "Contracting and Information Sharing Under Supply Chain Competition," *Management Science*, 54(4), 701–715.

Harris, M., C. H. Kriebel, and A. Raviv (1982): "Asymmetric Information, Incentives and Intrafirm Resource Allocation," *Management Science*, 28(6), 604–620.

Hirshleifer, J. (1957): "Economics of the Divisionalized Firm," *The Journal of Business*, 30(2), 96–108.

Hölmstrom, B., and P. Milgrom (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55(2), 303–328.

Katz, M. L. (1991): "Game-Playing Agents: Unobservable Contracts as Precommitments," *The RAND Journal of Economics*, 22(3), 307–328.

Laffont, J.-J., and D. Martimort (2002): *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press.

Li, L., and H. Zhang (2008): "Confidentiality and Information Sharing in Supply Chain Coordination," *Management Science*, 54(8), 1467–1481.

Maier, N., and M. Ottaviani (2009): "Information Sharing in Common Agency: When is Transparency Good?," *Journal of the European Economic Association*, 7(1), 162–187.

Martimort, D. (1996): "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory," *The RAND Journal of Economics*, 27(1), 1–31.

Maskin, E., Y. Qian, and C. Xu (2000): "Incentives, Information, and Organizational Form," *Review of Economic Studies*, 67(2), 359–378.

Milgrom, P. R. (1981): "Good News and Bad News: Representation Theorems and Applications," *The Bell Journal of Economics*, 12(2), 380–391.

Mirrlees, J. A. (1999): "The Theory of Moral Hazard and Unobservable Behaviour: Part I," *Review of Economic Studies*, 66(1), 3–21.

Novshek, W., and H. Sonnenschein (1982): "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release," *The Bell Journal of Economics*, 13(1), 214–218.

Piccolo, S., and M. Pagnozzi (2013): "Information sharing between vertical hierarchies," *Games and Economic Behavior*, 79, 201–222.

Qian, Y., G. Roland, and C. Xu (2006): "Coordination and Experimentation in M-Form and U-Form Organizations," *Journal of Political Economy*, 114(2), 366–402.

Roberts, R. W. (1992): "Determinants of corporate social responsibility disclosure: an application of stakeholder theory," *Accounting Organizations and Society*, 17(6), 595–612.

Rotemberg, J. J. (1999): "Process- versus Function-Based Hierarchies," *Journal of Economics & Management Strategy*, 8(4), 453–487.

Vives, X. (1984): "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, 34, 71–94.

Wettstein, D. (1994): "Allocation of resources in a divisionalized firm," *Mathematical Social Sciences*, 28, 51–58.

# Appendix

**Preliminaries.** Before proving the results stated in the main body of the paper, we detail the conditions that guarantee that equilibrium efforts, the cost and the marginal cost of effort are positive in every admissible outcome of the game. These conditions define the region of parameters to which we restrict the analysis.

Using the expressions for the equilibrium efforts, it can be easily verified that they are positive as long as the following assumption holds

$$\textbf{A1:} \qquad 1+\delta+r(\sigma_j^2\left(1-\rho^2\right)+\delta\sigma_i^2)>0 \quad \forall i,j=1,2,$$

which implies that an agent whose principal adopts a regime with transparency, but does not receive information, exerts positive effort.

Moreover, the effort cost, $\psi_i\left(a_i,a_j\right)$, is positive if, and only if, $2\delta<\frac{a_i}{a_j}$, while the marginal cost is positive if, and only if, $\delta<\frac{a_i}{a_j}$, which is implied by the former condition. Using the expression for the equilibrium efforts, a sufficient condition for the effort cost to be (strictly) positive is

$$\textbf{A2:} \qquad 2\delta<\frac{1+\delta+r(\delta\sigma_i^2+(1-\rho^2)\sigma_j^2)}{1+\delta+r(\sigma_i^2+\delta(1-\rho^2)\sigma_j^2)} \quad \forall i,j=1,2,$$

which implies that an agent whose principal does not disclose but receives information incurs in effort costs. More generally, **A2** guarantees that the cost of effort is always positive in equilibrium, even when there are positive externalities between the agent's choices so that the net cost of effort is attenuated. Note that, given assumption **A1**, condition **A2** is compelling only if $\delta>0$.

**Proof of Lemma 1.** The proof of this result obtains by combining the agents' first-order conditions (6). The comparative statics is immediate. ■

**Proof of Proposition 1.** Suppose that principals do not communicate. The first-order conditions in (10) rewrite as

$$1-\delta^2=\left(1-\delta^2\right)r\sigma_i^2\beta_i+\left[\beta_i+\delta\beta_j^n-\delta(\beta_j^n+\delta\beta_i)\right] \quad \forall i,j=1,2,$$

whose solution yields

$$\beta_i^n=\frac{1}{1+r\sigma_i^2},$$

which implies that $\beta_s^n=\beta_i^*$. Substituting $\beta_i^n$ and $\beta_j^n$ into $a_i\left(\beta_i,\beta_j\right)$, we obtain that

$$a_i^n=\frac{1+r\sigma_j^2+\delta(1+r\sigma_i^2)}{\left(1-\delta^2\right)\left(1+r\sigma_i^2\right)\left(1+r\sigma_j^2\right)}.$$

Differentiating with respect to $\sigma_i^2$ and $\sigma_j^2$, we have that

$$\frac{\partial a_i^n}{\partial\sigma_i^2}=-\frac{r}{(1-\delta^2)(1+r\sigma_i^2)^2}<0,$$

and

$$\frac{\partial a_i^n}{\partial \sigma_j^2} = -\frac{r\delta}{(1-\delta^2)(1+r\sigma_i^2)^2} \geq 0 \quad \Leftrightarrow \quad \delta \leq 0.$$

Direct comparison between $a_i^n$ and $a_j^n$ yields

$$a_i^n - a_j^n = \frac{(\sigma_i + \sigma_j)(\sigma_j - \sigma_i)\, r}{(1+\delta)\left(1+r\sigma_i^2\right)(1+r\sigma_j^2)} \geq 0 \quad \Leftrightarrow \quad \sigma_i \leq \sigma_j.$$

Finally, it can be easily verified that, under assumptions **A1** and **A2**, $a_i^n > 0$ and $a_i^n - \delta a_j^n > 0$ for each $i, j = 1, 2$, which concludes the proof. ■

**Proof of Corollary 1.** The result that $a_i^n \geq a_i^*$ when $\delta \geq 0$ follows from the definition of the function $a_i(\beta_i^n, \beta_j^n)$ and the fact that $\beta_i^n = \beta_i^*$. Consider thus the case with $\delta < 0$. Then

$$a_i^n \geq a_i^* \quad \Leftrightarrow \quad a_i^n = \frac{1 + r\sigma_j^2 + \delta(1+r\sigma_i^2)}{\left(1-\delta^2\right)\left(1+r\sigma_i^2\right)(1+r\sigma_j^2)} \geq \frac{1}{1+r\sigma_i^2}.$$

First, recall that $\frac{\partial a_i^n}{\partial \sigma_j^2} > 0$ for $\delta < 0$ by Proposition 1. Second, observe that

$$\lim_{\sigma_j \to +\infty} (a_i^n - a_i^*) = \frac{\delta^2}{\left(1-\delta^2\right)(1+r\sigma_i^2)} > 0,$$

$$\lim_{\sigma_j \to 0} (a_i^n - a_i^*) = \frac{\delta\left(1+\delta+r\sigma_i^2\right)}{\left(1-\delta^2\right)\left(1+r\sigma_i^2\right)} < 0.$$

Hence, by the mean-value theorem there exists a threshold $\sigma_j^* > 0$ such that $a_i^n \geq a_i^*$ if $\sigma_j \geq \sigma_j^*$. ■

**Proof of Proposition 2.** Suppose that both principals choose to be transparent. The first-order conditions in (10)-(11) rewrite as

$$\begin{aligned} 1-\delta^2 &= \left(1-\delta^2\right) r(\sigma_i^2\beta_i + \gamma_i\sigma_i\sigma_j\rho) + [\beta_i + \delta\beta_j^t - \delta(\beta_j^t + \delta\beta_i)] \quad \forall i, j = 1, 2, \\ 0 &= \sigma_j^2\gamma_i + \beta_i\sigma_i\sigma_j\rho \quad \forall i, j = 1, 2, \end{aligned}$$

whose solution yields

$$\begin{aligned} \beta_i^t &= \frac{1}{1+r\sigma_i^2\,(1-\rho^2)}, \\ \gamma_i^t &= -\frac{\sigma_i}{\sigma_j}\,\frac{\rho}{1+r\sigma_i^2\,(1-\rho^2)}. \end{aligned}$$

Hence, $\beta_i^t \geq \beta_i^n = \beta_i^*$ for all $\rho \in (-1, 1)$. Moreover, $\gamma_i^t \geq 0$ if, and only if, $\rho \leq 0$. Substituting $\beta_i^t$ and $\beta_j^t$ into $a_i(\beta_i, \beta_j)$, we then obtain

$$a_i^t = \frac{1 + \delta + r(1-\rho^2)(\sigma_j^2 + \delta\sigma_i^2)}{\left(1-\delta^2\right)(1+r\sigma_i^2\,(1-\rho^2))(1+r\sigma_j^2\,(1-\rho^2))}.$$

Differentiating with respect to $\sigma_i^2$ and $\sigma_j^2$, we find that

$$\begin{aligned}\frac{\partial a_i^t}{\partial \sigma_i^2} &= -\frac{\left(1-\rho^2\right) r}{(1-\delta^2)(1+r\sigma_i^2\left(1-\rho^2\right))^2} < 0,\\ \frac{\partial a_i^t}{\partial \sigma_j^2} &= -\frac{\left(1-\rho^2\right) r\delta}{(1-\delta^2)(1+r\sigma_i^2\left(1-\rho^2\right))^2} \geq 0 \quad \Leftrightarrow \quad \delta \leq 0.\end{aligned}$$

Direct comparison of $a_i^t$ and $a_j^t$ yields

$$a_i^t - a_j^t = \frac{\left(1-\rho^2\right)\left(\sigma_i+\sigma_j\right)\left(\sigma_j-\sigma_i\right) r}{(1+\delta)(1+r\sigma_i^2\left(1-\rho^2\right))(1+r\sigma_j^2\left(1-\rho^2\right))} \geq 0 \quad \Leftrightarrow \quad \sigma_i \leq \sigma_j.$$

Finally, it can be easily verified that under assumptions **A1** and **A2** $a_i^t > 0$ and $a_i^t - \delta a_j^t > 0$ for each $(i,j) = 1,2$, which concludes the proof. ■

**Proof of Corollary 2.** The proof of this result follows the same logic as that offered to prove Corollary 1. ■

**Proof of Proposition 3.** The proof of this result follows the same logic as in the proofs of Propositions 1 and 2. ■

**Proof of Proposition 4.** First, note that

$$\begin{aligned}a_i^t - a_i^{n,t} &\equiv \frac{1}{1-\delta^2}[\beta_i^t + \delta\beta_j^t - (\beta_i^t + \delta\beta_j^n)]\\ &= \frac{\delta}{1-\delta^2}[\beta_j^t - \beta_i^n] \geq 0 \quad \Leftrightarrow \quad \delta \geq 0.\end{aligned}$$

Similarly,

$$\begin{aligned}a_i^n - a_i^{t,n} &\equiv \frac{1}{1-\delta^2}[\beta_i^n + \delta\beta_j^n - (\beta_i^n + \delta\beta_j^t)]\\ &= \frac{\delta}{1-\delta^2}[\beta_j^n - \beta_j^t] \geq 0 \quad \Leftrightarrow \quad \delta \leq 0.\end{aligned}$$

Hence, the result. ■

**Proof of Proposition 5.** We want to prove that the equilibrium is unique and that it features full transparency when $\delta > 0$ and no communication when $\delta < 0$.

Consider first the incentive to stick to an equilibrium with fully transparency. This is given by the sign of $\pi_i^t - \pi_i^{n,t}$. We want to prove that $\pi_i^t - \pi_i^{n,t} < 0$ for at least one principal when $\delta < 0$, while $\pi_i^t - \pi_i^{n,t} > 0$ for both principals when $\delta > 0$. We can rewrite (15) as

$$\pi_i^t - \pi_i^{n,t} = a_i^t - \frac{(a_i^t)^2}{2} + \delta a_i^t a_j^t - \left[a_i^{n,t} - \frac{(a_i^{n,t})^2}{2} + \delta a_i^{n,t} a_j^{t,n}\right],$$

which, substituting the efforts as functions of the contracts — i.e., equation (7) — and simplifying, becomes

$$\pi_i^t - \pi_i^{n,t} = [\beta_j^t - \beta_j^n]\delta \frac{2 + \delta(\beta_j^n + \beta_j^t - 2\delta\left(1 - \beta_i^t\right))}{2\left(1 - \delta^2\right)^2}. \tag{A1}$$

Notice first that (A1) equals 0 when $\delta = 0$, while its sign depends, for given $\delta$, on the sign of the numerator — in fact, $\beta_j^t > \beta_j^n$ by Proposition 2. Define the numerator of (A1) as

$$\xi_i(\delta) \equiv 2 + \delta(\beta_j^n + \beta_j^t - 2\delta\left(1 - \beta_i^t\right)).$$

Consider first $\delta < 0$. Then, we need $\xi(\delta) > 0$ for at least one $i = 1, 2$. Because $\beta_i^t < 1$ by Proposition 2, it is clear that

$$\frac{\partial \xi_i(\delta)}{\partial \delta} = \beta_j^n + \beta_j^t - 4\delta\left(1 - \beta_i^t\right) > 0 \qquad \forall \delta \in (-1, 0).$$

Hence, in the range considered, $\xi_i(\delta)$ is minimized at $\delta \to -1$, taking value $\xi_i(-1) = 2\beta_i^t - \beta_j^t - \beta_i^n$. Let's now assume that $\sigma_j > \sigma_i$. Then, by Proposition 2, $\beta_i^t > \beta_j^t$, implying that $\xi(-1) > 0$ and, *a fortiori*, $\xi_i(\delta) > 0$ for all $\delta < 0$. This implies that (A1) is negative so that principal $P_i$ deviates from the equilibrium with full transparency. If $\sigma_j < \sigma_i$, then the argument applies unchanged to principal $P_j$ who breaks the equilibrium. This proves that there exists no equilibrium with full transparency when $\delta < 0$.

Consider now $\delta > 0$. Then, we want to show that $\xi_i(\delta) > 0$ for all $\delta > 0$ for both principals. Now, $\xi_i(\delta)$ is clearly concave and such that $\xi_i^{'}(0) = \beta_j^n + \beta_j^t > 0$. Hence, the minimum value of $\xi_i(\delta)$ over the range $[0, 1]$ is either at $\delta = 0$ or, if the function has a maximum for some $\delta^{\max} \in (0, 1)$, at $\delta = 1$. Because $\xi_i(0) = 2$ and $\xi_i(1) = \beta_j^n + \beta_j^t + 2\beta_i^t > 0$, we conclude that $\xi_i(\delta) > 0$ for all $\delta > 0$. Hence, (A1) is positive for both principals and full transparency is indeed an equilibrium.

Next, consider the incentive to stick to an equilibrium with no communication. We want to prove that $\pi_i^n - \pi_i^{t,n} < 0$ for at least one principal when $\delta > 0$, while $\pi_i^n - \pi_i^{t,n} > 0$ for both principals when $\delta < 0$. We can rewrite the profit difference as

$$\pi_i^n - \pi_i^{t,n} = a_i^n - \frac{(a_i^n)^2}{2} + \delta a_i^n a_j^n - \left[a_i^{t,n} - \frac{(a_i^{t,n})^2}{2} + \delta a_i^{t,n} a_j^{n,t}\right],$$

which, using (7) and simplifying, becomes

$$\pi_i^n - \pi_i^{t,n} = -[\beta_j^t - \beta_j^n]\delta \frac{2 + \delta(\beta_j^n + \beta_j^t - 2\delta\left(1 - \beta_i^n\right))}{2\left(1 - \delta^2\right)^2}. \tag{A2}$$

Again, (A2) equals 0 when $\delta = 0$, while its sign depends, for given $\delta$, on the sign of the numerator. Define the numerator as

$$\chi_i(\delta) \equiv 2 + \delta(\beta_j^n + \beta_j^t - 2\delta\left(1 - \beta_i^n\right)).$$

Consider first $\delta > 0$. By the same logic used above, it is easy to show that $\chi_i(\delta)$ is a concave function increasing at $\delta = 0$ and such that it has a minimum either at $\delta = 0$ or at $\delta = 1$. Because $\chi_i(0) = 2 > 0$ and $\chi_i(1) = \beta_j^n + \beta_j^t + 2\beta_i^n > 0$, it is clear that, when $\delta > 0$, it holds $\pi_i^n - \pi_i^{t,n} < 0$ for both principals. Together with the arguments made above, this implies that, whenever $\delta > 0$, the unique equilibrium features full transparency.

Consider now $\delta < 0$. We want to show that $\pi_i^n - \pi_i^{t,n} > 0$ for both principals. The profit differential can be rewritten as in equation (16)

$$\pi_i^n - \pi_i^{t,n} = a_i^n - \psi(a_i^n, a_j^n) - [a_i^{t,n} - \psi(a_i^{t,n}, a_j^n)] - \delta a_i^{t,n}[a_j^{n,t} - a_j^n].$$

We know from Section 5.3 that $a_j^{n,t} > a_j^n$ so that the last term above is clearly positive. We still have to show that the sum of the remaining addenda is also positive. We can rewrite it as

$$\pi_i^n - \pi_i^{t,n} + \delta a_i^{t,n}[a_j^{n,t} - a_j^n] = a_i^n \left[1 - \frac{a_i^n}{2} + \delta a_j^n\right] - a_i^{t,n}\left[1 - \frac{a_i^{t,n}}{2} + \delta a_j^n\right],$$

which, using (7), becomes

$$\pi_i^n - \pi_i^{t,n} + \delta a_i^{t,n}[a_j^{n,t} - a_j^n] = -(\beta_j^t - \beta_j^n)\delta \frac{2\left(1-\delta^2\right)\left(1-\beta_i^n\right) - \delta(\beta_j^t - \beta_j^n)}{2\left(1-\delta^2\right)^2}. \quad \text{(A3)}$$

The sign of (A3) depends, for given $\delta$, on the sign of the numerator. Which is positive when $\delta < 0$ for $i = 1, 2$, as $\beta_i^n < 1$ and $\beta_j^t \geq \beta_j^n$. Hence, $\pi_i^n - \pi_i^{t,n} > 0$ for both principals whenever $\delta < 0$. This, coupled with the arguments above establishes uniqueness of the no communication equilibrium when $\delta < 0$. ■

Highlights



- We study transparency and contract design in multidivisional organizations.
- Divisions communicate when they impose positive externalities on each other.
- By contrast, they don’t communicate with negative externalities.