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WORKING PAPER N. 12/1

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Abstract
A large literature on the regulation of banks has explained deposit rate control (ceilings) and capital requirements as alternative regulatory instruments for reducing moral hazard issues (i.e. the propensity of banks to take too large risks). Over the last 30-40 years, almost uniformly, regulators have moved from regimes relying on the former instrument to ones using the latter. Hitherto the theoretical literature does not seem to offer much support for this policy change whereas our contribution seeks to establish a real trade-off between the two regulatory regimes. In our model, which is an adaptation of that of Repullo (2004), the deadweight loss of capital control is its higher opportunity cost as compared to the returns derived from normal banking activities. There are several potential costs associated with deposit rate ceilings, but inspired by the observed consolidation of the banking sector after the liberalization that took place in the eighties, we focus on one: the tendency towards excess entry in the banking sector. While historically, and unlike in our stylized model, entry was not free for banks, we argue that the excess profits associated with deposit rate ceilings are likely to have put (political) pressure on regulators to allow an increased number of banks, with associated costs for society. We show that, with the trade-off as described above, and depending on the parameter configuration, each of the two regimes may welfare-dominate the other.

Keywords: Banking regulation, moral hazard, deposit rate control, capital requirements, Salop model.
JEL classification: D43, D82, G21, G28.

1 Introduction

The recent history of bank regulation may be divided into four phases. In the first phase, which in some countries lasted only until the early 70's while for others until the mid 80's, bank regulation was mostly performed at the national level and was many times very tight.[1] During this phase interest rates (deposit and/or lending) were often regulated, which had the negative side effect of impairing competition and conceivably also innovation in the banking sector. This and the increasing degree of internationalization, also of the capital markets, is likely to have been one important motive inducing regulators to move to a second phase where capital requirements on banks, ostensibly serving to deal with credit risks, worked to reduce moral hazard problems. In this phase, regulation also turned into an international matter, often with the Basel Committee on Banking Supervision as coordinator (and instigator), and thus we may date the beginning of this phase to be in 1988, the year of the first Basel Accord.

The intentions behind moving to the third phase (the beginning of which coincides with that of the second Basel accord in 2004) was to make regulation more flexible and to induce yet more competition among banks. In this phase, at least for some banks, regulation would not take place by means of standardized capital requirements, but be based on the banks' own risk analysis tools with a consequent potential for making regulation more cost efficient. The implementation of the second Basel accord was interrupted by the 2008 financial crisis which led to the formulation of yet another accord, the third Basel Accord (2010-2011). This fourth phase of bank regulation is characterized by a tightening of control and supervision, exemplified by an increase in capital ratio requirements and a reversal to more objective standards for defining the risk structure of the individual bank's assets.

[1] See Demirgüç-Kunt and Detragiache (1998) and Kaminsky and Schmukler (2003) for studies of the date of the unset of financial liberalization in various countries.

Noticeably absent from the third Basel accord, however, is any attempt to return to regulating deposit/lending interest rates.[2] Rather the intention seems to have been to sharpen the tool of capital requirement to make it adequate for an international scene of increased financial sophistication, innovation and competition. The rationale one may conjecture from this approach is that regulators agree that competition among banks has brought considerable benefits to the world economy and reversing to a regime with price controls would be a too costly way of avoiding the kind of excessive risk-taking by banks (and other financial institutions) that undoubtedly was one of the main causes of the near collapse of financial markets witnessed in late 2008.

The view that interest rate control is not an attractive regulatory instrument and that competition among financial institutions is overall beneficial to the general economy is far from equivocally supported by economists. One of the main theoretical arguments against it is that increased competition and in particular the ability to freely set interest rates tend to erode the franchise values of the individual bank which in turn leads to increased risk taking by banks. A prominent and often cited theoretical contribution is that of Hellman et. al. (2000), henceforth HMS. These authors argue that while pure deposit rate control regulation is Pareto optimal this is never so for pure capital requirements regulation: capital requirements can only be Pareto optimal in conjunction with deposit rate control - in fact they interprete their main Proposition as stating that "... the current policy regime practiced in most countries around the world (i.e. using just a capital requirement with no deposit-rate control) is a Pareto-inferior policy choice" (Repullo 2004, p. 156).

The model of HMS is essentially partial equilibrium and as Repullo (2004) showed, some of their results do not necessarily

[2]Some aspects of regulations implemented or in the process of being implemented do resemble aspects of earlier regulatory regimes. In particular laws that limit the scope of banks' activities (similar to "narrow banking") have been introduced (in the US) or are being discussed (by The Independent Commission on Banking in the UK).

hold in a general equilibrium context.[3] In fact Repullo (2004) can be seen as essentially an investigation into the soundness of some of the conclusions of HMS, using a full general equilibrium model (the Salop model of monopolistic competition).[4] Repullo only compares pure deposit rate control and pure capital requirements regimes, but does to some extend support the conclusion of HMS by finding that the former is effective whenever the latter is while the opposite does not hold (Hellman et al 2000, p. 175).

As is acknowledged by Repullo, his model suffers from some shortcomings, in particular that aggregate demand for bank deposits is fixed (independent from the deposit rate offered by the bank) - a shortcoming that, since we use Repullo's model, is also present in our study. However, it is worth noticing that this shortcoming is unlikely to work in favor of deposit rate control, since it ignores the financial repression that may be the outcome of keeping the deposit rate low.

In the absence of the possibility of financial repression or other undesired effects of deposit rate control the conclusion of Repullo's model (and that of HMS) is in a certain sense foregone. This is because there is a deadweight loss associated with using capital requirements stemming from the (realistic) assumption that equity capital is costly (i.e. the opportunity cost of capital is higher than the return on any investment the bank may undertake) while there is no deadweight cost associated with deposit rate control - it only shifts surplus from depositors to banks. Only because there are distinct agents in the models of Repullo and HMS (the ex-post and ex-ante identical banks, the (typical) depositor and

[3]In Repullo's model (as in ours) an increase in the capital requirements does not erode the franchise value of the banks since it is fully born by depositors. In our model with free entry this would hold even if supply of deposit was elastic. Repullo also shows the existence of a non-binding Pareto optimal deposit rate control as we also do.

[4]Repullo(2004) also briefly studies another regulatory option, that of risk-based capital requirements and finds it to be superior to the others. However, he does not formalize the issue of how the regulator can get to know the riskiness of an individual bank's asset portfolio.

(in HMS) the government, who is paying for deposit insurance) who benefit in different ways from regulation, does this fundamental difference between the two regulatory regimes not become decisive in ranking them. Here we ask whether there are possible negative consequences of a reduced level of competition stemming from deposit rate control that may be captured in a model in the tradition of HMS and Repullo (2004). Apart from the issue of financial repression, the most obvious answer relates to efficiency and innovation: one would expect that, if the tendency to increased risk taking could be checked, increased competition would lead to efficiency gains and to an improved offer of services - gains that the last Basel accord is probably trying to preserve.

Our analysis takes off from the observation that the removal of deposit rate controls was followed by a wave of consolidations in the banking sector.[5] Numerous studies show that during the 80's and 90's, after the liberalization of financial markets, there was a noticeable reduction in the number of banks which seems paradoxical since this would normally be taken to indicate a reduction in the level of competition.[6] Another stylized fact is that transfers from regimes with interest rate control to regimes with capital control seem to have been accompanied with an increase in the real rate offered to depositors. This indicates that the ceiling imposed on interest rates was binding. Since a non-binding ceiling is not costly (in terms of the number of banks) and hence to be preferred if feasible, this and the aforementioned (and probably not unrelated) consolidation of the banking sector indicate that

[5] There were other important factors that contributed to this consolidation and we shall not attempt to establish here which were the more important ones.

[6] Schildbach(2008) reports a 28% reduction in the number of banks in Western Europe from 1997 to 2006, Fiorentino et al.(2008) report a reduction in the number of banks in Germany of 54% and in Italy of 26% over the period 1990-2005, while Altunbaş, Y. and D. Marqués (2008) report a reduction of US bank to the tune of 50% over the period 1980 - 2003. See also Goddard et al.(2007) for country by country data for Europe that, with the exception of Greece, Ireland and Netherlands confirm this trend for the period 1985 – 2004.

controlling the deposit rate was costly and that there *may* have been savings associated with transferring to a regime with capital control.

We formalize the idea that in the presence of deposit rate control, leading to higher profits for banks shielded from competition, an excessive number of banks would be established at a cost to society at large. Taking this into account a real, fundamental trade-off between the two regimes appears: capital requirements are costly because outside capital is costly while deposit rate control is costly because it leads to a bloated banking sector (and it is costly to set up banks).

To formalize this idea we make only one change to Repullo's model: assuming free entry, that is, that the number of banks is endogenous (determined by a zero expected profit condition), and that establishing a bank is costly.[7] Together with the assumption that depositors pay for deposit insurance via taxes, this assumption leads to a simplification of the welfare analysis: banks make zero profit in equilibrium and to compare the different regulatory regimes we only need to compare the welfare of the (average) depositor.

As in Repullo (2004) we essentially only compare a pure deposit rate control regime with a pure capital requirement regime. Unlike Repullo we confine our attention to the following question: which regime is optimal (i.e. we ask which regime a government, seeking to maximize welfare, would implement). We find that, depending on parameters, whenever regulation is called for, there are three possibilities: that deposit rate control with a non-binding ceiling is effective which is then optimal, that deposit rate control with a binding ceiling is optimal and that binding capital requirements are optimal. The latter possibility means that there are

[7] An early contribution by Gehrig(1995) also studies the welfare consequences of free (and restricted) entry, however in a model quite different from ours: the model does not consider moral hazard issues and has only one period. On the other hand the model has a more satisfactory description of competition among banks than ours since banks compete both for borrowers and depositors.

circumstances under which deposit-rate control is not Pareto optimal (a possibility that was excluded in HMS).

In Section 2 we present the model which is essentially the same as that of Repullo (2004) and define the first-best outcome. In Section 3 we consider capital requirements on the banks and show that in the model such requirements may be Pareto improving over no regulation while in Section 4 we do the same for deposit rate control. Finally, in Section 5, we compare the two regimes and show that, depending on parameters, any of them may dominate in terms of welfare. Section 6 concludes.

2 Model and first-best benchmark

There are only three differences between our model and that of Repullo (2004): we assume that the number of banks is determined by free entry and a zero profit condition, that it is costly to set up a bank and that each depositor has an amount D to deposit (potentially, an extra parameter in our study). We also pay closer attention to participation in the credit market by introducing a constraint that all potential depositors actually are customers with some bank.

In this infinite-horizon model there is at each date a continuum of potential risk neutral bank consumers, identical except for where they are placed on the unit circle, each desiring to place a deposit D for one period.[8] These agents live for two periods, saving when young and consuming when old, and at each date a new identical generation enters. We shall assume that these consumers do not have any storage technology available, implying that they, barring transportation costs, are willing to accept any (expected) net interest rate which is ≥ -1. As in many models of bank reg-

[8] As is usual with the Salop model, the distance between banks can be interpreted literally or as indicating differences in the product mix and services offered. See Cerqueiro et. al. (2007) for a study of how the localization of banks intersect with the services offered and how consolidation has effected these.

ulation, the existence of deposit insurance implies that depositors only care about the interest rate offered by the bank. As in Repullo, the unit cost of "walking to the bank" is μ, where this cost can best be interpreted as a cost of having to buy a product which is some "distance" away from the ideal product.

In this monopolistic competition model entrance of banks, at a cost C, is endogenous and takes place if and only if expected profit is positive. The resulting n banks, placing themselves uniformly on the unit circle, each have zero expected profit.[9] As in HMS and Repullo (2004), in each period banks choose to invest the deposits received in either a prudent asset, yielding a net return $\alpha > 0$ with probability 1, or a gambling asset which has return net returns $\gamma > \alpha$ with probability $1 - \pi$ and $\beta > -1$ with probability π. They will not invest in both assets. Since, by assumption, $\alpha > (1 - \pi)\gamma + \pi\beta$, from the point of view of society the prudent asset is the desirable one. This assumption can also be written as

$$\pi > \underline{\pi} := \frac{\gamma - \alpha}{\gamma - \beta} \tag{1}$$

As in Repullo (2004), the (opportunity) cost of outside capital is $\rho > \alpha$ for the banks, and this rate is then also used to discount future profits.

Timing, inside each period, thus is as follows: first potential banks decide to enter or not, secondly, with the number n of banks being given, a symmetric, subgame perfect Nash equilibrium in terms of investment choice and deposit rate (possibly restricted) is reached. In the first period, $t = 1$, this is the game being played. At date $t > 1$, if a bank is declared bankrupt, a new bank immediately replaces it. In the second stage at date t, n is taken as given and the game is as the second stage at date 1.

The *demand function for deposits* of bank j with n banks in the market is derived following Repullo closely. If all other banks offer the (net) interest rate r and bank j offers r_j, a depositor at a distance z from bank j and thus at a distance $\frac{1}{n} - z$ from the

[9] We simplify the analysis by allowing that $n \in \mathbb{R}_+$ rather than $n \in \mathbb{N}$

other nearest bank will be indifferent between these two banks if

$$Dr_j - \mu z = Dr - \mu(\frac{1}{n} - z)$$

In other words, bank j will attract all depositors within a distance of

$$z = \frac{1}{2n} + \frac{r_j - r}{2\mu} D$$

meaning, since each depositor will place a deposit equal to D, that its total demand will be

$$\mathfrak{D}(r_j, r, n) = \frac{1}{n} D + \frac{r_j - r}{\mu} D^2 \tag{2}$$

Remark 1. *Participation.* The reasoning above, and thus the demand function, is only correct when there is full participation with the interest rate r (and with banks symmetrically placed on the circle). We shall below impose this full participation condition in equilibrium. A bank offering an interest rate r_j lower than r may not capture depositors whose participation constraint becomes binding. Suppose for example that $(1 + r)D = \frac{\mu}{2n}$ so that the depositor in the middle between the two banks is exactly indifferent between participating or not. If bank 1 lowers the interest rate to r_j, according to our reasoning the depositor located at a distance $z' = \frac{1}{2n} + \frac{r_j - r}{2\mu} D$ is indifferent between which bank to use. But this depositor will not want to use bank 2 (that kept the interest rate at r) and hence not bank 1 either. Formally we have that the transportation cost to bank 1 is

$$\begin{aligned} \mu(\frac{1}{2n} + \frac{r_j - r}{2\mu} D) &= \frac{\mu}{2n} + \frac{D(1 + r_j) - D(1 + r)}{2} \\ &= \frac{\frac{\mu}{2n} + D(1 + r_j)}{2} > D(1 + r_j) \end{aligned}$$

so that the depositor located at distance z' from bank 1 prefers not to walk to this bank. ■

First Best

It may be illuminating to calculate the first best outcome: here D is invested in the safe asset, i.e. the (net) return is $D\alpha$ at each date, and the number of banks is set to maximize average welfare. We assume investment can only take place through banks. Notice that in our model banks always have zero expected profit and the relevant measure of welfare is the expected welfare of the average (over the circle) depositor.

From this should be deducted the transportation costs (depending on n) and the set-up cost $n\rho C$ for this optimal n (where we are using the banks' discount rate, i.e. we assume that they have to borrow at rate ρ, but are compensated by consumers), which, because we are looking at the unit circle, is also the average set-up cost per depositor.

Transportation Costs with n banks

With n banks there are n sections (bordered by two banks), each having a distance of $\frac{1}{n}$. The consumer in the middle of that section has the longest distance to a bank: $\frac{1}{2n}$. Recalling that this consumer will only walk to the bank if his return satisfies $(1+r)D > \frac{\mu}{2n}$, and considering that bank profits $D\alpha - Dr - n\rho C$ are zero and set-up costs are shifted to consumers, yields the constraint

$$(1+\alpha)D > n\rho C + \frac{\mu}{2n} \tag{3}$$

This puts some restrictions on the parameters of the model.[10] We shall choose our parameters such that (3) is fulfilled.

The aggregate transportation costs for the consumers in any section is

[10] These restrictions are not explicitly considered in Repullo(2004). Note that if the agents have a storage technology, the participation constraint becomes $\alpha D > nC\rho + \frac{\mu}{2n}$. We are assuming that only consumers who use the bank pay for the set-up costs.

$$2\mu \int_0^{\frac{1}{2n}} m dm = 2\mu \left[\frac{1}{2} m^2 \right]_0^{\frac{1}{2n}} = \frac{1}{4} \frac{\mu}{n^2}$$

Thus average (which is also the aggregate) transportation costs for the unit circle is $n \frac{1}{4} \frac{\mu}{n^2} = \frac{1}{4} \frac{\mu}{n}$.

To maximize average welfare and ignoring for now (3) we therefore have to maximize

$$(1 + \alpha) D - \frac{\mu}{4n} - n\rho C$$

i.e. minimize $\frac{\mu}{4n} + n\rho C$, i.e. the optimal number of banks is $n^* = \frac{1}{2}\sqrt{\frac{\mu}{\rho C}}$. Now taking into account constraint (3) we require $(1 + \alpha)D > n^* \rho C + \frac{\mu}{2n^*}$.

The first-best average welfare of the consumers is (per period):

$$\begin{aligned} W^* &= (1 + \alpha)D - \tfrac{\mu}{4n^*} - n^* \rho C = (1 + \alpha)D - \tfrac{1}{4} \tfrac{\mu}{\frac{1}{2}\sqrt{\frac{\mu}{\rho C}}} - \tfrac{1}{2}\sqrt{\tfrac{\mu}{\rho C}}\rho C \\ &= (1 + \alpha)D - \tfrac{1}{2}\sqrt{\mu \rho C} - \tfrac{1}{2}\sqrt{\mu \rho C} = (1 + \alpha)D - \sqrt{\mu \rho C} \end{aligned}$$

For convenience we shall first study the model when capital requirements are imposed and then, in Section 4, study the model with interest rate control.

3 Capital requirements

We now consider the case where banks are required to have a capitalization equal to $k \geq 0$ times total deposits. Like Repullo we shall only study symmetric equilibria where either all banks use the prudent asset or all banks use the gambling asset. Suppose first the prudent asset is being used by all banks in the second stage and that all depositors on the unit circle use some bank. Then for a given number n of banks, the present value of being in the market is (see Appendix 2 or Repullo, 2004): $V_P(n) = \frac{\mu}{\rho n^2}$. Setting this value equal to C and solving for n, we get

$$n_P = \sqrt{\frac{\mu}{\rho C}} \tag{4}$$

which is always larger than $n^* = \frac{1}{2}n_P$, i.e. there are always too many banks in a prudent equilibrium. Also notice that n_P does neither depend on k nor on D. Essentially, the cost of the capital requirement is passed on to depositors and hence does not affect the present value, $V_P(n)$. When it comes to D, there are two opposite effects which exactly cancel out each other. Firstly, a higher D leads to greater competition since depositors are willing to travel longer. This will affect the value of being operating a bank negatively. Secondly, a higher D leads to a higher profit per depositor.

With this number determined, and continuing to assume that all depositors use some bank, the equilibrium deposit rate in a prudent equilibrium is (again, see Appendix 2 or Repullo, 2004, for details):

$$r_P(k, n_P) = \alpha - \frac{\mu}{n_P D} - (\rho - \alpha)k \tag{5}$$

Similarly, if the gambling asset is being used by all banks (and all potential depositors participate), the present value of a gambling bank is $V_G(n) = \frac{(1-\pi)\mu}{(\rho+\pi)n^2}$ (see Appendix 2 or Repullo, 2004) when there are n banks.[11]

With free entry, the number of banks, n_G, then solves $V_G(n) = \frac{(1-\pi)\mu}{(\rho+\pi)n^2} = C$, i.e.

$$n_G(\pi) = \sqrt{\frac{(1-\pi)\mu}{(\rho+\pi)C}} \tag{6}$$

[11]The present value formula assumes that gambling banks are bankrupt in the bad state. Notice that if this were not so, a bank would get a higher profit form investing in the prudent asset rather than in the gambling asset (see Appendix 2 for more details).

which again does not depend on k. To understand the relationship between n^* and n_G note that

$$\sqrt{\frac{1-\pi}{\rho+\pi}} = \frac{1}{2\sqrt{\rho}} \Leftrightarrow 4\rho\,(1-\pi) = \rho+\pi \Leftrightarrow \pi = \frac{3\rho}{1+4\rho}$$

i.e. there could be some $\pi \in (0,1)$ and ρ such that $n_G = n^*$. Note that since $\frac{1-\pi}{\rho+\pi} < \frac{1}{\rho}$, $n_G < n_P$ always. We shall throughout assume $n_G \geq 2$, i.e. $\frac{(1-\pi)\mu}{(\rho+\pi)C} \geq 4$ which we write

$$\pi \leq \frac{1-\frac{4C}{\mu}\rho}{1+\frac{4C}{\mu}} =: \overline{\pi}_G < 1 \tag{7}$$

so that there are always at least two banks, whether we are in the prudent or in the gambling equilibrium.

The equilibrium deposit rate in the gambling equilibrium is then (see Appendix 2)

$$r_G(k, n_G) = \gamma - \frac{\mu}{n_G D} - [\frac{1+\rho}{1-\pi} - (1+\gamma)]k \tag{8}$$

Full participation

We shall only study equilibria with full participation. All depositors participate if the depositor in the middle between two banks uses the bank. In the prudent equilibrium this is the case if:

$$(1 + r_P(k, n_P))D \geq \frac{\mu}{2n_P} \tag{9}$$

which, for $k = 0$, is equivalent to

$$(1+\alpha)D \geq \frac{3}{2}\sqrt{\mu\rho C} \tag{10}$$

while for the gambling equilibrium we require

$$(1 + r_G(k, n_G))D \geq \frac{\mu}{2n_G} \tag{11}$$

which, for $k = 0$, is equivalent to

$$(1+\gamma)D \geq \frac{3}{2}\sqrt{\frac{\mu(\pi+\rho)C}{1-\pi}} \tag{12}$$

In the latter requirement we have taken deposit insurance into account. Assuming that the tax paid to cover deposit insurance is independent of whether a potential depositor uses the banks or not the cost of this insurance is exogenous to any individual depositor.

Letting, with $\delta_P := \rho - \alpha$, $\delta_G := \frac{1+\rho}{1-\pi} - (1+\gamma)$ (so that $\delta_G - \delta_P > 0$) and $h := \sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} > 1$,

$$m_P(k) := \frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2(h-1)} \tag{13}$$

and

$$m_G(k) := hm_P(k)$$

this thus leads to the following definition (see Appendix 2 for explanations).[12]

Definition 1. *An equilibrium with full participation, for k given, is a pair $(n, r) \in \mathbb{R}^2$ s.t. either (i) $n = n_P \geq 2$, $\frac{\mu}{n_P} \geq m_P(k)D$, $r = r_P(k, n_P)$ and (9) holds, or (ii) $n = n_G \geq 2$, $\frac{\mu}{n_G} \leq m_G(k)D$, $r = r_G(k, n_G)$ and (11) holds.*

Lemma 1. *$\frac{\mu}{n_P} = m_P(k)D \Leftrightarrow \frac{\mu}{n_G} = m_G(k)D$.*

Proof:

$$\frac{\mu}{\sqrt{\frac{\mu}{\rho C}}} = m_P(k)D \Leftrightarrow \frac{\mu}{\sqrt{\frac{\mu}{C}}}\sqrt{\rho}\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} = \sqrt{\frac{\rho+\pi}{(1-\pi)\rho}}m_P(k)D$$

[12] $\delta_G - \delta_P > 0$ follows since $\alpha - \gamma + \frac{1+\rho}{1-\pi} - (1+\rho) = \alpha - \gamma + (1+\rho)\frac{\pi}{1-\pi} > \alpha - \gamma + (1+\rho)\frac{\gamma-\alpha}{\alpha-\beta}$ (since $\pi \geq \underline{\pi}(\gamma, \alpha, \beta)$) where the final expression is > 0 because $\frac{1+\rho}{\alpha-\beta} > 1$.

$$\Leftrightarrow \frac{\mu}{\sqrt{\frac{\mu}{C}}\sqrt{\frac{1-\pi}{\rho+\pi}}} = hm_P(k)D \Leftrightarrow \frac{\mu}{n_G} = m_G(k)D \blacksquare$$

Note that since $m_P(k)$ (and hence $m_G(k)$) is strictly decreasing in k, there is exactly one $\widetilde{k}$ so that $\frac{\mu}{n_P} = m_P(\widetilde{k})D$ (and hence $\frac{\mu}{n_G} = m_G(\widetilde{k})D$). In fact, from (13)

$$\widetilde{k}(\pi) = \frac{\frac{\mu}{n_P}2(h-1)\frac{1}{D} - (\gamma-\alpha)}{\delta_P - \delta_G} = \frac{\frac{\sqrt{\mu\rho C}}{D}2\left[\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} - 1\right] - (\gamma-\alpha)}{\rho - \alpha - \frac{1+\rho}{1-\pi} + (1+\gamma)} \tag{14}$$

This is the only value for which both equilibria exist simultaneously. For $k < \widetilde{k}$ we have $\frac{\mu}{n_G} < m_G(k)$ and hence a gambling equilibrium, while for $k > \widetilde{k}$, $\frac{\mu}{n_P} > m_P(k)$ gives us a prudent equilibrium.

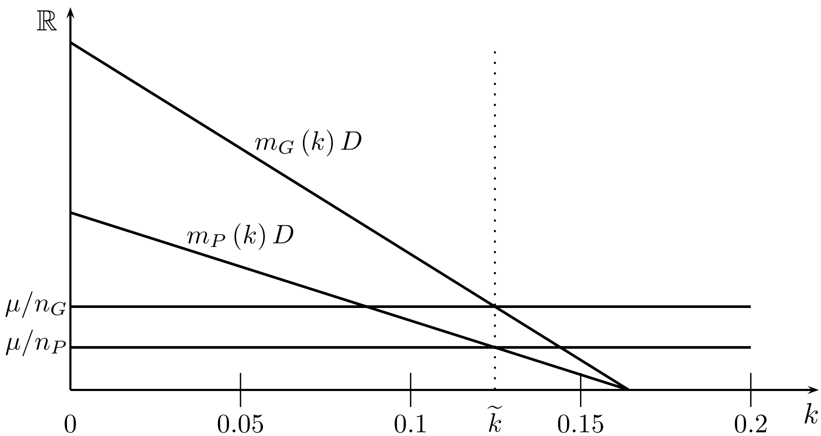

Fig. 1: Determination of $\tilde{k}$

In the above Figure, which is similar to Fig. 1 of Repullo (2004), we superimpose two horizontal lines, one at $\frac{\mu}{n_G}$ and one at $\frac{\mu}{n_P}$. The first line meets the graph of $m_G(k)D$ at $\tilde{k}$ and the second line meets the graph of $m_P(k)D$ at $\tilde{k}$ as well. To the left of $\tilde{k}$, $m_G(k)D > \frac{\mu}{n_G}$ and $m_P(k)D > \frac{\mu}{n_P}$ while to the right of $\tilde{k}$, $m_G(k)D < \frac{\mu}{n_G}$ and $m_P(k)D < \frac{\mu}{n_P}$. This is the essence of the lemma.

Analysis of $\tilde{k}$.

Notice that the denominator $\delta_P - \delta_G$ of $\tilde{k}$ is decreasing in π, positive for $\pi = 0$ and negative for $\pi \geq \underline{\pi}$. Thus its zero at $\widetilde{\pi}_\infty$, where $\widetilde{\pi}_\infty$ satisfies $\rho - \alpha - \frac{1+\rho}{1-\pi} + (1+\gamma) = 0$, i.e.

$$\widetilde{\pi}_\infty = 1 - \frac{1+\rho}{1+\rho+\gamma-\alpha}$$

is such that $\widetilde{\pi}_\infty < \underline{\pi}$.

Next consider the numerator of $\tilde{k}$, as given on the RHS of (14). It needs to be negative as well for $\tilde{k}$ to be positive. Since $\frac{d}{d\pi}\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} > 0$ (with $\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} = \infty$ at $\pi = 1$), it is strictly increasing in π and, since at $\pi = 0$ it is $\alpha - \gamma < 0$, there is a unique $\tilde{\pi}_0$ such that it is zero. Thus for $\pi > \max\{\widetilde{\pi}_\infty, \tilde{\pi}_0\}$, $\widetilde{k}(\pi) < 0$, and the only equilibrium is the prudent one. If $\underline{\pi} < \tilde{\pi}_0$, then there is a region where $\widetilde{k}(\pi) > 0$; moreover, it has a positive vertical asymptote at $\widetilde{\pi}_\infty$.

Solving for $\tilde{\pi}_0$ (14) yields

$$\frac{2\sqrt{\mu\rho C}}{D}\left[\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} - 1\right] - (\gamma - \alpha) = 0$$

$\Leftrightarrow$

$$\frac{\rho+\pi}{(1-\pi)\rho} = \left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}} + 1\right]^2$$

$\Leftrightarrow$

$$\rho + \pi = \left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}} + 1\right]^2 (1-\pi)\rho$$

$\Leftrightarrow$

$$\pi\left\{1 + \left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}} + 1\right]^2 \rho\right\} = \left\{\left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}} + 1\right]^2 - 1\right\}\rho$$

implying that

$$\tilde{\pi}_0 = \frac{\left(\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}}+1\right)^2-1}{\left(\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}}+1\right)^2+\frac{1}{\rho}} < 1 \tag{15}$$

Notice that $\tilde{\pi}_0 = 0$ for $D = 0$, $\partial\tilde{\pi}_0/\partial C < 0$, $\partial\tilde{\pi}_0/\partial D > 0$ and $\tilde{\pi}_0 \to 1$ when C becomes small and/or D becomes large. This implies the following

Lemma 2. *There exist $\overline{C}_0 > 0$ and $\underline{D}_0 > 0$ such that, for all $C < \overline{C}_0$ and $D > \underline{D}_0$, $\tilde{\pi}_0 > \widetilde{\pi}_\infty$. Then on the interval $(\widetilde{\pi}_\infty, \tilde{\pi}_0)$ $\widetilde{k}(\pi)$ is positive and strictly decreasing in π with $\widetilde{k}(\pi) \uparrow \infty$ for $\pi \downarrow \widetilde{\pi}_\infty$ and $\widetilde{k}(\tilde{\pi}_0) = 0$.*

<u>Proof</u>: There only remains to show that the derivative of $\widetilde{k}(\pi)$ with respect to π is negative on $(\widetilde{\pi}_\infty, \tilde{\pi}_0)$. To this end write $\widetilde{k}(\pi)$ from (14) as $f(\pi)/g(\pi)$. Then $\frac{d}{d\pi}(-\frac{1+\rho}{1-\pi}) < 0$ implies that

$$\frac{d\widetilde{k}}{d\pi} = \frac{2\frac{\sqrt{\mu\rho C}}{D}\frac{d}{d\pi}(\sqrt{\frac{\rho+\pi}{1-\pi}})g(\pi) - \frac{d}{d\pi}(-\frac{1+\rho}{1-\pi})f(\pi)}{g^2(\pi)}$$

is negative when $f(\pi) \leq 0$, i.e. on $[0, \tilde{\pi}_0]$. ■

<u>Existence of full participation equilibrium for $k = 0$.</u>

Proposition 1. *There are parameter values such that $\underline{\pi} < \overline{\pi}_G(C)$. In that case there exists $\underline{D}$ such that for $D \geq \underline{D}$ and for all $\pi \in [\underline{\pi}, \overline{\pi}_G]$ there is a full participation equilibrium when $k = 0$.*

<u>Proof</u>: Since $\underline{\pi} < 1$ and $\overline{\pi}_G \to 1$ when $C/\mu \to 0$ (see (7)), the claimed parameter values exist. Next, it is sufficient to show existence of a gambling equilibrium. Now for $\pi = \overline{\pi}_G$, (12) holds for D sufficiently large and thus for any π in the interval $[\underline{\pi}, \overline{\pi}_G]$. Furthermore, $m_G(0)$ attains as a function of π a positive minimum $\underline{m}$ on $[\underline{\pi}, \overline{\pi}_G]$. For large D then $\frac{\mu}{n_G} \leq \underline{m}D \leq m_G(0)D$. ■

Equilibrium Welfare

When calculating equilibrium welfare, we should bear in mind that strictly speaking this only makes sense when there are at least two banks.

Prudent Equilibrium

The average (per period) welfare of the consumer in the prudent equilibrium, with capital requirements k, is

$$\begin{aligned} W_{P,\pi}(k) &= (1+\alpha)(1+k)D - \frac{\mu}{4\sqrt{\frac{\mu}{C\rho}}} - \rho\sqrt{\frac{\mu}{C\rho}}C - (1+\rho)kD \\ &= \left[(1+\alpha) + (\alpha-\rho)k\right]D - \frac{5}{4}\sqrt{\mu\rho C} \qquad (16) \end{aligned}$$

To compare this with W^*, rewrite W_P as

$$\begin{aligned} W_P(k,\pi) &= (1+\alpha)D + (\alpha-\rho)kD - \left(\frac{5}{4} - 1 + 1\right)\sqrt{\mu\rho C} \\ &= (1+\alpha)D - \sqrt{\mu\rho C} + (\alpha-\rho)kD - \frac{1}{4}\sqrt{\mu\rho C} \\ &= W^* - (\rho-\alpha)kD - \frac{1}{4}\sqrt{\mu\rho C} \end{aligned}$$

This shows that the efficiency loss in the prudent equilibrium is due to two factors: the cost of capital and the cost of having too many banks. In Appendix 2 we show that there is another way to compute the welfare in the prudent equilibrium, namely by looking at the return of the average consumer.

Gambling Equilibrium

At the end of any period, with probability $1-\pi$, ρC has to be paid as interest on the loan, and with probability π (i.e. in case of low outcome) interest as well as the principal has to be paid. Thus expected payment is $(1-\pi)\rho C + \pi(1+\rho)C = (\rho+\pi)C$. Then

$$
\begin{aligned}
W_G &= (1+(1-\pi)\gamma+\pi\beta)(1+k)D \\
&\quad -\frac{\mu}{4n_G} - n_G(\rho+\pi)C-(1+\rho)kD \qquad (17)\\
&= [1+(1-\pi)\gamma+\pi\beta]D+[(1-\pi)\gamma+\pi\beta-\rho]kD \\
&\quad -\frac{1}{4}\sqrt{\mu C}\sqrt{\frac{\rho+\pi}{1-\pi}}-\left[\sqrt{(1-\pi)(\rho+\pi)}\sqrt{\mu C}\right] \\
&= [1+(1-\pi)\gamma+\pi\beta]D+[(1-\pi)\gamma+\pi\beta-\rho]kD \\
&\quad -\sqrt{\frac{\mu C(\rho+\pi)}{1-\pi}}\left(\frac{1}{4}+1-\pi\right) \\
&= [1+(1-\pi)\gamma+\pi\beta]D+[(1-\pi)\gamma+\pi\beta-\rho]kD \\
&\quad -\sqrt{\frac{\mu C(\rho+\pi)}{1-\pi}}\left(\frac{5}{4}-\pi\right) \\
&= :W_G(k,\pi)
\end{aligned}
$$

As we did with W_P, in Appendix 2 we also show, as a consistency check, for W_G that it equals the return of the average consumer.

As we already noted, n_G, the number of banks in a gambling equilibrium, may be closer to the optimal number of banks than is n_P, the number of banks in a prudent equilibrium. This advantage should be compared with the disadvantage of a lower return and the cost of establishing new banks after existing banks go bankrupt. The following lemma clarifies the comparison.

Lemma 3. *Assume $\rho \leq 1$. Then $W_G(0,\pi) \leq W_P(0)$ for all $\pi \geq \underline{\pi}$.*

Proof: Since $\alpha \geq (1-\pi)\gamma+\pi\beta$ for $\pi \geq \underline{\pi}$, it is sufficient to show that $\frac{5}{4}\sqrt{\mu\rho C} \leq \sqrt{\frac{\mu C(\rho+\pi)}{1-\pi}}\left(\frac{5}{4}-\pi\right)$ which, after squaring both sides, multiplying by $1-\pi$ and rearranging can be written as

$$
\pi\left(\pi^2-\frac{5}{2}\pi+\frac{25}{16}\right)+\rho\left(\pi^2-\frac{15}{16}\pi\right)\geq 0 \qquad (18)
$$

$\pi^2-\frac{5}{2}\pi+\frac{25}{16}$ has its minimum at $\pi=5/4$ with value zero while $\pi^2-\frac{15}{16}\pi$ is negative on $(0,15/16)$, non-negative elsewhere. It is thus clear that if (18) holds for all $\pi \in [0,1]$ and $\rho = 1$ it does so for all $\pi \in [0,1]$ and $\rho \le 1$, too. Inserting $\rho = 1$ in the LHS of (18) we get $\pi\left(\pi^2 - \frac{3}{2}\pi + \frac{5}{8}\right)$. $\pi^2 - \frac{3}{2}\pi + \frac{5}{8}$ has its minimum at $\pi = 3/4$ with value $1/16 > 0$. We conclude that (18) holds for all $\pi \in [\underline{\pi}, 1]$ and $\rho \in [0,1]$. ■

We are interested in comparing welfare in the gambling equilibrium when $k = 0$ and welfare in the prudent equilibrium with k positive, in particular, in finding $\widehat{k}(\pi)$ such that the two are equal. Thus $\widehat{k}(\pi)$ solves $W_P(k) - W_G(0,\pi) = 0$ where $W_P(k)$ is given by (16) and $W_G(0,\pi)$ by

$$W_G(0,\pi) = [1 + (1-\pi)\gamma + \pi\beta]D - \sqrt{\frac{(\rho+\pi)\mu C}{1-\pi}}\left(\frac{5}{4} - \pi\right) \quad (19)$$

This yields

$$\widehat{k}(\pi) = \frac{\alpha - (1-\pi)\gamma - \pi\beta}{\rho - \alpha} + \frac{\sqrt{\mu C}}{(\rho-\alpha)D}\left[\sqrt{\frac{\rho+\pi}{1-\pi}}\left(\frac{5}{4} - \pi\right) - \frac{5}{4}\sqrt{\rho}\right] \quad (20)$$

We summarize its properties in the following

Lemma 4. *Assume $\rho < 1$. Then for all $D > 0$ the following holds: (i) $\widehat{k}(\pi)$ is strictly increasing in π; (ii) $\widehat{k}(0) = (\alpha - \gamma)/(\rho - \alpha) < 0$ and $\lim_{\pi\to 1}\widehat{k}(\pi) = \infty$; (iii) there exists a unique $\widehat{\pi}_0$ such that $\widehat{k}(\widehat{\pi}_0) = 0$. Moreover, $\widehat{\pi}_0 < \underline{\pi}$.*

Proof: See Appendix 1. ■

We now ask if there are conditions under which capital control is meaningful. This gives rise to the following

Definition 2 *Meaningful regulation.* We say that regulation is meaningful when $\alpha, \beta, \gamma, \mu, \pi, \rho, C$ and D fulfil the following parameter requirements:

(i) $\gamma > \alpha > \beta$, $\rho > \alpha$.

(ii) $\pi \ge \underline{\pi}$.

(iii) $\widetilde{k}(\pi) > 0$, i.e. $\pi < \tilde{\pi}_0$; so without capital requirements the gambling equilibrium will prevail (assuming at least two banks and full participation), thus giving a reason to impose capital requirements.

(iv) $\pi \leq \overline{\pi}_G$; so there will be at least 2 banks in the gambling equilibrium, hence in the prudent equilibrium.

(v) $(1+r_P(\widetilde{k}(\pi), n_P))D \geq \frac{\mu}{2n_P}$; so that if we impose the capital requirement, the full participation requirement (9) that we imposed earlier holds. Notice that we do not exclude $r_P < 0$!

(vi) $(1 + r_G(0, n_G))D \geq \frac{\mu}{2n_G}$, the full participation requirement (12) we imposed earlier.

(vii) $W_G(0, \pi) \leq W_P(\widetilde{k}(\pi))$; so that it actually make sense, in terms of welfare for the agents, to impose capital requirements. The condition holds if $\widehat{k}(\pi) \geq \widetilde{k}(\pi)$.

Remark 2. *Participation*

In defining the optimal strategy of a potential deviator we ignore possible participation constraints on parts of the depositors, meaning that we only provide sufficient conditions for existence of prudent respectively gambling equilibria. However in Proposition 2 below we consider large D. It is easy to show that, in the equilibria we consider, large D, holding all other parameter values fixed, means that the participation constraint will not bind for any deviator.

Proposition 2. *There are $\alpha, \beta, \gamma, \rho, \mu, \overline{C} > 0$, $\underline{D} > 0$ and an open interval $I \subset [0, 1]$ s.t. for $C \leq \overline{C}$, $D > \underline{D}$ and $\pi \in I$ all parameter requirements are fulfilled.*

Proof: See Appendix 1.

Note that in the proof of this proposition all parameter values can be locally varied without invalidating the relations established. This means that there is an open ball (set) in the parameter space s.t. all parameter requirements are fulfilled.

Example 1. Consider the following parameter values: $\alpha = 0.06$, $\beta = -0.1$, $\gamma = \rho = 0.1$, $\mu = 1$, $C = 0.025$ and $D = 10$. Then the δ in (42) (see Appendix 1) is 2.5 so that condition (40) is satisfied, and we obtain $\widetilde{\pi}_\infty = 0.035088 < \widehat{\pi}_0 = 0.1817 < \underline{\pi} = 0.2 < \pi^* = 0.20503 < \tilde{\pi}_0 = 0.68571 < \overline{\pi}_G = 0.9$, and thus $I = (0.20503, 0.68571)$. These values are illustrated in Figure 2. The decreasing curve plots $\widetilde{k}$ while the increasing one $\widehat{k}$. The part of $\widetilde{k}$ between $\pi = 0.20503$ ($= \pi^*$, the abscissa of the intersection between the two curves) and $\pi = 0.68571$ ($= \tilde{\pi}_0$, the intersection of $\widetilde{k}$ with the horizontal axes) depicts the set of prudent equilibria (π, k) with a capital requirement $k = \widetilde{k}(\pi) > 0$ which are feasible and preferable to gambling equilibria.

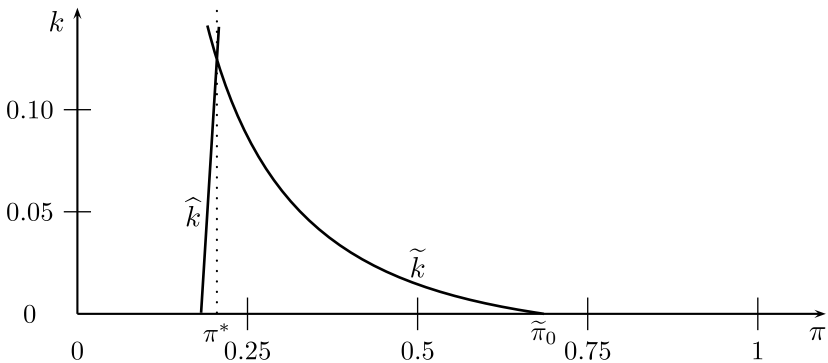

Fig. 2: Meaningful capital requirement

The number of firms in a prudent equilibrium is $n_P = 20$ while the optimal number is $n^* = 10$. The interest rate varies from $r_P(k^*, 20) = 0.050008$, where $k^* = \widetilde{k}(\pi^*) = 0.12479$, to $r_P(0, 20) = 0.055$, where $0 = \widetilde{k}(\tilde{\pi}_0)$. Regarding welfare, it varies from $W_P(k^*) = 10.488$ to $W_P(0) = 10.538$ while the optimum welfare is $W^* = 10.55$.

For pairs (π, k) with $\pi^* < \pi < \tilde{\pi}_0$ and $k < \widetilde{k}(\pi)$ gambling equilibria occur. However, they are all welfare-dominated by corresponding prudent equilibria $\left(\pi, \widetilde{k}(\pi)\right)$. In particular, welfare in the gambling equilibria $(\pi, 0)$ varies from $W_G(0, \pi^*) = 10.488$ to $W_G(0, \tilde{\pi}_0) = 9.4875$. The number of firms varies from $n_G = 10.21$ for $\pi = \pi^*$ to $n_G = 4$ for $\pi = \tilde{\pi}_0$ while the interest rate varies from $r_G(0, 10.21) = 0.090206$ to $r_G(0, 4) = 0.075$.

Note that there is an interval of admissible π's, namely $[\underline{\pi}, \pi^*)$, in which $W_G(0, \pi) > W_P\left(\widetilde{k}\,(\pi)\right)$, i.e. the gambling equilibrium is welfare-superior to the prudent equilibrium with capital control, in spite of the fact that the gambling asset has an expected return $(1 - \pi)\gamma + \pi\beta < \alpha$, i.e. smaller than the return of the safe asset. This is so because $W_G(0, \pi^*) = W_P\left(\widetilde{k}\,(\pi^*)\right)$ and $W_G(0, \pi) - W_P\left(\widetilde{k}\,(\pi)\right)$ is decreasing in π. The reason for this is that the number of firms in a gambling equilibrium is much closer to the first-best number than the one in the prudent equilibrium. This underlines the importance of letting the number of firms be an endogenous variable rather than an exogenous one as in Hellman et.al. (1997) and Repullo (2004).

Finally, it is easy to check that the participation constraints (v) and (vi) are by large margins satisfied.

4 Deposit rate control

Interest control, i.e. ceilings on deposit interest rates, may potentially induce banks to invest in the prudent asset because this allows them to reap the benefit of the low deposit interest rates for a longer period. As we shall assume that banks are free to enter the market the increased prospective profit resulting from interest rate ceilings may lead to more banks entering the market. Thus the cost of the interest rate ceiling is higher total set-up costs (with capital requirement the cost derives from the opportunity costs of capital put up by the banks). Obviously, there may be other costs associated with interest rate control, namely the reduced incentives to innovate when it is not possible to capture market shares by offering better rates than competitors. Our model does not capture such costs however.

The set-up cost for a bank is C. The n banks are placed symmetrically on the circle, i.e. for any bank the distance to any of its neighbors is the same as that of any other bank.

The game being played is as follows:

Date 0: The deposit rate ceiling is determined by the regulator.

At date $t \geq 1$ the game is as outlined in section 2.1.

In the following analysis we shall initially ignore participation constraints. Later in this section, we shall present parameter conditions that justify this.

In a prudent equilibrium with an interest ceiling $\overline{r}$ and n banks, the interest being offered is $\overline{R}_P(\overline{r}) = \min\{\overline{r}, r_P(0,n)\} = \min\{\overline{r}, \alpha - \frac{\mu}{nD}\}$. The value of being in the market in a prudent equilibrium with n banks is V_P where

$$V_P = \frac{1}{1+\rho}[\alpha - \overline{R}_P(\overline{r})]\frac{D}{n} + \frac{1}{1+\rho}V_P$$

So $V_P = (\alpha - \overline{r})\frac{D}{\rho n}$ if $\overline{R}_P(\overline{r}) = \overline{r}$ and $V_P = \frac{\mu}{nD}\frac{D}{\rho n}$ if $\overline{R}_P(\overline{r}) = r_P$. This means that the present value is

$$V_P = \max\left\{\frac{\mu}{\rho n^2}, \frac{\alpha - \overline{r}}{\rho n}D\right\}$$

With this value determined we can find the number of banks, $\overline{n}_P(\overline{r})$, namely that n that solves $V_P = C$. If $\overline{r} \leq r_P$, this equation becomes $\frac{\alpha - \overline{r}}{\rho n}D = C$, i.e.

$$\overline{n}_P(\overline{r}) = \frac{\alpha - \overline{r}}{\rho C}D \tag{21}$$

In the other case $\overline{n}_P(\overline{r}) = n_P = \sqrt{\frac{\mu}{\rho C}}$. We conclude that

$$\overline{n}_P(\overline{r}) = \max\left\{\sqrt{\frac{\mu}{\rho C}}, \frac{\alpha - \overline{r}}{\rho C}D\right\}$$

Notice that there will always be at least as many banks in a prudent equilibrium with interest rate control as in a prudent equilibrium with capital control.

We next turn to finding the number of banks in a gambling equilibrium with interest rate control. Here, with n banks, $\overline{R}_G(\overline{r}) = \min\{\overline{r}, \gamma - \frac{\mu}{nD}\}$ and the value of being in the market is

$$V_G = \frac{1-\pi}{1+\rho}[\gamma - \overline{R}_G(\overline{r})]\frac{D}{n} + \frac{1-\pi}{1+\rho}V_G$$

so that $V_G = [\gamma - \overline{r}]\frac{D}{n}\frac{1-\pi}{\pi+\rho}$ if $\overline{r} \leq r_G$, $\frac{\mu}{n^2}\frac{1-\pi}{\pi+\rho}$ else. Thus,

$$V_G = \max\left\{\frac{\mu}{n^2}\frac{1-\pi}{\pi+\rho}, [\gamma - \overline{r}]\frac{D}{n}\frac{1-\pi}{\pi+\rho}\right\}$$

As for the case of the prudent equilibrium, we find the number of banks in the gambling equilibrium with interest rate control to be the n s.t. $V_G = C$, that is, if $\overline{r} \leq r_G$ we solve $\frac{1-\pi}{\pi+\rho}\frac{\gamma-\overline{r}}{n}D = C$, i.e. $\overline{n}_G(\overline{r}) = \frac{1-\pi}{\pi+\rho}\frac{\gamma-\overline{r}}{C}D$, and else $\overline{n}_G(\overline{r}) = \sqrt{\frac{1-\pi}{\pi+\rho}\frac{\mu}{C}}$. Thus

$$\overline{n}_G(\overline{r}) = \max\left\{\sqrt{\frac{1-\pi}{\pi+\rho}\frac{\mu}{C}}, \frac{1-\pi}{\pi+\rho}\frac{\gamma-\overline{r}}{C}D\right\}$$

Existence of a prudent equilibrium

There are three potential deviations from a symmetric Nash equilibrium in which all the $\overline{n}_P(\overline{r})$ banks use the prudent asset and charge the interest rate $\overline{R}_P(\overline{r}) = \min\{\overline{r}, \alpha - \frac{\mu}{n_P D}\} = \min\{\overline{r}, \alpha - \frac{\sqrt{\mu\rho C}}{D}\}$:

(i) A bank already in the market charges another interest rate but continues to use the prudent asset.

(ii) A bank already in the market uses the gambling asset instead of the prudent asset and charges another interest rate.

(iii) A bank decides to enter the market, using either the prudent or the gambling asset.

In finding $\overline{n}_P(\overline{r})$ above we ruled out deviations of type (i). Note that deviations of type (iii) will not be profitable, if deviations of type (i) and (ii) are not: with $\overline{n}_P(\overline{r})$ banks in the market and at fixed positions the new entrant would be facing more competition than a deviating bank already in the market. Hence if charging a different interest rate and or investing in the gambling asset is not attractive for the latter, it is also not attractive for the former. And using the prudent asset and charging the same interest rate as those banks already in the market clearly would lead to negative profits. We shall now turn to conditions that rule out deviations of type (ii).

Given the interest rate $\overline{R}_P(\overline{r})$ charged by all the other $\overline{n}_P(\overline{r})-1$ banks (using the prudent asset) and given $\overline{r}$, a potential deviator solves:

$$\max_{r\leq\overline{r}} \frac{1-\pi}{1+\rho}(\gamma-r)\mathfrak{D}(r,\overline{R}_P(\overline{r}),\overline{n}_P(\overline{r}))+\frac{1-\pi}{1+\rho}C$$

(recall that C is the present value of using the prudent asset and $\mathfrak{D}(\cdot)$ the demand function for deposits, see (2)). If the value of this problem is $\leq C$ it is not profitable to deviate and the prudent equilibrium is a Nash equilibrium. The solution to this problem, i.e. to

$$\max_{r\leq\overline{r}} (\gamma-r)\left[\frac{1}{\overline{n}_P(\overline{r})}D+\frac{r-\overline{R}_P(\overline{r})}{\mu}D^2\right]$$

is

$$\min\left\{\overline{r}, \frac{\overline{R}_P(\overline{r})+\gamma}{2}-\frac{\mu}{2D\overline{n}_P(\overline{r})}\right\} \tag{22}$$

We thus consider three cases:

(a) $\overline{r}<\alpha-\frac{\sqrt{\mu\rho C}}{D}$, so $\overline{r}$ is binding.

(b) $\alpha-\frac{\sqrt{\mu\rho C}}{D}\leq\overline{r}<\frac{1}{2}\left\{\alpha+\gamma-2\frac{\sqrt{\mu\rho C}}{D}\right\}\equiv\overline{r}^*$ (where the upper limit is found by setting $\overline{R}_P(\overline{r})=r_P=\alpha-\frac{\sqrt{\mu\rho C}}{D}$ and $\overline{n}_P(\overline{r})=n_P=\sqrt{\frac{\mu}{\rho C}}$ in $\frac{\overline{R}_P(\overline{r})+\gamma}{2}-\frac{\mu}{2D\overline{n}_P(\overline{r})}$). In this case the limit is only binding for the potential deviator.

(c) $\overline{r}\geq\frac{1}{2}\left\{\alpha+\gamma-2\frac{\sqrt{\mu\rho C}}{D}\right\}$, so the limit is binding for no one.

In case (a), since $\overline{n}_P(\overline{r}) = \frac{\alpha-\overline{r}}{\rho C} D > \sqrt{\frac{\mu}{\rho C}}$, we have $\gamma - \frac{\mu}{D\overline{n}_P(\overline{r})} > \alpha - \frac{\mu}{D\sqrt{\frac{\mu}{\rho C}}} = \alpha - \frac{\sqrt{\mu\rho C}}{D} > \overline{r}$, so $\min\left\{\overline{r}, \frac{\overline{R}_P(\overline{r})+\gamma}{2} - \frac{\mu}{2D\overline{n}_P(\overline{r})}\right\} = \overline{r}$ and it is optimal for a potential deviator to set $r = \overline{r}$. Inserting this in the maximand we get: $\frac{1-\pi}{1+\rho}\frac{\gamma-\overline{r}}{\alpha-\overline{r}}\rho C + \frac{1-\pi}{1+\rho}C$ which is $\leq C$ if $\frac{\gamma-\overline{r}}{\alpha-\overline{r}}\rho \leq \frac{\rho+\pi}{1-\pi}$. This means that for the existence of a prudent equilibrium we require

$$\overline{r} \leq \frac{(\rho+\pi)\alpha - \rho(1-\pi)\gamma}{\pi(1+\rho)} \equiv \overline{r}_P \tag{23}$$

Repullo (2004, Prop. 3) found that independently of μ and n, whenever the interest ceiling $\overline{r}$ is $\leq \overline{r}_P$ there is a prudent equilibrium which is parallel to the inequality we just established.

Remark 3. *Analysis of* $\overline{r}_P$

Recall the following restrictions: $\gamma > \alpha > (1-\pi)\gamma + \pi\beta$, $0 \leq \alpha < \rho < 1$ and $\beta \geq -1$. Rewrite $\overline{r}_P$ as $\frac{\rho[\alpha-\gamma(1-\pi)]+\pi\alpha}{\pi(1+\rho)}$. Then, since $\alpha - \gamma(1-\pi) > \pi\beta \geq -\pi$, $\overline{r}_P > \frac{\rho(-\pi)+\pi\alpha}{\pi(1+\rho)} > \frac{-1+0}{1+1} = -\frac{1}{2}$. Notice that for $\rho = 1, \beta = -1$ and $\alpha = 0$, $\overline{r}_P = -\frac{1}{2}$ when $\alpha = (1-\pi)\gamma + \pi\beta$. Moreover, $\overline{r}_P < \frac{\rho[\alpha-\alpha(1-\pi)]+\pi\alpha}{\pi(1+\rho)} = \frac{\alpha\pi\rho+\alpha\pi}{\pi(1+\rho)} = \alpha$ and $\overline{r}_P$ approaches α as γ, ρ and α approach 1. We conclude that

$$\overline{r}_P \in (-\frac{1}{2}, \alpha) \;\blacksquare$$

Case (b): $\overline{R}_P(\overline{r}) = \alpha - \frac{\sqrt{\mu\rho C}}{D}$, $\overline{n}_P(\overline{r}) = \sqrt{\frac{\mu}{\rho C}}$ and the optimal r for a potential deviator would be $\frac{1}{2}\left\{\alpha + \gamma - 2\frac{\sqrt{\mu\rho C}}{D}\right\} > \overline{r}$, that is the potential deviator chooses $r = \overline{r}$. The requirement for the non-existence of a profitable deviation is (following a parallel reasoning to the one for capital requirements - see Appendix 3)[13]

$$(\gamma - \overline{r})\left(\frac{\overline{r}-\alpha}{\mu}D^2 + 2\sqrt{\frac{\rho C}{\mu}}D\right) \leq \frac{\rho+\pi}{1-\pi}C$$

which can be rewritten as

$$-\overline{r}^2 + \left[(\gamma+\alpha) - \frac{2}{D}\sqrt{\rho C\mu}\right]\overline{r} - \left[\alpha\gamma - \frac{2\gamma}{D}\sqrt{\rho C\mu} + \frac{\rho+\pi}{1-\pi}\frac{C\mu}{D^2}\right] \leq 0 \tag{24}$$

The maximum of the LHS of (24) is at $\overline{r}^* > \alpha - \frac{1}{D}\sqrt{\rho C\mu}$. Let, for the case where the maximum value is non-negative, $\overline{r}_1 \leq \overline{r}^* \leq \overline{r}_2$ be the solutions to the LHS of (24) being $= 0$. Thus if, for $\overline{r} = \alpha - \frac{1}{D}\sqrt{\rho C\mu}$, the LHS of (24) is > 0 we have $\overline{r}_1 < \alpha - \frac{1}{D}\sqrt{\rho C\mu}$ which implies that for $\alpha - \frac{\sqrt{\mu\rho C}}{D} \leq \overline{r} < \frac{1}{2}\left\{\alpha + \gamma - 2\frac{\sqrt{\mu\rho C}}{D}\right\}$ the LHS of (24) is > 0, i.e. a deviation pays, and a prudent equilibrium does not exist.

If, on the other hand, for $\overline{r} = \alpha - \frac{1}{D}\sqrt{\rho C\mu}$ the LHS of (24) is ≤ 0, we have either of the following cases:

- there is a prudent equilibrium for $\overline{r} \in (\alpha - \frac{1}{D}\sqrt{\rho C\mu}, \overline{r}_1]$ (this is the case when at $\overline{r}^*$ the LHS of (24) is ≥ 0).

or

- there is a prudent equilibrium for $\overline{r} \in (\alpha - \frac{1}{D}\sqrt{\rho C\mu}, \overline{r}^*]$ (this is the case when at $\overline{r}^*$ the LHS of (24) is < 0 and corresponds to the case where there is no need for regulation, since the bank will always use the prudent asset - see also Appendix 4, Fact 2).

Inserting $\overline{r} = \alpha - \frac{1}{D}\sqrt{\rho C\mu}$ in the LHS of (24) we get that for a prudent equilibrium to exist we must have

$$-\frac{\pi(1+\rho)}{1-\pi}\frac{\mu C}{D^2} + \frac{\gamma-\alpha}{D}\sqrt{\rho C\mu} \leq 0 \tag{25}$$

We summarize in the following

[13] We require that the expected discounted profit from a one-shot deviation is $\leq$ the expected discounted profit from no deviation (which is equal to C): $\frac{1-\pi}{1+\rho}\left((\gamma-\overline{r})\left(\frac{\overline{r}-\alpha}{\mu}D^2 + 2\sqrt{\frac{\rho C}{\mu}}D\right) + C\right) \leq C$.

Lemma 5. A necessary and sufficient condition for there to be a non-binding interest rate ceiling that induces the prudent equilibrium is that (25) holds ■

Remark 4. Suppose that

$$\overline{r}_P \geq \alpha - \frac{1}{D}\sqrt{\rho C \mu} \tag{26}$$

so that a prudent equilibrium with a binding interest rate ceiling exists, $\forall \overline{r} \leq \alpha - \frac{1}{D}\sqrt{\rho C \mu}$. Note then that since $\overline{r} = \alpha - \frac{1}{D}\sqrt{\rho C \mu}$ is also a non-binding ceiling, the condition for such an equilibrium to exist, (25), holds as well. Thus (26) implies (25). In fact, condition (25) is equivalent to $\frac{\pi(1+\rho)}{1-\pi}\frac{\sqrt{\mu C}}{D} \geq (\gamma - \alpha)\sqrt{\rho}$ while (26) says that $\frac{(\rho+\pi)\alpha - \rho(1-\pi)\gamma}{\pi(1+\rho)} \geq \alpha - \frac{1}{D}\sqrt{\rho \mu C}$, which is equivalent to $\frac{\pi(1+\rho)}{1-\pi}\frac{\sqrt{\mu C}}{D} \geq \frac{-\rho\alpha(1-\pi)+\rho(1-\pi)\gamma}{1-\pi}$ which is in turn equivalent to $\frac{\pi(1+\rho)}{1-\pi}\frac{\sqrt{\mu C}}{D} \geq (\gamma - \alpha)\sqrt{\rho}$. We therefore conclude that

$$\overline{r}_P < \alpha - \frac{\sqrt{\rho \mu C}}{D}$$

is equivalent to there being no prudent equilibrium with an interest ceiling that is non-binding except for potential deviators. Notice that this condition holds for D large. ■

Case (c) where the interest rate ceiling is non-binding for all corresponds to the case we have studied in Section 3 with $k = 0$.

The possibility of a prudent equilibrium with binding and non-binding interest ceiling will be illustrated in Panel (a), Figure 4, Section 5.

Existence of gambling equilibrium

Existence of a gambling equilibrium with a binding interest ceiling will not be of much relevance for our results, hence we do not provide a full analysis here. Below we present the general picture and in Appendix 3 we provide a few more details.

Given $\overline{r}$ and the interest rate $\overline{R}_G(\overline{r})$ charged by the $\overline{n}_G(\overline{r}) - 1$ other banks (using the gambling asset), the potential deviator considers the problem

$$\max_{r \leq \overline{r}} \frac{1}{1+\rho}(\alpha - r)\mathfrak{D}(r, \overline{R}_G(\overline{r}), \overline{n}_G(\overline{r})) + \frac{C}{1+\rho}$$

where C is the present value of being in the gambling equilibrium. If the value of this problem is $\leq C$ it does not pay to deviate and the gambling equilibrium with interest rate ceiling $\overline{r}$ exists. Using the definition (2) of the the demand function, the first-order condition to the unconstrained problem is

$$-\frac{r - \overline{R}_G(\overline{r})}{\mu}D^2 - \frac{D}{\overline{n}_G(\overline{r})} + \frac{\alpha - r}{\mu}D^2 = 0 \ ,$$

so the solution to the problem is

$$\min\left\{\overline{r}, \frac{\overline{R}_G(\overline{r}) + \alpha - \frac{\mu}{D\overline{n}_G(\overline{r})}}{2}\right\}$$

For $\overline{r} \leq \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$ (from (8) and (6)), $\overline{R}_G(\overline{r}) = \overline{r}$, and at $\overline{r} = \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$ the solution to the deviator's problem is $\frac{1}{2}\left\{\gamma + \alpha - \frac{2}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}\right\}$ which is $\leq \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$, so that at this point the ceiling is not binding. We now find the point ($\leq \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$) at which the ceiling becomes binding for the potential deviator, that is the solution to

$$\frac{\overline{r} + \alpha - \frac{\mu}{D\overline{n}_G(\overline{r})}}{2} = \overline{r}$$

or, using that $\overline{n}_G(\overline{r}) = \frac{1-\pi}{\pi+\rho}\frac{\gamma-\overline{r}}{C}D$,

$$\frac{\overline{r} + \alpha - \frac{\pi+\rho}{1-\pi}\frac{C\mu}{D^2(\gamma-\overline{r})}}{2} = \overline{r}$$

which can be rewritten as

$$\alpha - \frac{\pi + \rho}{1 - \pi} \frac{C\mu}{D^2 (\gamma - r)} = \overline{r} \tag{27}$$

This equation holds iff

$$\overline{r}^2 - (\gamma + \alpha)\overline{r} + \left(\alpha\gamma - \frac{\pi + \rho}{1 - \pi} \frac{C\mu}{D^2} \right) = 0$$

The determinant of this quadratic equation, $\Delta = (\gamma + \alpha)^2 - 4\left(\alpha\gamma - \frac{\pi+\rho}{1-\pi}\frac{C\mu}{D^2}\right) = (\gamma - \alpha)^2 + \frac{\pi+\rho}{1-\pi}\frac{C\mu}{D^2}$, is positive, so the solutions,

$$\overline{r}_1 = \frac{\gamma + \alpha}{2} - \sqrt{\Delta}/2 \text{ and } \overline{r}_2 = \frac{\gamma + \alpha}{2} + \sqrt{\Delta}/2$$

are real. Notice that $\overline{r}_2 > \frac{\gamma+\alpha}{2} + \frac{\sqrt{(\gamma-\alpha)^2}}{2} = \gamma > \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$ at which point $\overline{r}$ is no longer binding. As a consequence we divide up into three cases:

Case (a): $\overline{r} < \overline{r}_1$. The ceiling is binding for all, including the potential deviator.

Case (b): $\overline{r}_1 \leq \overline{r} < \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$. The ceiling is not binding for a potential deviator, but for all others it is.

Case (c): $\overline{r} \geq \gamma - \frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\rho}\mu C}$. This case corresponds to the case studied in Section 3 with $k = 0$.

5 Deposit rate control versus capital requirements

In this section we ask whether there are conditions under which capital control leads to higher welfare than an interest rate ceiling and vice versa. Before proceeding to this question we need however to study in more detail the welfare of a gambling equilibrium. Because welfare is influenced not only by the overall return but also by the number of banks, it is not a priori the case that a prudent equilibrium dominates, in terms of welfare, a gambling equilibrium when there is a deposit rate ceiling. Thus in Proposition 2 we were careful to check that a gambling equilibrium with $k = 0$ was actually dominated by a prudent equilibrium when k was sufficiently large to make the prudent equilibrium implementable. Since the welfare of a gambling equilibrium is strictly decreasing in k there was no need to check the welfare of a gambling equilibrium with $k > 0$. However, in the case of an interest rate ceiling, the cost of regulation is the total set-up cost, and we are no longer ensured that the welfare of a gambling equilibrium is maximized when there is no (binding) interest rate ceiling. We turn to finding the optimal number of banks when the gambling asset is being used, i.e. the solution to

$$\max_n \gamma D - \frac{\mu}{4n} - n(\rho + \pi)C$$

which is $n_G^* = \frac{1}{2}\sqrt{\frac{\mu}{(\rho+\pi)C}} < n^*$. When there is no interest rate ceiling, the number of banks is $\overline{n}_G(\infty) = \sqrt{\frac{(1-\pi)\mu}{(\pi+\rho)C}}$, so if $\sqrt{1-\pi} > 1/2$ i.e. $\pi \leq 3/4$, $n_G^* \leq \overline{n}_G(\infty)$. This condition is sufficient for welfare in a gambling equilibrium without interest rate control to be larger than welfare in any gambling equilibrium with interest rate control. This condition then implies that in comparing prudent equilibria (under some regulation) with gambling equilibria (under some regulation) we need only consider the gambling equilibria with no regulation (assuming that such an equilibrium exists).

Optimal regulation: interest rate ceiling.

We now find conditions under which it is possible and meaningful to impose an interest rate ceiling that is non-binding. As before this means that without regulation there would be a gambling equilibrium with a lower welfare than a potential prudent equilibrium with regulation and that the latter type of equilibria can actually be established. We thus require:

(i) Without regulation there is a gambling equilibrium (a) with full participation and (b) with the number of banks ≥ 2.
(ii) There exists a prudent equilibrium with a non binding interest rate ceiling (a) with full participation and (b) with the number of banks ≥ 2 .
(iii) Welfare in the prudent equilibrium with the non-binding interest ceiling is higher than welfare in any other possible equilibrium with or without regulation.

For (i) we require $\tilde{k}(\pi) > 0$, which is equivalent to:

$$\gamma - \alpha > \sqrt{\frac{\mu\rho C}{D}} 2 \left(\sqrt{\frac{\rho + \pi}{(1-\pi)\rho}} - 1 \right) \tag{28}$$

as well as $(1+\gamma)D > \frac{3}{2}\sqrt{\frac{\mu(\pi+\rho)C}{1-\pi}}$ (see (12) above), $0 < \pi < \overline{\pi}_G$ (see (7) above).

For (ii) we require (25) to hold which we repeat here for convenience:

$$-\frac{\pi(1+\rho)}{1-\pi}\frac{\mu C}{D^2} + \frac{\gamma - \alpha}{D}\sqrt{\rho C \mu} \leq 0$$

and (10) while condition (b) follows from (i).

As concerning (iii) we know from the definition of $W_P(k)$ that a prudent equilibrium with no regulation welfare dominates any prudent equilibrium with capital control. Since $\overline{n}_P(\overline{r}) \geq n_P > n^*$ it follows that a prudent equilibrium with no regulation also welfare dominates any prudent equilibrium with interest rate ceiling. Finally we know from Lemma 3 that a prudent equilibrium with no regulation welfare dominates a gambling equilibrium with no regulation which in turn welfare dominates all gambling equilibria with capital control (see definition of $W_G(k)$). If, furthermore, $\pi \leq \frac{3}{4}$, then we know that a gambling equilibrium with no regulation welfare dominates all gambling equilibria with interest rate control. Thus $\pi \leq \frac{3}{4}$ is sufficient to guarantee (iii). We can now state

Proposition 3. There are values of the parameters such that conditions (i)-(iii) above are fulfilled, i.e. such that the optimal feasible policy is to impose an interest rate ceiling that is non-binding.

Proof: Note that (28) holds if

$$\gamma - \alpha > \frac{\sqrt{\mu C}}{D} 2\sqrt{\frac{\rho + \pi}{1 - \pi}} \tag{29}$$

Pick $\gamma > 0$ and $\pi \leq \frac{3}{4}$ s.t. $(1-\pi)\gamma - \pi < 0$ and then a value V of $\frac{\sqrt{\mu C}}{D}$ so small that $\frac{1}{2}\gamma > \frac{\sqrt{\mu C}}{D} 2\sqrt{\frac{1+\pi}{1-\pi}}$ (so that (29) holds for all $\rho \leq 1$ and $\alpha \leq \gamma/2$), and that both $(1+\alpha)D \geq \frac{3}{2}\frac{\sqrt{\mu\rho C}}{1-\pi}$ and $(1+\gamma)D > \frac{3}{2}\sqrt{\frac{\mu(\pi+\rho)C}{1-\pi}}$ hold when $\alpha = 0$ and $\rho = 1$. We can then pick values of μ and C s.t. $\overline{\pi}_G = \frac{1-\frac{4C}{\mu}\rho}{1+\frac{4C}{\mu}} > \frac{3}{4}$ for $\rho = 1$ - hence also for $\rho < 1$. By picking $\rho > 0$ sufficiently small, we can then guarantee that (25) holds even with $\alpha = 0$. Setting finally $\alpha = \min\{\frac{\gamma}{2}, \frac{\rho}{2}\}$ we assure that $\gamma > \alpha > (1-\pi)\gamma + \pi\beta$ for some $\beta > -1$ and that $\rho > \alpha$, as well as all the other conditions. ■

Optimal regulation: capital control

For capital control to make sense we need the conditions (i) - (vii), outlined in "Meaningful Regulation" in Section 3, to hold and that the best prudent equilibrium with interest control is welfare dominated by the prudent equilibrium with capital control.

As we already found, welfare in a prudent equilibrium with capital control is

$$W_P(k) = D\left[1 + \alpha - (\rho - \alpha)k - \frac{5}{4}\frac{\sqrt{\mu\rho C}}{D}\right] \tag{30}$$

which, using (14), becomes

$$W_P(\widetilde{k}\,(\pi))$$

$$= D\left[1 + \alpha - (\rho - \alpha)\frac{\frac{\sqrt{\mu\rho C}}{D}2\left(\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} - 1\right) - (\gamma - \alpha)}{\rho - \alpha - \frac{1+\rho}{1-\pi} + (1+\gamma)} - \frac{5}{4}\frac{\sqrt{\mu\rho C}}{D}\right] \tag{31}$$

while in a prudent equilibrium with a binding interest rate ceiling $\overline{r}$ it is

$$\begin{aligned} \overline{W}_P(\overline{r}) &= D(1+\alpha) - \frac{\mu}{4\overline{n}_P(\overline{r})} - C\overline{n}_P(\overline{r})\rho \\ &= D(1+\alpha) - \frac{\mu\rho C}{4D(\alpha-\overline{r})} - D(\alpha-\overline{r}) \end{aligned}$$

that is

$$\overline{W}_P(\overline{r}) = D\left[1+\overline{r} - \frac{\mu\rho C}{4D^2(\alpha-\overline{r})}\right]. \tag{32}$$

By (23) this yields

$$\overline{W}_P(\overline{r}_P(\pi)) = D\left[1+\frac{(\rho+\pi)\alpha-\rho(1-\pi)\gamma}{\pi(1+\rho)} - \frac{\delta(\gamma-\alpha)\rho C}{4D(\alpha-\frac{(\rho+\pi)\alpha-\rho(1-\pi)\gamma}{\pi(1+\rho)})}\right].$$

Finally, if the ceiling is not binding, then $\overline{W}_P(\overline{r})$ is

$$W_P(0) = D\left[1+\alpha-\frac{5}{4}\frac{\sqrt{\mu\rho C}}{D}\right]. \tag{33}$$

The following example illustrates the various possible incidences.

Example 2. We take the same parameter values as in Example 1, i.e. $\alpha = 0.06$, $\beta = -0.1$, $\gamma = \rho = 0.1$, $\mu = 1$, $C = 0.025$ and $D = 10$, which yield $\widetilde{\pi}_\infty = 0.035088 < \widehat{\pi}_0 = 0.1817 < \underline{\pi} = 0.2 < \pi^*$ $= 0.20503 < \tilde{\pi}_0 = 0.68571 < \overline{\pi}_G = 0.9$ and $I = (0.20503, 0.68571)$. In Figure 3 we show welfare (panel (a)) and number of firms (panel (b)) depending on π corresponding to equilibria in four different regimes: (i) gambling equilibirum without regulation, W_G, (eq. 19), and n_G, (6), (ii) prudent equilibrium with capital requirement, W_P, (31), and n_P, (4), (iii) prudent equilibrium with interest control and binding ceiling, $\overline{W}_P$, (5), and $\overline{n}_P$, (21), and (iv) prudent equilibrium with interest control and non-binding ceiling, $W_P(0)$, (33), and n_P, (4).

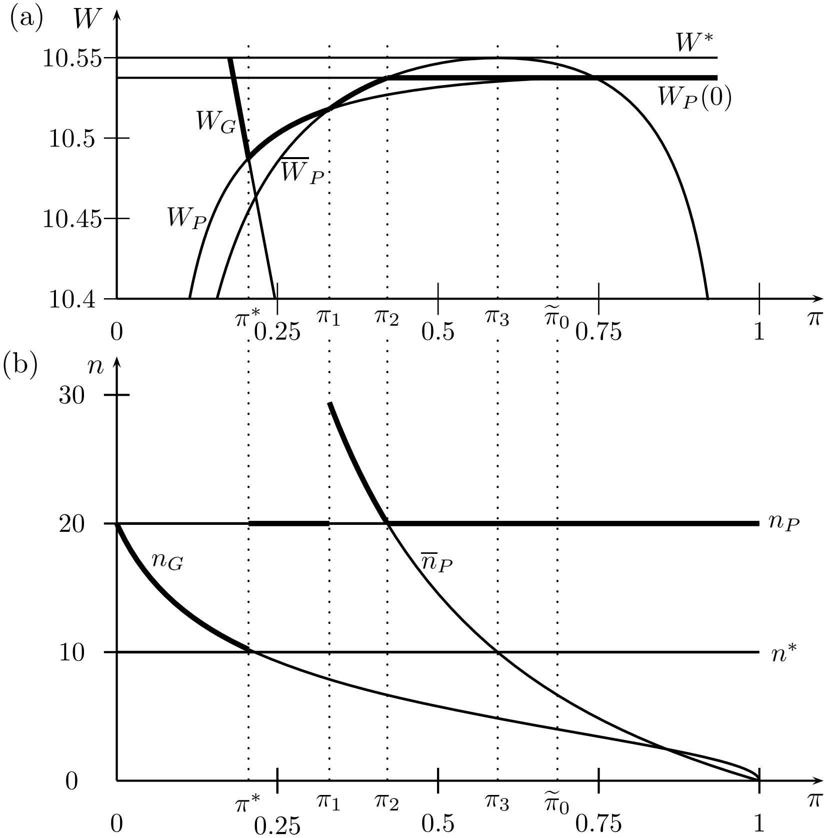

Fig. 3: Welfare and number of firms

The upper line in panel (a) shows the first-best value of welfare W^* using the prudent asset. In regime (i) where $\pi < \pi^*$, gambling equilibrium dominates the other equilibria. In (ii) ($\pi^* \leq \pi \leq 0.330822 =: \pi_1$) prudent equilibrium with capital control dominates both the gambling equilibrium and the prudent equilibrium with interest ceiling. Then for $\pi \leq 0.42105 =: \pi_2$ (regime (iii)), interest control with binding ceiling is best. Until here, in accordance with condition (25), a prudent equilibrium with non-binding interest ceiling does not exist. That changes at π_2 (regime (iv)) where the interest ceiling ceases to be binding and the equilibrium becomes one with non-binding interest ceiling until the end of the interval of relevant probabilities, i.e. $\pi \leq \widetilde{\pi}_0$.

Note that, if it were possible to impose a deposit interest rate floor, the optimum could be reached. It corresponds to $r = 0.0575 = \overline{r}_P(\pi)$ with $\pi = 0.5928 =: \pi_3$ in which case $\overline{n}_p = 10$, thus coinciding with the optimal number of firms n^*. This is also consistent with banks' total cost being equal to revenue, $Dr + n\rho C = 0.575 + 0.025 = 0.6 = D\alpha$, so that their profit is zero. However, if banks are free to choose an interest rate r smaller than or equal to $\overline{r} = 0.0575$, then they will indeed choose a smaller one and $r = \overline{r}$ is not an equilibrium rate.

Regarding the number of banks (panel (b)), in regime (i) it decreases from 20 to 10.21, jumps at π^* back to 20 (regime (ii)), jumps again at the boundary between (ii) and regime (iii), when there is a switch to binding interest control, to 29.424, which then decreases again steadily to 20 when there is the further switch to regime (iv). From this it is to be seen that the number of firms is larger when there is a binding interest ceiling, i.e. $\pi \in (\pi_1, \pi_2)$, so that deposit rate control leads to a bloated banking sector. Within (π_1, π_2), the smaller is π, the larger is the number of firms, which at π_1 becomes so costly that it is optimal to switch to capital control (regime (ii)) despite the cost of outside capital. ■

The above example illustrates the following general result:

Proposition 4. There are values of the parameters such that the best feasible policy is to impose capital requirements.

Proof: From (31) and (5) it follows that, by increasing D sufficiently, $W_P(\widetilde{k}(\pi)) > \overline{W}_P(\overline{r}_P(\pi))$ can be obtained from

$$\alpha - (\rho - \alpha)\frac{-(\gamma - \alpha)}{\rho - \alpha - \frac{1+\rho}{1-\pi} + (1+\gamma)} > \frac{(\rho + \pi)\alpha - \rho(1-\pi)\gamma}{\pi(1+\rho)} \qquad (34)$$

whenever that is satisfied. Now the RHS becomes for $\pi = \underline{\pi}$, since

$\alpha = (1 - \underline{\pi})\gamma + \underline{\pi}\beta,$

$$\frac{(\rho + \underline{\pi})\alpha - \rho(1 - \underline{\pi})\gamma}{\underline{\pi}(1 + \rho)} = \frac{(\rho + \underline{\pi})\alpha - \rho\,(\alpha - \underline{\pi}\beta)}{\underline{\pi}(1 + \rho)} = \frac{\alpha + \rho\beta}{1 + \rho}$$

which is stricly smaller than α as $\beta < \alpha$, which is equivalent to $\alpha + \rho\beta < (1 + \rho)\,\alpha$, i.e. $(\alpha + \rho\beta)\,/\,(1 + \rho) < \alpha$. But α is also the value of the LHS of (34) for $\rho = \alpha$. Thus ρ and π can be somewhat increased above α and $\underline{\pi}$, respectively, without invalidating (34). ■

In Figure 4 we summarize the relationship between π and the occurrence of the various types of equilibrium and regulatory regimes. The details of derivation are given in Appendix 4. The figure shows prudent equilibria with interest ceiling in panel (a) and combines it with prudent equilibria with capital control (panel (b), reproducing Fig. 2) and welfare (panel (c), reproducing panel (a) of Fig. 3).

Curve C_1C_1 in panel (a) shows where condition (24) is satisfied with equality while the inequality is violated inside the region delineated by C_1C_1. Similarly curve C_2C_2 depicts equality in condition (23), and the inequality is satisfied below it. C_2C_2 reaches the unconstrained equilibrium value r_P at π_2. Thus region A is the set of all pairs $(\pi, \overline{r})$ such that a prudent equilibrium with binding interest ceiling occurs while region B is the corresponding set of prudent equilibria with non-binding ceiling.

Panel (b) shows the optimal values of capital requirement for varying π. While in Fig. 2 only capital control was considered, now also interest control is an option. In fact, although capital control remains the best policy for probabilities between π^* and π_1, as can be seen from panel (c) this is no longer true for $\pi > \pi_1$. Between π_1 and π_2 interest control with binding interest ceiling is optimal while between π_2 and $\tilde{\pi}_0$ the same is true for interest control with non-binding interest ceiling. Beyond $\tilde{\pi}_0$ no regulation is required to obtain a prudent equilibrium.

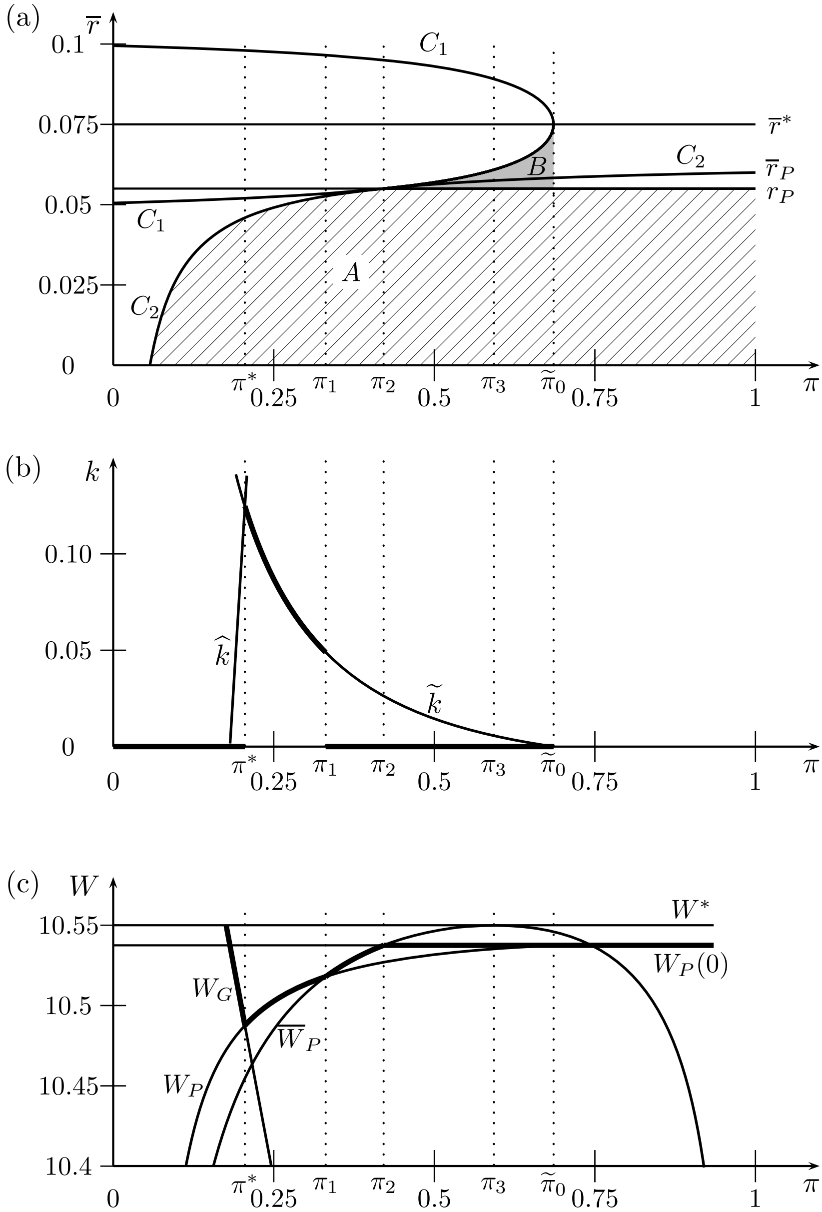

Fig. 4: Different equilibria depending on the probability of bankruptcy

It is well understood that capital requirements may alleviate the moral hazard problem of the bank choosing an asset that is excessively risky and possibly with a too low expected pay-off. The reason is that bank capital has a lower seniority than deposits, which induces the bank to seek a lower variance of pay-offs. On the other hand, deposit rate control is often thought to address moral hazard issues through another channel. By reducing the rate paid to depositors and thus increasing profits, the value of the bank as a going concern comes to include rents derived from the ceiling imposed. Whenever the bank chooses an investment strategy that may lead to it being bankrupt in an adverse scenario the bank risks losing the discounted sum of all future rents, its so-called franchise value, which in this sense also has a lower seniority than the deposits owed. There is however another, more direct effect of a deposit rate ceiling since it prevents a bank engaging in excessively risky activities from attracting more depositors by offering a higher interest rate. This means (assuming inelastic supply of deposits) that the bank cannot exploit increasing returns, present whenever it chooses an investment strategy that involves a positive probability that it will not be able to pay back its depositors and hence be declared bankrupt. Since entry is free in our model, only this direct effect is relevant when we compare the two regulatory tools.

It is natural to assume that capital requirements are costly because the opportunity cost of capital is higher than the returns offered by the bank's normal activities. What is then the cost of deposit rate ceilings? The most immediate answer is financial repression. Lowering the interest offered will reduce the flow of funds through banks and into investment with attractive returns. While this effect could potentially be studied in our model, by assuming full participation (that is that all potential depositors place all their savings with the banks) we have in effect assumed away this effect. In our model deposit rate ceilings are potentially costly because they lead to excessive entry of banks and hence excessive cost of setting up banks. Indeed when one compares

(30) and (32), the costs of setting up a bank, C, enters negatively in the welfare in a regime with interest rate control, while in the welfare of a regime with capital only the square root of C enters negatively.

6 Conclusion

There are two possible negative consequences of deposit rate control, namely financial repression (reduced savings) and inefficiencies in the organization and operation of the banking industry. As is clear from the model of Repullo (2004) (as well as from our results) the first negative effect may also be present when regulators use capital requirements (since the cost of these may be passed on to depositors in the form of lower deposit rates). The second effect is however more likely to be associated with a regime that restricts competition between banks. But when the number of banks is given, as in Repullo's model (and implicity in the model of Hellman et. al., 1997) this second effect cannot come to the fore. In fact, if financial repression is ignored (as is done here and in Repullo's model), any ambiguity about the welfare effects originates solely with distributional issues since there is no deadweight loss arising from deposit rate control.[14] We allow for a potential deadweight loss by letting the number of banks be endogenously determined and this means that now there is a real trade-off between the two regimes.

Two stylized facts seem to corroborate the relevance of our analysis. The first is the high number of banks present under regimes with interest rate ceilings, or equivalently the consolidation that took place in countries that changed from interest rate control to capital control. The other is that the transfer from a regime with interest rate control to a regime with capital control seem to have been accompanied with an increase in the real rate offered to depositors. This indicates that the ceiling imposed

[14] Such ambiguity due to distributional issues could incidentally be removed by assuming that depositors are the owners of all banks.

on interest rates was binding. Since a non-binding ceiling is not costly (in terms of the number of banks) and hence to be preferred if feasible, these two (not unrelated) observations indicate that controlling the deposit rate was costly and that there *may* have been savings associated with transferring to a regime with capital control.

We shall not claim that, like in our model, banks were free to establish themselves during the times of deposit rate control, but we think that there is likely to have been pressure on regulators (e.g. from local politicians) to allow the number of banks to increase. In our model, with free entry, the potential negative consequences of such excess entry then becomes clear. Like Repullo (2004), but unlike Hellman et. al. (1997), we have only compared the two extremes: deposit rate control only or capital requirements only. It would be surprising if no combination of these did better (in fact this is what Hellman et. al., 1997, to some extend find), but an evaluation of the optimal combination of the two instruments is outside the scope of our analysis. Rather, our contribution has been to make explicit possible societal costs of each type of regulation, which should, in turn, be helpful in understanding the trade-offs between them.

Appendix 1: Proofs of Lemma 4 and Proposition 2

Proof of Lemma 4

(i) By definition of $\widehat{k}(\pi)$ its derivative is given by

$$\frac{\partial \widehat{k}}{\partial \pi} = -\left(-\frac{\partial W_G(0,\pi)}{\partial \pi}\right) / \frac{\partial W_P\left(\widehat{k}(\pi)\right)}{\partial k}$$

Since (16) implies $\frac{\partial W_P(k)}{\partial k} = (\alpha - \rho) D < 0$ we thus get $sgn \frac{\partial \widehat{k}}{\partial \pi} = -sgn \frac{\partial W_G(0,\pi)}{\partial \pi}$.

Regarding $\frac{\partial W_G(0,\pi)}{\partial \pi}$, from (17) and $k = 0$ we obtain

$$W_G(0,\pi) = [1 + (1-\pi)\gamma + \pi\beta] D - \left[\frac{1}{4}\frac{\mu}{n_G} + n_G(\rho + \pi) C\right]$$

with n_G given by (6). Setting

$$A(\pi) = [1 + (1-\pi)\gamma + \pi\beta] D, B(\pi) = \left[\frac{1}{4}\frac{\mu}{n_G} + n_G(\rho+\pi) C\right]$$

this yields

$$\frac{\partial W_G(0,\pi)}{\partial \pi} = A'(\pi) - B'(\pi)$$

Clearly $A'(\pi) < 0$ as $\gamma > \beta$ while

$$B'(\pi) = -\frac{1}{4}\mu\frac{n_G'}{n_G^2} + n_G'(\rho+\pi) C + n_G C$$

and thus the sign of $B'(\pi)$ is undetermined. Setting in (6)

$$f(\pi) := \frac{1-\pi}{\rho+\pi}$$

we obtain

$$n_G' = \frac{1}{2}\sqrt{\frac{\mu}{C}}(f(\pi))^{-1/2} f'(\pi) = \frac{f'(\pi)}{2f(\pi)} n_G = g(\pi) n_G$$

where $g(\pi) := f'(\pi)/(2f(\pi))$. Therefore

$$B'(\pi) n_G = -\frac{1}{4}\mu g(\pi) + g(\pi) n_G^2 (\rho+\pi) C + n_G^2 C \tag{35}$$

Now

$$f'(\pi) = \frac{(\rho+\pi)(-1) - (1-\pi)}{(\rho+\pi)^2} = -\frac{1+\rho}{(\rho+\pi)^2}$$

and thus

$$g(\pi) = -\frac{1+\rho}{2(\rho+\pi)^2}\left(\frac{1-\pi}{\rho+\pi}\right)^{-1} = -\frac{1+\rho}{2(\rho+\pi)(1-\pi)}$$

Using (6), equation (35) becomes therefore

$$B'(\pi) n_G = \mu\frac{1+\rho}{8(\rho+\pi)(1-\pi)} - \mu\frac{1+\rho}{2(\rho+\pi)} + \mu\frac{1-\pi}{\rho+\pi}$$

$\Leftrightarrow$

$$\frac{B'(\pi)\, n_G\,(\rho+\pi)}{\mu} = \frac{1+\rho}{8\,(1-\pi)} - \frac{1+\rho}{2} + 1 - \pi$$

$$= \frac{1+\rho-4\,(1-\pi)\,(1+\rho)+8\,(1-\pi)^2}{8\,(1-\pi)} \tag{36}$$

From this it is clear that $B'(\pi) > 0$ iff (36) is positive which in turn is true iff its numerator is so. Now set $x := 1+\rho$ and $y := 1-\pi$. We know that $x \geq 1$ and $0 \leq y \leq 1$. Then consider the inequality $x - 4xy + 8y^2 > 0$. It is equivalent to

$$\begin{cases} (1) \quad x > \frac{-8y^2}{1-4y} \quad if \quad y < 1/4 \\ \\ (2) \quad 8y^2 > 0 \quad if \quad y = 1/4 \\ \\ (3) \quad x < \frac{-8y^2}{1-4y} \quad if \quad y > 1/4 \end{cases}$$

Case (1) is satisfied since x is positive while case (2) is trivial. Case (3) can be rewritten

$$x < \frac{8y^2}{4y-1} \quad if \quad y > 1/4 \tag{37}$$

To see what this implies for x minimize the RHS. Its derivative is

$$\frac{(4y-1)\,16y - 8y^2 \cdot 4}{(4y-1)^2} = \frac{32y^2-16y}{(4y-1)^2}$$

which is zero iff $y = 1/2 =: y_{\min}$. Then (37) yields for its RHS $8y_{\min}^2/\,(4y_{\min}-1) = 2$. Since $x = 1+\rho$, to satisfy (37) - and thus $\partial\widehat{k}/\partial\pi > 0$ - it is therefore sufficient to have $\rho < 1$ which is what we have assumed.

(ii) This follows from (20).

(iii) Setting in (20)

$$\widehat{k}_A(\pi) := \frac{\alpha-(1-\pi)\gamma-\pi\beta}{\rho-\alpha}$$

and

$$\widehat{k}_B(\pi) := \frac{\sqrt{\mu C}}{(\rho-\alpha)}\left[\sqrt{\frac{\rho+\pi}{1-\pi}}(\frac{5}{4}-\pi)-\frac{5}{4}\sqrt{\rho}\right]$$

we obtain

$$\widehat{k}(\pi) = \widehat{k}_A(\pi) + \widehat{k}_B(\pi)/D$$

Then $\widehat{k}'_A = \gamma - \beta > 0$, $\widehat{k}_A(0) = (\alpha-\gamma)/(\rho-\alpha) < 0$, $\widehat{k}_A(1) = (\alpha-\beta)/(\rho-\alpha) > 0$ and $\widehat{k}_A(\pi) = 0$ iff $\pi = (\gamma-\beta)/(y-\alpha) = \underline{\pi}(\gamma,\alpha,\beta)$.

Regarding $\widehat{k}_B(\pi)$, we have $\widehat{k}_B(0) = 0$ and $\lim_{\pi\to 1}\widehat{k}_B(\pi) = \infty$. To see what is the sign of $\widehat{k}_B(\pi)$ for $\pi \in (0,1)$ notice that $\widehat{k}_B(\pi) \geq 0$ iff

$$(\rho+\pi)\left(\frac{5}{4}-\pi\right)^2 - \frac{25}{16}\rho(1-\pi) \geq 0 \tag{38}$$

$\Leftrightarrow$

$$\frac{25}{16}\rho - \frac{5}{2}\rho\pi + \rho\pi^2 + \frac{25}{16}\pi - \frac{5}{2}\pi^2 + \pi^3 - \frac{25}{16}\rho + \frac{25}{16}\rho\pi \geq 0$$

$\Leftrightarrow$

$$\pi^3 + \left(\rho - \frac{5}{2}\right)\pi^2 + \left(\frac{25}{16} - \frac{15}{16}\rho\right)\pi \geq 0$$

and $\widehat{k}_B(\pi) = 0$ iff $\pi = 0$ or

$$\pi^2 + \left(\rho - \frac{5}{2}\right)\pi + \frac{25}{16} - \frac{15}{16}\rho = 0$$

Solving the last equation for ρ yields

$$\rho\left(\pi - \frac{15}{16}\right) = -\pi^2 + \frac{5}{2}\pi - \frac{25}{16} = -\left(\frac{5}{4}-\pi\right)^2$$

$\Leftrightarrow$

$$\rho = \frac{\left(\frac{5}{4}-\pi\right)^2}{\frac{15}{16}-\pi} =: \rho(\pi)$$

From this follows

$$\begin{aligned}
\frac{d\rho}{d\pi} &= \frac{-2\left(\frac{5}{4}-\pi\right)\left(\frac{15}{16}-\pi\right)+\left(\frac{5}{4}-\pi\right)^2}{\left(\frac{15}{16}-\pi\right)^2} \\
&= \left(\frac{5}{4}-\pi\right)\frac{-2\left(\frac{15}{16}-\pi\right)+\frac{5}{4}-\pi}{\left(\frac{15}{16}-\pi\right)^2} = \frac{\frac{5}{4}-\pi}{\left(\frac{15}{16}-\pi\right)^2}\left(\pi-\frac{5}{8}\right) .
\end{aligned}$$

For $\pi \in [0, 15/16]$ this yields $d\rho/d\pi = 0$ iff $\pi = 5/8$ which, since $\rho(\pi) \to \infty$ for $\pi \to 15/16 > 5/8$, is the unique minimizer of $\rho(\pi)$ in $[0, 15/16]$. But $\rho(5/8) = \left(\frac{5}{8}-\frac{5}{4}\right)^2 / \left(\frac{15}{16}-\frac{5}{8}\right) = 5/4$ which means that for all $\rho < 5/4$ $\widehat{k}_B(\pi) \neq 0$ for all $\pi \in [0, 15/16]$ and, since for $\rho = 1$ and $\pi = 5/8$ (38) is satisfied with strict inequality (LHS $= 25/8^3$), $\widehat{k}_B(\pi) > 0$ for all $\pi \in (0, 15/16)$ whenever $\rho < 5/4$.

To see what is the sign of $\widehat{k}_B(\pi)$ for $\pi \geq 15/16$ observe for the LHS of (38) that for $\pi > 0$

$$\begin{aligned}
& (\rho+\pi)\left(\frac{5}{4}-\pi\right)^2 - \frac{25}{16}\rho(1-\pi) \\
> \;& \rho\left(\frac{5}{4}-\pi\right)^2 - \frac{25}{16}\rho(1-\pi) \\
= \;& \rho\left(-\frac{5}{2}\pi+\pi^2+\frac{25}{16}\pi\right) = \rho\pi\left(\pi-\frac{15}{16}\right)
\end{aligned}$$

which is non-negative for $\pi \geq 15/16$. Thus $\widehat{k}_B(\pi) > 0$ for all $\pi \in (0, 1]$.

Regarding $\widehat{k}(\pi)$, since $\widehat{k}_A(0) < 0$, $\widehat{k}_B(0) = 0$, $\widehat{k}_A(\underline{\pi}) = 0$, $\widehat{k}_B(\underline{\pi}) > 0$ and $\widehat{k} = \widehat{k}_A + \widehat{k}_B/D$ is strictly increasing in π under our assumption $\rho < 1$, there exists a unique $\widehat{\pi}_0$ such that $\widehat{k}(\widehat{\pi}_0) = 0$ for any $D > 0$, and this $\widehat{\pi}_0$ must be smaller than $\underline{\pi}$. ■

Proof of Proposition 2

We proceed by selecting sequentially all parameters except π so that there will be an open interval I such that, for all parameters picked and any $\pi \in I$, all the parameter restrictions (i) - (viii) are fulfilled.

Pick $\alpha > 0$, $\beta \geq -1$, γ and $\rho < 1$ s.t. (i) holds. Then by (1) $\underline{\pi} < 1$ so that (ii) can be satisfied.

Next consider (iii). By Lemma 2 and since $\widetilde{\pi}_\infty < \underline{\pi}$, to obtain $\widetilde{k}(\pi) > 0$ it is sufficient to have $\widetilde{\pi}_0 > \underline{\pi}$ and $\pi \in (\underline{\pi}, \widetilde{\pi}_0)$. But since by (15) $\widetilde{\pi}_0 \to 1$ for $D/\sqrt{\mu\rho C} \to \infty$ there exists $\underline{L}_1 > 0$ s.t.

$$\widetilde{\pi}_0 > \underline{\pi} \text{ for all } C, D, \mu \text{ with } D/\sqrt{\mu\rho C} \geq \underline{L}_1 \tag{39}$$

Regarding (iv), from (iii), i.e. $\pi < \widetilde{\pi}_0$, it is sufficient to have $\overline{\pi}_G > \widetilde{\pi}_0$. Since by (15) and (7) both $\widetilde{\pi}_0$ and $\overline{\pi}_G$ tend to 1 for $C \to 0$, the question is which of two converges faster. To this end we show that, if μ and D are such that

$$\mu > (\gamma - \alpha)\, D \tag{40}$$

then there exists a threshold function $\overline{C}\,(D, \mu) > 0$ such that

$$\overline{\pi}_G > \widetilde{\pi}_0 \text{ for all } C < \overline{C}\,(D, \mu)\ .$$

Indeed, write $\widetilde{\pi}_0$ as

$$\tilde{\pi}_0 = \frac{\left(\frac{A}{\sqrt{C}} + 1\right)^2 - 1}{\left(\frac{A}{\sqrt{C}} + 1\right)^2 + B} < 1,\ \ A = \frac{(\gamma - \alpha)\, D}{2\sqrt{\mu\rho}}, B = \frac{1}{\rho}$$

and $\overline{\pi}_G$ as

$$\overline{\pi}_G = \frac{1 - aC}{1 + bC},\ \ a = 4\rho/\mu, b = 4/\mu.$$

Then

$$\begin{aligned} \tilde{\pi}_0 &= \frac{\frac{A^2}{C} + \frac{2A}{\sqrt{C}}}{\frac{A^2}{C} + \frac{2A}{\sqrt{C}} + 1 + B} = \frac{A^2 + 2A\sqrt{C}}{A^2 + 2A\sqrt{C} + (1+B)\, C} \\ &= \frac{1 + \frac{2}{A}\sqrt{C}}{1 + \frac{2}{A}\sqrt{C} + \frac{1+B}{A^2} C} \end{aligned}$$

and $\tilde{\pi}_0 < \overline{\pi}_G$ iff

$$\left(1 + \frac{2}{A}\sqrt{C}\right)(1 + bC) < \left(1 + \frac{2}{A}\sqrt{C} + \frac{1+B}{A^2} C\right)(1 - aC)$$

$\Leftrightarrow$

$$1 + bC + \tfrac{2}{A}\sqrt{C} + \tfrac{2b}{A}C\sqrt{C}$$
$$< 1 - aC + \tfrac{2}{A}\sqrt{C} - \tfrac{2a}{A}C\sqrt{C} + \tfrac{1+B}{A^2}C - \tfrac{a(1+B)}{A^2}C^2$$

$\Leftrightarrow$

$$\left(a + b - \frac{1+B}{A^2}\right)C + \frac{2\,(a+b)}{A}C\sqrt{C} + \frac{a\,(1+B)}{A^2}C^2 < 0 \quad (41)$$

Now since $C\sqrt{C}$ and C^2 converge faster to zero than C, this inequality is satisfied for some small enough $C > 0$ iff $a + b - (1+B)/A^2 < 0$. From the above definitions of the parameters a, b, A and B this amounts to

$$\begin{aligned}
\frac{4\rho}{\mu} + \frac{4}{\mu} - \frac{1+\frac{1}{\rho}}{\left(\frac{(\gamma-\alpha)D}{2\sqrt{\mu\rho}}\right)^2} &= \frac{4}{\mu}(\rho+1) - \frac{4\,(\rho+1)\,\mu}{(\gamma-\alpha)^2 D^2} \\
&= 4\,(\rho+1)\left[\frac{1}{\mu} - \frac{\mu}{(\gamma-\alpha)^2 D^2}\right]
\end{aligned}$$

being negative which, since $\rho > -1$, is equivalent to (40).

At this point, choose $\delta > 1$ and set

$$\mu = \delta\,(\gamma - \alpha)\,D \quad (42)$$

for the remainder of the proof, and write by a slight abuse of notation

$$\overline{C}\,(D) = \overline{C}\,(D, \delta\,(\gamma-\alpha)\,D)\,.$$

Then, since now $D/\sqrt{\mu\rho C} = \sqrt{D}/\sqrt{\delta\,(\gamma-\alpha)\,\rho C}$, (39) can be rephrased as follows: there exist $\overline{C}_1 > 0$ and $\underline{D}_1 > 0$ such that

$$\widetilde{\pi}_0 > \underline{\pi} \text{ for all } C \leq \overline{C}_1, D \geq \underline{D}_1$$

As regards (v), note that from (14) and (42)

$$\widetilde{k}(\pi) = \frac{\frac{\sqrt{\delta(\gamma-\alpha)D\rho C}}{D}2\left[\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}}-1\right]-(\gamma-\alpha)}{\rho-\alpha-\frac{1+\rho}{1-\pi}+1+\gamma}$$

$$= \frac{(1-\pi)\left\{2\sqrt{\delta(\gamma-\alpha)\rho C/D}\left[\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}}-1\right]-(\gamma-\alpha)\right\}}{(1-\pi)(\rho-\alpha+1+\gamma)-(1+\rho)}$$

$$= \frac{2\sqrt{\delta(\gamma-\alpha)\rho C/D}\left[\sqrt{\frac{(1-\pi)(\rho+\pi)}{\rho}}-(1-\pi)\right]-(1-\pi)(\gamma-\alpha)}{(1-\pi)(\gamma-\alpha)-\pi(1+\rho)}$$

$$\leq \frac{-(1-\pi)(\gamma-\alpha)}{(1-\pi)(\gamma-\alpha)-\pi(1+\rho)}$$

The last inequality follows from the fact that the denominator

$$(1-\pi)(\gamma-\alpha)-\pi(1+\rho) = (1-\pi)\left(\rho-\alpha-\frac{1+\rho}{1-\pi}+1+\gamma\right)$$

is negative for $\pi > \underline{\pi}$ and, moreover,

$$\sqrt{\frac{(1-\pi)(\rho+\pi)}{\rho}}-(1-\pi) \geq 0 \text{ for } \pi \leq 1.$$

Next, since

$$\alpha > (1-\pi)\gamma + \pi\beta \geq (1-\pi)\gamma - \pi$$

for $\pi > \underline{\pi}$, we have

$$(1-\pi)(\gamma-\alpha) = (1-\pi)\gamma-(1-\pi)\alpha < \alpha+\pi-(1-\pi)\alpha = \pi(1+\alpha) \tag{43}$$

This implies

$$\widetilde{k}(\pi) \leq \frac{-\pi(1+\alpha)}{(1-\pi)(\gamma-\alpha)-\pi(1+\rho)} = \frac{1+\alpha}{1+\rho-\frac{1-\pi}{\pi}(\gamma-\alpha)}$$

where now the denominator of the last fraction is positive for $\underline{\pi} \leq \pi < \widetilde{\pi}_0$. This yields

$$\begin{aligned} 1+\alpha-\widetilde{k}(\pi)(\rho-\alpha) &\geq (1+\alpha)\left[1-\frac{\rho-\alpha}{1+\rho-\frac{1-\pi}{\pi}(\gamma-\alpha)}\right] \\ &\geq (1+\alpha)\left[1-\frac{\rho-\alpha}{1+\rho-\frac{1-\underline{\pi}}{\underline{\pi}}(\gamma-\alpha)}\right] \\ &=: \underline{L}_2 > 0 \end{aligned}$$

which is positive since by (43)

$$1+\rho-\frac{1-\underline{\pi}}{\underline{\pi}}\left(\gamma-\alpha\right)-\left(\rho-\alpha\right)=1+\alpha-\frac{1}{\underline{\pi}}\left(1-\underline{\pi}\right)\left(\gamma-\alpha\right)>0$$

Consequently

$$(1+r_P(\widetilde{k}(\pi),n_P))D=[1+\alpha-(\rho-\alpha)\widetilde{k}(\pi)]D-\frac{\mu}{n_P}\geq \underline{L}_2D-\frac{\mu}{n_P}\ .$$

Thus to satisfy (v) it is sufficient to have $\underline{L}_2D\geq 3\mu/\left(2n_P\right)$. From (4) and (42) this amounts to

$$\underline{L}_2D\geq\frac{3\mu}{2\sqrt{\frac{\mu}{\rho C}}}=\frac{3}{2}\sqrt{\mu\rho C}=\frac{3}{2}\sqrt{\delta\left(\gamma-\alpha\right)\rho CD}$$

which can be achieved by increasing D and/or decreasing C appropriately. Thus there exist $\overline{C}_2>0$ and $\underline{D}_2>0$ such that

$$(1+r_P(\widetilde{k}(\pi),n_P))D\geq\frac{\mu}{2n_P}\ \text{for all}\ \ (C,D)\ \text{ with } C\leq\overline{C}_2,D\geq\underline{D}_2$$

Now consider (vi). By (8) it is equivalent to

$$\left(1+\gamma\right)D-\frac{\mu}{n_G}\geq\frac{\mu}{2n_G}$$

and thus by (6) to

$$\left(1+\gamma\right)D\geq\frac{3}{2}\sqrt{\frac{\mu\left(\rho+\pi\right)C}{1-\pi}}\ .$$

Since $\pi\leq\widetilde{\pi}_0$, this is implied by

$$\left(1+\gamma\right)D\geq\frac{3}{2}\sqrt{\frac{\mu\left(\rho+\widetilde{\pi}_0\right)C}{1-\widetilde{\pi}_0}} \tag{44}$$

which, using (42) and (15), becomes

$$(1+\gamma)\,D \geq \frac{3}{2}\sqrt{\frac{\delta\,(\gamma-\alpha)\,D\left(\rho+\frac{\left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\delta(\gamma-\alpha)D\rho C}}+1\right]^2-1}{\left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\delta(\gamma-\alpha)D\rho C}}+1\right]^2+\frac{1}{\rho}}\right)C}{1-\frac{\left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\delta(\gamma-\alpha)D\rho C}}+1\right]^2-1}{\left[\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\delta(\gamma-\alpha)D\rho C}}+1\right]^2+\frac{1}{\rho}}}}$$

$$= \tfrac{3}{2}\sqrt{\frac{\delta(\gamma-\alpha)D\left(\rho\left[\frac{\gamma-\alpha}{2}\sqrt{\frac{D}{\delta(\gamma-\alpha)\rho C}}+1\right]^2+\left[\frac{\gamma-\alpha}{2}\sqrt{\frac{D}{\delta(\gamma-\alpha)\rho C}}+1\right]^2\right)C}{\frac{1}{\rho}+1}}$$

$$= \tfrac{3}{2}\sqrt{\delta\,(\gamma-\alpha)\,D\rho\left(\tfrac{\gamma-\alpha}{2}\sqrt{\tfrac{D}{\delta(\gamma-\alpha)\rho C}}+1\right)^2 C}$$

$$= \tfrac{3}{2}\sqrt{\delta\,(\gamma-\alpha)\,D\rho\left(\left(\tfrac{\gamma-\alpha}{2}\right)^2\tfrac{D}{\delta(\gamma-\alpha)\rho C}+(\gamma-\alpha)\sqrt{\tfrac{D}{\delta(\gamma-\alpha)\rho C}}+1\right)C}$$

$$= \tfrac{3}{2}\sqrt{\delta\,(\gamma-\alpha)\,D\rho\left(\left(\tfrac{\gamma-\alpha}{2}\right)^2\tfrac{D}{\delta(\gamma-\alpha)\rho}+(\gamma-\alpha)\sqrt{\tfrac{CD}{\delta(\gamma-\alpha)\rho}}+C\right)}$$

For $C=0$ the last expression yields

$$\frac{3}{2}\sqrt{D^2\left(\frac{\gamma-\alpha}{2}\right)^2} = \frac{3}{4}\,(\gamma-\alpha)\,D$$

and thus at $C=0$ condition (44) becomes

$$1+\gamma \geq \frac{3}{4}\,(\gamma-\alpha)$$

which is always fulfilled with strict inequality. Thus there exists $\overline{C}_3$ such that (vii) is satisfied for all $C \leq \overline{C}_3$. At this point we can set $\underline{D} = \max\{\underline{D}_0, \underline{D}_1, \underline{D}_2\}$ and $\overline{C} = \min\left\{\overline{C}_0, \overline{C}_1, \overline{C}_2, \overline{C}_3, \overline{C}\left(\underline{D}\right)\right\}$. Then for $0 < C \leq \overline{C}$ and $D \geq \underline{D}$ the parameter restrictions (i) to (vi) are fulfilled.

Finally, to also satisfy (vii), i.e. $\widehat{k}(\pi) > \widetilde{k}(\pi)$, note that, as a consequence of Lemma 2 and Lemma 4, $\widehat{k}(\pi)$ and $\widetilde{k}(\pi)$ intersect for some $\pi^* \in (\widehat{\pi}_0, \widetilde{\pi}_0)$. If C and D are chosen such that $0 < C \leq \overline{C}$

and $D \geq \underline{D}$, then $\widehat{k}(\pi) > \widetilde{k}(\pi)$ for all π in the (non-empty and non-degenerate) interval $I = \left(\max\left\{\pi^*, \underline{\pi}\right\}, \widetilde{\pi}_0\right)$, and all parameter restrictions are met. ■

Appendix 2: Analysis regarding capital requirements

Equilibrium with the prudent asset

As in Repullo we study the symmetric equilibrium when all banks use the prudent asset. Corresponding to equation (4) in Repullo we have that each bank maximizes the present value of future profits, i.e. solves:

$$\max_{r_j} -k\mathfrak{D}(r_j, r, n) + \frac{1}{1+\rho}[\alpha - r_j + (1+\alpha)k]\mathfrak{D}(r_j, r, n) + \frac{1}{1+\rho}V_P \tag{45}$$

where V_P is the value for the individual bank of being in the prudent equilibrium. Using (2), the first order condition for this problem is

$$-k\frac{D^2}{\mu} - \frac{1}{1+\rho}\mathfrak{D}(r_j, r, n) + \frac{1}{1+\rho}[\alpha - r_j + (1+\alpha)k]\frac{D^2}{\mu} = 0$$

Since we are considering a symmetric equilibrium we set $r_j = r$ which, after multiplying through by $\frac{(1+\rho)\mu}{D^2}$, gives

$$-(1+\rho)k - \frac{\mu}{nD} + \alpha - r + (1+\alpha)k = 0$$

that is

$$r_P(n,k) = \alpha - \frac{\mu}{nD} - \delta_P k \tag{46}$$

where $\delta_P = \rho - \alpha$. Note that this equation puts some limits on k. Inserting this interest rate in the maximand above we get

$$\begin{aligned} V_P &= -k\frac{D}{n} + \frac{1}{1+\rho}[\frac{\mu}{nD} + (\rho-\alpha)k + (1+\alpha)k]\frac{D}{n} + \frac{1}{1+\rho}V_P \\ &= \frac{1}{1+\rho}\frac{\mu}{n^2} + \frac{1}{1+\rho}V_P \end{aligned}$$

so that

$$V_P = \frac{\mu}{\rho n^2} \tag{47}$$

just like in Repullo(2004). The reason for this equality is that D has two impacts on the banks: firstly an increase in D increases competition by making depositors willing to travel further, which has a negative impact on the value of being in the market. Secondly, it enhances the return for every depositor the bank has, which increases V_P. These two effects exactly offset each other. Setting $V_P = C$ we finally get

$$n_P = \sqrt{\frac{\mu}{\rho C}} \tag{48}$$

Equilibrium with the gambling asset

Here in a symmetric equilibrium each bank j is choosing its interest rate r_j to solve

$$\max_{r_j} -kD(r_j, r, n) + \frac{1-\pi}{1+\rho}[\gamma - r_j + (1+\gamma)k]D(r_j, r, n) + \frac{1-\pi}{1+\rho}V_G \tag{49}$$

where V_G is the value, to the bank, of the gambling equilibrium. The first order condition for this problem is

$$-k\frac{D^2}{\mu} - \frac{1-\pi}{1+\rho}D(r_j, r, n) + \frac{1-\pi}{1+\rho}[\gamma - r_j + (1+\gamma)k]\frac{D^2}{\mu} = 0$$

Setting, as before $r_j = r$, and multiplying through by $\frac{1+\rho}{1-\pi}\frac{\mu}{D^2}$ gives us

$$-k\frac{1+\rho}{1-\pi} - \frac{\mu}{nD} + \gamma - r + (1+\gamma)k = 0$$

that is

$$r_G(k, n) = \gamma - \frac{\mu}{nD} - \delta_G k \tag{50}$$

where

$$\delta_G = \frac{1+\rho}{1-\pi} - (1+\gamma)\ .$$

Inserting r_G into the objective function we find

$$V_G = -k\frac{D}{n}+\frac{1-\pi}{1+\rho}[\frac{\mu}{nD}+(\frac{1+\rho}{1-\pi}-(1+\gamma))k+(1+\gamma)k]\frac{D}{n}+V_G\frac{1-\pi}{1+\rho}$$

so that

$$V_G = \frac{1-\pi}{1+\rho}\frac{\mu}{n^2}+\frac{1-\pi}{1+\rho}V_G$$

i.e.

$$V_G = \frac{\mu}{n^2}\frac{1-\pi}{\rho+\pi}$$

which is the same as in Repullo. Setting this equal to C we get

$$n_G = \sqrt{\frac{\mu}{C}}\sqrt{\frac{1-\pi}{\rho+\pi}} \tag{51}$$

Viability of the prudent equilibrium

Like in Repullo(2004) we next find the condition for viability of the prudent equilibrium, i.e. we ask if, when the $n_P - 1$ other banks use the prudent equilibrium, it pays for an individual bank to deviate. Such a bank would solve

$$\begin{aligned}\max_{r_j} -kD(r_j, r_P(n_P,k), n_P) + \frac{1-\pi}{1+\rho}[\gamma - r_j + (1+\gamma)k] \\ \times D(r_j, r_P(n_P,k), n_P) + \frac{1-\pi}{1+\rho}V_P\end{aligned}$$

with first order condition

$-k\frac{D^2}{\mu} - \frac{1-\pi}{1+\rho}[\frac{r_j - r_P(n_P,k)}{\mu}D^2 + \frac{D}{n_P}] + \frac{1-\pi}{1+\rho}(\gamma - r_j + (1+\gamma)k)\frac{D^2}{\mu} = 0$

Multiplying through by $\frac{1+\rho}{1-\pi}\frac{\mu}{D^2}$ we get

$$-k\frac{1+\rho}{1-\pi} - (r_j - r_P(n_P,k)) - \frac{\mu}{Dn_P} + \gamma - r_j + (1+\gamma)k = 0$$

so that

$$r_j = \frac{r_G(n_P,k) + r_P(n_P,k)}{2} \tag{52}$$

Notice that we did not impose participation constraints on the deviator, i.e. we do not take into account that some depositors may choose not to walk to any bank, if the deviator offers an interest rate different from $r_P(n_P, k)$. The reason is that if deviation is not desirable without this constraint, it is certainly not desirable with the constraint. Thus we find sufficient conditions for the viability of the prudent equilibrium.

Remark 5. In the maximization problem we have assumed that a deviator chooses to invest entirely in the gambling asset. Let us briefly justify this. If the deviator may invest in both assets his problem becomes

$$\begin{aligned}
&\max_{r_j, z\in[0,]} -kD(r_j, r_P(n_P, k), n_P) \\
&+\left\{\frac{\pi}{1+\rho}\max\{0, z[\beta+(1+\beta)k]+(z-1)[\alpha+(1+\alpha)k]-r_j\}\right. \\
&\left.+\frac{1-\pi}{1+\rho}[z\gamma+(1-z)\alpha-r_j+(1+z\gamma+(1-z)\alpha)k]\right\} \\
&\times D(r_j, r_P(n_P, k), n_P) \\
&+[1_{\{z[\beta+(1+\beta)k]+(z-1)[\alpha+(1+\alpha)k]-r_j\geq 0\}}(z, r_j)\frac{\pi}{1+\rho}+\frac{1-\pi}{1+\rho}]V_P
\end{aligned}$$

where $1_{\{\cdot\}}(z, r)$ is the indicator function. Letting r^* and z^* being the solutions, if $z^*[\beta+(1+\beta)k]+(z^*-1)[\alpha+(1+\alpha)k]-r^*\geq 0$ (which, because $\alpha>\beta$ then also holds for $z\leq z^*$), z^* must solve

$$\begin{aligned}
&\max_{z\leq z^*} -kD(r^*, r_P(n_P, k), n_P) \\
&+\left\{\frac{\pi}{1+\rho}[z[\beta+(1+\beta)k]+(z-1)[\alpha+(1+\alpha)k]]\right. \\
&\left.+\frac{1-\pi}{1+\rho}[z\gamma+(1-z)\alpha-r^*+(1+z\gamma+(1-z)\alpha)k-r^*]\right\} \\
&\times D(r^*, r_P(n_P, k), n_P) \\
&+\frac{1}{1+\rho}V_P
\end{aligned}$$

However, since $\pi[\beta+(1+\beta)k]+(1-\pi)[\gamma+(1+\gamma)k] < \alpha+(1+\alpha)k$, this means that $z^* = 0$. If, on the other hand, $z^*[\beta + (1 + \beta)k] + (z^* - 1)[\alpha + (1 + \alpha)k] - r^* < 0$, z^* must solve

$$\max_{z \geq z^*} -kD(r^*, r_P(n_P, k), n_P) + \frac{1-\pi}{1+\rho}[z\gamma + (1-z)\alpha - r^*$$
$$+(1 + z\gamma + (1-z)\alpha)k]D(r^*, r_P(n_P, k), n_P) + \frac{1}{1+\rho}V_P$$

which, because $\gamma > \alpha$, means that $z^* = 1$ ■

Using (52), we now find the value of the deviation to the bank:

$$-k[\tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}D^2+\tfrac{D}{n_P}]+\tfrac{1-\pi}{1+\rho}[\gamma-\tfrac{r_G(n_P,k)+r_P(n_P,k)}{2}+(1+\gamma)k]$$
$$\times[\tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}D^2 + \tfrac{D}{n_P}] + \tfrac{1-\pi}{1+\rho}V_P$$
$$= \tfrac{1-\pi}{1+\rho}[\gamma - \tfrac{r_G(n_P,k)-r_P(n_P,k)}{2} + (1+\gamma)k - \tfrac{1+\rho}{1+\pi}k][\tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}$$
$$+\tfrac{D}{n_P}] + \tfrac{1-\pi}{1+\rho}V_P$$
$$= \tfrac{1-\pi}{1+\rho}[r_G(n_P,k) + \tfrac{\mu}{n_P D} - \tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}[\tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}$$
$$+\tfrac{D}{n_P}] + \tfrac{1-\pi}{1+\rho}V_P$$
$$= \tfrac{1-\pi}{1+\rho}[\tfrac{r_G(n_P,k)-r_D(n_P,k)}{2} + \tfrac{\mu}{n_P D}][\tfrac{r_G(n_P,k)-r_P(n_P,k)}{2\mu}$$
$$+\tfrac{D}{n_P}] + \tfrac{1-\pi}{1+\rho}V_P$$
$$= \tfrac{1-\pi}{1+\rho}[\tfrac{\{(r_G(n_P,k)-r_D(n_P,k))D\}^2}{4\mu} + \tfrac{(r_G(n_P,k)-r_D(n_P,k))D\}^2}{n_P}$$
$$+\tfrac{\mu}{n_P^2} + \tfrac{1-\pi}{1+\rho}V_P$$

Thus for a prudent equilibrium to exist we require

$$\tfrac{1-\pi}{1+\rho}\left[\tfrac{\{(r_G(n_P,k)-r_D(n_P,k))D\}^2}{4\mu} + \tfrac{(r_G(n_P,k)-r_D(n_P,k))D\}^2}{n_P} + V_P\right] + \tfrac{\mu}{n_P^2}$$
$$\leq V_P$$

This condition is parallel to the condition of Repullo (2004) in his proof of Proposition 1, and using a parallel reasoning this can be rewritten as

$$\frac{\{(r_G(n_P,k)-r_D(n_P,k))D\}^2}{2}+(r_G(n_P,k)-r_D(n_P,k))D\frac{\mu}{n_P}-(h^2-1)\left[\frac{\mu}{n_P}\right]^2 \leq 0$$

where $h = \sqrt{\frac{\rho+\pi}{(1-\pi)\rho}}$ which, following the argument of Repullo means that the requirement is

$$\frac{\mu}{n_P} \geq \frac{\gamma - \alpha - (\delta_G - \delta_P)k}{2(h-1)} D = m_P(k)D \tag{53}$$

and where, as in Repullo (2004), $m_P(k) = \frac{\gamma-\alpha-(\delta_G-\delta_P)k}{2(h-1)}$.

Viability of the gambling equilibrium

The problem of a deviator (one among n_G banks) is to choose an interest rate r_j to solve

$$\max_{r_j} -kD(r_j,r_G,n_G)+\frac{1}{1+\rho}[\alpha-r_j+(1+\alpha)k]D(r_j,r_G,n_G)+\frac{1}{1+\rho}V_G \tag{54}$$

The first order condition for this problem is

$$-k\frac{D^2}{\mu} - \frac{1}{1+\rho}[\frac{r_j-r_G(n_G,k)}{\mu}D^2 + \frac{D}{n}] + \frac{1}{1+\rho}[\alpha - r_j + (1+\alpha)k]\frac{D^2}{\mu} = 0$$

Multiplying through by $(1+\rho)\frac{\mu}{D^2}$ we get

$$-(1+\rho)k - (r_j - r_G(n_G,k)) - \frac{\mu}{n_G(n_G,k)D} + \alpha - r_j + (1+\alpha)k = 0$$

so that

$$r_j = \frac{r_G(n_G,k) + r_P(n_G,k)}{2} \tag{55}$$

Inserting (55) into (54) we find the value of the deviation:

$$\begin{aligned}
&\frac{1}{1+\rho}[\alpha - \frac{r_G(n_G,k)-r_P(n_G,k)}{2} \\
&+(1+\alpha)k - (1+\rho)k][\frac{r_P(n_G,k)-r_G(n_G,k)}{2\mu}D^2 + \frac{D}{n}] + \frac{1}{1+\rho}V_G \\
&= \frac{1}{1+\rho}[\frac{r_P(n_G,k)-r_G(n_G,k)}{2}+\frac{\mu}{nD}][\frac{r_P(n_G,k)-r_G(n_G,k)}{2\mu}D^2+\frac{D}{n}]+\frac{1}{1+\rho}V_G \\
&= \frac{1}{1+\rho}[\frac{[(r_P(n_G,k)-r_G(n_G,k))D]^2}{4\mu}+\frac{(r_P(n_G,k)-r_G(n_G,k))D}{n_G}+\frac{\mu}{n_G^2}]+\frac{1}{1+\rho}V_G
\end{aligned}$$

This means that deviating is not advantageous (i.e. all firms gambling is a Nash equilibrium) if

$$\frac{1}{1+\rho}\left[\frac{[(r_P(n_G,k)-r_G(n_G,k))D]^2}{4\mu}+\frac{(r_P(n_G,k)-r_G(n_G,k))D}{n_G}+\frac{\mu}{n_G^2}+V_G\right]\leq V_G$$

that is

$$\frac{[(r_P(n_G,k)-r_G(n_G,k))D]^2}{4\mu}-\frac{(r_P(n_G,k)-r_G(n_G,k))D}{n_G}+\frac{\mu}{n_G^2}-\rho V_G\leq 0$$

that is, using the definition of V_G, if

$$\frac{[(r_P(n_G,k)-r_G(n_G,k))D]^2}{4}-(r_P(n_G,k)-r_G(n_G,k))D\frac{\mu}{n_G}+(1-\rho\frac{1-\pi}{\rho-\pi})\left\{\frac{\mu}{n_G}\right\}^2 \leq 0$$

Noting that $1-\rho\frac{1-\pi}{\rho-\pi}=1-\frac{1}{h^2}$ and using a reasoning parallel to that of Repullo(2004) we then get the condition

$$\frac{\mu}{n_G}\leq\frac{h[\gamma-\alpha-(\delta_G-\delta_P)k]}{2(h-1)}D=hm_P(k)D=m_G(k)D \qquad (56)$$

Notice that, as was the case for the prudent equilibrium, we did not impose participation constraints on the deviator, the reasons being the same as stated there. Thus we find sufficient conditions for the viability of the gambling equilibrium.

Bankruptcy of a gambling bank in the bad state

We have assumed that a gambling bank will be bankrupt in the bad state and shall now explain why this will always be so. Consider first a prudent equilibrium and let

$$V_G(r_j)=-kD(r_j,r_P,n_P)+\frac{1-\pi}{1+\rho}[\gamma-r_j+(1+\gamma)k]D(r_j,r_P,n_P) + \frac{1-\pi}{1+\rho}V_P$$

Maximizing $V_G(r_j)$ gives $\hat{r}_j$. Suppose that $(1+\beta)(1+k)\geq\hat{r}_j+1$, so that the deviating bank is not bankrupt in the bad state. Then since in that case

$$
\begin{aligned}
V_G(\hat{r}_j) &< -k - kD(\hat{r}_j, r_P, n_P) + \tfrac{1-\pi}{1+\rho}[\gamma - \hat{r}_j + (1+\gamma)k]D(\hat{r}_j, r_P, n_P) \\
&\quad + \tfrac{\pi}{1+\rho}[\beta - \hat{r}_j + (1+\beta)k]D(\hat{r}_j, r_P, n_P) + \tfrac{1-\pi}{1+\rho}V_P \\
&\le -kD(\hat{r}_j, r_P, n_P) + \tfrac{1}{1+\rho}[\alpha - \hat{r}_j + (1+\alpha)k]D(\hat{r}_j, r_P, n_P) + \tfrac{1}{1+\rho}V_P \\
&\le V_P
\end{aligned}
$$

a deviation is not attractive. Notice that for any other choice of interest rate, r_j for a deviating bank, since $V_G(r_j) \le V_G(\hat{r}_j)$, if $(1+\beta)(1+k) \ge \hat{r}_j + 1$ this interest rate is not attractive. In other words, the only attractive deviations imply bankruptcy. From this we conclude that if $V_G(r_j) \ge V_P$ a deviating bank is bankrupt in the bad state.

Consider next the viability of a gambling equilibrium, (r_G, n_G). If the banks are not bankrupt in the bad state, then by the reasoning above, it pays to deviate to the prudent equilibrium. Thus, with $V_P(\hat{r}_j)$ being the value of an optimal deviation to a prudent strategy, starting from a gambling equilibrium, if $V_G \ge V_P(\hat{r}_j)$ the banks using the gambling strategy must be bankrupt in the bad state.

We found that there is a $\tilde{k}$ such that both a gambling equilibrium (r_G, n_G) and a prudent equilibrium, (R_P, n_P) exist.

In the gambling equilibrium a deviation by a single bank to a prudent strategy does not increase discounted profits, which implies that in the gambling equilibrium banks are bankrupt in the bad state. In the prudent equilibrium, a deviation by a single bank to the gambling strategy does not increase discounted expected profits, which is only possible if the deviating bank is bankrupt in the bad state.

As we increase k above $\tilde{k}$, only prudent equilibria are viable, which could either mean that a optimal deviation implies no bankruptcy or that it implies bankruptcy. As we lower k below $\tilde{k}$ the prudent equilibrium is not viable since there always exists a profit-increasing deviation to a gambling strategy (necessarily implying bankruptcy in the bad state) and a gambling equilibrium is viable (necessarily meaning that banks are bankrupt in the bad state).

Since the optimal regulation using capital requirements sets $k = \tilde{k}$ we have shown that assuming that gambling banks are bankrupt in the bad state does not invalidate our analysis.

Welfare in the prudent equilibrium as the return of the average consumer

There is another way to compute the welfare in the prudent equilibrium, namely by looking at the return of the average consumer, which is

$$(1 + r_p)D - \frac{\mu}{4n_P} \tag{57}$$

We check that this is indeed equal to W_P, thus we check whether

$$(1 + r_P)D - \tfrac{1}{4}\tfrac{\mu}{n_P} = (1 - \alpha)(1 + k)D - \tfrac{\mu}{4n_P} - n_P\rho C - (1 + \rho)kD$$

i.e. if

$$(1 + \alpha - \tfrac{\mu}{n_P D} - (\rho - \alpha)k)D - \tfrac{1}{4}\tfrac{\mu}{n_P}$$
$$= (1+\alpha)(1+k)D - \tfrac{\mu}{4n_P} - n_P\rho C - (1+\rho)kD$$

Since

$$(1+\alpha)(1+k)D - \rho n_P C - (1+\rho)kD = (1+\alpha)D - (\rho-\alpha)kD - n_P\rho C$$

we just have to check if $\frac{\mu}{n_P} = n_P\rho C$ which is the case since $\frac{\mu}{C\rho} = n_P^2$. ■

Welfare in the gambling equilibrium as the return of the average consumer

As we did with W_P, we may also perform a consistency check of W_G, i.e. check if

$$(1 - \pi)(1 + r_G)D + \pi(1 + \beta)(1 + k)D - \frac{1}{4}\frac{\mu}{n_G} = W_G(k, \pi) \tag{58}$$

Observing that

$$(1 - \pi)(1 + r_G)D + \pi(1 + \beta)(1 + k)D$$
$$= (1-\pi)\left\{1 + \gamma - \tfrac{\mu}{n_G D} - \left[\tfrac{1+\rho}{1-\pi} - (1 + \gamma)\right] k\right\} D + \pi(1+\beta)(1+k)D$$

$$= [1 + (1-\pi)\gamma + \pi\beta]D + [(1-\pi)(1+\gamma) + \pi(1+\beta)]kD$$
$$-(1+\rho)kD - (1-\pi)\tfrac{\mu}{n_G}$$

we only need to check if

$$n_G\,(\rho+\pi)\,C = (1-\pi)\frac{\mu}{n_G}$$

However the LHS of this equation is $\sqrt{(1-\pi)\,(\rho+\pi)\,\mu C}$ as is the RHS. So the two approaches to computing welfare in the gambling equilibrium are indeed equivalent. ■

Appendix 3: Analysis regarding deposit rate control

Remark 6. *Bankruptcy of gambling banks under deposit rate control.* As for capital requirements, it does not pay to deviate to the gambling strategy with an interest rate that does not imply bankruptcy in the bad state, hence we get the format of the problem presented in the main body of the text.

Further analysis of the gambling equilibrium with interest rate ceiling.

We analyse case (a) where $\overline{R}_G(\overline{r}) = \overline{r}$ and the optimal interest rate for the deviator is $\overline{r}$. For existence of gambling equilibrium we require $(\alpha - \overline{r})\mathfrak{D}(\overline{r}, \overline{r}, \overline{n}_G(\overline{r})) \leq \rho C$, i.e. $(\alpha - \overline{r})\frac{C}{\gamma - \overline{r}}\frac{\pi+\rho}{1-\pi} \leq \rho C$ which, recalling (23), can be simplified to $\overline{r}_P \leq \overline{r}$. If furthermore $\overline{r} < \overline{r}_1$, then there is a gambling equilibrium where the ceiling is binding in equilibrium and for any potential deviator.

We next analyze case (b) where $\overline{R}_G(\overline{r}) = \overline{r}$ and the optimal interest rate for the deviator is $r^* = \frac{1}{2}\left\{\overline{r} + \alpha - \frac{\mu}{D\overline{n}_G(\overline{r})}\right\} = \frac{1}{2}\left\{\overline{r} + \alpha - \frac{1}{D^2}\frac{\pi+\rho}{1-\pi}\frac{\mu C}{\gamma - \overline{r}}\right\}$. For existence we require

$$\Pi(\overline{r}) \equiv (\alpha - r^*)\mathfrak{D}(r^*, \overline{r}, \overline{n}_G(\overline{r}) = (\alpha - r^*)\left\{\frac{r^* - \overline{r}}{\mu}D^2 + \frac{D}{\overline{n}_G(\overline{r})}\right\} \leq \rho C$$

that is

$\frac{1}{2}\left\{\alpha-\overline{r}+\frac{1}{D^2}\frac{\pi+\rho}{1-\pi}\frac{\mu C}{\gamma-\overline{r}}\right\}\left\{\frac{1}{2}\left[\alpha-\frac{1}{D^2}\frac{\pi+\rho}{1-\pi}\frac{\mu C}{\gamma-\overline{r}}-\overline{r}\right]\frac{D^2}{\mu}+\frac{\pi+\rho}{1-\pi}\frac{C}{\gamma-\overline{r}}\right\}$

$\leq \rho C$

which can be simplified to

$\left\{(\alpha-\overline{r})(\gamma-\overline{r})+\frac{1}{D^2}\frac{\pi+\rho}{1-\pi}\mu C\right\}$

$\times\left\{\left[(\alpha-\overline{r})(\gamma-\overline{r})-\frac{1}{D^2}\frac{\pi+\rho}{1-\pi}\mu C\right]\frac{D^2}{\mu}+2\frac{\pi+\rho}{1-\pi}C\right\}\leq 4\rho C(\gamma-\overline{r})^2$

which cannot hold for large D. However this inequality is difficult to analyze further. We make one more potentially useful observation. Using the envelope theorem we have

$\frac{d\Pi}{d\overline{r}}=(\alpha-r^*)\left[-\frac{D^2}{\mu}-\frac{D}{\overline{n}_G(\overline{r})^2}\frac{d\overline{n}_G(\overline{r})}{d\overline{r}}\right]=(\alpha-r^*)\left[-\frac{D^2}{\mu}+\frac{\pi+\rho}{1-\pi}C\frac{1}{(\gamma-\overline{r})^2}\right]$

which is <0 if $\frac{D^2}{\mu}>\frac{\pi+\rho}{1-\pi}C\frac{1}{(\gamma-\overline{r})^2}\Leftrightarrow(\gamma-\overline{r})^2>\frac{\pi+\rho}{1-\pi}\frac{C\mu}{D^2}\Leftrightarrow\gamma-\frac{1}{D}\sqrt{\frac{\pi+\rho}{1-\pi}C\mu}>\overline{r}$ which is the case. Hence as $\overline{r}$ increases the profit of a deviation decreases and hence a gambling equilibrium becomes more likely to exist. This is of course what we would expect - the whole point of an interest rate ceiling is to make a gambling equilibrium less likely.

Appendix 4: Derivation of Figure 4

Fact 1: In panel (a) the curve C_1C_1, defined by equality of (24), and the curve C_2C_2, defined by equality of (23), are such that C_1C_1 lies above C_2C_2 except at the point (π_2, r_P) where the curves are tangent.

Proof: Let

$$f(r_j,r,n,\pi):=\frac{1-\pi}{1+\rho}(\gamma-r_j)\mathfrak{D}(r_j,r,n)+\frac{1-\pi}{1+\rho}C$$

and

$$V(\overline{r},r,n,\pi):=\max_{r_j\leq\overline{r}}f(r_j,r,n,\pi)\,,\ \overline{V}(\overline{r},r,n,\pi):=f(\overline{r},r,n,\pi)\ .$$

Then $\overline{V}(\overline{r},r,n,\pi)=V(\overline{r},r,n,\pi)$ if the constraint $r_j\le\overline{r}$ is binding, which is the case we are interested in. Define moreover

$$\overline{V}_b(\overline{r},\pi):=\overline{V}(\overline{r},\overline{r},\overline{n}_P(\overline{r}),\pi),\ \text{with}\ \overline{n}_P(\overline{r})=\frac{\alpha-\overline{r}}{\rho C}D,$$

and

$$\overline{V}_{nb}(\overline{r},\pi):=\overline{V}(\overline{r},r_P,n_P,\pi),\ \text{with}\ r_P=\alpha-\frac{\sqrt{\mu\rho C}}{D},\ n_P=\sqrt{\frac{\mu}{\rho C}}\,.$$

We know that $\overline{r}\le r_P\Leftrightarrow\overline{n}_P(\overline{r})\ge n_P$ and $\overline{r}=r_P\Leftrightarrow\overline{n}_P(\overline{r})=n_P$. Since the curves C_1C_1 and C_2C_2 are by (24) and (23) the level curves $\overline{V}_{nb}(\overline{r},\pi)\equiv C$ and $\overline{V}_b(\overline{r},\pi)\equiv C$, respectively, and both $\overline{V}_{nb}(\cdot)$ and $\overline{V}_b(\cdot)$ are differentiable functions, the claim will follow from the following

Lemma 6. *$\overline{V}_b(\overline{r},\pi)\ge\overline{V}_{nb}(\overline{r},\pi)$ for all $\overline{r},\pi$, and $\overline{V}_b(\overline{r},\pi)=\overline{V}_{nb}(\overline{r},\pi)$ if and only if $\overline{r}=r_P$.*

Proof: $\overline{V}_b\ge\overline{V}_{nb}\Leftrightarrow\mathfrak{D}(\overline{r},\overline{r},\overline{n}_P(\overline{r}))\ge\mathfrak{D}(\overline{r},r_P,n_P)$ which by (2) is equivalent to

$$\frac{D}{\overline{n}_P(\overline{r})}+(\overline{r}-\overline{r})\frac{D^2}{\mu}\ge\frac{D}{n_P}+(\overline{r}-r_P)\frac{D^2}{\mu}$$

$\Leftrightarrow$

$$\frac{1}{\frac{\alpha-\overline{r}}{\rho C}D}\ge\frac{1}{\sqrt{\frac{\mu}{\rho C}}}+\left(\overline{r}-\alpha+\frac{\sqrt{\mu\rho C}}{D}\right)\frac{D}{\mu}$$

$\Leftrightarrow$

$$\frac{\mu\rho C}{(\alpha-\overline{r})D}\ge\sqrt{\mu\rho C}-(\alpha-\overline{r})D+\sqrt{\mu\rho C}$$

$\Leftrightarrow$

$$\mu\rho C-2\sqrt{\mu\rho C}(\alpha-\overline{r})D+(\alpha-\overline{r})^2D^2\ge0$$

$\Leftrightarrow$

$$\left(\sqrt{\mu\rho C}-(\alpha-\overline{r})D\right)^2\ge0$$

which is true. Moreover, we get equality iff $\sqrt{\mu\rho C}=(\alpha-\overline{r})D\Leftrightarrow\sqrt{\frac{\mu}{\rho C}}=\frac{(\alpha-\overline{r})D}{\rho C}\Leftrightarrow n_P=\overline{n}_P(\overline{r})$. ■

Remark 7. There is a second way of proving tangency which is the following. Let $(d\overline{r}/d\pi)_b$ and $(d\overline{r}/d\pi)_{nb}$ be defined implicitly by $\overline{V}_b(\overline{r},\pi) \equiv C$ and $\overline{V}_{nb}(\overline{r},\pi) \equiv C$, respectively. From the definitions of $\overline{V}_b$ and $\overline{V}_{nb}$ we get that $(d\overline{r}/d\pi)_b = (d\overline{r}/d\pi)_{nb}$ iff

$$-\frac{\partial\overline{V}_b/\partial\pi}{\partial\overline{V}_b/\partial\overline{r}} = -\frac{\partial\overline{V}_{nb}/\partial\pi}{\partial\overline{V}_{nb}/\partial\overline{r}} \Leftrightarrow \frac{\partial\overline{V}/\partial\pi}{\frac{\partial\overline{V}}{\partial\overline{r}}+\frac{\partial\overline{V}}{\partial r}+\frac{\partial\overline{V}}{\partial n}\overline{n}'_P(\overline{r})} = \frac{\partial\overline{V}/\partial\pi}{\frac{\partial\overline{V}}{\partial\overline{r}}}$$

$$\Leftrightarrow \frac{\partial\overline{V}}{\partial r} + \frac{\partial\overline{V}}{\partial n}\overline{n}'_P(\overline{r}) = 0 \Leftrightarrow \frac{\partial\mathfrak{D}}{\partial r} + \frac{\partial\mathfrak{D}}{\partial n}\overline{n}'_P(\overline{r}) = 0$$

$$\Leftrightarrow -\frac{D^2}{\mu} - \frac{D}{\overline{n}^2_P(\overline{r})}\overline{n}'_P(\overline{r}) = 0 \Leftrightarrow -\frac{D}{\left(\frac{\alpha-\overline{r}}{\rho C}\right)^2 D^2}\left(-\frac{D}{\rho C}\right) = \frac{D^2}{\mu}$$

$$\Leftrightarrow \alpha - \overline{r} = \frac{\sqrt{\mu\rho C}}{D} \Leftrightarrow \overline{r} = \alpha - \frac{\sqrt{\mu\rho C}}{D} = r_P\ .$$

Fact 2: The value of π in panel (a) at which curve $\mathrm{C}_1\mathrm{C}_1$ turns vertical is the same at which in panel (b) $\widetilde{k}(\pi)$ intersects the horizontal axis, i.e. at $\pi = \tilde{\pi}_0$.

<u>Proof</u>: We know that $\widetilde{k}(\tilde{\pi}_0) = 0$ with $\tilde{\pi}_0$ given by (15). Write $\tilde{\pi}_0 = (A-1)/(A+1/\rho)$ where $A = \left(\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}}+1\right)^2$. Then the term $(\rho+\pi)/(1-\pi)$ in (15) for $\pi = \tilde{\pi}_0$ becomes
$(\rho A + 1 + A - 1)/(A + 1/\rho - A + 1) = \rho A$.
The point on $\mathrm{C}_1\mathrm{C}_1$ where the curve turns vertical is where the two roots $\overline{r}_1$ and $\overline{r}_2$ of the equality case of (24) coincide. This means that the corresponding discriminant $b^2 - 4ac$ becomes

$$\left[(\gamma+\alpha) - \frac{2}{D}\sqrt{\rho C\mu}\right]^2 - 4\left[\alpha\gamma - \frac{2\gamma}{D}\sqrt{\rho C\mu} + \rho\left(\frac{\gamma-\alpha}{2}\frac{D}{\sqrt{\mu\rho C}}+1\right)^2\frac{C\mu}{D^2}\right]$$

and must be zero. To see the latter, rewrite the expression as

$$(\gamma+\alpha)^2 - 4\frac{\gamma+\alpha}{D}\sqrt{\rho C\mu} + \frac{4}{D^2}\rho C\mu - 4\alpha\gamma + \frac{8\gamma}{D}\sqrt{\rho C\mu}$$

$$-4\frac{\rho C\mu}{D^2}\left[\frac{(\gamma-\alpha)^2}{4}\frac{D^2}{\mu\rho C} + (\gamma-\alpha)\frac{D}{\sqrt{\mu\rho C}} + 1\right]$$

$$= (\gamma+\alpha)^2 + \sqrt{\rho C\mu}\left(\frac{8\gamma}{D} - 4\frac{\gamma+\alpha}{D}\right) - 4\alpha\gamma - (\gamma-\alpha)^2 - 4(\gamma-\alpha)\frac{\sqrt{\rho C\mu}}{D}$$

$$= \gamma^2 + 2\alpha\gamma + \alpha^2 - 4\alpha\gamma - \gamma^2 + 2\alpha\gamma - \alpha^2 + \frac{\sqrt{\rho C\mu}}{D}(8\gamma - 4\gamma - 4\alpha - 4\gamma + 4\alpha)$$

$$= 0\ .$$

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