

Strategic Behavior and Economic Dynamics in ICT Markets: An Agent-Based Framework

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General Introduction

State-space models (SSMs) provide a widely used framework for analyzing dynamic systems where hidden states evolve over time based on observed variables. Lux & Zwinkels (2018) summarizes the framework of a general state-space model as:

$$\begin{aligned}x_t &= f(x_{t-1}, \varepsilon_t) \\ y_t &= g(x_t, \eta_t)\end{aligned}$$

where x_t represents the stochastic evolution in time of a vector of states, ε_t represents a summary for all stochastic factors that enter into the dynamics of x_t , y_t represents a vector of measurements, and η_t represents a summary for all stochastic factors that make the vector of measurements a noisy signal of the states x_t . While SSMs are traditionally applied in econometrics and control theory, this dissertation explores their broader applicability in economic modeling.

Specifically, although the primary models developed in this work are small-scale agent-based models (ABMs) rather than classical SSMs, they share key methodological features. Tesfatsion (2006) illustrates that agent-based modeling specifies the initial state of an economic system by specifying each agent’s initial data and behavioral methods and the degree of accessibility of these data and methods to other agents. This approach shares similar features with general SSMs. Lux & Zwinkels (2018) further summarizes that the estimation of ABMs could profit substantially from the methods for state-space models or slightly more general latent variable models. Moreover, many ABMs, including the models in this thesis, to a large extent satisfy the general framework of a general SSM mentioned above. In particular, ABMs capture system dynamics through agent interactions, which collectively define an evolving state over time. This conceptual similarity allows for the application of statistical inference techniques inspired by SSMs, such as parameter estimation and model validation, to ABMs. By adopting this perspective, the dissertation aims to bridge elements of state-space analysis and agent-based modeling, while recognizing the fundamental differences between the two approaches.

Based on the “bottom-up” nature, ABMs consider the microfoundations of the economic system and have the potential to show emergent behavior in a complex world. A substantial body of research suggests the value of these models,

especially in macroeconomic analysis and Industrial Organization studies (Dawid & Delli Gatti 2018, Dawid et al. 2021, Axtell & Farmer 2022). From theory to data, research related to calibrating ABMs includes frequentist approaches, e.g., method of moments, maximum likelihood, and recent Bayesian inference methods, e.g., Bayesian Monte Carlo and Bayesian kernels. Other advanced methods including neural networks have also been applied (Grazzini & Richiardi 2015, Lux & Zwinkels 2018, Delli Gatti & Grazzini 2020, Platt 2022). Nevertheless, it is important to acknowledge that at the meso-level of economics—particularly when examining competitive dynamics within specific industries—existing approaches may overlook critical complexities. The Information and Communications Technology (ICT) industry provides a compelling example, characterized by intricate dynamics such as platform competition, data restrictions, user experience, and capacity constraints. These complexities have not been sufficiently addressed in prior literature, either at the theoretical level or within empirical studies that primarily emphasize structural parameter estimation.

The dissertation consists of three main chapters. The first two chapters are theoretical, developing models to analyze firm strategies and market dynamics. While both focus on theoretical foundations, they differ in scope: Chapter 1 establishes the core theoretical framework of an industrial organization model, examining how data restriction policies influence firm profits and social welfare. Chapter 2 extends the analysis by incorporating consumer networks and consumer congestion effects, exploring their impact on market outcomes and firm competition. The final chapter (Chapter 3) is empirical, applying the theoretical foundations of the first two chapters to real-world data. By integrating firm-level data, it assesses the practical relevance and applicability of the proposed models, bridging the gap between theory and empirical validation.

Chapter 1 examines the impact of data restrictions on the software market, focusing on how varying levels of regulation influence the economic activities and welfare over time. Unlike traditional goods, software products are inherently intangible, rely heavily on information systems, and operate under complex licensing mechanisms. Their pricing and availability differ across regions due to variations in local data regulations. To investigate these dynamics, this chapter adopts a two-fold approach, combining game theory and agent-based modeling to simulate interactions among key market participants—software owners and developers, consumers, and governments. The proposed models capture firms’ strategic behavior, consumers’ decision-making processes, and the dynamic pricing mechanisms shaped by evolving regulatory landscapes. Findings from computational experiments highlight that data restrictions play a crucial role in shaping price formation and market structures. Stricter regulations can distort prices, alter competitive strategies, and reshape market dynamics, underscoring the need for a balanced regulatory

framework that supports both market efficiency and consumer protection. Chapter 2 assesses the interplay between firm capacity selection, network effects, and market competition in the ICT industry. Traditional industrial organization theory highlights R&D investment as the principal driver of firm profitability through innovation. In ICT markets with capacity constraints, however, firms face a more complex strategic environment. Service capacity investments, which determine how much customer demand firms can accommodate, play a critical role in shaping both operational efficiency and service quality. When firms operate beyond their capacity limits, service quality deteriorates, leading to customer loss and diminished profits. Additionally, consumer behavior, which is strongly shaped by digital community interactions, plays a crucial role in determining the perceived value of R&D-driven quality improvements. The model in this chapter captures firms' investment strategies, the decision-making processes of consumers, and the broader influence of consumer networks on market outcomes. The findings provide new insights into the interdependencies between service capacity, R&D investment effectiveness, and consumer engagement. By investigating how firms navigate these challenges, this chapter contributes to a deeper understanding of strategic capacity selection in competitive environments and the role of network effects in shaping firm performance. The results highlight the importance of balancing capacity constraints, investment efficiency, and consumer relationships to sustain long-term profitability in the ICT sector.

Chapter 3 explores advancements in parameter estimation and calibration in agent-based models (ABMs). While ABMs have significant potential to uncover emergent phenomena and inform policy decisions, the estimation and calibration of model parameters remains a major challenge. This chapter addresses this issue by analyzing the model setup introduced in Chapter 2, where strategic firm behavior and network effects play a central role. To enhance model validity, this chapter employs state-of-the-art estimation techniques, including the Simulated Method of Moments (SMM) and Bayesian Monte Carlo methods, integrating financial data from major U.S. telecommunication firms (AT&T and T-Mobile) to calibrate parameters and validate the model. By systematically comparing these methodologies, this chapter evaluates their suitability, effectiveness, and practical implications for agent-based economic modeling. Beyond methodological advancements, this chapter contributes to the broader discourse on improving the validity and applicability of computational economic models. The findings provide key insights into overcoming industry-specific challenges, enhancing model reliability, and bridging the gap between theoretical modeling and real-world applications.

Chapter 1

Heterogeneous Consumers, Firm Competition and the Data Value Chain: An Agent-based Approach

The advancement of Information and Communications Technologies (ICTs) and the trend towards digitalization have significantly fueled the growth of software applications. These applications are distinct from traditional goods due to their reliance on information systems, the intangible nature of products, and the critical role of licensing. The pricing and availability of these applications differ internationally due to varying local data restrictions. In this paper, we aim to answer how the level of data restriction influences the geographical distribution of economic activities over time. To decipher the impact of data restriction levels on the software market, we employ a game-theoretical approach and an agent-based approach involving entities such as firms, consumers, and governments. Our model includes the interactions between software owners and developers, the selection process of diverse consumers, the tactics of software owners, and the fluctuation of prices. Our computational experiments indicate that the intensity of data restrictions can influence the price formation process. We recommend that governments in different regions should work collaboratively to minimize data regulation in the public software market and that firms should better understand consumer preferences, so that the related policies can benefit both consumers and local businesses.

Keywords: Data Value Chain; Software product; Data restriction level; Heterogeneous agents; Computational simulations

1.1 Introduction

With the swift advancement of Information and Communications Technologies (ICTs), software products, particularly mobile applications (Apps), have seen widespread adoption and integration into production. Viewed from the standpoint of individual consumers, software platforms (for example, Google Play Store, Apple Store, etc.) offer an array of Apps serving diverse functions, ranging from work to leisure activities.¹² On the business and research front, corporations and research institutions demand more specialized tools tailored to their professional needs, often possessing greater financial capacity to invest in these products and their associated services. A prime example of this is using database applications for data analysis. Furthermore, examining the situation on the macroeconomic level reveals a significant surge in the growth of Artificial Intelligence (AI) companies, underscoring the critical importance of the software industry in large economies (Figure 1.1).

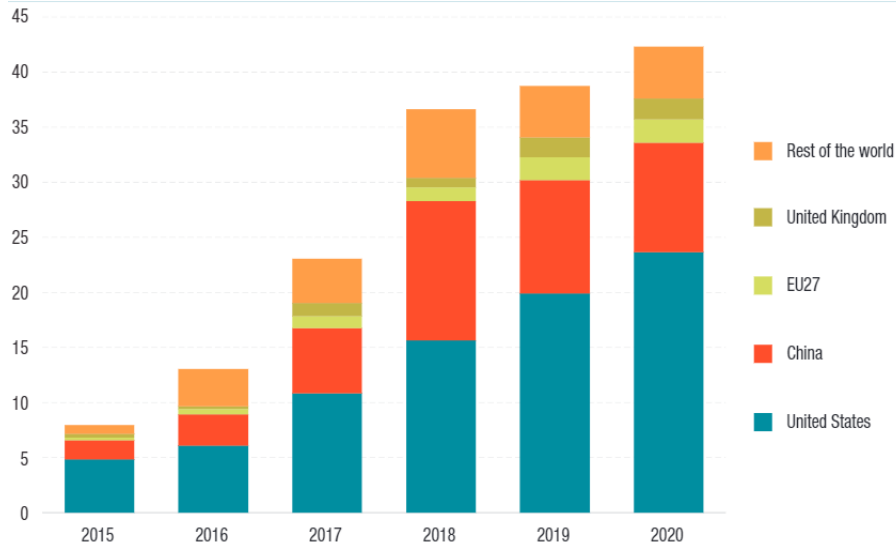


Figure 1.1: Private investment in AI companies, by economy, 2015–2020 (Billions of dollars). *Source: UNCTAD calculations, based on the publicly available database of NetBase Quid – 2021 AI Index Report (Zhang et al. 2021, UNCTAD 2021).*

In the software product market, we observe two notable trends. Firstly, not every App is accessible in every geographic location. For example, the Apple Store mandates customers to verify their residential region, and Apps are only available

¹<https://play.google.com/store/>

²<https://www.apple.com/store>

for purchase and download if they comply with the data restriction policies of the user’s region. Secondly, Apps designed for similar purposes may vary in pricing and functionalities, such as Apps offering business solutions. Since Apps and other software products are fundamentally data-driven, it is evident that data restriction policies play a crucial role in this market. When a government in one region implements stringent data restriction regulations, it may hinder consumers in another region from using Apps originating from the restricted area. This limitation can arise due to many factors, such as political concerns, local business protection measures, and local data protection laws.³

There have been essential policy discussions about data restriction policies. For instance, in 2021, the United Nations Conference on Trade and Development (UNCTAD) raised a key question: Should governments promote the free flow of data, or should they bolster domestic data storage localization to fulfill various national objectives (UNCTAD 2021)? The debate typically follows one of two paths: increasing local data restrictions to promote data localization, or easing them to enable freer cross-regional data flows. Prior research has not conclusively determined which approach is more advantageous for local economic development and societal welfare. Furthermore, the objectives of local governments may diverge from those of global-maximizers, which can be regarded as entities operating at a higher organizational level. This divergence results in a complex interplay of trade-offs between local data policies and global promotion strategies.⁴

While there exists an extensive array of documented data policies at the conceptual level, our economic analysis focuses primarily on one key element: the data restriction level. It is feasible to draw comparisons regarding data restrictions across various countries and regions (Figure 1.2). Ferracane et al. (2018) highlight that, generally, emerging economies tend to impose more stringent data restrictions. Additionally, within the European Union, France (with a restriction level of 0.45) and Germany (0.41) are noted for having the most restrictive data policies. Besides, the EU average (0.24) is more restrictive than that of the United States (0.15). This gradation in data restriction levels provides a valuable framework for understanding the varying degrees of data openness and control across different geopolitical regions.

³A specific example is a temporary block of ChatGPT in Italy on March 31, 2023 (Bertomeu et al. 2023).

⁴For instance, the European Union plays the role of the global-maximizer regarding the EU countries.

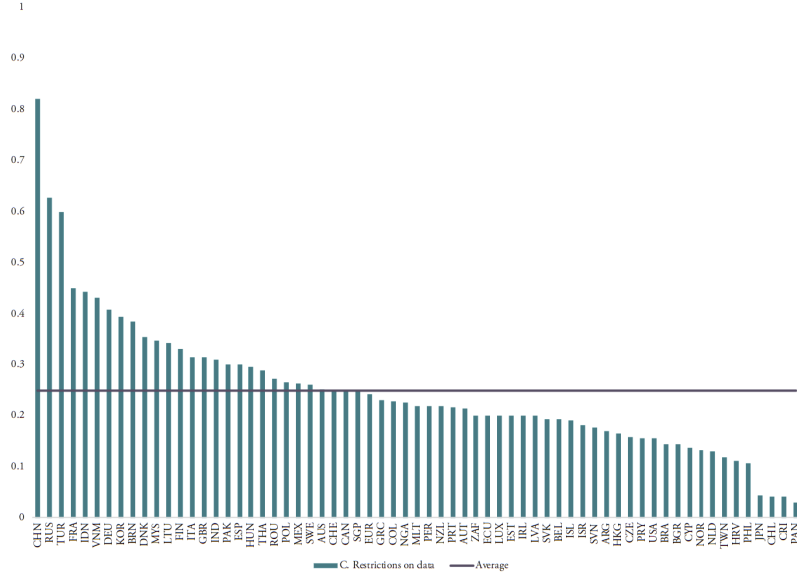


Figure 1.2: Restrictions on data of the Digital Trade Restrictiveness Index (Fer-racane et al. 2018).

Meanwhile, it is necessary to understand the software market structure. A software firm may opt for in-house development or employ individuals or other companies for software development. Following the initial access to raw data and the setup of the product’s framework, developers must ensure its functionality, which involves running and testing the preliminary version. Subsequently, the software firm conducts market analysis to determine the product’s pricing. After completing these stages, the software firm releases the product on a platform for consumer purchase. This process forms a “Data Value Chain” (DVC), a concept that originates from Porter’s 1985 definition. Porter delineated a value chain as a sequence of discrete activities a firm undertakes in the design, production, marketing, and distribution of the product (Porter 1985). Evidently, major digital platforms such as Apple, Tencent, and Alibaba have been actively investing in the global DVC, reaping profits from digital products. Previous literature (Pussep et al. 2011, UNCTAD 2021) has attempted to elucidate the DVC structure using flowcharts (Figure 1.3, Figure 1.4).



Figure 1.3: Stages of the Data Value Chain.



Figure 1.4: Activities in a unified software value chain.

Distinct economic properties characterize software products. As outlined by Pussep (2017), these properties include reliance on information systems, a preference for software over hardware implementations, and the role of customer involvement in value creation, among others. Additionally, these products are intangible, and exempt from transportation and inventory costs. When consumers acquire a software product, their purchase typically involves obtaining a license and installing the product on one or multiple devices. Moreover, there is substantial evidence highlighting the heterogeneity within software products. This diversity is not only in terms of functionality and application but also in pricing, target user groups, and compatibility with various hardware and operating systems. Such heterogeneity is crucial for understanding market dynamics and consumer choices in the software industry. The diversity of software products can be visualized in Figure 1.5.

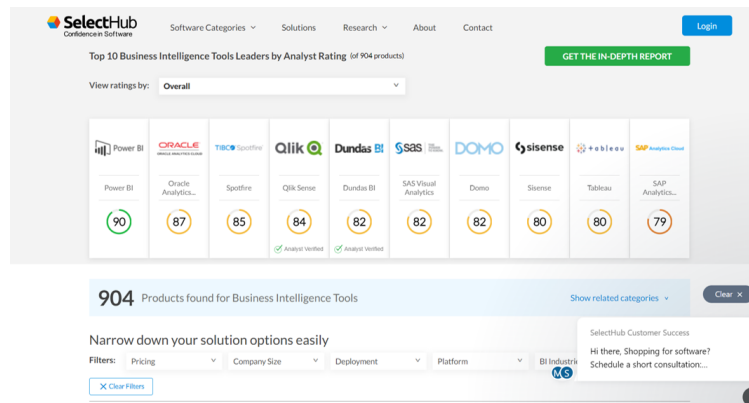


Figure 1.5: Top 10 Business Intelligence Tools Leaders by Analyst Rating of 904 products. Retrieved 15.11.2023. from <https://www.selecthub.com/c/business-intelligence-tools/>.

We aim to answer the following questions. Firstly, how does the level of local data restriction impact the spatial distribution of economic activities? This inquiry seeks to understand the extent to which data policies at a regional level influence the geographical dispersion and concentration of economic operations,

particularly in the context of the software industry. Secondly, what are the policy implications of data restrictions when considering both local economic profitability and global welfare? This question concerns the broader implications of data restriction policies, weighing local economic benefits against their overall impact on global economic and social well-being. Lastly, what strategies can firms employ to generate profits during firm competition? This question focuses on the tactical approaches that firms can adopt in response to the evolving landscape of data restrictions and market dynamics, aiming to maintain profitability and market outcomes across different time frames.

In this paper, our primary focus is on examining the impact of local data restrictions on the economic activities. The Data Value Chain (DVC), established through contracts between software copyright owners and developers, facilitates the production and value creation of software products. This research is driven by two main objectives: Firstly, while the topic of Global Value Chains (GVCs) has received considerable attention, a detailed analysis specifically addressing the added value associated with software products can provide a novel perspective to the discussion. Given the rapid evolution of digital economics, we posit that modeling the DVC will offer deeper insights into the interplay between data restriction level and the profitability of firms. Secondly, from a methodological standpoint, the valuation of data has predominantly been discussed in qualitative terms. Integrating the effects of digitalization into a concrete economic model presents a challenge. This perspective not only bridges the gap between theory and practice but also enhances the understanding of the dynamics between digitalization and economic models.

1.2 Literature review

In formulating our model, we clarify the concept and variations of the value chain. To this end, we draw upon key academic works that significantly help us understand Global Value Chains (GVCs) and their evolution. Kano et al. (2020) conducted a thorough review of GVC research, focusing on highly cited conceptual frameworks. They notably pointed out that the Data Value Chain (DVC) represents a fundamentally new business model, which has yet to be thoroughly understood by scholars in the field of international business. Antràs & Chor (2021) surveyed the recent body of economic literature that highlights the critical role of GVCs in influencing international trade flows and production patterns. This survey provides a broad overview of the economic implications of GVCs. Furthermore, Dawid et al. (2021) examined the impact of digitization, particularly in terms of product innovation, on the location of economic activities and value extraction within global supply chains. Their research also explores the effects of digitization

on industry dynamics, including market concentration and firm heterogeneity, using an agent-based model. These studies collectively inform our approach to modeling, providing a foundational understanding of both the traditional and digital aspects of value chains, which is crucial for our analysis of the software market in the context of data restrictions.

Analyzing software products necessitates a thorough understanding of the impact of digitalization on their economic properties. This perspective is crucial in evaluating how technological advancements shape the software industry. Goldfarb & Tucker (2017) underscored the shifts in various costs brought about by digitalization, including those related to search, replication, transportation, tracking, and verification processes. Their insights are instrumental in understanding the cost dynamics of digital products, including software. Connell et al. (2018) explored the influence of technology and innovation on the flexibility and sustainability of Global Value Chains (GVCs). This exploration is particularly relevant in assessing how digitalization affects the structure and function of value chains in the software industry. Schlüter & Hetterscheid (2017) provided concrete examples of how Industry 4.0 and digitalization reshape various sectors. Their work offers practical insights into the real-world implications of these technological shifts. Moreover, Dawid & Muehlheusser (2022) employed a dynamic model to analyze the effects of liability regimes on innovation, as well as the timing of market introduction and penetration of smart products. This study is particularly relevant for understanding the life cycle of smart products in the context of rapidly evolving digital technologies. While most literature on digitalization does not specifically focus on the role of software products, these works collectively suggest that digitalization is ushering in new economic phenomena. Understanding these changes is vital for understanding the evolving landscape of the software industry and the broader digital economy.

Our research is grounded in two fundamental areas: Industrial Organization and Digital Economics. The pioneering work of Hotelling (1929) raised critical questions about understanding the stability of competitive situations and the impact of slight deviations in product offerings to attract buyers. This concept has been pivotal in exploring market dynamics and consumer behavior. Building on Hotelling's model, Gabszewicz & Thisse (1982) expanded the framework to analyze more complex structures in market competition. Simultaneously, Graitson (1982) surveyed the literature from 1929 to 1981, summarizing extensions of the Hotelling model by relaxing assumptions regarding the number of firms, demand curve shapes, and types of market spaces. Tabuchi & Thisse (1995) further investigated location-price games in markets focusing on the sequential entry of firms. Shy (1996) summarized the concept of vertical differentiation and its market impacts. Larralde et al. (2006) analyzed consumer choice influenced by additional and unobservable

factors using a two-dimensional logit function. They expanded this model in 2009 to include consumer heterogeneity in multiple dimensions (Larralde et al. 2009). More recent research has incorporated additional market aspects and employed numerical simulations to manage market complexity. Guo et al. (2015) included production technology and labor input in their models, highlighting the significant role of production technology in determining equilibrium locations. Peng et al. (2020) developed a three-dimensional discrete dynamical system reflecting bounded rationality and demonstrated the emergence of complex dynamics, including cycles and chaos, in market equilibrium. In our study, we adopt the structure of the two-dimensional Hotelling model, focusing specifically on consumer behavior driven by product functionality and associated data restrictions. This approach allows us to concentrate on the essential aspects of consumer choice in the context of software products within digital economics.

In exploring cross-border data flows, researchers have employed various approaches to analyze related business activities, particularly in the context of the digital economy and software industry. Kephart et al. (2000) examined the role of software agents in the economy, focusing on their utilization in dynamic pricing algorithms. This perspective is crucial for understanding automated pricing strategies in digital markets. Dasgupta & Hashimoto (2004) explored the mechanisms used by intelligent agents to assist sellers in online economies, highlighting the increasing reliance on artificial intelligence in e-commerce. Pussep et al. (2011) presented a case study illustrating how to apply the software value chain to gauge the level of vertical integration within the software industry. Their work provides insights into the complexities of software production and distribution. Castro & McQuinn (2015) discussed the significance of cross-border data flows for various industries, advocating against protectionist policies that hinder international data exchange due to their long-term detrimental effects. Bauer et al. (2016) studied the economic impacts of regulations on the free flow of data and data localization, emphasizing the broader economic implications of such policies. Aaronson (2018) highlighted the distinct characteristics of data products compared to traditional goods, shedding light on unique challenges and opportunities. GSM Association (2018) outlined the framework, value creation, issues, and policy implications of the Data Value Chain (DVC), providing an overview of the field. Casalini & López-González (2019) examined the levels of data restrictiveness, storage requirements, and the digital thread in modern manufacturing, along with the estimated costs of personal data separation. Nguyen & Paczos (2020) discussed aspects of data monetization, categories of data, cross-border data flows, and global data center traffic, offering a nuanced understanding of data's economic value. Azmeh et al. (2021) investigated policy strategies for digital development, emphasizing the need for strategic policy interventions in the digital sphere. Finally, UNCTAD (2021)

reviewed data restrictions and analyzed their implications in the realms of ICTs and e-commerce, providing a global perspective on data policy. These studies collectively offer a diverse and thorough understanding of the various aspects and implications of cross-border data flows, crucial for understanding the current and future landscape of the digital economy.

Researchers employed various methodologies to analyze global value chains (GVCs) and digital trade. In our research, we aim to blend these diverse approaches to focus specifically on the interactions among firms, governments and consumers. De Backer & Miroudot (2013) utilized the OECD Inter-Country Input-Output Model to illustrate the integration and positioning of countries within GVCs, offering detailed assessments across numerous industries. Their method provides a macroeconomic perspective on the role of different countries in the global market. Ferracane et al. (2018) developed the Digital Trade Restrictiveness Index (DTRI), assessing digital trade restrictions across 64 countries. This index is critical for understanding the regulatory landscape affecting digital trade. Chalaby (2019) discussed the implications of outsourcing, including the vertical disintegration and formation of GVCs in media industries. This approach offers insights into the structural changes in industries due to globalization. Asian Development Bank et al. (2021) examined value creation in GVCs and the distribution of this value among participants, while Frontier Economics Ltd. (2021) focused on modeling the role of cross-border data flows in international trade and quantifying their economic impact.

In our paper, we employ a game theoretical approach within one period, in line with the principles of Industrial Organization literature. Across multiple periods, we refer to the heuristic design suggested by Nagle & Müller (2018) and Dawid et al. (2019), along with the validation methods from Delli Gatti & Reissl (2022) and Wang (2022). Both static and dynamic approaches in our analysis necessitate computational simulations. We anticipate revealing insightful results by observing the choices of heterogeneous consumers derived from these simulations, which sheds light on consumer behavior in the face of varying data restrictions.

1.3 Model design

1.3.1 General structure

In our model, we delineate the distinct categories of agents, each playing a critical role in the digital market landscape:

- **Firms:** These are firms that hold the copyright and license of the data product. In our basic model, there are two Owners, referred to as firms a and b ,

located in regions A and B respectively. Each Owner finds software developers, which are groups comprising individuals or organizations responsible for developing the data product. In our model, there are two Developers, firms c and d , which are contractually associated with Owners a and b respectively. Our model features a duopoly between Owners, where Developers do not play actively.

- **Consumers:** These are users who are potential buyers of the data product. In the model, Consumers are numerically identified, ranging from user 1 to user n , and are situated either within one of the two regions (A or B) or outside these regions.
- **Governments:** Local governments in each region are responsible for controlling the levels of local data restrictions. In the basic model, there are two such Governments, each governing the data policies in their respective regions.

The architecture of our model is outlined in Figure 1.6, providing a clear representation of the interactions and relationships among these diverse agents. This structured approach allows for a nuanced understanding of the dynamics within the digital market, particularly focusing on the effects of data restrictions and consumer behavior.

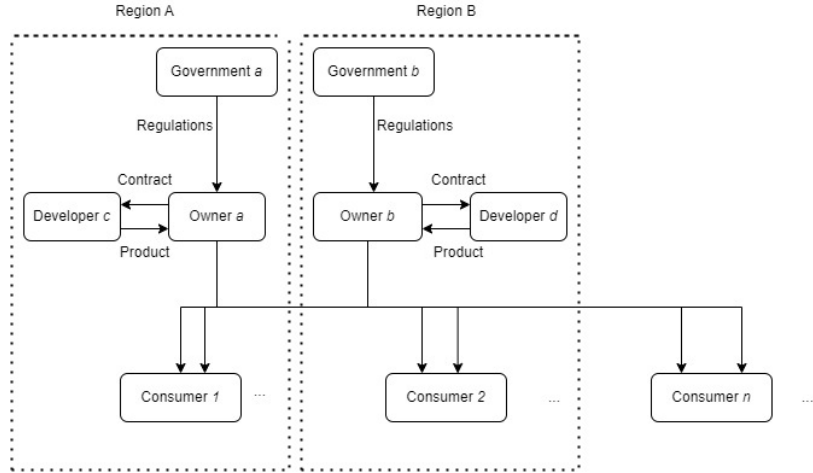


Figure 1.6: Model architecture. Active agents include: Governments, Owners, and Consumers.

Our model is structured to mimic the sequence of events occurring within a single period, delineating the interactions among Owners, Developers, Consumers, and Governments. The process unfolds as follows:

1. Contract Negotiation: Owners enter into contracts with Developers. Given that Owners possess significantly more bargaining power, they ultimately decide the payments to Developers.
2. Product Development: Developers engage in the creation of the data product. This phase involves designing algorithms and applying intellectual labor to finalize the product.
3. Payments and Sales: Owners compensate Developers for their work and spend data restriction costs, assuming they are in the same region. Owners then market and sell their products across borders through online platforms, reaching customers globally.
4. Consumer Decision-Making: Consumers evaluate the products offered by different Owners. Their decision process is twofold: (i) deciding whether to purchase a data product at all and if so, (ii) selecting which specific product to buy. Consumers might refer to product descriptions and user reviews, or opt for free-trial versions. Besides, consumers pay no transaction costs in this process.
5. Transaction Completion and Profit Calculation: Once all transactions are completed, each Owner gets its profit, based on the decisions made by Consumers during the period.

We design this model to capture the structure of the software market, focusing on the roles of data restriction policies and consumer behavior in shaping market outcomes. The assumption of no transaction costs for consumers and the emphasis on the bargaining power imbalance among firms influence the interactions and outcomes within this framework.

To streamline our analysis while maintaining its general applicability, we adopt a set of fundamental assumptions. These assumptions are centered around the nature of data products and their development process: We assume that data products in the market are vertically differentiated, meaning that Consumers can discern differences in functionality among the products (Shy 1996). We denote the value added by Developers to a product owned by Owner i (i being either a or b) as Δp_i . The Owner chooses an investment level $c_{0,i}$ for product development, which directly determines the value created through the production function $\Delta p_i(c_{0,i})$, where $\frac{d\Delta p_i}{dc_{0,i}} > 0$. We assume the Owner pays the Developer exactly the value contributed, so the investment $c_{0,i}$ represents both the payment to Developers and the resulting value added to the product. This assumption eliminates bargaining considerations and simplifies the Owner–Developer transaction. The software product’s functionality θ_i combines a baseline quality $\bar{\theta}_i$ (determined by

the Owner’s resources, raw data quality, etc.) with the Developer’s contribution: $\theta_i(c_{0,i}) = \bar{\theta}_i + \Delta p_i(c_{0,i})$.⁵

1.3.2 Consumers

In our model, we incorporate the diverse preferences of Consumers towards both the functionality of the software product and the data restriction levels in their region. This diversity is captured through two variables for each Consumer: The taste towards software functionality x_j and the taste towards the data restriction level y_j . x_j is modeled as a uniformly distributed variable between 0 and 1 ($x_j \sim U[0, 1]$). Here, a value of 0 indicates a lack of concern for quality, while a value of 1 signifies a high concern for the quality of the software. y_j is uniformly distributed between -1 and 1 ($y_j \sim U[-1, 1]$). A value of -1 suggests an aversion to data regulations, 0 indicates an indifference to such regulations, and 1 shows a preference for having data regulations in place. Additionally, we make two further assumptions related to Consumers and their interactions with the software products: Firstly, all Consumers of a product from Owner i (where $i \in \{a, b\}$) are assumed to adhere to the data policies of the region where Owner i is located. This assumption simplifies the model by aligning Consumer behavior with regional data policies. Secondly, the price of the same product is assumed to be identical for Consumers, regardless of their region. This uniform pricing strategy implies that geographical location does not influence the cost of the software product for Consumers.

Building on the concept of ranking utility as formulated by Gabszewicz & Thisse (1979), our model incorporates a straightforward decision-making rule for Consumers regarding their software purchases. This rule is centered around the concept of reserved utility ($\bar{u}_j$). Each Consumer j ’s reserved utility is defined as the minimum level of satisfaction needed to make a purchase. It is assumed to be uniformly distributed between a lower limit $\bar{u}_{j,low}$ and an upper limit $\bar{u}_{j,high}$, such that $\bar{u}_j \sim U[\bar{u}_{j,low}, \bar{u}_{j,high}]$. A Consumer will opt to buy software i if her reserved utility ($\bar{u}_j$) is less than or equal to the utility she derives from obtaining that software ($u_{j,i}$). In other words, a Consumer purchases a product only if it meets or exceeds her threshold of utility: $\bar{u}_j \leq u_{j,i}$. In scenarios where a Consumer considers two software products, she will buy the one offering higher utility. If the utilities derived from both software products a and b are equal (i.e., $u_{j,a} = u_{j,b}$), and both are greater than her reserved utility ($\bar{u}_j \leq u_{j,a} = u_{j,b}$), she will make a random choice between the two. The model also includes $c_{res,i}$, which represents the unit cost of data restriction within the region of Owner i . This cost encompasses tariffs

⁵In the codes, we specify a linear relationship $\Delta p_i(c_{0,i}) = 2c_{0,i}$ and set equal baseline qualities for both Owners ($\bar{\theta}_a = \bar{\theta}_b = 0.8$).

and any other cross-regional taxes that may apply. Each Consumer takes this cost into account when making their purchase decision. Through these mechanisms, our model captures the decision-making process of Consumers in a market where software products are differentiated not only by their inherent qualities but also by the data restriction policies of their respective regions. This approach allows us to analyze how these factors collectively influence Consumer behavior in the software product marketplace.

The utility function of Consumer j ($j = 1, \dots, n$) for the software of Owner i is defined as follows:

$$u_{j,i}(\theta_i, c_{res,i}, p_i) = \bar{u}_{j,i} + x_j\theta_i + y_jc_{res,i} - p_i$$

In this equation:

- $\bar{u}_{j,i}$ represents the baseline utility or the fundamental attitude of Consumer j towards the Owner i . It's a constant for each Consumer-Owner pair.
- θ_i signifies the software functionality.
- $c_{res,i}$ denotes the cost of data restriction within the region of Owner i , as perceived by the consumer.
- p_i is the sale price of the product from Owner i . We assume a constant marginal disutility of price.⁶
- x_j and y_j are Consumer j 's attitudes towards the software's functionality and the data restriction level, respectively.

The decision-making process for each Consumer involves choosing the software that provides the highest utility. Hence, for Consumer j , if she opts for the software from Owner a over that from Owner b , it implies that $u_{j,a} \geq u_{j,b}$. Mathematically, this preference can be expressed as:

$$\bar{u}_{j,a} + x_j\theta_a + y_jc_{res,a} - p_a \geq \bar{u}_{j,b} + x_j\theta_b + y_jc_{res,b} - p_b$$

This utility-based selection process underscores that each Consumer's choice is influenced by a combination of their personal preferences (x_j and y_j), the inherent properties of the products (θ_i), the regional data restriction policies ($c_{res,i}$), and the pricing strategies of the Owners (p_i). The population of Consumers is characterized by the distribution of their attitudes towards software functionality and data restriction policies. Each agent (Consumer) is represented by a pair of attitudes

⁶This setting has been widely used in discrete choice models (McFadden 1974, Berry et al. 1995).

(x_j, y_j) , and these pairs are assumed to follow a bivariate statistical distribution $F(x, y)$. This distribution captures the diversity of Consumer preferences in the market, allowing for analyzing market behavior and product selection.

To streamline the analysis, we impose the following assumptions on market dynamics and agent behavior:

1. **Equal Reputation of Owners:** It's assumed that all Consumers perceive the Owners to have the same reputation, leading to no inherent preference for one Owner over the other. This is mathematically represented as $\bar{u}_{j,a} = \bar{u}_{j,b}$ for all Consumers.
2. **Government-controlled data restriction levels:** The level of data restriction in a region, represented by $c_{res,i}$, is determined solely by the local Government, not by the Owners. This reflects real-world scenarios where regional data policies are typically set by governmental entities.
3. **Owners as price makers in the Bertrand competition:** The Owners are considered price makers and engage in Bertrand competition. In the Bertrand competition, each firm sets its product price with the assumption that the other firm's prices will remain constant. This type of competition typically leads to price wars, with each Owner trying to undercut the other's prices to capture a larger market share.

Under these assumptions, the model aims to capture the interplay between software quality, data restriction policies, pricing strategies, and consumer preferences. By considering a scenario where Owners have equal reputations and compete on prices while Governments control data restriction policies, the model reflects a market where the primary differentiated terms are the software's functionality, the level of data restrictions, and the price set by each Owner. To investigate the consumer decision-making process and market dynamics, we will examine a specific scenario within this framework.

If regulations (data restriction levels) and product functionalities differ between the two software products offered by Owners a and b , it would create a situation where Consumer preferences can be divided based on their attitudes towards these factors. In this scenario, the critical points on the x-axis (reflecting attitudes towards product functionality) and the y-axis (reflecting attitudes towards data restriction levels) will delineate the consumer market into segments (Figure 1.7).

$$\begin{aligned}
\bar{u}_{j,a} + x_j\theta_a + y_jc_{res,a} - p_a &\geq \bar{u}_{j,b} + x_j\theta_b + y_jc_{res,b} - p_b \\
&\Leftrightarrow \\
x_j\theta_a + y_jc_{res,a} - p_a &\geq x_j\theta_b + y_jc_{res,b} - p_b \\
&\Leftrightarrow \\
x_j(\theta_a - \theta_b) + y_j(c_{res,a} - c_{res,b}) &\geq p_a - p_b \\
&\Leftrightarrow \\
x_j\Delta\theta + y_j\Delta c_{res,R} &\geq \Delta p_s
\end{aligned}$$

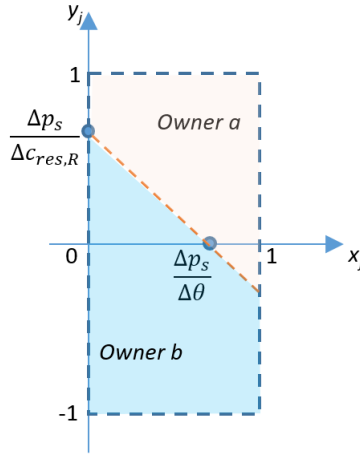


Figure 1.7: Critical points and consumer types.

The intuitive notion that consumers prefer products with superior functionality, lower regulation costs, and lower prices forms the basis of our model's market segmentation. This preference leads to the partitioning of the support of the bivariate distribution, reflecting the diverse consumer attitudes towards product functionality and data restriction levels.

The choice of the specific distribution to represent consumer attitudes can vary. While we might start with uniform distributions for simplicity, as indicated by the uniform distribution of x_j (attitude towards functionality) and y_j (attitude towards data restriction levels), other distributions can also be considered. For instance, normal distributions with different means, variances, or correlations could provide a more nuanced representation of consumer preferences.

Conducting simulations can visually reinforce this argument, showing how consumers are segmented based on their preferences. Even if we generalize the joint distributions, the presence of critical points on the x-axis and y-axis, along with a linear separation line, is anticipated. It suggests that the model's results are

robust; despite changes in the underlying distributions of consumer preferences, we can still identify a clear decision boundary that separates the market based on consumer choices.

In more complex scenarios with other distributions that allow for correlation between x_j and y_j , the fundamental concept remains: there will be a boundary separating the market into segments, with consumers in each segment preferring one product over the other based on their combined attitudes. The robustness of these findings underscores the model’s capacity to capture consumer behavior (Figure 1.8).

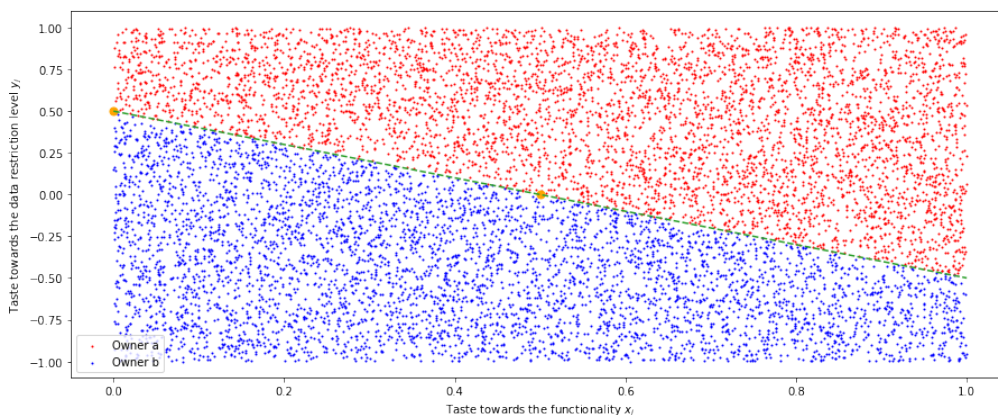


Figure 1.8: Example simulation graph of the Consumers’ tastes when regulations and product functionalities differ ($c_{res,a} = 25, c_{res,b} = 5, c_{0,a} = 70, c_{0,b} = 60, p_a = 50, p_b = 40$).

While our initial discussion has been based on the assumption that the bivariate statistical distribution of consumer attitudes is a simple two-dimensional uniform distribution without correlation, real-world consumer behavior is often more complex. As suggested by Madden & Rainie (2015), people’s attitudes towards data security and data-driven functionalities are diverse. Some consumers may prioritize data security and be careful of data insecurity, while others may hold different opinions. This heterogeneity in consumer attitudes indicates that the simple assumption of a uniform distribution may not fully capture the nuances of consumer preferences. As consumer attitudes towards functionality and data restriction levels are varied, we may need a more complex distribution to model these preferences. In such cases, traditional analytical methods might struggle to provide clear insights due to the complexity of analyzing multidimensional distributions. This is where computational simulations become particularly valuable. Computational simulations offer a flexible and powerful tool to model complex systems, allowing for the exploration of various scenarios and the understanding of market dynamics

that might be intractable with analytical methods alone.

Hence, our emphasis on a computational approach is grounded in its capacity to model complex consumer behaviors and market dynamics efficiently and effectively. By leveraging computational simulations, we can accommodate a wide range of distribution assumptions, explore the impact of different consumer attitudes on market outcomes, and gain nuanced and robust insights.

1.3.3 Firms

The firms are defined as the software license owners. The profit of Owner i of one period can be calculated as:

$$\Pi_i = m_i(u_{j,i}(\theta_i, c_{res,i}, p_i), u_{j,-i}(\theta_{-i}, c_{res,-i}, p_{-i})) \cdot (p_i - c_{res,i}) - f_i(c_{0,i})$$

where m_i is the number of Consumers who buy the software of Owner i . Under the assumption of a joint distribution of consumer tastes, market demand can be derived from simulations. These simulations allow for the modeling of complex consumer behaviors and preferences, capturing the diversity of consumer attitudes towards product functionality and data restriction levels. As the user base of a software product expands, the complexities and requirements for maintaining data security and adhering to data compliance regulations also increase. In this context, the cost of data restriction can be seen as a variable cost that changes with the scale of the product's user base.

The maximization problem of Owner i :

$$\begin{aligned} \max_{c_{0,i}, p_i} \quad & \Pi_i = m_i p_i - c_{total,i} \\ \text{s.t.} \quad & \theta_i = \bar{\theta}_i + \Delta p_i(c_{0,i}) \\ & c_{total,i} = f_i(c_{0,i}) + m_i c_{res,i} \\ & (x_j, y_j) \sim F(x, y) \\ & \Omega = \{(x, y) : u_{x,y,i} \geq u_{x,y,-i}\} \\ & m_i = \iint_{\Omega} dF(x, y) \end{aligned}$$

where Δp_i is the added value in functionality by the investment, f_i is the cost for contract, maintenance and upgrades, and $F(x, y)$ is the joint distribution of the Customer j 's tastes x_j and y_j . In our computational model, rather than calculating the number of purchases directly, we employ the following formulae:

$$m_i = \sum_{j=1}^n \mathbb{1}_{u_{j,i} > u_{j,-i}} + \sum_{j=1}^n \beta \mathbb{1}_{u_{j,i} = u_{j,-i}}$$

where β follows a Bernoulli distribution with the probability of 0.5.⁷

Since the market demand is indirectly modeled, our computational approach involves inputting a range of prices into the model and observing the resulting profits for each Owner. This method allows us to explore how different pricing strategies impact the Owner’s profitability under various market conditions. By varying the prices and analyzing the outcomes, we can identify the price points that maximize profits for each Owner, given the specific circumstances, such as the level of data restrictions, consumer preferences, and cost structures.

1.3.4 Government

In our model, each Government in the respective regions implements local data restriction policies that directly impact the Owners operating within their jurisdiction. The level of data restriction in the region i ($c_{res,i}$) is quantified by a numerical value of at least zero. This value is comparable across different regions, with a larger number indicating a more stringent local data restriction policy. These restrictions, in turn, increase the operational costs for the Owners based in the respective regions, as they have to invest more in ensuring compliance with these policies. The primary objective of a Government in this context is the local welfare, comprising the local consumer surplus and the profit of local firms. The balance between these two terms can be complex, as stricter data restriction policies might protect consumer privacy and data security (contributing to consumer surplus to some extent) but could also increase operational costs for Owners (potentially hindering the economic development of local firms). This relationship highlights the trade-offs and decision-making complexities faced by Governments in setting data restriction policies. It also underscores the importance of carefully considering the implications of these policies on both the business environment and the broader societal welfare. The welfare can be formulated as

$$W_i = \sum_{j=1}^n u_{j,i} \mathbb{1}_{u_{j,i} > u_{j,-i}} + \sum_{j=1}^n \beta u_{j,i} \mathbb{1}_{u_{j,i} = u_{j,-i}} + \Pi_i$$

where $i \in \{a, b\}$.⁸

⁷ $\beta \sim \text{Bernoulli}(0.5)$.

⁸From the perspective of the global-maximizer, it is important to consider the aggregate welfare $W = \sum_{i \in \{a,b\}} W_i$. In real-world contexts, a global-maximizer might influence market dynamics through international regulations, trade agreements, or global standards, impacting how local Governments set data restriction policies or how Owners and Developers approach the global market. However, in the scope of this model, the global-maximizer’s influence is not explicitly modeled as a direct input.

1.3.5 Basic setup and demand functions

We design the computational program with *Python* and *Julia*, two powerful programming languages well-suited for data analysis and numerical computations (Van Rossum & Drake 2009, Bezanson et al. 2015). Initially, the model is run with a set of baseline parameters. This baseline scenario serves as a reference point, providing initial results against which subsequent adjustments can be compared. Each adjustment (in product prices, investment costs, or costs of data restrictions) is conducted while holding all other factors constant. This “ceteris paribus” (all else equal) approach allows for a clear understanding of the direct impact of each variable on the market outcomes.

We adopt a static game framework for price competition between Owner a and Owner b . This framework assumes that both players make their pricing decisions simultaneously without knowing the other’s decision. This setup is characteristic of the Bertrand model of oligopoly, where firms compete by setting prices simultaneously (Tremblay & Tremblay 2019). The cost of the contract with the Developer (investment in product development) and the level of data restriction are held constant across the two regions. This assumption simplifies the model by ensuring that the only variable affecting competition is the price set by each Owner. If the joint distribution of Consumers’ tastes is straightforward (e.g., a bivariate uniform distribution), then it’s possible to solve the problem analytically. However, to accommodate more complex and realistic distributions of consumer preferences, we approach the problem through computational methods. The approach outlined is designed to find the Nash equilibrium in our model by systematically exploring the price strategies of Owners a and b . Here’s a detailed breakdown of the steps:

1. For given $c_{res,a}$, $c_{res,b}$, $c_{0,a}$, $c_{0,b}$, vary p_a from 0 up to a predetermined upper limit (200 in the codes). For each value of p_a , compute Owner b ’s profit for all possible prices of Owner b ’s product (p_b). Identify the price p_b^* that gives Owner b the maximum profit for that particular p_a and save these optimal responses as coordinate pairs (p_a, p_b^*) .
2. Vary p_b from 0 up to a predetermined upper limit (200 in the codes). For each value of p_b , compute Owner a ’s profit for all possible prices of Owner a ’s product (p_a). Identify the price p_a^* that gives Owner a the maximum profit for that particular p_b and save these optimal responses as coordinate pairs (p_a^*, p_b) .
3. Find the intersection of the two sets of coordinates obtained from the steps above. The intersection point represents the price pair where neither Owner a nor Owner b can unilaterally change their price to increase their profit, given the price set by the other owner. (p_a^*, p_b^*) is the Nash equilibrium in the context of this price competition.

When the exact Nash equilibrium prices are not readily identifiable due to the discrete nature of the step size in the price range, we try to shorten the steps and estimate the equilibrium prices. By utilizing computational simulations and visualizing profit grids, this approach offers a flexible framework for analyzing complex market scenarios. It helps in identifying stable pricing strategies and provides valuable insights into how different market factors interact to shape the competitive landscape.

Comparing outcomes across various scenarios is an effective method for understanding the economic effects of investment and data restriction levels. We consider six representative scenarios (Table 1.1).

Scenario	$c_{0,a}$	$c_{0,b}$	$c_{res,a}$	$c_{res,b}$
1'	6	6	0	0
2'	6	0	0	0
3'	6	6	0	10
4'	6	0	0	10
5'	6	6	10	10
6'	6	0	10	10

Table 1.1: Initial setup of the scenarios.

Among the six scenarios, Scenarios 1' and 2' consider regions with low data restrictions, contrasting standard versus varied initial investments by Owners. Scenarios 5' and 6' depict the opposite end, with high data restrictions, again comparing the standard against varied investments. Scenarios 3' and 4' represent mixed environments where data restriction levels differ across regions, examining both uniform and distinct investment levels. These scenarios collectively offer insights into how data policies and investment strategies shape market outcomes, providing valuable perspectives for Owners in decision-making and for policymakers in understanding the broader impact of data regulations on the market and consumer welfare.

We show the graphs of the demand function of Owner a in each scenario by running simulations (Figure 1.9). In each round, we let p_a range from 1 to 100 (step = 1) while holding all other parameters equal. The selling price of Owner b 's product is noted as "Price of Opponent" in each sub-figure. Intuitively, the demand function in this market is also downward-sloping. Apart from the graphs that support this argument, we point out that in the symmetric cases, the curves appear to be horizontal in a wide range of quantities. We could also observe that fewer consumers decide to purchase the product under the same prices and higher data restrictions.

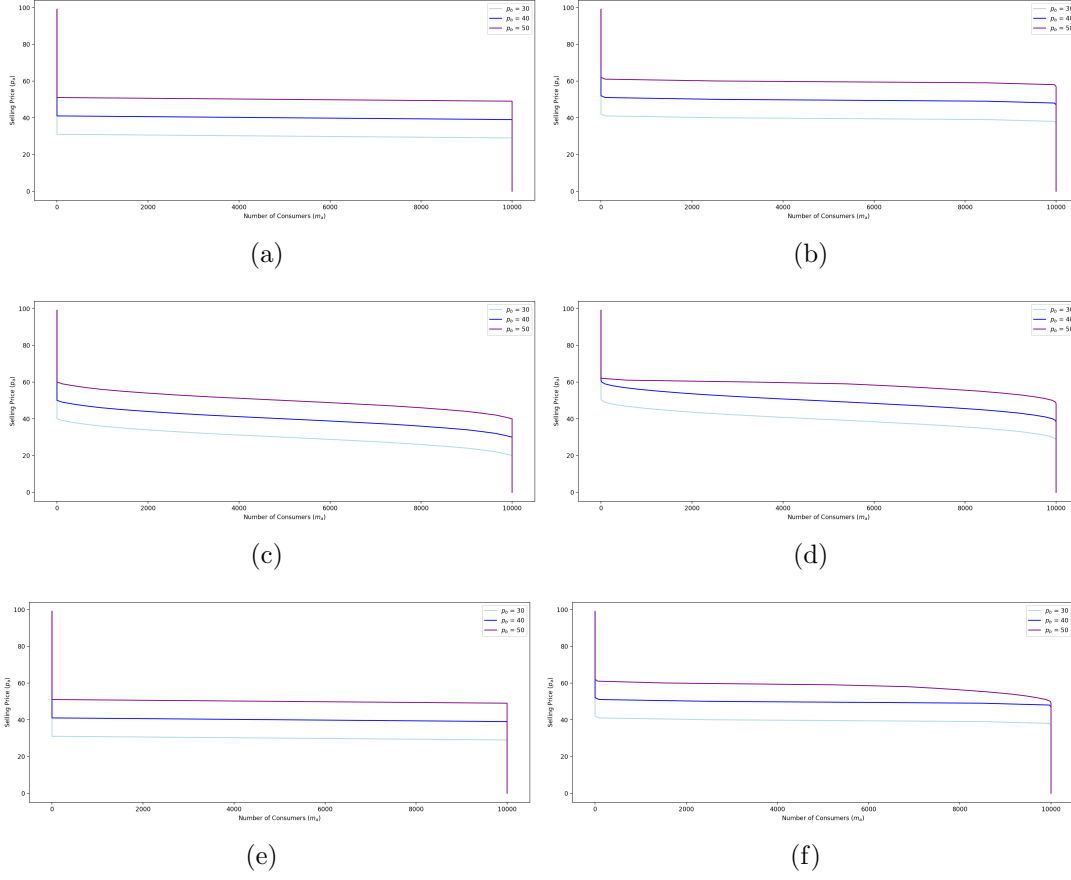


Figure 1.9: Demand functions of Owner a . Sub-figures (a) - (f) represent Scenarios 1' - 6'.

1.4 Multi-stage game

1.4.1 Price competition

Under our setting, there are three types of variables: (a) The data restriction level ($c_{res,i}$, set by the government at Owner i 's location), (b) the investment level ($c_{0,i}$, set by Owner i), and (c) the prices of the product (p_i , set by Owner i). Before we consider the price and profit changes across periods, we need to specify the following questions: If everything is symmetric, i.e., the Owners are under the same data restriction level and the Owners have the same investment level, will there be an equilibrium when the market evolves? Furthermore, if the Owners collude, for example, they agree on the same price which is higher than the equilibrium price, would either of them have an incentive to have a deviation?

First of all, under the symmetry, Consumers focus only on the price, and the better strategy for an Owner is always to have a good price offer to attract more Consumers. Assuming the Consumers' attitudes follow a bivariate uniform distribution, we can analytically calculate the market share of the Owners.

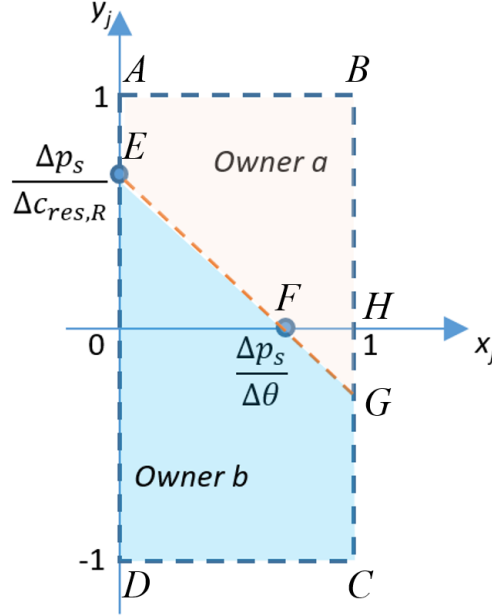


Figure 1.10: Critical points and consumer types.

In Figure 1.10, we assume that $0 \leq \frac{\Delta p_s}{\Delta c_{res,R}} \leq 1$ and that $0 \leq \frac{\Delta p_s}{\Delta \theta} \leq 1$. All Consumers in the right trapezoid $ABGE$ select the product of Owner a , while the other Consumers in the right trapezoid $EGCD$ select the product of Owner b . Since $\triangle OEF \sim \triangle HGF$, we have:

$$\begin{aligned} \frac{OE}{GH} &= \frac{OF}{FH} \\ \frac{\frac{\Delta p_s}{\Delta c_{res,R}}}{GH} &= \frac{\frac{\Delta p_s}{\Delta \theta}}{1 - \frac{\Delta p_s}{\Delta \theta}} \\ GH &= \frac{\frac{\Delta \theta}{\Delta c_{res,R}}}{1 - \frac{\Delta p_s}{\Delta \theta}} \end{aligned}$$

Therefore, the area of Owner a and that of Owner b can be formulated as:

$$\begin{aligned} S_a &= S_{\square AOHB} - S_{\triangle OEF} + S_{\triangle FHG} \\ S_b &= S_{\square ODC H} + S_{\triangle OEF} - S_{\triangle FHG} \end{aligned}$$

The according market shares can also be formulated:

$$\begin{aligned} MktShare_a &= \frac{S_a}{S_{\square ABCD}} \\ MktShare_b &= \frac{S_b}{S_{\square ABCD}} \end{aligned}$$

Through the calculations, we find:

$$\begin{aligned} S_a &= 1 - \frac{\Delta p_s}{\Delta c_{res,R}} + \frac{1}{2} \frac{\Delta \theta}{\Delta c_{res,R}} \\ S_b &= 1 + \frac{\Delta p_s}{\Delta c_{res,R}} - \frac{1}{2} \frac{\Delta \theta}{\Delta c_{res,R}} \\ MktShare_a &= \frac{1}{2} \left(1 - \frac{\Delta p_s}{\Delta c_{res,R}} + \frac{1}{2} \frac{\Delta \theta}{\Delta c_{res,R}} \right) \\ MktShare_b &= \frac{1}{2} \left(1 + \frac{\Delta p_s}{\Delta c_{res,R}} - \frac{1}{2} \frac{\Delta \theta}{\Delta c_{res,R}} \right) \end{aligned}$$

In the situation where $-1 \leq \frac{\Delta p_s}{\Delta c_{res,R}} \leq 0$, we calculate the results and find that they are the same as before. Here we clarify the relationship between the market share and the bivariate distribution at the same period: If the distribution is a bivariate uniform distribution, the market shares resulting from the simulations are approximately the same as what we calculated, since the points representing the Consumers' tastes are uniformly distributed in the region of each Owner. Otherwise, the market shares differ, and through the simulations, we can estimate the market shares by generating a certain number of points (Consumers) and counting the number of points falling in each region.

Under the symmetric situation, since $\Delta c_{res,R} = \Delta \theta = 0$, the point $\frac{\Delta p_s}{\Delta c_{res,R}}$ and the point $\frac{\Delta p_s}{\Delta \theta}$ cannot be shown in the graph. In this case, we let $c_{0,a} = c_{0,b} = \bar{c}_0$, $c_{res,a} = c_{res,b} = \bar{c}_{res}$, $p_a = p_b = \bar{p}$. Given the population n and assuming that all Consumers purchase one product, we have $m_a = m_b = \frac{1}{2}n$. We check the profit of Owner a :

$$\begin{aligned} \Pi_a &= m_a \bar{p} - c_{total,a} \\ &= m_a \bar{p} - m_a c_{res,a} - c_{0,a} \\ &= \frac{1}{2}n(\bar{p} - \bar{c}_{res}) - \bar{c}_0 \end{aligned}$$

For Owner a , to get a non-negative profit, since $n > 0$ and $\bar{c}_0 \geq 0$, $\bar{p} > \bar{c}_{res}$. We have:

$$\begin{aligned} \Pi_a &\geq 0 \\ \frac{1}{2}n(\bar{p} - \bar{c}_{res}) &\geq \bar{c}_0 \\ \bar{p} &\geq \bar{c}_{res} + \frac{2\bar{c}_0}{n} \end{aligned}$$

We show that the minimum price offered by an Owner is $\bar{c}_{res} + \frac{2\bar{c}_0}{n}$. Via collusion, Owners could agree on a price that is higher than this "minimum price", and get more profits than before, as long as the Consumers could still bear the price changes, i.e., the expected utility levels of the Consumers still exceed their reserved utility levels.

Now we discuss if one of the Owners would deviate from the collusion. If Π_a becomes higher than before after deviation (assuming the price offered by Owner b holds the same as before), then Owner a would probably deviate. In a symmetric case, after Owner a does a price reduction of ε units, all Consumers would purchase the product of a since it is less expensive. The new profit of Owner a becomes $\Pi'_a = n(\bar{p} - \bar{c}_{res} - \varepsilon) - \bar{c}_0$, while the new profit of Owner b becomes $\Pi'_b = 0(\bar{p} - \bar{c}_{res}) - \bar{c}_0 = -\bar{c}_0 \leq 0$. Owner a would deviate if:

$$\begin{aligned}\Pi_a &< \Pi'_a \\ \frac{1}{2}n(\bar{p} - \bar{c}_{res}) - \bar{c}_0 &< n(\bar{p} - \bar{c}_{res} - \varepsilon) - \bar{c}_0 \\ \varepsilon &< \frac{1}{2}(\bar{p} - \bar{c}_{res})\end{aligned}$$

When Owners agree on the “minimum price”, i.e. $\bar{p} = \bar{c}_{res} + \frac{2\bar{c}_0}{n}$, there would be deviation if $\varepsilon < \frac{1}{2}(\bar{p} - \bar{c}_{res}) = \frac{\bar{c}_0}{n}$. If Owner a deviates, its profit becomes $\Pi'_a = n(\bar{p} - \bar{c}_{res} - \varepsilon) - \bar{c}_0$. To make sure that $\Pi'_a \geq 0$ after deviation, there is:

$$\begin{aligned}n(\bar{p} - \bar{c}_{res} - \varepsilon) - \bar{c}_0 &\geq 0 \\ \bar{p} - \bar{c}_{res} - \varepsilon &\geq \frac{\bar{c}_0}{n} \\ \varepsilon &\leq \bar{p} - \bar{c}_{res} - \frac{\bar{c}_0}{n}\end{aligned}$$

Therefore, each Owner would deviate, if both conditions hold, i.e. $\varepsilon < \frac{1}{2}(\bar{p} - \bar{c}_{res})$ and $\varepsilon \leq \bar{p} - \bar{c}_{res} - \frac{\bar{c}_0}{n}$. During competition between two Owners, they may initially agree upon the same selling price. However, each Owner may have an incentive to deviate from this agreement by reducing their price to attract additional Consumers, continuing this price reduction process until the incentive to deviate (ε) reaches zero. As this model also incorporates investment competition and data regulation settings, we analyze this economy using a multi-stage game framework to clarify its structure and implications.

1.4.2 Subgame perfect equilibrium analysis

Following our assumptions and the basic settings of Bertrand competition, we could define a multi-stage game where Governments and Owners decide sequentially the related decision variables, namely the data restriction level, the investment, and the selling price. In the first stage, each Government sets the data restriction level for its respective region. This decision lays the foundation for the market environment in which Owners operate, impacting their cost structures and

potentially influencing consumer preferences. Subsequently, with the data restriction policies in place, Owners decide on their level of investment in product development. This investment influences the functionality of their products, impacting their appeal to consumers and their production costs. This stage allows Owners to strategically adjust their investments in response to the regulatory environment and competitive landscape. In the final stage, Owners engage in price competition, setting the selling prices for their products. This stage mirrors the Bertrand competition model, where Owners compete for market share by strategically pricing their products, considering both the competitive pricing of their opponent and the cost structures shaped by their investment decisions and the prevailing data restriction policies. The main difference from the original Bertrand analysis is that we allow each Owner to adjust its investment and allow each Government to adjust the data restriction level within the region, while in the former case Owners' investments and Governments' regulations were exogenous in each scenario.

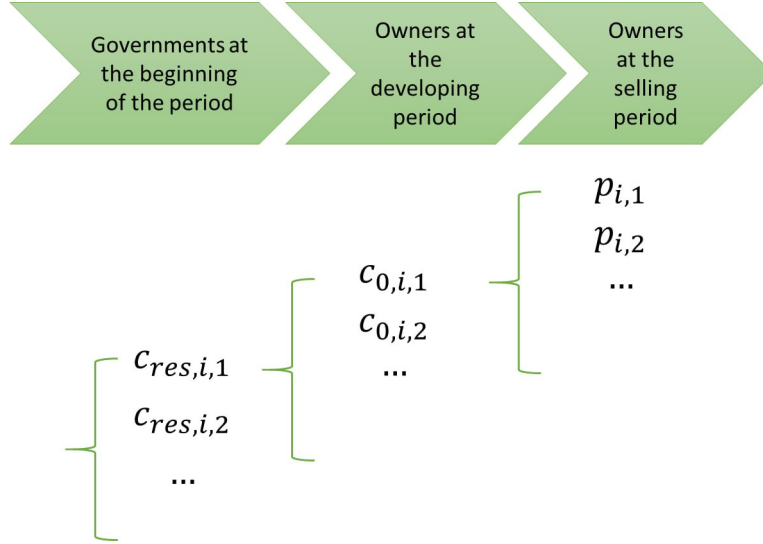


Figure 1.11: Three stages of the game.

To identify a potential subgame perfect equilibrium in our dynamic multi-stage game, we employ backward induction. This method is a widely recognized approach for analyzing behavior in dynamic decision problems, especially in the context of sequential games where players make decisions at different stages (Ke 2019). We could complete this task according to the following steps:

1. Select certain local data restriction levels $(c_{res,a}^{(1)}, c_{res,b}^{(1)})$.
2. Given certain investment combinations, visualize the approximate location

of the equilibrium point and find the specific points with a smaller searching range and shorter searching steps.

3. Visualize the profit of each Owner based on each investment combination. Plot the coordinates of the best responses of Owners and use the smoothing splines to find potential Nash equilibria.⁹
4. Select other local data restriction levels $(c_{res,a}^{(1)}, c_{res,b}^{(2)})$ and $(c_{res,a}^{(2)}, c_{res,b}^{(2)})$ (in our case $c_{res,b}^{(1)} = 0, c_{res,b}^{(2)} = 10$). Follow the same steps to find the Nash equilibria.
5. Compare the results and find the optimal data restriction level for each government.

In the end, we find the equilibrium prices and profits of both Owners under the given data restriction levels of different regions. Here we define a pair of intersections in the form of

$$\{(c_{res,a}, c_{res,b}), (c_{0,a}^*, c_{0,b}^*), (p_a^*, p_b^*), (\Pi_a^*, \Pi_b^*)\}$$

In our analysis, we adopt a specific bivariate distribution to model consumer tastes, assuming that the preference for product functionality follows a normal distribution $x_j \sim N(0.8, 0.05^2)$, indicating most consumers favor a product that adequately meets their needs. Tastes towards data restriction levels are normally distributed as $y_j \sim N(0, 0.2^2)$, reflecting a general neutrality with an equal split between those advocating for and against data restrictions. The correlation between these distributions is assumed to be zero, suggesting that a Consumer's preference for product functionality is independent of their view on data restriction policies. This choice of distribution underscores the originality of our analysis method, which can be adapted to various real-world scenarios (Figure 1.12).

⁹This step is to check whether there are any neglected Nash equilibria resulting from the discreteness of computations.

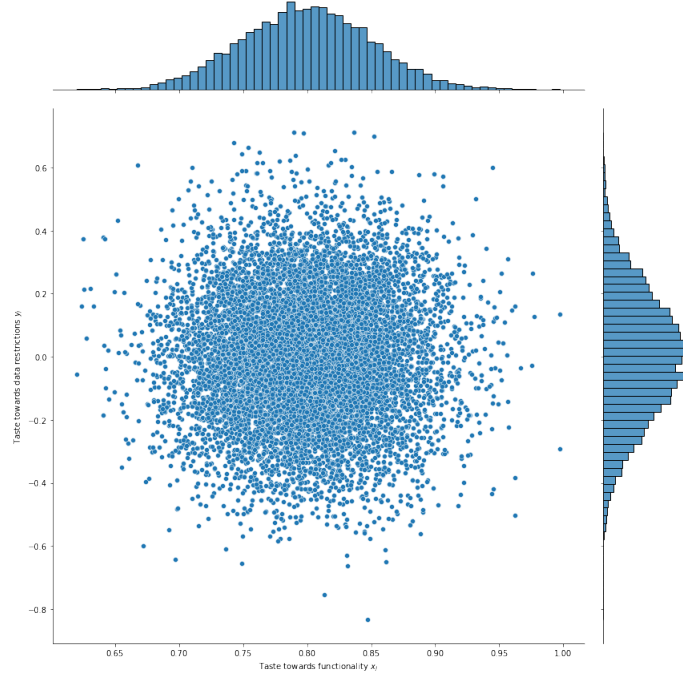


Figure 1.12: Consumers' tastes under a certain bivariate normal distribution. $x_j \sim N(0.8, 0.05^2)$, $y_j \sim N(0, 0.2^2)$, $\rho = 0$.

In our computational model, we explore how varying data restriction levels impact market outcomes by simulating scenarios with different pairs of data restriction levels. To manage computational resources efficiently, we select a sample of 100 Consumers from the joint distribution to represent the broader market. We set data restriction levels as given pairs, and get the equilibrium prices based on the best responses and the corresponding profits and welfare under each given pair of investments.

We refine our understanding of the total cost functions to better align with the complexities inherent in software development. While we established a direct correlation between investment and total cost, we now recognize the need for a more nuanced approach that reflects the reality of escalating costs associated with higher investments. We maintain that the functionality improvement (Δp_i) of the software has a linear relationship with the investment ($c_{0,i}$), represented as $\Delta p_i(c_{0,i}) = 2c_{0,i}$. This suggests that any increase in investment directly enhances the software's functionality. Meanwhile, we introduce a more convex relationship between the investment and the total cost related to contracts, maintenance, and upgrades. This adjustment is based on the understanding that the costs associated with software development escalate more rapidly than the investment itself due to factors such as high demand for and costs of hiring skilled developers, the

increasing complexity of managing larger projects, and the exponential growth in maintenance and support requirements as software becomes more sophisticated. From a technical standpoint, incorporating these total cost functions into the model is essential for obtaining an interior solution to the Nash Equilibrium. This ensures that our analysis adequately reflects the complex economic interactions and strategic considerations faced by Owners in a competitive environment. We introduce the following total cost functions into our model and proceed with the analysis accordingly:

1. $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$
2. $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$

Based on different pairs of data restriction levels, we let the Owners find the best response prices based on specific pairs of investments and note the profits under these best response prices in the grids. We pick the pairs' data restriction levels as (0, 0), (0, 10), and (10, 10), and set the step size of the investments as 0.5.

Utilizing our computational codes, we meticulously identify Nash equilibria for each pair of data restriction levels by plotting tuples of best response investments from Owners a and b (e.g., the tuple $(c_{0,a}^*, c_{0,b})$ signifies Owner a's optimal investment given Owner b's investment). The intersection points of these best response tuples represent investment equilibria $(c_{0,a}^*, c_{0,b}^*)$. Blue points represent tuples $(c_{0,a}^*, c_{0,b})$, while red crosses indicate tuples $(c_{0,a}, c_{0,b}^*)$. Intersections where a blue point and a red cross coincide indicate an equilibrium pair.

Given the discrete nature of simulation results and the fixed-step increase in investment, smoothing curves are drawn to identify additional potential equilibrium pairs. This approach, particularly useful for handling complex joint distributions of consumer tastes, illustrates why computational methods are preferred in our analysis; they generalize results across different distributions, a task often challenging for analytical methods. We use the *Python* function *scipy.interpolate* to obtain spline representations in the two-dimensional plane defined by $c_{0,a}$ and $c_{0,b}$ (Virtanen et al. 2020).

B-splines are chosen for data smoothing due to their desirable characteristics, such as positive partition of unity, knot averages, and local linear independence. In each sub-figure, we plot curves with minimal or no smoothing (parameter $s \leq 2$) and curves representing complete smoothing ($s = 21$). This approach ensures both the precision of our model in capturing the nuances of investment strategies and the robustness of our findings across varying levels of data restriction and investment increments.

Following the results shown in Appendix 1.7.1, the pairs of intersections under the setting of $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$ are ¹⁰

¹⁰The equilibria marked with * are the results based on the intersection of the two smoothing

- $\{(0, 0), (0.6, 6.7), (1, 9), (-0.2, 599.2)\}^*$
- $\{(0, 0), (0, 7), (1, 11), (6, 691)\}$
- $\{(0, 0), (1, 6.5), (1, 9), (7, 553.4)\}$
- $\{(0, 0), (6.5, 1), (9, 1), (553.4, 7)\}$
- $\{(0, 0), (6.7, 0.6), (9, 1), (599.2, -0.2)\}^*$
- $\{(0, 0), (7, 0), (11, 1), (691, 6)\}$
- $\{(0, 10), (7.3, 0.4), (18, 11), (1411, -0.1)\}^*$
- $\{(0, 10), (8, 0), (20, 11), (1428, 3)\}$
- $\{(10, 10), (0, 7), (11, 21), (6, 691)\}$
- $\{(10, 10), (0.5, 7.5), (11, 21), (5.9, 612.1)\}$
- $\{(10, 10), (1, 5.5), (11, 17), (0, 526.6)\}$
- $\{(10, 10), (5.5, 1), (17, 11), (526.6, 0)\}$
- $\{(10, 10), (7.5, 0.5), (21, 11), (612.1, 5.9)\}$
- $\{(10, 10), (7, 0), (21, 11), (691, 6)\}$

Under the case where $(c_{res,a}, c_{res,b}) = (0, 0)$, the numerical analysis identifies multiple pure-strategy Nash equilibria. These equilibria can be grouped into mirror-image asymmetric outcomes in which one region emerges as the high-investment, high-profit winner, while the other remains in a low-investment, low-profit position. Hence, the game exhibits equilibrium multiplicity and path dependence: even under identical policy conditions, different market allocations may arise depending on expectations, initial conditions, or adjustment dynamics. Therefore, the Nash equilibrium concept alone does not uniquely pin down the realized outcome, and additional equilibrium-selection criteria are needed for prediction.

Under the case where $(c_{res,a}, c_{res,b}) = (0, 10)$, the equilibrium set becomes considerably less ambiguous. For this asymmetric policy pair, all identified Nash equilibria display the same qualitative pattern: the less restricted region chooses a high level of investment, charges a higher price, and captures nearly all profits, whereas the

curves. Same for the setting of $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$. The equilibria marked with $\dagger$ represent fully symmetric behavior between the two firms. While simulation results may exhibit slight differences in profits, we report the outcomes where profits are equal by averaging the generated values.

more restricted region invests little and earns approximately zero profit. The small differences across equilibria appear to be numerical rather than economically substantive. Thus, asymmetric data regulation acts as an equilibrium-selection device that shifts the market toward a highly uneven competitive outcome.

Under the case where $(c_{res,a}, c_{res,b}) = (10, 10)$, the numerical analysis again identifies multiple pure-strategy Nash equilibria. These equilibria take the form of mirror-image asymmetric pairs: one region chooses high investment and captures most of the profits, while the other remains in a low-investment, low-profit position. Hence, equalizing data regulation at a high level does not restore a unique or symmetric market outcome. Instead, the game continues to be characterized by equilibrium multiplicity and path dependence.

The pairs of intersections under the setting of $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$ are

- $\{(0, 0), (1, 3), (1, 4), (16, 251)\}$
- $\{(0, 0), (2.3, 2.3), (1, 1), (22, 22)\}^{*\dagger}$
- $\{(0, 0), (3, 1), (4, 1), (251, 16)\}$
- $\{(0, 10), (3, 1), (12, 11), (999, 9)\}$
- $\{(0, 10), (3.4, 0.6), (13, 11), (1062.4, 7.9)\}^*$
- $\{(10, 10), (0, 3), (10, 14), (0, 315)\}$
- $\{(10, 10), (2.3, 2.3), (11, 11), (22, 22)\}^{*\dagger}$
- $\{(10, 10), (3, 0), (14, 10), (315, 0)\}$
- $\{(10, 10), (1, 4), (11, 15), (0, 239)\}$
- $\{(10, 10), (4, 1), (15, 11), (239, 0)\}$

When the investment cost function is changed from a cubic to a quartic form, the equilibrium structure changes qualitatively under the case where $(c_{res,a}, c_{res,b}) = (0, 0)$. In addition to the two mirror-image asymmetric equilibria, the model yields a fully symmetric equilibrium in which both firms choose the same investment level and price and earn identical profits. This suggests that greater convexity of investment costs weakens the incentive for aggressive strategic divergence and can sustain a balanced coexistence outcome.

Under the case where $(c_{res,a}, c_{res,b}) = (0, 10)$, changing the investment cost function from cubic to quartic does not qualitatively alter the equilibrium ranking of the two regions. In all identified Nash equilibria, the less restricted region chooses a substantially higher investment level and captures nearly all profits, while the

more restricted region remains in a weak competitive position. The differences across equilibria are quantitatively small, suggesting that the asymmetric policy configuration robustly selects a one-sided market outcome.

Under the case where $(c_{res,a}, c_{res,b}) = (10, 10)$, the high-restriction symmetric policy pair yields not only mirror-image asymmetric Nash equilibria but also a fully symmetric interior equilibrium. In that equilibrium, both firms choose the same investment level and price and earn identical profits. Relative to the cubic-cost specification, this indicates that greater convexity of investment costs reduces the incentive for aggressive divergence and makes a balanced coexistence outcome feasible. Nevertheless, asymmetric winner–loser equilibria remain possible, implying that stronger cost convexity mitigates, but does not eliminate equilibrium multiplicity.

Our results show that higher data restriction levels are generally associated with higher equilibrium prices, as Owners raise prices to offset the additional regulatory burden. When data restriction levels are identical across regions, the model may generate symmetric equilibrium prices, which are associated with two possible patterns: either one Owner invests profitably while the other earns a loss, or both Owners invest and obtain similar profits. The resulting profit configuration depends on the specific form of the total cost function. In cases where an equilibrium implies negative profits, such outcomes are not economically plausible, since both Owners could instead choose not to invest and secure a payoff of zero.

When data restriction levels are uniformly high across regions, equilibrium prices are also higher; however, this does not necessarily imply higher profits for Owners. Rather, the results highlight the delicate trade-off between increased regulatory costs and firms’ pricing responses. By contrast, when data restriction levels differ across regions, the Owner located in the more heavily restricted region tends to earn substantially lower profits than the Owner in the less restricted region. This imbalance provides a strong incentive for local governments to reduce data restriction levels in order to improve the competitive position of domestic firms.

From the perspective of local governments, when Consumers are relatively insensitive to differences in local data restriction levels, maintaining generally low data restriction levels appears to be beneficial. Such a policy can support the profitability of local Owners while also fostering a more competitive market environment. Overall, this finding underscores the importance of careful policy design and strategic decision-making by both Owners and policymakers in shaping competition in the software product market.

1.4.3 Distributions of consumers’ tastes

To find out the effects brought by different joint distributions of Consumers’ tastes, we draw other samples and compare the results based on different samples. Our

distributions include:

- Distribution 1: Joint normal distribution. $x_j \sim N(0.8, 0.05^2)$, $y_j \sim N(0, 0.2^2)$, correlation $\rho = 0$.¹¹
- Distribution 2: Joint normal distribution. $x_j \sim N(0.5, 0.05^2)$, $y_j \sim N(-0.5, 0.2^2)$, correlation $\rho = 0$.
- Distribution 3: Joint normal distribution. $x_j \sim N(0.5, 0.05^2)$, $y_j \sim N(0.5, 0.2^2)$, correlation $\rho = 0$.

The figure presented in Appendix 1.7.1 shows samples drawn according to Distributions 1–3, using $n = 10,000$. However, for computational efficiency and without loss of generality, simulations in our analysis are performed with a reduced sample size of $n = 100$.

The pairs of intersections based on Distribution 2 include:

- $\{(0, 0), (0.5, 5), (1, 5), (12.9, 310)\}$
- $\{(0, 0), (1.2, 5.6), (1, 5), (14.3, 244.4)\}^*$
- $\{(0, 0), (3.6, 3.6), (1, 1), (3.3, 3.3)\}^\dagger$
- $\{(0, 0), (5, 0.5), (5, 1), (310, 12.9)\}$
- $\{(0, 0), (5.6, 1.2), (5, 1), (244.4, 14.3)\}^*$
- $\{(0, 10), (5, 0), (19, 11), (1699, 4)\}$
- $\{(0, 10), (5.5, 0.5), (19, 11), (1657.6, 3.9)\}$
- $\{(0, 10), (5.3, 0.4), (19, 11), (1656.1, 4.9)\}^*$
- $\{(10, 10), (0.5, 6), (11, 16), (15.9, 288)\}$
- $\{(10, 10), (6, 0.5), (16, 11), (288, 15.9)\}$
- $\{(10, 10), (1.7, 4.7), (11, 13), (-4.9, 196.2)\}^*$
- $\{(10, 10), (4.7, 1.7), (13, 11), (196.2, -4.9)\}^*$

The pairs of intersections based on Distribution 3 include:

- $\{(0, 0), (0.5, 5), (1, 5), (12.9, 310)\}$
- $\{(0, 0), (5, 0.5), (5, 1), (310, 12.9)\}$

¹¹In the previous analysis, we show the results based on this distribution.

- $\{(0, 0), (1.2, 5.6), (1, 5), (14.3, 244.4)\}^*$
- $\{(0, 0), (5.6, 1.2), (5, 1), (244.4, 14.3)\}^*$
- $\{(0, 0), (3.6, 3.6), (1, 1), (3.3, 3.3)\}^{*\dagger}$
- $\{(0, 10), (5.5, 0), (10, 11), (753.6, 8)\}$
- $\{(0, 10), (6, 0.5), (10, 11), (704, 7.9)\}$
- $\{(0, 10), (5.6, 0.5), (9, 11), (697.4, 2.9)\}^*$
- $\{(10, 10), (0.5, 6), (11, 16), (15.9, 288)\}$
- $\{(10, 10), (6, 0.5), (16, 11), (288, 15.9)\}$
- $\{(10, 10), (1.8, 4.8), (10, 12), (-5.8, 89.4)\}^*$
- $\{(10, 10), (4.8, 1.8), (12, 10), (89.4, -5.8)\}^*$

Within the subgame framework, our results indicate that asymmetric data restriction levels lead to asymmetric investment and profit outcomes: the Owner located in the less restricted region consistently invests more and earns higher profits than its counterpart. This pattern accords with the intuitive expectation that a more favorable regulatory environment encourages investment and generates higher returns. By contrast, when data restriction levels are identical across regions, the Nash equilibrium often takes an asymmetric form in which one Owner invests while the other does not.

Accordingly, even under identical data restriction levels, the static game does not necessarily yield a symmetric equilibrium. This finding highlights the strategic uncertainty each Owner faces regarding the rival's investment decision. It also motivates the dynamic analysis developed in Section 1.5, which is needed to better understand the strategic interaction and decision-making processes in this market. The dynamic framework makes it possible to examine how investment strategies evolve in response to market conditions and competitive behavior, thereby providing a richer account of the strategic landscape in the software product market.

1.4.4 Welfare analysis

According to the definition of local welfare and that of global welfare, we depict the payoff matrices based on the expectation value of local welfare and global welfare under equilibrium prices respectively.

		$c_{res,a}$				$c_{res,a}$			
			0	10				0	10
$c_{res,b}$	0	<u>5313.8</u> , <u>5313.8</u>			$c_{res,b}$ <th>0</th> <td colspan="3"><u>5522.1</u>, <u>5522.1</u></td>	0	<u>5522.1</u> , <u>5522.1</u>		
	10	<u>10484.8</u> , 282.3				10	<u>9588.6</u> , 854		
		282.3, <u>10484.8</u>					4733, <u>4733</u>		
		4765.1, 4765.1							

		$c_{res,a}$				$c_{res,a}$			
			0	10				0	10
$c_{res,b}$	0	<u>5152.1</u> , <u>5152.1</u>			$c_{res,b}$ <th>0</th> <td colspan="3"><u>5152.1</u>, <u>5152.1</u></td>	0	<u>5152.1</u> , <u>5152.1</u>		
	10	<u>9950</u> , 392.5				10	<u>9585.1</u> , 775.2		
		392.5, <u>9950</u>					4869.9, <u>4869.9</u>		
		4369.9, 4369.9							

Figure 1.13: Payoff matrices of the multi-stage game. Payoff tuple in each square: $(\mathbb{E}(W_a^*), \mathbb{E}(W_b^*))$. Top-left and Top-Right are based on Distribution 01, Bottom-Left is based on Distribution 02, and Bottom-Right is based on Distribution 03. The total cost function in Top-Right is $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$, while in others, the function is $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.

		$c_{res,a}$				$c_{res,a}$			
			0	10				0	10
$c_{res,b}$	0	10627.6			$c_{res,b}$	0	<u>11044.2</u>		
	10	<u>10767.1</u>				10	10442.6		
		<u>10767.1</u>					9466		
		9530.2							

		$c_{res,a}$				$c_{res,a}$			
			0	10				0	10
$c_{res,b}$	0	10304.2			$c_{res,b}$	0	10304.2		
	10	<u>10342.5</u>				10	<u>10360.3</u>		
		<u>10342.5</u>					<u>10360.3</u>		
		8739.8					9739.8		

Figure 1.14: Payoff matrices of the multi-stage game. Payoff tuple in each square: $\sum_i \mathbb{E}(W_i^*)$. Top-left and Top-Right are based on Distribution 01, Bottom-Left is based on Distribution 02, and Bottom-Right is based on Distribution 03. The total cost function in Top-Right is $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$, while in others, the function is $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.

The analysis of the payoff matrices sheds light on the contrasting strategic equilibria from the perspectives of a global-maximizer and local governments within the market (Figure 1.13, Figure 1.14). The findings highlight a nuanced landscape of strategic preferences and welfare implications:

1. From the perspective of the global-maximizer, the pure-strategy Nash Equilibrium from a global-maximizer's standpoint often involves differentiated data restriction levels across regions. This differentiation suggests that a global-maximizer can accommodate varying levels of data restrictions, balancing the diverse regulatory environments to optimize global welfare. The global-maximizer's tolerance for differentiated data restriction levels indicates a broader perspective that seeks to harmonize local policies with global welfare objectives.
2. From the perspective of a local government, the pure-strategy Nash Equilibrium from the perspective of any local government typically advocates for low data restriction levels within its jurisdiction. This preference is driven by the incentive to enhance local welfare, suggesting that local governments perceive reduced data restrictions as a means to stimulate local economic activity and consumer satisfaction. The local government's strategy aligns with fostering a business-friendly environment that encourages investment, innovation, and consumer choice.

The discrepancy between the strategic preferences of the global-maximizer and local governments underlines the complex interplay between global welfare optimization and local welfare considerations. Although a higher level of data restriction may appeal to certain consumer segments due to privacy concerns or other factors, the overall impact on welfare tends to be negative. This is because stringent data restrictions can hinder market efficiency, limit consumer options, and constrain economic growth. The analysis suggests that aligning data restriction policies with broader welfare goals requires a delicate balance. Policymakers need to consider not only the local economic and consumer welfare but also the implications of these policies on the global market and overall welfare.

1.5 Agent-based computations

1.5.1 The usage of heuristics

As the field of behavioral economics evolved, researchers recognized that agents in the economy exhibit bounded rationality, focusing more on individual decision-making rather than perfectly rational behavior. This shift led to an emphasis on

understanding the beliefs and heuristics that guide economic agents. Early work by Tversky and Kahneman (1974) revealed that decision-makers often rely on a limited set of heuristic principles, which can lead to systematic errors in judgment (Tversky & Kahneman 1974). This observation marked a significant departure from the notion of fully rational agents, highlighting the role of cognitive biases and heuristics in decision-making.

In the realm of economic modeling, the incorporation of heuristics offers a more nuanced and realistic representation of how individuals make decisions (Delli Gatti et al. 2011). These models acknowledge that agents may not always optimize in the traditional sense but instead use simple, rule-of-thumb strategies to navigate complex decision-making landscapes. The integration of heuristics into economic models provides a more robust microeconomic foundation, enhancing our understanding of market dynamics and consumer behavior. It allows economists to explain and predict economic phenomena more accurately by acknowledging the cognitive limitations and biases inherent in human decision-making.

The reasons of implementing heuristics include: Firstly, real market situations often involve complexities and uncertainties that challenge the traditional assumptions of economic models (Arthur 1992). For instance, firms in a market have limited information about their competitors, making utility maximization a complex endeavor. Owners aim to attract Consumers by offering products that enhance utility, but their decision-making is hindered by incomplete knowledge about their opponent's strategies, including pricing and product quality. Besides, recognizing these limitations, Owners may resort to heuristic principles to make decisions. This might involve using rule-of-thumb strategies or basing decisions on partial information. Moreover, while Owners cannot directly observe which Consumers buy which product, they can monitor market outcomes, such as the total number of purchases for each product. This feedback mechanism provides Owners with indirect information about market responses to their strategies and those of their competitors. To conclude, incorporating heuristics into the analysis of policy-makers and market players addresses the limitations of traditional economic models, particularly in complex, real-world scenarios. Traditional models often rest on assumptions of complete information, consistency, and well-defined preferences, which might not hold in intricate market environments.

In the economic landscape, even when agents can maximize their utility among limited information, they often gravitate towards actions that align more closely with their beliefs and experiences. This tendency is particularly pronounced in complex real-world scenarios where making accurate predictions is challenging, and adhering to behavioral rules or heuristics becomes a more manageable strategy (Axtell & Farmer 2022). Owners operating in markets, especially those engaged in continuous sales across multiple periods, can learn and adapt based on past

outcomes. This learning process allows them to refine their strategies over time, drawing on the lessons from previous periods to enhance profitability and market performance in subsequent ones. The use of heuristics plays a critical role in this dynamic adjustment process, offering a practical framework for decision-making in the face of uncertainty and incomplete information. Our analysis is based on the following assumptions:

1. Both Owners compete with each other within every period.
2. Both Owners apply the same way of market analysis and investment updates despite randomness.
3. Both Owners are myopic, i.e. they only care about their profits within one period instead of considering discounted future profits.
4. No entry or exit in the market.

In this case, we allow Owners to update the prices of their software products across different versions. Following this idea, we consider price and investment dynamics and enhance the model design.

1.5.2 Price and investment dynamics

Simulations with price updates

For both Owners, the information on the data restriction level is observable and exogenous because of the local regulators (Governments). The Owners cannot directly change the data restriction level, yet they could adjust two other variables across different periods: the selling prices and the investments in their products, while the prices could be adjusted more frequently than the investments since the investments are paid to Developers and the developing process takes time to make the product sufficiently more functional than the previous version. In general, the product update frequency varies across different companies, for instance, it could be twice a year or semi-annually. Here we assume that an Owner updates the investment every τ months and $\tau \geq 2$. We claim that the decision towards prices is more flexible than that towards investments, and we use a time axis to show the frequency of the updates (Figure 1.15).

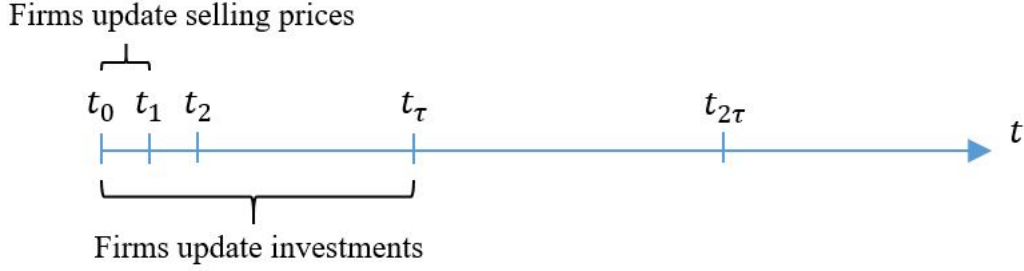


Figure 1.15: Description of update frequency.

The heuristic pricing rule we refer to comes from the idea of strategic pricing mentioned in Dawid et al. (2019). The idea is based on the description of Nagle & Müller (2018) and implies that firms could explore the market via strategic pricing. The price setting could be divided into three stages, namely a preliminary segment pricing stage, an optimization stage, and the final implementation stage. A firm could take a sequence of activities (market analysis, output analysis, cost analysis, and profit analysis) to estimate the possible effects of price changes on profitability.

Under the market analysis, the firm selects a representative sample of households and considers whether the consumers would buy the product after specific price increases. The expectation of the competitor's selling price, the consumer's sensitivity concerning price differences of the available goods and the expected market shares play essential roles. In our case, the Owners design surveys and deliver them to a sample of Consumers at the end of one period. Out of the total number of Consumers n (in the codes $n = 10,000$), Owner i selects a sample of n_s Consumers (if n_s is 100 then $\frac{n_s}{n} = 1\%$) and asks them whether they are willing to purchase the software of Owner i under different prices.

In our model, we simulate a survey mechanism for Owners to gauge consumer willingness to purchase their products at varying price points. This mechanism aims to provide Owners with insights into potential market shares and inform their pricing strategies for the upcoming period. Here's a breakdown of the process:

1. Prices presented in the survey are chosen from a range centered around the product's price in the previous period. For instance, if the price was $p_{i,t}$ in the last period, the survey might include prices from a uniform distribution $U[p_{i,t} - 5, p_{i,t} + 5]$. This range offers a spectrum of prices, allowing Owners to assess consumer sensitivity to price changes.
2. Owner i formulates a survey question, essentially asking, "Would you still consider buying our product at prices ..., while the price of the competitor's

product remains the same as in the last period?” The survey aims to estimate the Owner’s market share at different price points, enabling strategic adjustments for future pricing.

3. Consumers are presented with a list of alternative prices (p_{n_v} , $v = 1, 2, \dots, V$) and are asked to indicate whether they would consider purchasing the product at each price. For simplicity, consumer responses ($r_{i,j,t}$) are binary: 1 for ‘yes’ (would buy) and 0 for ‘no’ (would not buy). While the actual methodology for conducting the survey (sequential, random, or all prices at once) might influence consumer responses, we assume for the model’s simplicity that this influence is negligible in affecting the Owner’s decision-making process.
4. Owners analyze the survey data to understand consumer preferences and price sensitivities. Based on this analysis, Owners adjust their pricing strategy for the next period, aiming to optimize their market share and profitability.

After the Consumers respond to the Owners, each Owner estimates the possible order that each price alternative brings, i.e., the number of Consumers purchasing the product in the next period. For a certain price alternative p_{n_v} , in the sample, there are always some Consumers m_{n_v} who consider buying the product ($v = 1, 2, \dots, V$). Then, based on the information, each Owner calculates the “market share” (defined as $\frac{m_{n_v}}{n_s}$) and estimates the number of orders under each market share. The equation can be written as follows:

$$\tilde{m}_{i,t+1,n_v} = n \cdot \frac{m_{i,t,n_v}}{n_s}$$

Plugging $\tilde{m}_{i,t+1,n_v}$ into $\Pi_i(\theta_i, c_{0,i}, c_{res,i}, p_i)$, each Owner gets an estimated profit $\tilde{\Pi}_{i,t+1,n_v}$ under each price alternative p_{n_v} ($v = 1, 2, \dots, V$). By comparing the estimated profits, each Owner finds the price that brings the maximal estimated profit and sets this price as the product price for the next period. In our codes, if multiple prices fulfill the requirement simultaneously, we let each Owner select the lowest alternative price. If the maximal estimated profit is positive, the Owner adjusts the price, otherwise, the Owner keeps the price unchanged. The equation can be written as follows:

$$p_{i,t+1} = \min_{p_{i,t,n_v}} \{ \argmax \tilde{\Pi}_{i,t+1,n_v} \}$$

In subsequent periods, Owners repeat this market analysis based on the new prices, ensuring a continuous process of learning and adjustment. This dynamic approach allows Owners to adapt their strategies based on direct feedback from the market, aligning their pricing with consumer preferences and competitive pressures.

Simulations with price and investment updates

Owners can update their investments every τ periods, therefore, we can improve the framework as follows: Based on the market analysis for deciding on the selling price of the product, we allow Owners to update the investments in the products according to certain heuristics. The steps for the decision-making for the investment updates include:

1. At the beginning of the τ -th period ($\tau = 6$ in the codes), we assume that both Owners increase their investment by Δ units.
2. At the beginning of the 2τ -th period, each Owner calculates its average profit over the last five months. We define the amount of each Owner as $\overline{\Pi_{i,2\tau}}$ ($i \in \{a, b\}$). Owner i compares $\overline{\Pi_{i,\tau}}$ and $\overline{\Pi_{i,2\tau}}$. If $\frac{\overline{\Pi_{i,2\tau}} - \overline{\Pi_{i,\tau}}}{|\overline{\Pi_{i,\tau}}|} > \vartheta$ (ϑ is the updating threshold), Owner i continues the same direction of investment by Δ units; If $\frac{\overline{\Pi_{i,2\tau}} - \overline{\Pi_{i,\tau}}}{|\overline{\Pi_{i,\tau}}|} < -\vartheta$ Owner i adjusts investment reversely by Δ units; Otherwise, Owner i does not change its investment.
3. At the beginning of the $k\tau$ -th period ($k > 2$), each Owner compares $\overline{\Pi_{i,k\tau}}$ and $\overline{\Pi_{i,(k-1)\tau}}$ and adjusts the investment in the same way. We note that if the investment holds constant continuously twice, the Owner randomly increases or decreases the investment by ψ units at the beginning of the next τ -th period (ψ is drawn from $\{-\psi_0, \psi_0\}$ ($\psi_0 \geq 0$) with probability 50%).

We summarize the economic meaning of each parameter:

- $\psi_0 > 0$ guarantees the adjustments of the investments. A larger ψ_0 indicates that Owners take the random updates to a greater extent.
- Δ can be regarded as the learning rate. A larger Δ indicates that Owners take stronger measures regarding their investments.
- ϑ is the update threshold. A larger ϑ indicates that Owners only update their investments with a higher threshold.
- τ represents at which time unit an Owner updates its investment and shows the update frequency of the investment, i.e. an Owner updates its investment every τ months. A larger τ means slower updates in investment.

In the single-run simulations, we keep the same bivariate normal distribution as before and keep the initial prices of both products as 20, and we change the parameters Δ , ψ_0 , and ϑ and check how the Owners perform in the market. To check whether the investments, in the long run, converge around some specific

values (here we extend the time range to 1,000 months), we also draw the dynamics of the investments in separate figures.

From the simulation results shown below, we observe that when the initial investments are different, in most cases, the Owner with a high initial investment earns a higher profit than its opponent in almost every period. Exceptions occur in the cases where $\Delta = 0.25, \psi_0 = 0.05, \vartheta = 1$ and $\Delta = 0.5, \psi_0 = 0.1, \vartheta = 1$. Among the exceptions, we observe that the Owner with a larger initial investment decreases its investment while its opponent increases the investment accordingly, which eventually leads to the result that the Owner with a smaller initial investment earns more in the later periods. Since the phenomenon does not appear in the cases where $\vartheta > 1$, we show that when the threshold of updating investments is larger, i.e. when the Owners only update their investments if their profits are “sufficiently” larger or smaller than before, the Owner with a larger initial investment continues to have a “beginner advantage” in most periods.

Besides, we can also find arguments to answer the essential question of whether investments and prices converge in the long run. When the updating threshold is small ($\vartheta = 1$), with different parameters of Δ and ψ_0 and different initial investments, the investments vary largely. Some of the investments converge but others do not. In this case, the initial investments matter much in the market dynamics. When the updating threshold gets larger ($\vartheta = 1.5$ or $\vartheta = 2$), the investments eventually fluctuate around zero, which means one of the Owners invests a very small amount while its opponent does not invest. In this case, we believe that when Owners only adjust the investments when there is enough gain or loss, after periods of investment and price updates, Owners eventually focus more on price updates than investment updates.

We also observe that when the updating threshold is large ($\vartheta = 2$), when the data restriction levels are the same, the profits of Owners are pushed to values around zero after many periods. This does not happen when the data restriction levels are different, and both Owners get positive profits in most periods. The phenomenon seems to contrast with the result of the subgame perfect equilibrium analysis (the less regulated Owner gains a large profit while its opponent gains a profit around zero). The results here provide evidence that the market may not reach an equilibrium under the settings of heuristics and both Owners may be better off after a large number of periods.

Furthermore, introducing investment dynamics seems to bring more possibility of profit for the Owner under a higher restriction level. At certain periods, the profits of both Owners could be close to each other.

Statistical analysis of dynamics

From the single runs of simulations, we can already find the differences under different scenarios in the market, however, the information may be inaccurate because of the randomness in the market.¹² To find statistical features out of the individual iterations, we design an algorithm to collect the generated time series data. The steps of the algorithm include:

1. Under a certain distribution, in each scenario, given the initial prices ($p_a^{ini} = 20, p_b^{ini} = 20$), run simulations under 50 different random seeds and take the mean to get the initial outcome ($\Pi_a^{ini}, \Pi_b^{ini}, W_a^{ini}, W_b^{ini}$) of the 1st period.
2. For the later periods (in the codes we have 1,000 in total), run simulations under 30 different random seeds to generate 30 time series of price, investment, profit, and welfare.
3. Save the time series. Adjust the parameters and return to the 1st step.

We collect the data from the data generation to conduct statistical tests and confidence bands around the considered variables under the different policy variants, which coincides with a standard in the literature (Dawid & Delli Gatti 2018). To make reliable statements about the significance of the effects of parameter changes, we need a statistical analysis, relying either on statistical tests comparing distributions of variables at certain periods generated by different batch runs (e.g. a Wilcoxon Signed Rank Test) or on the estimation of dynamic statistical models (Dawid et al. 2019). Example statistics of generated time series include the statistical moments, maximum and minimum, confidence intervals, etc.¹³

In the case when most Consumers prefer a higher data restriction level, the Owner located in the region with the higher data restriction level can earn a higher profit. It shows the importance for the firms to have the information of the tastes of the consumers. Moreover, being located in an area with a high data restriction level may not be a disadvantage.

¹²In the codes, this may result from different random seeds.

¹³For confidence intervals we follow the assumption that at every period, the values under different random seeds approximately follow a t-distribution (95% confidence level, two-sided test).

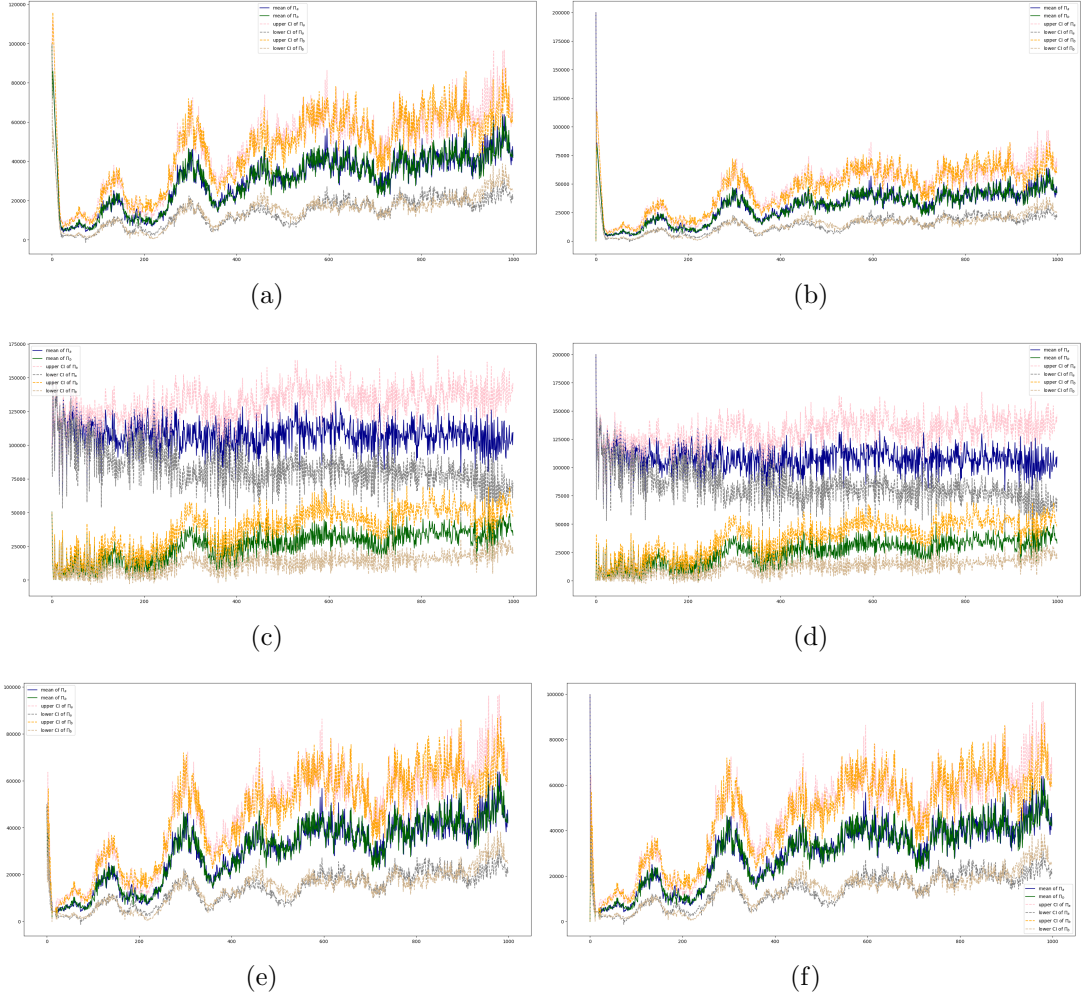


Figure 1.16: Statistics of Π_a and Π_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

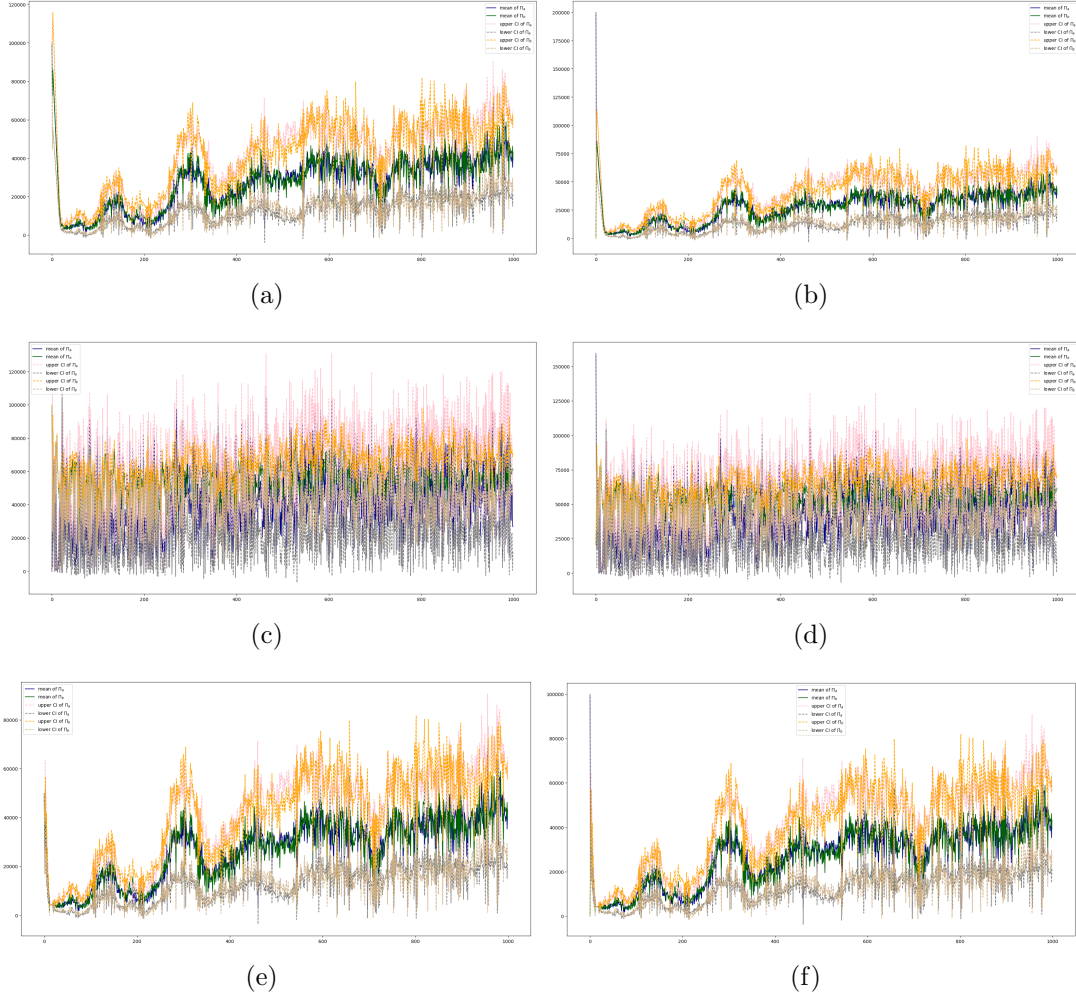


Figure 1.17: Statistics of Π_a and Π_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 3). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

From the aspect of global welfare, we find that on average, the trends of global welfare under different scenarios look similar, while the high data restriction level indeed decreases the amount of global welfare.

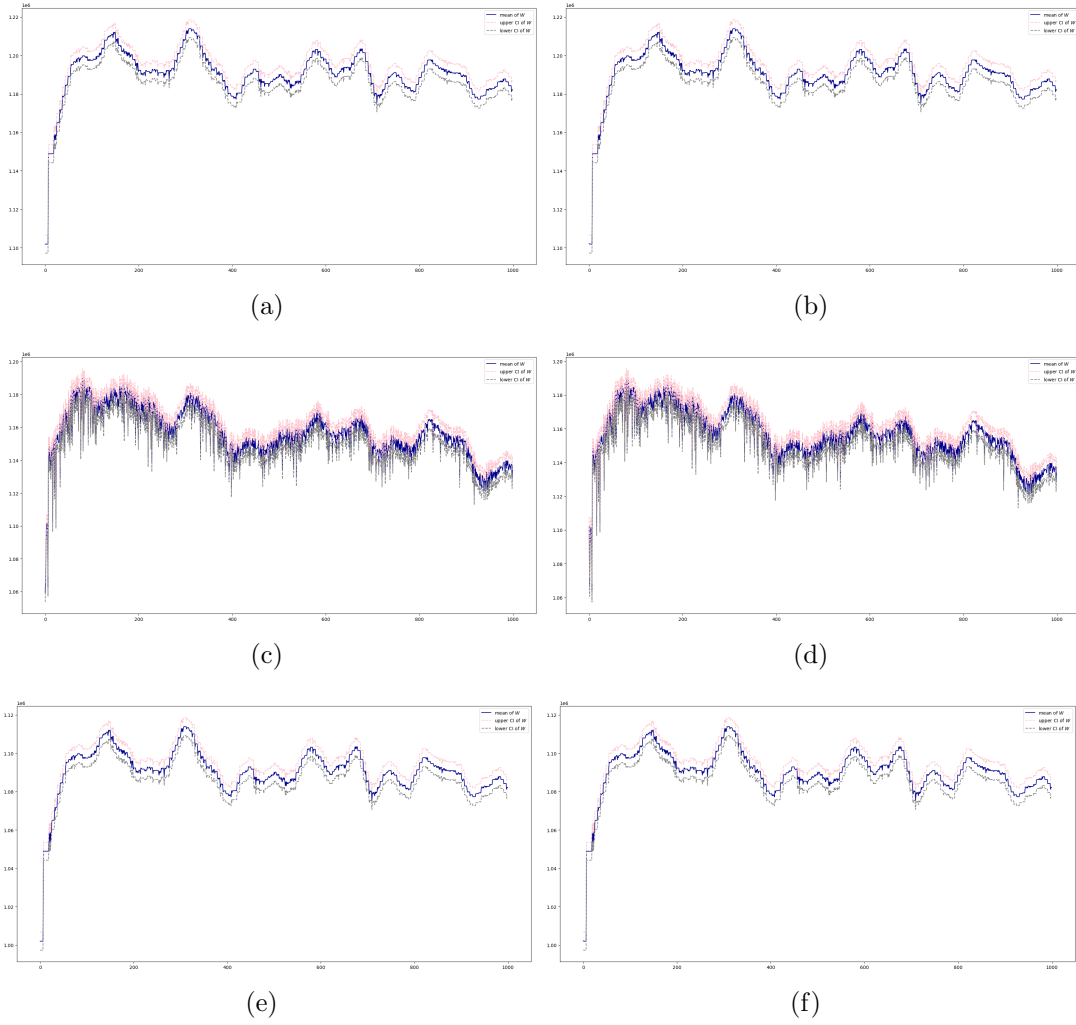


Figure 1.18: Statistics of W according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

Using the kernel density estimates, we find that at certain periods there could be a value of high density, but as time passes, the curves appear to become flatter, showing a divergent trend of the distributions of profit and welfare, which also indicates the uncertainty in the outcomes.

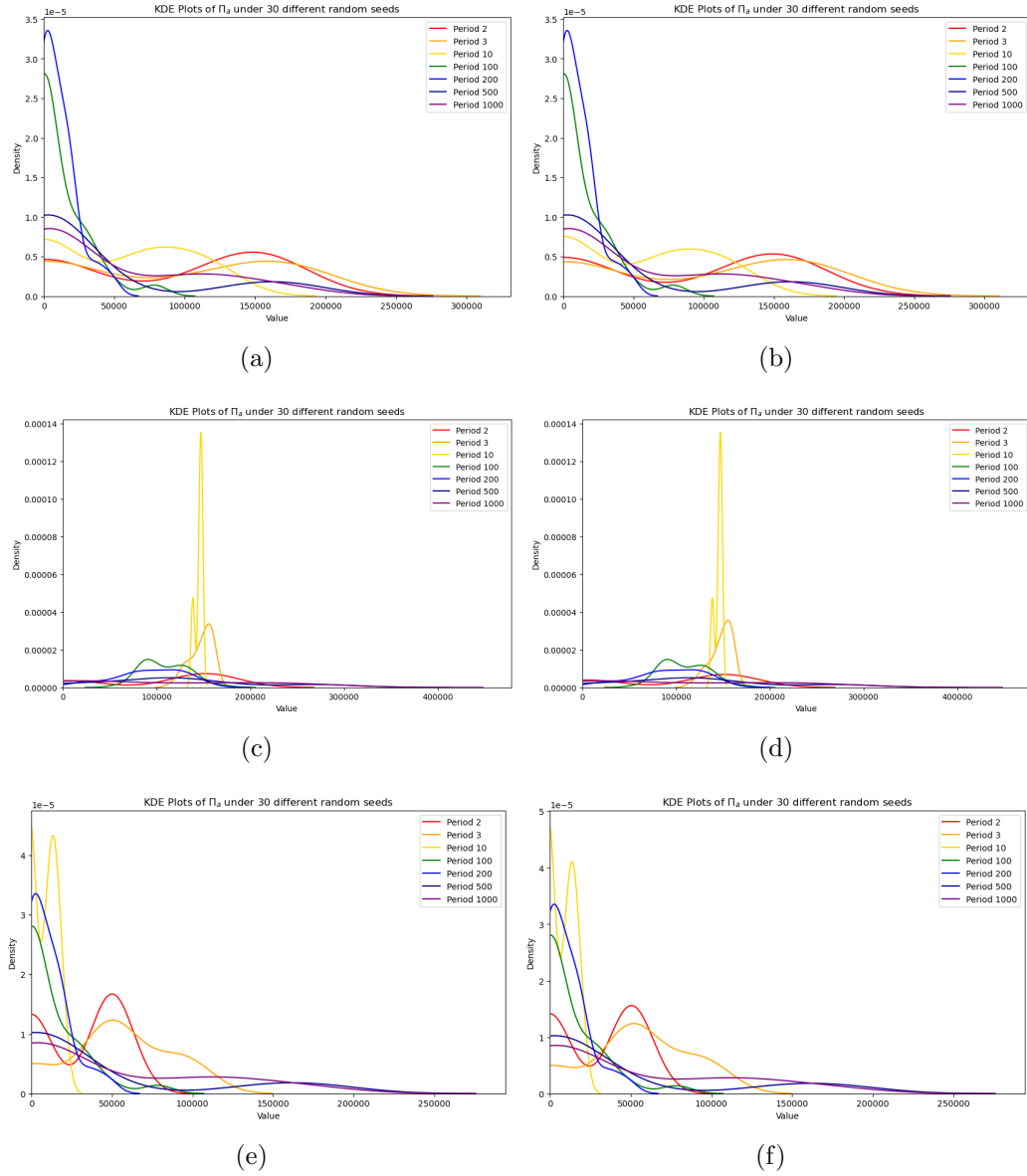


Figure 1.19: KDE plots according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

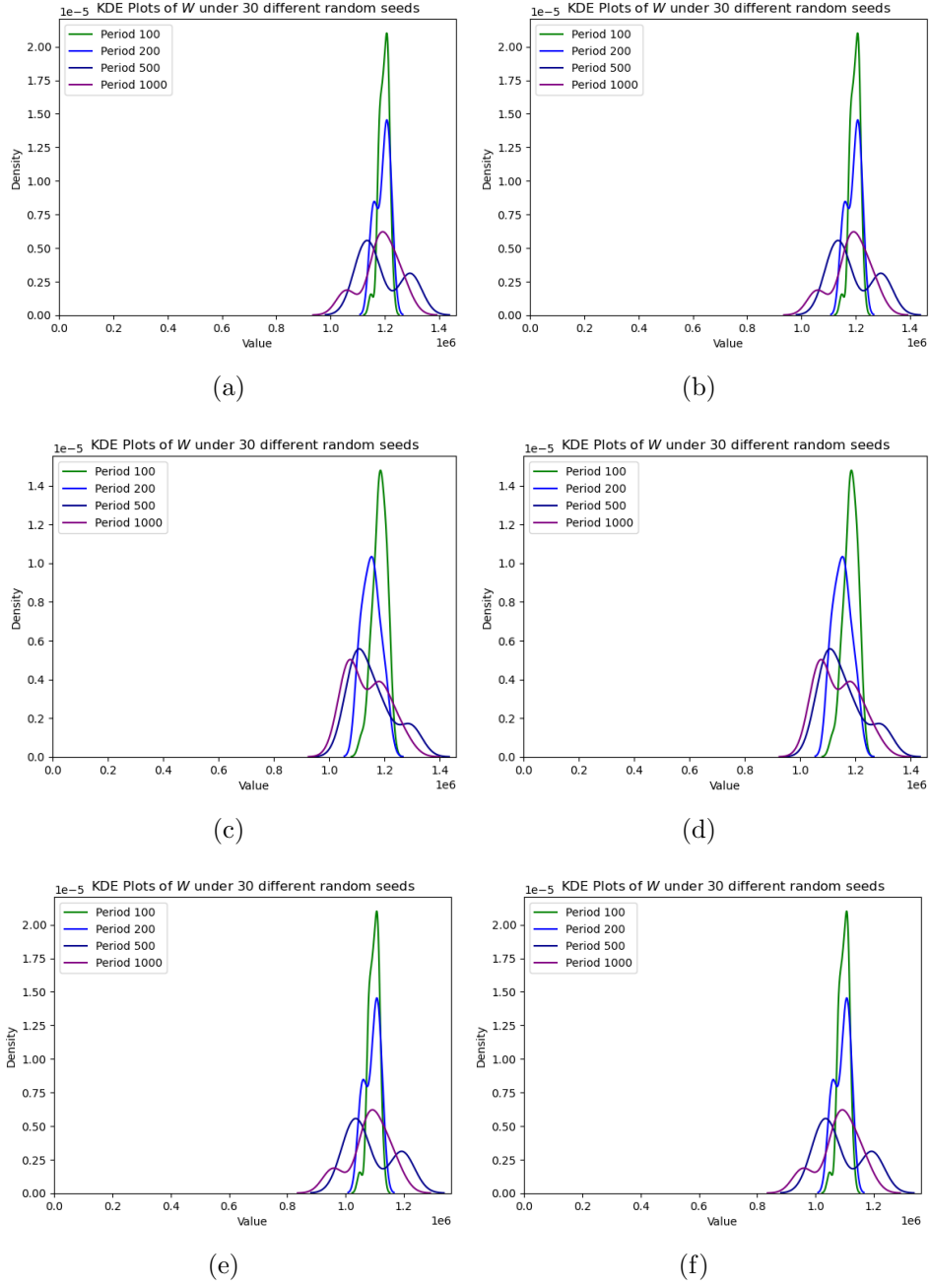


Figure 1.20: KDE plots according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

From the figures showing the difference in profits, we find that the initial investment difference directly influences the profits of the firms in the initial periods,

but does not have a great influence in the long run. When the two Owners are under different data restriction levels, the sample distribution can have a great impact on the firm competition. If the sample is based on a distribution where most Consumers show an indifferent attitude towards the data restriction level, then on average, the Owner under the higher data restriction level offers a product with a higher price and earns less profit. If the sample is based on a distribution where most Consumers show a positive attitude towards the data restriction level, then on average, the Owner under the higher data restriction level can earn a higher profit than its opponent.

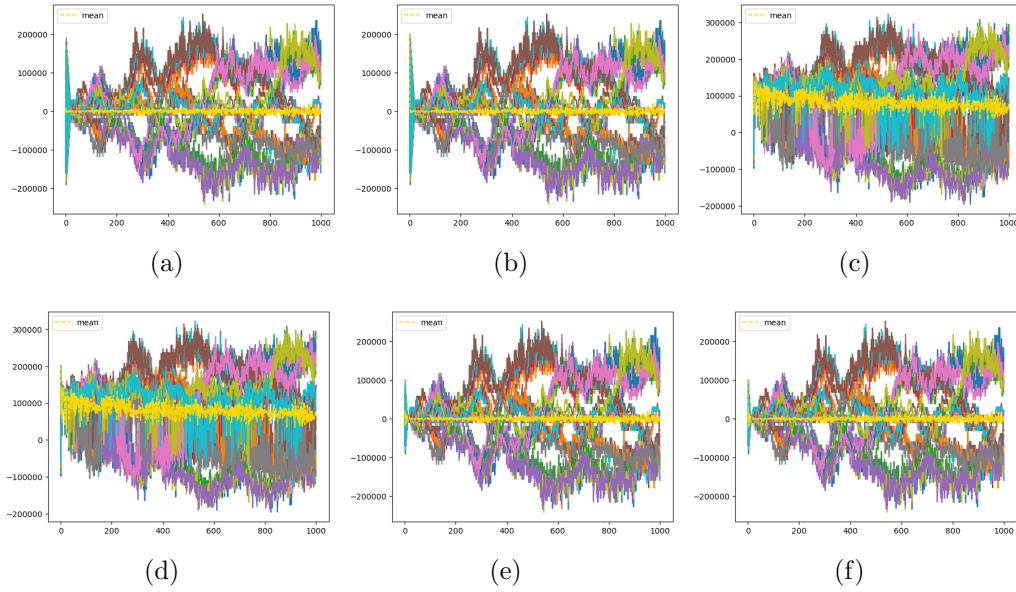


Figure 1.21: Difference of Π_a and Π_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

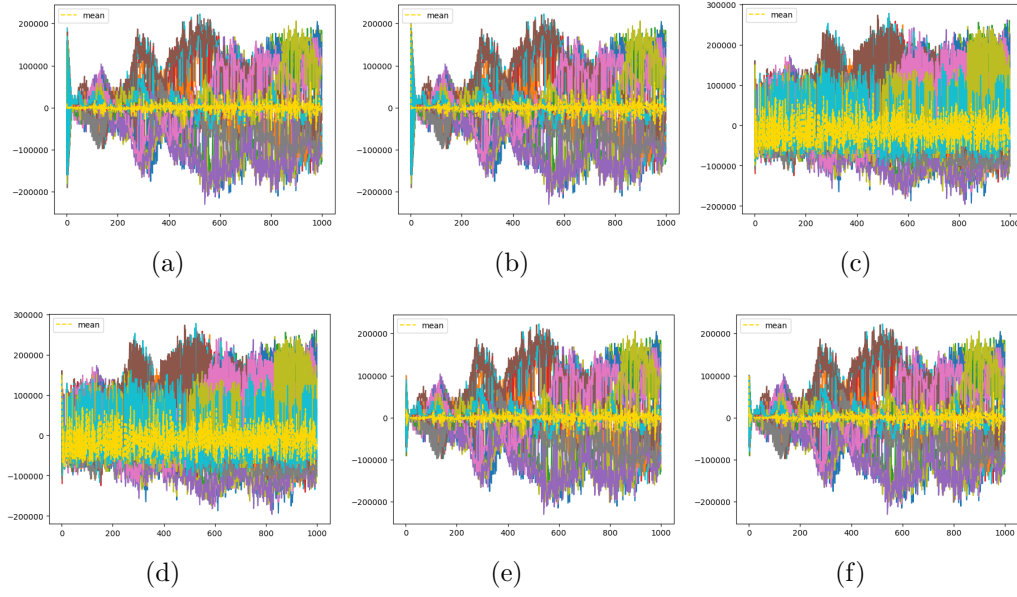


Figure 1.22: Difference of Π_a and Π_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 3). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

The autocorrelation of the prices shows some feature of periodicity (the period is about τ). Under different random seeds, it is not always the case that the first order autocorrelation is larger than the others. Besides, the extent of autocorrelation can be very different under different random seeds, showing the complexity of the market.

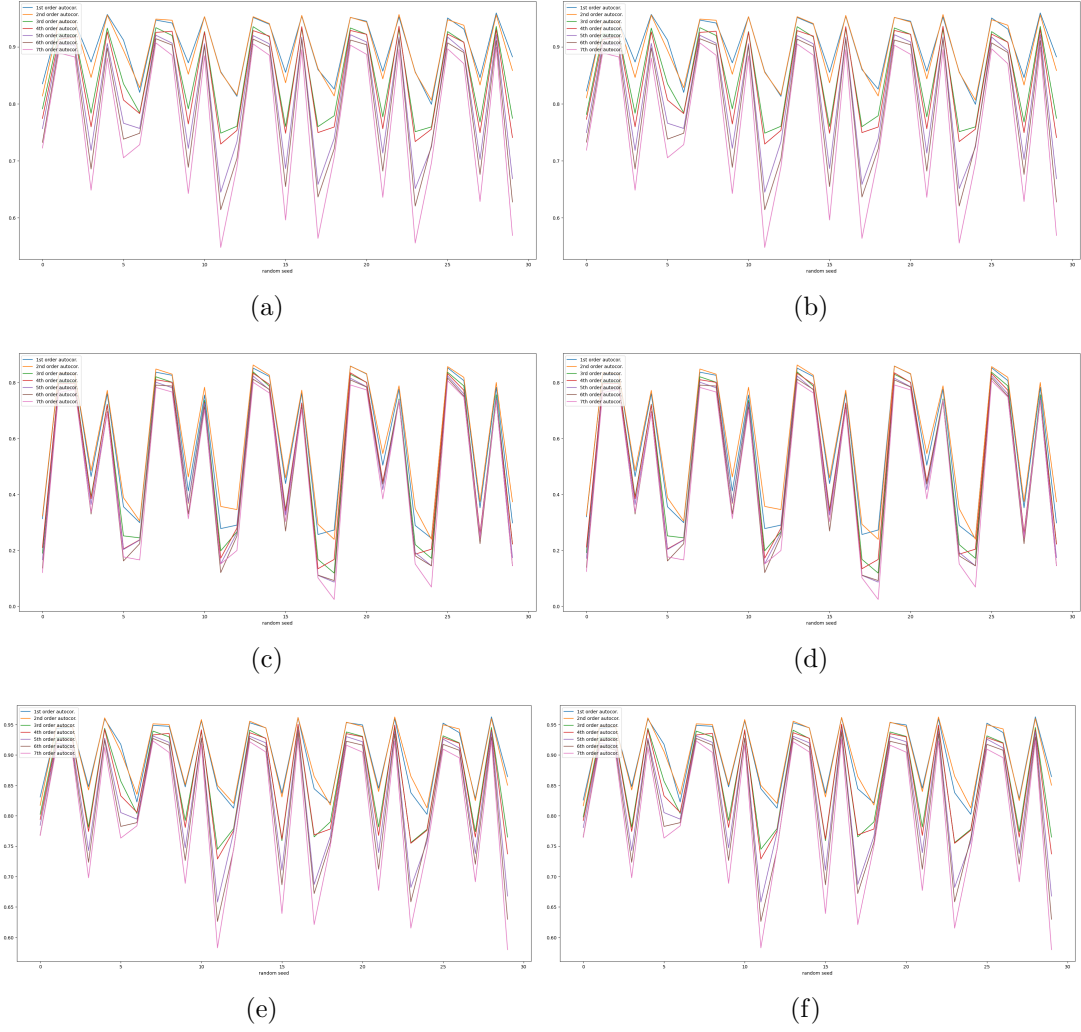


Figure 1.23: Autocorrelation of p_a according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

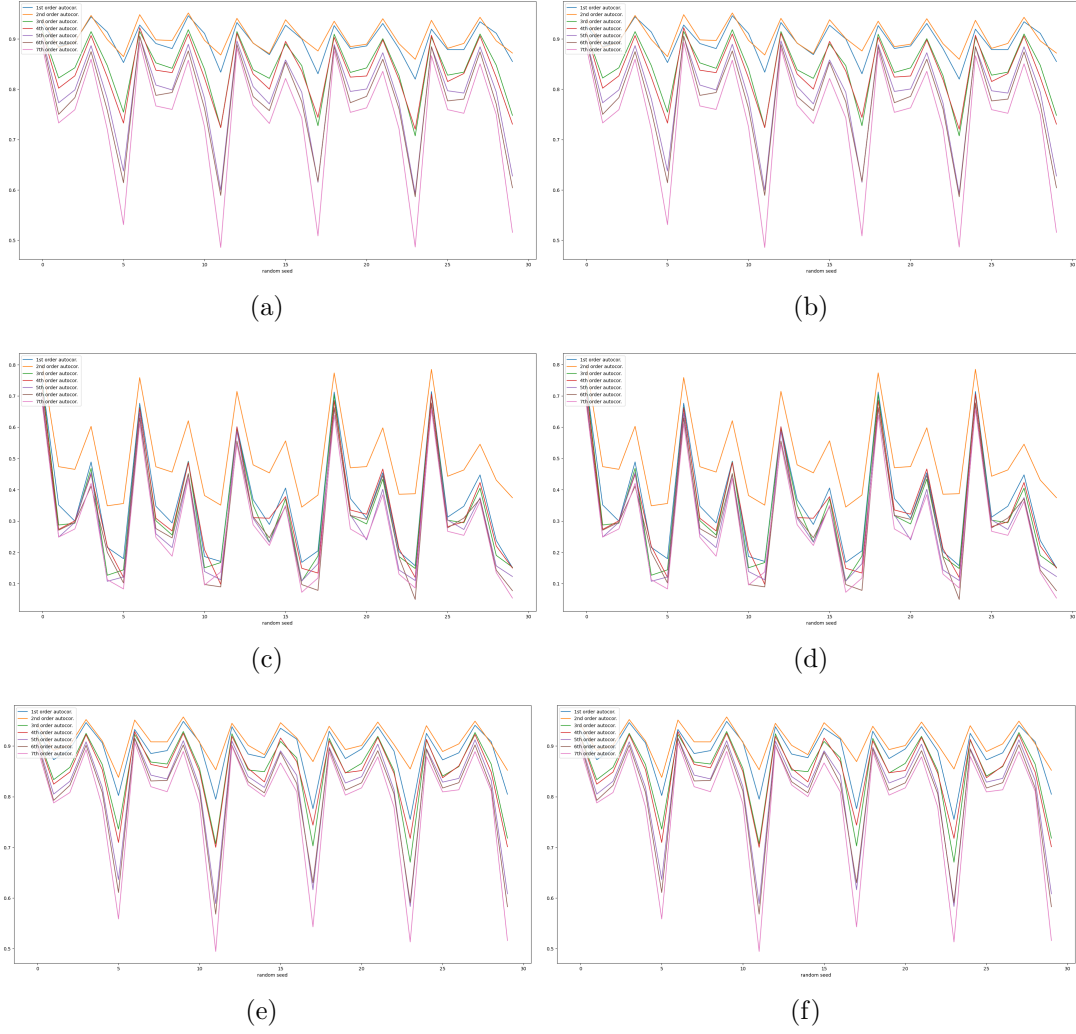


Figure 1.24: Autocorrelation of p_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

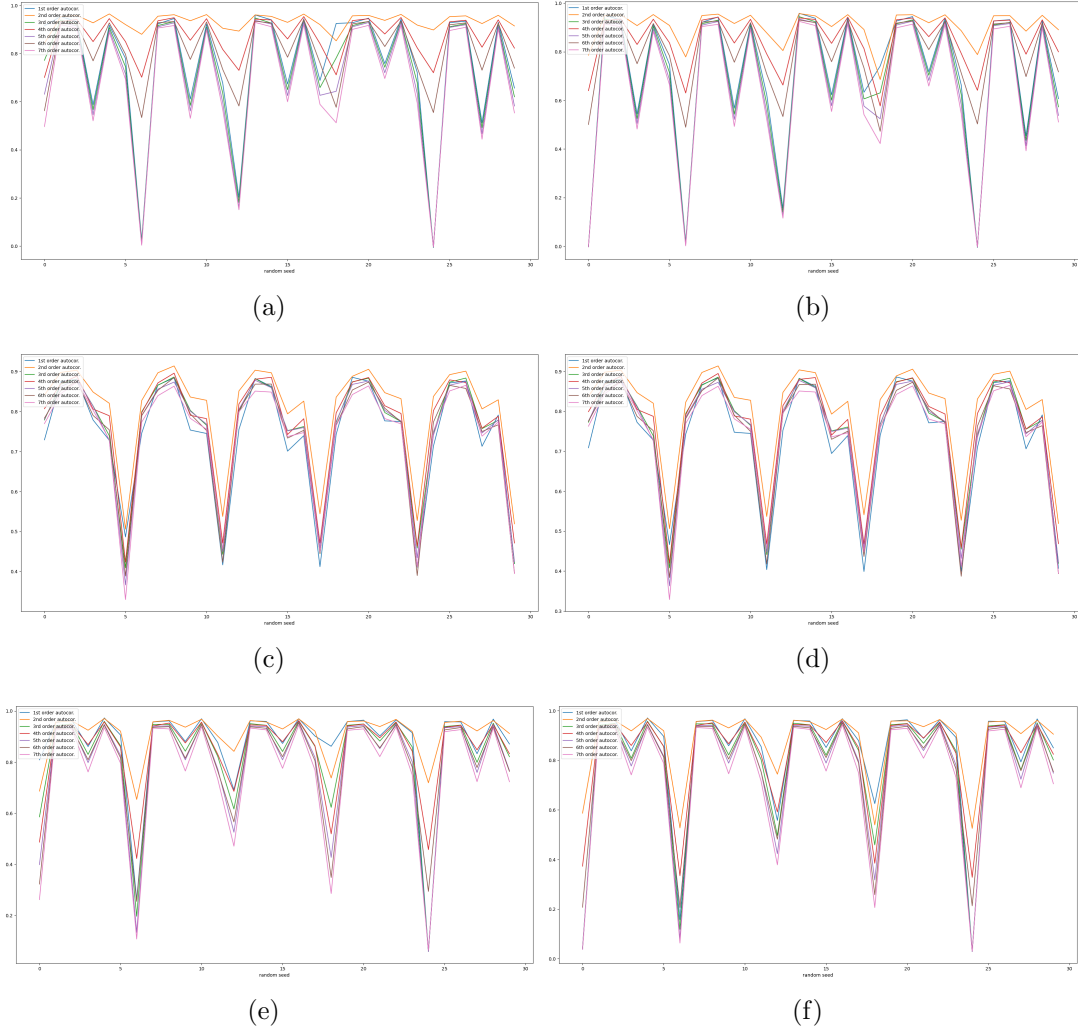


Figure 1.25: Autocorrelation of Π_a according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

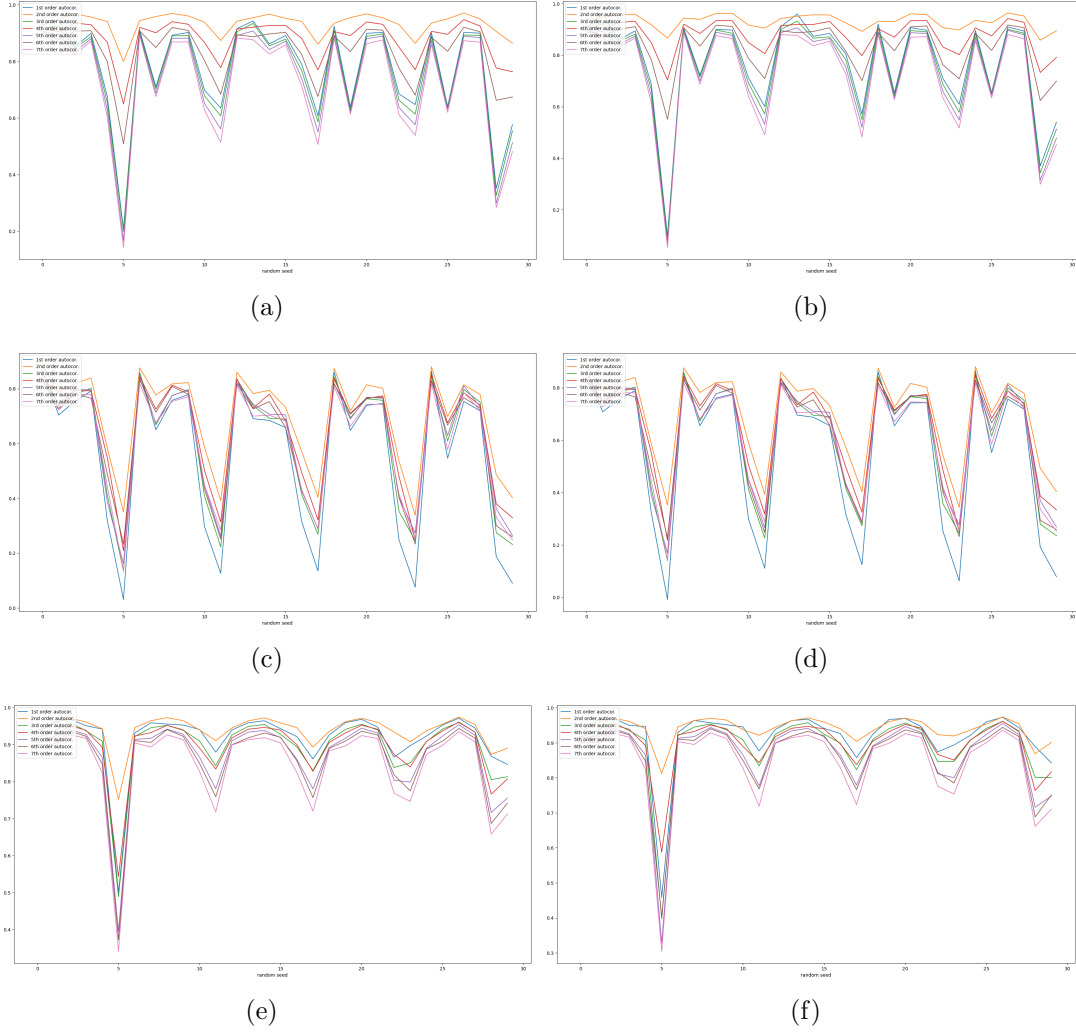


Figure 1.26: Autocorrelation of Π_b according to 30 time series. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

To study the correlation of variables, we can check the cross-correlation matrix. We calculate the correlation matrix under one random seed and take the average of the values for all the random seeds, which is the “mean correlation matrix among random seeds”. We find that under the same data restriction level, the mean prices are positively correlated with each other, while under different data restriction levels, the correlation of prices is around 0. Besides, under the same data restriction level, the correlation between the profit of an Owner and the investment is positive. Under different data restriction levels, this correlation is stronger for the Owner under a lower data restriction level. We also find that

the welfare of regions shows a pattern of “zero-sum”, and the global welfare is positively correlated with one of the Owners. For each Owner, the welfare of its region is positively correlated with its profit.

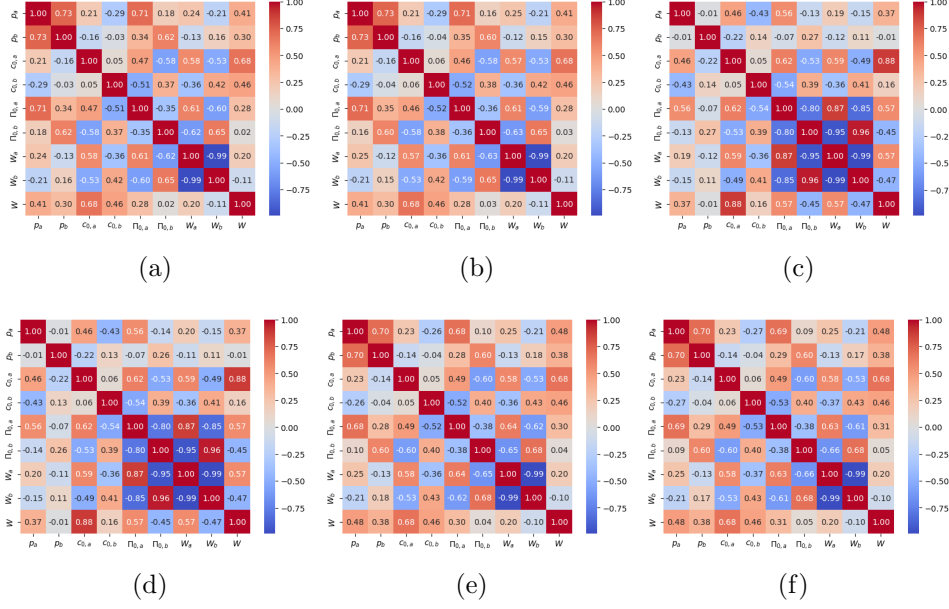


Figure 1.27: Mean correlation matrices among random seeds. Tastes of Consumers follow a bivariate normal distribution (Dist. 1). Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

From the correlations between the lagged investments and the profit, we observe that when most Consumers are indifferent towards the data restriction level, the scenarios with differentiated data restriction levels show the relatively largest correlations. Under scenarios with differentiated data restriction levels, the investment and the profit of Owner a at the same period are more correlated, while under scenarios with the same data restriction levels, the investment at one period and the profit before the period are more correlated. When most Consumers show a positive attitude towards the data restriction level, the correlation between the investment and the profit of the firm under a lower data restriction level becomes much lower than that of the case where most Consumers are indifferent to the data restrictions. This may suggest that when a firm is taking advantage of differentiated data policies, the effectiveness of the investment changes is also higher.

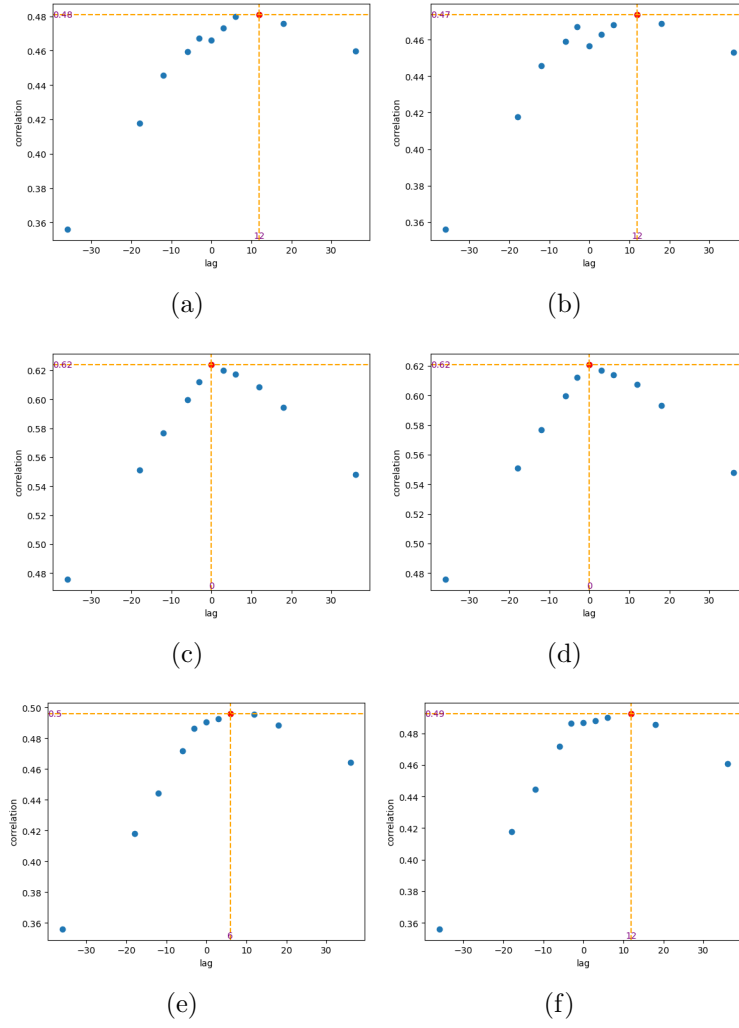


Figure 1.28: Correlation between the lagged investments and the profit Π_a . Negative values of lags show the correlation between the profit of one period and the investment before that period. Tastes of Consumers follow Dist. 1. Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

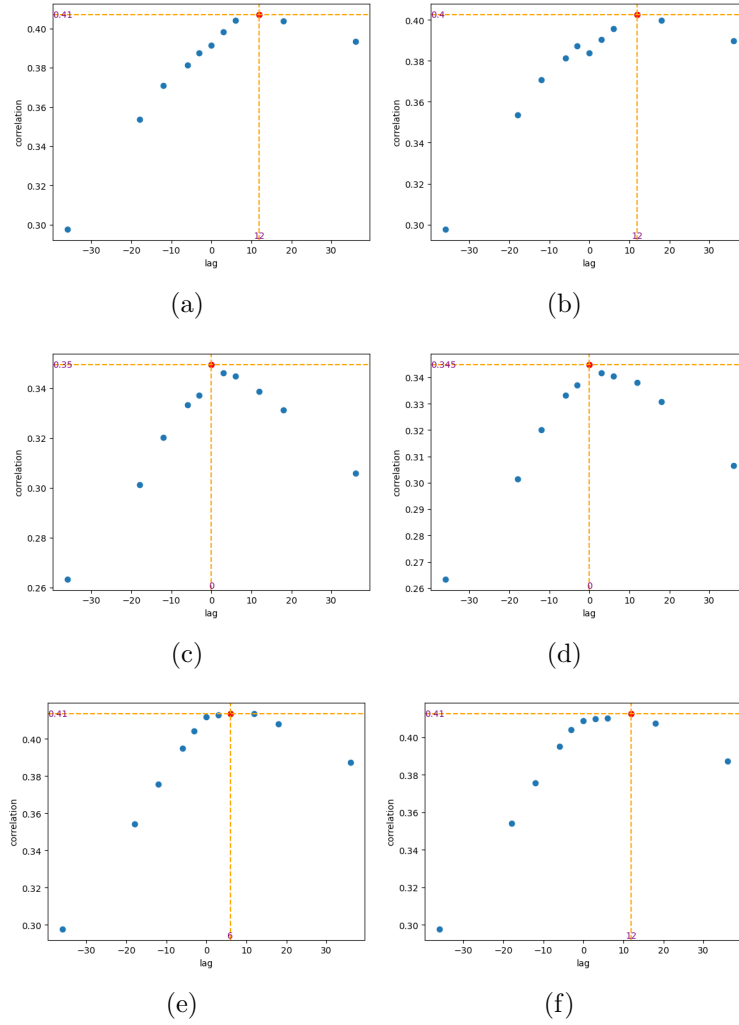


Figure 1.29: Correlation between the lagged investments and the profit Π_a . Negative values of lags show the correlation between the profit of one period and the investment before that period. Tastes of Consumers follow Dist. 3. Sub-figures (a) - (f) represent Scenarios 1' - 6'. $\Delta = 3$, $\psi_{0,a} = \psi_{0,b} = 1$, $\vartheta = 0.01$, $\tau = 6$.

To check the effects of the selection of random seeds, we conduct the Wilcoxon Signed Rank Test in each experiment.

1. Select one certain setup, take the mean under each period under 30 random seeds to build up the “mean time series”.
2. Calculate the first four moments of the “mean time series” to get the reference values (for p_a , p_b , $c_{0,a}$, $c_{0,b}$).

3. Compare each data point from the price and investment time series against the reference values respectively and record the p-values. If a p-value is smaller than 0.05, we reject the hypothesis H_0 (H_0 : “There is no significant difference between the data points and the reference value”).
4. Adjust the parameters and return to the 1st step.

From the test statistics, we find that the selection of random seeds does not make the mean of the price or investment time series significantly different, while for higher moments, it makes a difference, which further shows that randomness can cause very different fluctuations. In different scenarios, the influence of the randomness may also be different (for example, the kurtosis). Moreover, changes in parameters (τ, ϑ) may cause variations in the influence, which may provide the importance of conducting a sensitivity analysis.

Exp	Wilcoxon p-value - Mean	Wilcoxon p-value - Variance	Wilcoxon p-value - Skewness	Wilcoxon p-value - Kurtosis
1	0.98	0.0	0.04	0.24
2	0.98	0.0	0.04	0.26
3	0.97	0.0	0.00	0.03
4	0.98	0.0	0.00	0.03
5	0.98	0.0	0.00	0.00
6	0.98	0.0	0.00	0.00
7	0.76	0.0	0.00	0.19
8	0.76	0.0	0.00	0.19
9	0.95	0.0	0.00	0.00
10	0.95	0.0	0.00	0.00
11	0.75	0.0	0.00	0.00
12	0.75	0.0	0.00	0.00
13	0.98	0.0	0.04	0.24
14	0.98	0.0	0.04	0.26
15	0.97	0.0	0.00	0.03
16	0.98	0.0	0.00	0.03
17	0.98	0.0	0.00	0.00
18	0.98	0.0	0.00	0.00

Figure 1.30: P-value of Wilcoxon ranking test of moments of p_a in each experiment.

Exp	Wilcoxon p-value - Mean	Wilcoxon p-value - Variance	Wilcoxon p-value - Skewness	Wilcoxon p-value - Kurtosis
1	0.20	0.00	0.98	0.07
2	0.20	0.00	0.97	0.08
3	0.08	0.00	0.00	0.79
4	0.08	0.00	0.00	1.00
5	0.18	0.00	0.00	0.00
6	0.18	0.00	0.00	0.00
7	0.66	0.24	0.00	0.00
8	0.66	0.24	0.00	0.00
9	0.66	0.00	0.00	0.00
10	0.66	0.00	0.00	0.00
11	0.66	0.50	0.00	0.00
12	0.66	0.50	0.00	0.00
13	0.20	0.00	0.98	0.07
14	0.20	0.00	0.97	0.08
15	0.08	0.00	0.00	0.79
16	0.08	0.00	0.00	1.00
17	0.18	0.00	0.00	0.00
18	0.18	0.00	0.00	0.00

Figure 1.31: P-value of Wilcoxon ranking test of moments of p_b in each experiment.

Exp	Wilcoxon p-value - Mean	Wilcoxon p-value - Variance	Wilcoxon p-value - Skewness	Wilcoxon p-value - Kurtosis
1	0.43	0.0	0.0	0.00
2	0.43	0.0	0.0	0.00
3	0.43	0.0	0.0	0.00
4	0.43	0.0	0.0	0.00
5	0.43	0.0	0.0	0.00
6	0.43	0.0	0.0	0.00
7	0.97	0.0	0.0	0.66
8	0.97	0.0	0.0	0.66
9	0.97	0.0	0.0	0.66
10	0.97	0.0	0.0	0.66
11	0.97	0.0	0.0	0.66
12	0.97	0.0	0.0	0.66
13	0.43	0.0	0.0	0.00
14	0.43	0.0	0.0	0.00
15	0.43	0.0	0.0	0.00
16	0.43	0.0	0.0	0.00
17	0.43	0.0	0.0	0.00
18	0.43	0.0	0.0	0.00

Figure 1.32: P-value of Wilcoxon ranking test of moments of $c_{0,a}$ in each experiment.

Exp	Wilcoxon p-value - Mean	Wilcoxon p-value - Variance	Wilcoxon p-value - Skewness	Wilcoxon p-value - Kurtosis
1	0.44	0.0	0.00	0.00
2	0.44	0.0	0.35	0.00
3	0.44	0.0	0.00	0.00
4	0.44	0.0	0.35	0.00
5	0.44	0.0	0.00	0.00
6	0.44	0.0	0.35	0.00
7	1.00	0.0	0.00	0.70
8	1.00	0.0	0.00	0.07
9	1.00	0.0	0.00	0.70
10	1.00	0.0	0.00	0.07
11	1.00	0.0	0.00	0.70
12	1.00	0.0	0.00	0.07
13	0.44	0.0	0.00	0.00
14	0.44	0.0	0.35	0.00
15	0.44	0.0	0.00	0.00
16	0.44	0.0	0.35	0.00
17	0.44	0.0	0.00	0.00
18	0.44	0.0	0.35	0.00

Figure 1.33: P-value of Wilcoxon ranking test of moments of $c_{0,b}$ in each experiment.

Sensitivity analysis

By adjusting each parameter, we observe changes in different extents and directions. In each specific case, we calculate a ratio r_a which represents the frequency of Owner a having a larger profit than (or the same profit as) Owner b . The formula is shown as

$$r_a = \frac{1}{n_r} \sum_{\iota=1}^{n_r} \frac{T_{a,\iota}}{T_{total}}$$

where n_r represents the total number of random seeds, $T_{a,\iota}$ represents the number of times Owner a has a profit at least as large as that of Owner b under random seed ι and T_{total} represents the total number of periods observed. A larger r_a indicates that Owner a comes out more frequently as the winner in the market.

Setting	Δ	ψ_0	ϑ	τ	E (x_j)	SD (x_j)	E (y_j)	SD (y_j)	p_{n_V} $- p_{n_1}$	
1		3	1.0	0.01	6	0.8	0.05	0.0	0.2	10
2		3	1.0	0.01	3	0.8	0.05	0.0	0.2	10
3		3	1.0	0.10	6	0.8	0.05	0.0	0.2	10
4		3	0.1	0.01	6	0.8	0.05	0.0	0.2	10
5		3	1.0	0.01	6	0.5	0.05	0.5	0.2	10
6		3	1.0	0.01	6	0.8	0.05	0.0	0.2	4
7		3	1.0	0.01	6	0.8	0.05	0.0	0.5	4

Figure 1.34: Settings for sensitivity analysis

The following results show the sensitivity of adjusting parameters:¹⁴

- Distribution of the Consumers' tastes: Comparing Settings 1 and 5 as well as Settings 6 and 7, we find that when most Consumers have a positive attitude towards a higher data restriction level, the Owner under a higher data restriction level tends to get more profit than its opponent. Besides, if there is more variance in Consumers' tastes in data restrictions, the effect of heterogeneity in the data restriction level to r_a may be slightly higher and the effect of randomness may be smaller, i.e. there is less variation across the mean of the simulated time series.
- τ : Comparing Settings 1 and 2, we find that as τ decreases from 6 to 3, r_a decreases when there are asymmetric data restriction levels, and the standard deviation of the mean values of simulated time series largely increases. It implies that under a market with most Consumers indifferent towards the data restriction level, through frequent updates, the Owner under a higher data restriction level can have more probability of getting a higher profit, yet the realizations can be more volatile.
- ϑ : Comparing Settings 1 and 3, we find that as ϑ increases from 0.01 to 0.1, we do not observe significant changes, thus, we do not have enough evidence to explain the influence of the update threshold (at least under the range from 0.01 to 0.1).
- ψ_0 : Comparing Settings 1 and 4, we find that as ψ_0 decreases from 1 to 0.1, we observe that in general, r_a and the mean of the mean values of simulated time series has a large increase, while the standard deviation of the mean values of simulated time series significantly decreases. It implies that if the amount of random updates across the periods is small, the effect of heterogeneity in the data restriction level may be large, and with less

¹⁴Here we also list the range of price alternatives in the market analysis ($p_R = p_{n_V} - p_{n_1}$) as a parameter.

variation, the Owner under a lower data restriction level always tends to be the winner in the market.

- p_R : Comparing Settings 1 and 6, we find that as the range of the price alternatives decreases, there is a slight increase in r_a , mean and variance of the mean values of simulated time series, which implies that when Owners adjust the prices within a smaller range, the Owner under a lower data restriction level may be slightly more likely earn more profit with slightly more variations.

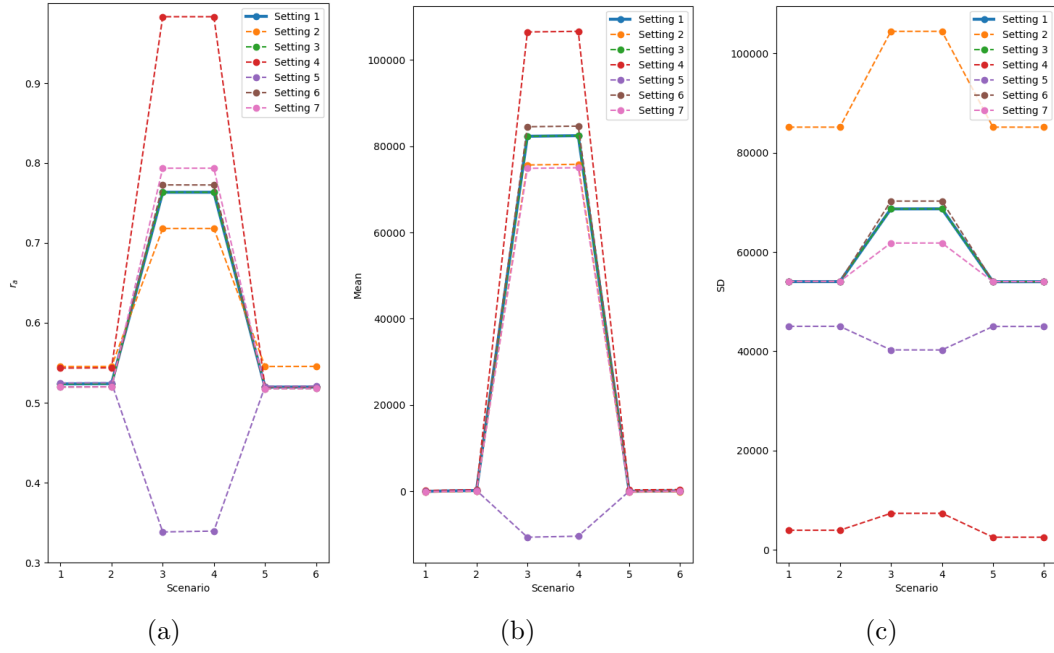


Figure 1.35: Statistics for sensitivity analysis. Sub-figures (a) - (c) represent r_a , the mean and standard deviation of the mean values of all simulated random series under different random seeds, respectively.

1.6 Conclusion

We consider this paper a typical agent-based approach involving the Data Value Chain, the software market, and the possible implications that contribute to the understanding of the interactions between the agents in the economy. Based on the foundation of product differentiation, we follow the principle of the IO literature and show the market performance by running simulations under a set of scenarios. Particularly, this approach could be more convenient than analytical

approaches when there is a complicated joint distribution of consumers' tastes, and the distribution indeed has a large impact on the firm competition.

We show that the customers' attitudes towards the products and their actions based on their utility levels could play a fundamental role in the market of software products. Our dynamic approach shows that from the perspective of a global-maximizer who only cares about social welfare, when the data restriction level is higher, there would be a loss in social welfare. This clarifies the reason why the global-maximizer is motivated to suggest to the local governments that they should set an agreement and try to decrease the data restriction level in both regions. If the governments cooperate in setting up the data restrictions and decrease the restriction costs, generally, the customers could be better off regarding their welfare. There could be more freedom for customers to select the products, and they would be more likely to purchase the products, which increases the profits of the firms.

From the perspective of the software product Owners, i.e. firms that own the copyright (and the license) of the product, the way of raising their profits lies in analyzing the consumer preferences in the market, observing the price that competitors offer and making strategies, increasing the research investments to increase their product's functionality, and conducting market research for more information. The results of the sensitivity analysis further point out the importance of consumer preferences and the firms' update frequency.

From the methodological view, the dynamic approach, compared with the static approach, brings more essential information and economic insights. If both Owners do not exit the market, differences in data restriction levels across regions can bring both Owners profits (the differentiation of products can be beneficial for both firms). The market may not reach an equilibrium under the settings of heuristics and both Owners may be better off after a large number of periods, even if they are under different levels of data restrictions. Furthermore, introducing investment dynamics seems to bring more possibilities of profit for the Owner under a higher restriction level. More effective investment updates bring opportunities to the Owners.

In future research, the model in this paper could be developed in different directions. One possible extension is to consider the case of monopolistic competition with more firms in the market and to check the market power across multiple periods. Regarding the model structure, the relationship between the Owners and Developers can be designed with more details, and more economic facts could be explained on the side of the Developers. The financing structure of the Owners may also be important when the business models of various software products are taken into account. Ideally, the gap between the theoretical model and the real-world data will also be needed for meticulous research: To bridge the gap

between the theoretical model and reality, we could pick two typical firms located in two different regions.¹⁵ The Digital Service Restrictiveness Index (OECD 2022) could be considered one index for the data restriction level, and from the financials of the two companies, we could thereby collect Research and Development, Revenue, Gross Profit, etc. For the step of parameter estimation, we could consider the Bayesian methods and distinguish between the prior and the posterior distributions.

Code availability

To support transparency and reproducibility, the code underlying the results presented in this study is publicly accessible at https://github.com/ZHongli626/Heterog_Consumers_DVC.

¹⁵For instance, Zoom in the US and Tencent Meeting in China.

1.7 Appendix

1.7.1 Multi-stage game

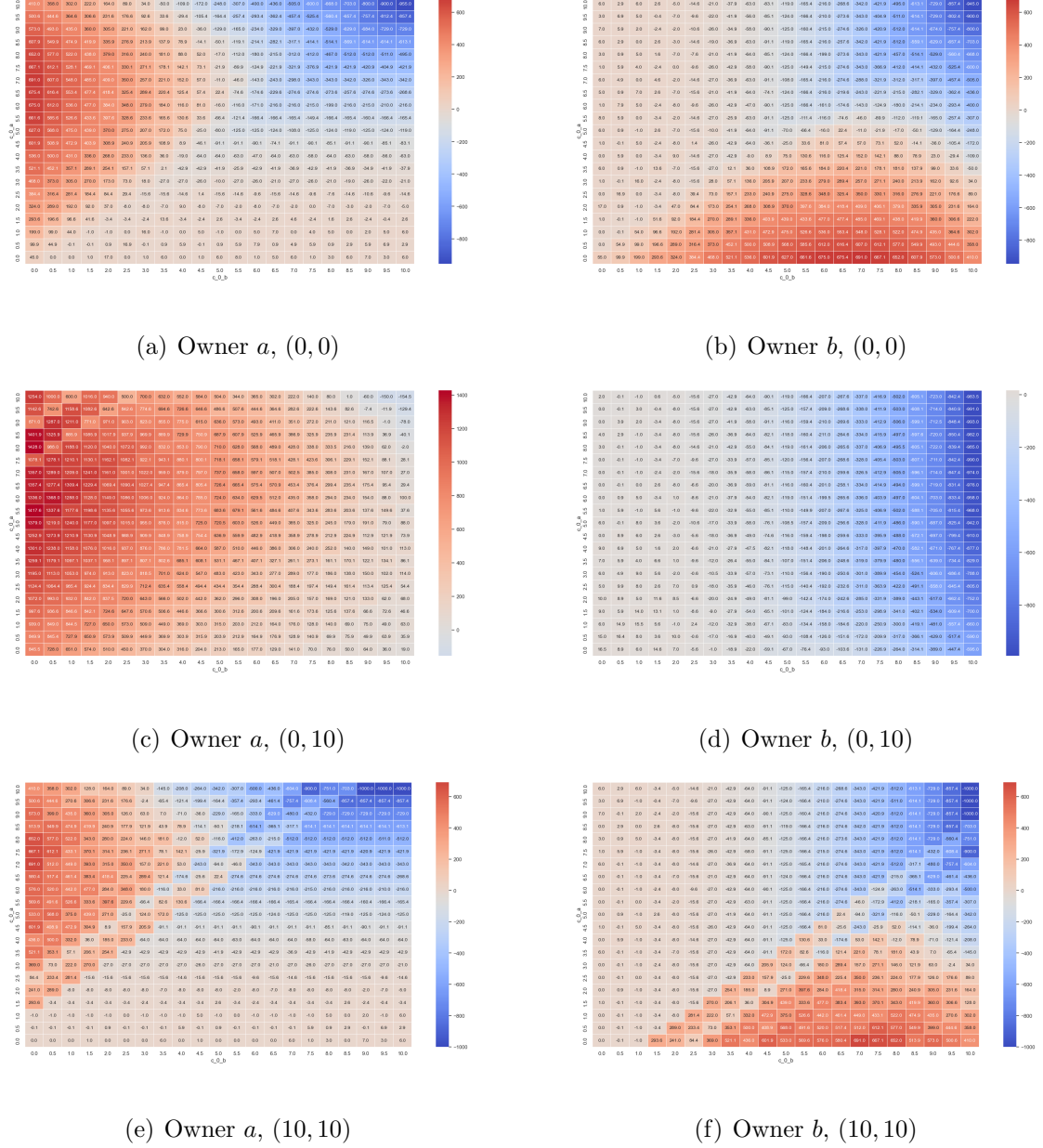
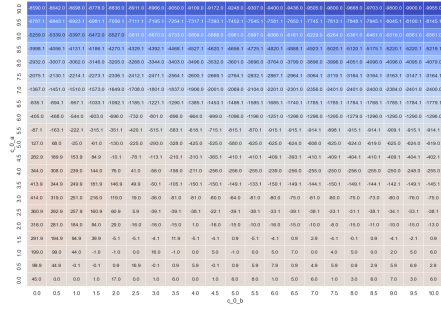
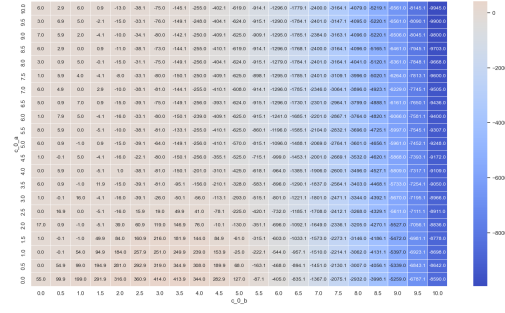


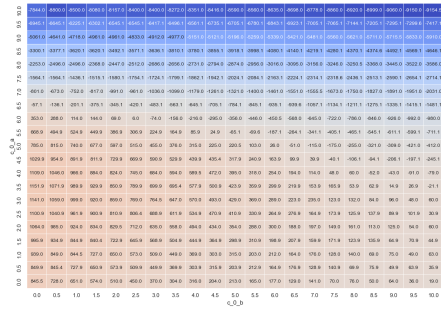
Figure 1.36: Profits of Owners based on equilibrium prices under different investment combinations. Numbers show Π_a^* and Π_b^* for each scenario $(c_{res,a}, c_{res,b}) = (0,0), (0,10), (10,10)$, respectively, with $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.



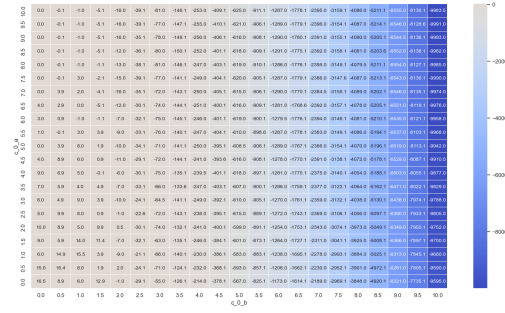
(a) Owner a , $(0,0)$



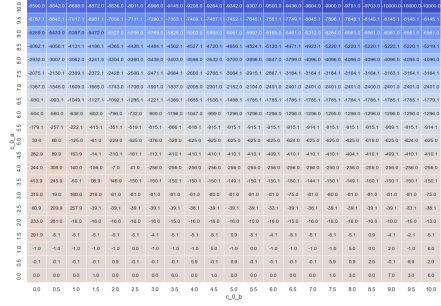
(b) Owner b , $(0,0)$



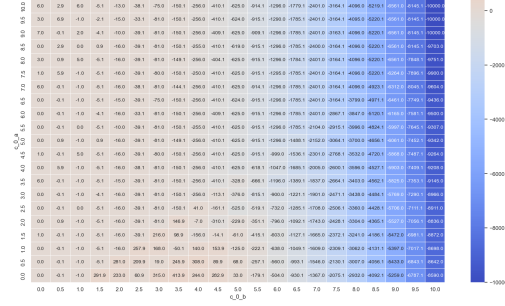
(c) Owner a , $(0,10)$



(d) Owner b , $(0,10)$

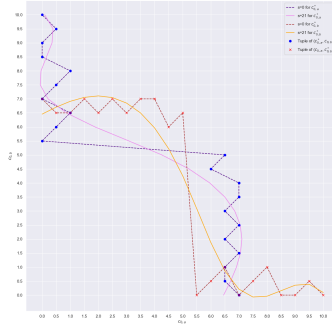


(e) Owner a , $(10,10)$

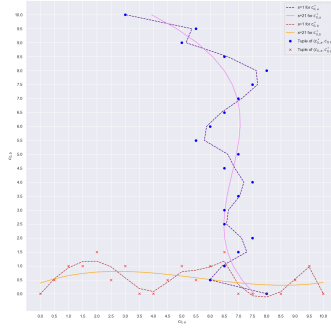


(f) Owner b , $(10,10)$

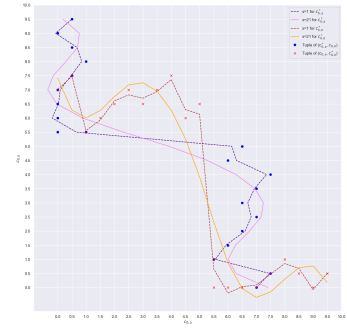
Figure 1.37: Profits of Owners based on equilibrium prices under different investment combinations. Numbers show Π_a^* and Π_b^* for each scenario $(c_{res,a}, c_{res,b}) = (0,0), (0,10), (10,10)$, respectively, with $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$.



(a) (0,0)

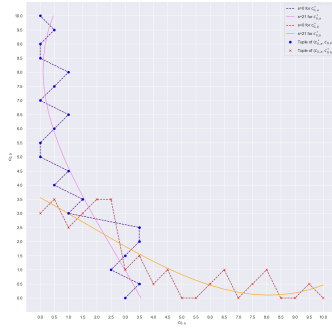


(b) (0,10)

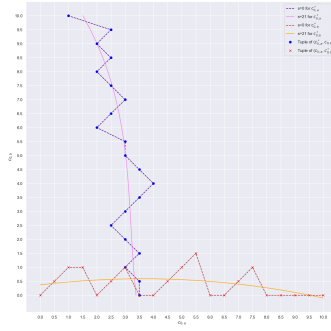


(c) (10,10)

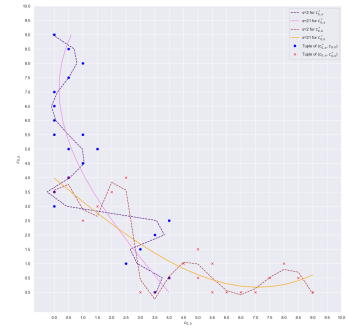
Figure 1.38: Best responses of Owners under different investment combinations. Sub-figures (a)–(c) show p_a^* and p_b^* for $(c_{res,a}, c_{res,b})$ equal to (0,0), (0,10), and (10,10), respectively, with $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.



(a) (0,0)

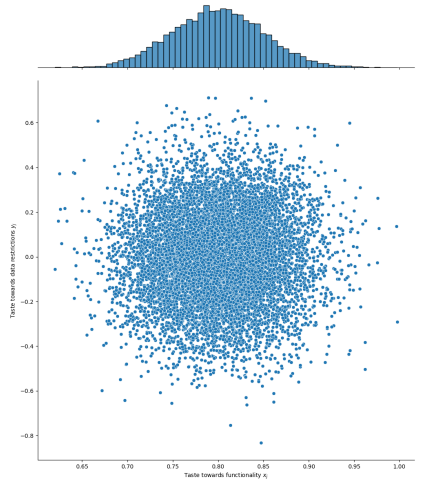


(b) (0,10)

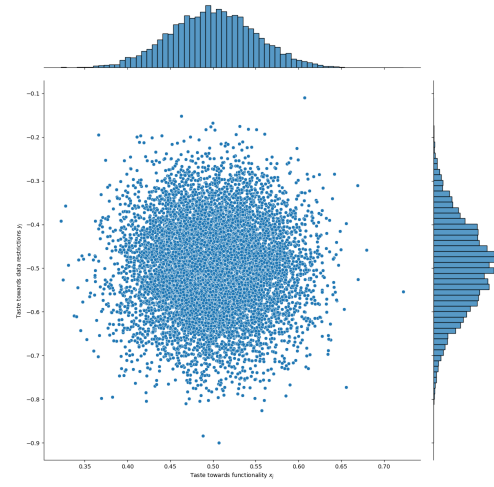


(c) (10,10)

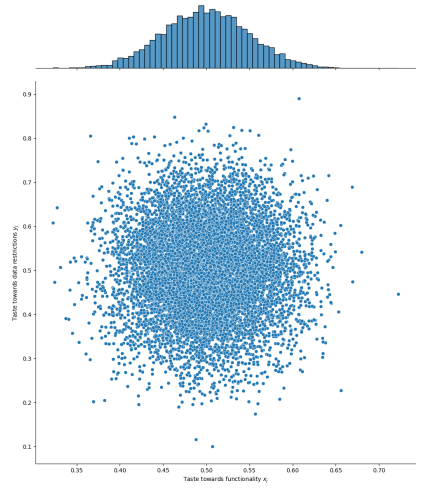
Figure 1.39: Best responses of Owners under different investment combinations. Sub-figures (a)–(c) show p_a^* and p_b^* for $(c_{res,a}, c_{res,b})$ equal to (0,0), (0,10), and (10,10), respectively, with $c_{total,i} = c_{0,i}^4 + m_i c_{res,i}$.



(a) Distribution 1

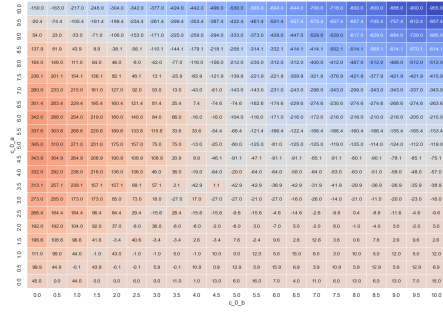


(b) Distribution 2

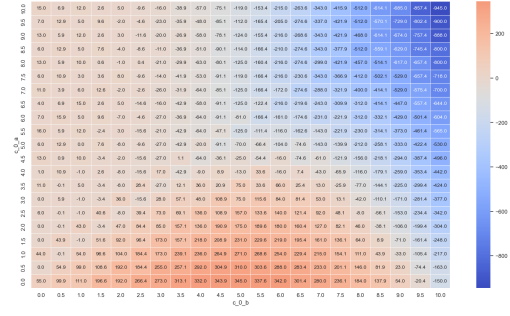


(c) Distribution 3

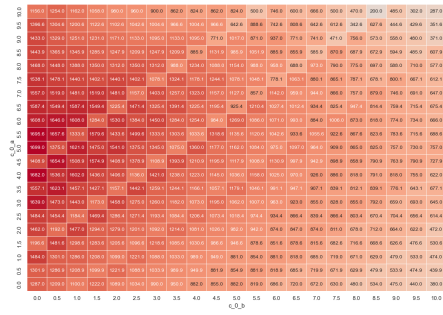
Figure 1.40: Example samples for different distributions ($n = 10,000$).



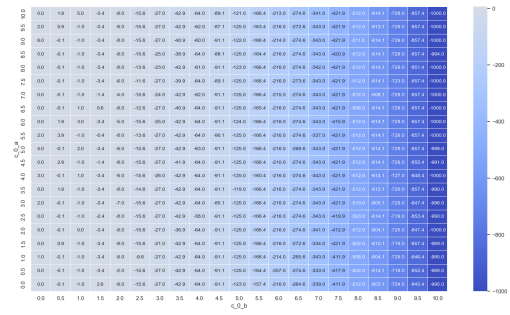
(a) Owner a , $(0,0)$



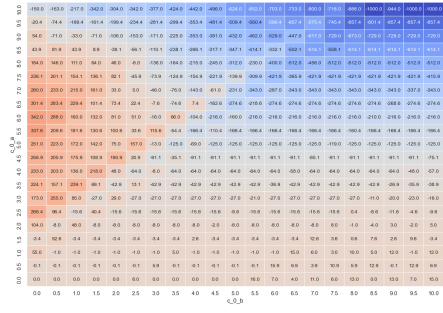
(b) Owner b , $(0,0)$



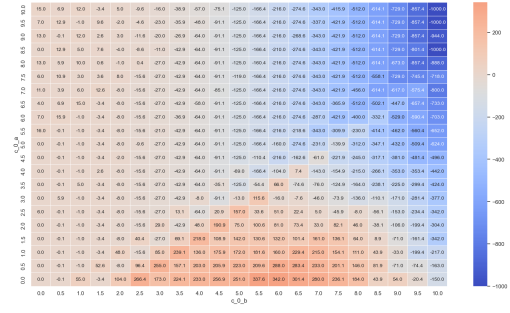
(c) Owner a , $(0,10)$



(d) Owner b , $(0,10)$

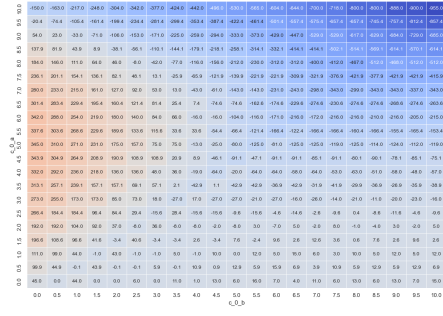


(e) Owner a , $(10,10)$

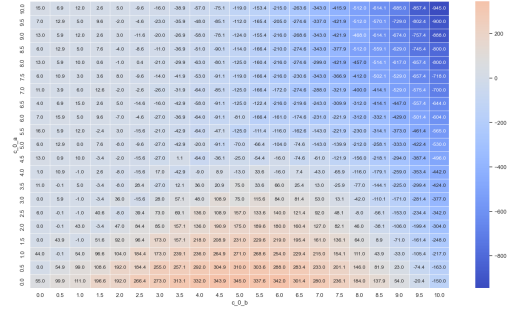


(f) Owner b , $(10,10)$

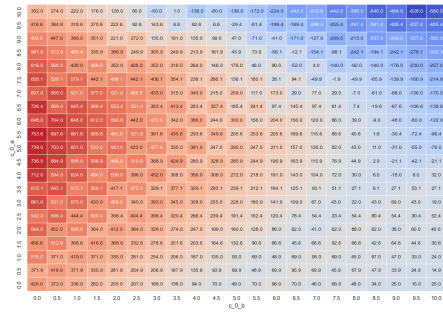
Figure 1.4f: Profits of Owners based on equilibrium prices under different investment combinations (Distribution 2). Numbers show Π_a^* and Π_b^* for each scenario with $(c_{res,a}, c_{res,b})$ equal to $(0,0)$, $(0,10)$, and $(10,10)$, respectively, and $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.



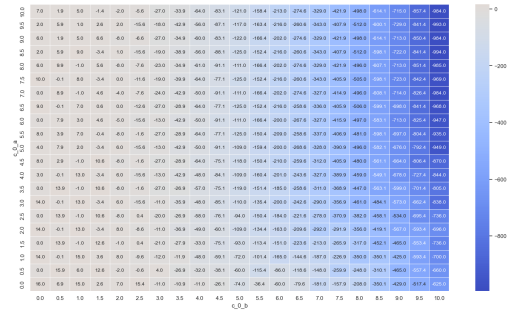
(a) Owner a , $(0,0)$



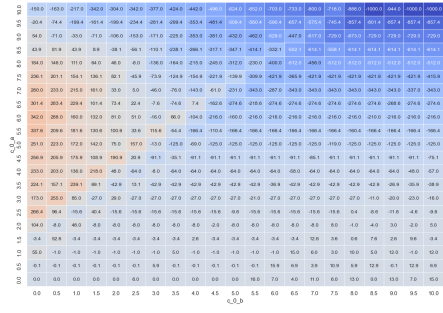
(b) Owner b , $(0,0)$



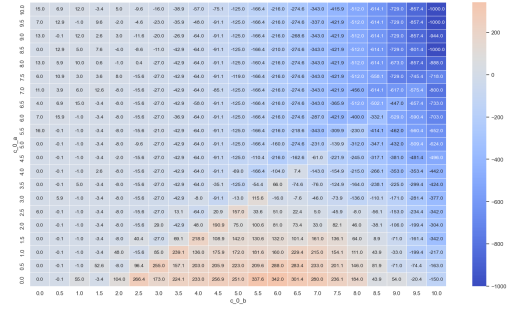
(c) Owner a , $(0,10)$



(d) Owner b , $(0,10)$



(e) Owner a , $(10,10)$



(f) Owner b , $(10,10)$

Figure 1.42: Profits of Owners based on equilibrium prices under different investment combinations (Distribution 3). Numbers show Π_a^* and Π_b^* for each scenario with $(c_{res,a}, c_{res,b})$ equal to $(0,0)$, $(0,10)$, and $(10,10)$, respectively, and $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.

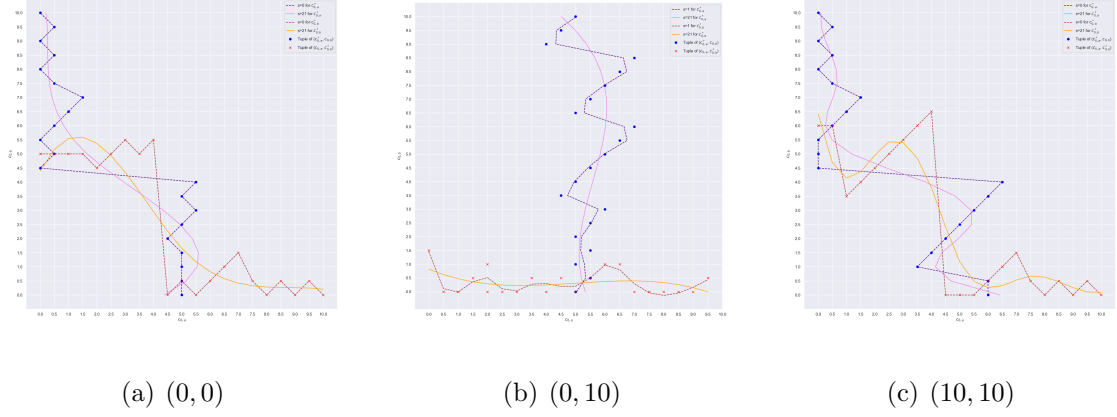


Figure 1.43: Best responses of Owners under different investment combinations for Distribution 2. Sub-figures (a)–(c) show p_a^* and p_b^* where $(c_{res,a}, c_{res,b})$ equals $(0,0)$, $(0,10)$, and $(10,10)$, respectively, with $c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$.

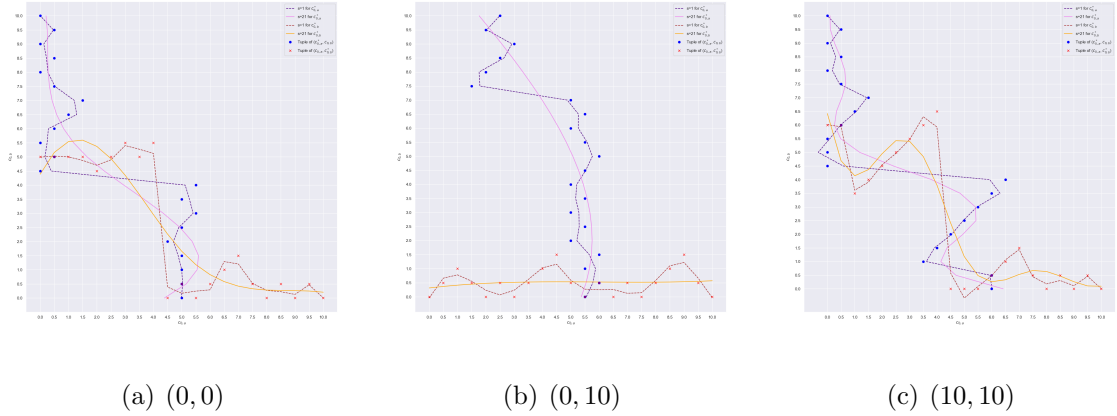
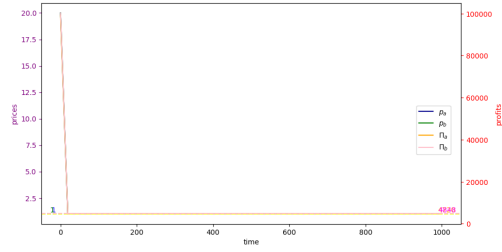


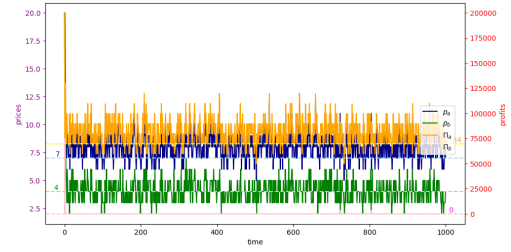
Figure 1.44: The best responses of Owners under different investment combinations following Distribution 3. Sub-figures (a)–(c) show p_a^* and p_b^* when $(c_{res,a}, c_{res,b})$ equals $(0,0)$, $(0,10)$, and $(10,10)$, respectively ($c_{total,i} = c_{0,i}^3 + m_i c_{res,i}$).

1.7.2 Agent-based computations

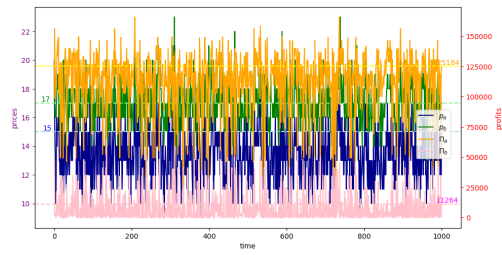
Simulations on price and investment updates



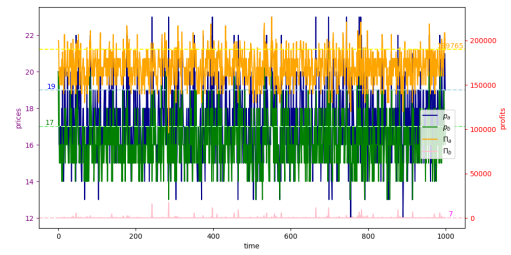
(a) Scenario 1'



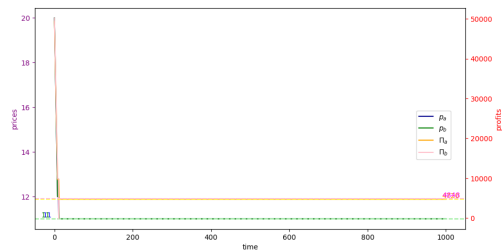
(b) Scenario 2'



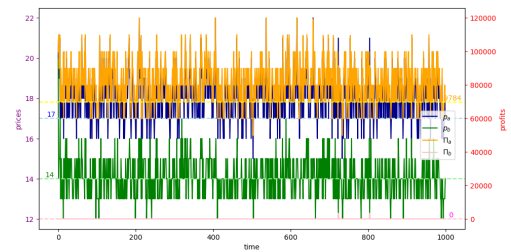
(c) Scenario 3'



(d) Scenario 4'

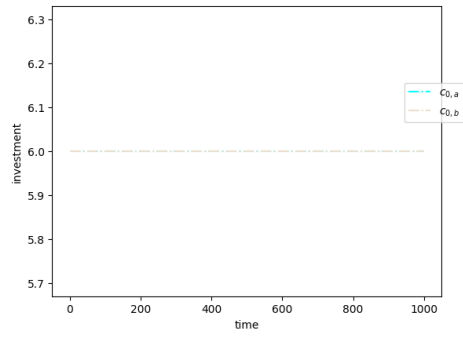


(e) Scenario 5'

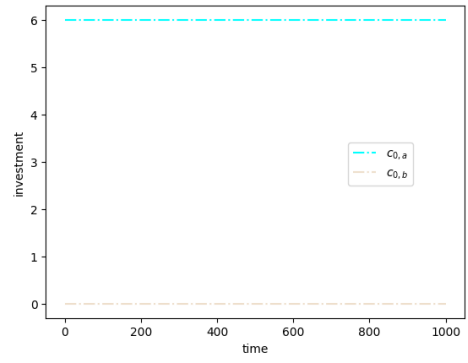


(f) Scenario 6'

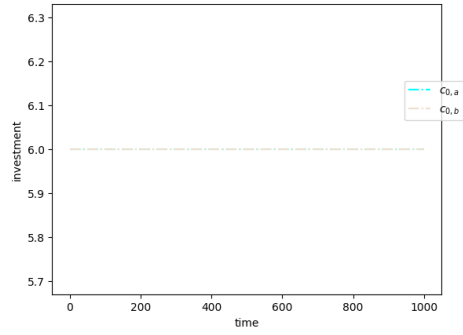
Figure 1.45: Examples of dynamics under Scenarios 1'–6' with price updates and constant investments.



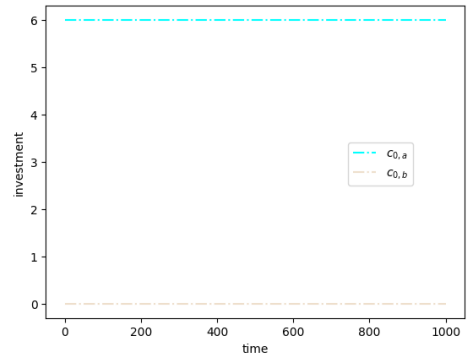
(a) Scenario 1'



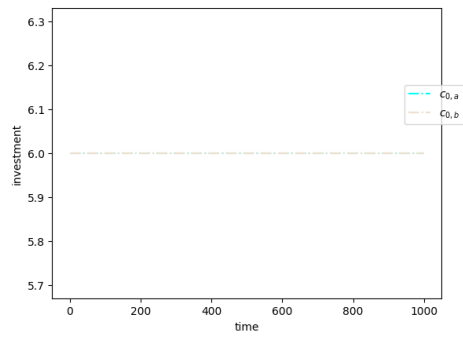
(b) Scenario 2'



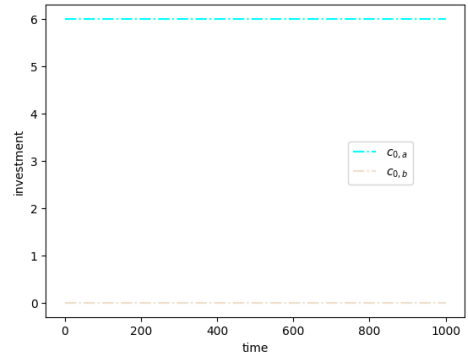
(c) Scenario 3'



(d) Scenario 4'

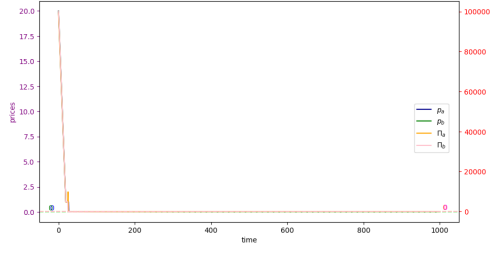


(e) Scenario 5'

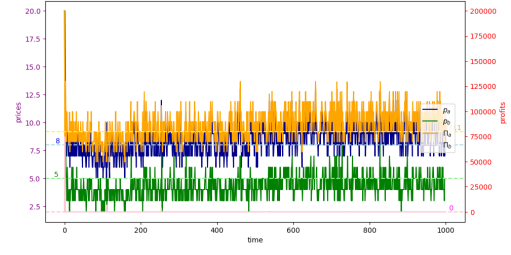


(f) Scenario 6'

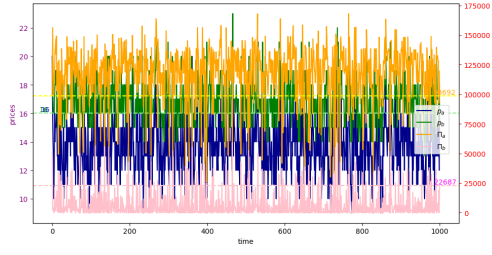
Figure 1.46: Examples of dynamics under Scenarios 1'–6' with price updates and constant investments (continued).



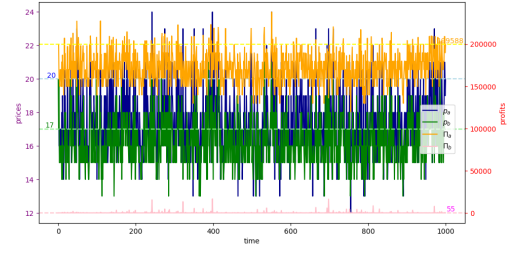
(a) Scenario 1'



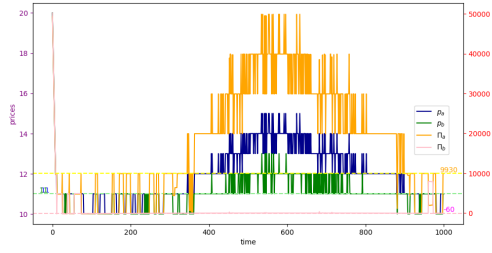
(b) Scenario 2'



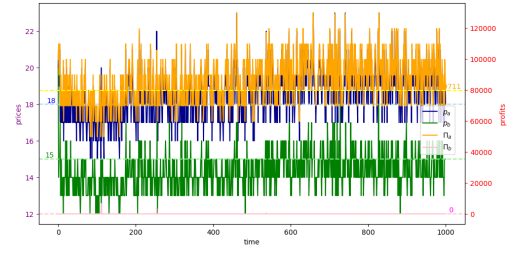
(c) Scenario 3'



(d) Scenario 4'

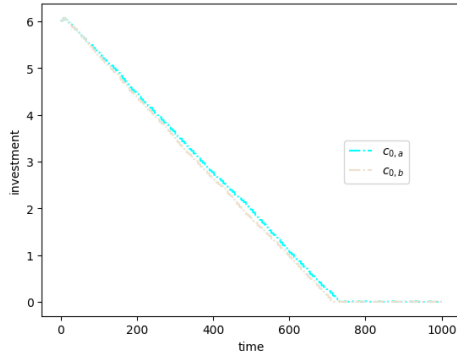


(e) Scenario 5'

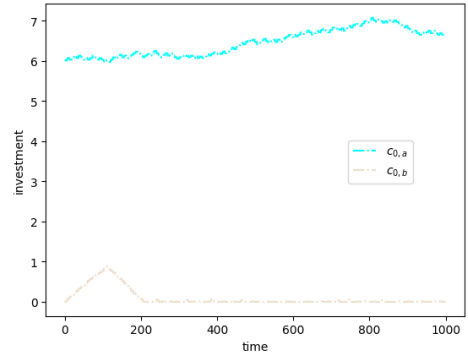


(f) Scenario 6'

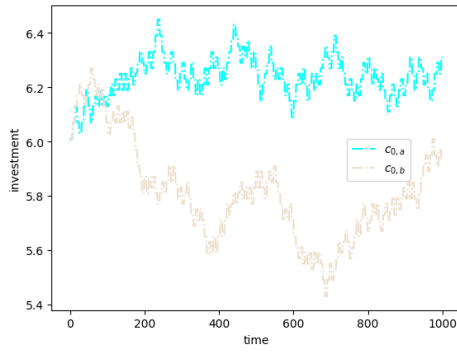
Figure 1.47: Examples of dynamics under Scenarios 1'–6'. $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 1$.



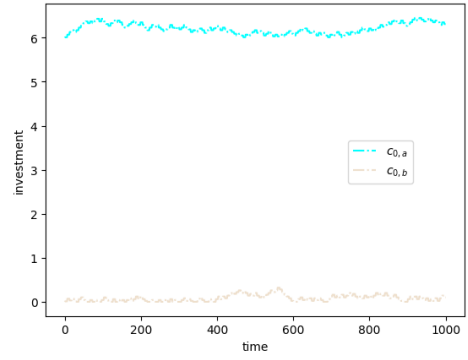
(a) Scenario 1'



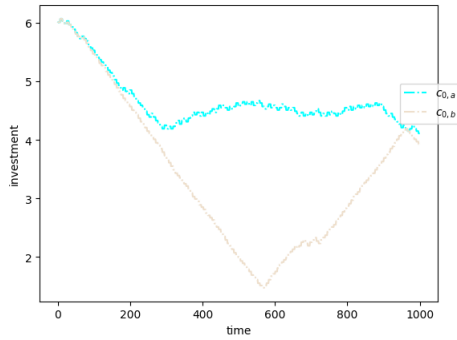
(b) Scenario 2'



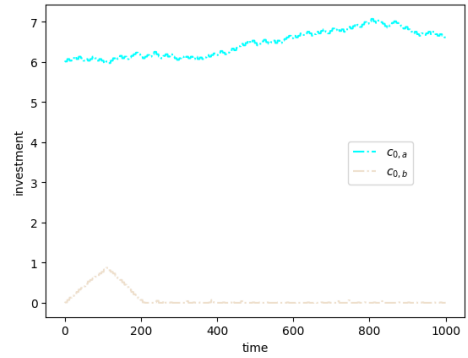
(c) Scenario 3'



(d) Scenario 4'

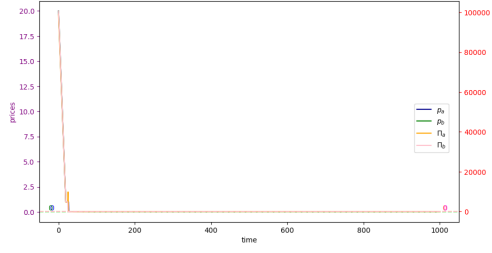


(e) Scenario 5'

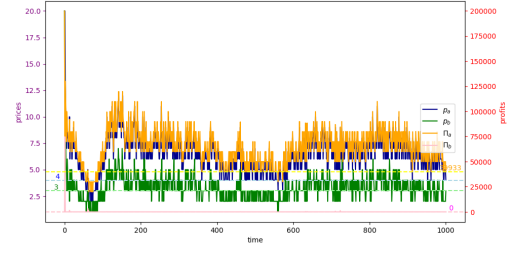


(f) Scenario 6'

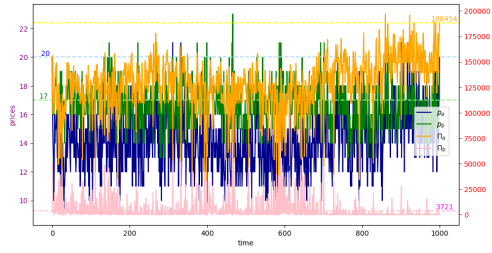
Figure 1.48: Examples of dynamics under Scenarios 1'–6' (continued). $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 1$.



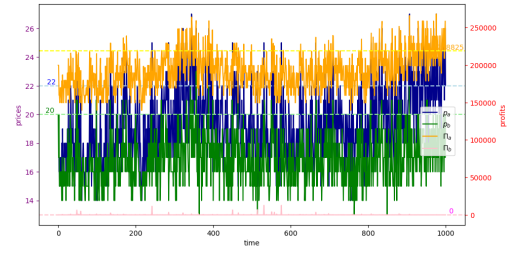
(a) Scenario 1'



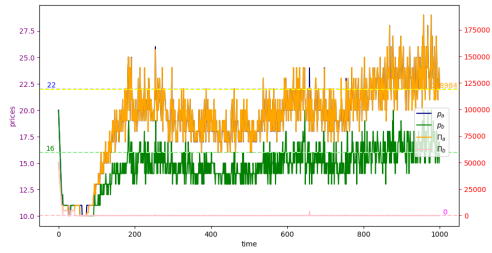
(b) Scenario 2'



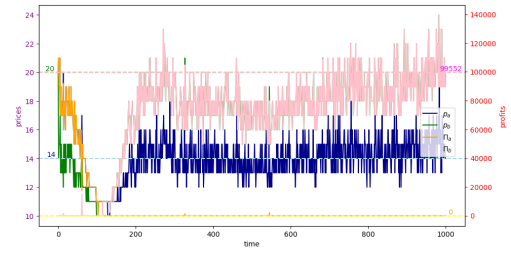
(c) Scenario 3'



(d) Scenario 4'

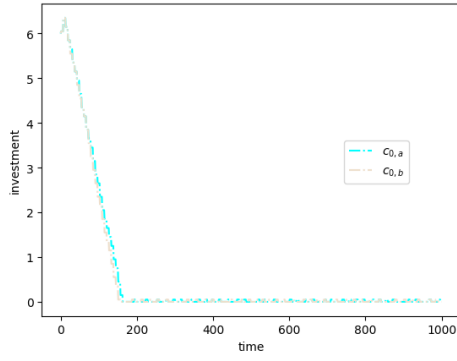


(e) Scenario 5'

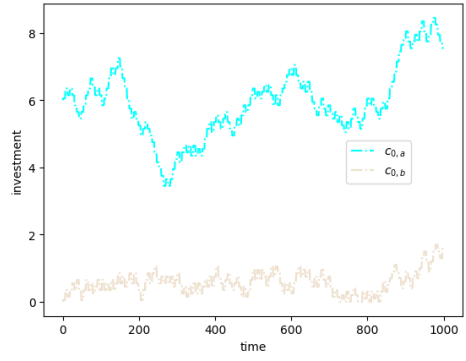


(f) Scenario 6'

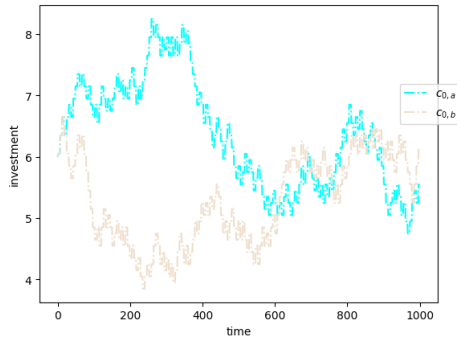
Figure 1.49: Examples of dynamics under Scenarios 1'–6'. $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 1$.



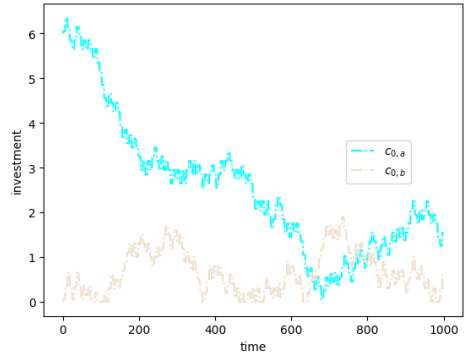
(a) Scenario 1'



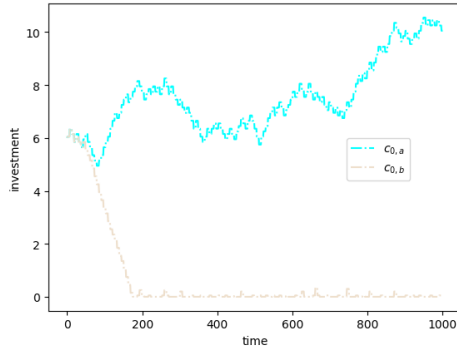
(b) Scenario 2'



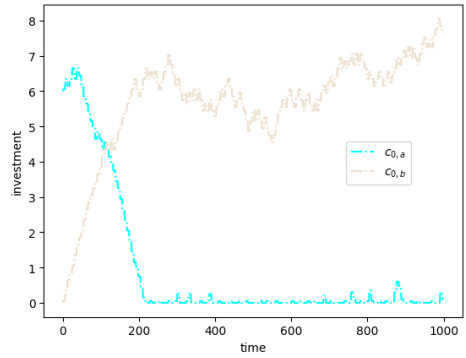
(c) Scenario 3'



(d) Scenario 4'

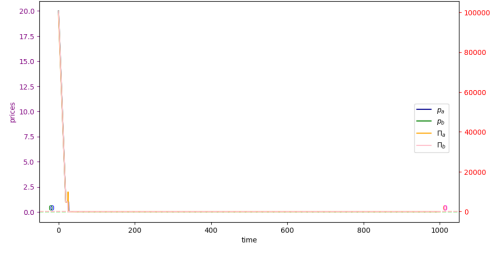


(e) Scenario 5'

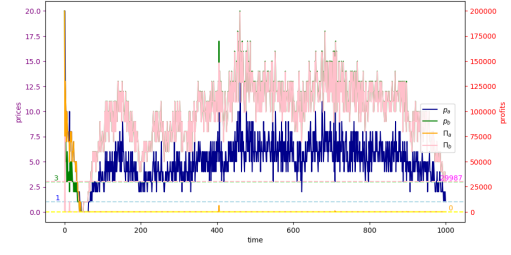


(f) Scenario 6'

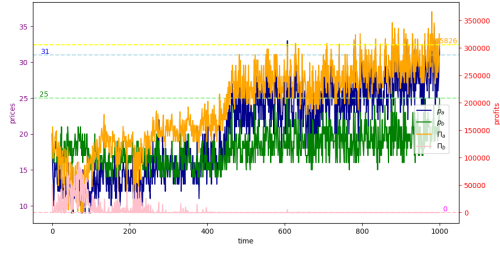
Figure 1.50: Examples of dynamics under Scenarios 1'–6' (continued). $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 1$.



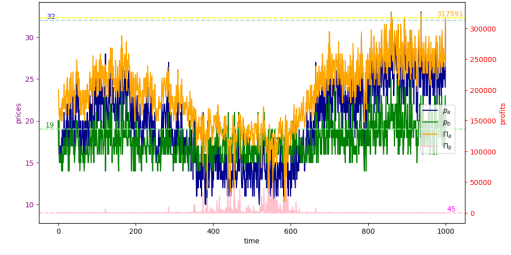
(a) Scenario 1'



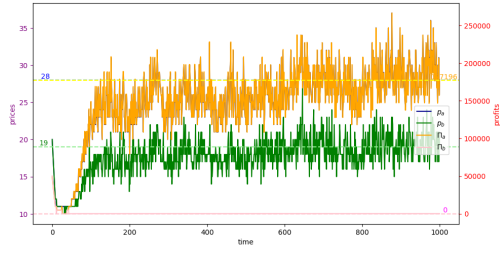
(b) Scenario 2'



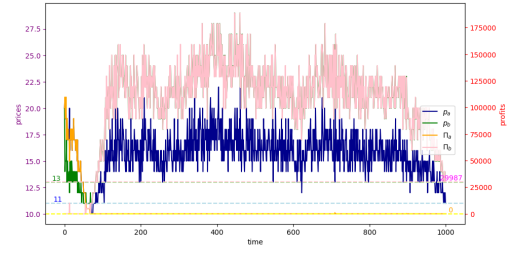
(c) Scenario 3'



(d) Scenario 4'

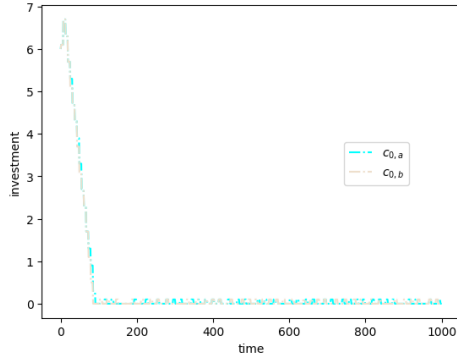


(e) Scenario 5'

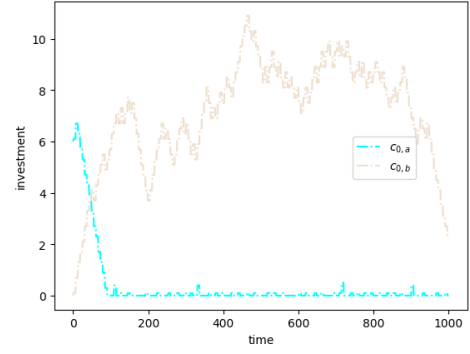


(f) Scenario 6'

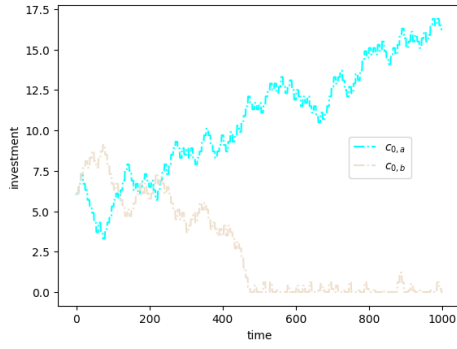
Figure 1.51: Examples of dynamics under Scenarios 1'–6'. $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 1$.



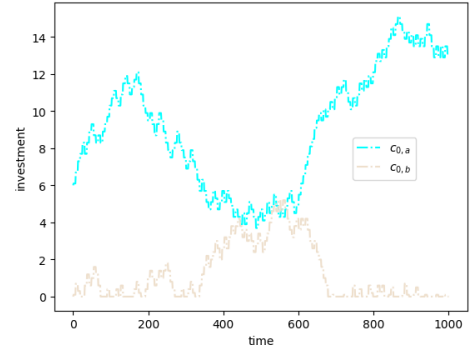
(a) Scenario 1'



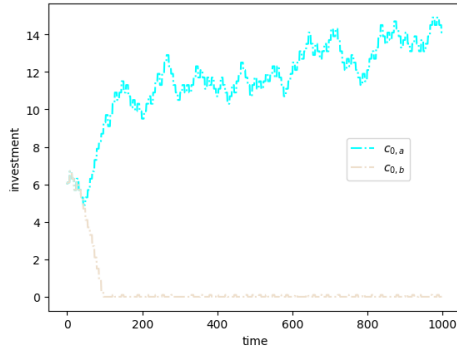
(b) Scenario 2'



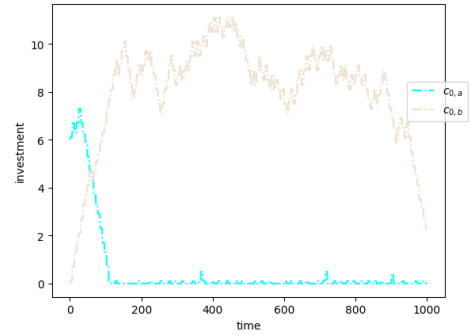
(c) Scenario 3'



(d) Scenario 4'

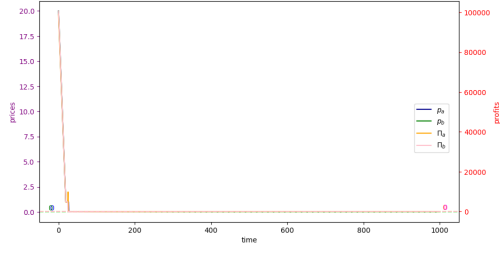


(e) Scenario 5'

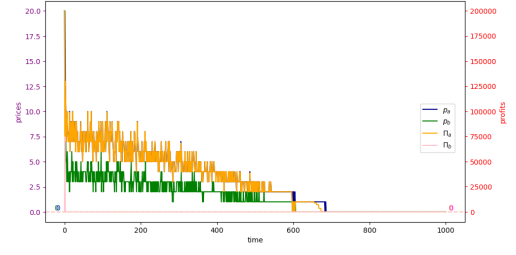


(f) Scenario 6'

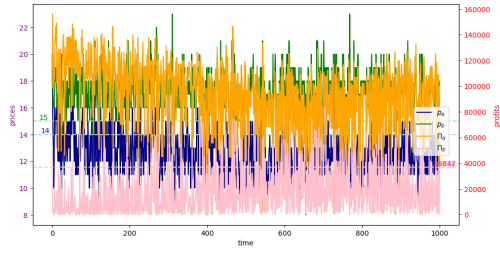
Figure 1.52: Examples of dynamics under Scenarios 1'–6' (continued). $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 1$.



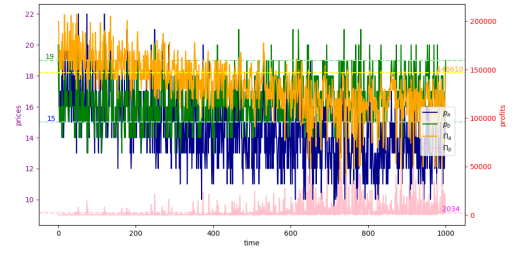
(a) Scenario 1'



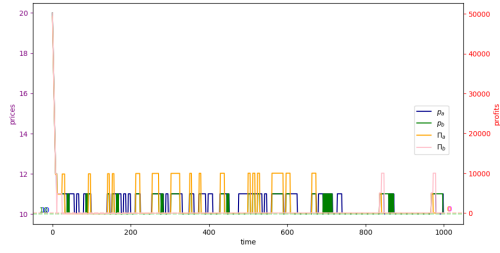
(b) Scenario 2'



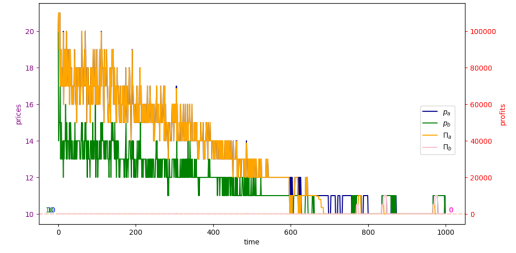
(c) Scenario 3'



(d) Scenario 4'

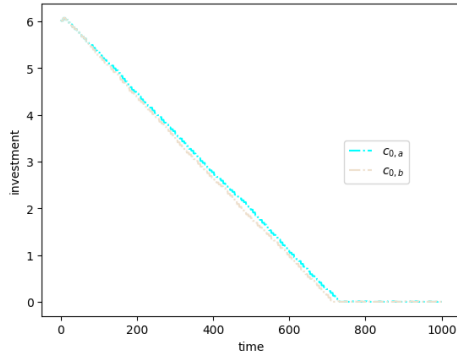


(e) Scenario 5'

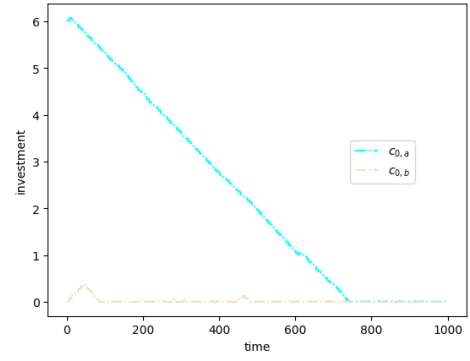


(f) Scenario 6'

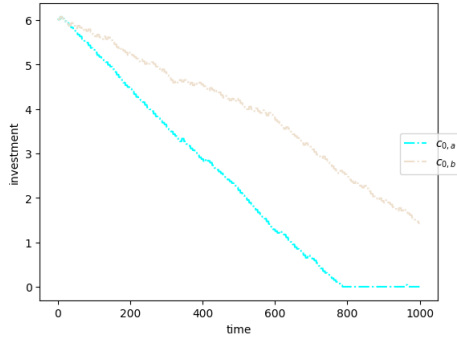
Figure 1.53: Examples of dynamics under Scenarios 1'–6'. $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 1.5$.



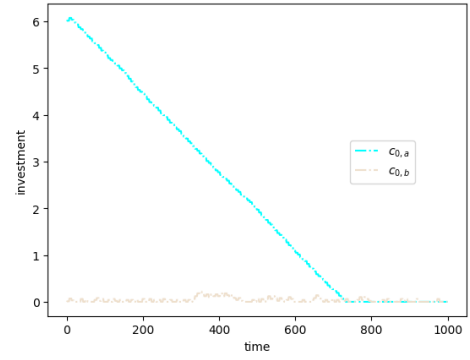
(a) Scenario 1'



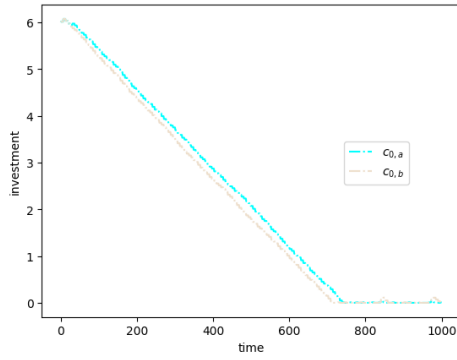
(b) Scenario 2'



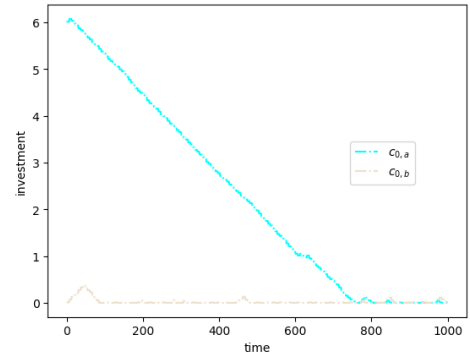
(c) Scenario 3'



(d) Scenario 4'

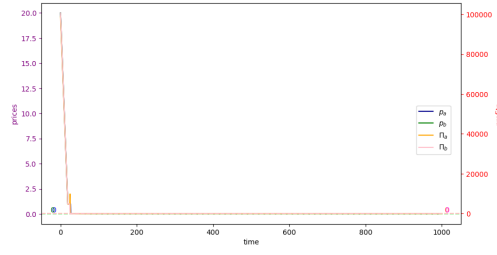


(e) Scenario 5'

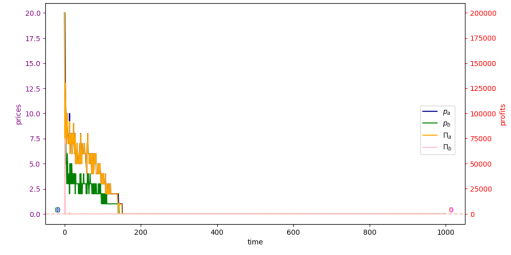


(f) Scenario 6'

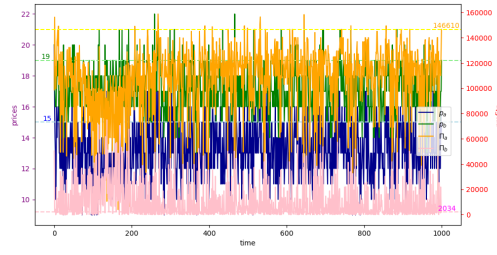
Figure 1.54: Examples of dynamics under Scenarios 1'–6' (continued). $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 1.5$.



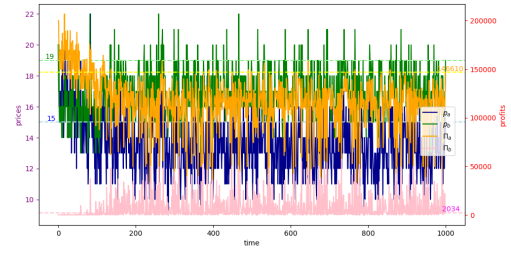
(a) Scenario 1'



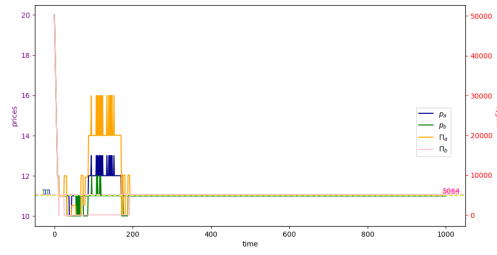
(b) Scenario 2'



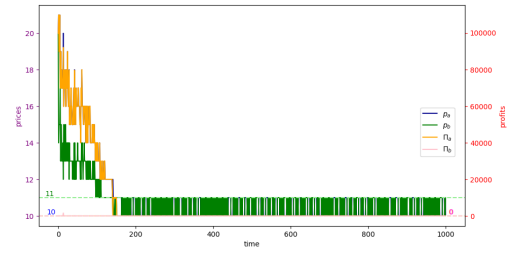
(c) Scenario 3'



(d) Scenario 4'

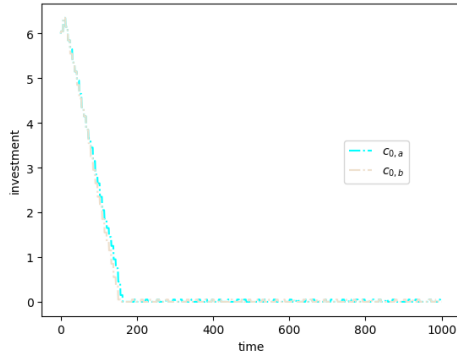


(e) Scenario 5'

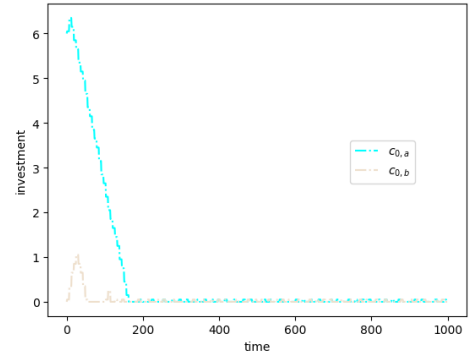


(f) Scenario 6'

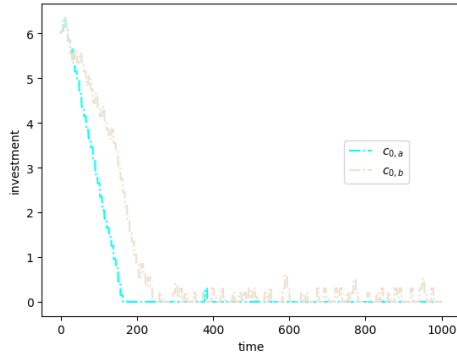
Figure 1.55: Parameter set: $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 1.5$.



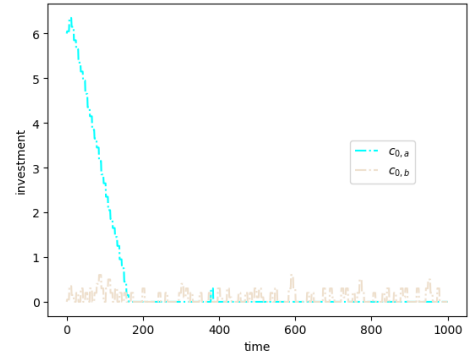
(a) Scenario 1'



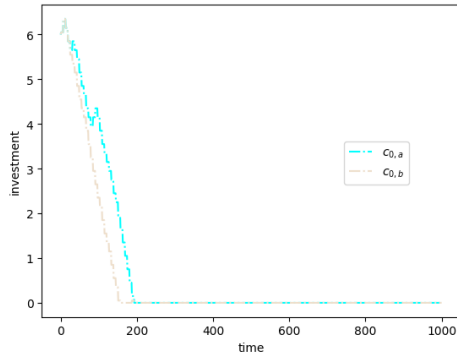
(b) Scenario 2'



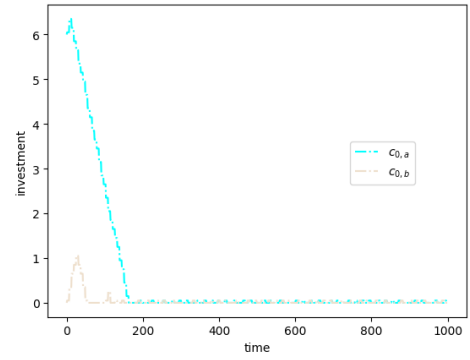
(c) Scenario 3'



(d) Scenario 4'

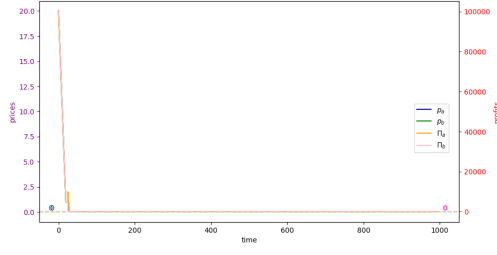


(e) Scenario 5'

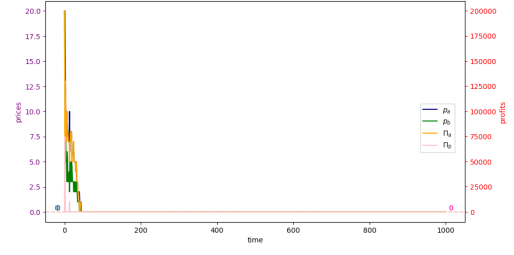


(f) Scenario 6'

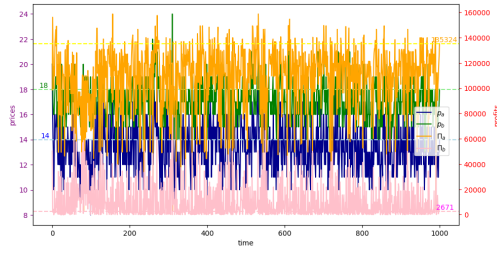
Figure 1.56: Parameter set: $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 1.5$ (continued).



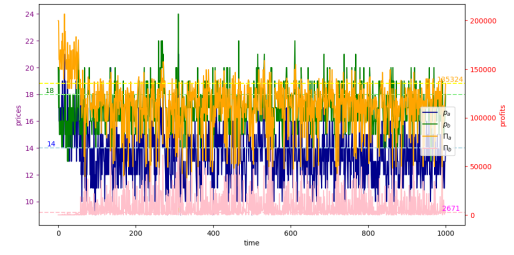
(a) Scenario 1'



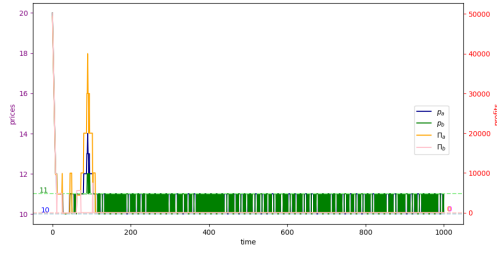
(b) Scenario 2'



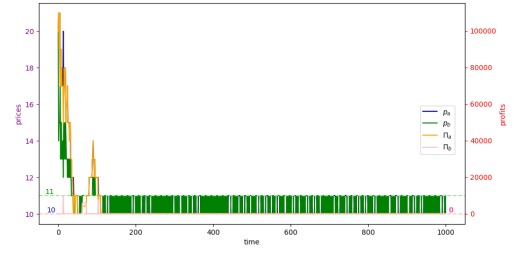
(c) Scenario 3'



(d) Scenario 4'

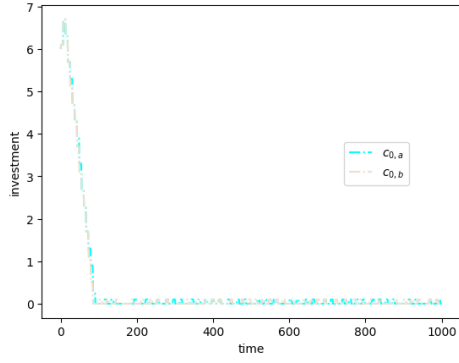


(e) Scenario 5'

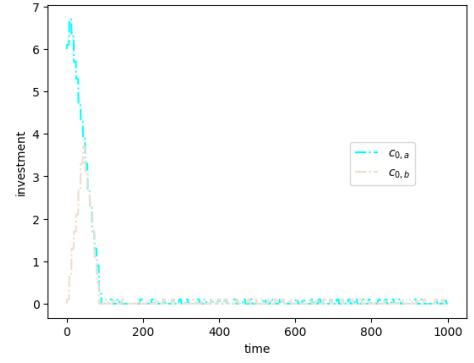


(f) Scenario 6'

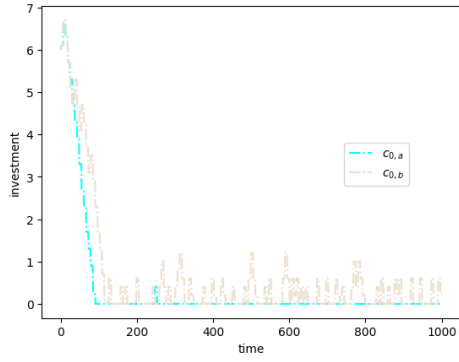
Figure 1.57: Parameter set: $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 1.5$.



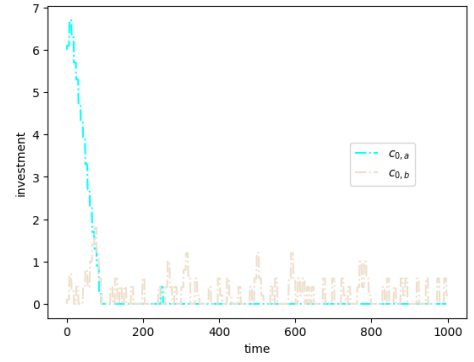
(a) Scenario 1'



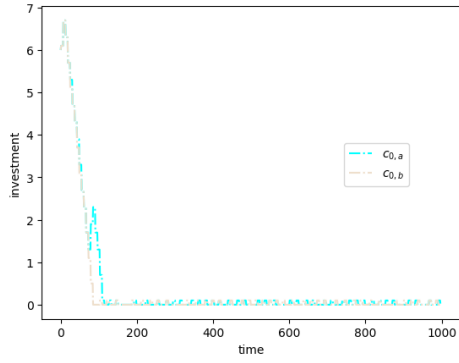
(b) Scenario 2'



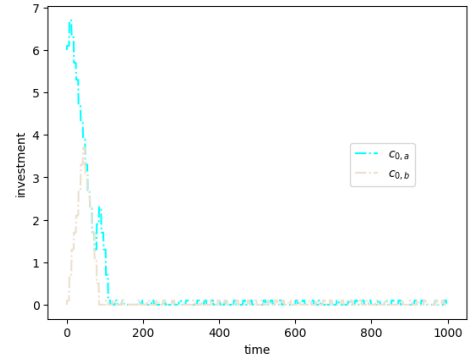
(c) Scenario 3'



(d) Scenario 4'

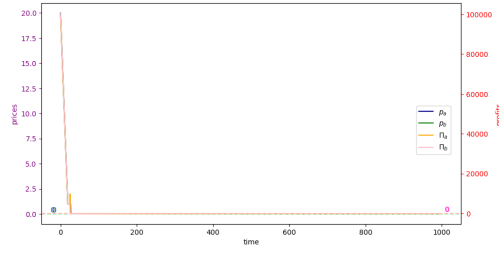


(e) Scenario 5'

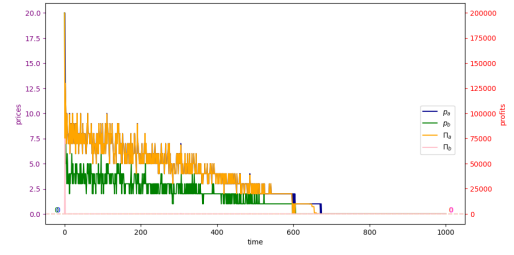


(f) Scenario 6'

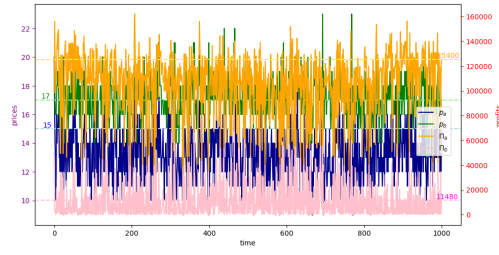
Figure 1.58: Parameter set: $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 1.5$ (continued).



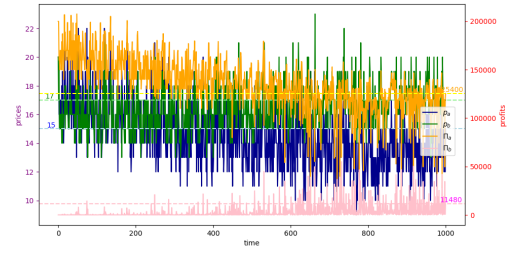
(a) Scenario 1'



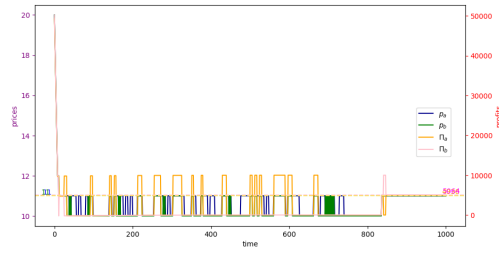
(b) Scenario 2'



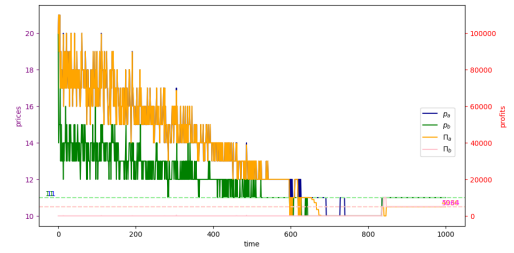
(c) Scenario 3'



(d) Scenario 4'

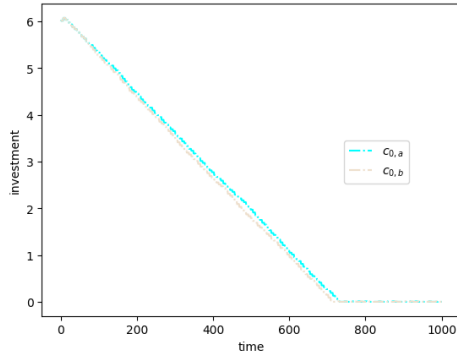


(e) Scenario 5'

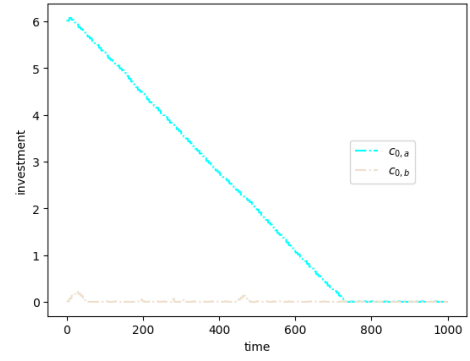


(f) Scenario 6'

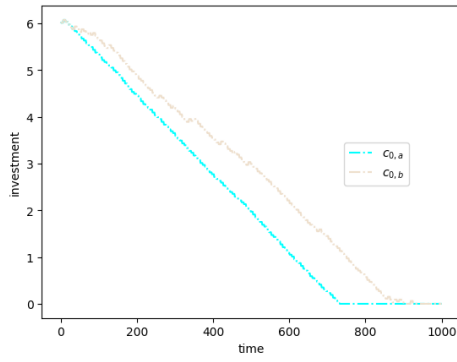
Figure 1.59: Parameter set: $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 2$.



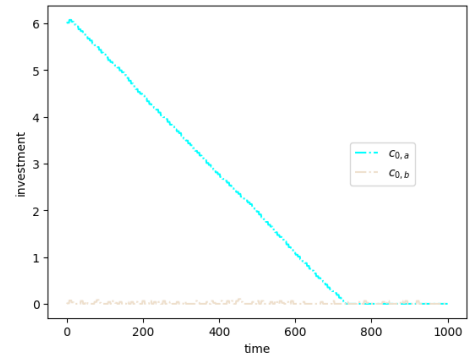
(a) Scenario 1'



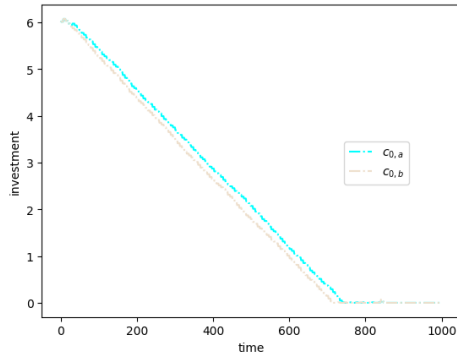
(b) Scenario 2'



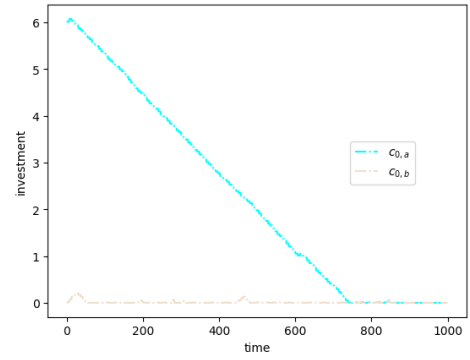
(c) Scenario 3'



(d) Scenario 4'

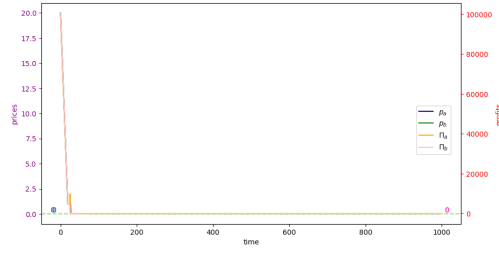


(e) Scenario 5'

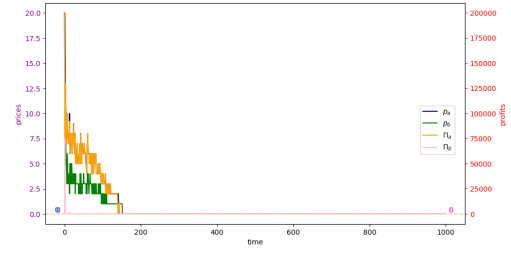


(f) Scenario 6'

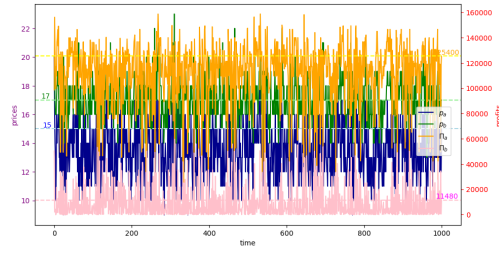
Figure 1.60: Parameter set: $\Delta = 0.05$, $\psi_0 = 0.01$, $\vartheta = 2$ (continued).



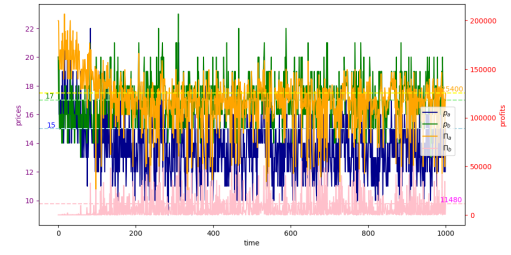
(a) Scenario 1'



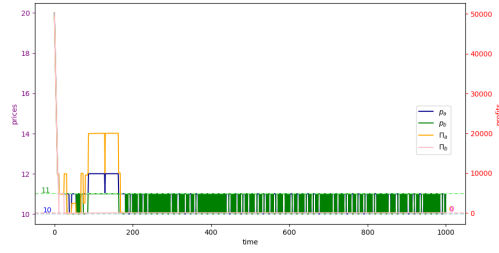
(b) Scenario 2'



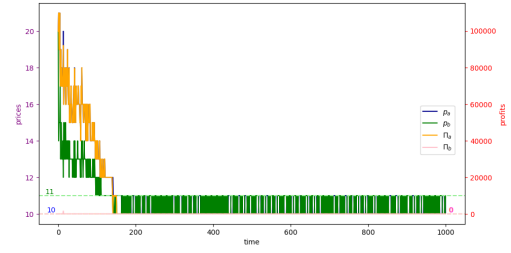
(c) Scenario 3'



(d) Scenario 4'

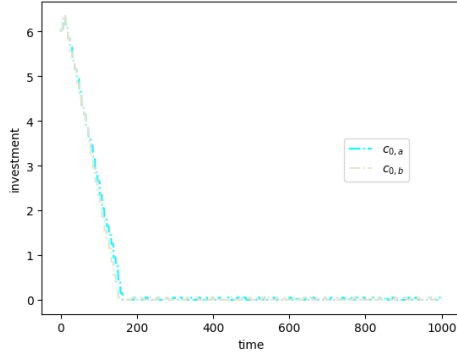


(e) Scenario 5'

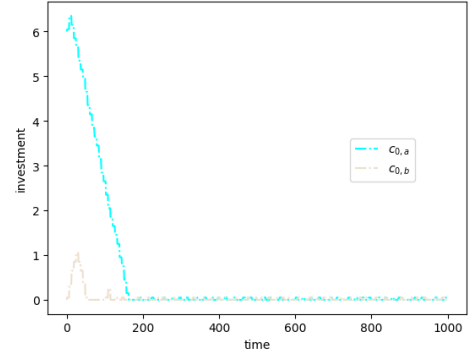


(f) Scenario 6'

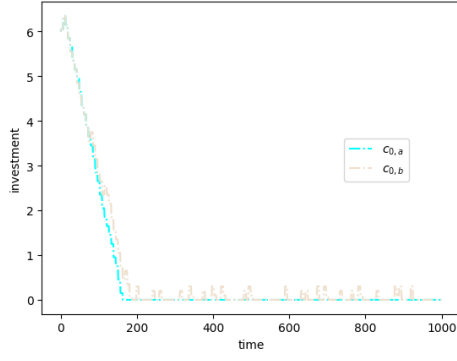
Figure 1.61: Parameter set: $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 2$.



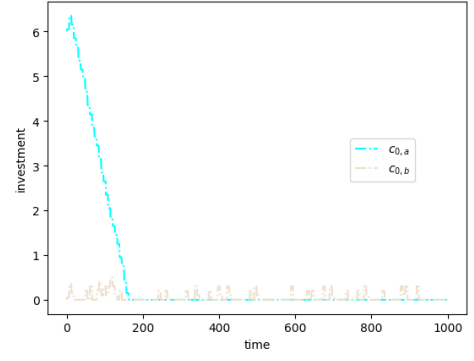
(a) Scenario 1'



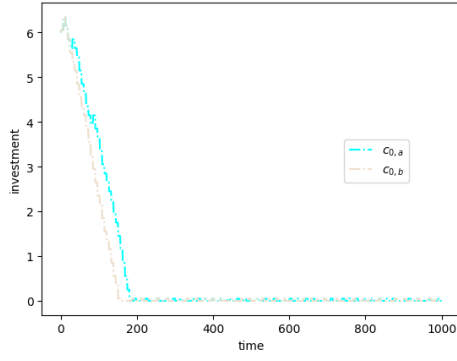
(b) Scenario 2'



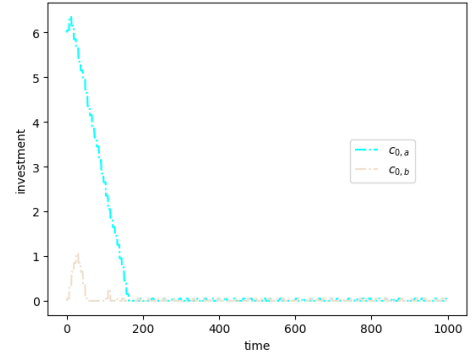
(c) Scenario 3'



(d) Scenario 4'

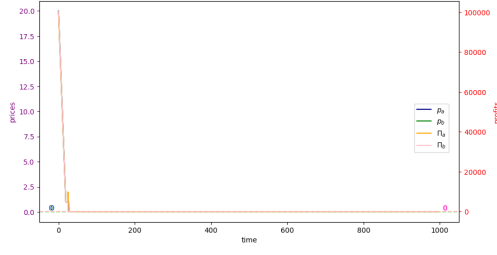


(e) Scenario 5'

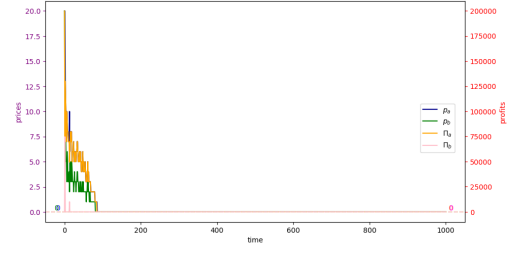


(f) Scenario 6'

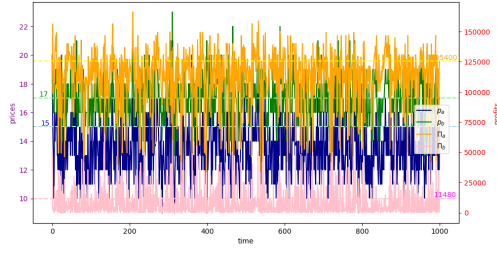
Figure 1.62: Parameter set: $\Delta = 0.25$, $\psi_0 = 0.05$, $\vartheta = 2$ (continued).



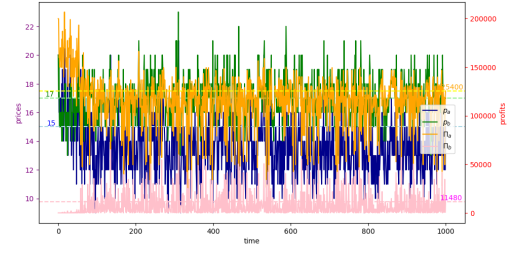
(a) Scenario 1'



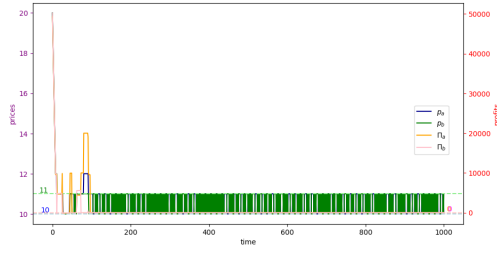
(b) Scenario 2'



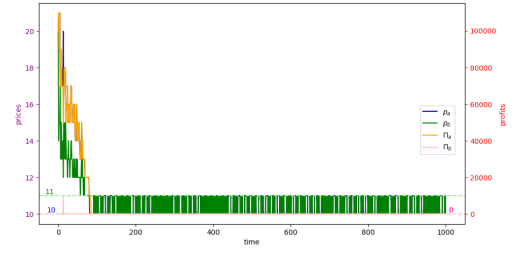
(c) Scenario 3'



(d) Scenario 4'

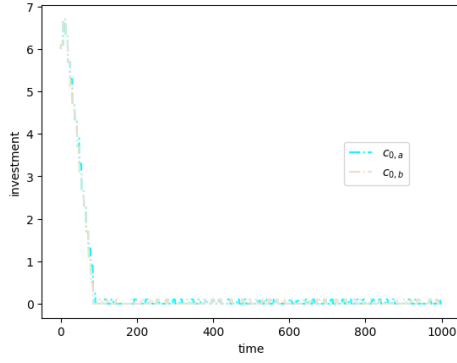


(e) Scenario 5'

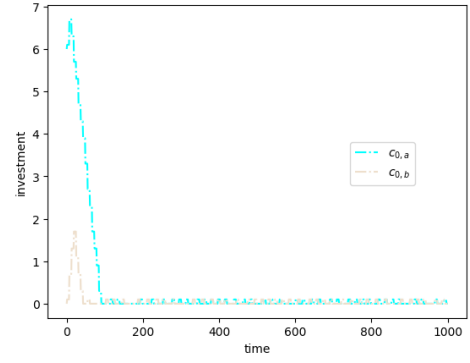


(f) Scenario 6'

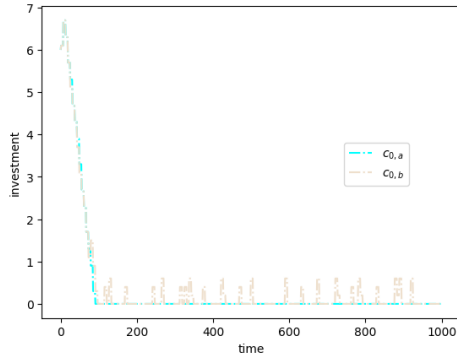
Figure 1.63: Parameter set: $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 2$.



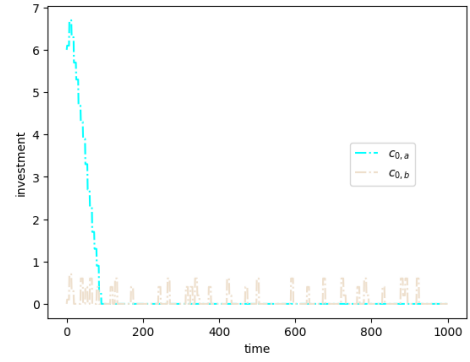
(a) Scenario 1'



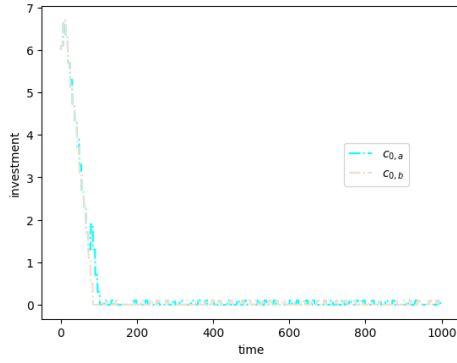
(b) Scenario 2'



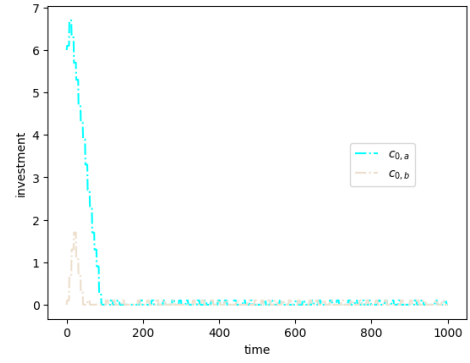
(c) Scenario 3'



(d) Scenario 4'



(e) Scenario 5'



(f) Scenario 6'

Figure 1.64: Parameter set: $\Delta = 0.5$, $\psi_0 = 0.1$, $\vartheta = 2$ (continued).

Chapter 2

Strategic Firm Capacity Selection under the Network Effects: A Computational Model of Capacity Investment Dynamics in the ICT Industry

Traditional industrial organization theory often interprets R&D investment as a driver of innovation and technological progress. In contrast, this paper focuses on a different but equally critical dimension of R&D—the physical investment in production and service capacity, which determines a firm’s ability to deliver high-quality services. In many ICT markets, profitability is not limited by the absence of innovation but by the constraints of infrastructure and capacity utilization. When firms exceed their optimal service capacity, product quality deteriorates and consumer satisfaction declines. Moreover, consumer behavior is increasingly shaped by digital community interactions and network effects, which can amplify or dampen the market value of these physical investments. To capture these mechanisms, we develop an agent-based model of competing ICT firms that strategically select their capacity levels while interacting with consumers embedded in a social network. The model uncovers how network effects, congestion, and capacity adjustments jointly shape market outcomes, profitability, and consumer engagement. This paper contributes to a deeper understanding of capacity management and network-driven competition in digital service industries.

Keywords: Firm Service Capacity; Network Effects; Computational Model; Capacity Investment; ICT Industry

2.1 Introduction

The traditional framework within the field of Industrial Organization (IO) emphasizes the critical role of Research and Development (R&D) investments in enhancing a firm’s profitability through product or process innovation. However, the assumption of a straightforward, positive relationship between R&D investment and profitability often breaks down in markets characterized by service capacity constraints. In such environments, even well-designed or innovative products may fail to deliver value if they cannot adequately support growing consumer demand. A key illustration of this issue arises from computer networks, where the problem of congestion control has long been recognized. As Peterson & Davie (2012) explain, when too many packets compete for the same transmission link, the queue overflows, leading to packet loss and degraded performance—a condition described as network congestion. From an economic standpoint, a similar logic applies: as demand surges, simultaneous usage of a service can exceed its capacity, resulting in congestion that harms the user experience. In this context, higher market demand does not necessarily translate into greater profitability. On the contrary, when congestion significantly degrades service quality, it can trigger a feedback loop—deteriorating user satisfaction reduces future demand, which in turn affects the firm’s market position and revenue. This dynamic underscores the importance of explicitly modeling service capacity and congestion in markets where product value is co-determined by usage levels and infrastructure constraints.

Standard economic theory approaches this problem by analyzing the interaction between market supply and demand. From the supply side, a firm in the Information and Communication Technology (ICT) industry that can control or at least adjust its service capacity should be aware of this fact. Crawford (1995) showed that some firms could use the strategy of withholding service capacity to generate revenue, which can be compared with the monopolist’s strategy of selecting a smaller output quantity. If a firm does not pay enough attention to the service capacity, the service quality can be greatly affected and the consumers could suffer a loss. In 2017, a service disruption occurred in Amazon S3 in the Northern Virginia (US-EAST-1) Region (Amazon 2024). Numerous companies and services (for example, Adobe’s services, Coursera, GitHub, etc.) were affected by the disruption (Helmke 2019). Amazon explained the reason as an error in the algorithm and claimed that “a larger set of servers was removed than intended”, and other technical writers urged that companies should do scenario tests and learn from the failures.

Additionally, the demand side also affects the market to a great extent. There has been evidence that different consumers have heterogeneous preferences. For instance, in the “INDEX” experiment, Varian (2002) observed that administrative and technical users were willing to pay significantly more than an average

user regarding the bandwidth. Apart from the cases where consumers make purchase decisions independently, consumer behavior can be significantly influenced by intracommunity communications, particularly within digital platforms. Such interactions, prevalent in virtual communities associated with telecommunication services, can amplify or mitigate the perceived value of product quality improvements made through R&D investments.

R&D investment in the ICT industry can be broadly categorized into two layers: the technological layer, which involves algorithmic and software innovations, and the physical layer, which focuses on the infrastructure development. Investments in the technological layer aim to improve the efficiency, prioritization, and management of existing network capacity. This includes mechanisms such as dynamically adjusting data transmission rates based on network conditions, optimizing bandwidth usage, and allocating resources more effectively to maintain service quality under congestion. In contrast, investments in the physical layer focus on expanding and upgrading the underlying infrastructure. These include the deployment of fiber-optic networks and 5G towers, acquisition of advanced routing hardware, expansion of data centers, and the development of new product lines or access points to support increased user demand. While both layers are important for sustaining service quality, this paper focuses on physical R&D investments. We emphasize this dimension because it follows a more deterministic investment logic and directly affects the firm’s capacity frontier. Algorithmic innovations, while impactful, often involve higher uncertainty and are better suited for future extensions of this work.

Type of Problem	Technological	Physical
Managing network congestion through data flow control (e.g., software-based rate adjustments)	✓	
Prioritizing service quality across users and applications (e.g., software-based resource allocation)	✓	
Limited buffer space in routing devices	✓	✓
Insufficient bandwidth on network links (e.g., overloaded fiber or wireless channels)		✓
Limited access in remote or underserved areas (e.g., rural connectivity issues)		✓
Hardware constraints in processing speed or capacity of network devices		✓

Table 2.1: Mapping of network-related challenges to the technological and physical layers of R&D.

Considering the complexities above, we introduce a comprehensive model designed

to elucidate the strategic decision-making process regarding firm capacity selection within the context of the ICT industry. Our model integrates the concept of network effects to reflect the multifaceted dynamics between firm strategies and consumer behaviors. By incorporating these elements, our proposed framework aims to shed light on the nuanced interplay between firm capacity constraints, R&D investment outcomes, and the pivotal role of consumer networks. This approach promises to unveil novel insights into the evolving dynamics of firm profitability and consumer engagement in the ICT sector. We aim to answer the following questions (or at least provide some insightful thoughts from the theoretical perspective): Firstly, how does a firm in the ICT industry decide on its R&D investment on the physical infrastructure considering the service capacity during the duopolistic competition and what are the possible market outcomes? Secondly, what role does the network effect among the consumers play in this market and how does it impact the firms' service capacity selection? Finally, compared with the traditional production capacity selection, how do the consumer network effect and the consumer congestion effect contribute to the market outcomes in the context of firm competition?

2.2 Literature review

2.2.1 R&D and product design

Investment in Research and Development (R&D) and product design plays a crucial role in the ICT industry, with extensive literature exploring its strategic importance. Aghion & Tirole (1994) provide a seminal analysis of R&D organization within an incomplete contract framework, offering key insights into the strategic management of R&D investments and their impact on innovation and firm dynamics. Building upon this framework, Bessen (2006) examines the economic incentives driving firms to contribute to Free/Open Source Software (F/OSS) despite the availability of proprietary alternatives. His findings highlight how F/OSS can address specialized and complex needs more effectively, influencing firms' innovation strategies. Further extending this discussion, Casadesus-Masanell & Ghemawat (2006) analyze the competitive dynamics between non-profit OSS initiatives, such as Linux, and for-profit alternatives like Microsoft Windows. Their dynamic mixed duopoly model demonstrates how pricing strategies shape market positioning and long-term competitive interactions. Chayet et al. (2011) highlight how congestion within production systems constrains product variety, emphasizing the importance of economies of scale in optimizing production decisions. In the context of software quality, survey studies by Colakoglu et al. (2021), Galli et al. (2020), and Haindl & Plösch (2022) provide comprehensive reviews of software quality metrics

and evaluation methodologies. These works contribute to a deeper understanding of software product quality assessment and its significance for development practices. Cao (2021) offers insights into the rise of Zoom video communications, analyzing market trends and consumer preferences in the evolving digital communication landscape. Macmillan et al. (2021) build on this by investigating how application-layer performance metrics adapt under varying network conditions, providing valuable findings on optimizing video conferencing technologies to enhance user experience. Decarolis & Li (2023), along with two separate studies by Decarolis, Li & Paternollo (2023), Decarolis, Rovigatti, Rovigatti & Shakhgildyan (2023), explore competitive dynamics in digital markets. Their research examines how default settings influence market penetration in the search engine industry and investigates the interaction between AI algorithms and data flow. These studies underscore the relevance of empirical design and auction theory in analyzing market competition, highlighting the potential for regulatory interventions to promote fairness and consumer choice. Alikhan et al. (2023) contribute to the discussion by focusing on cloud computing efficiency, proposing a model that significantly reduces data processing times. Their findings demonstrate the critical role of cloud optimization in improving data upload and access speeds, thereby enhancing overall system performance.

Overall, the literature underscores the intricate relationship between product quality, investment decisions, and firm sales in the ICT sector. It highlights a multifaceted approach to fostering growth and competitiveness, emphasizing the interplay of technological innovation, market structure, and regulatory considerations. However, much of the existing literature in economics tends to overlook the effects of consumer congestion and its implications for user experience. In practice, congestion can deteriorate consumer experience by reducing product availability, slowing service delivery, and increasing inefficiencies—factors that can lead to dissatisfaction, lower willingness to pay, and eventually shifts in consumer behavior such as reduced usage or switching to competitors. One notable exception is Courcoubetis & Weber (2003), who incorporate congestion into consumer utility by specifying it as

$$u_i(x_i, y) - px_i,$$

where x_i is the quantity of service consumed by individual i , p is the price, and $y = \sum_i x_i/k$ represents the system load relative to its capacity k . The utility function u_i is assumed to be strictly increasing and concave in x_i , and strictly decreasing and convex in y , capturing the diminishing returns from consumption and the disutility from congestion. While this functional form effectively introduces congestion as a negative externality, it does not fully account for the dynamic consequences of congestion within a structured model of firm-consumer interaction over time.

Therefore, existing models do not systematically incorporate the trade-off between network benefits and congestion costs in consumer decision-making. Moreover, most studies do not explicitly model how congestion impacts the competitive advantages of technologically superior firms. This omission creates a gap in understanding how high-tech firms sustain profitability when scalability constraints come into play.

2.2.2 Firm capacity selection

The literature on firm strategy regarding production capacity and technology selection is extensive, providing valuable insights into firms' competitive decision-making processes. Katz & Shapiro (1986) offer a foundational analysis of the strategic incentives for pursuing technical compatibility in dynamic markets. Their work highlights how firms navigate evolving industry structures through compatibility choices, shaping competitive advantages in technology adoption. Herk (1993) examines duopolistic markets where firms are capacity-constrained, demonstrating that Cournot strategies in capacity and pricing emerge naturally when consumers exhibit near risk neutrality toward service reliability. These findings extend the relevance of Cournot behavior to markets where pricing models introduce consumer switching costs, influencing firm strategy. Chand et al. (1996) explore firms' trade-offs in allocating production capacity between short-term operational efficiency and long-term process innovation. Their analysis underscores the strategic balance between immediate output and sustained technological improvements. Similarly, Besanko & Doraszelski (2004) investigate how seemingly identical firms can diverge in size and market dominance due to firm-specific shocks and investment irreversibility. Their work highlights the long-term implications of price competition and asset depreciation, showing how firms can evolve into asymmetric competitors despite starting with similar capabilities. Dai et al. (2005) contribute to the discussion by addressing the intricacies of pricing strategies in revenue management, analyzing how multiple firms compete for customers through dynamic pricing mechanisms. Their findings emphasize the importance of price optimization in maintaining market position. In the context of network industries, Turan et al. (2014) examine the challenges broadband providers face in managing network capacity under market uncertainties. Their research provides a structural analysis of the interplay between firms and consumers, shedding light on how firms adjust capacity to fluctuations in demand. Nakamura (2014) further explores capacity choice dilemmas in duopolistic markets with substitutable goods, revealing how firms adopt different strategic frameworks depending on whether they prioritize consumer welfare or profit maximization. Wen et al. (2015) analyze how market growth rates and uncertainty influence firms' investment and capacity utilization strategies. Their findings reveal that production decisions are heav-

ily shaped by expectations of future demand and the risks associated with over- or under-investment. Ahuja et al. (2016) extend this discussion to network performance, demonstrating how firms adapt dynamically to bandwidth fluctuations to maintain seamless connectivity. Levistky et al. (2022) evaluate algorithms for channel bandwidth and sub-channel selection using the NS-3 simulation platform. Their work advances strategic bandwidth allocation, optimizing network efficiency in competitive settings. Collectively, these studies provide a comprehensive view of how firms strategically manage production capacity and technology selection. They emphasize the need for adaptability, strategic foresight, and a nuanced understanding of market dynamics, investment decisions, and capacity constraints in maintaining a competitive edge.

It is essential to conceptualize “capacity selection” when analyzing ICT products. Traditionally, in manufacturing sectors, production capacity primarily represents a quantitative limit on output, where a larger capacity directly enhances a firm’s ability to meet consumer demand. However, this notion does not fully capture the role of capacity in the ICT industry, particularly for software products. Unlike physical goods, where production constraints are well-defined, capacity in ICT markets is more complex—it is not merely a ceiling on output but also a determinant of product performance and user experience. One could define service capacity as “the technical capability of an ICT product—encompassing its infrastructure and architecture—to support a certain number of users, transactions, or operations without degrading performance or user experience.” This conceptualization acknowledges that, in ICT markets, capacity is intrinsically linked to product quality. For instance, in telecommunications, bandwidth constraints limit service quality, while in cloud computing, server capacity affects response times and system reliability. Specific examples include bandwidth limitations in telecommunications networks and server capacity constraints in cloud-based services.

2.2.3 Network applications, consumer inertia and network effects

The role of networks in shaping consumer behavior and market dynamics has been widely studied. Jackson & Yariv (2007) analyze games on social networks where individuals make binary choices—such as adopting a new technology—based on past behavior. Their study examines the diffusion of behaviors and decision-making under incomplete information, exploring the multiple equilibria that emerge in networked environments. Galeotti et al. (2010) extend this analysis by examining strategic interactions within localized neighborhoods, where individual payoffs depend on an underlying network of connections. Their research highlights how degree correlations within a network structure influence strategic decisions, ulti-

mately shaping equilibrium behavior and market outcomes. Farrell & Klemperer (2007) investigate competition in markets characterized by switching costs and network effects, demonstrating why incompatible competition often proves more profitable than compatible competition. They emphasize that switching costs and network externalities create consumer lock-in, reducing market fluidity and potentially stifling competition. Kemper (2010) explores the valuation of network effects in software markets, summarizing various models that examine technology adoption, diffusion dynamics, and consumer network structures. The study underscores the importance of network topologies—such as random networks, small-world networks, and scale-free networks—in determining how adoption spreads within customer networks. Furthermore, it highlights how network dynamics influence market penetration and the competitive positioning of software firms. The customer churn rate has also been discussed for keeping the right consumers from the perspective of firms (Gallo 2014).

Existing studies on networks within the field of IO provide strong evidence that market participants are interconnected and that the behavior or status of their connections can significantly influence individual choices. However, most of these studies overlook the role of consumer heterogeneity—such as variations in preferences for product characteristics—and how these differences shape market outcomes and competitive dynamics. Addressing this heterogeneity is crucial for understanding how diverse consumer preferences interact with network effects, potentially leading to varied adoption patterns, demand shifts, and firm strategies.

2.2.4 Firm competition

Firm competition is shaped by a variety of external pressures, strategic interactions, and market dynamics. Marasco et al. (2016) investigate the evolution of competitive strategies in response to external shocks and unforeseen events, refining autonomous Lotka-Volterra models. Their research integrates market prediction, dynamic rivalry, and non-linear demand dynamics, offering an approach closely linked to the logit demand model. Killingback & Killingback (2021) examine the zero-profit dilemma within Bertrand competition, addressing how evolutionary dynamics impact the stability of Bertrand duopolies. Their findings suggest that in a smooth Bertrand game, firms face less severe competitive penalties, which destabilizes the classic zero-profit equilibrium. MacKay & Remer (2022) analyze oligopolistic competition with dynamic pricing strategies, focusing on consumer inertia, market influence, and demand dynamics. Their research applied to a hypothetical merger in the retail gasoline sector demonstrates that static models tend to overestimate price increases compared to dynamic models. They also highlight how the method of merger implementation significantly influences equilibrium prices, particularly in markets where consumer inertia weakens competitive

pressure. Geng et al. (2022) distinguish between two types of social interactions that influence firm competition: Market-expansion effects, where overall market demand grows as both firms increase sales, and value-enhancement effects, where a consumer derives additional benefits from purchasing from a firm with a strong sales history. Their study examines optimal pricing and quality differentiation strategies in a socially influenced market, introducing a two-stage duopoly competition model that integrates analytical and numerical experiments. Their findings emphasize that firms must adapt their strategies based on the dominant type of social interaction, as this directly affects profitability, product positioning, and consumer surplus.

While existing research has extensively analyzed firms’ strategic decision-making concerning pricing and market demand, it is crucial to account for bounded rationality when examining firm behavior. Information asymmetry is not limited to the relationship between firms and consumers—it also exists between competing firms, influencing strategic interactions. Instead of pursuing global optimization, which is often impractical, firms make decisions based on historical performance and observed consumer behavior over past periods. In such settings, firms rely on heuristic-based behavioral rules to adapt their strategies dynamically, rather than engaging in exhaustive optimization—a more realistic approach given the complexity and uncertainty of real-world markets.

2.3 Model design

2.3.1 General structure

We consider two types of agents in the model: firms and consumers. “Firms” refer to the firms that hold the copyright and license of the product. In our basic duopolistic competition model, there are two firms, referred to as firms a and b . Since our focus is not on the local data restriction, we leave the data restriction level exogenous and uniform for now. “Consumers”, or “households”, refer to users who are potential buyers of the product. In the model, consumers are numerically identified, ranging from user 1 to user N , and are situated either within one of the two regions (A or B) or outside these regions. The architecture of our model is outlined in Figure 2.1, providing a clear representation of the interactions and relationships among these diverse agents.

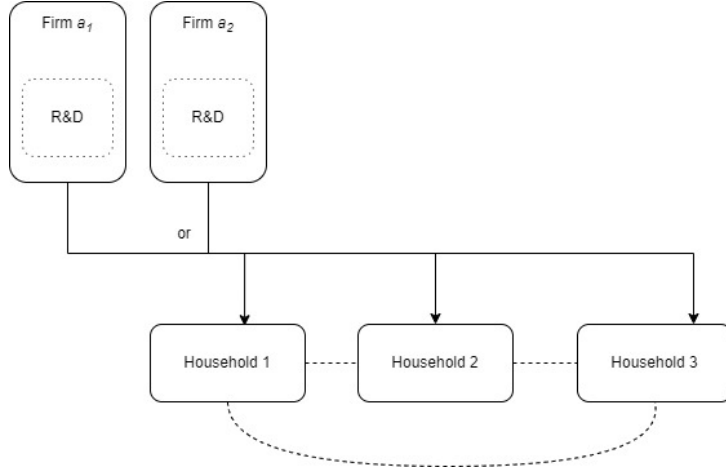


Figure 2.1: Model architecture

Our model captures the sequence of interactions between Firms and Consumers within a single period, illustrating the dynamic process from contract negotiation to consumer network evolution. The stages are as follows:

1. **Contract Negotiation:** Firms engage with R&D teams to outline contracts and determine the investment in capacity.
2. **Product Development:** During this phase, R&D teams concentrate on product innovation, involving algorithm design and the application of intellectual effort to finalize the product's features.
3. **Market Analysis:** If firms already have previous sales data, they draw a sample of consumers and check their willingness to pay in order to set the price of product in the current period.
4. **Investment Updates:** If firms already have previous sales data, they compare their own historical profits and adjust their investments in their infrastructure and on technology.
5. **Payments and Sales:** Firms market their offerings on online platforms, thereby reaching a global consumer base.
6. **Consumer Decision-Making:** Consumers assess the available products based on detailed descriptions, user reviews, and prior experiences. They may also explore free-trial versions. This decision-making process entails two steps: firstly, determining whether to make a purchase, and secondly, choosing which product to buy. Notably, consumers face no transaction costs.

7. **Transaction Completion and Profit Calculation:** Following the completion of transactions, each Firm calculates its profit, which is contingent on the choices made by Consumers.
8. **Product Consumption:** Consumers begin to use the products they have purchased and share their experiences within their social networks, shaping their overall user experience.

2.3.2 Consumers

Consumers evaluate, make decisions on purchases, and consume purchased products in every period. In the benchmark model, we assume that consumers are not communicating with each other and they make their decisions independently. In the next step, we propose a model involving social networks. The introduction of social network theory into the model is motivated by its ability to accurately represent the intricate dynamics of consumer interactions within the ICT market. In this market, consumers often base their purchasing decisions and subscription continuities not solely on individual preferences or product attributes, but significantly on the influences of their community.¹ This suggests a strong interdependency in decision-making processes among consumers, where the selection of one consumer can be influenced by the choices and opinions of others within their network.

By integrating a social network structure into the model, we aim to capture this complex web of consumer interactions effectively. This structure allows for a better understanding of how information, opinions, and trends are disseminated through networks and impact the collective decision-making process. It acknowledges that consumers are not isolated agents making decisions in a vacuum but are embedded in a social fabric that significantly shapes their preferences and actions. Thus, the inclusion of social network theory offers a more comprehensive and realistic approach to analyzing consumer behavior in the ICT market, providing insights into how social influences contribute to the dynamics of product adoption and subscription sustainability.

The foundational network structure is established as follows: At the outset, the market consists of a core group of initial consumers, while the remainder of potential consumers remains unconnected. It is posited that a consumer will only consider purchasing a product if they are part of at least one connection. This initial network configuration is modeled on a small-world network paradigm. The rationale behind this choice is supported by Kemper (2010), who argues that complex customer networks in software markets typically exhibit small-world characteristics. In this context, interactions between consumers are conceptualized as links, and their purchasing decisions are represented as node status.

¹The community here may refer to an immediate social circle or an online user community.

As the network progresses over time, consumers may form new connections or experience a decrease in interactions. This dynamic is modeled by the addition or removal of links. Importantly, for connected consumers, the decisions made by their connections in the previous period can influence their current purchasing choices. In essence, the network is subject to evolution over time, with the decision-making processes of consumers being significantly shaped by their network connections.

2.3.3 Firms

Consider a telecommunications company that provides services via cable to households. This company primarily improves its service capacity by building additional server parks and other physical infrastructural improvements. We let $c_{0,i,t}^{Phy}$ denote the physical R&D investment of firm i in period t .² Physical capacity expansions result in an additive increase in capacity, reflecting a direct increase in resources. We denote firm i 's capacity in period t as $\chi_{i,t}$. In the initial period ($t = 0$), firm i 's capacity is supposed to be

$$\chi_{i,0} = S_{i,0}A_{i,0}$$

where S_0 represents the physical capacity (for instance, the server capacity) in the initial period and A_0 represents the technological productivity in the initial period. During the period, firms may design the product by themselves or outsource the product design to other research teams (firms or groups of individuals). In either way the firm carries out an investment. In a single period, firms can choose whether they invest in the product, and once they decide to invest, they would purchase the physical infrastructure for their production.

The updated physical capacity can be formulated as:³

$$\begin{aligned} S_{i,t+\tau} &= (1 - \beta^D)S_{i,t} + \Delta S_{i,t} \\ &= (1 - \beta^D)S_{i,t} + f(c_{0,i,t}^{Phy}) \end{aligned}$$

where the depreciation rate $\beta^D \in [0, 1]$. We assume for now that the increased complexity in managing and integrating large-scale server operations, the higher energy costs, and the need for advanced cooling systems do not scale linearly with the number of servers, and the rise in physical capacity should be a concave function. Further, we assume that the function has the form:

$$\Delta S_{i,t} = f(c_{0,i,t}^{Phy}) := \beta_i^C \ln(c_{0,i,t}^{Phy} + 1)$$

²We note a slight abuse of notation: unless otherwise specified, the physical R&D investment is the firm's sole investment in period t , i.e., $c_{0,i,t} = c_{0,i,t}^{Phy}$.

³We assume that firms update the service capacity every τ periods ($\tau \geq 1$).

with $c_{0,i,t}^{Phy} \geq 0$ and $\beta_i^C > 0$. β_i^C represents the effectiveness of the updates of firm i . With a larger β_i^C , the updates are more effective.

It is easy to show that $f'(c_{0,i,t}^{Phy}) = \frac{1}{1+c_{0,i,t}^{Phy}} > 0$ and $f''(c_{0,i,t}^{Phy}) = \frac{-1}{(1+c_{0,i,t}^{Phy})^2} < 0$. The results satisfy our assumption of concavity. Thus, firm i 's capacity in period $t + \tau$ is

$$\chi_{i,t+\tau} = S_{i,t+\tau} A_{i,t+\tau} = ((1 - \beta^D)S_{i,t} + \Delta S_{i,t}) A_{i,t+\tau}$$

In the initial period, firms find some potential consumers to use and test the product, thus, the observed functionality is determined and the number of consumers is an estimated number. Here we assume that this number is a constant $\bar{m}$ ($\bar{m} > 0$). As we have explained, the observed functionality increases in the product service capacity and decreases in the number of consumers, yet how can we describe the partial relationship between service capacity and the number of consumers? To answer this question, we refer to the research on telecommunications for evidence. Qamar et al. (2019) designed a MATLAB simulation for testing the overall network performance with CI (close-in) and FI (floating intercept) propagation models. They observed that as the number of user increases, the average user throughput (the amount of data received by a connected user on average in a cellular network) decreases (Figure 2.2). From this observation, we propose that in our theoretical model it is reasonable to have the following properties.

- The observed product functionality increases as the service capacity increases ($\frac{\partial \theta_i}{\partial \chi_i} > 0$).
- The observed product functionality decreases as the number of users increases ($\frac{\partial \theta_i}{\partial m_i} < 0$).
- As more users share the service, the negative effect on the observed product functionality diminishes, or at least holds constant ($\frac{\partial^2 \theta_i}{\partial m_i^2} \geq 0$).

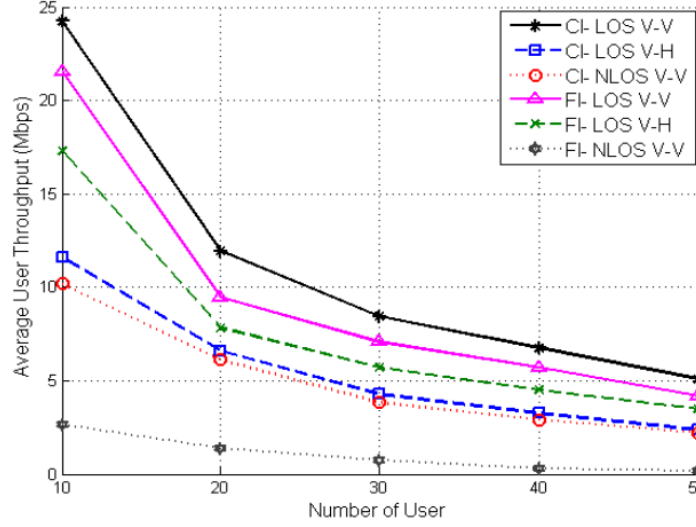


Figure 2.2: Average user throughput vs number of users. Source: Qamar et al. (2019).

For the simplicity of the analysis, we start with a linear model describing the basic observed functionality of firm i 's product, namely

$$\theta_{i,0} = \beta_2 \chi_{i,0} - \beta_0 \bar{m} + \beta_3$$

The observed functionality of firm i in period $(t + \tau)$ is then

$$\theta_{i,t+\tau}(\chi_{i,t+\tau}, m_{i,t}) = \beta_2 \chi_{i,t+\tau} - \beta_0 m_{i,t} + \beta_3$$

where $m_{i,t}$ is the number of consumers purchasing product of firm i in period t , parameters $\beta_0, \beta_2, \beta_3 > 0$.

To investigate the peer effect among consumers, we assume that there is a fixed consumer network in which consumers share information about the products with each other. The structure of the fixed network will be elaborated in Section 2.5.2. By now, we denote the network as $G = (N, E(G))$ where the number of nodes is N and the set of edges as $E(G)$.⁴ In our context, each node represents a single Consumer and each link represents a relation between two Consumers. Furthermore, we denote the set of all neighbors of node j in the network as $\mathcal{N}_j(E(G)) = \{j' | (j, j') \in E(G)\}$. The degree (the number of neighbors) of j is $d_j(E(G)) = |\mathcal{N}_j(E(G))|$. The utility of Consumer j is then⁵

$$u_{j,i,t}(x_j, \theta_{i,t}, p_{i,t}, \mu_{i,j,t-1}) = \bar{u} + x_j \theta_{i,t} + \beta_4 \mu_{i,j,t-1} - \beta_5 p_{i,t}$$

⁴Jackson (2011) mentioned that most of the terminology origins from standard graph theory.

⁵The utility function here is similar to the design of Mussa & Rosen (1978): $u = x + \theta q$, where x is a composite commodity, θ refers to the customer type and q is the product quality.

where $\mu_{i,j,t-1}$ is the number of j 's neighbors who purchase the product from firm i in period $t-1$ (by assumption $\mu_{i,j,t-1} \leq d_j(E(G))$), x_j is consumer j 's taste towards functionality with $x_j \sim U[0, 1]$, parameters $\bar{u}, \beta_4, \beta_5 > 0$.⁶

Consumer j would only buy a product if at least the product fulfills her reserved utility $u_{j,R}$. For simplicity, we assume that all consumers have a homogeneous and time-invariant reserved utility ($u_{j,R} = u_R, \forall j \in J$ with $J = \{1, 2, \dots, N\}$ representing the set of consumer ID). Assuming that in the initial period, only a subset $J_0 \in J$ of the consumers get the information of the products, we denote $n_t^{net} = |J_{t-1}|$ for $t \geq 1$ as the set of consumers who have made purchasing decisions.⁷ The product information diffusion is therefore

$$J_t = J_{t-1} \cup \{j | \mathcal{N}_j(E(G)) \cap J_{t-1} \neq \emptyset\}$$

For consumers whose utility from at least one product exceeds the reservation utility u_R , the number of consumers purchasing from firm i is given by⁸

$$m_{i,t} = \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,i,t} > u_{j,-i,t}} + \sum_{j \in J_{t-1}} \beta_{j,i,t} \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}}$$

where $\beta_{j,i,t}$ follows a Bernoulli distribution with the probability of 0.5 ($\beta_{j,i,t} \stackrel{\text{i.i.d.}}{\sim} \text{Bernoulli}(0.5)$). In the case of indifference, each consumer is assigned to exactly one of the two firms, so that $\beta_{j,i,t} + \beta_{j,-i,t} = 1$. The total number of consumers in period t can be formulated as

$$\begin{aligned} m_t &= m_{i,t} + m_{-i,t} \\ &= \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,i,t} > u_{j,-i,t}} + \sum_{j \in J_{t-1}} \beta_{j,i,t} \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}} \\ &\quad + \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,-i,t} > u_{j,i,t}} + \sum_{j \in J_{t-1}} \beta_{j,-i,t} \mathbb{1}_{u_{j,-i,t} = u_{j,i,t}} \\ &= \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,i,t} \neq u_{j,-i,t}} + \sum_{j \in J_{t-1}} (\beta_{j,i,t} + \beta_{j,-i,t}) \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}} \\ &= \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,i,t} \neq u_{j,-i,t}} + \sum_{j \in J_{t-1}} \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}} \\ &= |J_{t-1}| = n_t^{net} \end{aligned}$$

The total cost of firm i in period t is then

$$c_{total,i,t} = c_{0,i,t}^{Phy} + m_{i,t} c_{res,i}$$

⁶A higher value of x_j represents a higher preference for functionality. $U[0, 1]$ represents a uniform distribution from 0 to 1. $\mu_{i,j,t-1}$ should also include node j if j also bought the same product as her neighbors did in period $t-1$.

⁷ $n_0^{net} = 0$ since no consumer buys any products in the initial period.

⁸ $m_{i,t}$ can also be interpreted as the market demand and can be derived from simulations.

as we assume that investment is an expenditure for the firms. $c_{\text{res},i}$ represents the marginal cost of production.

Thus, the profit of firm i in period t can be calculated as

$$\Pi_{i,t}(m_{i,t}, p_{i,t}, c_{0,i,t}, c_{\text{res},i}) = m_{i,t}p_{i,t} - c_{\text{total},i,t}$$

2.4 Game-theoretical analysis

Under the assumption that a single time period encompasses a sequence of events, including decision-making processes of both firms and consumers, we propose to examine the underlying mechanisms within this context, intentionally omitting the consideration of investment and price adjustments over multiple periods. Our analysis is initially structured as a game, with the objective of identifying the Nash equilibrium in the steady state. To facilitate this, we design the game setting as follows:

Consider a market with two competing firms, each selling a single product to multiple consumers. In this model, each consumer purchases at most one product from these firms. The decision-making process for the firms is modeled as a game within a single period, so we omit time notation (t) for simplicity. The outline of the game can be summarized as follows: Firm i ($i \in \{a, b\}$) sets the price for its product p_i under complete information of the service capacity conditions (χ_a, χ_b) . There are two different situations, where (a) the observed product functionality is exogenous with respect to consumers' tastes, i.e. the observed functionalities of both firms (θ_a, θ_b) are given and are not influenced directly by x_j , and (b) the observed product functionality is endogenous with respect to consumers' tastes, i.e. the observed functionalities of both firms (θ_a, θ_b) are influenced directly by x_j . Although our primary contribution lies in the analysis of Situation (b), the analysis of Situation (a) offers benchmark results that serve as a basis for comparison with the findings discussed in the literature on Bertrand competition. The main method of our analysis is considering a representative consumer who selects at most one of the two products offered by different firms, and when the consumer shows an indifferent attitude, i.e. $u_{j,i} = u_{j,-i}$, then the specific taste of the consumer is the threshold dividing the market.

2.4.1 Strategic price decisions under exogenous product functionality

Suppose there is only one period and that the observed quality is given for both firms, then the setting appears to be similar to a Bertrand duopoly on the prices. In this case, time t does not play a role and the updates of the physical and

technological service capacity do not play a role at all. Besides, we assume that consumers are distributed on a continuum $[0,1]$ and they are marked by their preference towards functionality x_j . In the continuous setting, the tie between the utilities in the “indifference case” can be tolerated. We rewrite the utility function of consumer j as

$$u_{j,i}(x_j, \theta_i, p_i) = \bar{u} + x_j \theta_i - p_i$$

To discuss the decision-making process of consumer j , we follow the selection rule as defined. Assuming that both firms produce products which satisfy consumer j 's reserved utility u_R ($u_{j,i} \geq u_R$ for $i \in \{a, b\}$), then consumer j selects the product of firm a if $u_{j,a} \geq u_{j,b}$, which can be formulated as⁹

$$\begin{aligned} \bar{u} + x_j \theta_a - p_a &\geq \bar{u} + x_j \theta_b - p_b \\ \Leftrightarrow \\ x_j \theta_a - p_a &\geq x_j \theta_b - p_b \\ \Leftrightarrow \\ x_j (\theta_a - \theta_b) &\geq p_a - p_b \\ \Leftrightarrow \\ x_j \Delta \theta &\geq \Delta p \end{aligned}$$

Therefore, for products with asymmetric observed functionality, if $\Delta \theta > 0$, consumer j selects a product from firm a if $x_j \geq \frac{\Delta p}{\Delta \theta}$. Similarly, if $\Delta \theta < 0$, consumer j selects a product from firm a if $x_j \leq \frac{\Delta p}{\Delta \theta}$. For products with symmetric observed functionality, i.e. $\Delta \theta = 0$, consumer j always considers the product with a lower price, and the threshold $\frac{\Delta p}{\Delta \theta}$ is no longer valid.

Figure 2.3 simulates the decision-making process of 100 consumers in the market. Each consumer is marked by her taste towards functionality. Sub-figure (a) shows the case in which the products have asymmetric observed functionality and clearly the threshold $\frac{\Delta p}{\Delta \theta}$ divides the aggregate market demand.¹⁰ Sub-figures (b) and (c) show the case in which the products have symmetric observed functionality, and the results suggest that in these cases, market demand is no longer divided by a certain threshold. We also point out that by comparing Sub-figures (b) and (c), we find that in the case where the product prices are generally higher, there are fewer consumers buying the products, i.e. the aggregate market demand is smaller.

⁹Here we define $\Delta \theta = \theta_a - \theta_b$ and $\Delta p = p_a - p_b$.

¹⁰In the codes, $\frac{\Delta p}{\Delta \theta} = 0.5$.

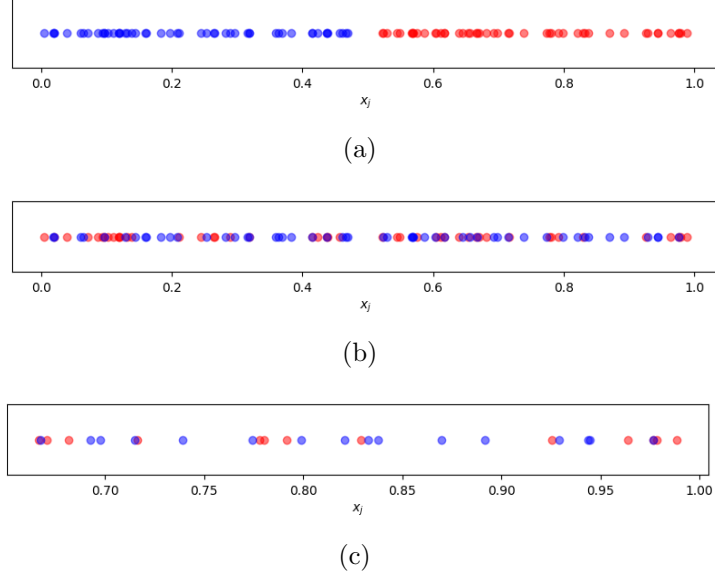


Figure 2.3: Example simulations of the consumers' decision-making process. Red points represent “selecting firm a 's product” and blue points represent “selecting firm b 's product”.

From the firms' perspective, since the focus is to have a profit as much as possible, it is crucial to consider the investment and price strategies to achieve the goal. We can rewrite the objective function of firm i as¹¹

$$\begin{aligned}
 \Pi_i(m_i, p_i, c_{0,i}, c_{\text{res},i}) &= m_i p_i - c_{\text{total},i} \\
 &= m_i p_i - m_i c_{\text{res},i} - c_{0,i} \\
 &= m_i (p_i - c_{\text{res},i}) - c_{0,i}
 \end{aligned}$$

Since the investment $c_{0,i}$ is only related to the profit in future periods, firms would not take any actions to increase the profit for the current period. Thus, $c_{0,i} = 0$ for $i \in \{a, b\}$. Suppose that $u_{j,i} \geq u_R, \forall j \in J$, i.e. all consumers would like to buy one product, would the firms compete over the prices à la Bertrand, or would they cooperate with each other and divide the market? To answer this question, first we need to check if the firms' products have symmetric functionality, then we need to discuss the outcomes of two scenarios—competition and collusion. Figure 2.4 shows the market demand under these scenarios.¹²

¹¹We assume the marginal costs of both firms are the same, i.e. $c_{\text{res},a} = c_{\text{res},b}$.

¹²In this figure, we denote θ_a with θ .

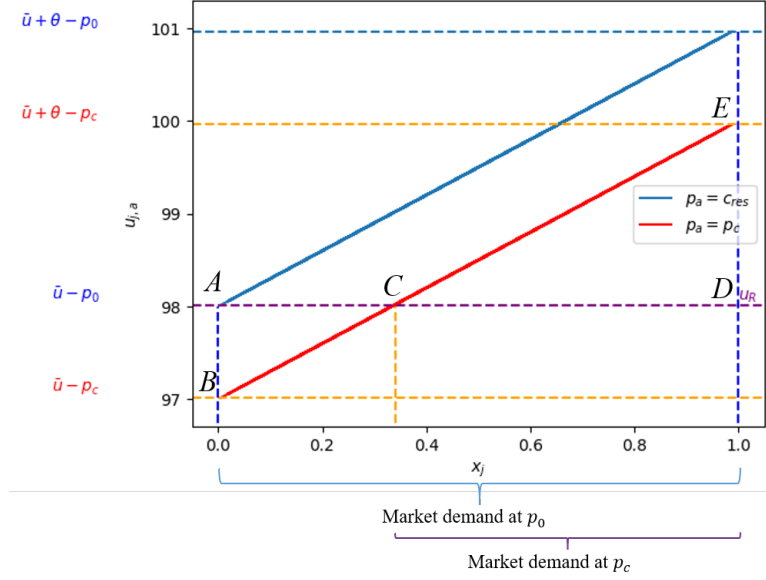


Figure 2.4: Market demand before and after the collusion of the firms. The blue curve represents the utility of every consumer when $p_a = p_b = c_{res}$, and the red curve represents the utility of every consumer when $p_a = p_b = p_c$.

For products with asymmetric functionality, we assume firms are aware that there exists a difference in the product functionality, so they probably have to price their products differently. Previously we point out that if $\Delta\theta > 0$ and $0 \leq \frac{\Delta p}{\Delta\theta} \leq 1$, consumer j with $x_j \in \{\frac{\Delta p}{\Delta\theta}, 1\}$ buys a product from firm a , therefore, the maximization problem of the optimal price selection for firms can be formulated as

$$\begin{aligned} \max_{p_a} \Pi_a &= (1 - \frac{p_a - p_b}{\Delta\theta})(p_a - c_{res}) \\ \max_{p_b} \Pi_b &= (\frac{p_a - p_b}{\Delta\theta})(p_b - c_{res}) \end{aligned}$$

We can solve for the optimal prices p_a^* and p_b^* by taking partial derivatives and let $\frac{\partial \Pi_a}{\partial p_a} = 0$ and $\frac{\partial \Pi_b}{\partial p_b} = 0$. We have

$$\begin{aligned} \frac{\partial \Pi_a}{\partial p_a} &= \frac{-2p_a + p_b + c_{res} + \Delta\theta}{\Delta\theta} = 0 \\ \frac{\partial \Pi_b}{\partial p_b} &= \frac{p_a - 2p_b + c_{res}}{\Delta\theta} = 0 \end{aligned}$$

By solving the equations above, we derive that

$$\begin{aligned} p_a^* &= c_{res} + \frac{2}{3}\Delta\theta \\ p_b^* &= c_{res} + \frac{1}{3}\Delta\theta \end{aligned}$$

The optimal profits are

$$\begin{aligned} \Pi_a^* &= \frac{2}{3}\left(\frac{2}{3}\Delta\theta\right) = \frac{4}{9}\Delta\theta \\ \Pi_b^* &= \frac{1}{3}\left(\frac{1}{3}\Delta\theta\right) = \frac{1}{9}\Delta\theta \end{aligned}$$

Now we discuss the situation in which firms have the products with symmetric functionality. Under the scenario of competition, both firms set the price as the marginal cost c_{res} , and no firm earns a profit. Clearly, no firm would set a price even less than c_{res} because this generates a loss. If one firm, say firm a , deviates from this price by adding a mark-up $\varepsilon > 0$ ($p'_a = c_{res} + \varepsilon$), then all consumers turn to firm b because of this price increase. The following result would be $\Pi'_a = 0(p'_a - c_{res}) = 0\varepsilon = 0$ and $\Pi'_b = 1(c_{res} - c_{res}) = 0$. Thus, a price which is equal to the marginal cost ($p_0 = p_a = p_b = c_{res}$) is indeed a Nash equilibrium, and no single firm has the incentive to deviate from this price.

Under the scenario of collusion, both firms raise the product prices to p_c ($p_c = p_0 + \varepsilon$ with $\varepsilon > 0$) and divide the aggregate profit. Although for both firms the market demand is smaller than the case in which $p_0 = c_{res}$, there are still consumers buying from each firm. A price increase from p_0 to p_c leads to a general decrease of the consumer utility by ε units. If the reserved consumer utility is equal to the consumer who is indifferent with the product functionality in the case of pure price competition ($u_R = \bar{u} - p_0$), the increase in the price leads to a decrease in the demand (this is depicted by the decrease from line segment AD to CD). From the property of similar triangles ($\triangle ACB \sim \triangle DCE$), we can derive that

$$\begin{aligned} \frac{AC}{DC} &= \frac{AB}{DE} \\ \frac{AC}{1 - AC} &= \frac{\varepsilon}{\theta - \varepsilon} \\ AC &= \frac{\varepsilon}{\theta} \end{aligned}$$

Therefore, when the market price rises from p_0 to $p_c = p_0 + \varepsilon$, the aggregate market demand decreases from 1 to $(1 - \frac{\varepsilon}{\theta})$. When ε is large, the aggregate market demand $(1 - \frac{\varepsilon}{\theta})$ is small, indicating that more mark-up leads to fewer consumers in the market. When firms collude and divide the market, firm i 's demand becomes

$\frac{1}{2}(1 - \frac{\varepsilon}{\theta})$, and the profit becomes

$$\begin{aligned}\Pi_c &= (p_c - c_{res}) \cdot \frac{1}{2}(1 - \frac{\varepsilon}{\theta}) \\ &= \frac{1}{2}\varepsilon(1 - \frac{\varepsilon}{\theta}) \\ &= -\frac{1}{2\theta}\varepsilon^2 + \frac{1}{2}\varepsilon\end{aligned}$$

Firms have the incentive to collude, if $\Pi_c \geq \Pi_0 \Leftrightarrow 0 \leq \varepsilon \leq \theta$, i.e. if the mark-up is less than the value of the functionality, there is indeed space for collusion. To find a mark-up bringing the largest profit under collusion, we let $\frac{\partial \Pi_c}{\partial \varepsilon} = 0$, then we have

$$\begin{aligned}-\frac{1}{2\theta}\varepsilon^2 + \frac{1}{2}\varepsilon &= 0 \\ \varepsilon &= \frac{1}{2}\theta\end{aligned}$$

The market demand of each firm becomes $\frac{1}{2}(1 - \frac{1}{2}) = \frac{1}{4}$, and the profit under collusion of each firm is then $-\frac{1}{2\theta}(\frac{1}{2}\theta)^2 + \frac{1}{2}(\frac{1}{2}\theta) = \frac{1}{8}\theta > 0$. The result shows that firms would like to collude, compared with the pure price competition. Our analysis also shows the effect of the position of the reserved utility (so far we assume that $u_R = \bar{u} - p_0$). If $u_R < \bar{u} - p_0$, there is more space for collusion. Similarly, if $u_R > \bar{u} - p_0$, there is less space for collusion.

Comparing the results, we have two findings: Firstly, if firms compete with each other, under the situation of asymmetric observed functionality, both firms earn a positive profit, while under the situation of symmetric observed functionality, both firms earn zero profit. This indicates that product differentiation can bring profits for both firms. Secondly, when we compare the market outcomes under the situation of symmetric observed functionality, we find that the optimal strategy for a firm is to collude with the other firm, and the case of pure price competition is less likely to appear in the market.

2.4.2 Strategic price decisions under endogenous product functionality

So far, we assume that in the game, the observed functionality of each product is exogenous. To move one step further for analyzing the role of the consumers, we need to consider the role of number of consumers ($m_{i,t}$) in the game. One intuitive question is, would there be a steady state Nash equilibrium in this game? Similarly, through the analysis of the representative consumer and the profit maximization, we can find some useful results to answer this question. In the steady state,

regardless of the time, it always holds that

$$\theta_i(\chi_i, m_i) = \beta_2 \chi_i - \beta_0 m_i + \beta_3$$

with $\beta_0, \beta_2, \beta_3 > 0$. Assuming that there is product differentiation, firm a has a higher service capacity ($\chi_a > \chi_b$) and asks for a higher price ($p_a > p_b$). Consumer j would buy from firm a if the following condition holds:

$$\begin{aligned} u_{j,a} &\geq u_{j,b} \\ \Leftrightarrow \\ x_j \theta_a - p_a &\geq x_j \theta_b - p_b \\ \Leftrightarrow \\ x_j(\theta_a - \theta_b) &\geq p_a - p_b > 0 \end{aligned}$$

Since we assume that $x_j \sim U[0, 1]$, it requires that $\theta_a - \theta_b > 0$, which can be formulated as

$$\begin{aligned} \beta_2 \chi_a - \beta_0 m_a + \beta_3 - (\beta_2 \chi_b - \beta_0 m_b + \beta_3) &> 0 \\ \Leftrightarrow \\ \beta_2(\chi_a - \chi_b) - \beta_0(m_a - m_b) &> 0 \\ \Leftrightarrow \\ m_a - m_b &< \frac{\beta_2}{\beta_0} \cdot (\chi_a - \chi_b) \end{aligned}$$

Since $\frac{\beta_2}{\beta_0} \cdot (\chi_a - \chi_b) > 0$, the condition above always holds if $m_a \leq m_b$. If $m_a > m_b$, the intuition could be as follows: From the perspective of parameters, when $\frac{\beta_2}{\beta_0}$ is relatively large, i.e., when the importance of service capacity is to some extent larger than the importance of the number of consumers, it is more likely that the condition can be satisfied; when $\chi_a - \chi_b$ is relatively large, i.e., when firm a has a service capacity which is “larger enough” than that of firm b , it is more likely that the condition can be satisfied.

We denote the consumer threshold by $\hat{x}$.¹³ Since the user at the consumer threshold should be indifferent toward products offered by the two firms, we let $\hat{x}$ satisfy $u_{j,a} = u_{j,b}$. Thus, we need to have $x_j = \hat{x}$ which satisfies

$$\begin{aligned} u_{j,a} &= u_{j,b} \\ \Leftrightarrow \\ x_j(\theta_a - \theta_b) &= p_a - p_b \\ \Leftrightarrow \\ x_j(\beta_2(\chi_a - \chi_b) - \beta_0(m_a - m_b)) &= p_a - p_b \end{aligned}$$

¹³This could be compared with a standard Hotelling model, while in our model, the location of consumers represents the preference on the functionality and the consumer threshold divides the consumers' decisions (Hotelling 1929).

Assuming that both firms provide products that satisfy the reserved utility for every consumer ($u_R \leq u_{j,i} \forall j \in J, i \in \{a, b\}$), it is clear that all consumers sum up to 1. In the following steps, we check if we can get interior solutions for equilibrium prices. Since under market equilibrium, all consumers in $[\hat{x}, 1]$ (consumers who are more aware of the product functionality) buy from firm a and all consumers in $[0, \hat{x}]$ buy from firm b , we know that $m_a = 1 - \hat{x}$ and $m_b = \hat{x}$. To find the location of $\hat{x}$, we substitute m_a and m_b with terms related to $\hat{x}$ and solve the following equation for $\hat{x}$:

$$\hat{x}(\beta_2(\chi_a - \chi_b) - \beta_0(1 - \hat{x} - \hat{x})) = p_a - p_b$$

We denote $(p_a - p_b)$ and $(\chi_a - \chi_b)$ with Δp and $\Delta\chi$. Now we rearrange the equation as a quadratic equation with the form of $A\hat{x}^2 + B\hat{x} + C = 0$, with

$$\begin{aligned} A &= 2\beta_0 \\ B &= \beta_2\Delta\chi - \beta_0 \\ C &= -\Delta p \end{aligned}$$

The existence of a solution requires $B^2 - 4AC \geq 0$, so we check whether the condition holds. The condition $B^2 - 4AC \geq 0$ can be formulated as

$$\begin{aligned} (\beta_2\Delta\chi - \beta_0)^2 &\geq 4(2\beta_0)(-\Delta p) \\ &\Leftrightarrow \\ (\beta_2\Delta\chi - \beta_0)^2 + 8\Delta p\beta_0 &\geq 0 \end{aligned}$$

Since $(\beta_2\Delta\chi - \beta_0)^2 \geq 0$ and $8\Delta p\beta_0 > 0$, we know that the discriminant

$$\delta = (\beta_2\Delta\chi - \beta_0)^2 + 8\Delta p\beta_0 > 0$$

Thus, there exists two different solutions for $\hat{x}$, which can be formulated as

$$\hat{x} = \frac{-\beta_2\Delta\chi + \beta_0 \pm \delta^{\frac{1}{2}}}{4\beta_0}$$

with

$$\delta = (\beta_2\Delta\chi - \beta_0)^2 + 8\Delta p\beta_0$$

To align the analysis with our assumption, we need to discuss whether $\hat{x}$ fulfills the condition $0 \leq \hat{x} \leq 1$. Thus, we discuss the two different solutions of $\hat{x}$ separately: Firstly, we discuss whether $0 \leq \frac{-\beta_2\Delta\chi + \beta_0 - \delta^{\frac{1}{2}}}{4\beta_0} \leq 1$ holds. To fulfill $\frac{-\beta_2\Delta\chi + \beta_0 - \delta^{\frac{1}{2}}}{4\beta_0} \geq 0$, we need to have

$$\delta^{\frac{1}{2}} \leq \beta_0 - \beta_2\Delta\chi$$

If $\beta_0 - \beta_2\Delta\chi \geq 0$ holds, then the condition is equivalent to

$$\begin{aligned}\delta &\leq (\beta_0 - \beta_2\Delta\chi)^2 \\ &\Leftrightarrow \\ \Delta p\beta_0 &\leq 0\end{aligned}$$

which contradicts to our assumption that $\Delta p\beta_0 > 0$. Therefore, $\frac{-\beta_2\Delta\chi + \beta_0 - \delta^{\frac{1}{2}}}{4\beta_0} < 0$. We then discuss whether $0 \leq \frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} \leq 1$ holds. To fulfill $\frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} \geq 0$, we need to have

$$\delta^{\frac{1}{2}} \geq \beta_2\Delta\chi - \beta_0$$

The condition holds for sure if $\beta_2\Delta\chi - \beta_0 \leq 0$. If $\beta_2\Delta\chi - \beta_0 > 0$, then the condition is equivalent to

$$\begin{aligned}\delta &\geq (\beta_2\Delta\chi - \beta_0)^2 \\ &\Leftrightarrow \\ 8\Delta p\beta_0 &\geq 0\end{aligned}$$

which also holds for sure. Thus, $\frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} \geq 0$ holds.

To fulfill $\frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} \leq 1$, we need to have

$$\begin{aligned}\frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} &\leq 1 \\ &\Leftrightarrow \\ \delta^{\frac{1}{2}} &\leq 3\beta_0 + \beta_2\Delta\chi \\ &\Leftrightarrow \\ \delta &\leq (3\beta_0 + \beta_2\Delta\chi)^2 \\ &\Leftrightarrow \\ \Delta p &\leq \beta_0 + \beta_2\Delta\chi\end{aligned}$$

Therefore, the consumer threshold is

$$\hat{x} = \frac{-\beta_2\Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0}$$

with

$$\begin{aligned}\delta &= (\beta_2\Delta\chi - \beta_0)^2 + 8\Delta p\beta_0 \\ \Delta p &\leq \beta_0 + \beta_2\Delta\chi\end{aligned}$$

With the endogenous threshold $\hat{x}$, we can write the maximization problem of firm a and firm b as

$$\begin{aligned}\max_{p_a} \Pi_a &= (1 - \hat{x})p_a \\ \max_{p_b} \Pi_b &= \hat{x}p_b\end{aligned}$$

where

$$\hat{x} = \frac{-\beta_2 \Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0}$$

with

$$\begin{aligned}\delta &= (\beta_2 \Delta\chi - \beta_0)^2 + 8\Delta p \beta_0 \\ \Delta p &\leq \beta_0 + \beta_2 \Delta\chi\end{aligned}$$

The consumer threshold provides us information about the number of consumers of the two firms. When firm a has a larger number of consumers, we have

$$\begin{aligned}\hat{x} &< \frac{1}{2} \\ \Leftrightarrow \\ \frac{-\beta_2 \Delta\chi + \beta_0 + \delta^{\frac{1}{2}}}{4\beta_0} &< \frac{1}{2} \\ \Leftrightarrow \\ \delta^{\frac{1}{2}} &< \beta_0 + \beta_2 \Delta\chi \\ \Leftrightarrow \\ \Delta\chi &> \frac{2\Delta p}{\beta_2}\end{aligned}$$

Intuitively, from the perspective of firm a , holding all others equal, a large difference in the product service capacity ($\Delta\chi$) should lead to a higher number of consumers.

By taking the partial derivatives for solving the optimal prices for each firm, we let $\frac{\partial \Pi_a}{\partial p_a} = 0$ and $\frac{\partial \Pi_b}{\partial p_b} = 0$ and solve for the equilibrium prices p_a^* and p_b^* . Since it is complicated to show the analytical solutions, we assign different values on parameters $\beta_0, \beta_2, \chi_a, \chi_b$ and apply *Mathematica* to find the numerical solutions (Wolfram Research Inc. 2024).

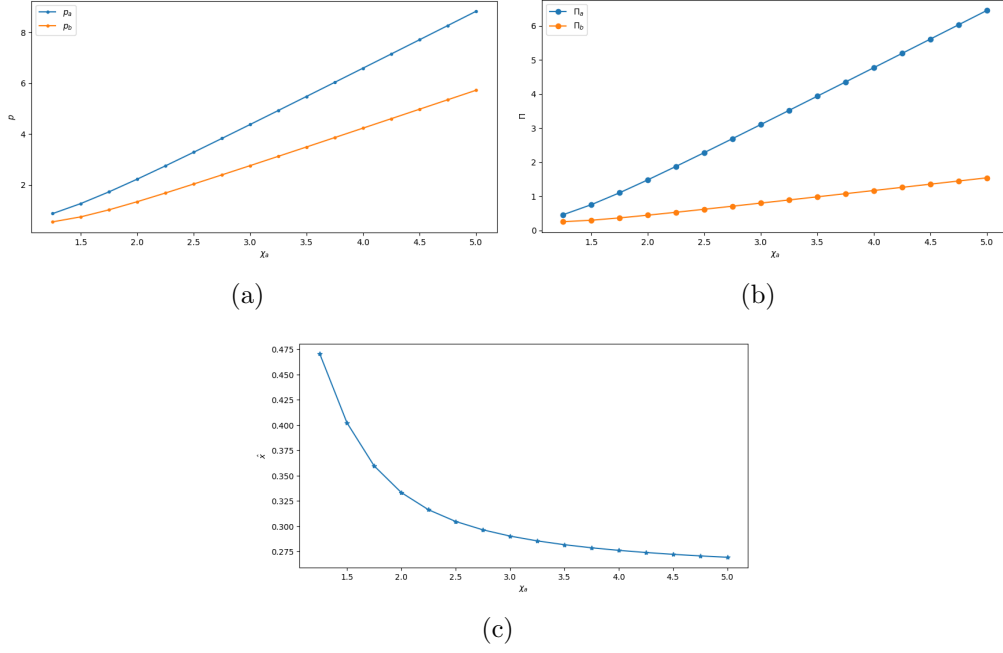


Figure 2.5: Sub-figures include: (a) Equilibrium prices, (b) profits under equilibrium, and (c) consumer threshold (or the number of consumers of firm b). Model parameters: $\beta_0 = 1, \beta_2 = 3, \chi_b = 1, \chi_a \in [1.25, 5]$.

From Figure 2.5, we know that holding all others equal, when there is a larger difference in service capacity ($\Delta\chi$), in equilibrium, the difference in price and profit of the firms becomes larger, and the firm with an advantage in service capacity has more consumers. Both firms set prices higher than the marginal cost, and the “mark-up” on the marginal cost becomes higher as $\Delta\chi$ becomes larger. As $\Delta\chi$ becomes larger, the decreasing speed in the consumer threshold $\hat{x}$ diminishes.

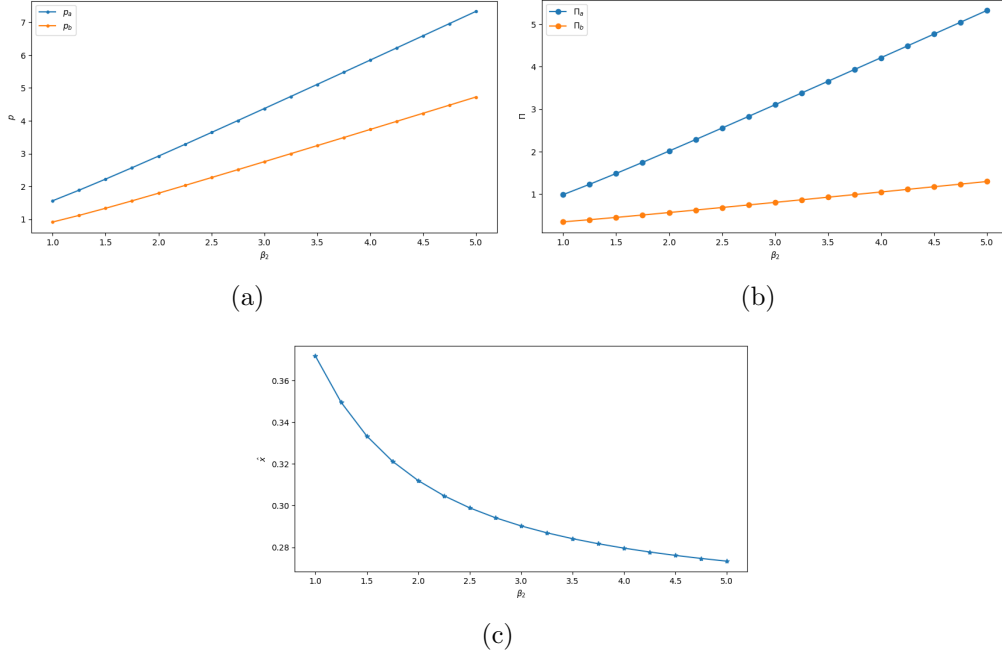


Figure 2.6: Sub-figures include: (a) Equilibrium prices, (b) profits under equilibrium, and (c) consumer threshold (or the number of consumers of firm b). Model parameters: $\beta_0 = 1, \chi_a = 3, \chi_b = 1, \beta_2 \in [1, 5]$.

From Figure 2.6, we know that holding all others equal, when β_2 is much larger than β_0 , i.e., when the effect of the service capacity is much higher than the effect of the number of consumers in the observed functionality, in equilibrium, the difference in price and profit of the firms becomes larger. The firm with an advantage in service capacity has more consumers. Both firms set prices higher than the marginal cost, and the “mark-up” on the marginal cost becomes higher as $\frac{\beta_2}{\beta_0}$ becomes larger. As $\frac{\beta_2}{\beta_0}$ becomes larger, the decreasing speed in the consumer threshold $\hat{x}$ diminishes.

To sum up, under our model where the observed product functionality is influenced by endogenous factors, we find that there are viable numerical solutions for the consumer threshold, allowing for the coexistence of both firms in the market. Particularly, in scenarios where there is significant disparity in service capacity or when the impact of service capacity substantially outweighs the influence of consumer number on product functionality, the equilibrium shifts. More consumers are inclined to purchase from the firm with superior service capacity. This assumes a uniform distribution of consumers’ preferences across a continuum from 0 to 1, where each point represents a unique consumer. This result can be compared to the rivalry between a smaller ICT firm, offering less functional yet affordably

priced products, and a larger ICT firm, providing highly functional products at a premium. The interplay between quality and price disparities culminates in differentiated pricing and profit margins, yet neither firm is entirely edged out of the market in numerous instances.

In this game design, we have highlighted the feasibility of the consumer threshold. Nevertheless, when we consider the interdependent decision-making of consumers and the role of the R&D investment across different periods, we cannot get much information from the model. When analyzing the decision-making process of consumers within this game, it is crucial to determine the presence and influence of a consumer network. Given our assumption that a consumer's utility function is impacted by the decisions of their connections, the introduction of a network structure undeniably alters the consumer selection. In the dynamic approach with consumer networks, we further point out the role of network effect and the outcomes of the R&D investment.

2.4.3 Extension to a two-stage game

Based on our analysis of equilibrium prices, we are aware of the importance of asymmetry between firms. One crucial question is how could the Nash equilibrium be distorted if the firms actively take action before the price competition. Therefore, we extend our static analysis to a two-stage game. The structure of the game is defined as follows: Consider a market with two competing firms, each selling a single product to multiple consumers. In this model, each consumer purchases at most one product from these firms. The decision-making process for the firms is modeled as a two-stage game within a single time period, hence the omission of time notation (t) for simplicity:

- **Stage 1:** Firm i ($i \in \{a, b\}$) decides on its R&D investment on the physical infrastructure, denoted by $c_{0,i}^{Phy}$.
- **Stage 2:** Firm i ($i \in \{a, b\}$) sets the price of its product p_i .

Suppose firm i has the initial physical infrastructure $S_{i,0}$ and the initial technology $A_{i,0}$. In **Stage 1**, it tries to carry out a non-negative investments, namely $c_{0,i}^{Phy} \geq 0$. Then its physical infrastructure in this period should be

$$S_i = S_{i,0} + \log(c_{0,i}^{Phy} + 1)$$

Therefore, the product service capacity of firm i can be represented as

$$\chi_i = (S_{i,0} + \log(c_{0,i}^{Phy} + 1))A_{i,0}$$

In **Stage 2**, firm i decides on the price p_i . The maximization problem is similar to what we have shown in the previous subsection despite the consideration of the R&D investment, namely¹⁴

$$\begin{aligned}\max_{p_a} \Pi_a &= (1 - \hat{x})p_a - c_{0,a}^{Phy} \\ \max_{p_b} \Pi_b &= \hat{x}p_b - c_{0,b}^{Phy}\end{aligned}$$

Since the model involves a complex optimization over continuous R&D investments, we discretize the strategy space to do numerical analysis. In **Stage 1**, each firm i selects an R&D investment level (covering both physical infrastructure and technology) from a discrete grid. For illustration, we first present a simplified game tree with high and low investment levels, following Church & Ware (2000) and implemented using the *istgame* package (Cho 2022). For the equilibrium analysis, we adopt a finer grid in which both firms' investments range from 0 to 5 with a step size of 0.25. For each investment pair, we solve the second-stage pricing game and compute the corresponding equilibrium prices. The resulting equilibrium prices determine the associated profits and consumer thresholds. These equilibrium outcomes serve as continuation payoffs for the first-stage investment game. We then identify best responses for each firm over the discrete investment grid and locate mutual best-response pairs. The resulting strategy profile constitutes a pure-strategy subgame perfect Nash equilibrium (SPNE) of the discretized two-stage game.

¹⁴Here we also assume that $\Delta\chi = \chi_a - \chi_b > 0$.

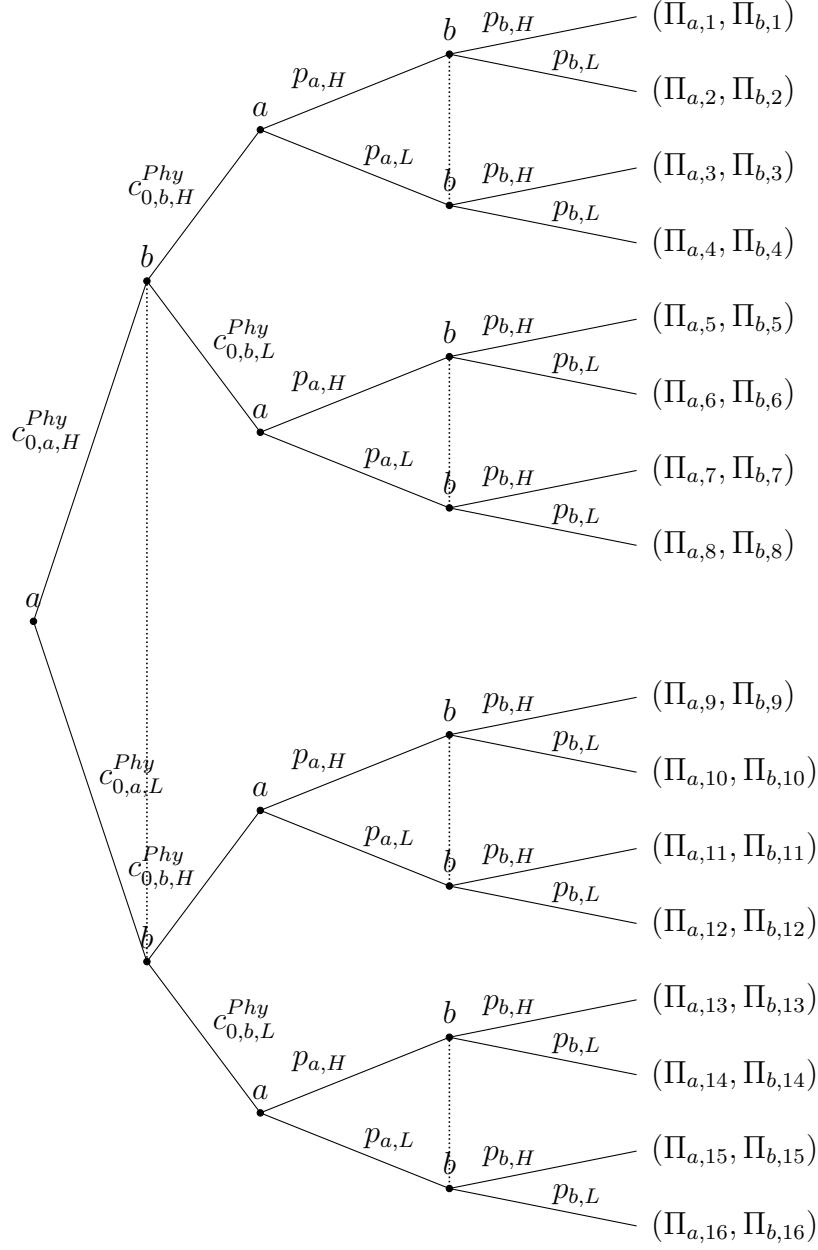


Figure 2.7: Example game tree of firm a and b in a two-stage game assuming that each firm has two choices at each decision node. $\Pi_{i,c}$ denotes firm i 's profit under the c -th circumstance. $c_{0,i,H}^{Phy}$ and $c_{0,i,L}^{Phy}$ denote a high physical investment and a low physical investment of firm i . p_H and p_L denote a high price and a low price.

We denote a subgame perfect pure-strategy Nash equilibrium by

$$\left\{ (c_{0,a}^{Phy*}, c_{0,b}^{Phy*}), (p_a^*, p_b^*), (x_a^*, x_b^*), (\Pi_a^*, \Pi_b^*) \right\}.$$

The equilibrium is identified numerically by backward induction over a discrete strategy grid. For each pair of first-stage capacity choices $(c_{0,a}^{Phy}, c_{0,b}^{Phy})$, we first solve the second-stage pricing game and obtain the corresponding Nash equilibrium prices (p_a^*, p_b^*) . These equilibrium prices determine the associated demands, consumer thresholds, and profits, which serve as continuation payoffs for the first-stage investment game. More specifically, for each firm $i \in \{a, b\}$ and for each fixed capacity choice of its rival $c_{0,-i}^{Phy}$, we compute the equilibrium profit for all feasible values of $c_{0,i}^{Phy}$ on the grid. Firm i 's best response is then defined as the grid point that maximizes its equilibrium profit conditional on the rival's choice. A pure-strategy subgame perfect Nash equilibrium is obtained at any grid point for which the capacity choices of both firms are mutual best responses, given the continuation payoffs derived from the second-stage pricing equilibria. In other words, if the best-response correspondence of firm a to firm b 's capacity choice and that of firm b to firm a 's capacity choice intersect at the same grid point, that point is identified as a subgame perfect Nash equilibrium of the discretized two-stage game. The corresponding equilibrium prices, demands, and profits are then retrieved from the same grid position, as determined by the second-stage pricing equilibrium associated with that investment pair.

Our calculation leads to the following results (see Appendix 2.7.1):

- SPNE under original product quality combination $(\chi_{a,0}, \chi_{b,0}) = (10, 1)$:
 $\{(0.75, 0), (21.308, 14.039), (0.742, 0.258), (15.064, 3.620)\}$
- SPNE under original product quality combination $(\chi_{a,0}, \chi_{b,0}) = (5, 1)$:
 $\{(0.75, 0), (10.078, 6.553), (0.733, 0.267), (6.639, 1.748)\}$
- SPNE under original product quality combination $(\chi_{a,0}, \chi_{b,0}) = (2, 1)$:
 $\{(0.75, 0), (3.410, 2.114), (0.697, 0.303), (1.628, 0.640)\}$

A few results can be drawn from the analysis. Firstly, when there is a gap between the original product functionality, the firm with an advantage in basic product functionality could have an incentive to invest in its physical service capacity. Secondly, in equilibrium, through investment, the firm with higher original product functionality can charge a higher price, attract more consumers, and earn a higher profit than its opponent. When there is a larger gap between the original product functionality, the advantage of this firm becomes larger. Besides, in each Nash equilibrium under asymmetric settings, both firms can earn a positive profit. This shows that the vertical differentiation of the products can bring a better outcome

in equilibrium than the outcome in a standard Bertrand competition. Moreover, focusing on the R&D intensity under each Nash equilibrium, we find that under a larger gap in original product quality, the R&D intensity of the firm with higher original product functionality is smaller, while its opponent's R&D intensity holds at zero.¹⁵ This shows that a firm with higher original product functionality does not have to carry out R&D investment so much concerning its revenue in equilibrium, also indicating an advantage for the firm.

This competition can be described as an asymmetric duopoly. Both ICT firms coexist in the market, while the large firm invests more in its product and earns more, the small firm invests less and earns less. This phenomenon can be observed across various markets. For instance, regarding the U.S. telecommunications market, there have been three firms dominating the shares of subscriptions, namely AT&T, T-Mobile and Verizon. Taking the first two firms for example, we can observe that AT&T has larger share of subscriptions and earns significantly more than T-Mobile. Although T-Mobile earns a smaller net profit in most of the years, Conde & Lunsford (2021) note that it has still been attracting many customers by centering its business strategy on two pillars—customers and affordability.

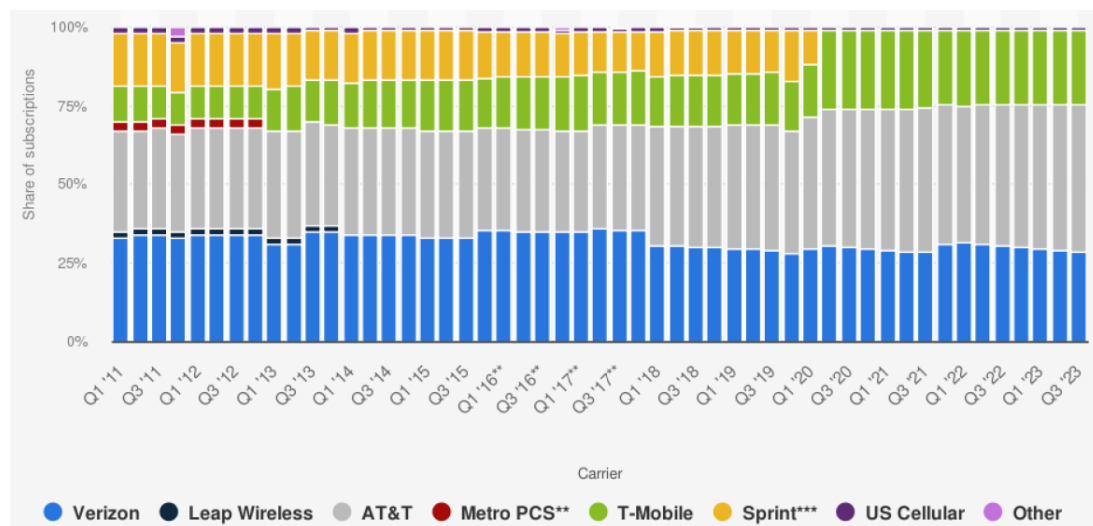


Figure 2.8: Wireless subscriptions market share by carrier in the U.S. from 1st Quarter 2011 to 3rd Quarter 2023. Source: Statista 2024. Retrieved May 13th, 2024 from <https://www.statista.com/statistics/199359/market-share-of-wireless-carriers-in-the-us-by-subscriptions/>.

¹⁵The equation of the R&D intensity of firm i is $RD_i^{int} = \frac{c_{0,i}^{Phy}}{p_i m_i}$. It is a common measure in the literature (Coff 2003).

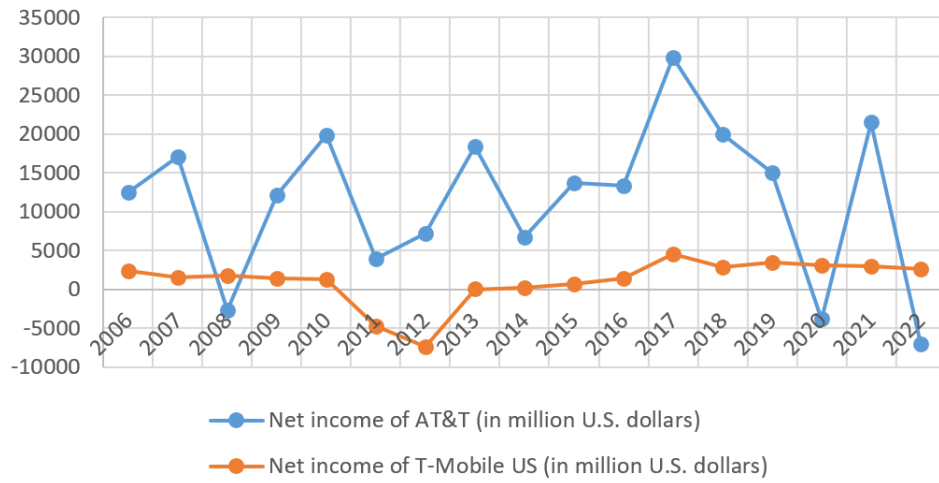


Figure 2.9: Net income of AT&T and T-Mobile from 2006 to 2022. Data source: Statista 2024. Retrieved May 13th, 2024 from <https://www.statista.com/markets/418/technology-telecommunications/>.

2.5 Agent-based analysis

Model setup	Game-theoretical analysis	Agent-based analysis
Consumer taste distribution	Uniform	Uniform/Normal/...
Number of consumers	Multiple, or infinity	Multiple
Consumer network	No network	Specific network structure
Switching cost	No switching cost	Specific switching cost
Product prices	Single period Nash equilibrium	Dynamic prices
R&D investments	High, Medium or Low	Dynamic R&D investments
Information of the market	Complete information	Complete/Partial information

Table 2.2: The main differences between the game-theoretical and the agent-based model design.

Applying dynamic analysis with computational simulations has multiple advantages:

- In the market of ICT products, the consumer taste distribution may not be a uniform distribution. While various forms of distribution may be hard to analyze analytically, computational simulations can reduce the calculation burden by doing experiments on different sample data drawn from statistical distributions.
- In reality, not all consumers know the information of the market and the product at the first periods. The dynamic analysis can capture the advertising effort of the firms and the information diffusion within the consumer network.
- The agent-based analysis provides the opportunity of adding more features and can provide more economic insights compared to the game-theoretical analysis. For instance, through the dynamic analysis, we can get more information of the effect of the consumer network, the effect of the switching cost, the price and R&D investment dynamics, etc.

2.5.1 Design of the model dynamics

Now we incorporate dynamics into the model, along with an evolving network structure, to examine the strategic behaviors of firms and their resulting implications. Through this analysis, we aim to gain deeper insights into the intricacies of

market structure, the distribution of market shares among competing firms, and the overall welfare of consumers. Such an endeavor is anticipated to shed light on the complex interplay between firm strategies and market dynamics.

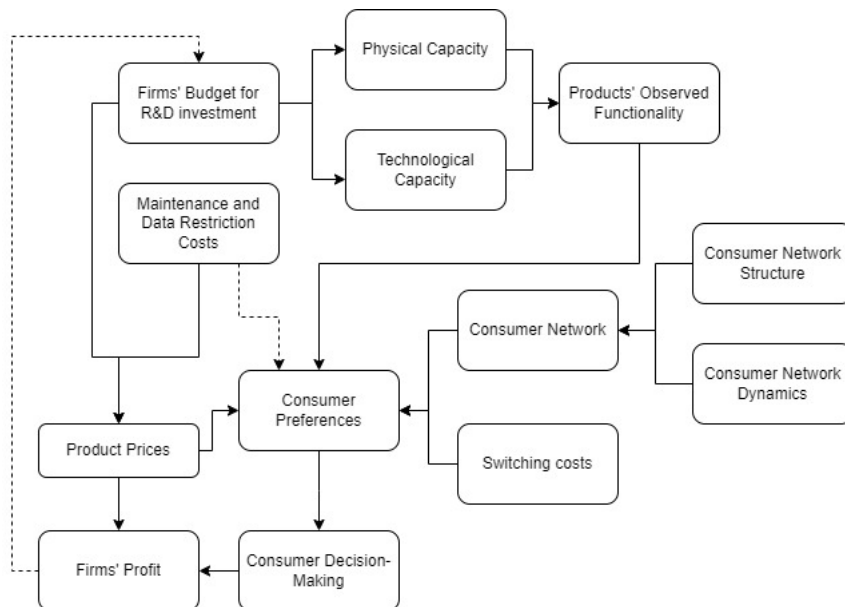


Figure 2.10: Mechanism of the model involving social networks. Solid arrows represent the analyzed influences, while dashed arrows represent the non-analyzed but possible influences.

Before consumers purchase a product, they would normally seek information from their acquaintances. It is essential to decide on the initial state of the social network before we dig into the network dynamics, because different types of networks have various properties. To mimic the real society, we apply a specific type of network, namely the small-world network.

To explore the market size of firms, we assume that at the beginning, only a proportion of consumers have the information of the firms, and only the consumers with the information can decide whether to buy a product. The idea of information diffusion is similar to the model in Banerjee et al. (2013). We depict the network dynamics as follows:

1. In the initial period, consumers (nodes) locate in a network and communicate with their neighbors through the information flows (undirected edges).¹⁶

¹⁶The “neighbors” do not necessarily mean geographical neighbors. All acquaintances of a consumer are her neighbors.

2. Before the firms sell their products, they post the information through marketing channels and get a proportion of consumers informed.¹⁷
3. Informed consumers make decisions about the products offered by two firms. At the end of the period, they share the information of the product with their neighbors.
4. At the beginning of the period t ($t > 1$), consumers build new connections and drop old connections with certain probabilities. We assume that consumers using the same product tend to connect with each other, while the consumers using different products tend to disconnect. The process from 2. to 4. follows.

Between different periods, if a consumer finds out that the product from firm $-i$ that she did not choose in the last period is more preferable in this period, she may switch to $-i$ based on a probability p_{sw} , i.e.

$$D_{j,t} = \begin{cases} 1 & \text{with probability } p_{sw} \\ 0 & \text{with probability } (1 - p_{sw}) \end{cases}$$

where $D_{j,t}$ represents the decision of consumer j in period t (1 represents switching, 0 else). This may be due to the loss of old connections, the cost of familiarizing the new product, the cost of coordination, etc. Intuitively, we assign each node a status, with 1 indicating “not informed”, 2 indicating “informed but not buying anything”, 3 indicating “informed and buying from a ” and 4 indicating “informed and buying from b ”. To represent the status of being informed and making a purchase, we let ζ_i denote the purchase status for firm i . Accordingly, $\zeta_a \equiv 3$ and $\zeta_b \equiv 4$. Once informed, a consumer cannot change to the “not informed” status. Thus, the possible status changes can be shown as:



Figure 2.11: Possible transitions of node status. Numbers 1-4 represent Status 1-4.

¹⁷Marketing effort is assumed to be of small significance on the firms' profit and the proportion of informed consumers is assumed to be fixed over time.

Therefore, we rewrite the consumer utility function as:

$$u_{j,i,t,s_{j,t}}(x_j, \theta_{i,t}, p_{i,t}, \mu_{i,j,t-1}) = \bar{u} + x_j \theta_{i,t} - p_{i,t} + \beta_4 \mu_{i,j,t-1}$$

where $\mu_{i,j,t-1}$ is the number of j 's neighbors who purchase the product from firm i in period $t-1$, x_j is consumer j 's taste towards functionality with $x_j \sim U[0, 1]$, parameter $\bar{u}, \beta_4 > 0$.

After introducing the switching probability, the dynamics of market demand also change. Some consumers may choose not to switch to a different status even if they perceive the user experience of an alternative product to be superior, due to the “lock-in” effect. In cases of utility indifference, consumers may also remain with the product they purchased in the previous period. Thus, the market demand in period t can be formulated as:

$$m_{i,t} = \sum_{j \in J_{t-1}} \left[\mathbb{1}_{u_{j,i,t} > u_{j,-i,t}} \cdot (D_{j,t} \cdot \mathbb{1}_{s_{j,t-1} = \zeta_{-i}} + \mathbb{1}_{s_{j,t-1} \neq \zeta_{-i}}) \right. \\ \left. + \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}} \cdot (\mathbb{1}_{s_{j,t-1} = \zeta_i} + \beta_{j,i,t} \cdot \mathbb{1}_{s_{j,t-1} \notin \{\zeta_i, \zeta_{-i}\}}) \right]$$

Figure 2.12 shows an example of the status transitions for three periods in the network with a few nodes. In period 1, yellow nodes represent the consumers who receive the information of the products of two firms, while the green nodes represent the consumers who are not informed. In this period, no consumer purchases any product. In period 2, the informed consumers in period 1 make purchase decisions and buys a product (the orange nodes represent consumers who buy from firm a and the blue nodes represent those who buy from firm b), and their connections get informed of the products and turned to yellow nodes. In period 3, the same transitions as those in period 2 take place. With time passes, more consumers get informed and make purchase decisions, and the process of market expansion can be simulated.

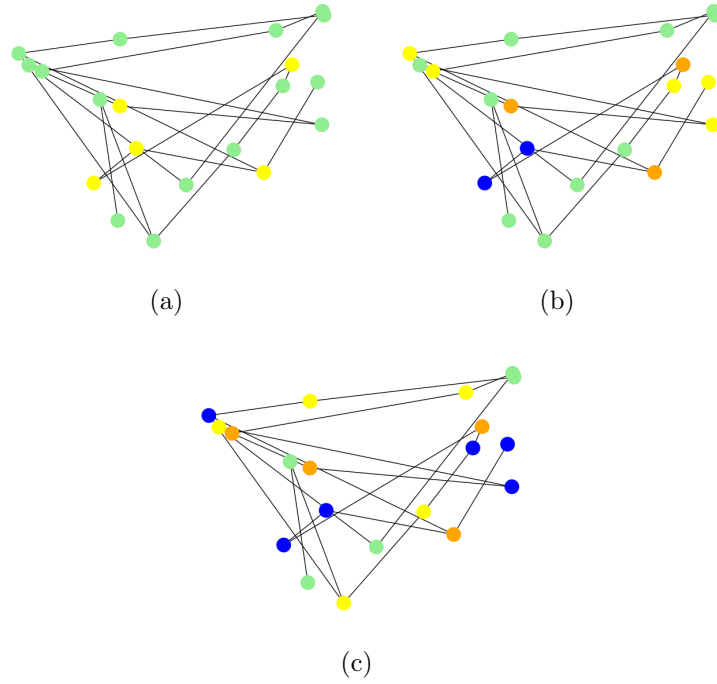


Figure 2.12: Example network dynamics.

To construct the initial network structure, we follow the idea of Watts & Strogatz (1998) and use the Python package *NetworkX* (Hagberg et al. 2008). We can show that the distribution of number of neighbors becomes more right-skewed as the probability of rewiring each edge increases, which accords with the result of Watts & Strogatz (1998) that the increasing probability leads to more irregularity and randomness of the graph.

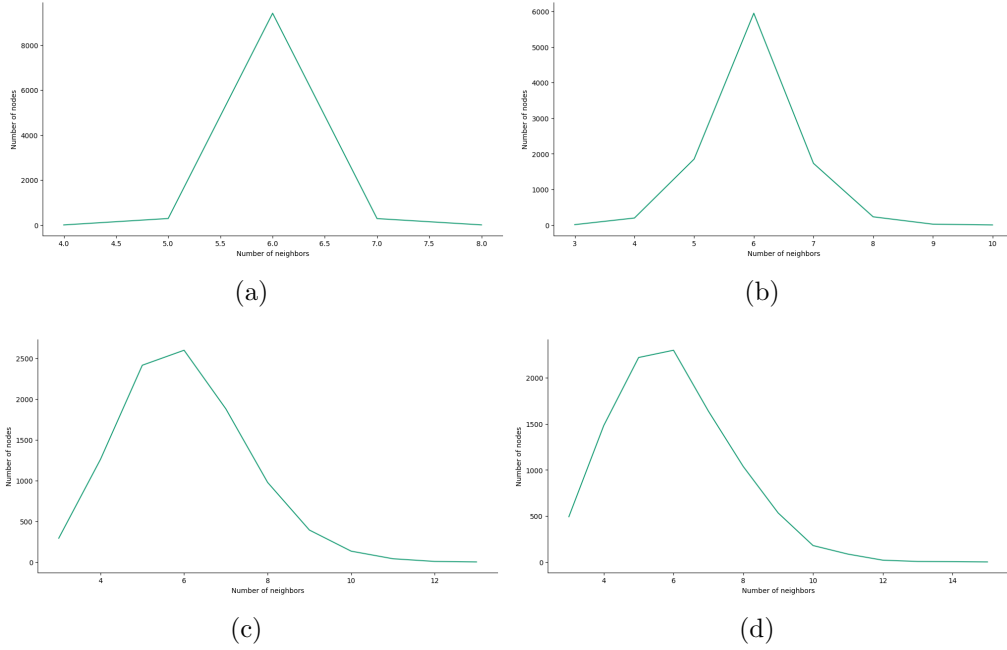


Figure 2.13: Distribution of the number of neighbors. Parameters of Watts–Strogatz small-world graph: Number of nodes: 10,000; Each node is connected to 6 nearest neighbors in ring topology; Sub-figure (a) to (d) shows the scenario in which the probability of rewiring each edge is 0.01, 0.1, 0.5 and 0.9 respectively.

2.5.2 Price and R&D investment dynamics under the network structure

In the model, the firms are able to control the R&D investment and the price of its product. Since they are uncertain about their opponent's strategies over time, it is complicated to conduct a maximization upon the profit. To deal with this problem, we tend to heuristics and assume that firms select a set of accessible behavioral rules for decision-making. We apply the idea of strategic pricing and let the firms learn from the market to take measures. The idea originates from the market exploration mentioned by Nagle & Müller (2018) and Dawid et al. (2019). Our analysis is based on a number of assumptions:

- Firms compete with each other over time.
- Firms conduct the same way of market analysis and investment updates despite randomness.

- Firms are myopic within each period, i.e. they focus on the profit within one period instead of considering discounted future profits.
- No entry or exit in the market. It is always the duopolistic competition.

To show how the firms deal with the control variables, we list the steps of the market analysis and those of the investment updates separately. A breakdown of the pricing strategies is shown as follows.

1. Firms choose price alternatives from a range centered around the product's price in the previous period for conducting the survey. For instance, if the price was $p_{i,t}$ in the last period, the survey can include price alternatives from a uniform distribution $U[p_{i,t} - 2, p_{i,t} + 2]$.¹⁸ This range offers a spectrum of prices, allowing Owners to assess consumer sensitivity to price changes.
2. Firm i formulates a survey question, essentially asking: "Would you still consider buying our product at prices ..."¹⁹ The survey aims to estimate the Owner's market share at different price points, enabling strategic adjustments for future pricing.
3. Consumers are presented with a list of alternative prices (p_{nv} , $v = 1, 2, \dots, V$) and are asked to indicate whether they would consider purchasing the product at each price. For simplicity, consumer responses ($r_{i,j,t}$) are binary: 1 for "yes" (would buy) and 0 for "no" (would not buy).²⁰
4. Owners analyze the survey data to understand consumer preferences and price sensitivities. Based on this analysis, Owners adjust their pricing strategy for the next period, aiming to optimize their market share and profitability.

According to the market analysis, firm i selects the price alternative which brings the relatively largest profit based on the expected market share.

For the R&D investment, firm i applies a behavioral rule of learning its historical profits. The basic steps are shown as follows.

1. At the beginning of the τ -th period, we assume that both Owners increase their investment by Δ units.

¹⁸In this case, the five integers in the range are the price alternatives.

¹⁹Firm i expects that consumers believe that the price of its competitor's product remains the same as in the last period.

²⁰While the actual methodology for conducting the survey (sequential, random, or all prices at once) might influence consumer responses, we assume for the model's simplicity that this influence is negligible in affecting the Owner's decision-making process.

2. At the beginning of the 2τ -th period, each Owner calculates its average profit over the last five months. We define the amount of each Owner as $\overline{\Pi_{i,2\tau}}$ ($i \in \{a, b\}$). Owner i compares $\overline{\Pi_{i,\tau}}$ and $\overline{\Pi_{i,2\tau}}$. If $\frac{\overline{\Pi_{i,2\tau}} - \overline{\Pi_{i,\tau}}}{|\overline{\Pi_{i,\tau}}|} > \vartheta$ (ϑ is the update threshold), Owner i continues the same direction of investment by Δ units; If $\frac{\overline{\Pi_{i,2\tau}} - \overline{\Pi_{i,\tau}}}{|\overline{\Pi_{i,\tau}}|} < -\vartheta$ Owner i adjusts investment reversely by Δ units; Otherwise, Owner i does not change its investment.

We show the parameter values in the benchmark cases in **Appendix 2.7.2**. We fix the initial prices of the firms and keep the structure of the consumer network as a fixed small-world network. Considering the model setup, the product service capacity $\chi_{i,t} > 0$, for the consistency of the model, we need to have a large enough β_3 to balance the negative effect of the consumer congestion effect component $(-\beta_0 m_i)$.²¹

We apply Monte Carlo simulations with 30 random seeds for 103 periods (three initial periods before the investment dynamics take place) to mimic randomness in specific events, for example, the consumer decision-making facing indifference in utility levels, the exact participants in the sample during firms' market analysis, etc. In **Appendix 2.7.4** we show the single runs of economic variables of the benchmark cases.

2.5.3 Market outcome indices

Since we aim to analyze the market outcomes based on possible parameter combinations, the crucial question is how to define the market outcome indices for comparing the performance of firms in the market. Considering the duopolistic competition, we define the indices with the economic variables covered in the model, including product prices, physical R&D investments, profits, number of consumers, product functionality, and consumer utility levels.

From the perspective of simulation techniques, we consider each run of simulations (decided by random seed) into analysis. Suppose that we have a fixed number of random seeds and ω represents the order of a random seed ($\omega \in \{1, 2, \dots, \Omega\}$) and that we simulate for a fixed number of periods with T representing the total number of periods, then we should have a realization matrix from the simulations (element $p_{i,t,\omega}$ represents the product price offered by firm i in period t under random seed ω). Taking the product price of firm i as an example.²² The price

²¹Technically, to guarantee a positive value of $\theta_{i,t}$.

²²The simulated data of physical R&D investments, profits, number of consumers, product functionality share the same structure. For the consumer utility levels, we take the mean of all consumers' utilities as the "representative utility level" in each period, i.e. under each random seed, we have $u_{i,t} = \frac{\sum_{j \in J_{t-1}} u_{j,i,t}}{|J_{t-1}|}$ for $t > 1$.

realization matrix can be formulated as:

$$\mathbf{P}_{i,t} = \begin{bmatrix} p_{i,t=1,\omega=1} & p_{i,t=2,\omega=1} & \cdots & p_{i,t=T,\omega=1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,t=1,\omega=\Omega} & p_{i,t=2,\omega=\Omega} & \cdots & p_{i,t=T,\omega=\Omega} \end{bmatrix}$$

All realizations in period 1 is defined as:

$$p_{i,t=1,\omega} = (p_{i,t=1,\omega=1}, p_{i,t=1,\omega=2}, \dots, p_{i,t=1,\omega=\Omega})^\top$$

Therefore, the price realization matrix can also be formulated as:

$$\mathbf{P}_{i,t} = [p_{i,t=1,\omega} \quad p_{i,t=2,\omega} \quad \cdots \quad p_{i,t=T,\omega}]$$

To compare the performance of the two firms, we can also define:

$$\begin{aligned} \Delta \mathbf{P}_t &= \mathbf{P}_{a,t} - \mathbf{P}_{b,t} \\ &= [\Delta p_{t=1,\omega} \quad \Delta p_{t=2,\omega} \quad \cdots \quad \Delta p_{t=T,\omega}] \end{aligned}$$

From matrix $\Delta \mathbf{P}_t$ we further define the statistical moments of the extracted values as follows:²³

$$\begin{aligned} \mathbb{E}[\mathbf{P}_t] &= [\mathbb{E}[\Delta p_{t=1,\omega}] \quad \mathbb{E}[\Delta p_{t=2,\omega}] \quad \cdots \quad \mathbb{E}[\Delta p_{t=T,\omega}]] \\ SD[\mathbf{P}_t] &= [SD[\Delta p_{t=1,\omega}] \quad SD[\Delta p_{t=2,\omega}] \quad \cdots \quad SD[\Delta p_{t=T,\omega}]] \\ \Delta p &= \mathbb{E}[\mathbb{E}[\mathbf{P}_t]] \\ \sigma_{\Delta p} &= \mathbb{E}[SD[\mathbf{P}_t]] \end{aligned}$$

The defined statistics Δp and $\sigma_{\Delta p}$ indicate the price difference between two firms and the volatility of the results across different random seeds. The same procedure is also performed for other economic variables expect for the consumer utility level. Instead of comparing the direct utility levels of the representative consumer, we propose an index RUI (Relative Utility Index) representing the relative utility to capture the utility changes over time. The index for each firm under each random seed in period t can be formulated as:

$$RUI_t = \frac{u_t}{u_1}$$

In summary, this step condenses the simulated realization matrices into specific values, facilitating the interpretation of market outcomes and enhancing the clarity and efficiency of the sensitivity analysis.

²³ $\mathbb{E}$ and SD represent the mean and the standard deviation of a vector.

2.5.4 Results of benchmark cases

The figure in **Appendix 2.7.2** provides information on the benchmark cases. In cases with symmetric technologies (Case 2 and Case 4), most market outcome indices are not statistically significant, which aligns with expectations given the symmetric competition in our model. Since both firms operate under identical conditions, neither has a clear advantage over the other, making it difficult to determine which firm outperforms its rival. In contrast, cases with asymmetric technologies (Case 1 and Case 3) exhibit statistically significant differences, allowing us to derive meaningful economic insights. The high-tech firm, firm *a*, consistently achieves higher profits than its lower-tech competitor, firm *b*. Furthermore, in scenarios where consumer network effects are present, this competitive advantage is amplified, reinforcing the importance of technology leadership in network-driven markets.

A key finding from these benchmark cases is the shift in economic mechanisms when consumer congestion is introduced. In most asymmetric cases, firm *a* typically charges a higher price than its competitor, delivers a product with greater observed functionality, and leads to higher average consumer satisfaction. However, the firm's investment and market demand strategies diverge depending on whether consumer congestion is considered: When there is no consumer congestion, firm *a* adopts a lower investment strategy while experiencing significantly lower market demand compared to its rival. When there is consumer congestion, firm *a* increases investment and experiences a market demand only slightly lower than its competitor. This shift suggests that consumer congestion may act as a strategic driver, which motivates the high-tech firm to invest more in product development. Consequently, instead of a high-price, low-investment strategy, the firm may transition to a high-price, high-investment approach in an attempt to capture greater market demand despite congestion constraints.

2.5.5 Sensitivity analysis

To discover the parameters' sensitivity and our model's robustness, we performed sensitivity analysis for essential parameters. We conduct simulations based on specific parameter variations, keeping all other parameters constant to isolate the effect of each change. The variations are shown as follows:

- Parameters related to consumer perception and decision-making
 - Lower price effect: $\beta_5 = 30 \rightarrow 20$
 - Higher switching probability: $p_{sw} = 0.1 \rightarrow 0.3$
 - Higher functionality coefficient: $\beta_2 = 3 \rightarrow 10$

- Parameters related to firm strategies and capacity management
 - Asymmetric initial physical R&D investments: $(c_{0,a,t=0}, c_{0,b,t=0}) = (10000, 10000) \rightarrow (12000, 10000)$
 - Asymmetric investment learning steps: $(\Delta_a, \Delta_b) = (1000, 1000) \rightarrow (2000, 1000)$
 - Higher investment update frequency (investment updates occur more frequently, reducing the interval between updates): $\tau = 12 \rightarrow 6$
 - Higher investment update threshold: $\vartheta = 0.05 \rightarrow 0.5$
 - Positive investment exploration step: $\psi = 0 \rightarrow 1$
 - Smaller depreciation rate $\beta^D = 0.2 \rightarrow 0.15$

Based on the results of benchmark cases, we add the market outcomes of asymmetric cases (Case 1 and Case 3) for comparison.

Parameters related to consumer perception and decision-making

(a) Results under a lower price effect

When the price effect is reduced, profit indices remain significant and even exceed the values observed in the benchmark cases. This suggests that a weaker price sensitivity allows firms to extract greater profits, particularly benefiting the high-tech firm, i.e. the high-tech firm can charge higher prices while maintaining positive investment levels, indicating that its market power strengthens when consumers are less responsive to price changes. Additionally, market demand for the high-tech firm's product surpasses that of its competitor, suggesting that in a low price-sensitivity environment, product differentiation (e.g., technological advantage) plays a larger role in consumer choice. A notable observation is that the consumer network effect reduces the investment level in this setting. This could be because, when price sensitivity is lower, firms focus more on leveraging existing network advantages rather than aggressively investing in capacity expansion.

In contrast to the benchmark cases, consumer congestion now hurts the high-tech firm's profitability. While the firm has greater flexibility to raise prices due to lower consumer price sensitivity, its ability to fully capitalize on this is constrained by congestion effects. If congestion limits accessibility or service quality, the firm may not achieve the profit gains it expects, as some consumers could be discouraged from purchasing despite lower price concerns.

(b) Results under a higher switching probability

When consumers become less sticky, meaning they switch their choices more flexibly, the high-tech firm tends to gain more profits than in the benchmark cases.

This suggests that higher consumer mobility benefits the technologically superior firm, as more consumers can reallocate toward higher-quality products. However, price increases are not the only driver of higher profits. In Case 1 of Group 2, the high-tech firm sets both a lower price and lower investment compared to the benchmark cases, yet it still captures a substantially larger market demand. This indicates that when consumers are more flexible, market share shifts can be a major determinant of firm profitability, even without aggressive pricing or investment strategies.

Similar to the benchmark cases, consumer network effects continue to enhance the high-tech firm's profitability. This suggests that as more consumers adopt the high-tech firm's product, positive feedback loops emerge, reinforcing its dominant position. While consumer congestion may limit the high-tech firm's ability to raise prices or increase investment, the overall profit can still increase. This implies that even if congestion discourages aggressive expansion, the firm benefits from a larger consumer base that remains profitable under constrained capacity.

(c) Results under a higher functionality coefficient

When the functionality coefficient increases, the weight of the service capacity component in the observed functionality rises, while the influence of consumer congestion becomes relatively weaker. This shift means that firms with higher service capacity gain a stronger competitive edge, as product functionality is now more dependent on capacity investment rather than congestion effects. This change leads to a substantial rise across all market outcome indices, indicating that higher functionality elasticity enhances market performance. The high-tech firm becomes the clear market leader, reinforcing its dominant position. Its ability to provide superior product functionality results in higher consumer adoption, increased revenue, and greater overall profitability. The dominance of the high-tech firm reflects a reinforcing competitive advantage, where superior capacity leads to sustained market leadership, even if full market exclusion of the competitor does not occur. The high-tech firm's investment levels rise significantly, as firms now have greater incentives to expand capacity to maintain their functionality advantage. However, consumer congestion remains a limiting factor. Despite the firm's high investment, congestion erodes profitability, suggesting diminishing returns when network constraints are severe. The consumer network effect continues to positively impact firm profits, reinforcing the idea that firms benefit from larger consumer bases when functionality is valued highly.

Parameters related to firm strategies and capacity management

(a) Results under asymmetric initial physical R&D investments

When the high-tech firm starts with a higher initial investment, most cases do not exhibit significant differences in profitability compared to the benchmark scenario.

However, in Case 3 of Group 1, the high-tech firm’s profit increases substantially, suggesting that an early investment advantage can reinforce market dominance, even if it does not result in a complete exclusion of the competitor. A key observation is that the consumer network effect plays a decisive role when consumer congestion is absent. In such cases, higher initial investment leads to higher market share and higher profit, as the firm benefits from positive demand spillovers. However, when both consumer network effects and congestion are present, the profit impact becomes more ambiguous—potentially falling below the benchmark case. This suggests that consumer congestion imposes limitations on the firm’s ability to fully leverage early investment advantages, leading to diminishing returns on initial R&D spending.

To sum up, early investment matters most in less constrained markets. When congestion effects are weak, the firm with a head start in investment can lock in a dominant position. If early investment strengthens the firm’s competitive edge, late-entering competitors may struggle to catch up, reinforcing market concentration.

(b) Results under asymmetric investment learning steps

When the high-tech firm adopts a larger investment learning step, its profit index tends to be lower in cases where the consumer network effect is present. This suggests that aggressively increasing investment does not necessarily translate into higher profitability under network-driven competition. Despite higher investment levels, the observed product functionality does not show a substantial increase, leading to only minor changes in market demand. This indicates that larger investment adjustments may not always be an effective strategy for improving firm performance. Since the network effect amplifies the value of existing users, the high-tech firm may not need to rely on large investment swings to secure demand. Instead, gradual improvements may be sufficient.

(c) Results under a higher investment update frequency

When firms increase the frequency of investment updates, the profitability of the high-tech firm declines in cases without consumer network effects but may increase when network effects are present. This suggests that the impact of frequent investment updates is highly dependent on market structure and demand dynamics. In cases without network effects, the high-tech firm invests significantly more than in the benchmark cases, yet market demand remains relatively unchanged. This indicates that frequent investment adjustments alone do not necessarily generate higher consumer adoption or profitability, especially when demand is primarily driven by factors other than investment. Conversely, in cases where consumer network effects are present, investment indices appear statistically insignificant, suggesting that the effectiveness of frequent investment updates diminishes. This implies that when firms already benefit from strong demand spillovers, adjusting

investment levels too frequently may yield limited additional gains.

(d) Results under a higher investment update threshold

When firms implement a larger investment update threshold, the high-tech firm's profit index tends to increase in most cases. This outcome is primarily driven by lower investment levels, suggesting that firms benefit from avoiding excessive investment adjustments. This pattern is similar to the case where firms operate with a lower investment update frequency, reinforcing the idea that less frequent, more deliberate investment decisions can enhance profitability. However, in cases without consumer congestion or network effects, the high-tech firm's profit index declines. Compared to the benchmark, the firm charges a higher price, invests more, and serves approximately the same market demand. This implies that when firms lack external demand reinforcements (such as the network effect), a high investment threshold alone does not guarantee higher profitability, especially if the additional investment does not significantly expand market share.

(e) Results under positive investment exploration steps

When firms adopt a small investment exploration step, making minor adjustments when profits remain unchanged for several periods, there are only small variations in investment levels, and most market outcome indices do not exhibit significant differences. The lack of substantial changes indicates that small, random investment adjustments do not meaningfully alter market dynamics or competitive positioning. This could imply that firms operating in relatively stable environments may not benefit from exploratory investment adjustments, as their existing strategies already align well with market conditions. On the other hand, we expect that more pronounced effects might emerge at larger exploration step sizes, where firms experiment with riskier investment strategies, potentially leading to greater variation in performance.

(f) Results under a smaller depreciation rate

When firms experience a lower depreciation rate, meaning their infrastructure does not decay as quickly over time, the technological advantage of the high-tech firm becomes the dominant factor shaping market outcomes. In this scenario, the high-tech firm charges significantly higher prices and offers a product with distinctly superior functionality, reinforcing its competitive edge. As a result, the high-tech firm captures a much larger share of market profits, leaving its competitor at a significant disadvantage. This suggests that when service capacity depreciates slowly, firms with an initial technological advantage can sustain their dominance more effectively, as they require less reinvestment to maintain their competitive position.

Overview of the parameter changes

From the benchmark cases, our main results include:

1. A higher level of technology should enable the high-tech firm to set a higher price relative to the low-tech firm.
2. A technologically advanced firm should develop products with superior observed functionality compared to those of the low-tech firm.
3. Greater technological capabilities should lead to relatively higher consumer satisfaction for the high-tech firm.
4. A higher level of technology should enhance the profitability of the high-tech firm relative to the low-tech firm.
5. The consumer network effect should help sustain the high-tech firm's market demand relative to the low-tech firm.
6. The consumer network effect should amplify technological advantages, further increasing the high-tech firm's profitability relative to the low-tech firm.
7. The consumer congestion effect should help sustain the high-tech firm's market demand relative to the low-tech firm.
8. The consumer congestion effect should enhance the profitability of the high-tech firm relative to the low-tech firm.
9. The interacting network and congestion effects should help sustain the high-tech firm's market demand relative to the low-tech firm.
10. The interacting network and congestion effects should enhance the profitability of the high-tech firm relative to the low-tech firm.

We obtain Results 1. - 4. by comparing the selected indices with zero, Results 5. - 6. are derived by analyzing the changes in the selected indices from Case 1 to Case 3 while holding $\beta_0 = 0$. Results 7. - 8. are determined by examining the changes in the selected indices when β_0 increases from 0 to 0.1 in Case 1. Results 9. - 10. are obtained by assessing the changes in the selected indices from $\beta_0 = 0$ in Case 1 to $\beta_0 = 0.1$ in Case 3.

Among the parameter changes in the sensitivity analysis, we cover nine essential changes, including: (i) lower price effect, (ii) higher switching probability, (iii) higher functionality coefficient, (iv) asymmetric initial physical R&D investments, (v) asymmetric investment learning steps, (vi) higher investment update frequency, (vii) higher investment update threshold, (viii) positive investment exploration steps, and (ix) smaller depreciation rate. To sum up, Table 2.3 shows the effects of parameters:

Results	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
1.	+	+	+	+	+	+	+	+	+
2.	+	+	+	+	+	+	+	+	+
3.	-	-	+	-	-	-	-	+	+
4.	+	+	+	+	+	+	+	+	+
5.	-	+	+	+	+	+	+	+	+
6.	0	+	+	+	+	+	+	+	+
7.	-	+	-	+	+	+	+	+	-
8.	-	+	-	+	+	+	+	+	-
9.	-	+	-	+	+	+	+	+	+
10.	0	+	0	+	+	+	+	+	+

Table 2.3: Sensitivity analysis results. + indicates that the parameter adjustment supports the expected result, - indicates that the parameter adjustment shows an opposite result, and 0 indicates that results are inconclusive due to small differences or high variability.

According to the table, Results 1., 2., and 4. are the most robust, indicating that the differentiation in technologies usually supports that the high-tech firm sets relatively higher prices, serves a product of higher observed functionality and earns a higher profit. Some consumers may not be as satisfied as in other cases (since Result 3. does not stay robust in some scenarios), but it does not alter the main market outcomes to a large extent.

Except for (i), Results 5. and 6. stay robust, indicating that the consumer network effect in most scenarios supports the technological advantage of the high-tech firm. In most scenarios, Results 7. and 8. stay robust, except for (i), (iii), and (ix), showing that the influence of the consumer congestion effect may have more complex implications for market outcomes; it does not always reinforce the high-tech firm's advantage. In terms of the interacting network and congestion effects, apart from (i) and (iii), the results stay robust, indicating that the two essential effects may mitigate each other when both are present.

2.5.6 Policy analysis

Based on the benchmark scenario with asymmetric technologies $(A_a, A_b) = (1.2, 1)$, we introduce a set of policy variations for firms, holding all other parameters constant. We examine differences in the following configurations:²⁴

- Price adjustment step $\xi_i \in \{1, 3, 5\}$

²⁴Parameters in the benchmark scenario are in **boldface**.

- Investment learning step $\eta_i \in \{1000, 5000, 9000\}$
- Investment exploration step $\psi_i \in \{0, 100, 500, 1000\}$
- Investment update frequency parameter $\tau_i \in \{12, 24, 36\}$
- Investment update threshold $\vartheta_i \in \{0.05, 0.1, 0.5\}$

Statistical tests

We include the simulation results in **Appendix 2.7.5**.²⁵ The crucial questions for policy suggestions lie in two directions: Firstly, how can we judge the performance of a policy? Secondly, do the policies lead to robust results? To answer both questions, we extract the mean profit series under each set of parameters. We define the mean profit series across all random seeds (denoted as Π_i for firm i) of certain parameter values as the benchmark setting, and by alternating one type of parameter, *ceteris paribus*, we get the mean profit series of the treatment setting. To answer the first question, we introduce two metrics for judging the firms' performance for firm $i \in \{a, b\}$, namely:

(a) The mean profit across the last 30 periods

$$\Pi_i^{(a)} = \frac{1}{30} \sum_{t=T-30}^T \Pi_{i,t}$$

and (b) the discounted profit²⁶

$$\Pi_i^{(b)} = \sum_{t=1}^T [\Pi_{i,t}(1 - \delta)^{t-1}]$$

We design both metrics, since the firm may consider both the long-run performance (concerning (a)) and the overall profitability (concerning (b)). For a certain firm, compared to the benchmark case, if in the treatment case, the two metrics go to different directions (or one stays the same), we consider the policy to be a “mixed” policy. Otherwise, if both metrics increase, we consider the policy a “good” policy, and if both metrics decrease, we consider it a “bad” policy.

To answer the second question, we apply statistical tests to judge whether there is any qualitative difference between the moments of the data in the benchmark cases (with the original parameters) and treatment cases (with adjusted parameters). Previous research applied Levene's test for the judgment of variance equality and t-tests for the judgment of mean equality (Levene 1960, Brown & Forsythe 1974, Lim

²⁵We use the bootstrapping method to calculate the confidence interval (Efron 1979).

²⁶In the code, we assume that the discount rate is equal to 20%.

& Loh 1996). Nevertheless, these statistical tests require independent variables and samples that are not heavily skewed. The t-tests also require approximately normally distributed samples. In our profit time series, when most consumers join the market, the profit may come to a peak after a few periods. Then, through the market competition, the profit of a firm decreases. Therefore, there is a skewed distribution, which may cause problems in interpreting the results. Thus, we apply nonparametric tests, which do not require strict assumptions, to judge the validity of the policies (Seabold & Perktold 2010, Virtanen et al. 2020). The whole process can be defined as follows:

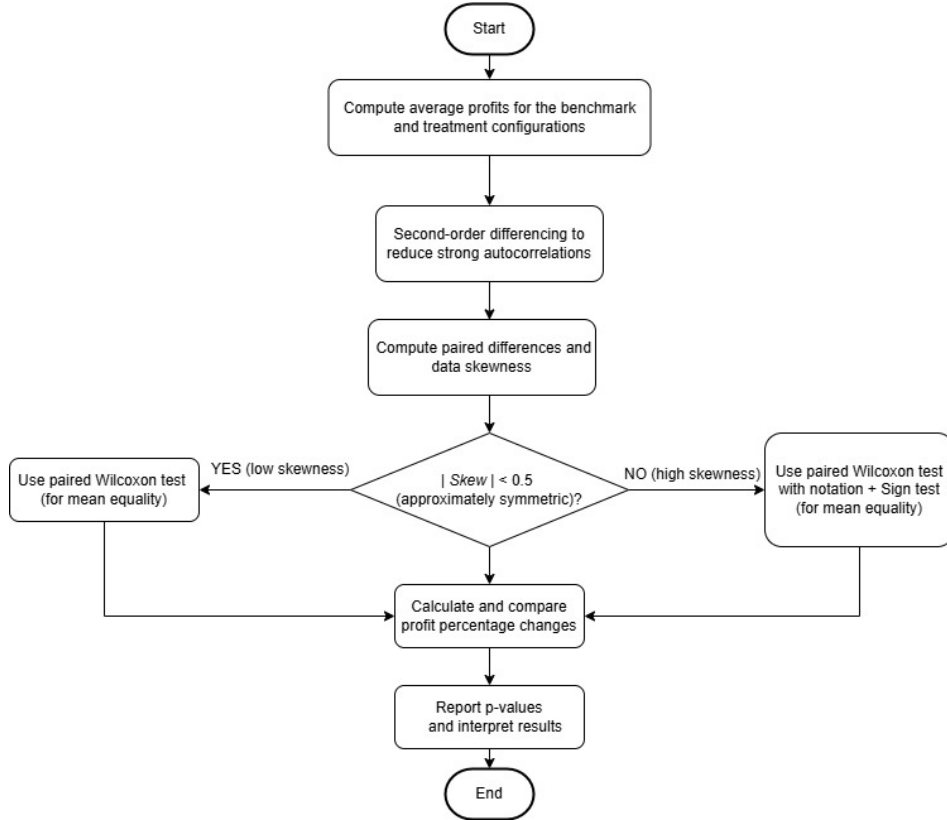


Figure 2.14: Flowchart of policy analysis.

Suppose we have two arrays for firm i :

- Benchmark case: $\{\Pi_t^{\text{BE}}\}, t = 1, 2, \dots, T$
- Treatment case: $\{\Pi_t^{\text{TR}}\}, t = 1, 2, \dots, T$

Due to the strong autocorrelation in the data, we apply first- or second-order differencing to mitigate its effect. The first and second order differences of Π_i are

defined as ($t > 0$)

$$D_{i,t} = \Pi_{i,t+1} - \Pi_{i,t}$$

$$d_{i,t} = D_{i,t+1} - D_{i,t} = (\Pi_{i,t+2} - \Pi_{i,t+1}) - (\Pi_{i,t+1} - \Pi_{i,t}) = \Pi_{i,t+2} - 2\Pi_{i,t+1} + \Pi_{i,t}$$

To test the autocorrelation, we apply the Durbin-Watson test (Durbin & Watson 1971). The second-order differencing significantly reduces autocorrelation. Next, we assess the skewness of the data. The paired Wilcoxon test assumes that samples are paired, the differences between pairs are symmetrically distributed, and observations are independent. If the data has high skewness, the assumption of symmetrically distributed differences can be violated (Wilcoxon 1945, Howell 2010, Hollander et al. 2014). Thus, for the skewed samples, we also apply the Sign test (a binomial test on the sign of differences) to ensure the robustness of the results. The logic of judgments can be demonstrated as follows:

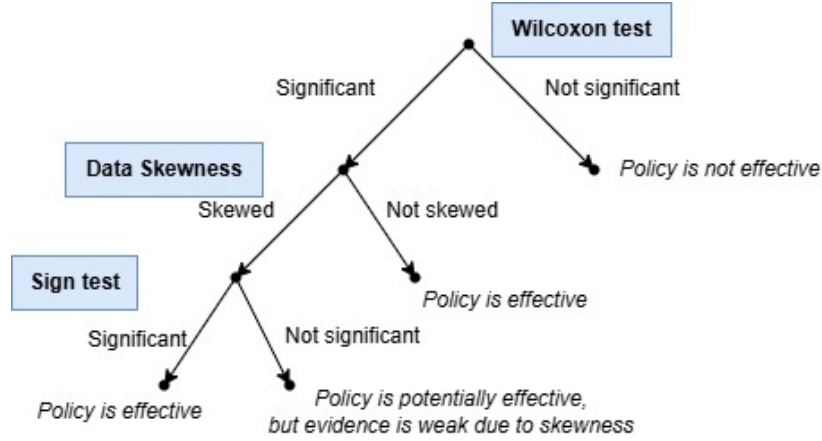


Figure 2.15: Logic of the statistical tests.

Policy matrix

We distinguish the parameter settings into four categories: (a) no network + no congestion ($\beta_4 = 0, \beta_0 = 0$), (b) no network + with congestion ($\beta_4 = 0, \beta_0 = 0.1$), (c) with network + no congestion ($\beta_4 = 50, \beta_0 = 0$), and (d) with network + with congestion ($\beta_4 = 50, \beta_0 = 0.1$). For each policy (parameter set), we assess the effect and the statistical significance.

Scenario	Firm	Parameters	Profit Change (Last 30)	Discounted Profit Change	Good Policy?	Skew?	Significance_Wilcoxon	Significance_Sign
eta_high_a	A	eta_a = 5000, eta_b = 1000	-0.93	-0.12	No	1	Significant (95%)	Significant (95%)
eta_high_a	B	eta_a = 5000, eta_b = 1000	-3.87	0.00	Mixed	1	Significant (95%)	Significant (95%)
eta_high_b	B	eta_a = 1000, eta_b = 5000	3.08	-0.06	Mixed	0	Significant (95%)	Unnecessary
eta_high_b	A	eta_a = 1000, eta_b = 5000	0.20	0.00	Mixed	1	Significant (95%)	Significant (95%)
eta_high_both	B	eta_a = 5000, eta_b = 5000	0.33	-0.06	Mixed	1	Marginal (90%)	Not significant
eta_high_both	A	eta_a = 5000, eta_b = 5000	-0.73	-0.13	No	0	Significant (95%)	Unnecessary
psi_high_a	A	psi_a = 100, psi_b = 0	0.00	0.00	Mixed	0	Significant (95%)	Unnecessary
psi_high_a	B	psi_a = 100, psi_b = 0	0.00	0.00	Mixed	1	Significant (95%)	Significant (95%)
psi_high_b	A	psi_a = 0, psi_b = 100	0.00	0.00	Mixed	0	Not significant	Unnecessary
psi_high_b	B	psi_a = 0, psi_b = 100	0.00	0.00	Mixed	0	Significant (95%)	Unnecessary
psi_high_both	A	psi_a = 100, psi_b = 100	0.00	0.00	Mixed	0	Significant (95%)	Unnecessary
psi_high_both	B	psi_a = 100, psi_b = 100	0.00	0.00	Mixed	0	Significant (95%)	Unnecessary
tau_high_a	B	tau_a = 24, tau_b = 12	0.30	0.00	Mixed	0	Significant (95%)	Unnecessary
tau_high_a	A	tau_a = 24, tau_b = 12	-0.22	0.03	Mixed	1	Significant (95%)	Significant (95%)
tau_high_b	A	tau_a = 12, tau_b = 24	-0.02	0.00	Mixed	1	Significant (95%)	Significant (95%)
tau_high_b	B	tau_a = 12, tau_b = 24	-0.14	0.01	Mixed	0	Not significant	Unnecessary
tau_high_both	B	tau_a = 24, tau_b = 24	0.12	0.01	Yes	0	Significant (95%)	Unnecessary
tau_high_both	A	tau_a = 24, tau_b = 24	-0.22	0.03	Mixed	0	Significant (95%)	Unnecessary
varthetaeta_high_a	A	varthetaeta_a = 0.1, varthetaeta_b = 0.05	-0.07	0.00	No	1	Significant (95%)	Significant (95%)
varthetaeta_high_a	B	varthetaeta_a = 0.1, varthetaeta_b = 0.05	0.06	0.00	Mixed	0	Significant (95%)	Unnecessary
varthetaeta_high_b	A	varthetaeta_a = 0.05, varthetaeta_b = 0.1	0.00	0.00	Mixed	1	Significant (95%)	Not significant
varthetaeta_high_b	B	varthetaeta_a = 0.05, varthetaeta_b = 0.1	-0.08	0.00	Mixed	0	Marginal (90%)	Unnecessary
varthetaeta_high_both	A	varthetaeta_a = 0.1, varthetaeta_b = 0.1	-0.07	0.00	No	1	Significant (95%)	Significant (95%)
varthetaeta_high_both	B	varthetaeta_a = 0.1, varthetaeta_b = 0.1	-0.02	0.00	Mixed	0	Significant (95%)	Unnecessary
xi_high_a	A	xi_a = 3, xi_b = 1	1.02	3.87	Yes	1	Significant (95%)	Significant (95%)
xi_high_a	B	xi_a = 3, xi_b = 1	-2.27	-2.13	No	1	Significant (95%)	Significant (95%)
xi_high_b	B	xi_a = 1, xi_b = 3	0.40	0.35	Yes	0	Significant (95%)	Unnecessary
xi_high_b	A	xi_a = 1, xi_b = 3	-1.15	-5.56	No	1	Significant (95%)	Not significant
xi_high_both	A	xi_a = 3, xi_b = 3	0.04	-1.31	Mixed	1	Not significant	Not significant
xi_high_both	B	xi_a = 3, xi_b = 3	1.63	-1.47	Mixed	1	Significant (95%)	Significant (95%)

Figure 2.16: Example assessment of policies.

Based on our results, we summarize the policy matrix as follows:

Overall, as both economic effects are introduced into the economic system, more policy parameters can be observed significant. In terms of strategic recommendations, both high-tech and low-tech firms generally exhibit consistent directions in price adjustment, but tend to diverge significantly in their investment learning strategies.

Based on the benchmark cases without considering any economic effects, for the high-tech firm, a more cautious approach to investment adjustments is typically recommended, reflecting the higher risks associated with rapid capacity expansion and the potential for overexposure in competitive, technology-driven markets. In contrast, the low-tech firm is encouraged to adopt a bolder investment strategy, allowing it to capture market share more aggressively.

Additionally, the consumer congestion effect tends to promote more conservative pricing, as firms must carefully balance price adjustments to avoid deterring consumers in saturated markets. In contrast, the consumer network effect encourages more aggressive price adjustments, leveraging the self-reinforcing benefits of network externalities to rapidly expand the customer base. The investment exploration also becomes more important when the consumer network effect exists.

Game on price adjustments

The policy matrix indicates that both firms are generally encouraged to adopt large price adjustment steps when the consumer network effect exists. Ceteris paribus, this pattern can be interpreted as the outcome of a strategic game in

Market Condition	High-Tech Firm Strategy	Low-Tech Firm Strategy
No Network + No Congestion	• Adopt a small investment learning step • Adopt a low investment update frequency, only considering the discounted profit • Adopt a low investment update threshold • Adopt a large price adjustment step	• Adopt a large investment learning step, only considering the last periods • Adopt a low investment update threshold • Adopt a large price adjustment step
No Network + With Congestion	• Adopt a small investment learning step • Adopt a low investment update frequency • Adopt a high investment update threshold • Adopt a small price adjustment step	• Adopt a small investment learning step • Adopt a small investment exploration step • Adopt a low investment update threshold • Adopt a small price adjustment step
With Network + No Congestion	• Adopt a small investment learning step • Adopt a small investment exploration step • Adopt a low investment update frequency • Adopt a high investment update threshold, only considering the last periods • Adopt a large price adjustment step	• Adopt a large investment learning step, only considering the last periods • Adopt a large investment exploration step, only considering the last periods • Adopt a low investment update threshold • Adopt a large price adjustment step

Market Condition	High-Tech Firm Strategy	Low-Tech Firm Strategy
With Network + With Congestion	• Adopt a small investment learning step • Adopt a low investment update frequency • Adopt a high investment update threshold, only considering the last periods • Adopt a large price adjustment step	• Adopt a large price adjustment step, only considering the last periods • Adopt a large investment exploration step, only considering the last periods • Adopt a low investment update frequency • Adopt a low investment update threshold • Adopt a large price adjustment step

Table 2.4: Strategic summary for high-tech and low-tech firms under varying market conditions.

which each firm selects its optimal price adjustment strategy in response to the other. Therefore, it is informative to analyze these strategies from the perspective of each firm, in order to better understand the underlying game structure and identify potential equilibria.

We present a 3×3 strategic matrix comparing the average profits (last 30 periods) and the discounted profits of the two competing firms a and b , under different levels of price adjustment steps: Low (L), Medium (M), and High (H) with $\{L, M, H\} = \{1, 3, 5\}$. For each block, we show the profit tuple $(\Pi_a^{(m)}, \Pi_b^{(m)})$ based on the metrics $(m)(m \in \{a, b\})$, and we underline the best responses for each firm.

NW + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(53782.76, 10469.13)	(3959.71, <u>24561.83</u>)	(3694.38, 18213.25)
	M	(88151.25, 866.68)	(<u>57857.44</u> , 11267.90)	(20791.97, <u>13623.39</u>)
	H	(<u>89565.98</u> , 94.62)	(55101.47, 3845.09)	(<u>42146.90</u> , <u>6744.24</u>)

NW + discounted profit		Firm b		
		L	M	H
Firm a	L	(48790.11, 6696.28)	(-29681.77, <u>62238.47</u>)	(-32525.96, 50596.02)
	M	(<u>99838.82</u> , -28256.04)	(24043.18, -4870.67)	(-15750.23, <u>12511.96</u>)
	H	(93618.54, -25914.62)	(<u>24609.42</u> , -15860.06)	(<u>1308.22</u> , <u>-13078.19</u>)

Figure 2.17: Profits based on the cases with only network effect. $A_a = 1.2, A_b = 1$.

Both + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(46404.00, 6907.62)	(22356.16, <u>17275.44</u>)	(35711.66, 8473.7)
	M	(<u>55309.21</u> , -849.29)	(<u>40150.97</u> , <u>8941.57</u>)	(<u>45536.23</u> , 4297.04)
	H	(51218.51, -215.56)	(32011.67, <u>12367.59</u>)	(43499.12, 5008.56)

Both + discounted profit		Firm b		
		L	M	H
Firm a	L	(37542.82, 3393.88)	(-10699.20, <u>14266.49</u>)	(3112.22, 9250.83)
	M	(<u>49514.05</u> , -26883.12)	(<u>4627.61</u> , <u>-3805.84</u>)	(<u>7016.01</u> , -17928.37)
	H	(46326.01, -24100.86)	(-262.23, <u>-14996.95</u>)	(-9349.82, -23196.92)

Figure 2.18: Profits based on the cases with both network effect and consumer congestion effect. $A_a = 1.2, A_b = 1$.

In the cases with only network effect, although (H, H) is stable in the game-theoretic sense (no firm has an incentive to deviate unilaterally), it is not socially or mutually optimal. Both firms would be better off coordinating on a more conservative pricing strategy such as (L, L) . These findings highlight the need for

coordination mechanisms or policy interventions to discourage overly aggressive pricing.

In the cases with both economic effects, once the congestion effect is introduced, the strategic landscape changes. The equilibrium moves to (M, M) —a moderate price adjustment strategy for both firms. This shift suggests that excessive responsiveness (i.e., high adjustment steps) becomes detrimental under congestion. This finding aligns well with the logic embedded in our model: Consumer congestion reduces observed product functionality. As firms increase the frequency of price changes, consumers—especially in congested settings—become more sensitive and disoriented, leading to a decline in the utility level and, consequently, market performance. To be more specific, we show the market dynamics under different price adjustment steps:

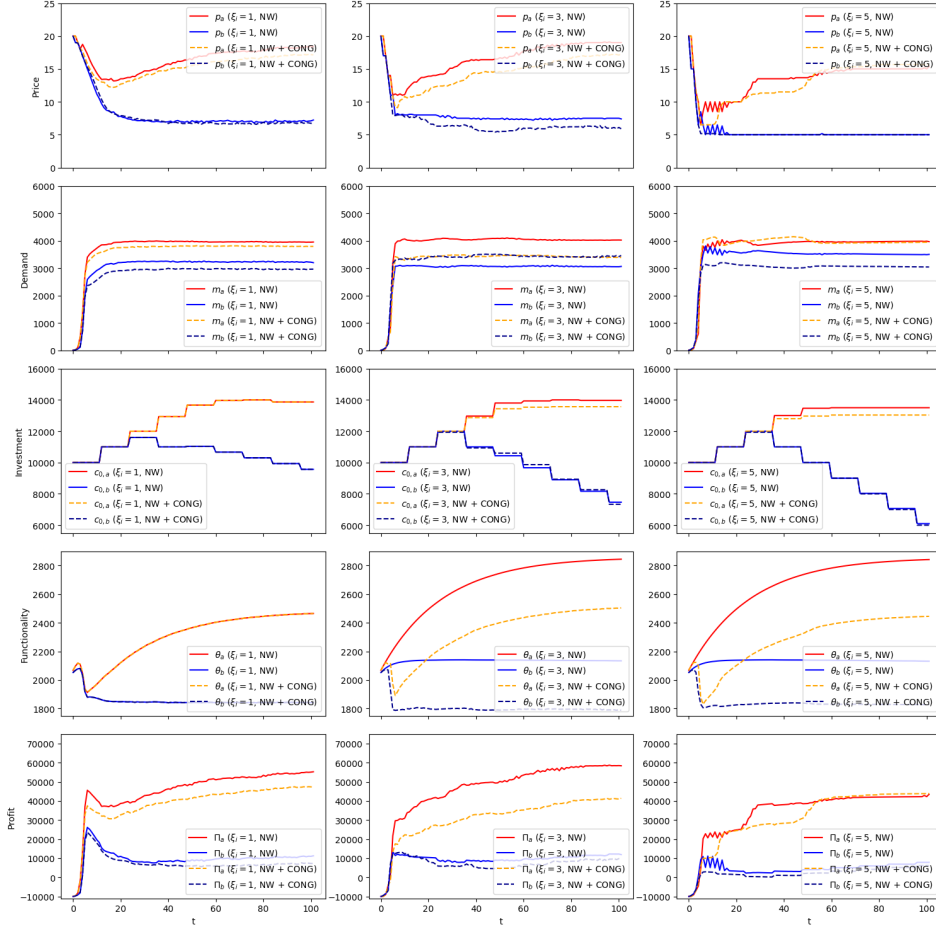


Figure 2.19: Market dynamics under different ξ_i ($i \in \{a, b\}$). “NW” represents the market setting with only network effect, and “NW+CONG” represents the market setting with both network effect and congestion effect. $A_a = 1.2$, $A_b = 1$.

When consumer congestion arises, the overall price level tends to decline. This shift in the Nash equilibrium can be understood through the following mechanism: as more consumers enter the market, the utility of individual consumers decreases due to the decreasing functionality caused by congestion effects. This leads to a reduction in market share for firms that aggressively raise prices, undermining the strategic advantage they previously held. As a result, the payoff from large price adjustments diminishes. Under these conditions, a medium price adjustment step becomes the dominant strategy for each firm, as it better balances profitability and consumer retention.

It is important to note that the policy suggestions are derived from the perspective of one firm at a time. If both firms were to follow these recommendations simul-

taneously, the resulting outcome could either correspond to a Nash equilibrium or deviate from it. This presents a potential limitation of the policy framework. Therefore, each firm should evaluate its strategy not in isolation, but in anticipation of the likely responses of its competitor, taking into account the behavior of the best response of the opponent.

To illustrate the impact of price adjustment flexibility more generally, we examine both the payoff matrices and the market dynamics in cases where the technological levels are identical ($A_a = A_b = 1$) but firms differ in their pricing flexibility ($\xi_i \in \{1, 3, 5\}$). To eliminate the influence of stochastic fluctuations, we compute the average profits for each firm under symmetric conditions. Based on these denoised outcomes, we derive the following results.

NW + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(23628.24, 23628.24)	(1391.66, 46870.77)	(11951.43, 54424.13)
	M	(43331.13, 6122.99)	(23033.62, 23033.62)	(11705.96, 27285.41)
	H	(54921.43, 11793.09)	(27476.14, 10872.47)	(18008.30, 18008.30)

NW + discounted profit		Firm b		
		L	M	H
Firm a	L	(20610.13, 20610.13)	(-34808.31, 87580.54)	(-16149.4, 65285.47)
	M	(75101.20, -25010.23)	(4929.18, 4929.18)	(-12260.23, 12832.83)
	H	(64416.08, -16395.18)	(13602.85, -14125.86)	(3939.72, 3939.72)

Figure 2.20: Profits based on the cases with only network effect. $A_a = A_b = 1$.

Both + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(19532.28, 19532.28)	(3756.52, 15848.91)	(8247.13, 15932.08)
	M	(15983.43, 4183.86)	(13509.20, 13509.20)	(15236.66, 13337.69)
	H	(16890.22, 9523.68)	(13305.17, 15514.52)	(11896.30, 11896.30)

Both + discounted profit		Firm b		
		L	M	H
Firm a	L	(18639.61, 18639.61)	(-26125.44, 22552.91)	(-16396.48, 19899.89)
	M	(22134.76, -25418.97)	(-9886.67, -9886.67)	(-7684.97, -10379.91)
	H	(19095.05, -15458.32)	(-10475.14, -6714.96)	(-13316.34, -13316.34)

Figure 2.21: Profits based on the cases with both network effect and consumer congestion effect. $A_a = A_b = 1$.

In the presence of only network effect, the unique Nash equilibrium occurs at (H, H) . When consumer congestion is introduced, the equilibrium shifts. Based on the mean profit over the last 30 periods, the Nash equilibria are (L, L) and (M, M) . However, when considering discounted profits, the unique Nash equilibrium is at (M, M) . Thus, similar to the asymmetric technology scenario, the symmetric case also demonstrates that introducing consumer congestion leads to a shift in equilibrium. This indicates that more flexible price adjustments by both firms do not always yield better outcomes in competitive markets.

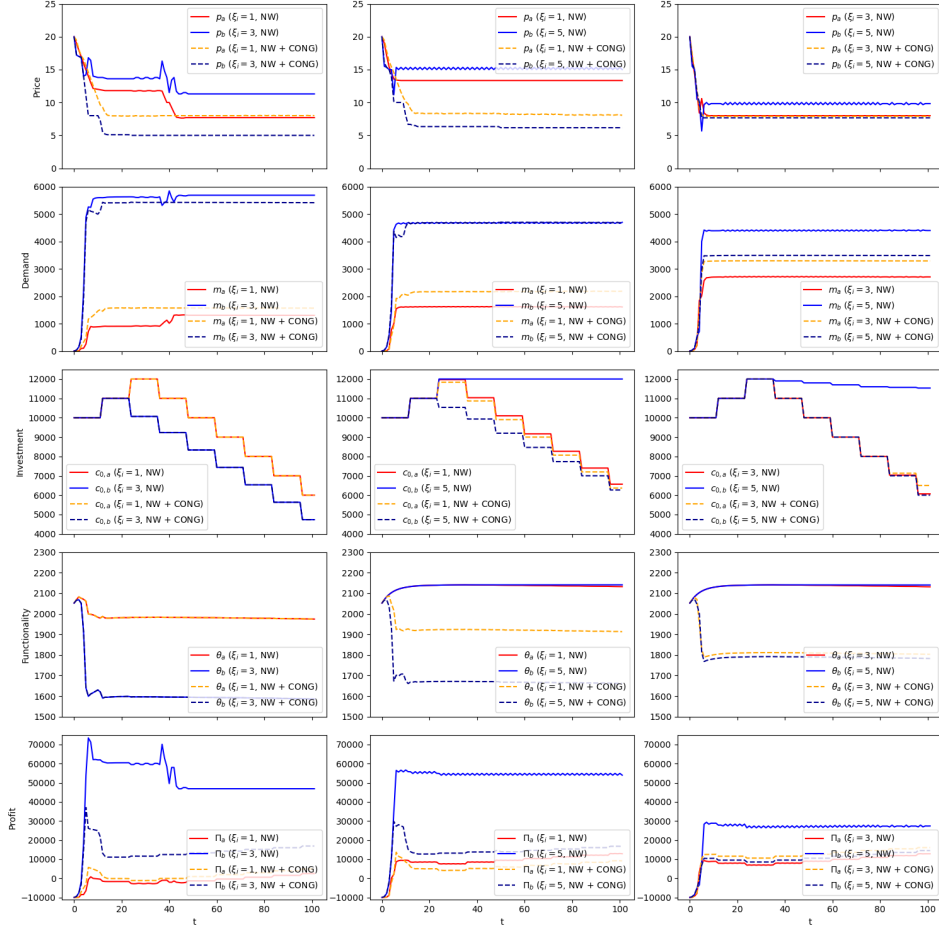


Figure 2.22: Market dynamics under different ξ_i ($i \in \{a, b\}$). “NW” represents the market setting with only network effect, and “NW+CONG” represents the market setting with both network effect and congestion effect. $A_a = A_b = 1$.

Ceteris paribus, when consumer congestion is not considered, the firm with more flexible price adjustments (i.e., higher ξ_i) consistently outperforms its competitor. This performance advantage is evident in both the price and demand dynamics: In the early periods, the flexible firm is able to aggressively lower its price, thereby capturing a larger share of the market. Once a sizable consumer base is locked in, the firm can gradually raise prices in later periods without suffering significant losses in demand. However, when consumer congestion effects are introduced, this advantage is significantly weakened. The reason lies in the congestion mechanism: As market demand becomes excessively concentrated in certain periods, consumers’ responsiveness to lock-in decreases, reducing the long-term benefit of early aggressive pricing. Consequently, the firm’s ability to sustain elevated profits

through flexible pricing is constrained under congestion.

2.5.7 Further robustness check

Given the potential influence of initial conditions on market outcomes due to path dependency, we vary the initial prices of both firms to assess the extent of this effect.²⁷ This robustness check illustrates a distinct scenario compared to previous experiments, as the reduced initial price may limit firms' ability to further lower prices during the simulation. Keeping the same design in firms' competition in price adjustments, we show the payoff matrices and the market outcomes under both asymmetric and symmetric technologies.

Payoffs and market dynamics

(a) Firms with asymmetric technologies

NW + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(<u>64857.52</u> , 8388.91)	(<u>49805.89</u> , 12634.13)	(<u>54135.23</u> , <u>15117.23</u>)
	M	(56900.32, 6807.91)	(27972.97, 15684.61)	(35923.14, <u>19072.27</u>)
	H	(33593.50, 12953.23)	(44105.32, 13484.53)	(49818.66, <u>15965.30</u>)

NW + discounted profit		Firm b		
		L	M	H
Firm a	L	(<u>9848.02</u> , -23542.07)	(<u>-2347.28</u> , -16861.42)	(-1063.44, <u>-16078.38</u>)
	M	(5732.98, -24588.40)	(-13177.77, -8965.35)	(-11050.77, <u>-7717.23</u>)
	H	(-10996.33, -18349.93)	(-3990.63, -20304.99)	(<u>3881.51</u> , <u>-16765.49</u>)

Figure 2.23: Profits based on the cases with only network effect. $A_a = 1.2$, $A_b = 1$.

²⁷In the code implementation, we replace the default initial price of 20 with a lower value of 3.

Both + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(46879.63, 8826.31)	(<u>45363.05</u> , 7539.76)	(48080.92, <u>13384.49</u>)
	M	(46730.89, 6058.59)	(37187.46, 11376.23)	(37197.40, <u>18179.91</u>)
	H	(<u>49162.76</u> , 6281.40)	(35142.63, 11856.73)	(35424.64, <u>15771.49</u>)

Both + discounted profit		Firm b		
		L	M	H
Firm a	L	(-3656.47, -21635.95)	(<u>-5005.32</u> , -20890.10)	(-5317.94, -19980.44)
	M	(<u>-2973.27</u> , -24873.68)	(-12301.39, -24655.85)	(-11992.53, <u>-21226.27</u>)
	H	(-4097.20, -25472.82)	(-13700.49, -24837.29)	(-12278.53, <u>-18848.62</u>)

Figure 2.24: Profits based on the cases with both network effect and consumer congestion effect. $A_a = 1.2, A_b = 1$.

In the cases with only network effect, under a smaller initial price, if firms only consider the mean profit of the last periods, the Nash Equilibrium should be (L, H) ; if firms only consider the discounted profit, the Nash Equilibrium should be (H, H) . This indicates that different initial prices may alter the pricing strategy of the high-tech firm. If both firms start the firm competition at a low price, the high-tech firm should have a more cautious strategy in price adjustments. In the cases with both effects, the Nash equilibrium should be (L, H) . The consumer congestion further reinforce the selection of a “cautious strategy” for the high-tech firm.

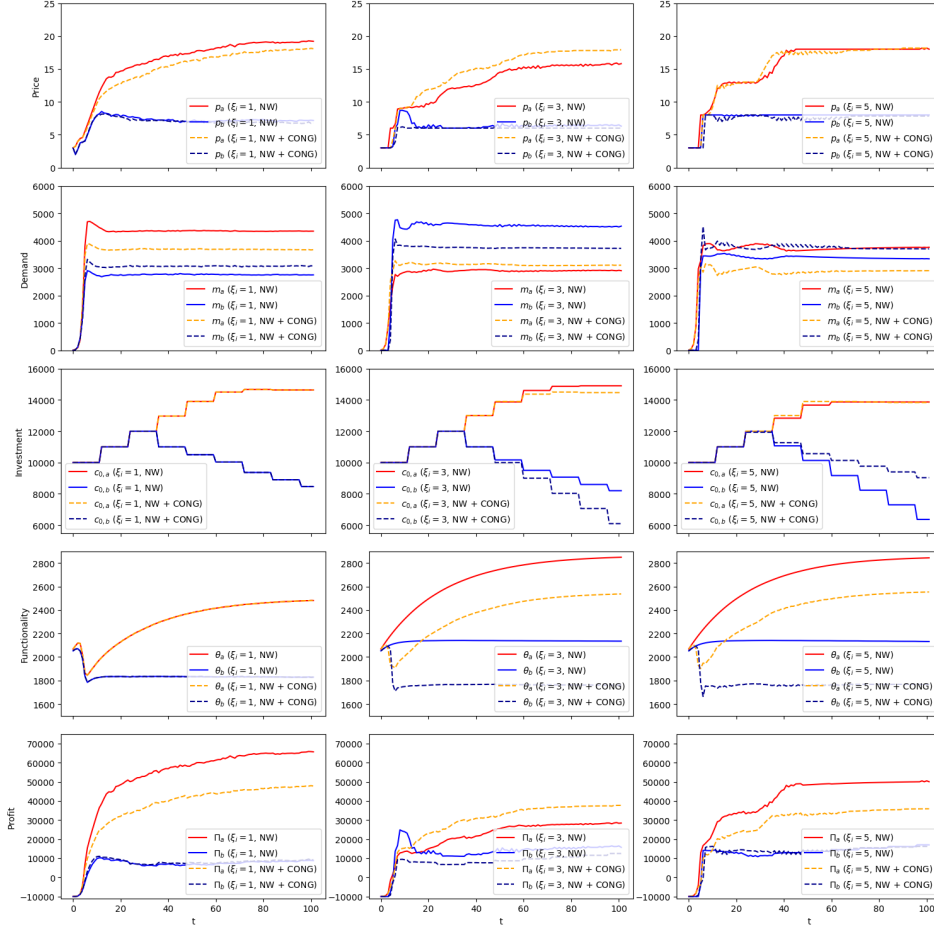


Figure 2.25: Market dynamics under different ξ_i ($i \in \{a, b\}$). “NW” represents the market setting with only network effect, and “NW+CONG” represents the market setting with both network effect and congestion effect. $A_a = 1.2, A_b = 1$.

When firms begin with a low initial price, they tend to raise prices as more consumers enter the market, taking advantage of the consumer network effect. However, this price growth eventually slows, as most consumers have already entered the market, and further price increases reduce consumer utility, potentially decreasing overall demand. The simulation results indicate that greater pricing flexibility for both firms can reduce the extent of market dominance by the high-tech firm, enabling the low-tech firm to occasionally attract a larger share of consumers. Nevertheless, in the long run, the high-tech firm maintains a competitive advantage, primarily driven by its ability to set higher prices and attract a larger consumer base, which together contribute to superior profitability.

(b) Firms with symmetric technologies

NW + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(<u>23610.98</u> , <u>23610.98</u>)	(25347.06, 12508.65)	(3604.73, 25019.09)
	M	(12508.65, <u>25347.06</u>)	(<u>19681.78</u> , <u>19681.78</u>)	(10129.52, 22499.20)
	H	(<u>25019.09</u> , 3604.73)	(22499.20, 10129.52)	(<u>19642.72</u> , <u>19642.72</u>)

NW + discounted profit		Firm b		
		L	M	H
Firm a	L	(<u>-6395.10</u> , <u>-6395.10</u>)	(-8025.52, -19232.33)	(-34769.87, <u>3928.17</u>)
	M	(-19232.33, -8025.52)	(<u>-5731.88</u> , <u>-5731.88</u>)	(-24150.35, <u>-2576.35</u>)
	H	(<u>3928.17</u> , -34769.87)	(<u>-2576.35</u> , -24150.35)	(<u>-13011.48</u> , <u>-13011.48</u>)

Figure 2.26: Profits based on the cases with only network effect. $A_a = A_b = 1$.

Both + mean profit of last 30 periods		Firm b		
		L	M	H
Firm a	L	(<u>17363.03</u> , <u>17363.03</u>)	(16256.84, 17006.71)	(14604.53, <u>18660.29</u>)
	M	(17006.71, <u>16256.84</u>)	(<u>15932.46</u> , <u>15932.46</u>)	(<u>16452.03</u> , 14248.10)
	H	(<u>18660.29</u> , 14604.53)	(14248.10, <u>16452.03</u>)	(<u>16355.94</u> , <u>16355.94</u>)

Both + discounted profit		Firm b		
		L	M	H
Firm a	L	(<u>-16001.14</u> , <u>-16001.14</u>)	(-18864.06, -17243.01)	(-21482.32, <u>-15048.65</u>)
	M	(-17243.01, -18864.06)	(<u>-17690.17</u> , <u>-17690.17</u>)	(-16969.80, -19183.24)
	H	(<u>-15048.65</u> , -21482.32)	(-19183.24, -16969.80)	(<u>-16314.52</u> , <u>-16314.52</u>)

Figure 2.27: Profits based on the cases with both network effect and consumer congestion effect. $A_a = A_b = 1$.

In the presence of only network effect, the unique Nash equilibrium occurs at (H, H) , and firms' strategies remain unchanged compared to scenarios with higher

initial prices. However, when consumer congestion is introduced, the outcome depends on the profit evaluation criterion. If firms base their decisions solely on the mean profit over the last periods, no pure-strategy Nash equilibrium appears to exist. In contrast, when decisions are based on discounted profit, two Nash equilibria emerge: (M, M) and (H, H) . This indicates that with the presence of consumer congestion, there seems to be more discrepancy in the firms' pricing strategies.

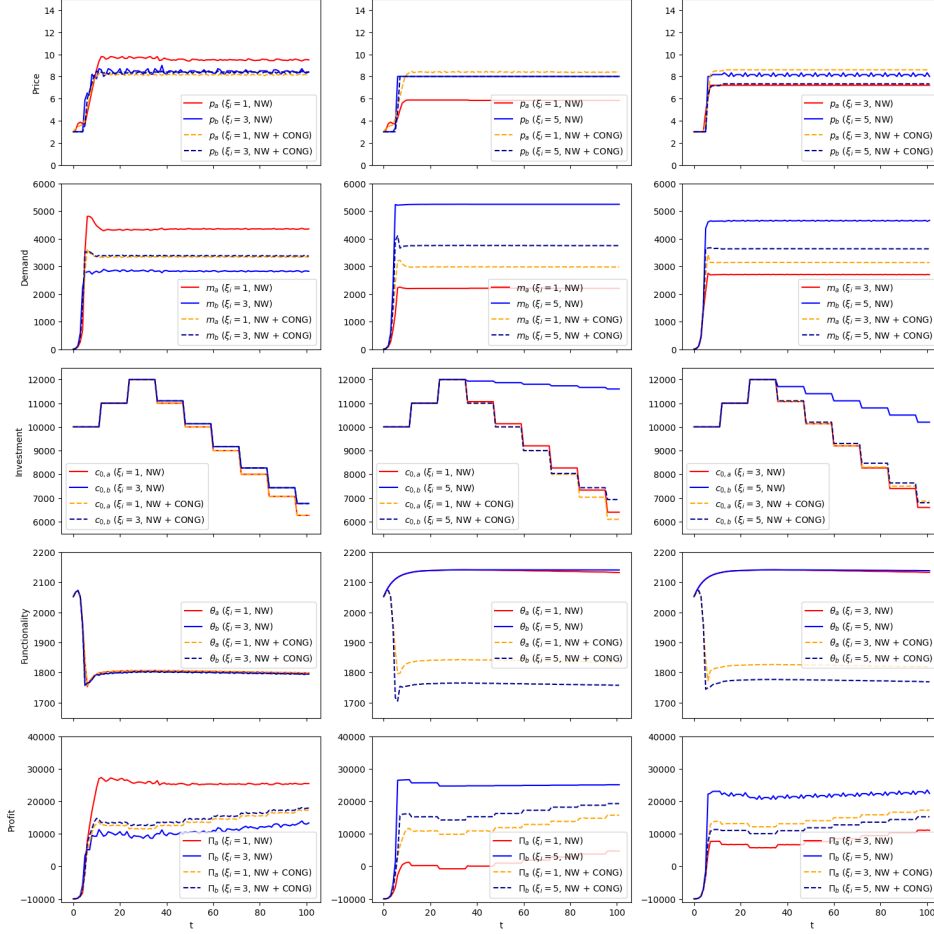


Figure 2.28: Market dynamics under different ξ_i ($i \in \{a, b\}$). “NW” represents the market setting with only network effect, and “NW+CONG” represents the market setting with both network effect and congestion effect. $A_a = A_b = 1$.

The simulation results indicate that ξ_i , which governs the firms' price adjustment flexibility, does influence market dynamics but is not the only critical factor. Firms that adopt a lower initial price in the initial periods and successfully attract more

consumers early on gain a long-term advantage in profitability, particularly under strong network effects. This advantage persists even when price flexibility is lower than that of the competitor. Thus, early strategic positioning can outweigh marginal adjustment capability, highlighting the importance of path-dependent dynamics in the market.

Multiple-run stability

We conduct a multiple-run stability analysis to examine whether the model exhibits path-dependent dynamics, i.e., whether different initial price levels can lead to structurally distinct long-run market outcomes, even under identical parameter settings.

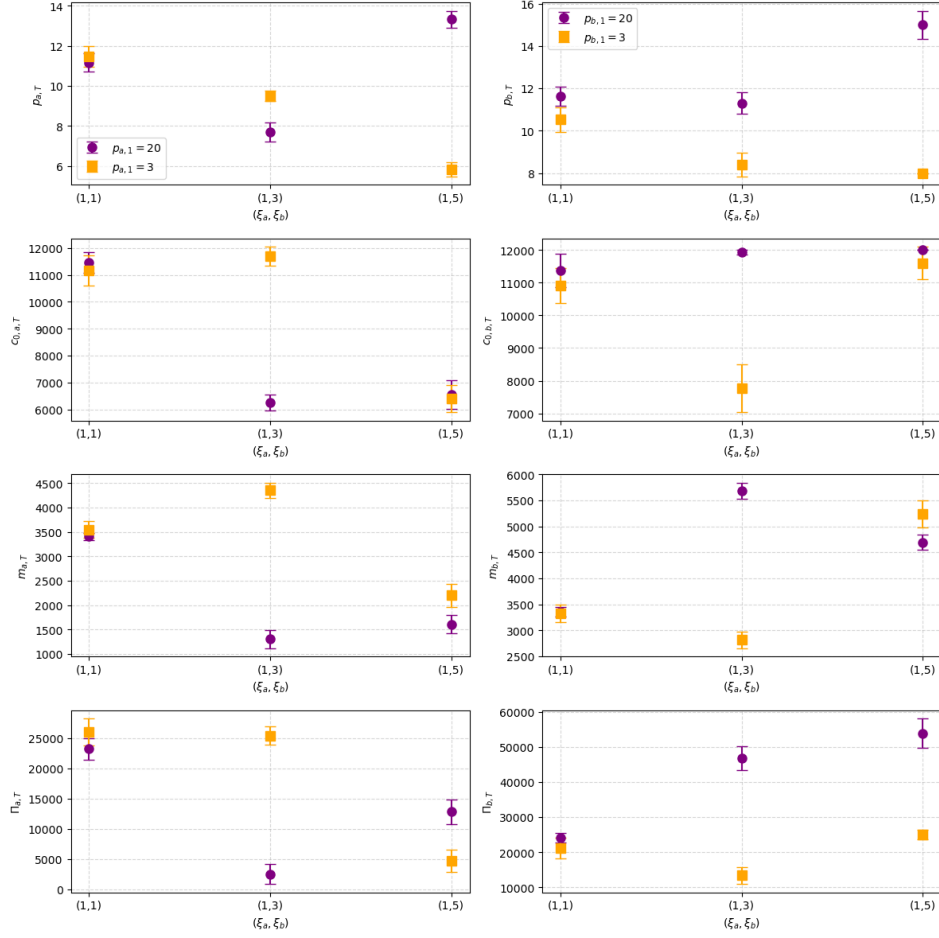


Figure 2.29: Final-period outcomes (mean and 95% confidence interval based on bootstrapping) under different price adjustment regimes (ξ_a, ξ_b) . Results are shown for two different initial price conditions. Only the network effect is considered. $A_a = A_b = 1$.

When price adjustment is asymmetric, the resulting market outcomes show pronounced divergence across initial conditions—as reflected in the non-overlapping confidence intervals. This indicates path dependence in the final-period market structure. Symmetric adjustment settings yield more consistent outcomes across initializations.²⁸

²⁸Under the cases with asymmetric technologies ($A_a = 1.2, A_b = 1$), final-period outcomes also show pronounced divergence across initial conditions.

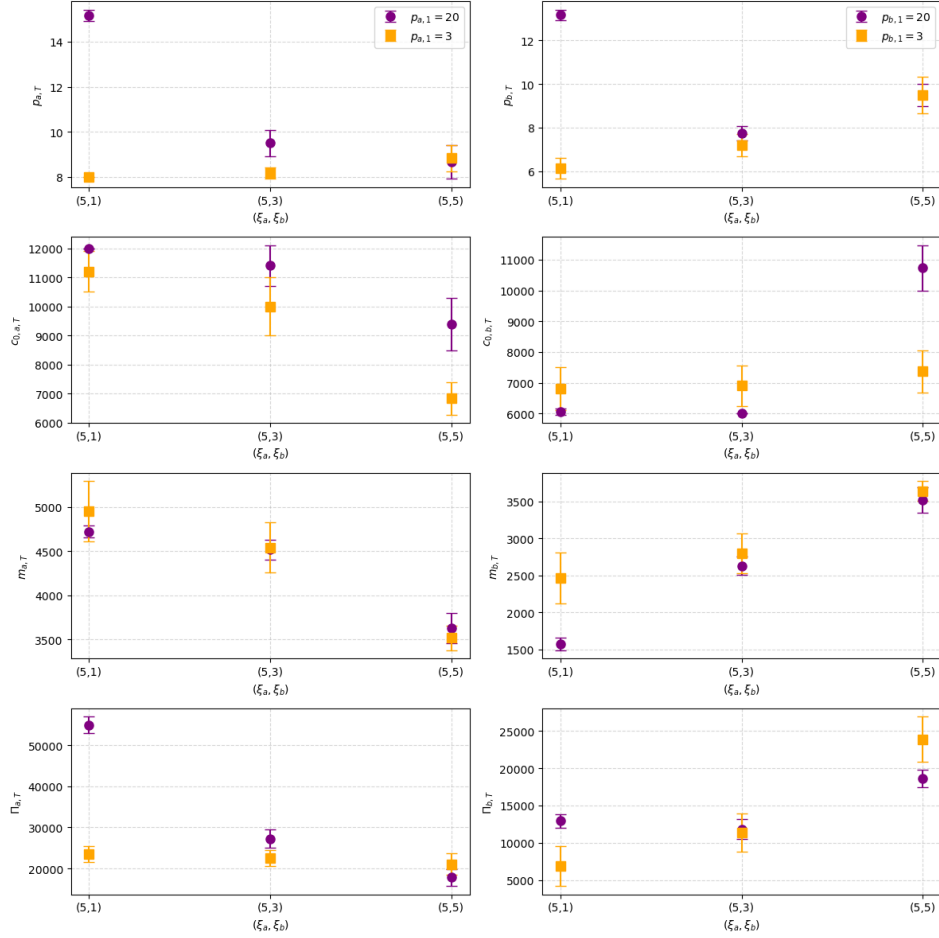


Figure 2.30: Final-period outcomes (mean and 95% confidence interval based on bootstrapping) under different price adjustment regimes (ξ_a, ξ_b) . Results are shown for two different initial price conditions. Only the network effect is considered. $A_a = A_b = 1$.

While the firms share identical technological capabilities, the results reveal that price and investment levels can still diverge structurally, especially as price adjustment becomes more flexible. This indicates that price flexibility acts as a behavioral amplifier, altering firms' responses to market conditions even in structurally symmetric settings. Importantly, the divergence in outcomes is observed across different initial price settings, further suggesting that non-linear feedback mechanisms—shaped by price dynamics—can lead to persistent heterogeneity in firm behavior despite symmetric fundamentals.²⁹

²⁹In the scenarios that incorporate both consumer congestion and network effects, the core findings remain robust. However, the divergence in final profits appears less pronounced—possibly

Final-stage randomness check

To assess whether the simulated time series reflect structured behavior or stochastic fluctuations, we conduct the Wald-Wolfowitz runs test for randomness. Figure 2.31 reports the fraction of simulation runs in which the null hypothesis of randomness is rejected at the 5% significance level.³⁰

Variable	Large initial price	Small initial price	Max. price adjustment	Tech	Case
p_a	66.67%	53.33%	(1, 1)	(1.2, 1)	NW
p_b	50.00%	53.33%	(1, 1)	(1.2, 1)	NW
$c_{0,a}$	100.00%	100.00%	(1, 1)	(1.2, 1)	NW
$c_{0,b}$	100.00%	100.00%	(1, 1)	(1.2, 1)	NW
m_a	60.00%	60.00%	(1, 1)	(1.2, 1)	NW
m_b	56.67%	63.33%	(1, 1)	(1.2, 1)	NW
Π_a	73.33%	63.33%	(1, 1)	(1.2, 1)	NW
Π_b	66.67%	73.33%	(1, 1)	(1.2, 1)	NW
p_a	53.33%	60.00%	(1, 1)	(1.2, 1)	Both
p_b	43.33%	50.00%	(1, 1)	(1.2, 1)	Both
$c_{0,a}$	100.00%	100.00%	(1, 1)	(1.2, 1)	Both
$c_{0,b}$	100.00%	100.00%	(1, 1)	(1.2, 1)	Both
m_a	56.67%	53.33%	(1, 1)	(1.2, 1)	Both
m_b	46.67%	46.67%	(1, 1)	(1.2, 1)	Both
Π_a	63.33%	66.67%	(1, 1)	(1.2, 1)	Both
Π_b	56.67%	56.67%	(1, 1)	(1.2, 1)	Both

Figure 2.31: Proportion of simulation runs in which the null hypothesis of randomness is rejected at the 5% significance level. The maximum price adjustments are denoted by (ξ_a, ξ_b) and the technology parameters by (A_a, A_b) .

A majority of economic variables exhibit high rejection rates, typically above 60%, and even reaching 100% for investments, indicating the presence of consistent, non-random patterns in the simulated dynamics. Notably, these results are stable across different initial price settings, suggesting that the model’s long-run behavior is not driven by transient initialization effects. The particularly strong non-randomness observed in investment-related variables further confirms that firm strategies evolve in a structured and payoff-responsive manner, which supports the robustness of our simulation framework.

due to the mitigating influence of congestion, which dampens firms’ ability to fully capitalize on early strategic advantages. This result holds robust when we check the cases with asymmetric technologies.

³⁰ “Large initial price” indicates an initial price of 20, while “small initial price” indicates an initial price of 3.

Variable	Large initial price	Small initial price	Max. price adjustment	Tech	Case
p_a	66.67%	86.67%	(3, 1)	(1.2, 1)	NW
p_b	50.00%	60.00%	(3, 1)	(1.2, 1)	NW
$c_{0,a}$	100.00%	100.00%	(3, 1)	(1.2, 1)	NW
$c_{0,b}$	100.00%	100.00%	(3, 1)	(1.2, 1)	NW
m_a	63.33%	83.33%	(3, 1)	(1.2, 1)	NW
m_b	60.00%	63.33%	(3, 1)	(1.2, 1)	NW
Π_a	60.00%	83.33%	(3, 1)	(1.2, 1)	NW
Π_b	93.33%	66.67%	(3, 1)	(1.2, 1)	NW
p_a	90.00%	90.00%	(3, 1)	(1.2, 1)	Both
p_b	70.00%	60.00%	(3, 1)	(1.2, 1)	Both
$c_{0,a}$	100.00%	100.00%	(3, 1)	(1.2, 1)	Both
$c_{0,b}$	100.00%	100.00%	(3, 1)	(1.2, 1)	Both
m_a	73.33%	83.33%	(3, 1)	(1.2, 1)	Both
m_b	70.00%	60.00%	(3, 1)	(1.2, 1)	Both
Π_a	76.67%	86.67%	(3, 1)	(1.2, 1)	Both
Π_b	83.33%	70.00%	(3, 1)	(1.2, 1)	Both

Figure 2.32: Proportion of simulation runs in which the null hypothesis of randomness is rejected at the 5% significance level. The maximum price adjustments are denoted by (ξ_a, ξ_b) and the technology parameters by (A_a, A_b) .

In scenarios where firms are allowed larger price adjustments (e.g., up to 3 units per update), the rejection rates increase upon the introduction of congestion. This indicates that strategic flexibility enables firms to effectively counter congestion, preserving or even enhancing the structure in market outcomes. These findings highlight that the impact of consumer congestion is conditional: it interacts with firms' adjustment capabilities and behavioral rules. Rather than destabilizing the system universally, congestion exerts context-dependent pressure, whose effect may be mitigated by greater adaptability in firm strategies.

Variable	Large initial price	Small initial price	Max. price adjustment	Tech	Case
p_a	93.33%	96.67%	(5, 5)	(1.2, 1)	NW
p_b	100.00%	100.00%	(5, 5)	(1.2, 1)	NW
$c_{0,a}$	100.00%	100.00%	(5, 5)	(1.2, 1)	NW
$c_{0,b}$	100.00%	100.00%	(5, 5)	(1.2, 1)	NW
m_a	100.00%	96.67%	(5, 5)	(1.2, 1)	NW
m_b	100.00%	96.67%	(5, 5)	(1.2, 1)	NW
Π_a	93.33%	96.67%	(5, 5)	(1.2, 1)	NW
Π_b	100.00%	100.00%	(5, 5)	(1.2, 1)	NW
p_a	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
p_b	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
$c_{0,a}$	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
$c_{0,b}$	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
m_a	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
m_b	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
Π_a	100.00%	100.00%	(5, 5)	(1.2, 1)	Both
Π_b	100.00%	100.00%	(5, 5)	(1.2, 1)	Both

Figure 2.33: Proportion of simulation runs in which the null hypothesis of randomness is rejected at the 5% significance level. The maximum price adjustments are denoted by (ξ_a, ξ_b) and the technology parameters by (A_a, A_b) .

In the cases where both firms have very flexible price adjustments ($\xi_a = \xi_b = 5$), across both scenarios—with and without consumer congestion—virtually all key economic variables reject the null hypothesis of randomness in over 90% of the simulation runs. This confirms that greater strategic flexibility amplifies the structural consistency of firm and market behavior. These results suggest that the internal decision rules and market feedback mechanisms of the model emerge more clearly when firms are allowed to adapt rapidly. In turn, this reinforces the robustness of our simulation framework: even in the presence of consumer-side frictions like congestion, allowing firms wider behavioral ranges restores equilibrium-seeking dynamics and eliminates erratic behavior.

Variable	Large initial price	Small initial price	Max. price adjustment	Tech	Case
p_a	53.33%	36.67%	(1, 1)	(1, 1)	NW
p_b	56.67%	36.67%	(1, 1)	(1, 1)	NW
$c_{0,a}$	100.0%	100.0%	(1, 1)	(1, 1)	NW
$c_{0,b}$	100.0%	100.0%	(1, 1)	(1, 1)	NW
m_a	50.0%	46.67%	(1, 1)	(1, 1)	NW
m_b	40.0%	43.33%	(1, 1)	(1, 1)	NW
Π_a	56.67%	53.33%	(1, 1)	(1, 1)	NW
Π_b	60.0%	50.0%	(1, 1)	(1, 1)	NW
p_a	63.33%	76.67%	(1, 1)	(1, 1)	Both
p_b	56.67%	83.33%	(1, 1)	(1, 1)	Both
$c_{0,a}$	100.0%	100.0%	(1, 1)	(1, 1)	Both
$c_{0,b}$	100.0%	100.0%	(1, 1)	(1, 1)	Both
m_a	66.67%	83.33%	(1, 1)	(1, 1)	Both
m_b	63.33%	80.0%	(1, 1)	(1, 1)	Both
Π_a	56.67%	73.33%	(1, 1)	(1, 1)	Both
Π_b	56.67%	76.67%	(1, 1)	(1, 1)	Both

Figure 2.34: Proportion of simulation runs in which the null hypothesis of randomness is rejected at the 5% significance level. The maximum price adjustments are denoted by (ξ_a, ξ_b) and the technology parameters by (A_a, A_b) .

Finally, we investigate how symmetry in firm capabilities affects the structure of simulation outcomes. Compared with asymmetric technology settings, the rejection rates are lower across most economic variables. This suggests that symmetry in firm capabilities introduces more randomness or indeterminacy in the model dynamics. Without clear asymmetries, firms' strategies tend to neutralize each other, and market outcomes such as prices, profits, and investment evolve in a less structured and more oscillatory way. In contrast, asymmetric technology induces more predictable patterns as the stronger firm establishes a persistent advantage. This reinforces the idea that heterogeneity in firm behavior is a key driver of structural dynamics in competitive environments.

2.6 Conclusion

Our research is among the first to systematically analyze the interplay between consumer congestion effects and consumer network effects in shaping firm competition within ICT markets, such as telecommunications, cloud services, and digital platforms. Unlike conventional R&D analyses focused on innovation probability, this paper highlights the role of physical R&D as an investment in capacity expansion and service quality within ICT markets. By incorporating heuristic-based decision-making under uncertainty in pricing and capacity related investment dynamics, we provide insights into the effectiveness of firm strategies across different market conditions. Our findings offer valuable implications for firms seeking to op-

timize pricing and investment policies and for policymakers evaluating the impact of network externalities and congestion effects on market efficiency.

Our game-theoretical analysis highlights that firms must consider basic product functionality from the outset. A firm with higher baseline functionality should invest more aggressively, while a technologically weaker competitor may sustain a presence by controlling costs and leveraging service capacity to maintain market share. Our agent-based approach further explores how firms navigate investment cycles, pricing strategies, and competitive positioning over time. We demonstrate when and why a firm with a technological edge can dominate a market, leading to highly concentrated industries, and identify conditions under which smaller firms can still compete, particularly when pricing flexibility and adaptive investment strategies come into play.

Our study advances the literature in four key directions: Firstly, concerning R&D and product design, we provide a dynamic perspective on investment strategies, showing that frequent or aggressive investment does not always translate into higher profits. We suggest firms to balance investment frequency, learning steps and update thresholds to align technological advancements with market conditions effectively. Secondly, concerning firm capacity selection, we redefine service capacity in ICT markets as a factor influencing service quality and product differentiation, unlike traditional models where capacity is merely a production constraint. Our results suggest that capacity decisions must account for congestion constraints, as over-investing in capacity does not necessarily drive higher demand if performance deteriorates. Thirdly, concerning network applications and consumer inertia, we incorporate consumer heterogeneity, considering differences in preferences and switching behaviors. Our findings show that the network effect strengthens competitive advantages, while congestion effects introduce counteracting pressures, altering firm profitability and pricing flexibility. Finally, concerning firm competition and strategic adaptation, our model moves beyond equilibrium approaches by incorporating bounded rationality, where firms rely on historical performance and adaptive heuristics rather than global optimization. We also reveal how firms must dynamically adjust pricing and investment to sustain competitive advantages in markets where network and congestion effects interact.

Our findings underscore the importance of network and congestion effects in shaping market outcomes. The consumer network effect can amplify market concentration by allowing past investments to generate sustained demand, thereby potentially reducing the need for frequent investment updates. Conversely, the consumer congestion effect limits the profitability of dominant firms, discouraging over-investment in capacity and weakening the effectiveness of high-pricing strategies when congestion lowers perceived service quality. Notably, network and congestion effects do not simply offset each other; their interaction creates complex

competitive dynamics that must be carefully managed. In the further robustness check, we show that the consumer congestion (a) may lead to changes in firms' pricing strategies, (b) may mitigate the divergence in final profits, and (c) may affect the structured behavior of economic variables.

Overall, firms must navigate strategic trade-offs in decision-making under market uncertainty. A firm must assess its market position before implementing pricing or investment updates, ensuring that investment decisions align with demand elasticity, competitive conditions, and cost constraints. While higher functionality elasticity enhances firm performance, it only leads to sustained advantages if firms effectively manage costs and ensure that additional investment translates into consumer adoption. Moreover, congestion constraints diminish the benefits of network effects, requiring firms to carefully balance service capacity with pricing and investment decisions.

Based on our current approach, we expect that future researchers can expand on this work in the following ways: First of all, studies can adopt causal inference techniques to better understand the underlying mechanisms driving firm behavior. Structural estimation or experimental designs could help isolate how investment decisions affect consumer behavior, firm profitability, and market structure. Secondly, industries such as telecommunications, digital platforms, and cloud services are ideal case studies for assessing the interplay between investment, pricing, and consumer demand elasticity. Cross-industry comparisons could reveal whether these mechanisms are more pronounced in network-driven markets compared to traditional industries. Thirdly, studies can allow firms to adjust pricing and investment dynamically based on long-term objectives, rather than reacting myopically to past observations, and examine how regulatory constraints may alter investment incentives. Finally, extending the model to multiple competing firms (rather than two) could provide insights into market fragmentation, firm differentiation strategies, and potential monopolization risks. Incorporating long-term firm entry and exit dynamics could also enhance understanding of how market competition evolves under different investment and congestion conditions.

By integrating network externalities, consumer heterogeneity, and congestion effects into a unified framework, our study provides a more comprehensive and dynamic perspective on firm competition in ICT markets. Future research could further explore regulatory interventions and policy frameworks that address the dual challenges of market concentration and congestion management, fostering a more competitive and efficient digital economy.

Code availability

To support transparency and reproducibility, the code underlying the results presented in this study is publicly accessible at <https://github.com/ZHongli626/Strategic-Firm-Capacity-Selection-under-the-Network-Effects>.

2.7 Appendix

2.7.1 Nash equilibrium outcomes in the two-stage game

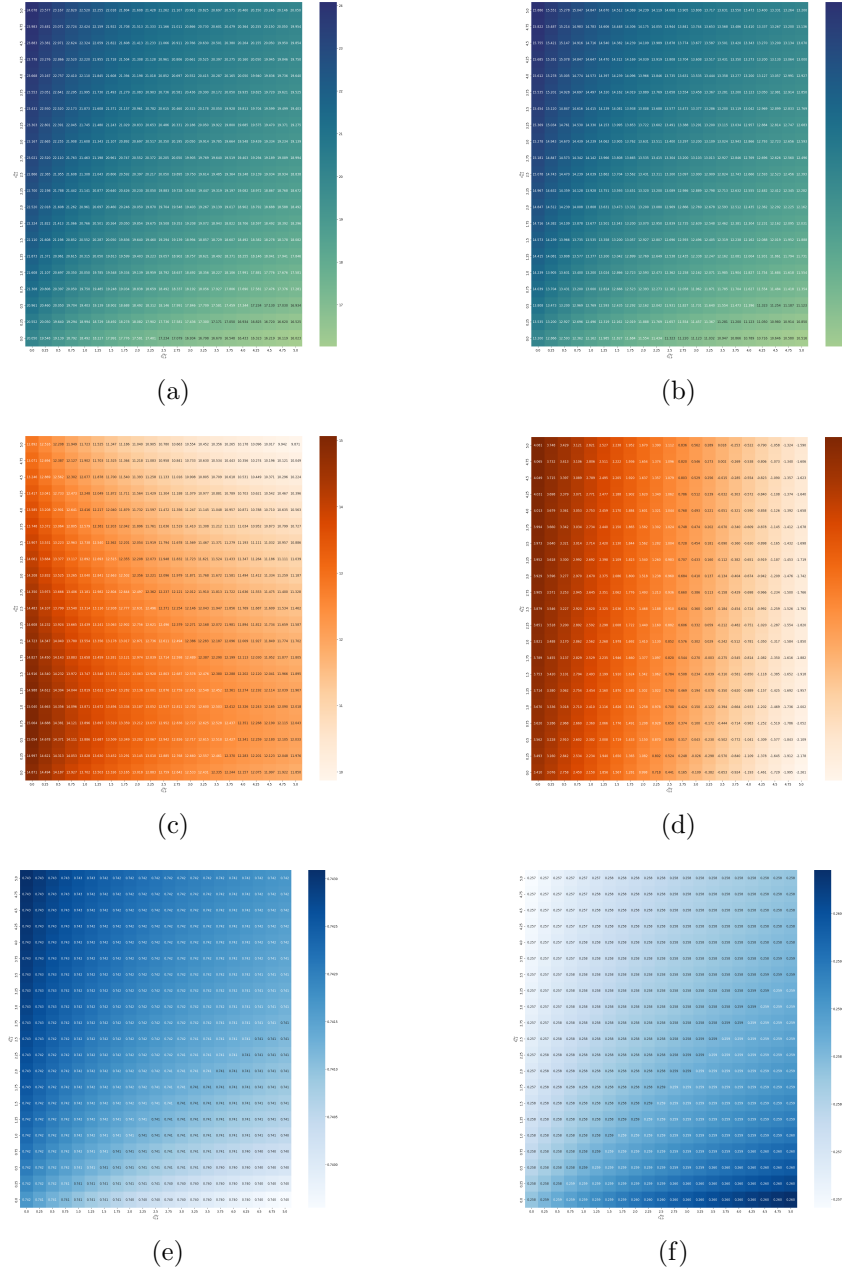


Figure 2.35: The grids within the Sub-figures include: (a) p_a^* , (b) p_b^* , (c) Π_a^* , (d) Π_b^* , (e) x_a^* , and (f) x_b^* . Model parameters: $S_{a,0} = 10, A_{a,0} = 1, S_{b,0} = 1, A_{b,0} = 1, \alpha = 0.05, \beta_0 = 1, \beta_2 = 3$.

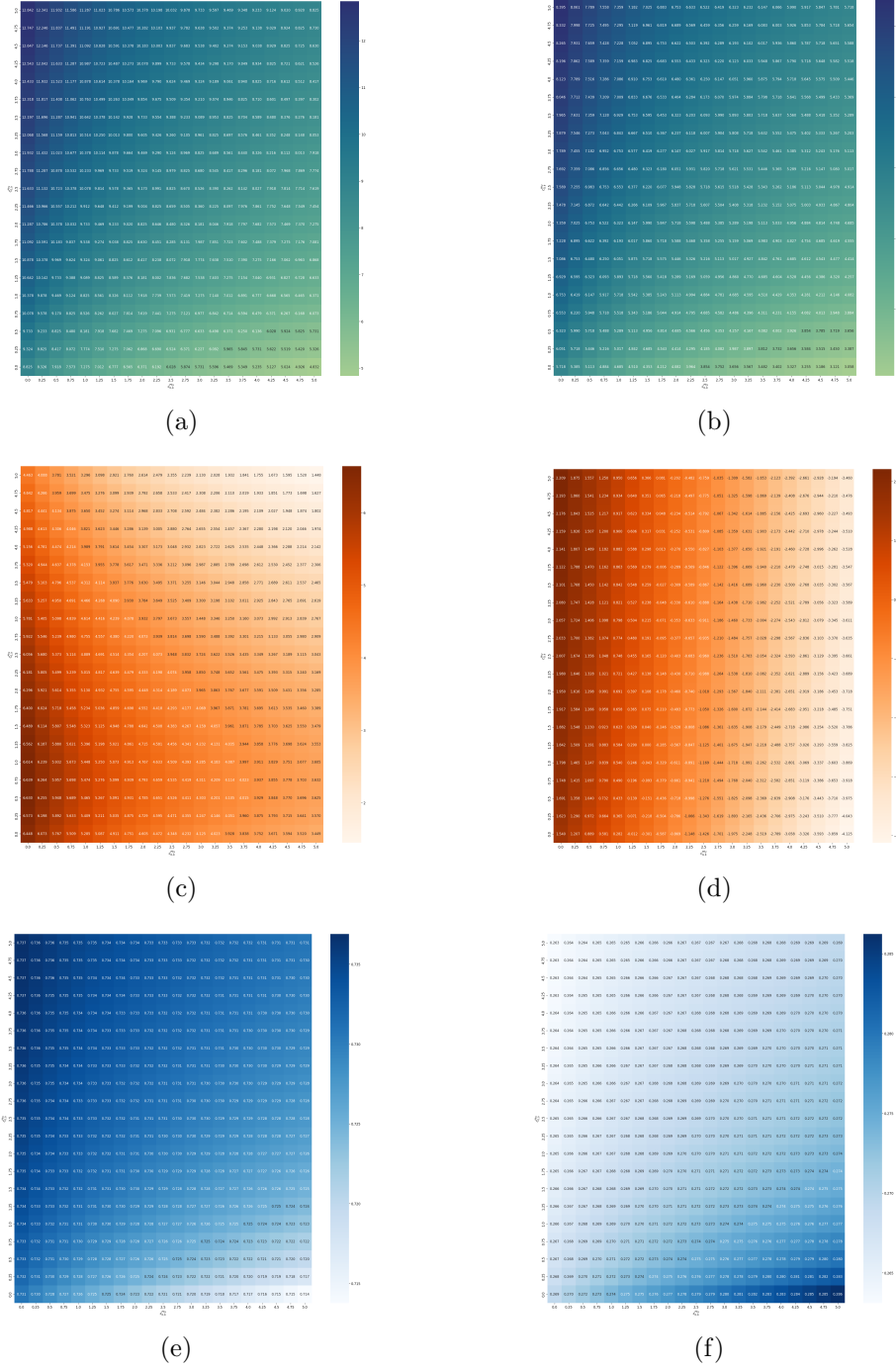
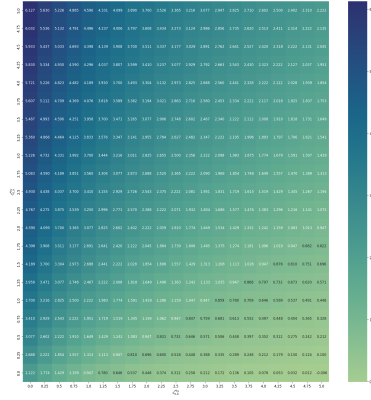
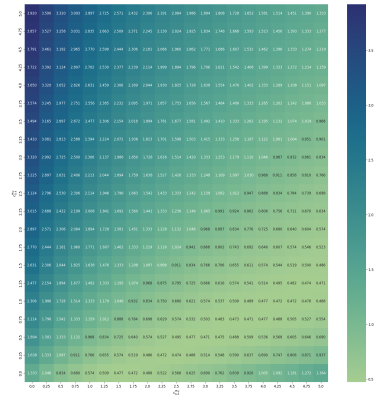


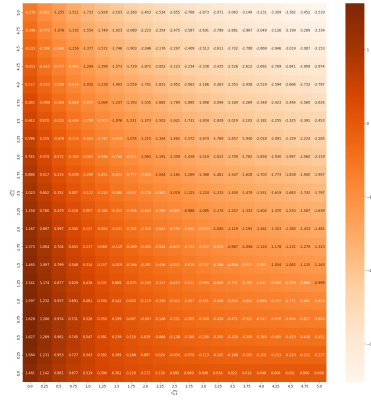
Figure 2.36: The grids within the Sub-figures include: (a) p_a^* , (b) p_b^* , (c) Π_a^* , (d) Π_b^* , (e) x_a^* , and (f) x_b^* . Model parameters: $S_{a,0} = 5, A_{a,0} = 1, S_{b,0} = 1, A_{b,0} = 1, \alpha = 0.05, \beta_0 = 1, \beta_2 = 3$.



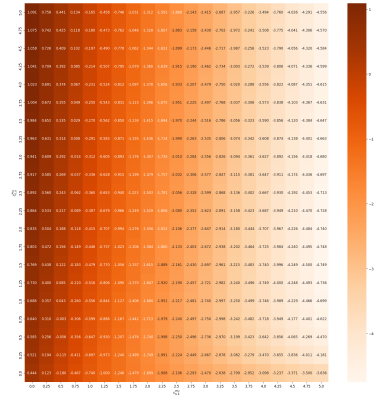
(a)



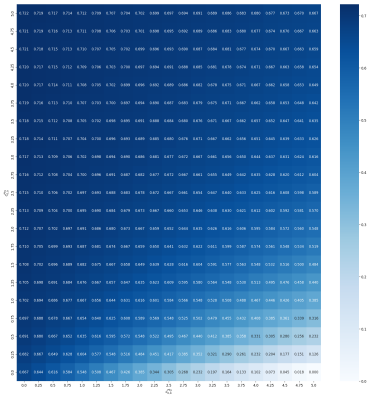
(b)



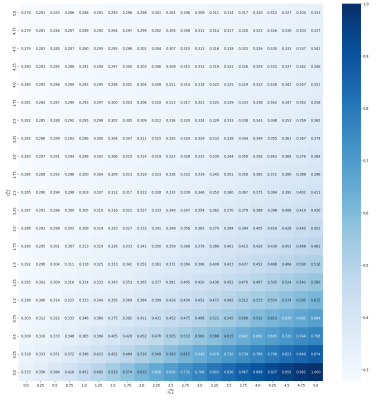
(c)



(d)



(e)



(f)

Figure 2.37: The grids within the Sub-figures include: (a) p_a^* , (b) p_b^* , (c) Π_a^* , (d) Π_b^* , (e) x_a^* , and (f) x_b^* . Model parameters: $S_{a,0} = 2, A_{a,0} = 1, S_{b,0} = 1, A_{b,0} = 1, \alpha = 0.05, \beta_0 = 1, \beta_2 = 3$.

2.7.2 Benchmark cases in simulations

The parameter settings of the benchmark scenario:

- **Depreciation rate:** $\beta^D = 0.2$
- **Firm technologies:**
 - * Asymmetric cases: $A_a = 1.2, A_b = 1$
 - * Symmetric cases: $A_a = 1, A_b = 1$
- **Investment effectiveness:** $\beta_a^C = 1, \beta_b^C = 1$
- **Parameters in product functionality:**
 - * $\beta_2 = 3$ (coefficient of service capacity $\chi_{i,t}$)
 - * β_0 (coefficient of the number of consumers in period t $m_{i,t}$; set to 0 when the consumer congestion effect is not considered, and 0.1 when it is)
 - * $\beta_3 = 2,000$ (constant, basic product functionality)
- **Parameters in consumer utility function:**
 - * $\bar{u} = 100$ (utility constant)
 - * β_4 (coefficient of the number of connections of a consumer; set to 0 when the consumer network effect is not considered, and 50 when it is)
 - * $\beta_5 = 30$ (coefficient of price effect)
- **Switching probability:** $p_{sw} = 0.1$
- **Reservation utility:** $u_R = 600$
- **Initial number of informed consumers:** $n_{\text{ini}} = 10$
- **Small-world network parameters:**
 - * $N = 10,000$ (total population)
 - * $k_G = 6$ (each node is connected to k nearest neighbors)
 - * $p_G = 0.3$ (probability of rewiring each edge)
- **Initial physical capacity:** $S_{a,t=0} = 1, S_{b,t=0} = 1$

- **Initial product prices:** $p_{a,t=0} = 20, p_{b,t=0} = 20$
- **Initial physical R&D investments:** $c_{0,a,t=0} = 10,000, c_{0,b,t=0} = 10,000$
- **Marginal cost:** $c_{\text{res},a} = 1, c_{\text{res},b} = 1$
- **Parameters related to price dynamics:**
 - * Maximum number of consumers in the sample: $n_s = 1,000$
 - * Number of price alternatives: $V = 3$
- **Parameters related to investment dynamics:**
 - * Investment update frequency: $\tau = 12$
 - * Update threshold: $\vartheta = 0.05$
 - * Learning step: $\eta_a = 1,000, \eta_b = 1,000$
 - * Random updates of investment (exploration step): $\psi = 0$

We categorize the simulated results into two groups: Group 1 ($\beta_0 = 0$, no consumer congestion effect) and Group 2 ($\beta_0 = 0.1$, with consumer congestion effect).³¹ In each group, there are four cases:

- Case 1: $A_a = 1.2, A_b = 1, \beta_4 = 0$. (Asymmetric technologies + No consumer network effect)
- Case 2: $A_a = 1, A_b = 1, \beta_4 = 0$. (Symmetric technologies + No consumer network effect)
- Case 3: $A_a = 1.2, A_b = 1, \beta_4 = 50$. (Asymmetric technologies + Consumer network effect)
- Case 4: $A_a = 1, A_b = 1, \beta_4 = 50$. (Symmetric technologies + Consumer network effect)

In **Sensitivity Analysis**, we focus on the market outcomes with adjusted parameter values based on the cases with firms under asymmetric technologies, namely Case 1 and Case 3.

³¹Since the population size is already large ($N = 10,000$ in the codes), $\beta_0 = 0.1$ is sufficient for observing the consumer congestion.

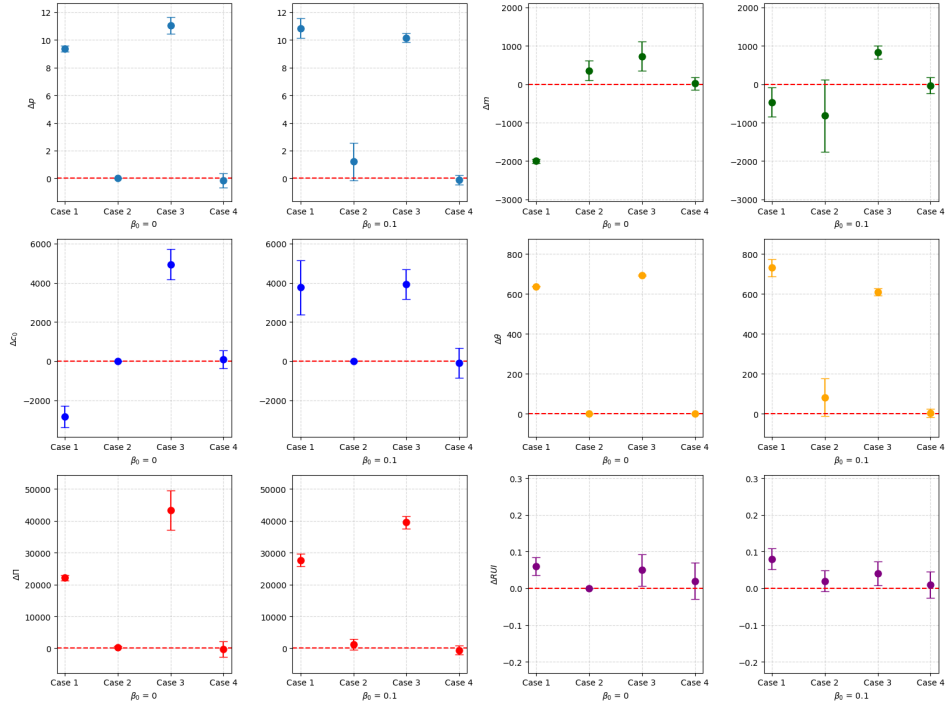


Figure 2.38: Benchmark cases illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test.

2.7.3 Sensitivity Analysis

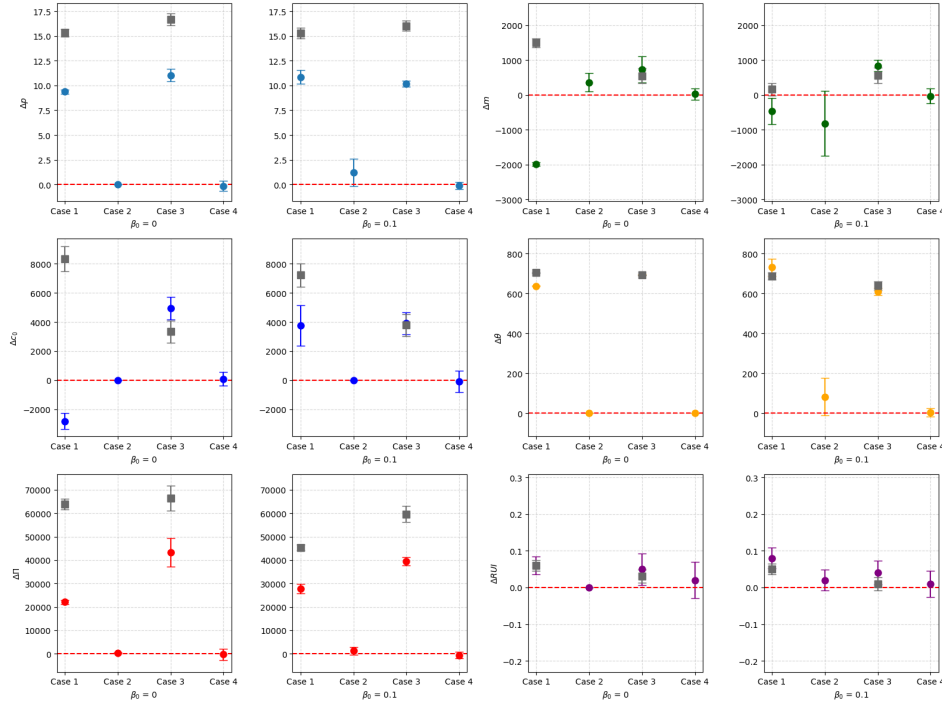


Figure 2.39: Cases of variation in price effect parameter β_5 illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

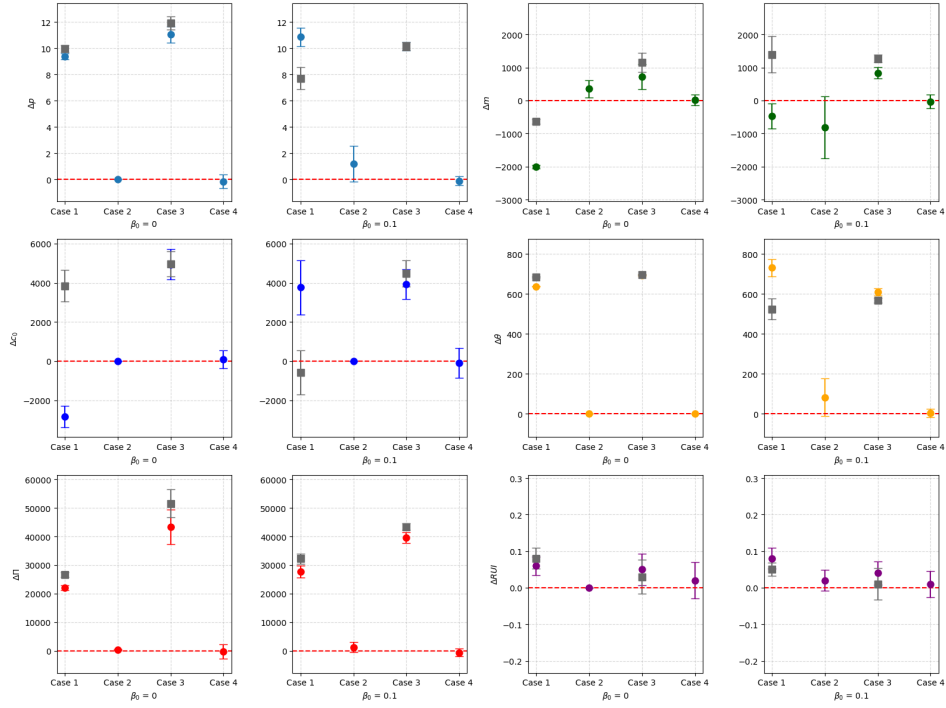


Figure 2.40: Cases of variation in switching probability parameter p_{sw} illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

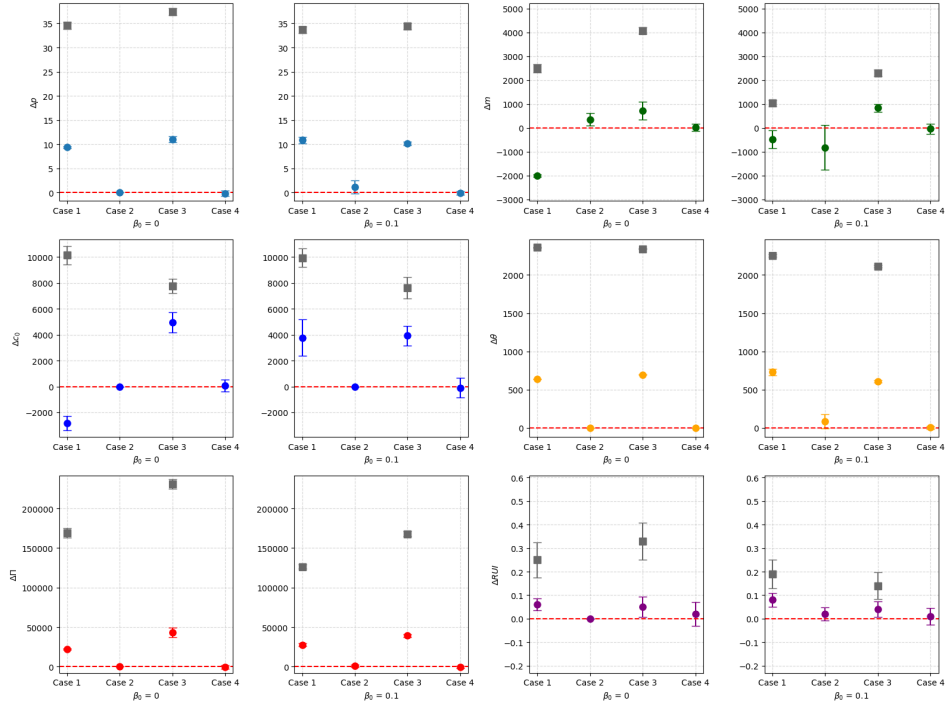


Figure 2.41: Cases of variation in functionality coefficient β_2 illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

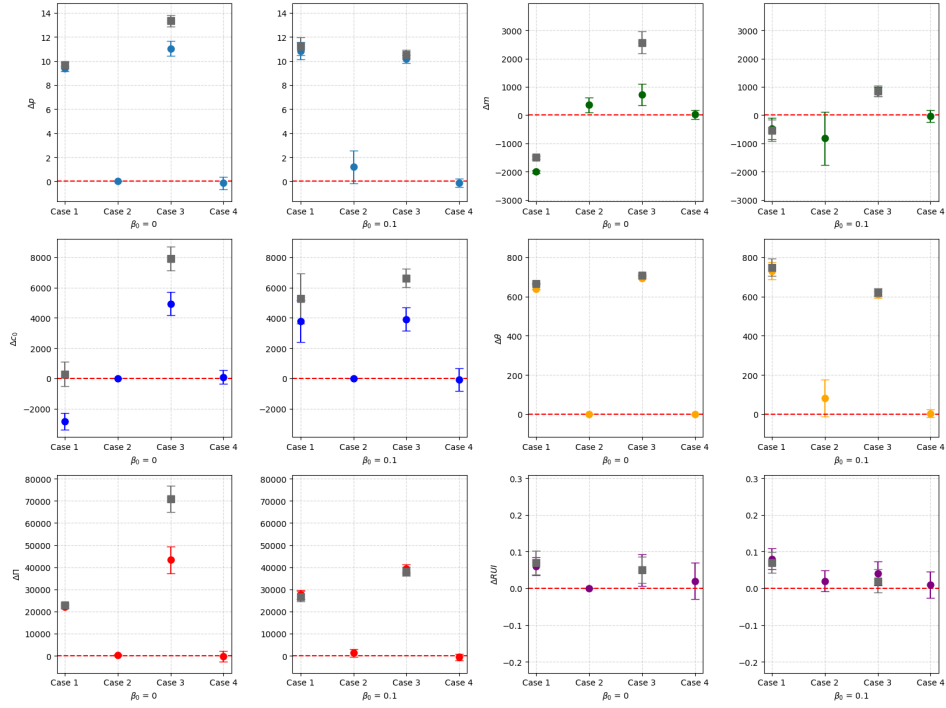


Figure 2.42: Cases of variation in initial physical R&D investment $c_{0,a,t=0}$ illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

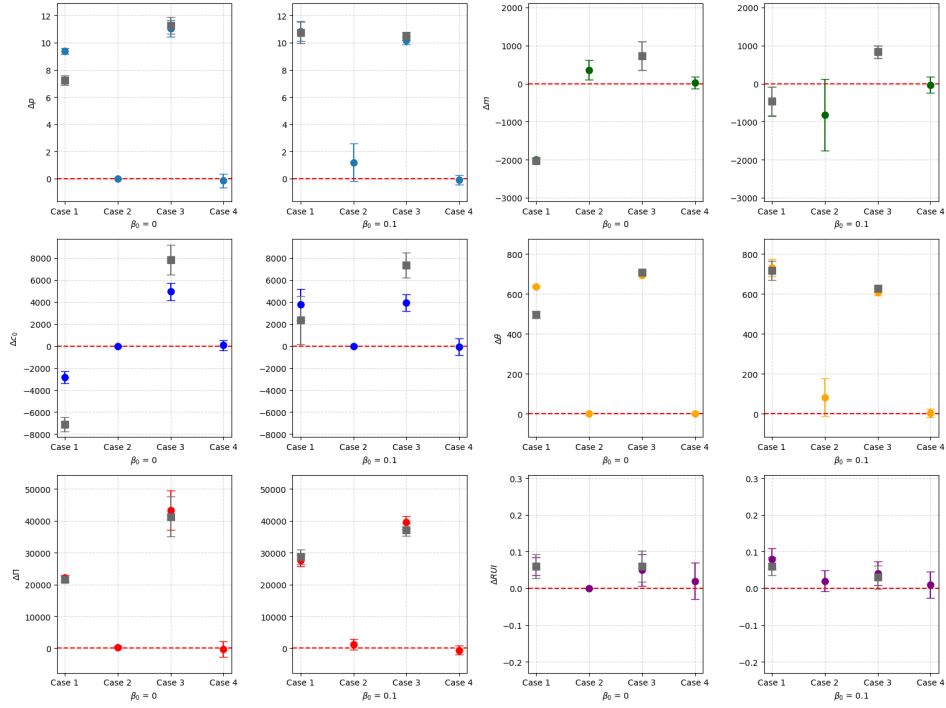


Figure 2.43: Cases of variation in investment learning step Δ_a illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

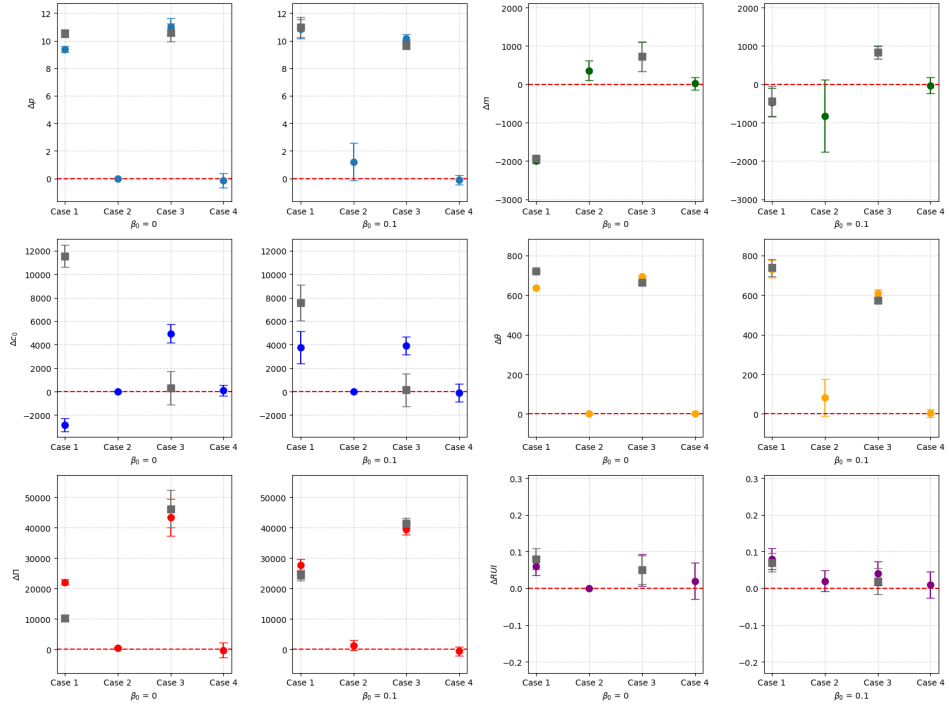


Figure 2.44: Cases of variation in investment update frequency τ illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

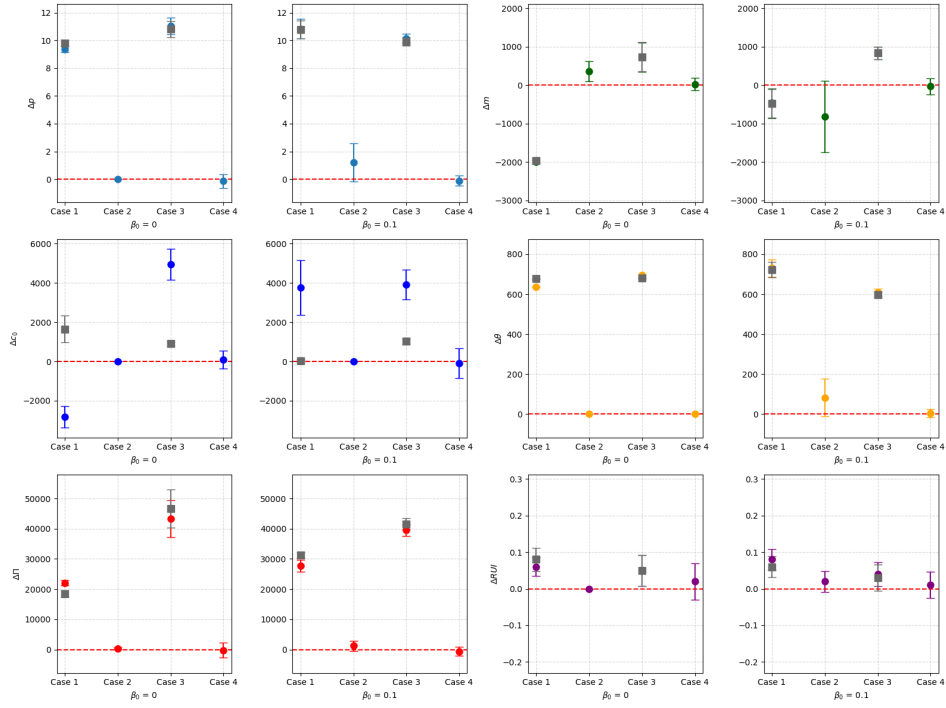


Figure 2.45: Cases of variation in investment update threshold ϑ illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

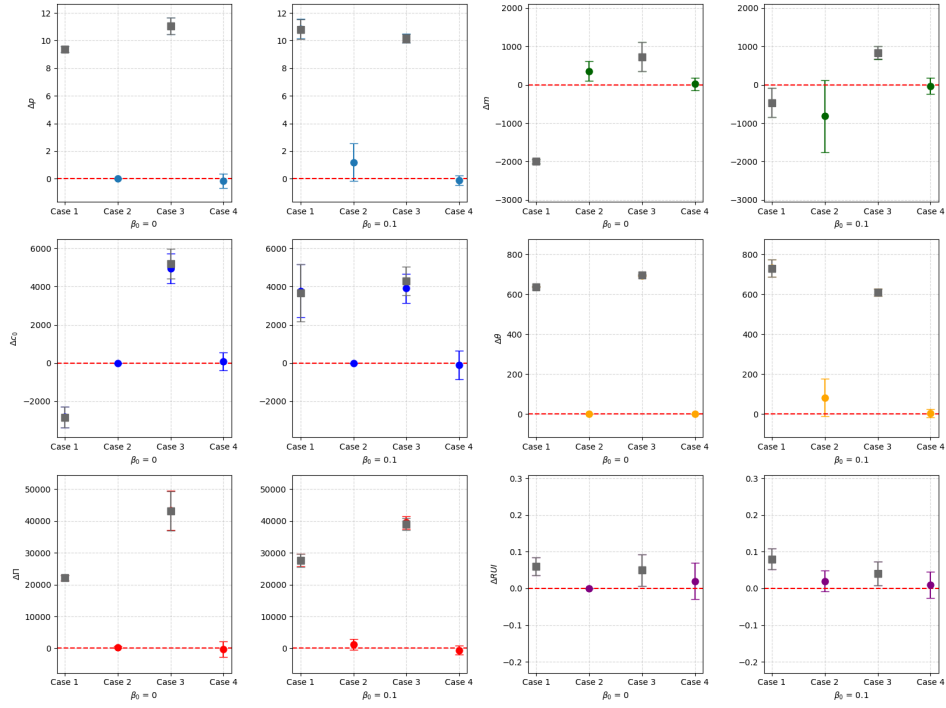


Figure 2.46: Cases of variation in investment exploration step ψ illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

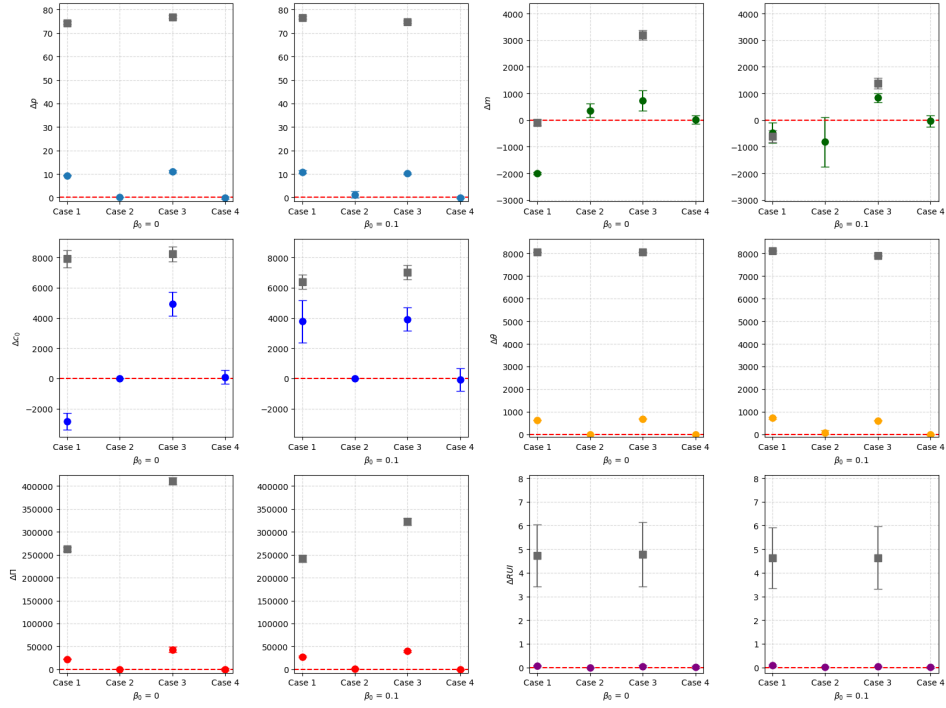
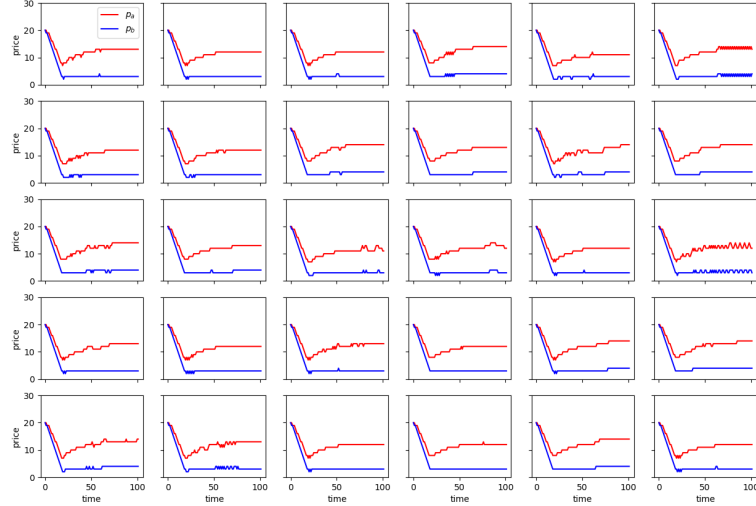


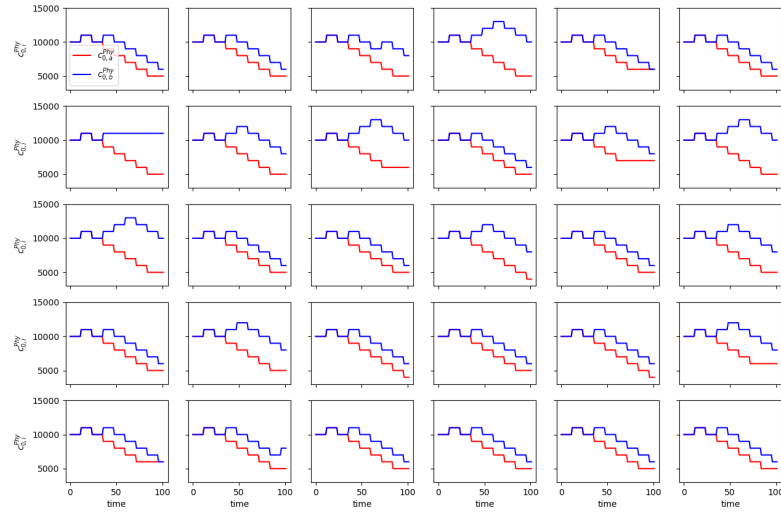
Figure 2.47: Cases of variation in depreciation rate β^D illustrating market outcome indices over the last 30 periods of simulations. In each sub-figure, each dot represents the mean across simulation runs, while each error bar represents the 95% confidence interval derived from a t-test. Squares and error bars in grey show the results of the asymmetric cases after the parameter adjustment.

2.7.4 Single runs of simulations for benchmark cases

A. Firms with asymmetric technologies

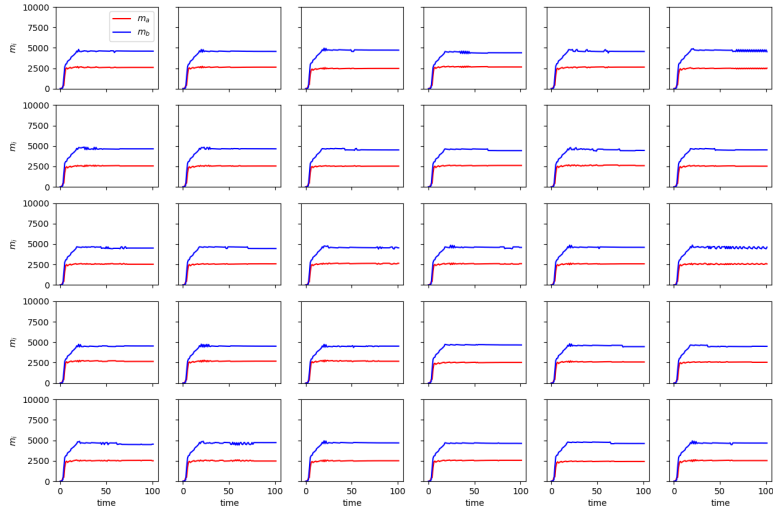


(a)

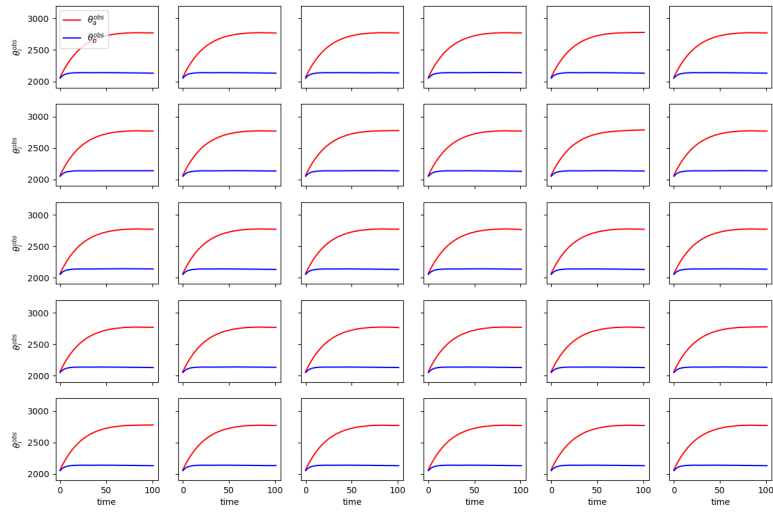


(b)

Figure 2.48: Single runs of simulations for price and investment dynamics. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

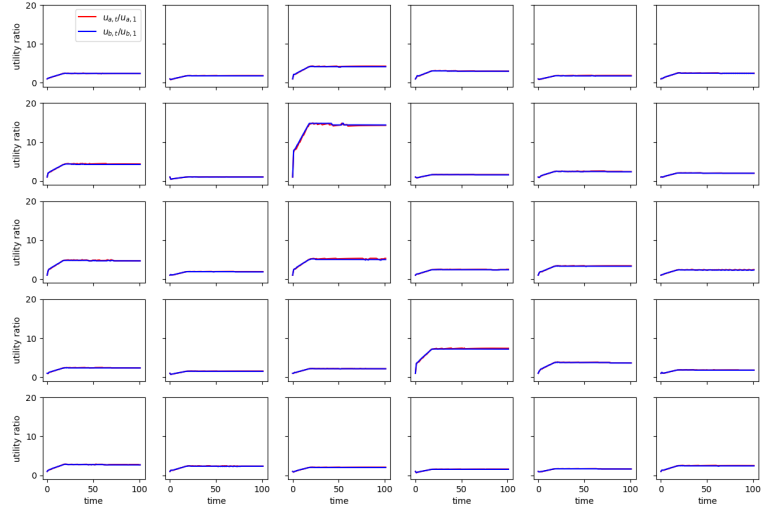


(a)

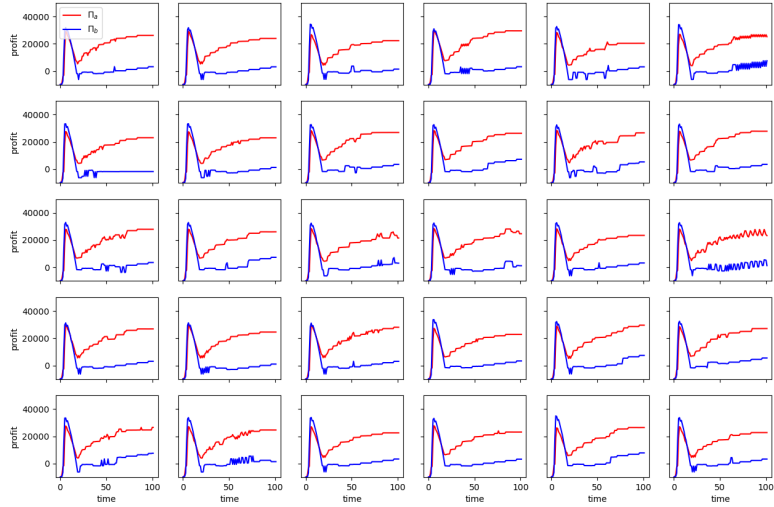


(b)

Figure 2.49: Single runs of simulations for market demand and observed product functionality. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

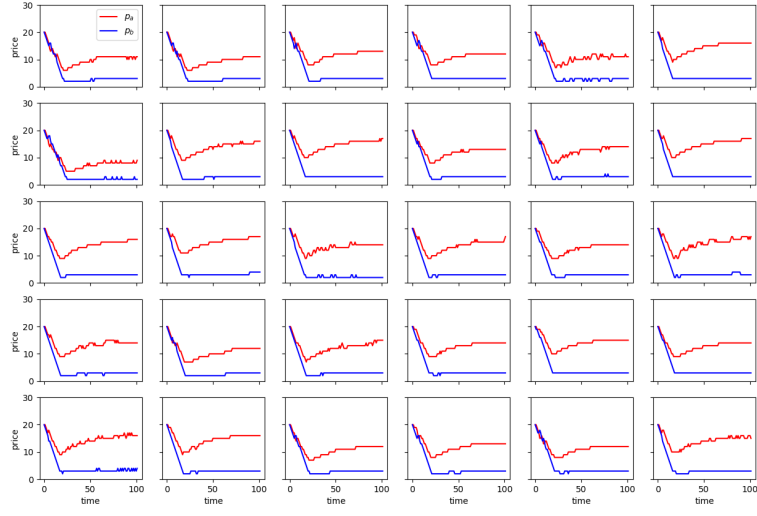


(a)

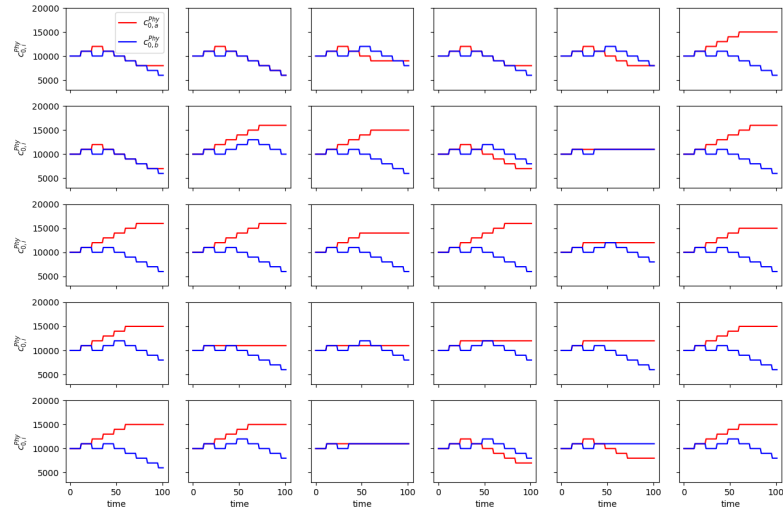


(b)

Figure 2.50: Single runs of simulations for Relative Utility Index and profit. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

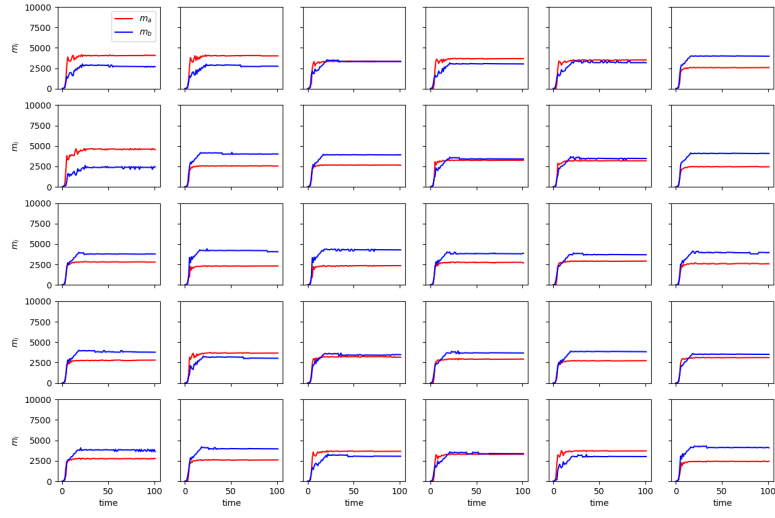


(a)

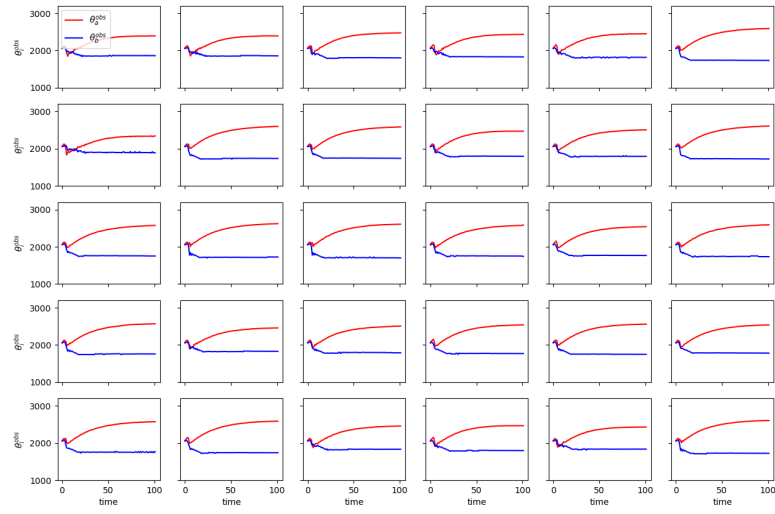


(b)

Figure 2.51: Single runs of simulations for price and investment dynamics. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

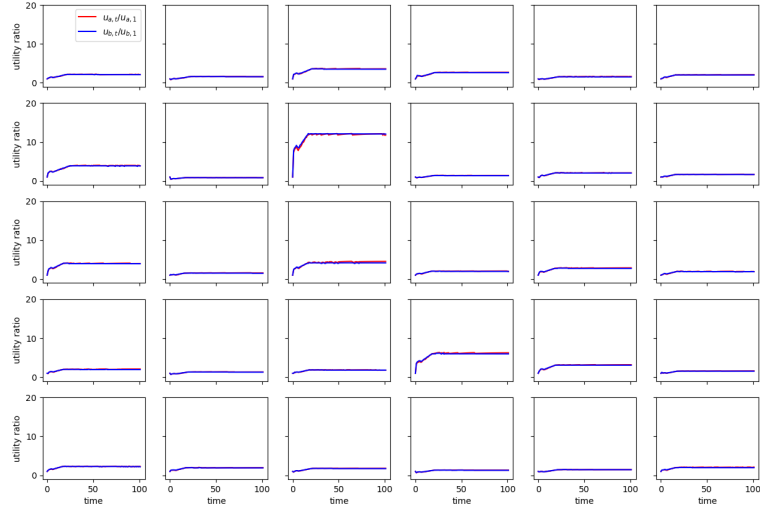


(a)

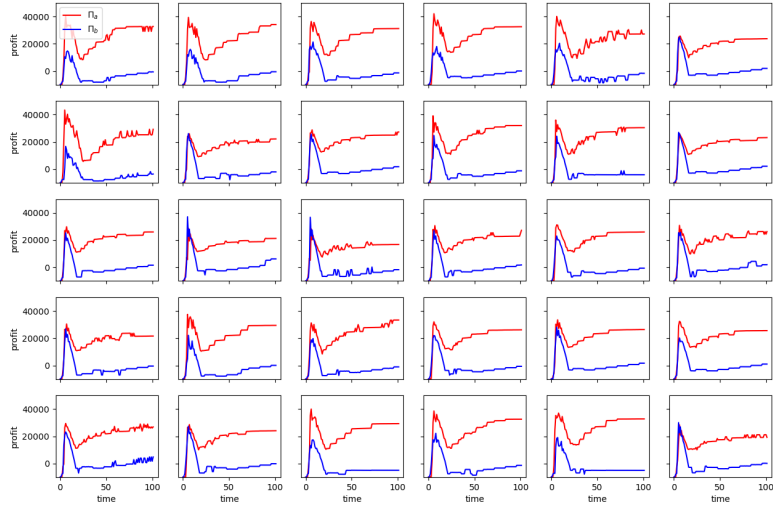


(b)

Figure 2.52: Single runs of simulations for market demand and observed product functionality. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

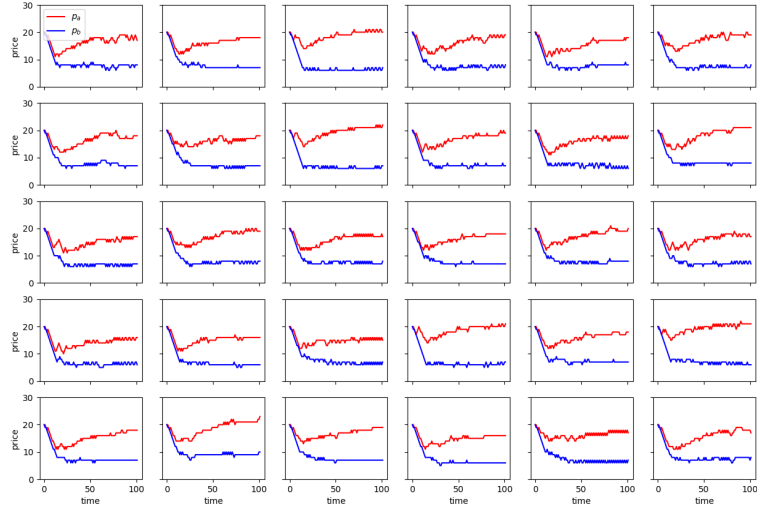


(a)

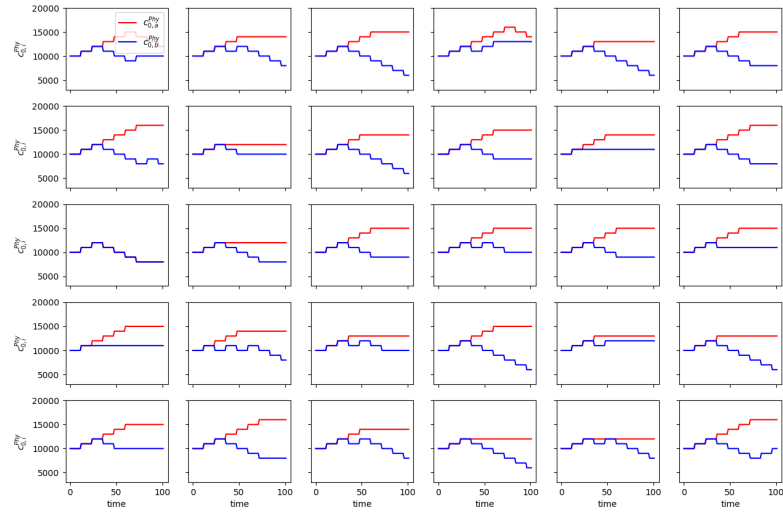


(b)

Figure 2.53: Single runs of simulations for Relative Utility Index and profit. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

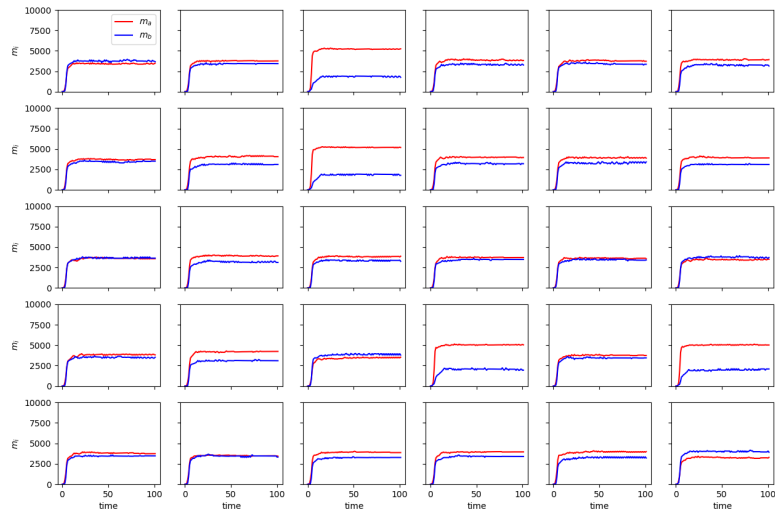


(a)

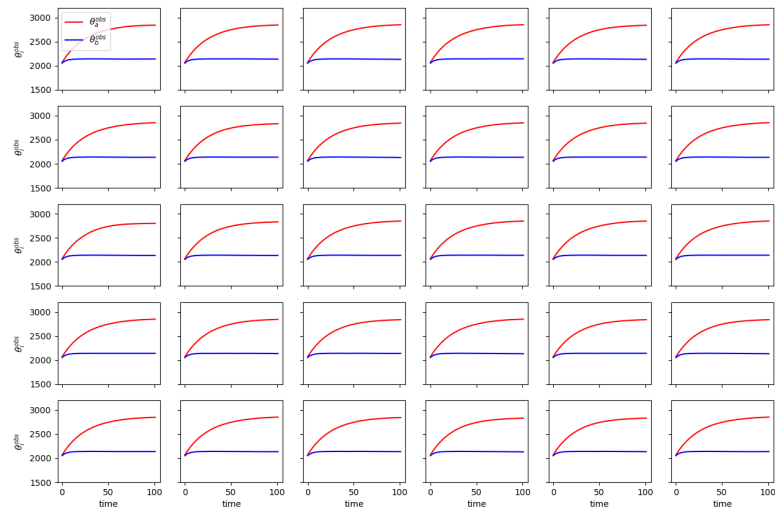


(b)

Figure 2.54: Single runs of simulations for price and investment dynamics. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

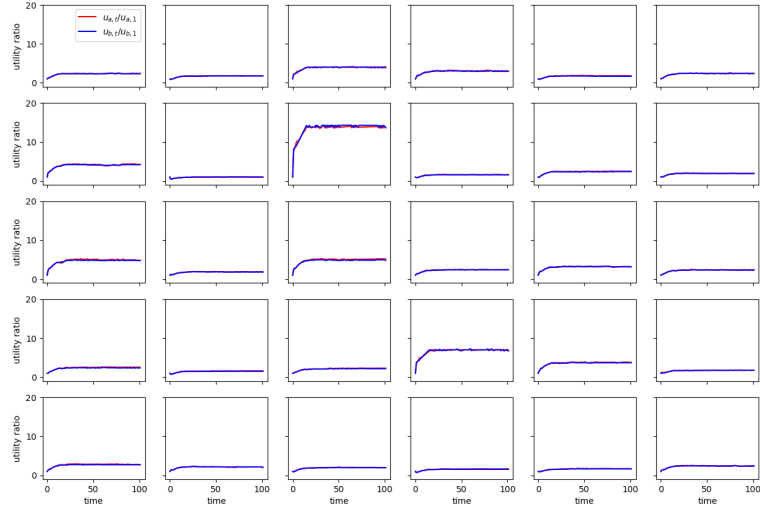


(a)

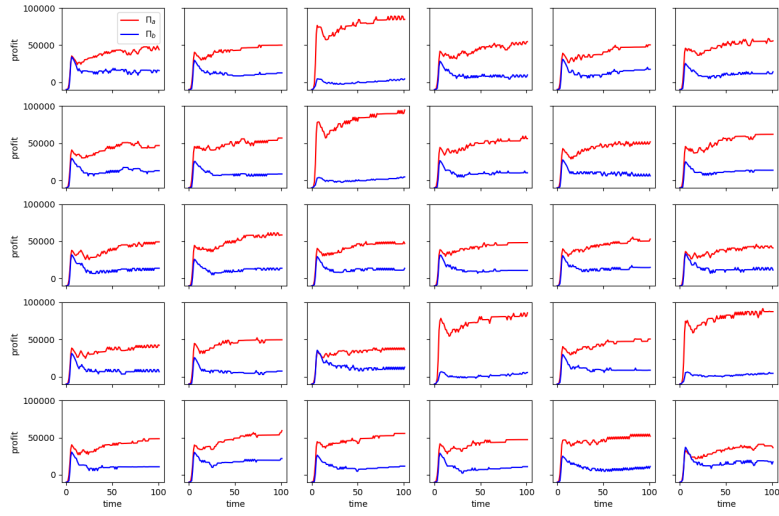


(b)

Figure 2.55: Single runs of simulations for market demand and observed product functionality. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

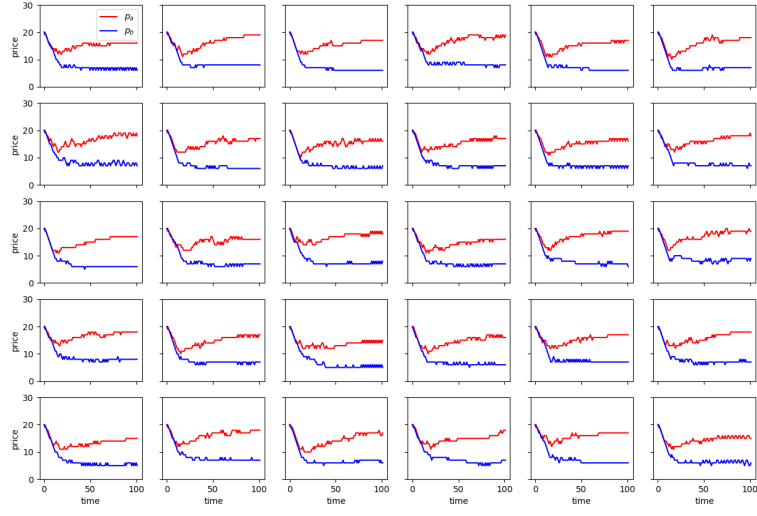


(a)

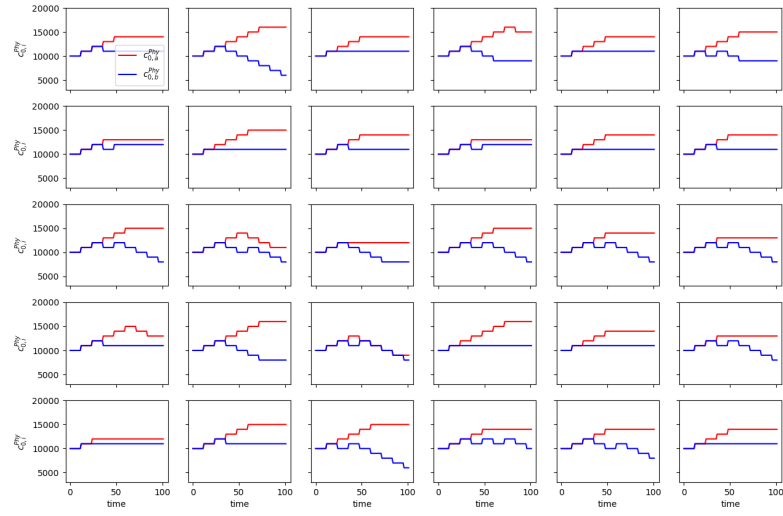


(b)

Figure 2.56: Single runs of simulations for Relative Utility Index and profit. $A_a = 1.2, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

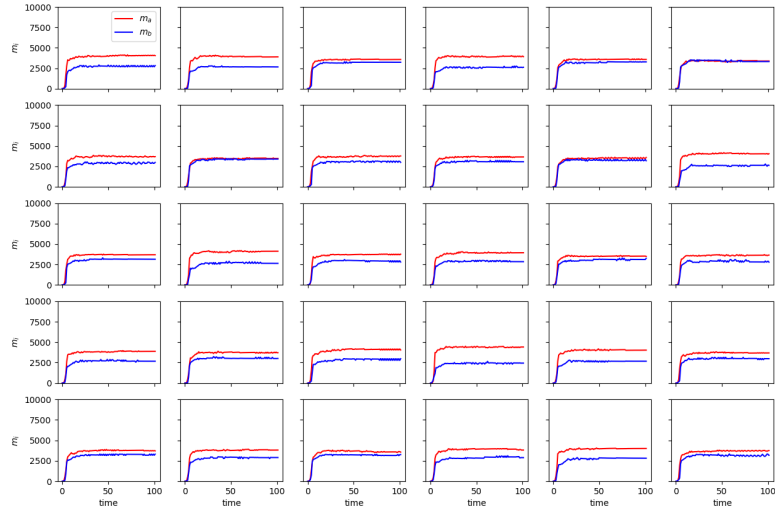


(a)

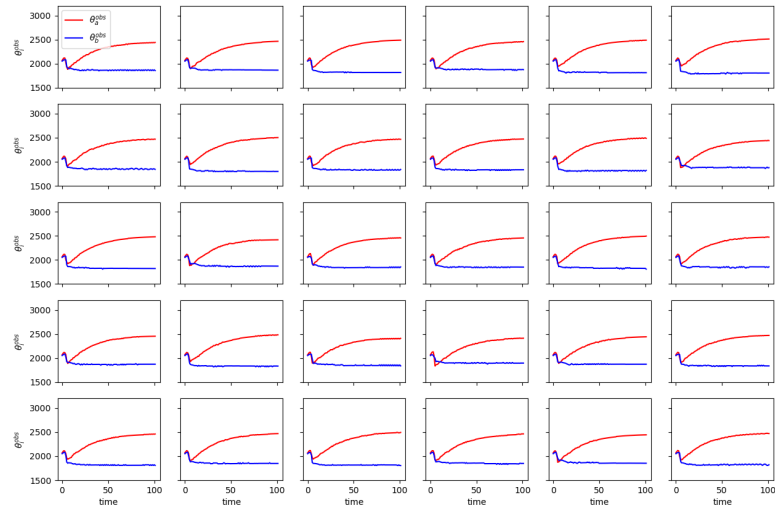


(b)

Figure 2.57: Single runs of simulations for price and investment dynamics. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.

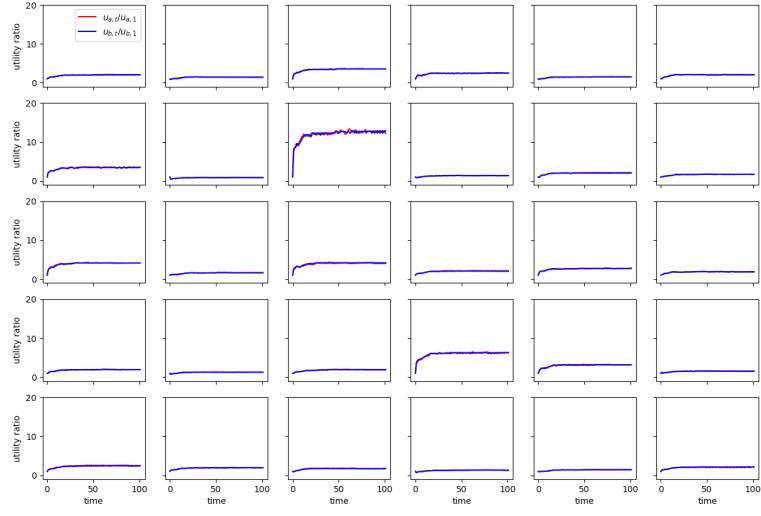


(a)

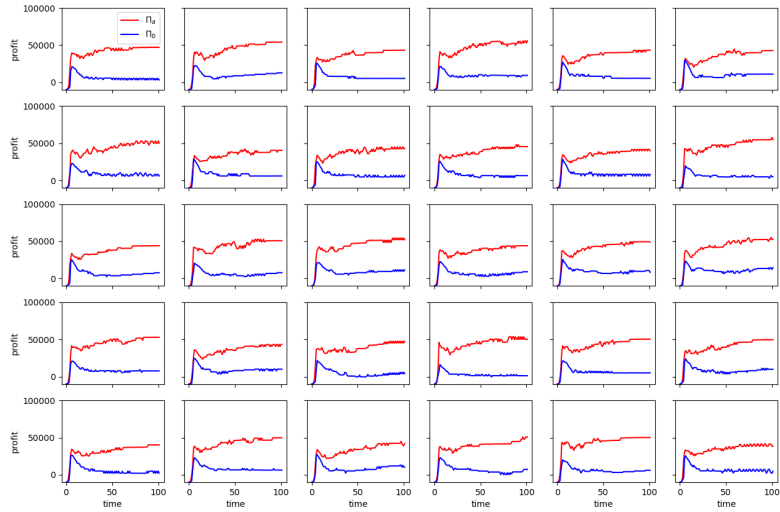


(b)

Figure 2.58: Single runs of simulations for market demand and observed product functionality. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.



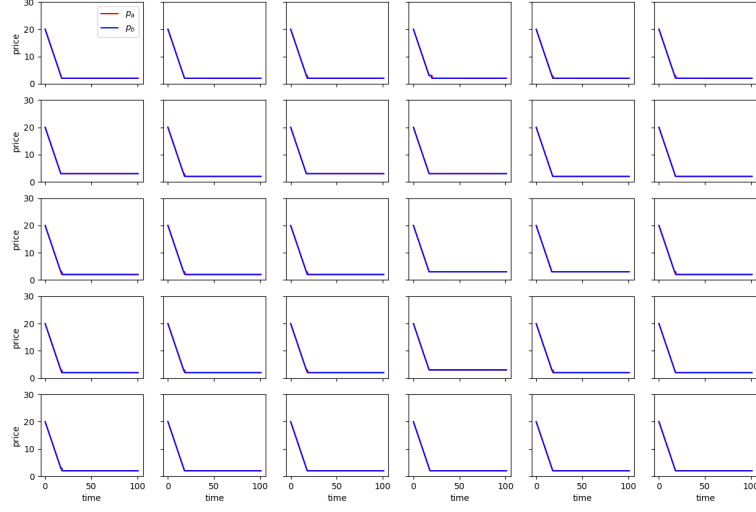
(a)



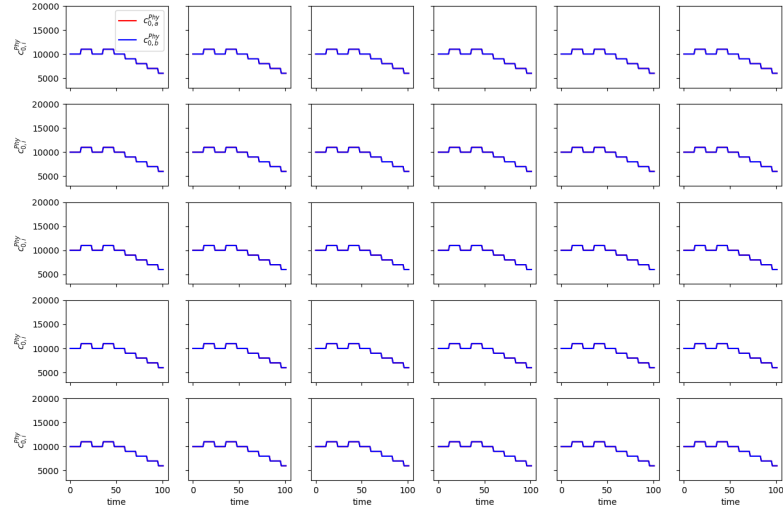
(b)

Figure 2.59: Single runs of simulations for Relative Utility Index and profit. $A_a = 1.2, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.

Firms with symmetric technologies

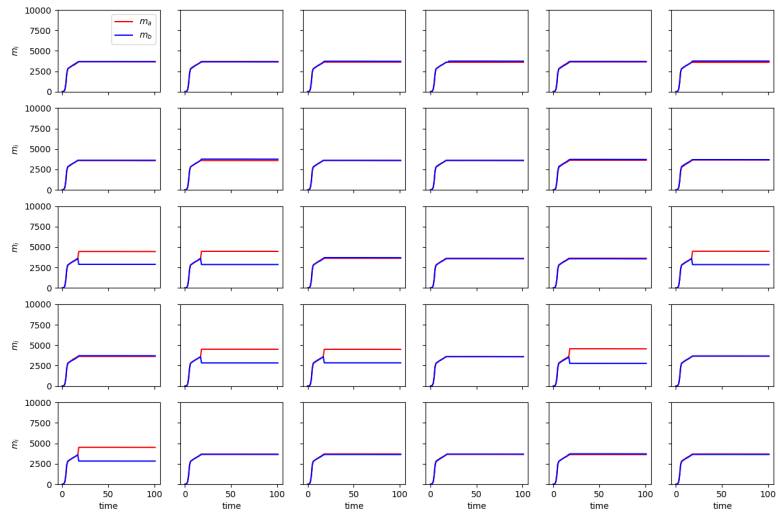


(a)

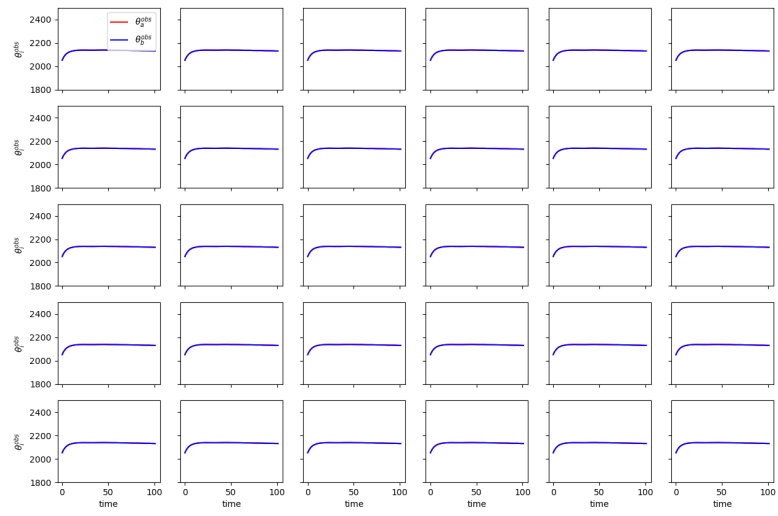


(b)

Figure 2.60: Single runs of simulations for price and investment dynamics. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

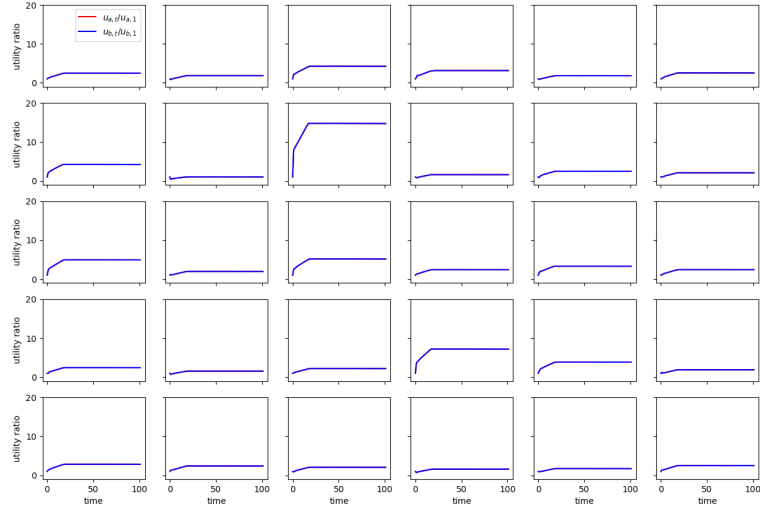


(a)

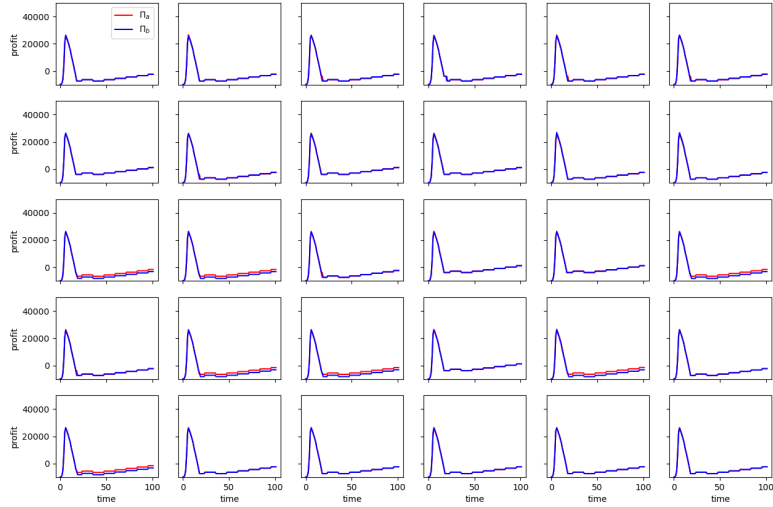


(b)

Figure 2.61: Single runs of simulations for market demand and observed product functionality. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

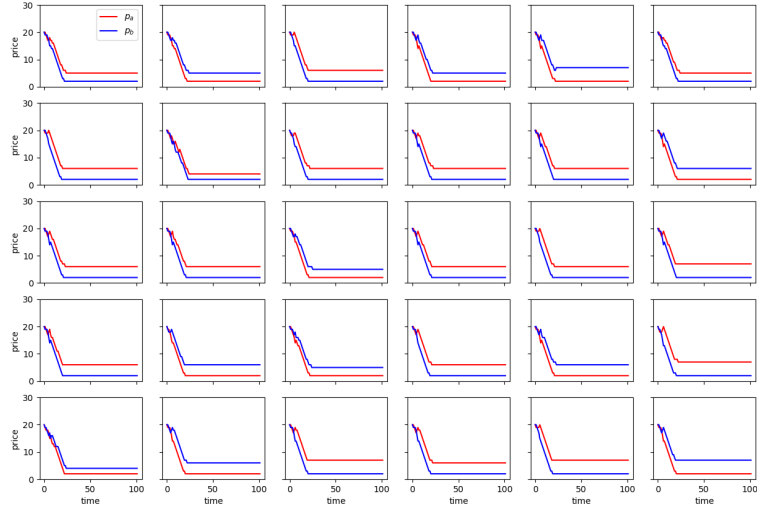


(a)

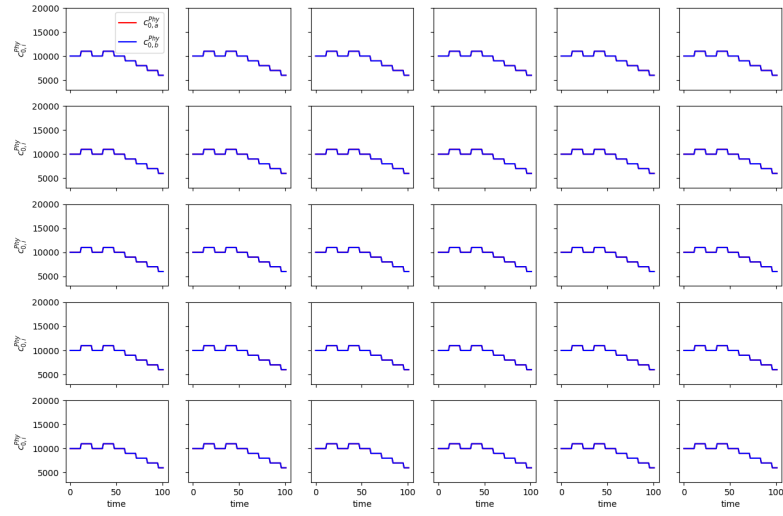


(b)

Figure 2.62: Single runs of simulations for Relative Utility Index and profit. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 0$.

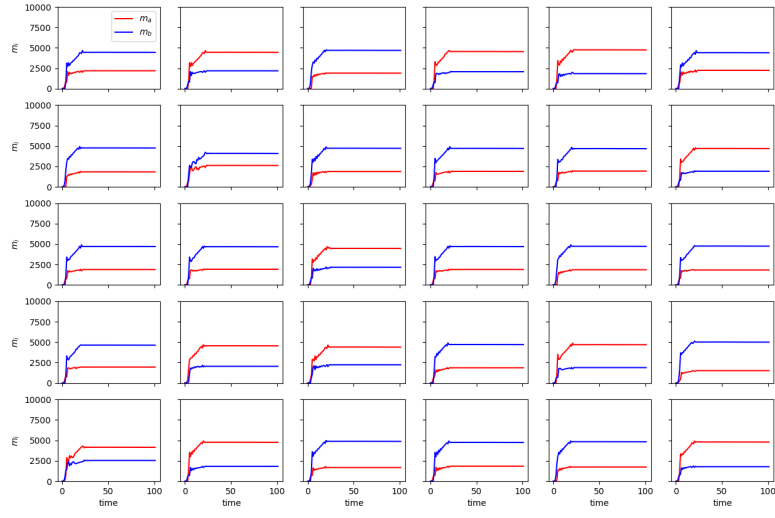


(a)

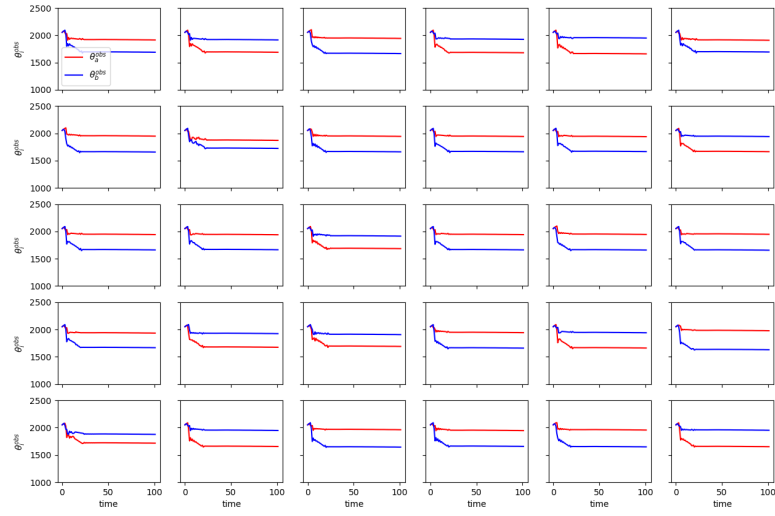


(b)

Figure 2.63: Single runs of simulations for price and investment dynamics. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

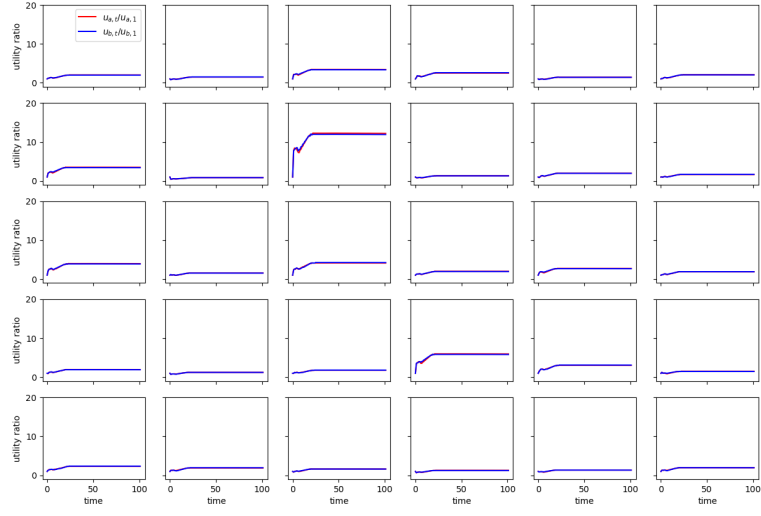


(a)

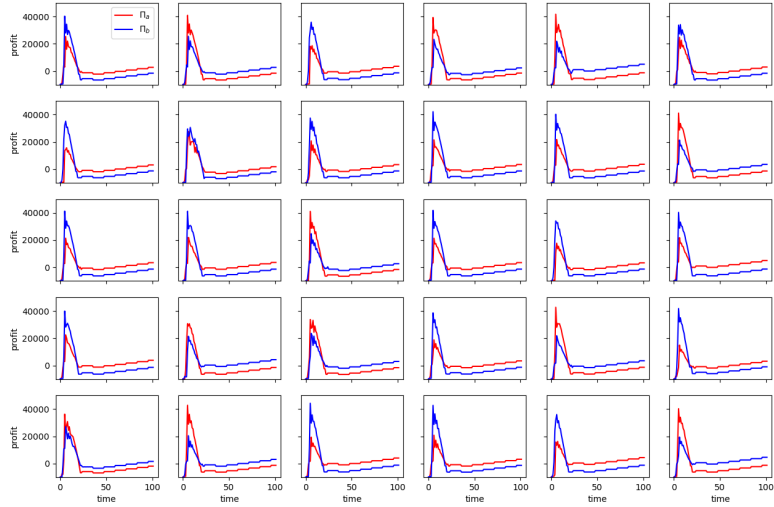


(b)

Figure 2.64: Single runs of simulations for market demand and observed product functionality. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

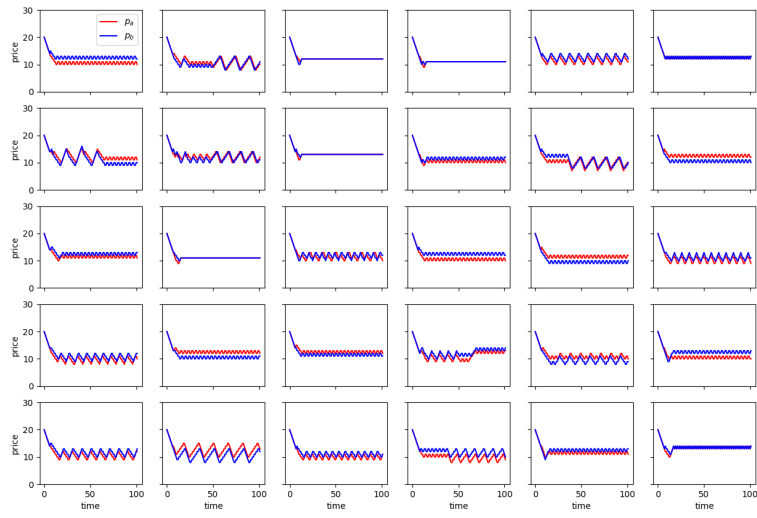


(a)

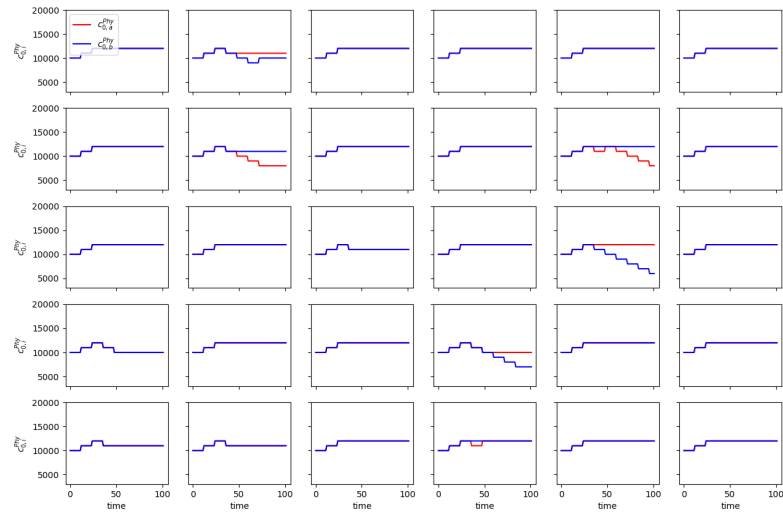


(b)

Figure 2.65: Single runs of simulations for Relative Utility Index and profit. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 0$.

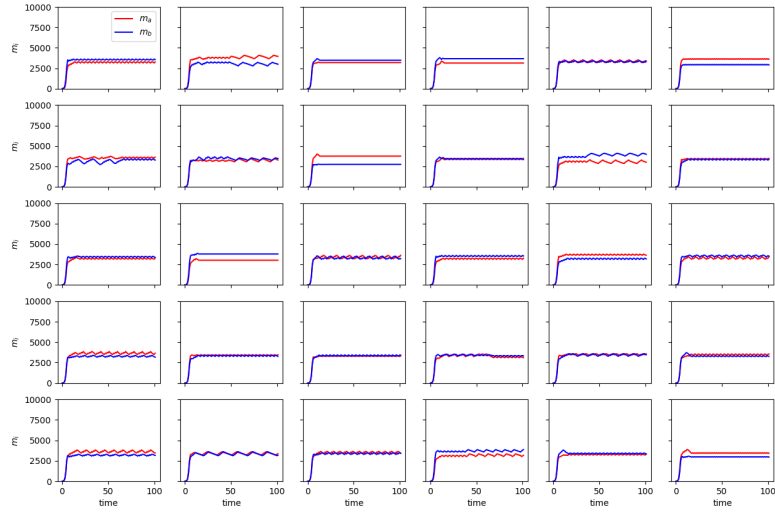


(a)

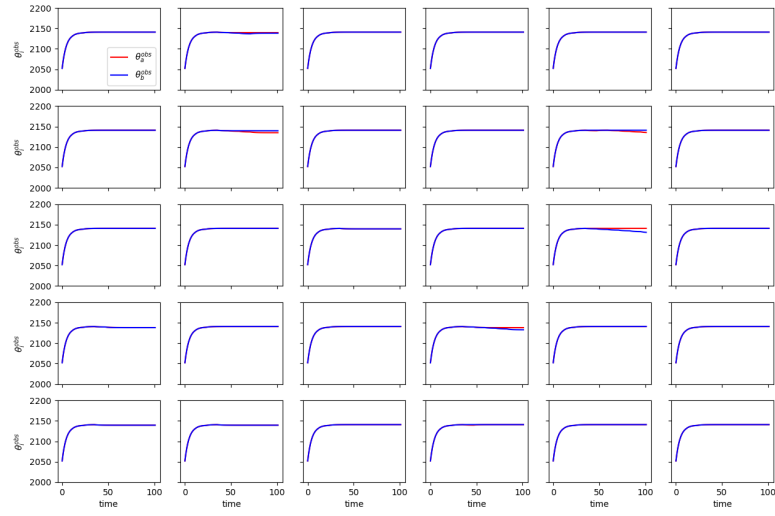


(b)

Figure 2.66: Single runs of simulations for price and investment dynamics. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

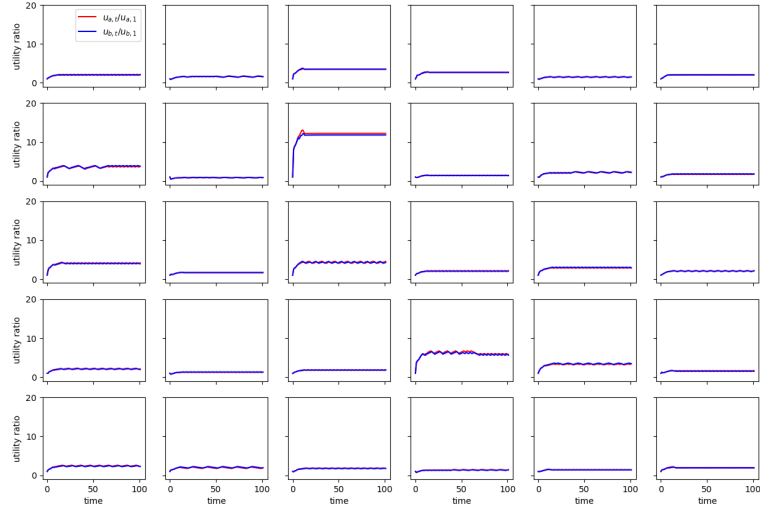


(a)

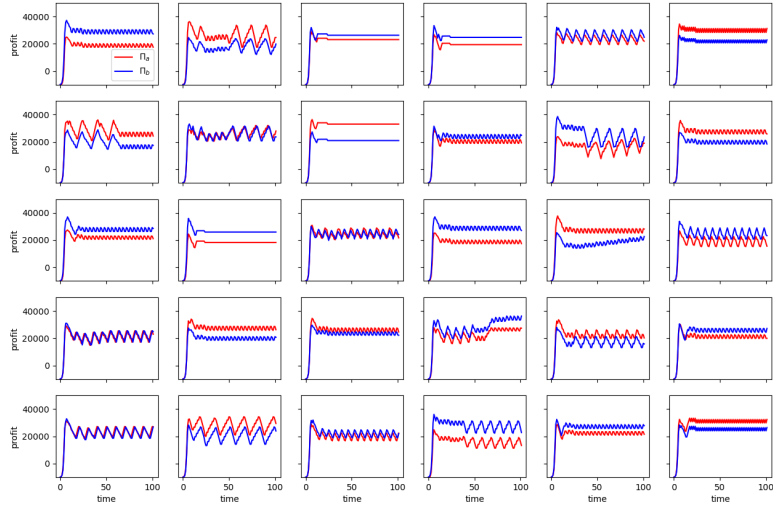


(b)

Figure 2.67: Single runs of simulations for market demand and observed product functionality. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

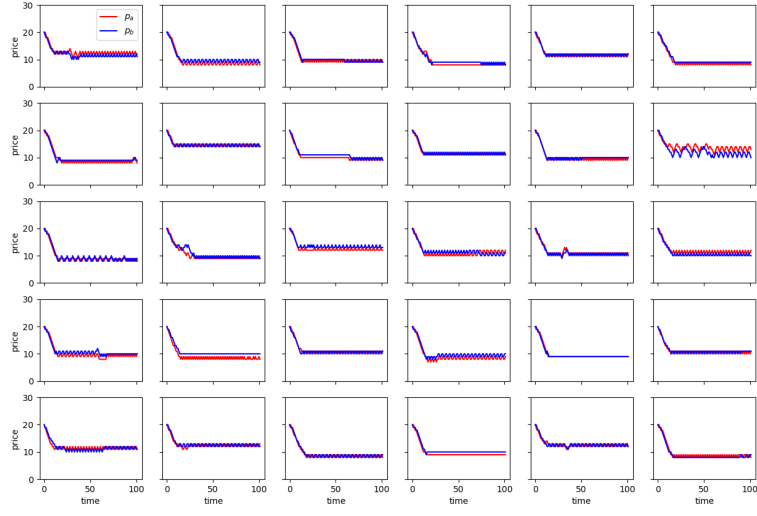


(a)

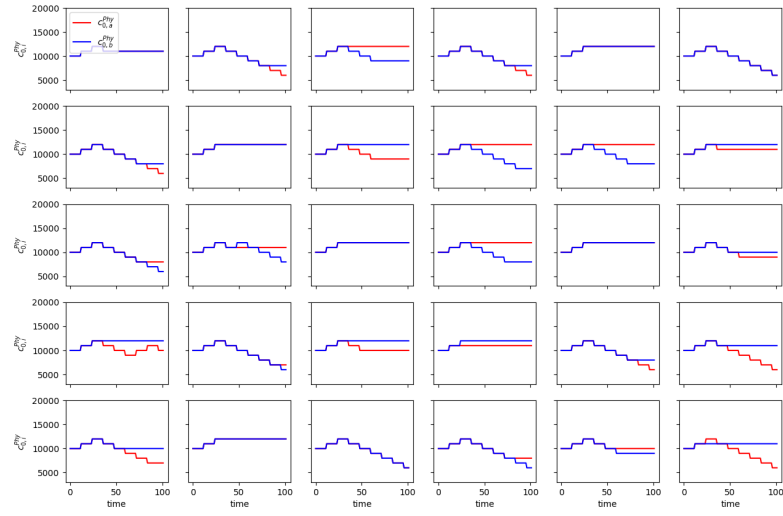


(b)

Figure 2.68: Single runs of simulations for Relative Utility Index and profit. $A_a = 1, A_b = 1, \beta_0 = 0, \beta_4 = 50$.

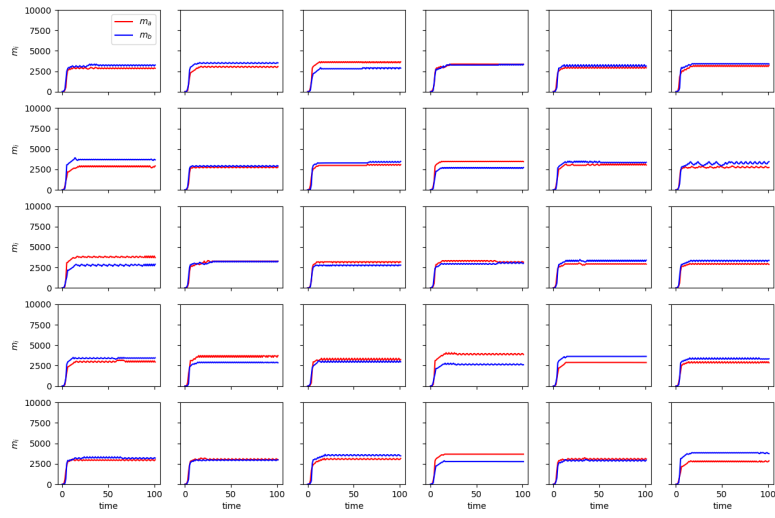


(a)

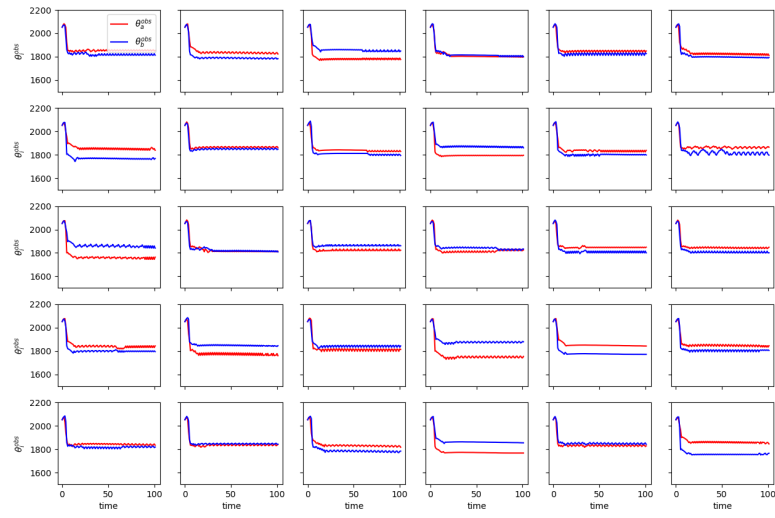


(b)

Figure 2.69: Single runs of simulations for price and investment dynamics. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.

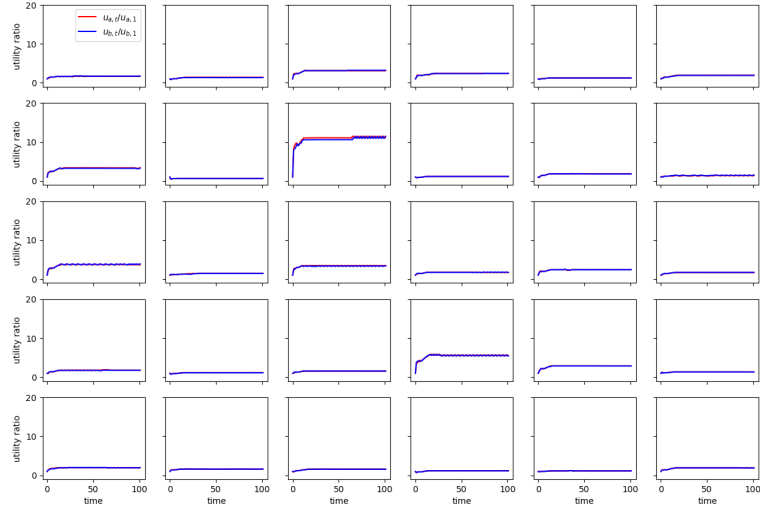


(a)

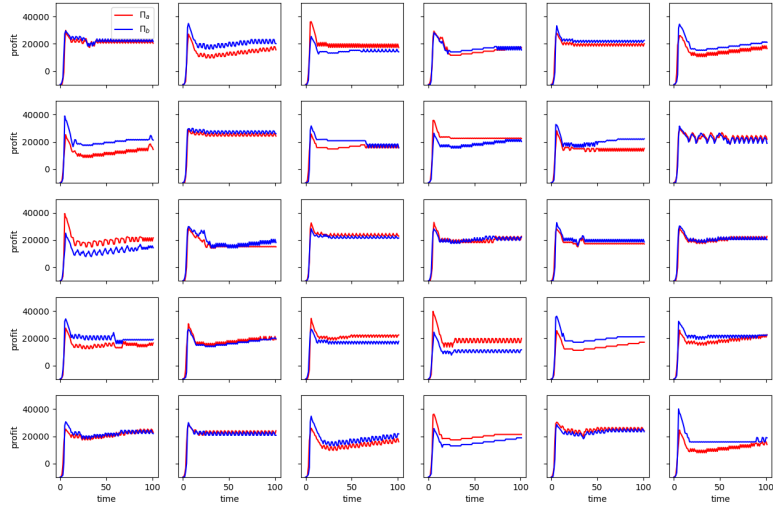


(b)

Figure 2.70: Single runs of simulations for market demand and observed product functionality. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.



(a)



(b)

Figure 2.71: Single runs of simulations for Relative Utility Index and profit. $A_a = 1, A_b = 1, \beta_0 = 0.1, \beta_4 = 50$.

2.7.5 Example cases in policy analysis

A. Without congestion or network effects ($\beta_0 = 0, \beta_4 = 0$)

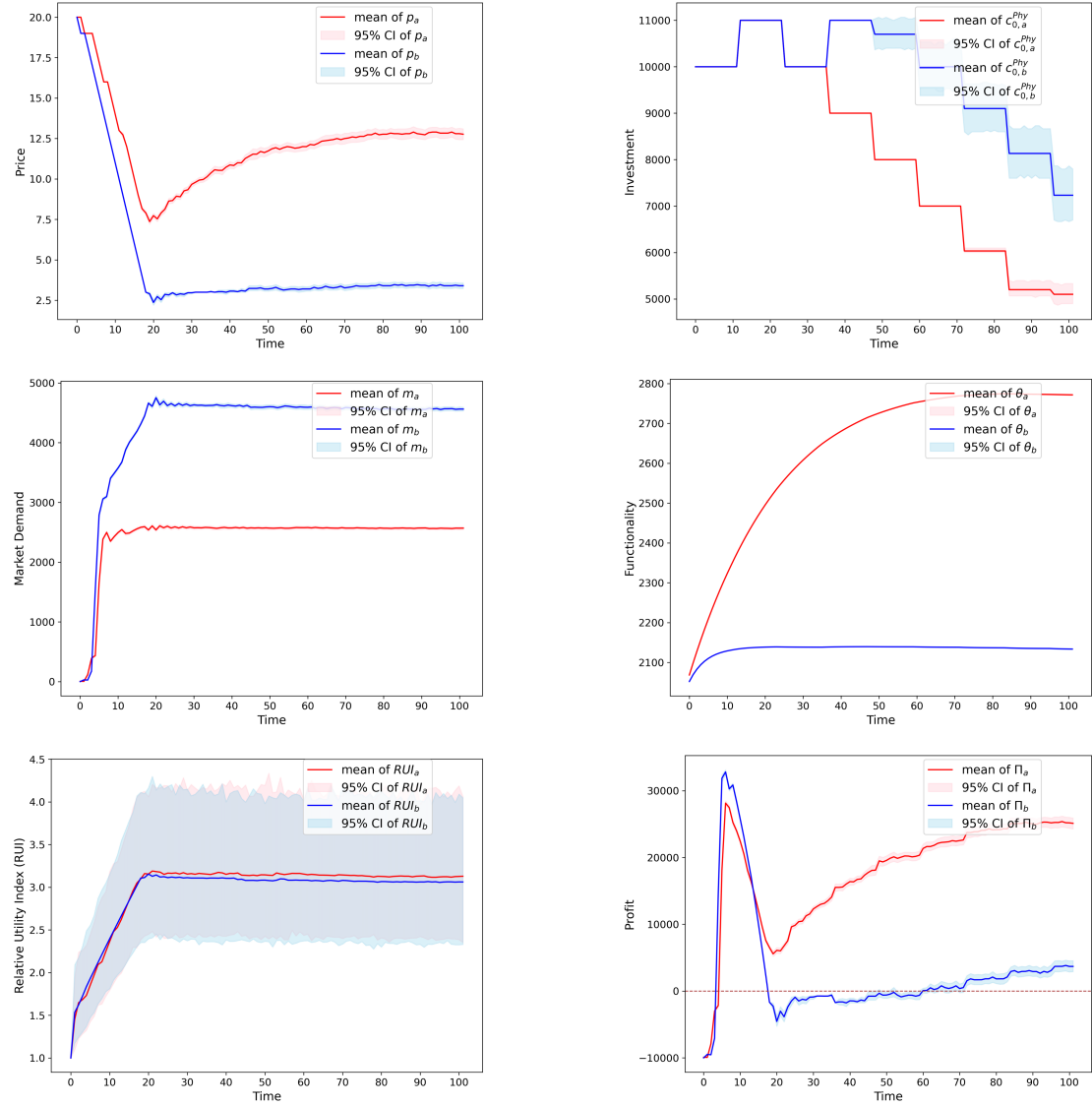


Figure 2.72: Mean and confidence interval of simulations. Benchmark case.

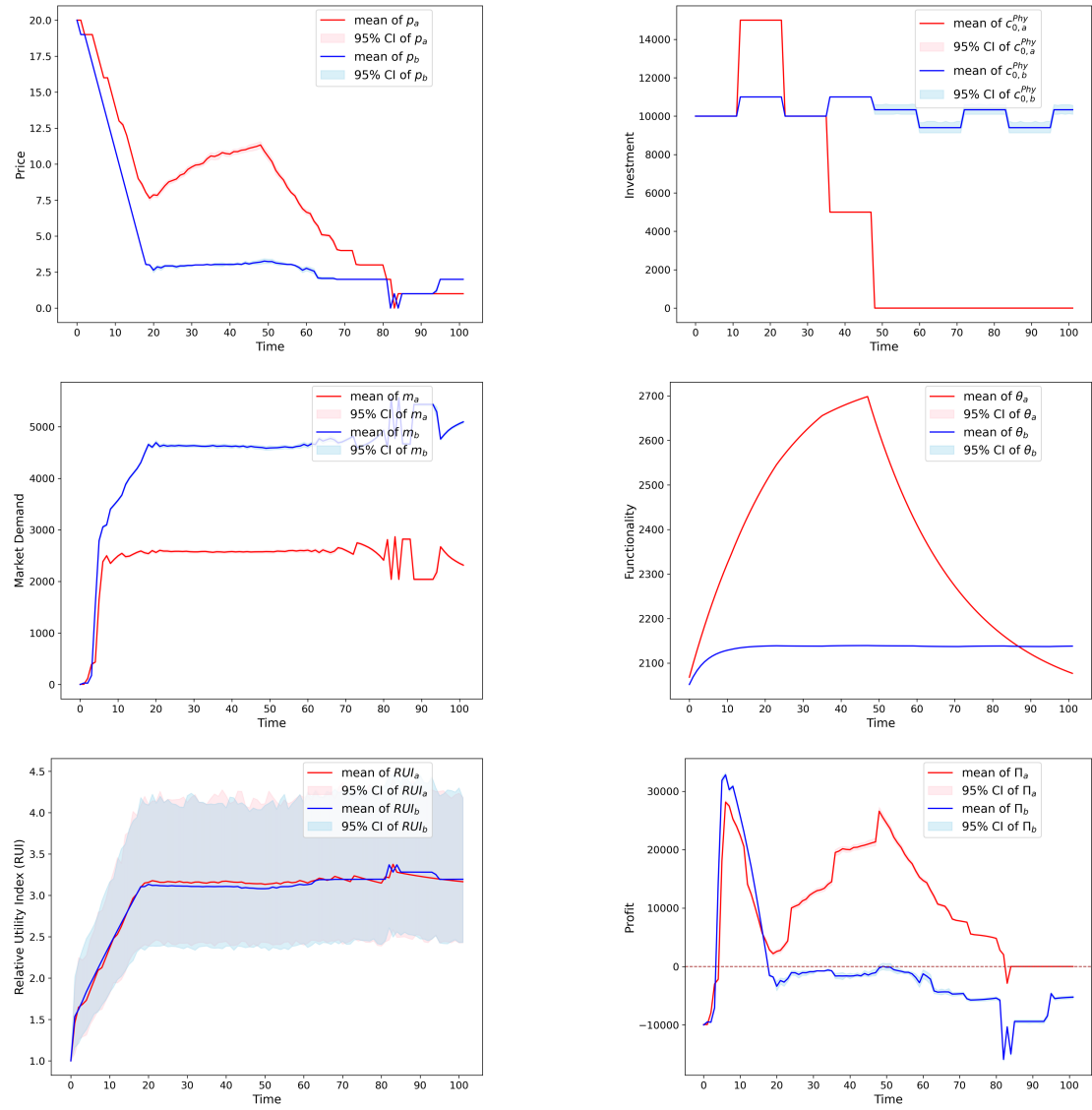


Figure 2.73: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (5000, 1000)$.

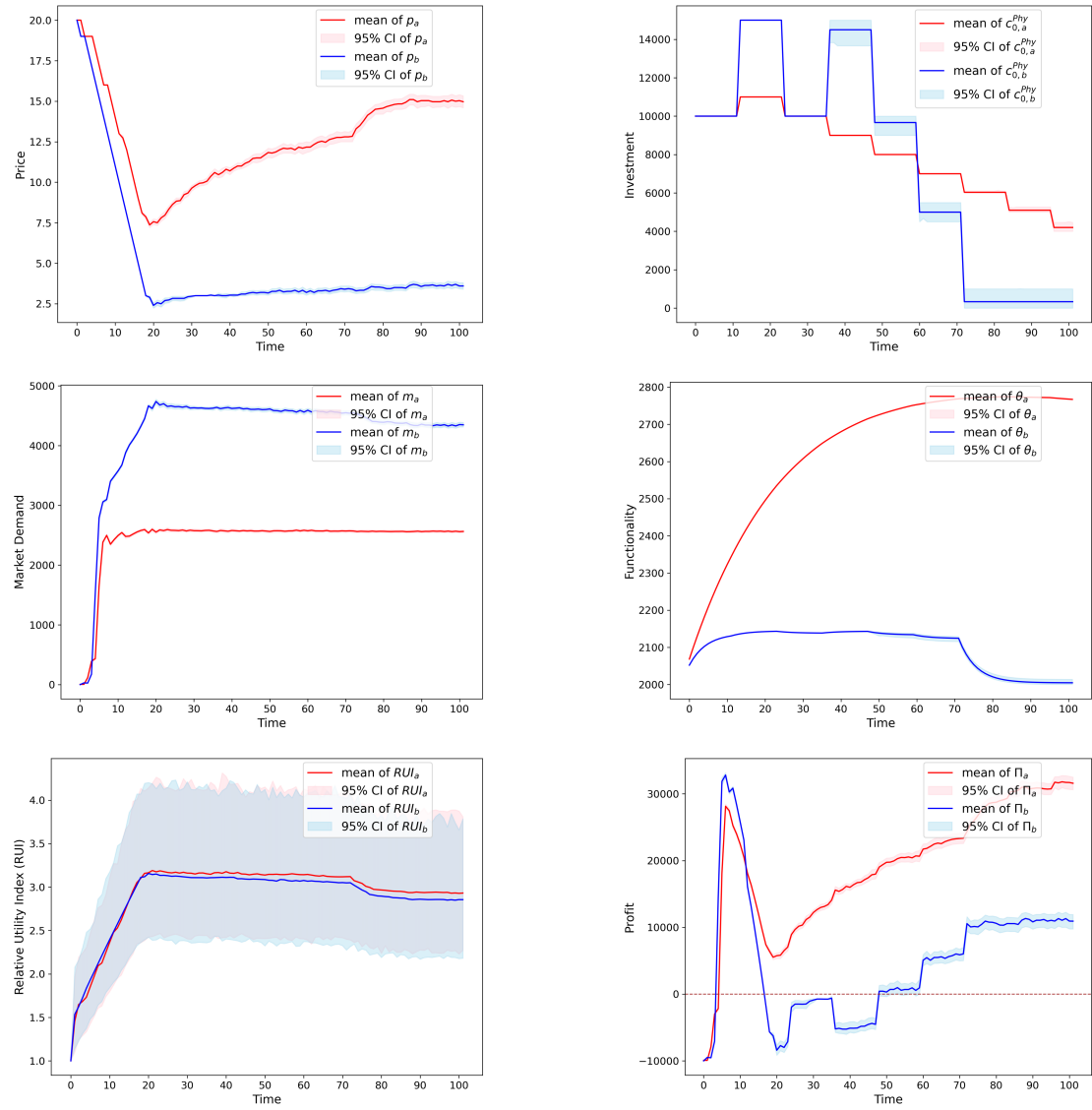


Figure 2.74: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (1000, 5000)$.

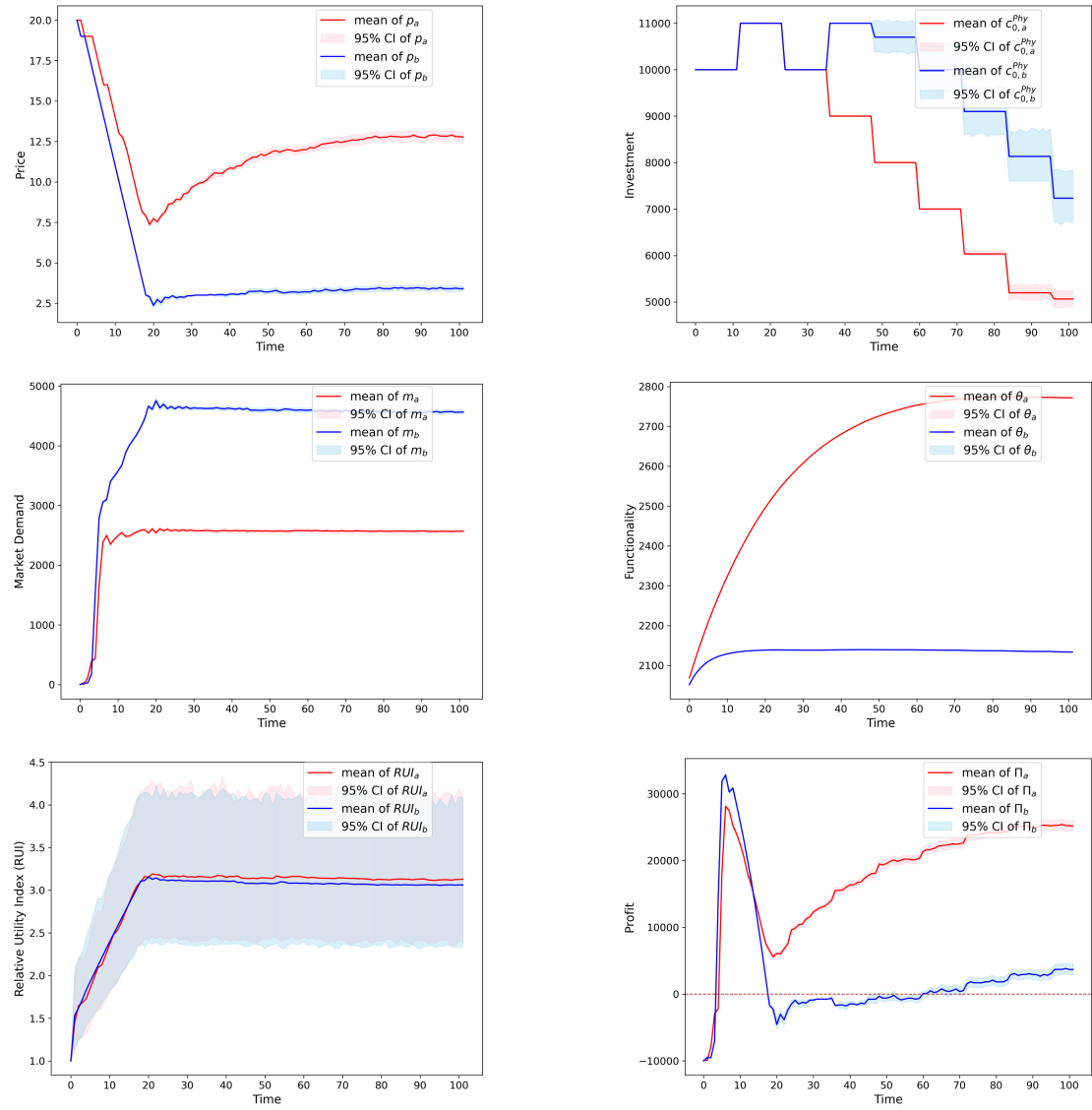


Figure 2.75: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (100, 0)$.

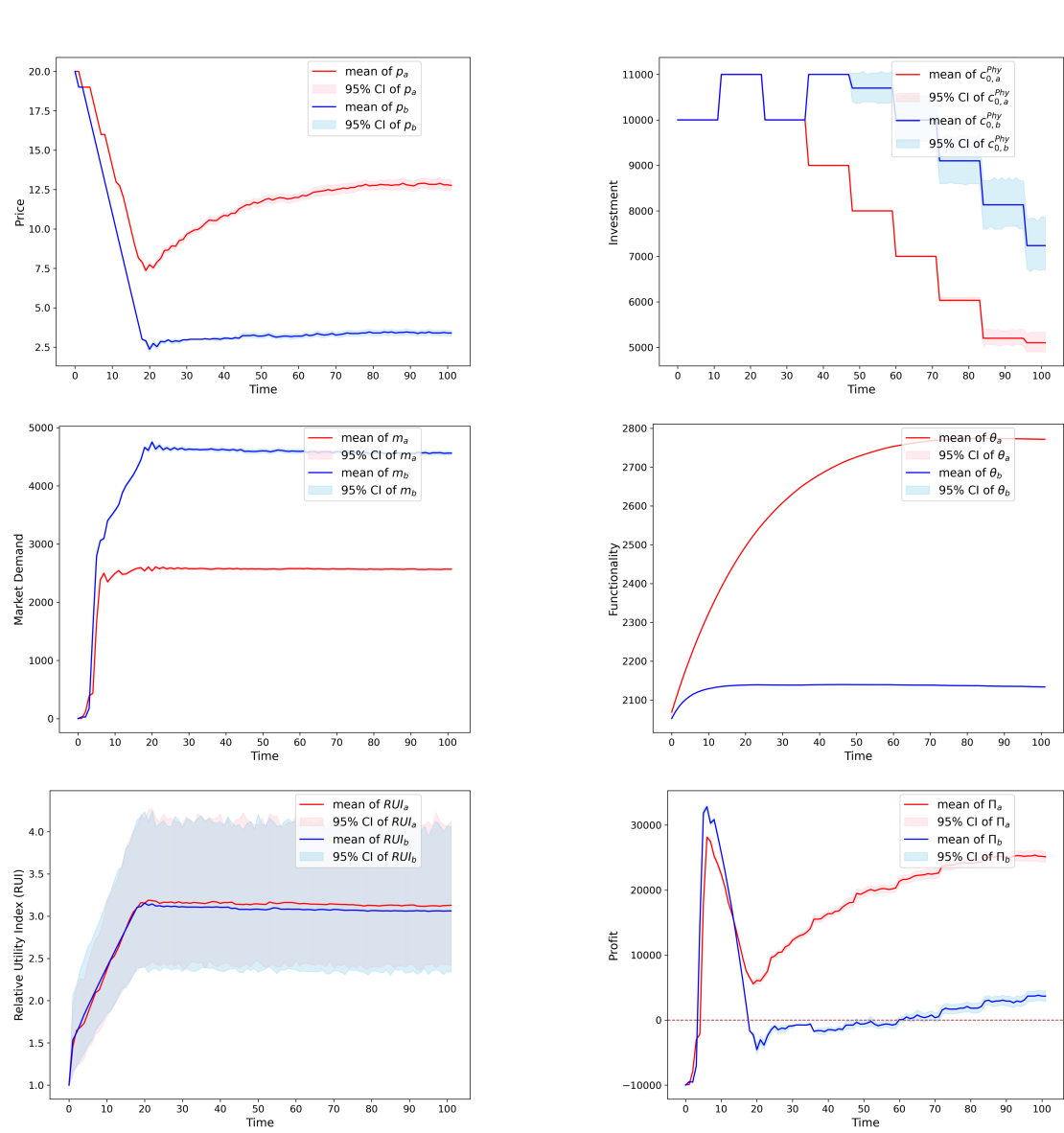


Figure 2.76: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (0, 100)$.

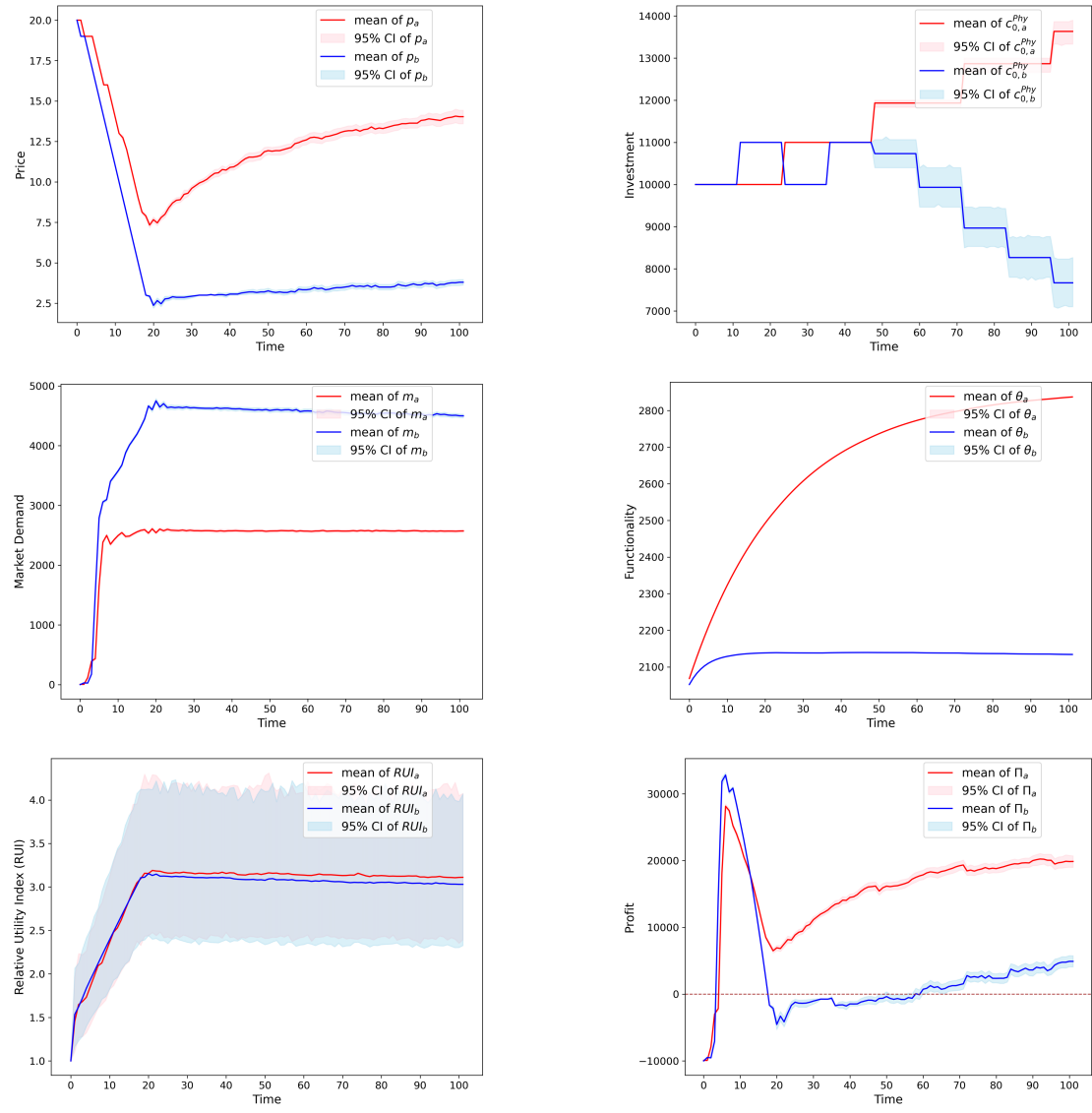


Figure 2.77: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (24, 12)$.

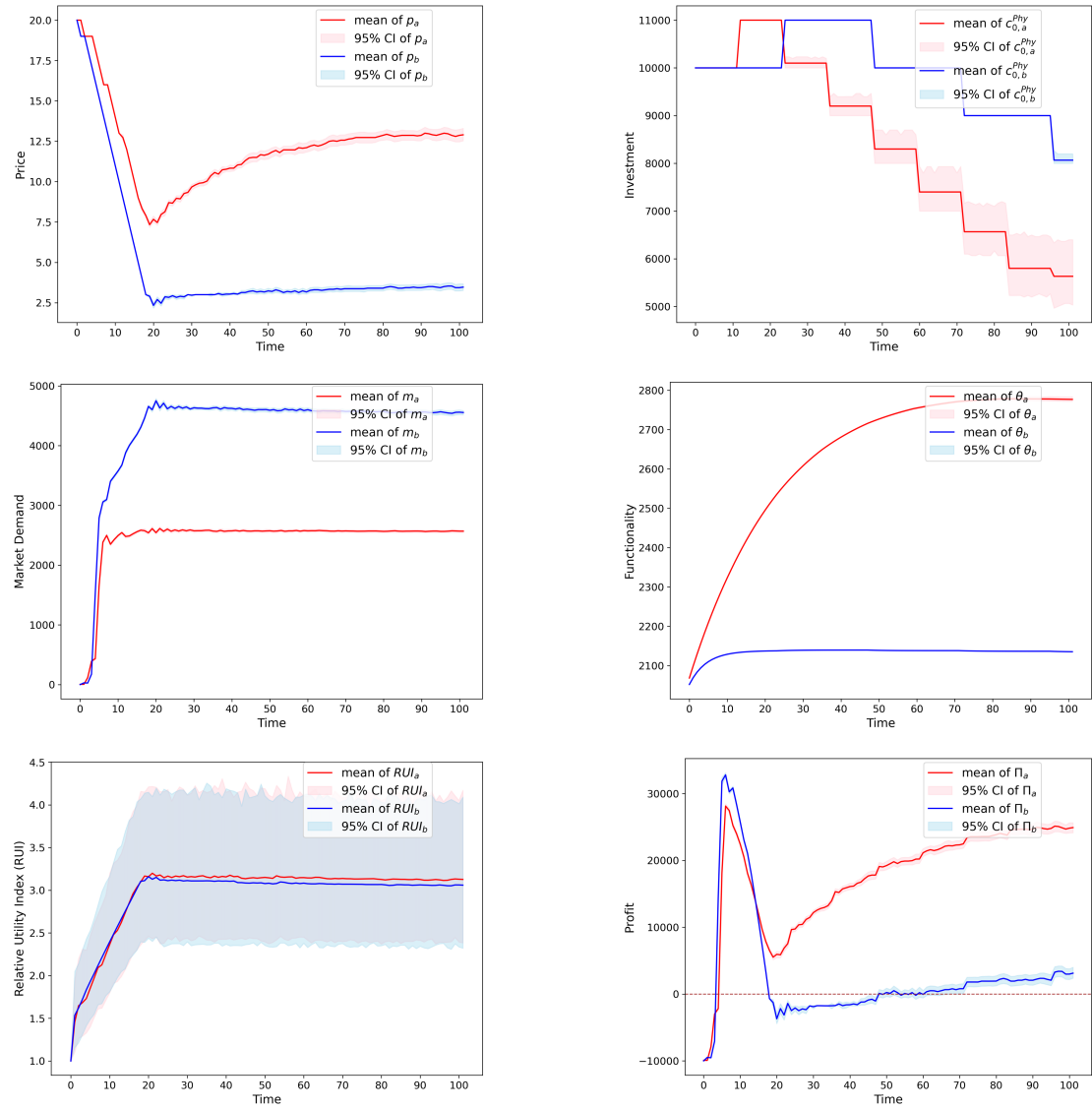


Figure 2.78: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (12, 24)$.

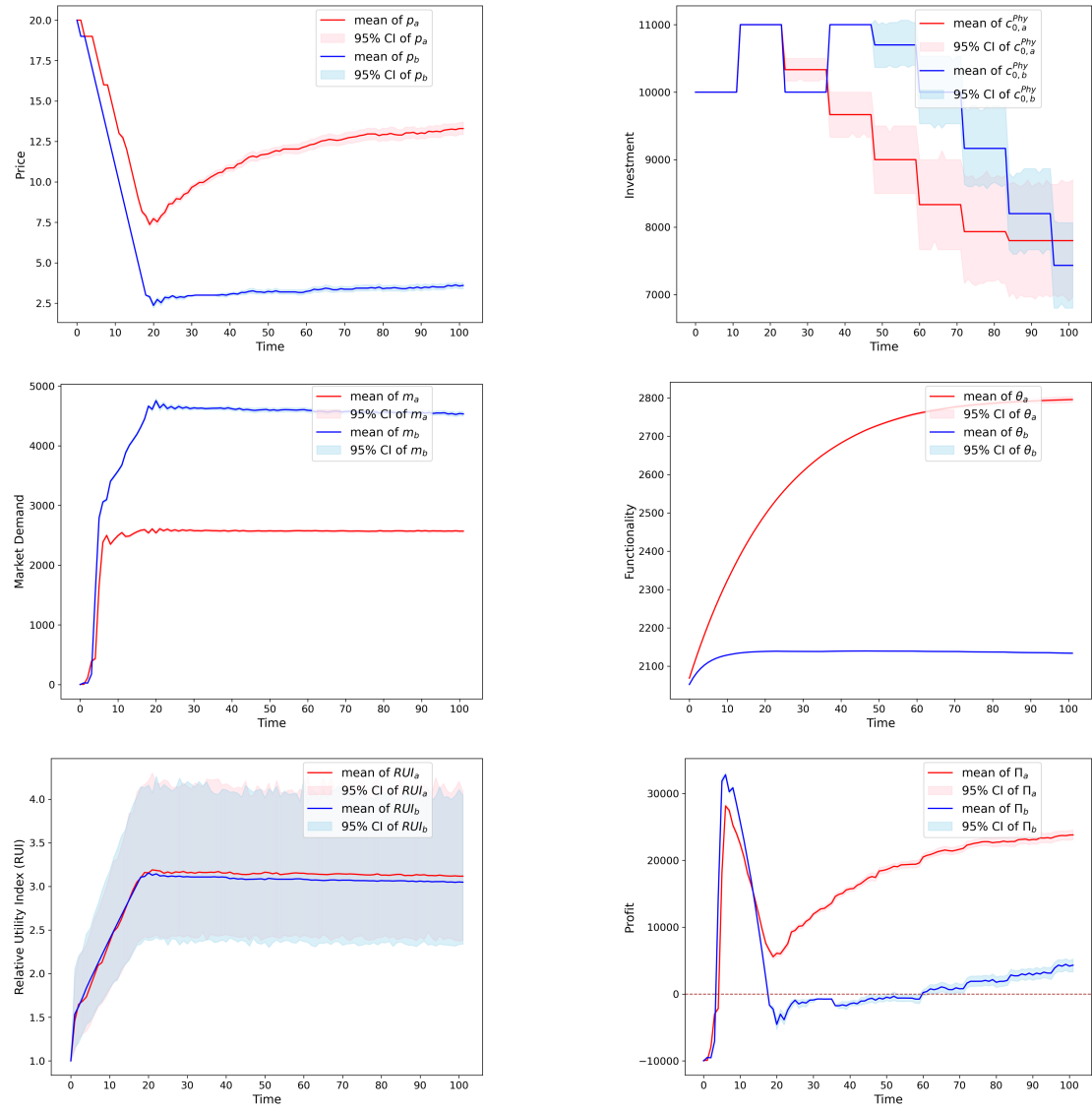


Figure 2.79: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.1, 0.05)$.

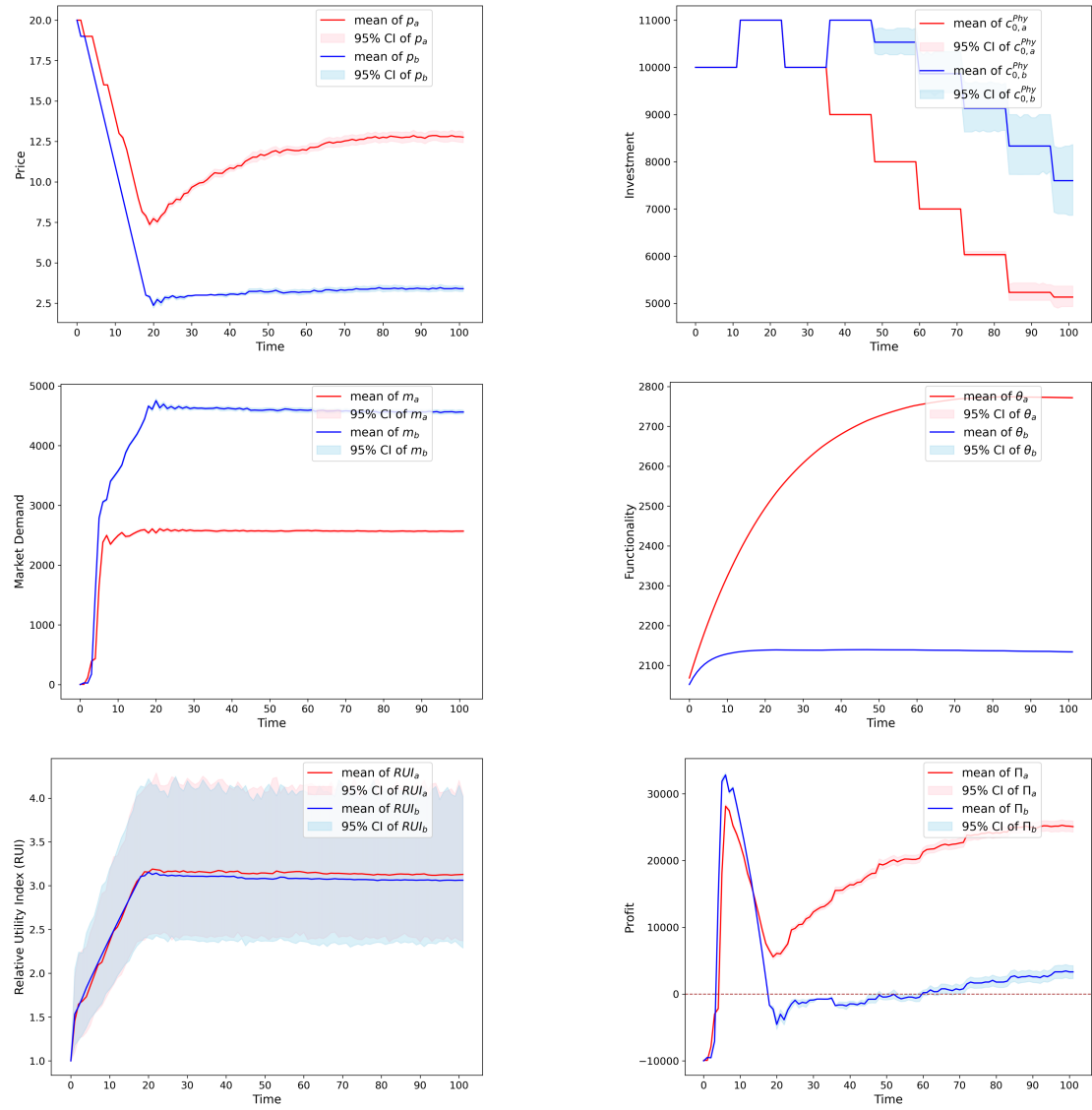


Figure 2.80: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.05, 0.1)$.

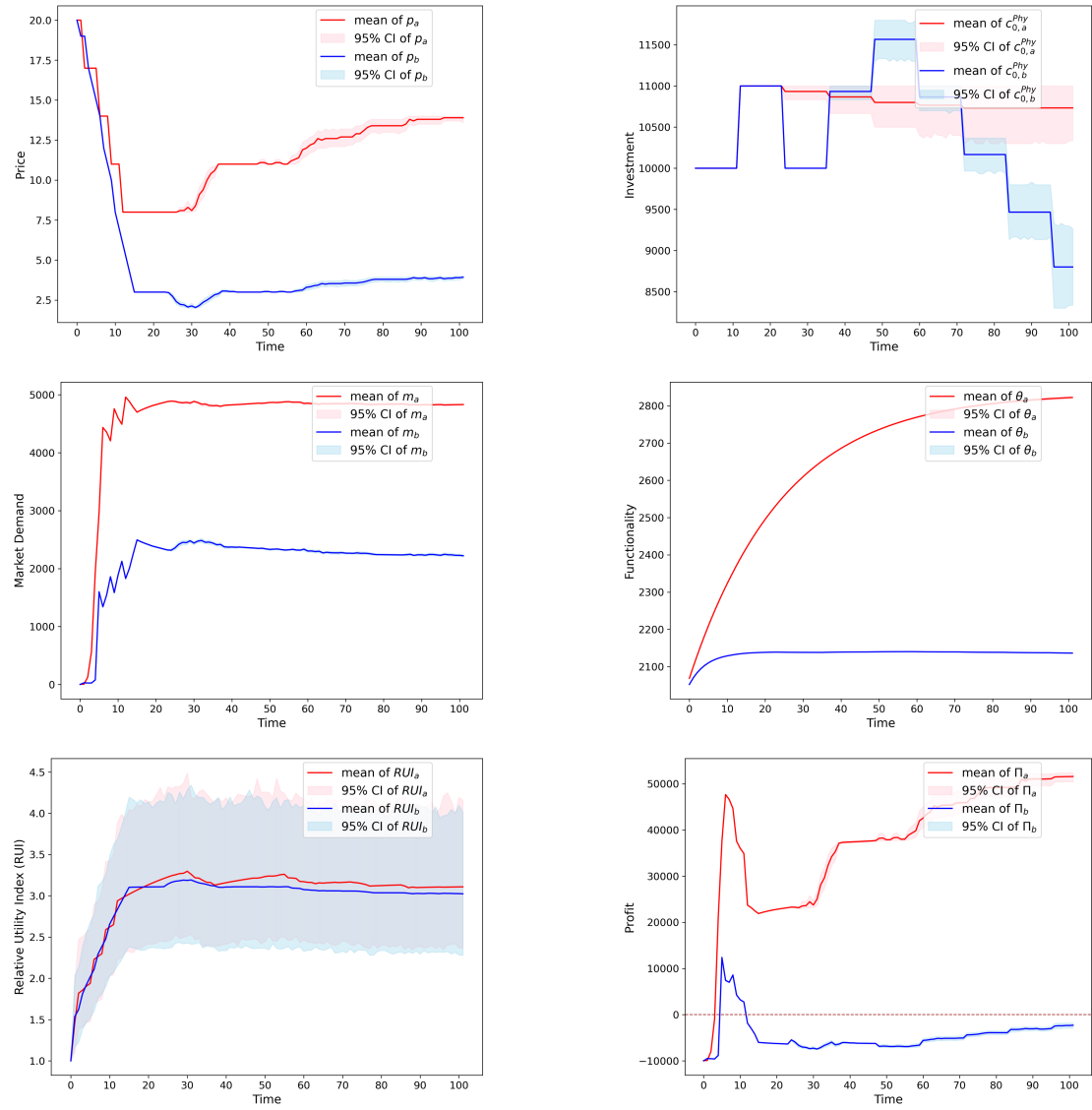


Figure 2.81: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (3, 1)$.

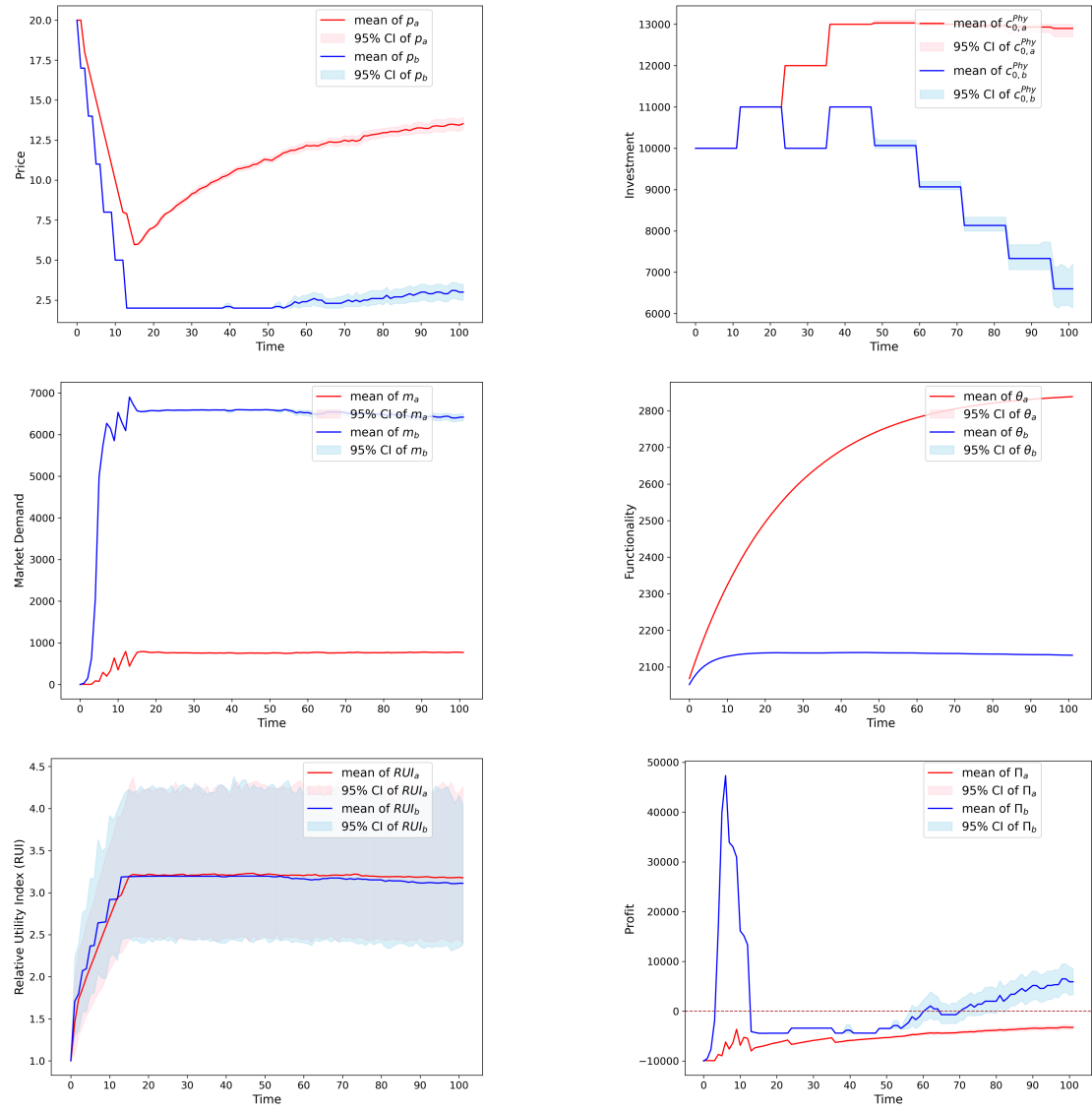


Figure 2.82: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (1, 3)$.

With congestion effect + No network effect ($\beta_0 = 0.1, \beta_4 = 0$)

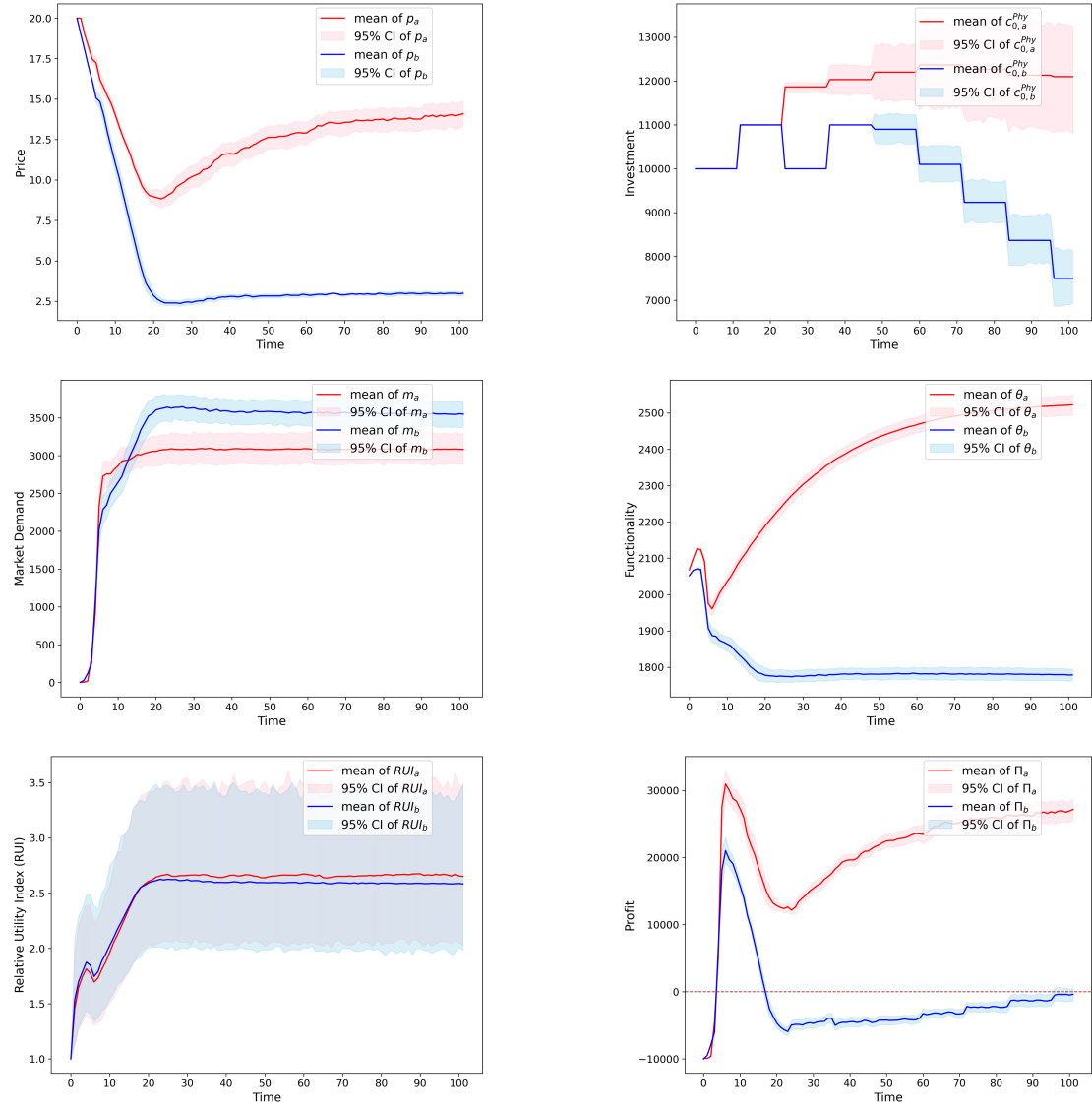


Figure 2.83: Mean and confidence interval of simulations. Benchmark case.

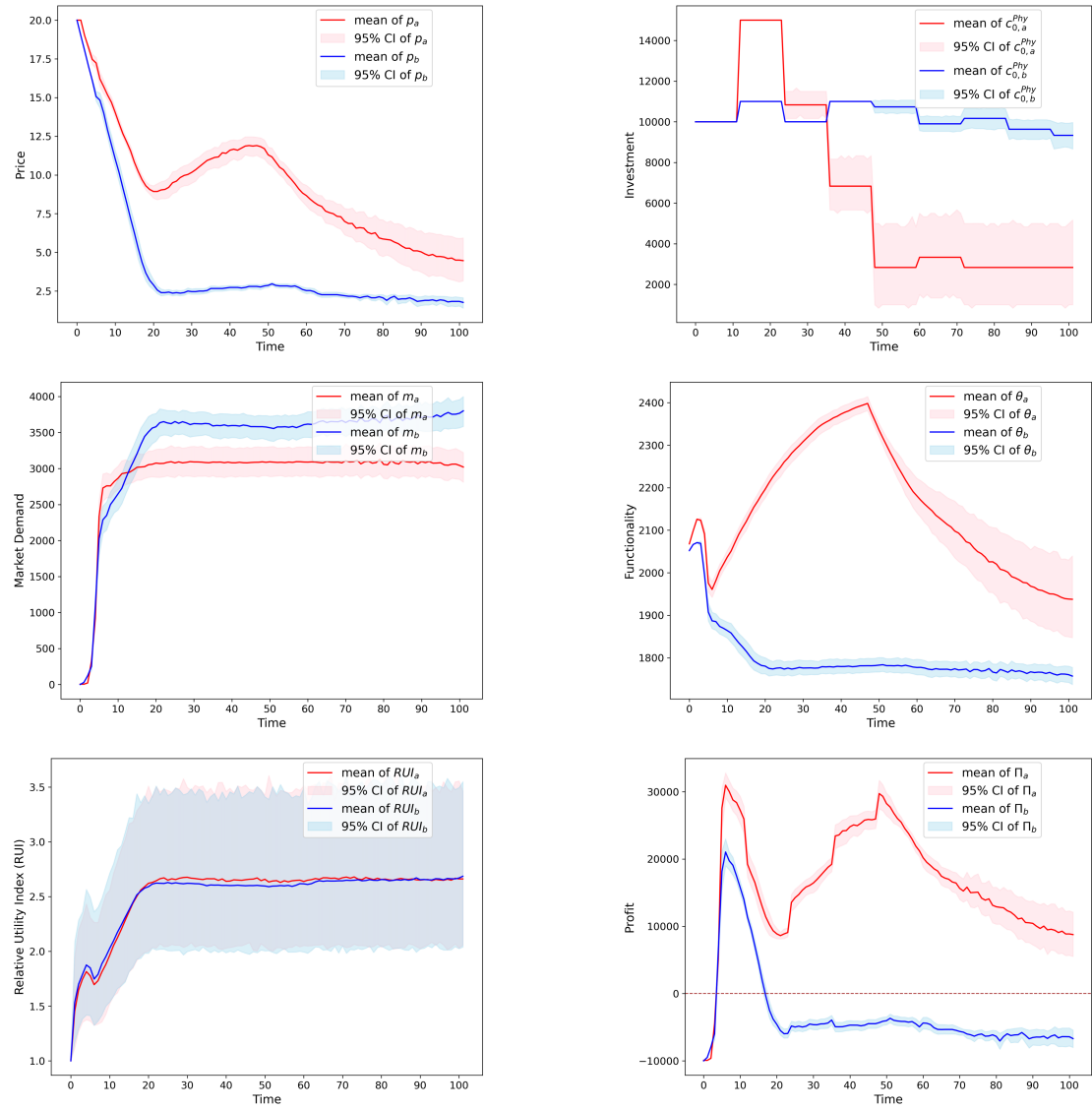


Figure 2.84: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (5000, 1000)$.

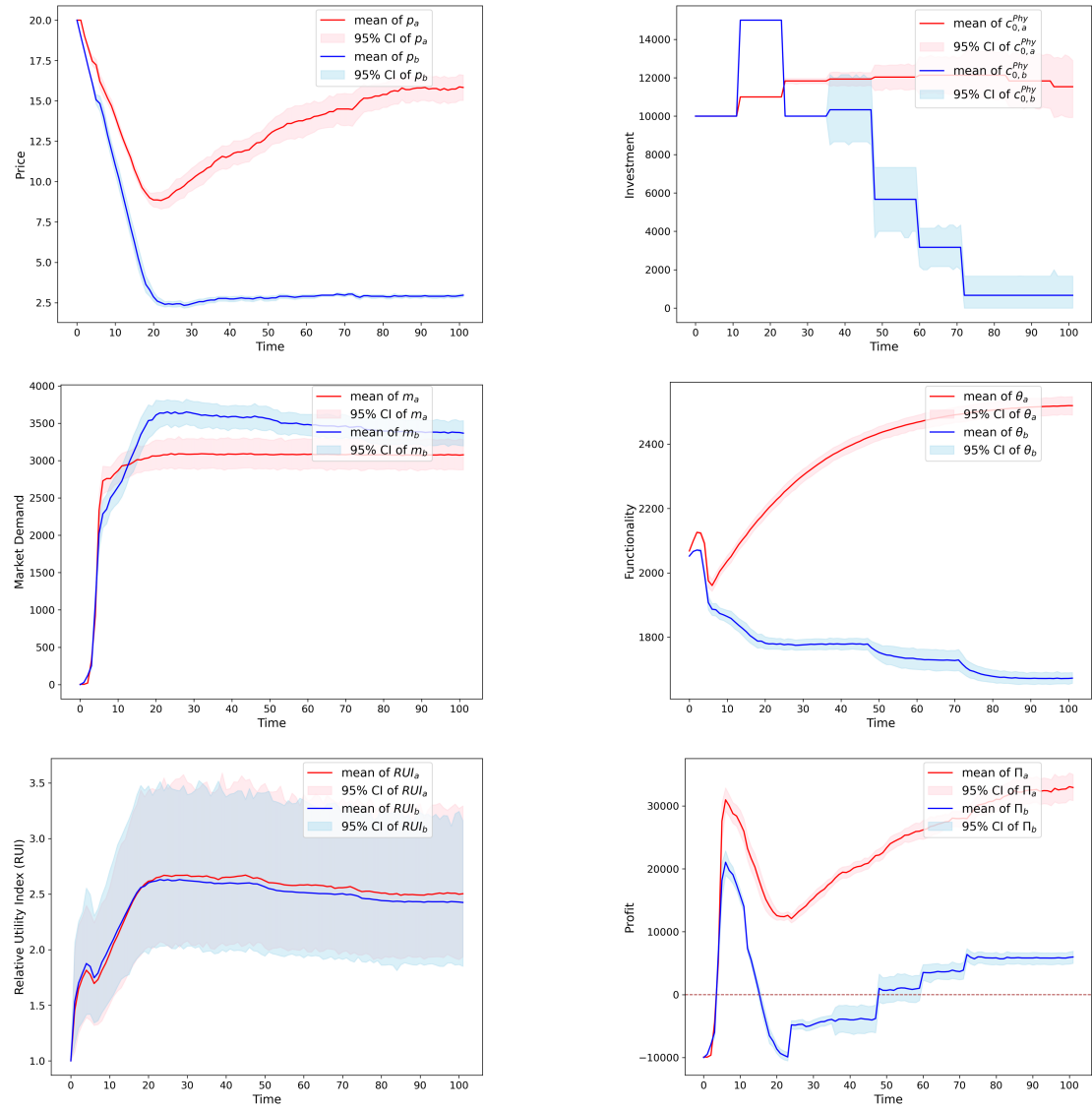


Figure 2.85: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (1000, 5000)$.

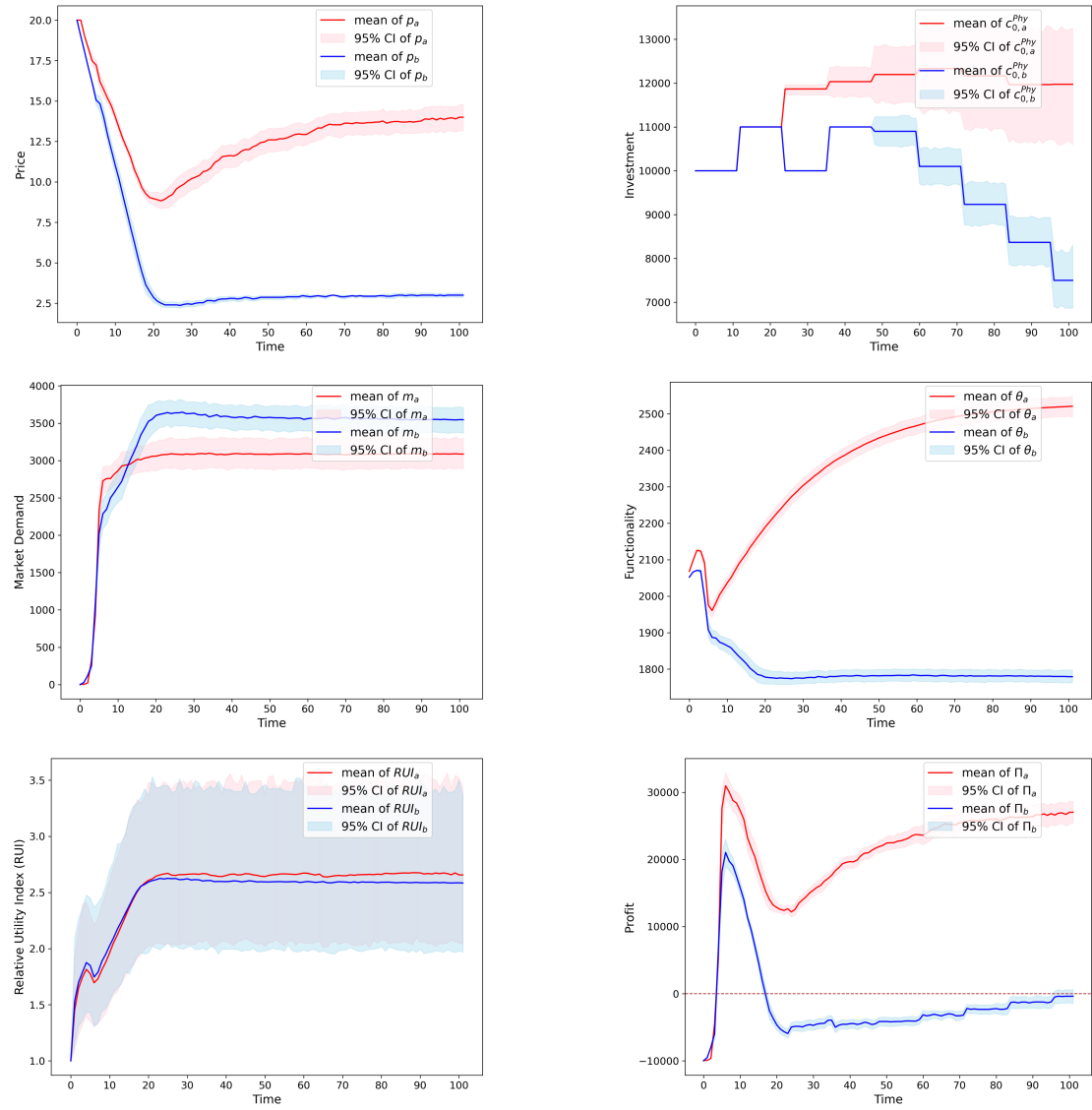


Figure 2.86: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (100, 0)$.

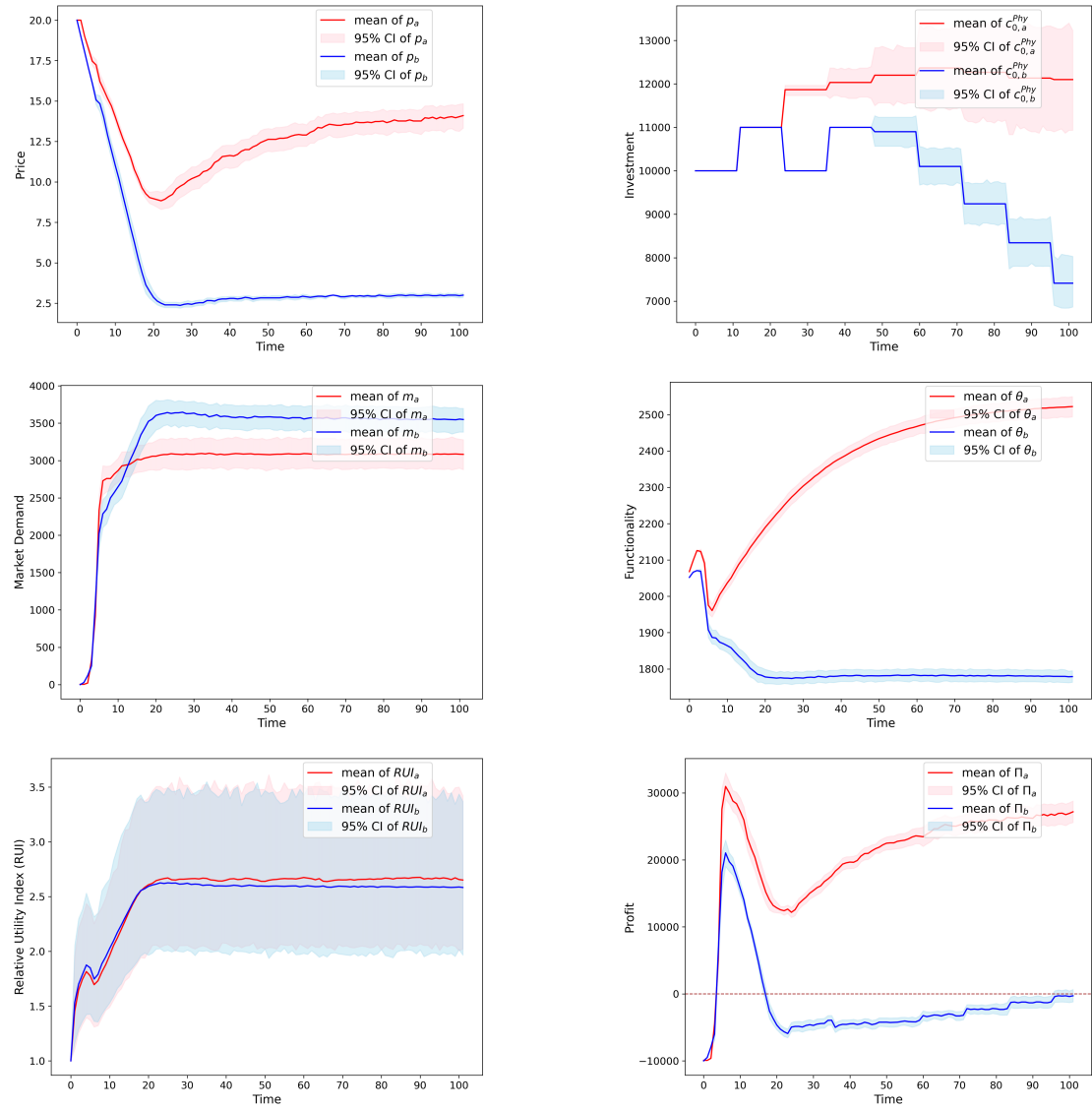


Figure 2.87: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (0, 100)$.

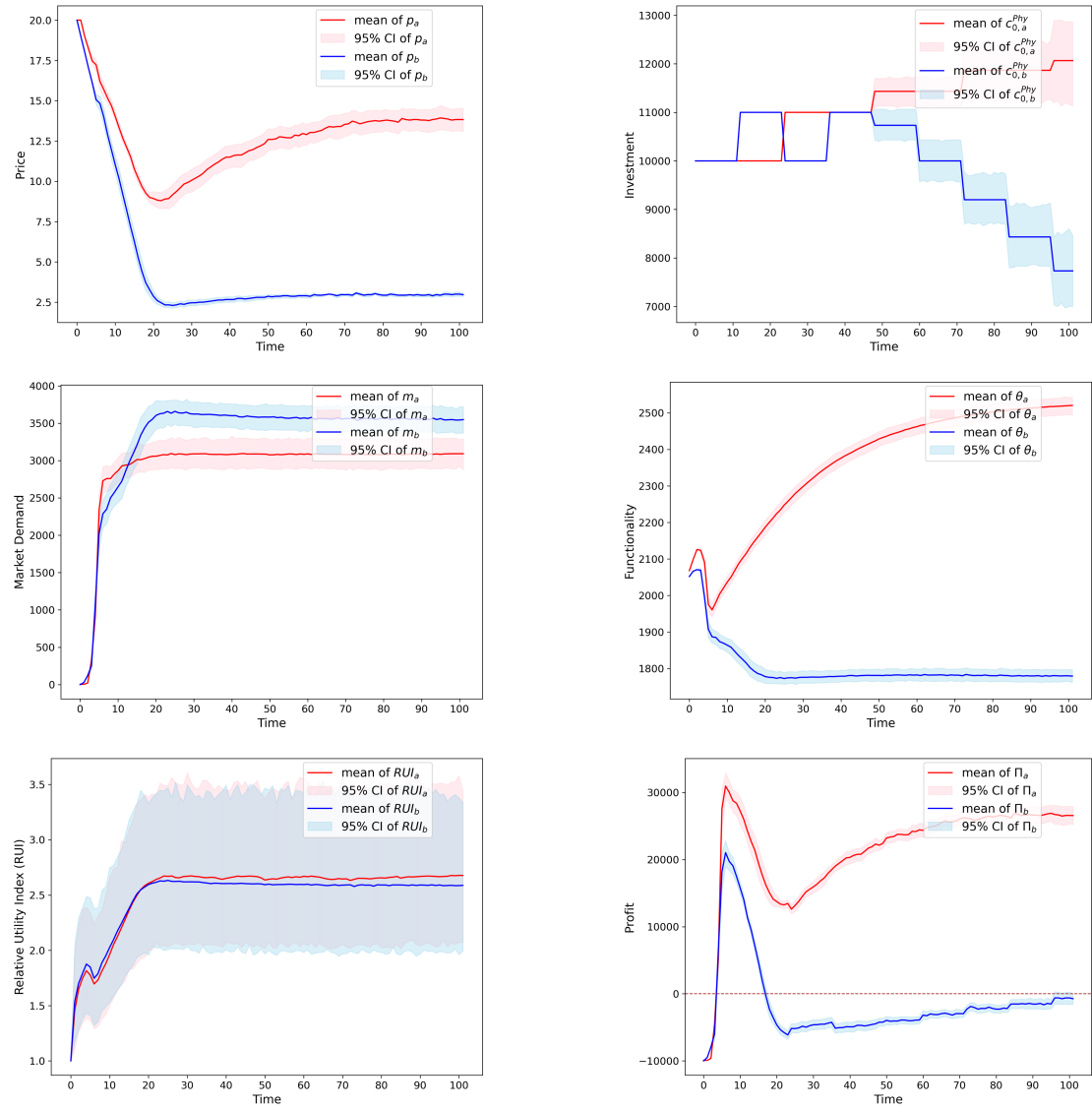


Figure 2.88: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (24, 12)$.

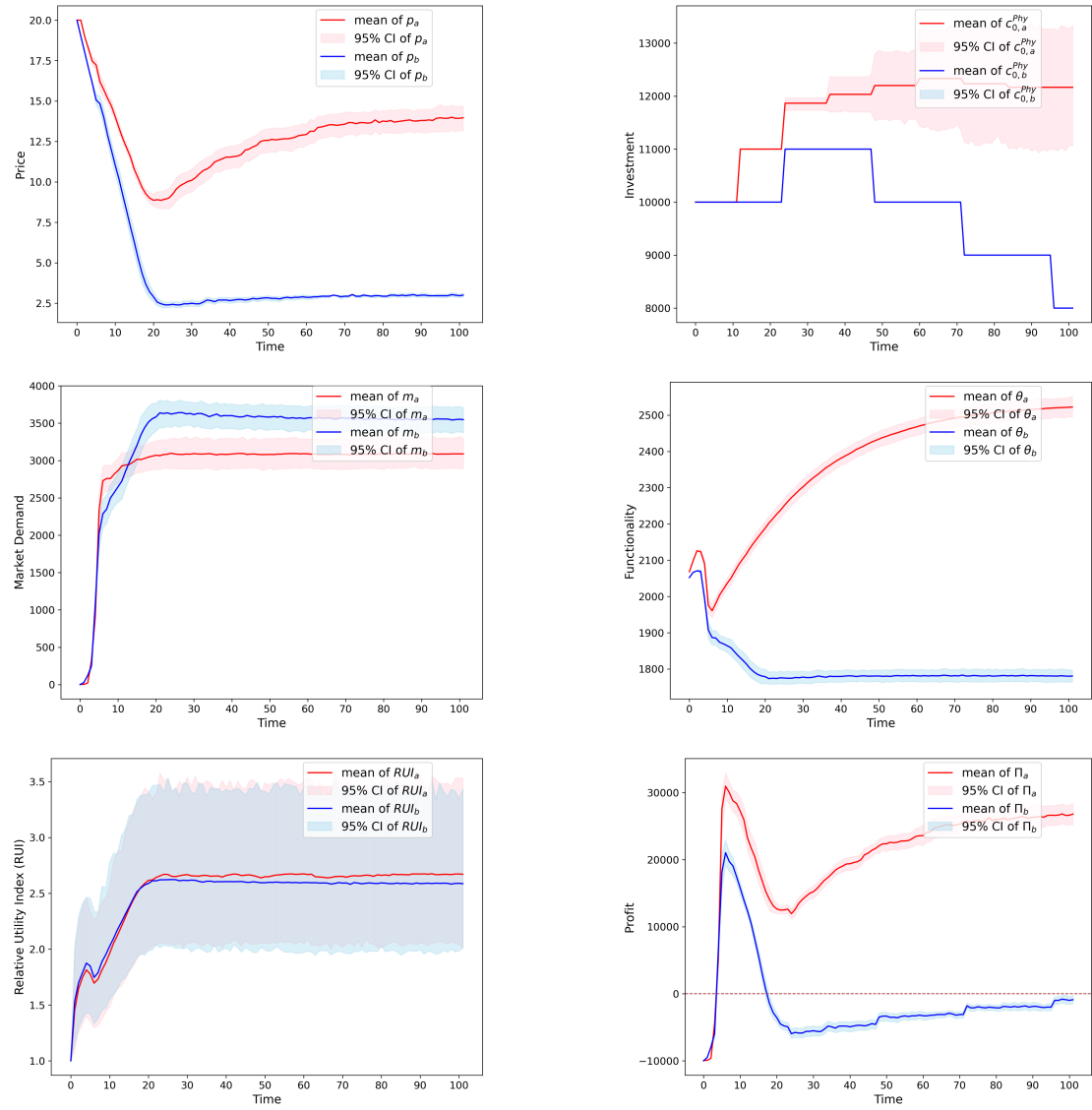


Figure 2.89: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (12, 24)$.

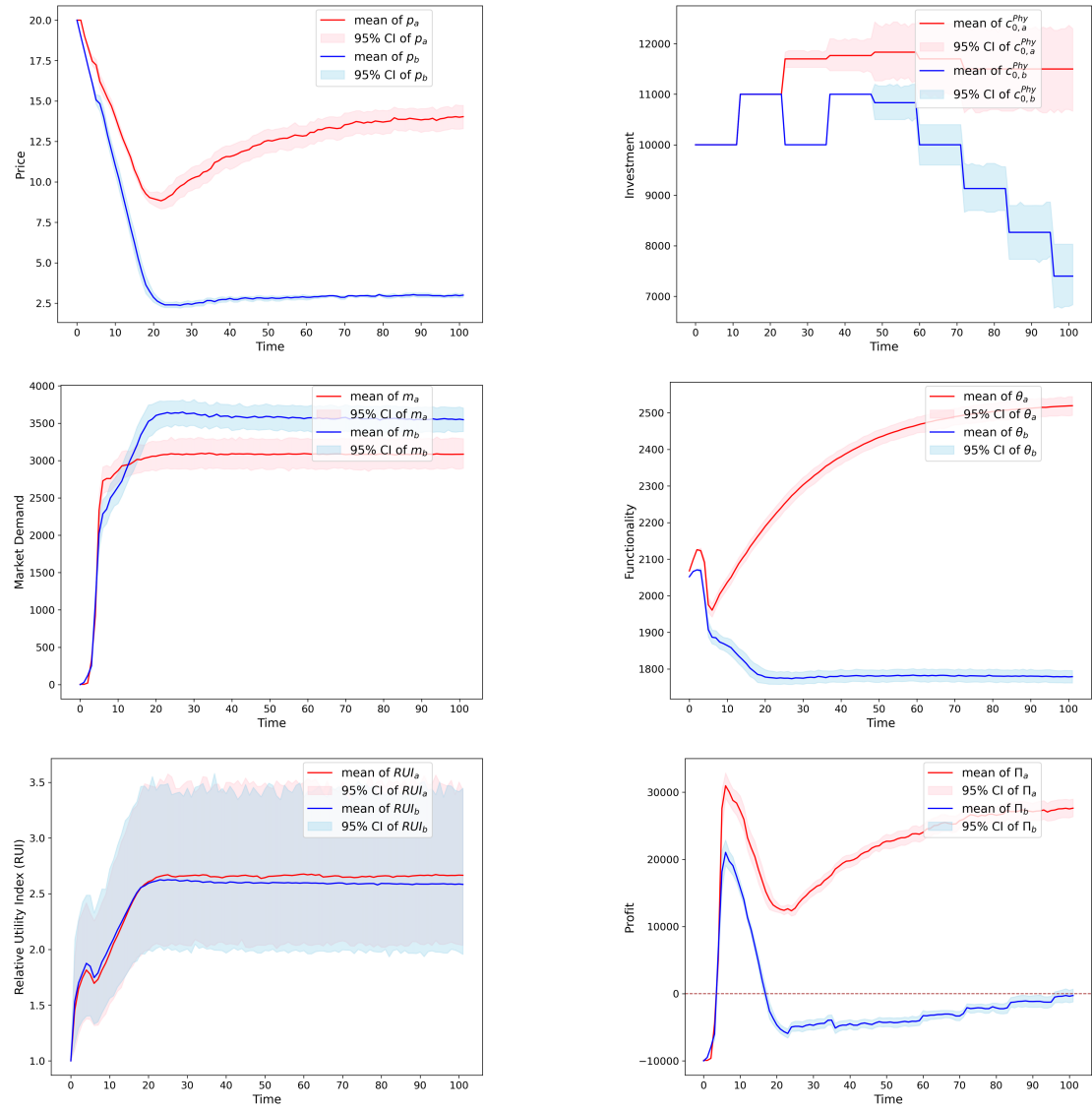


Figure 2.90: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.1, 0.05)$.

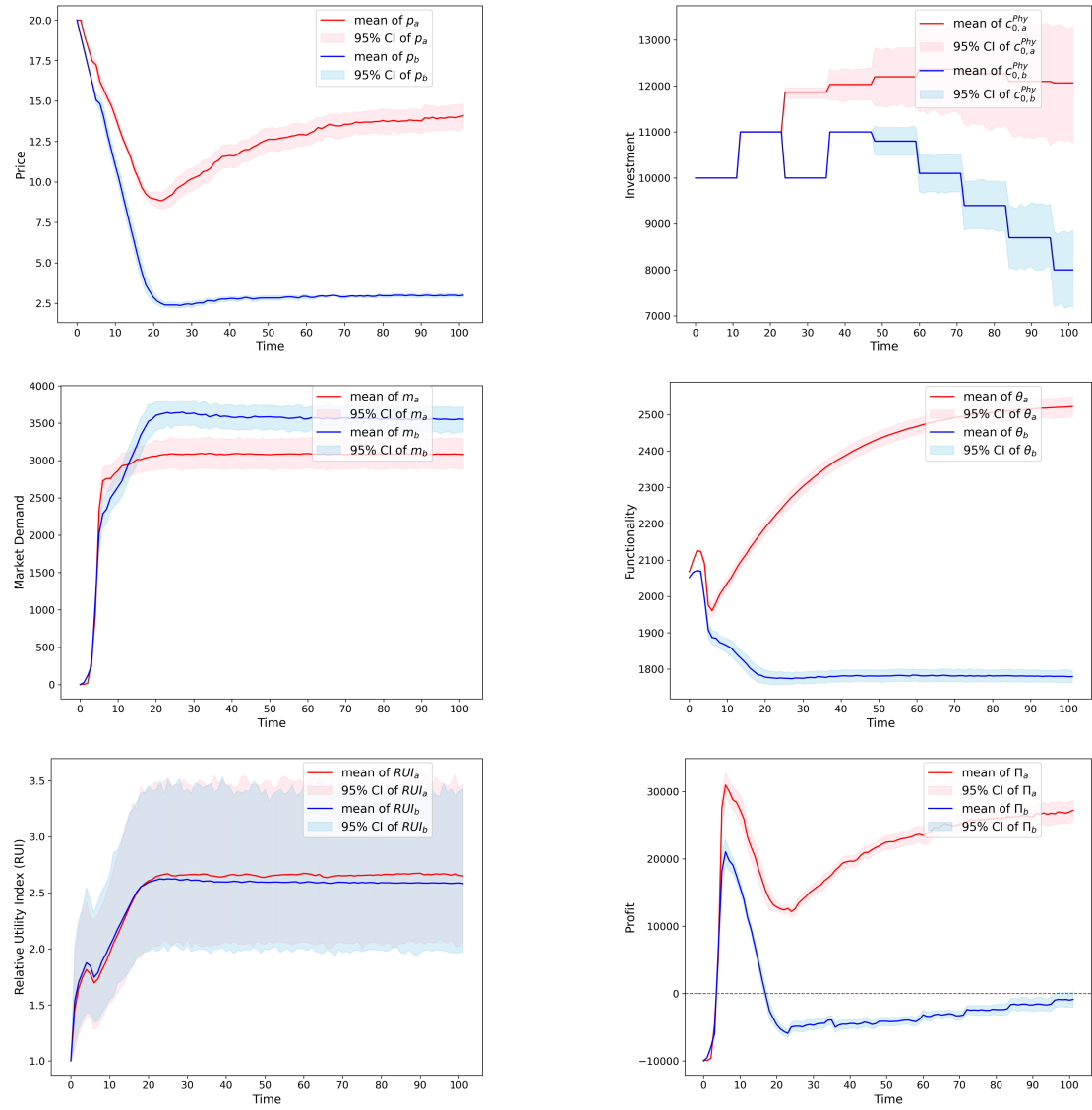


Figure 2.91: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.05, 0.1)$.

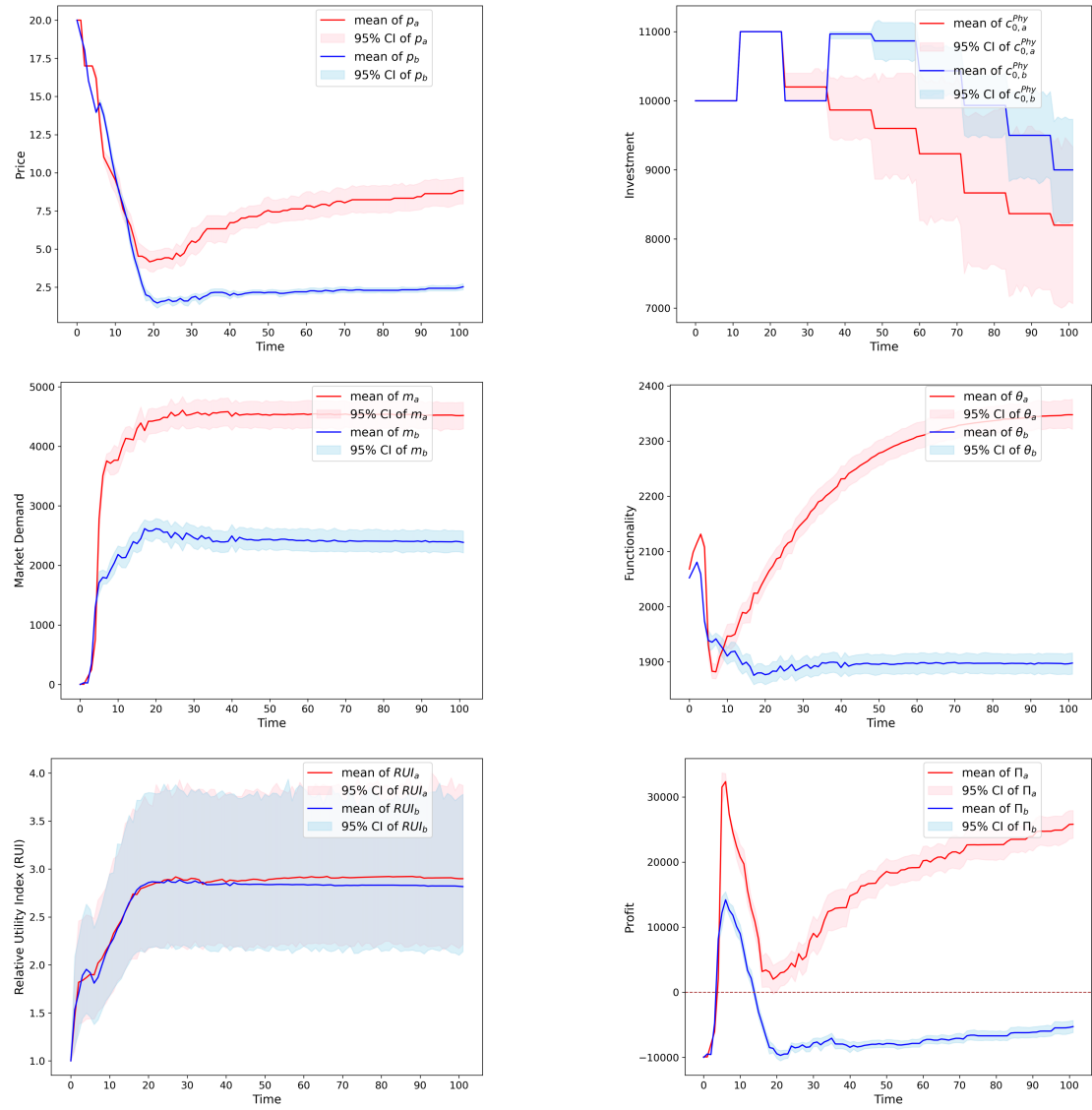


Figure 2.92: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (3, 1)$.

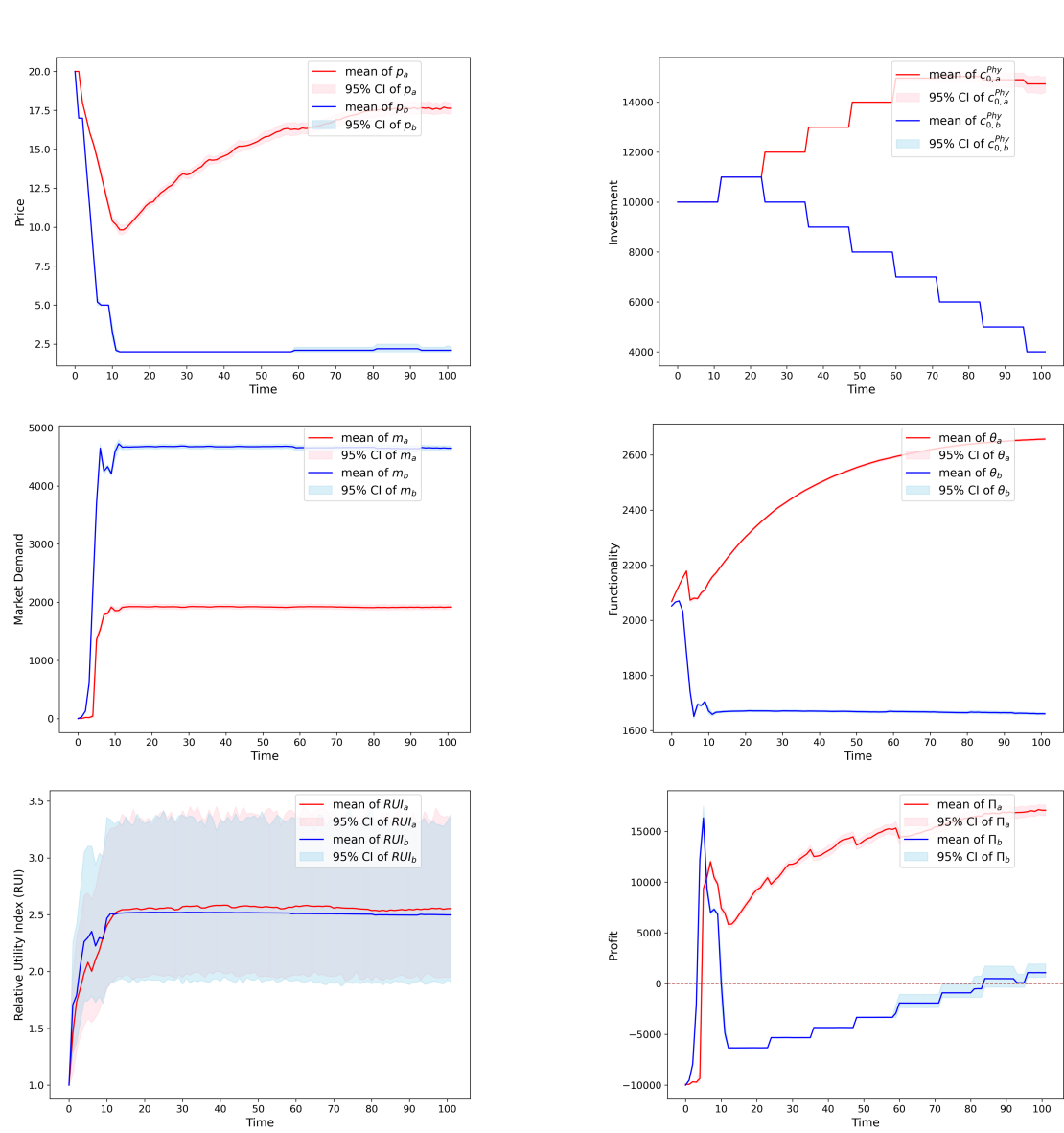


Figure 2.93: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (1, 3)$.

With network effect + No congestion effect ($\beta_0 = 0, \beta_4 = 50$)

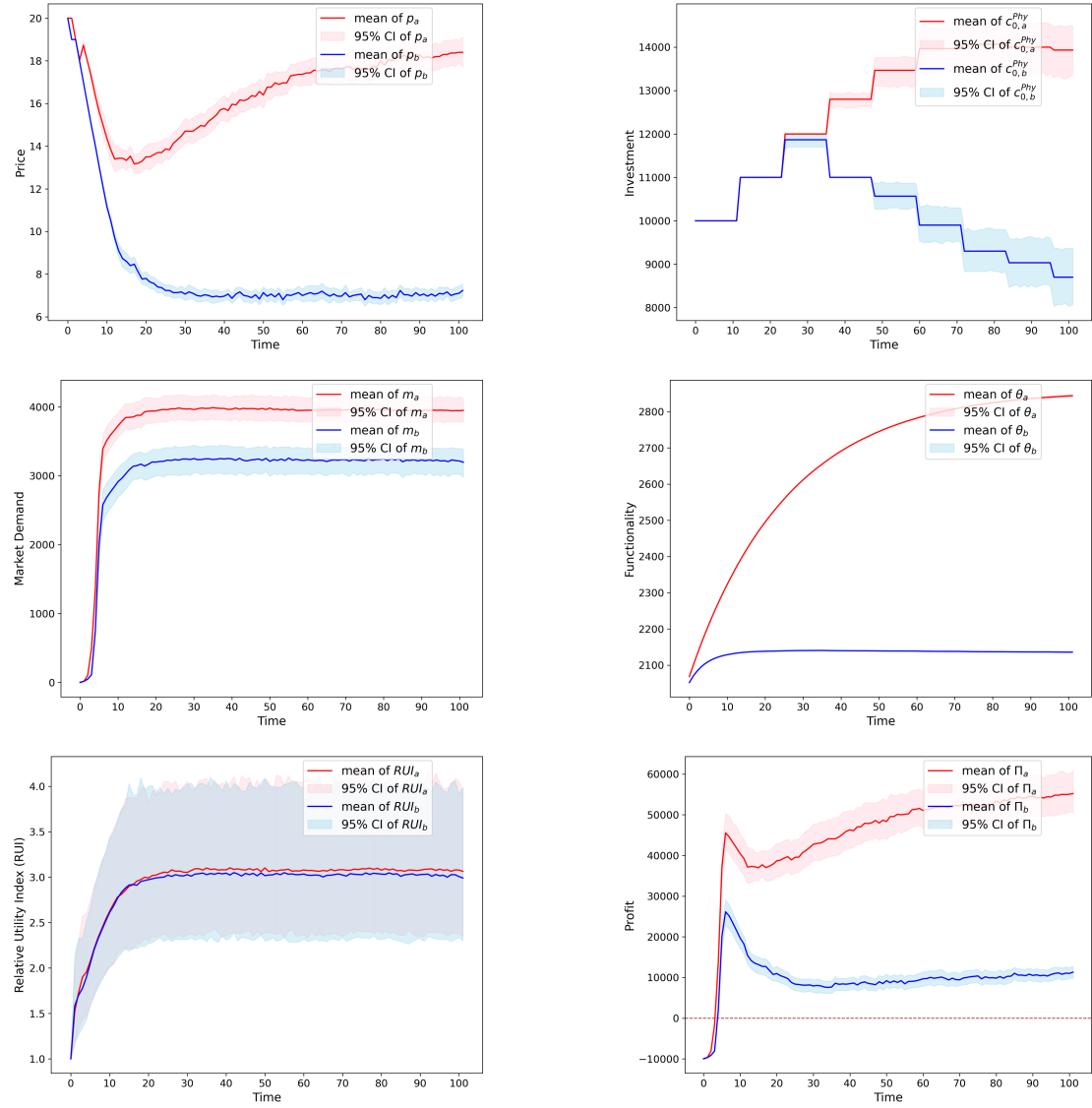


Figure 2.94: Mean and confidence interval of simulations. Benchmark case.

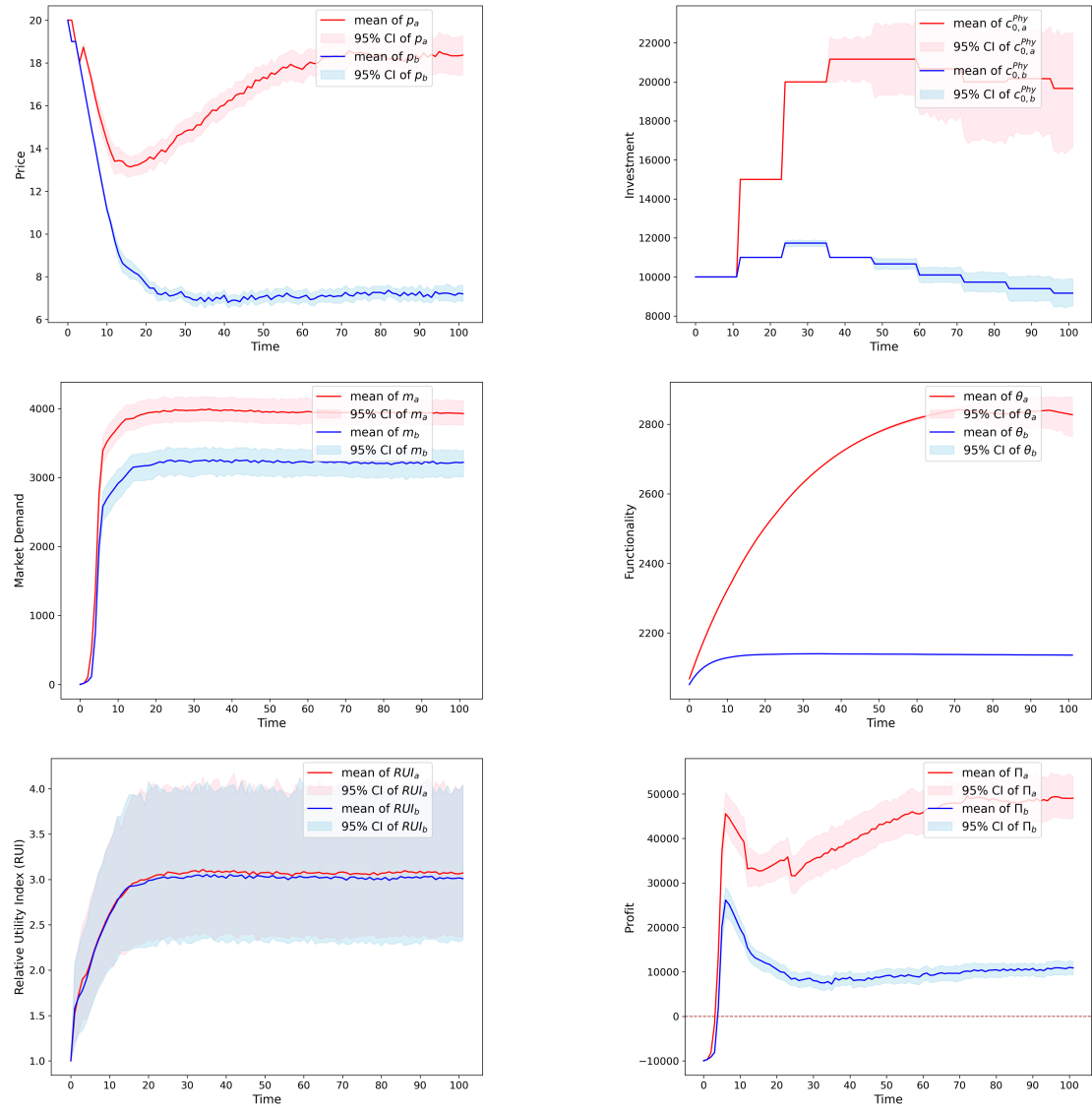


Figure 2.95: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (5000, 1000)$.

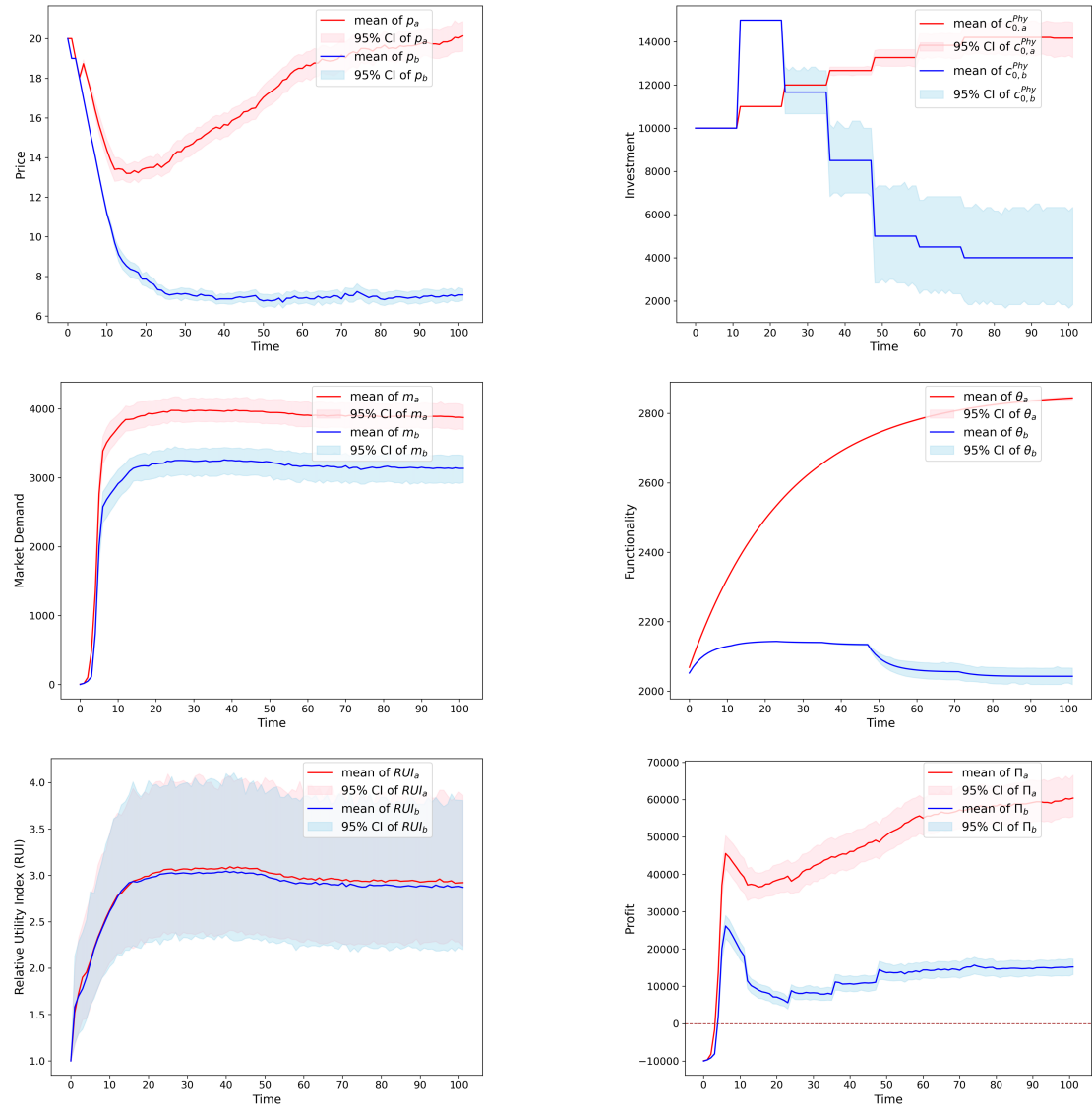


Figure 2.96: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (1000, 5000)$.

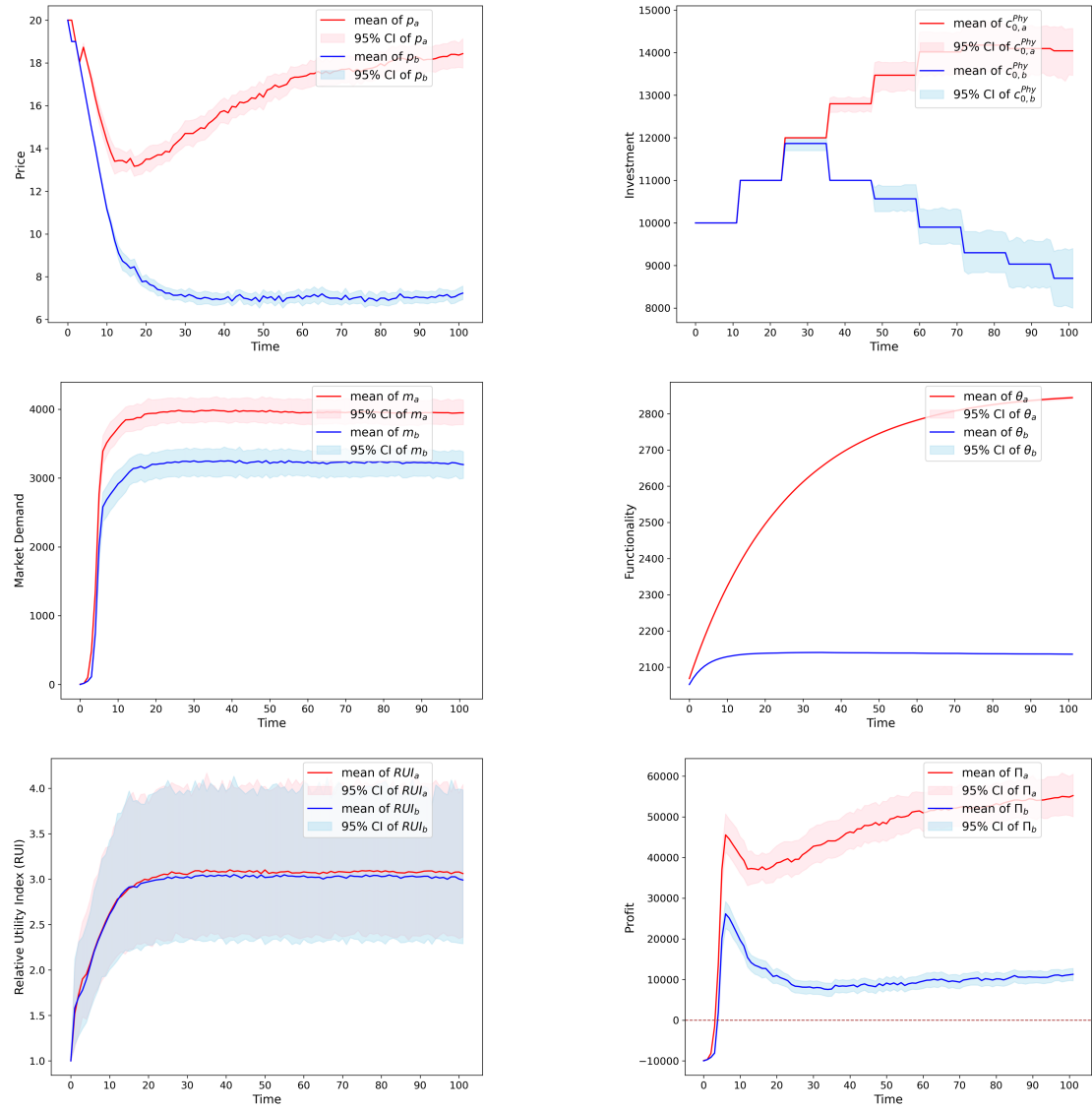


Figure 2.97: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (100, 0)$.

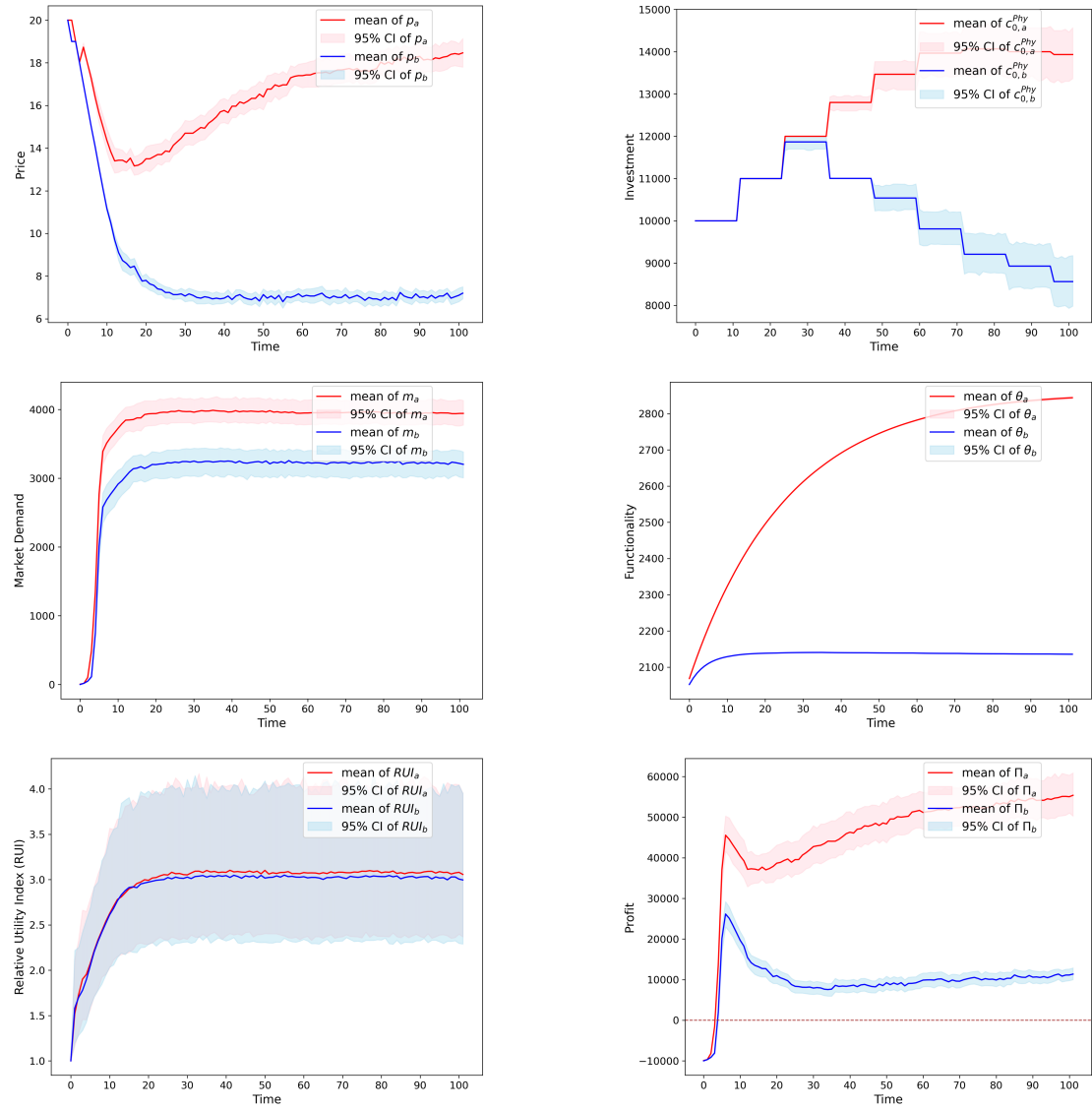


Figure 2.98: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (0, 100)$.

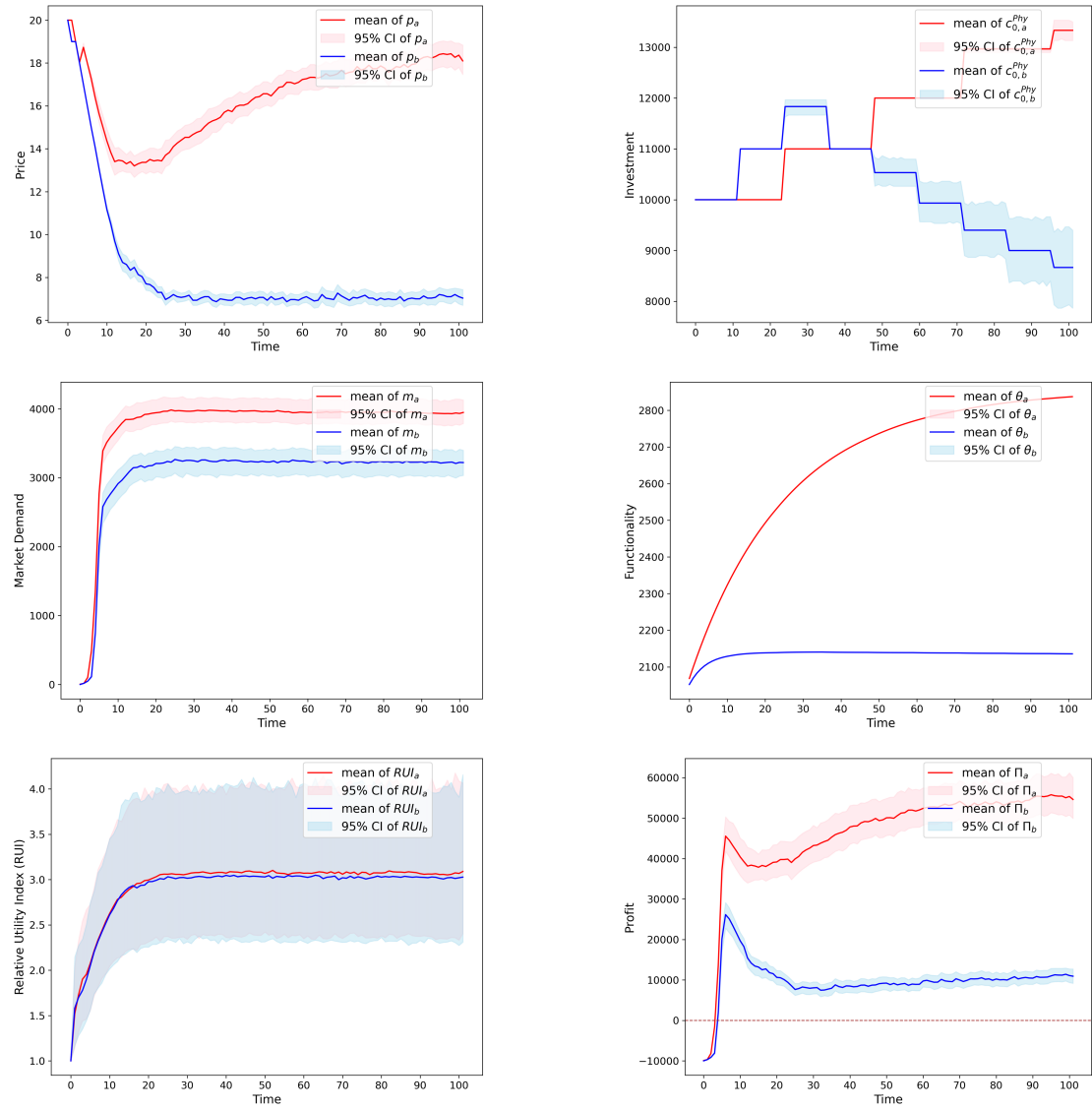


Figure 2.99: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (24, 12)$.

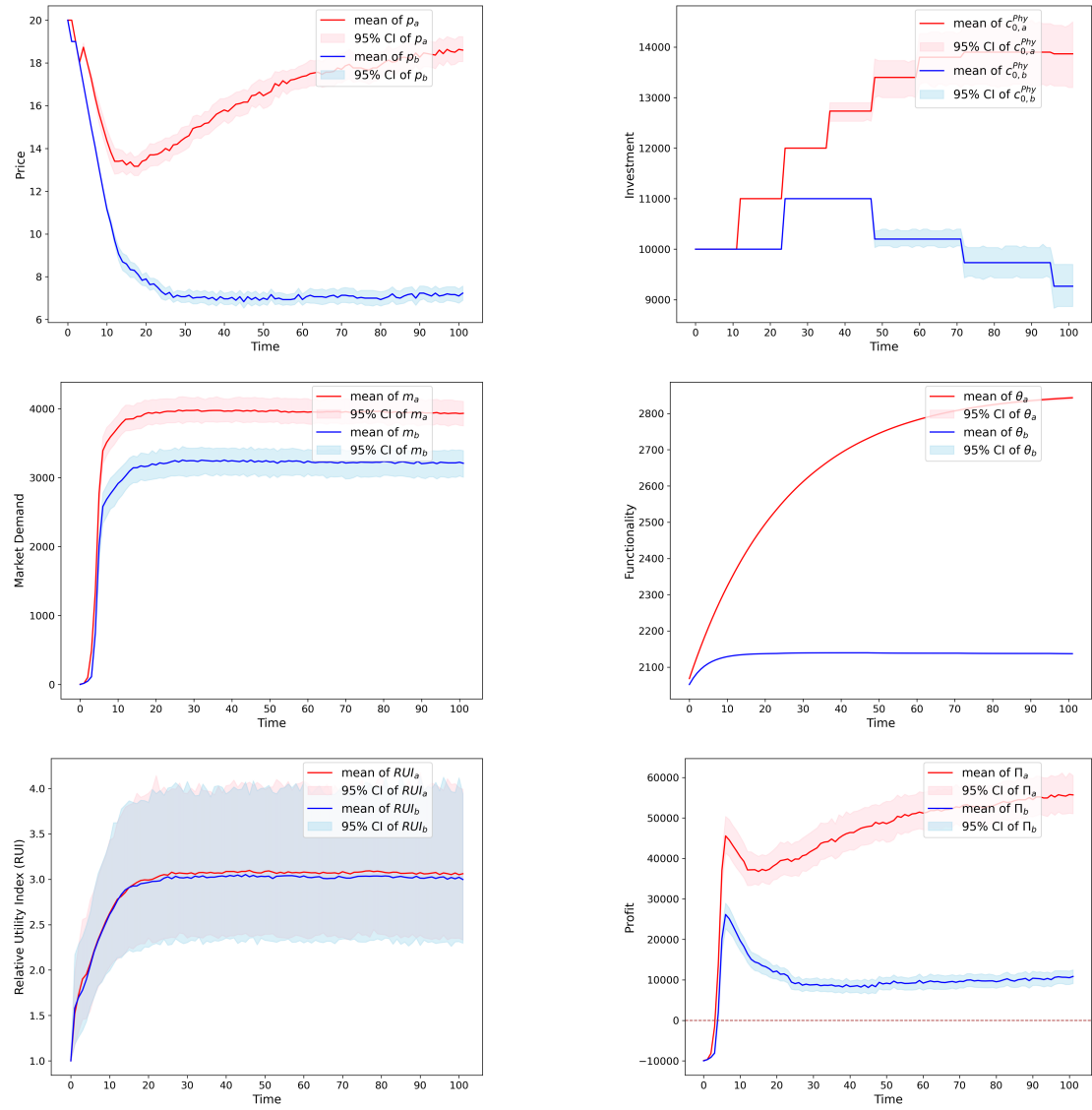


Figure 2.100: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (12, 24)$.

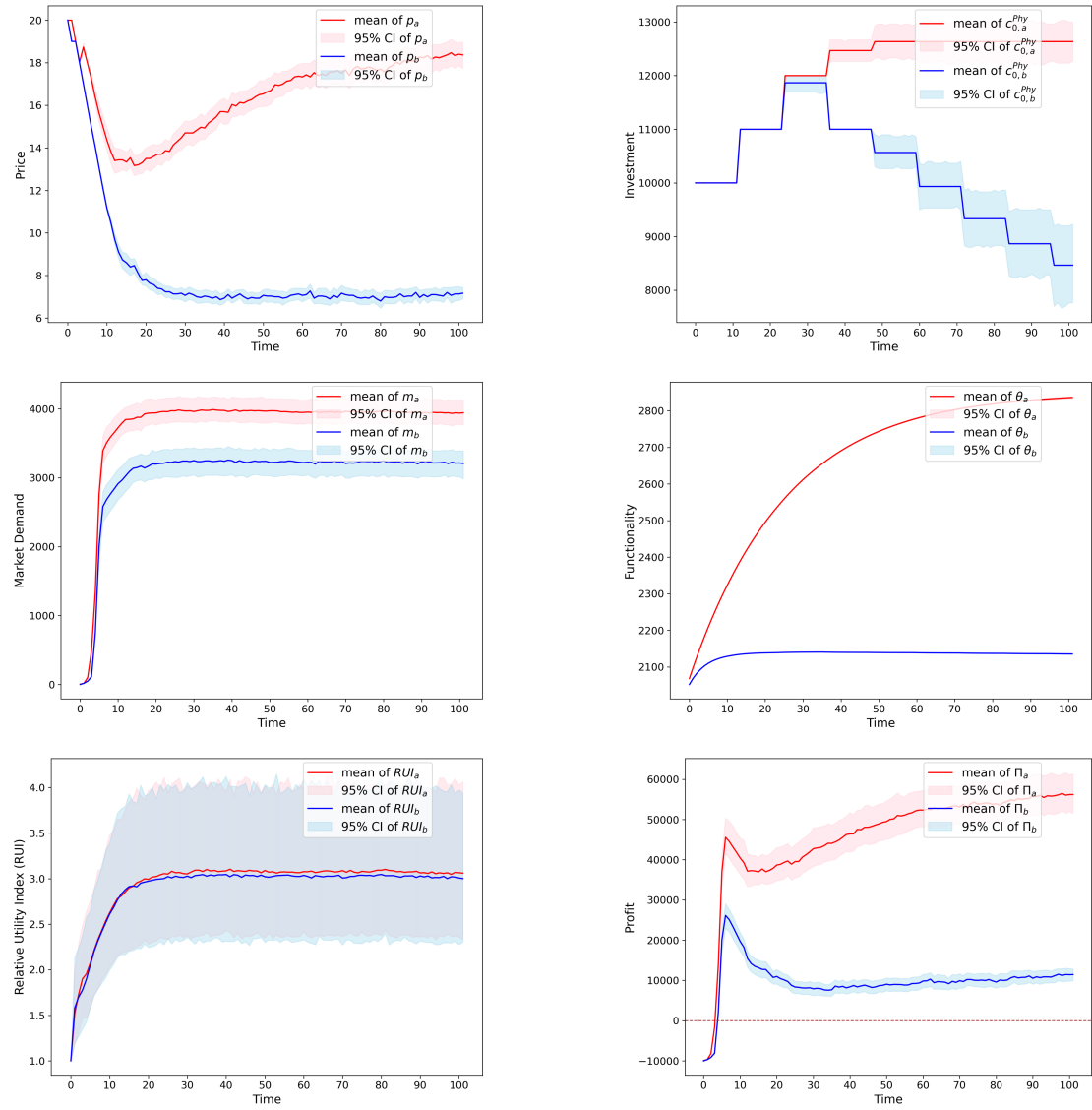


Figure 2.101: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.1, 0.05)$.

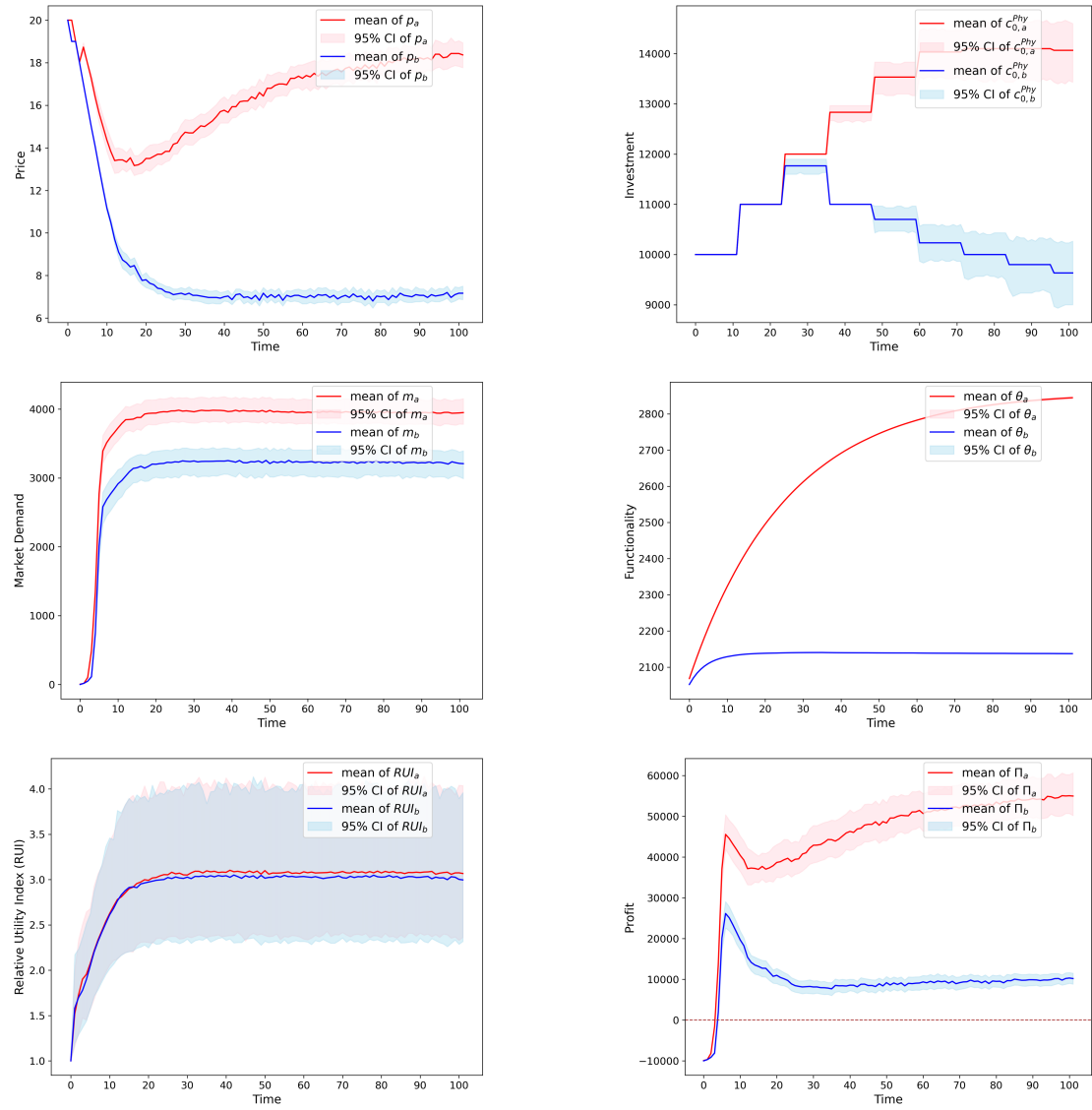


Figure 2.102: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.05, 0.1)$.

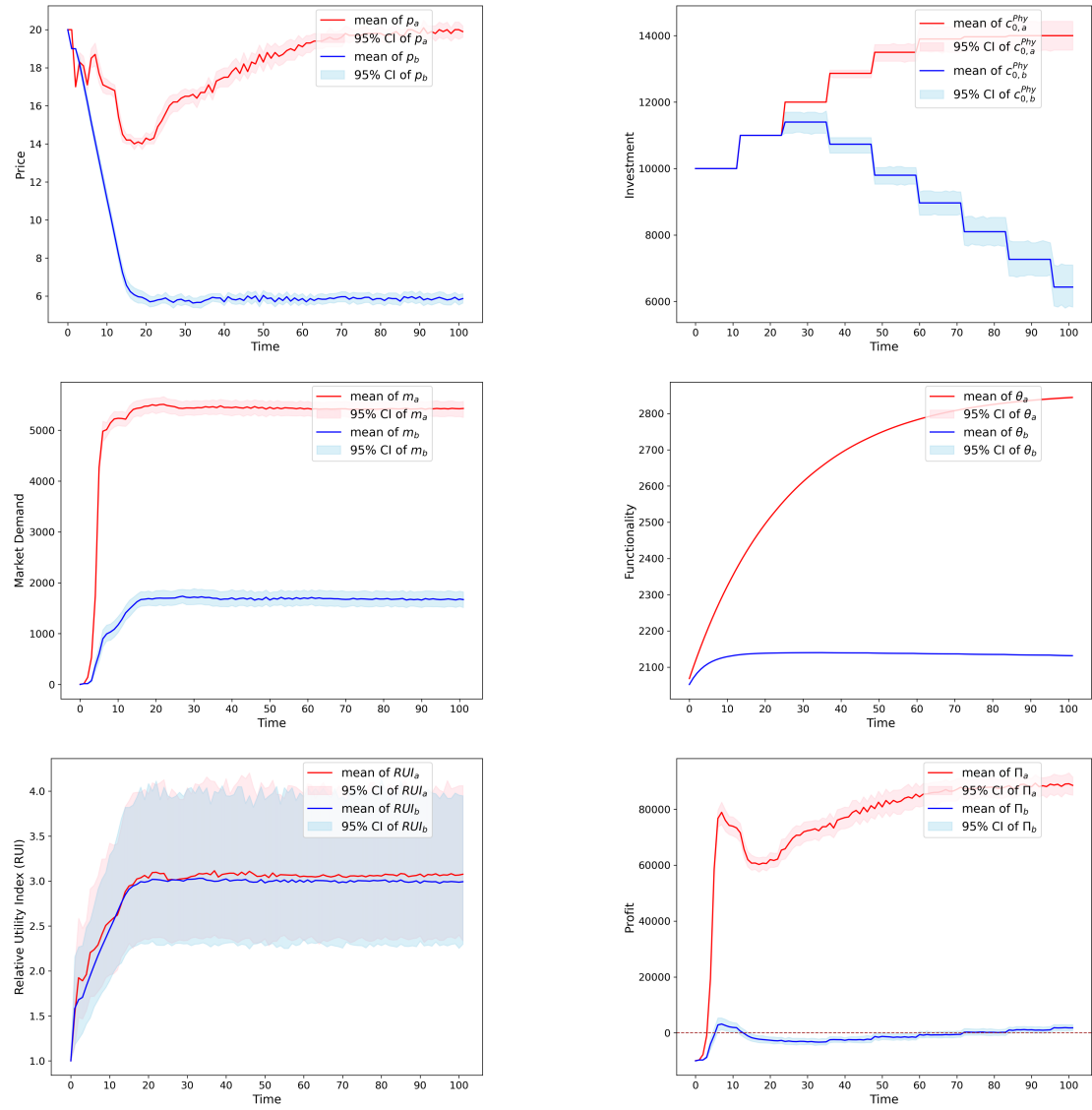


Figure 2.103: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (3, 1)$.

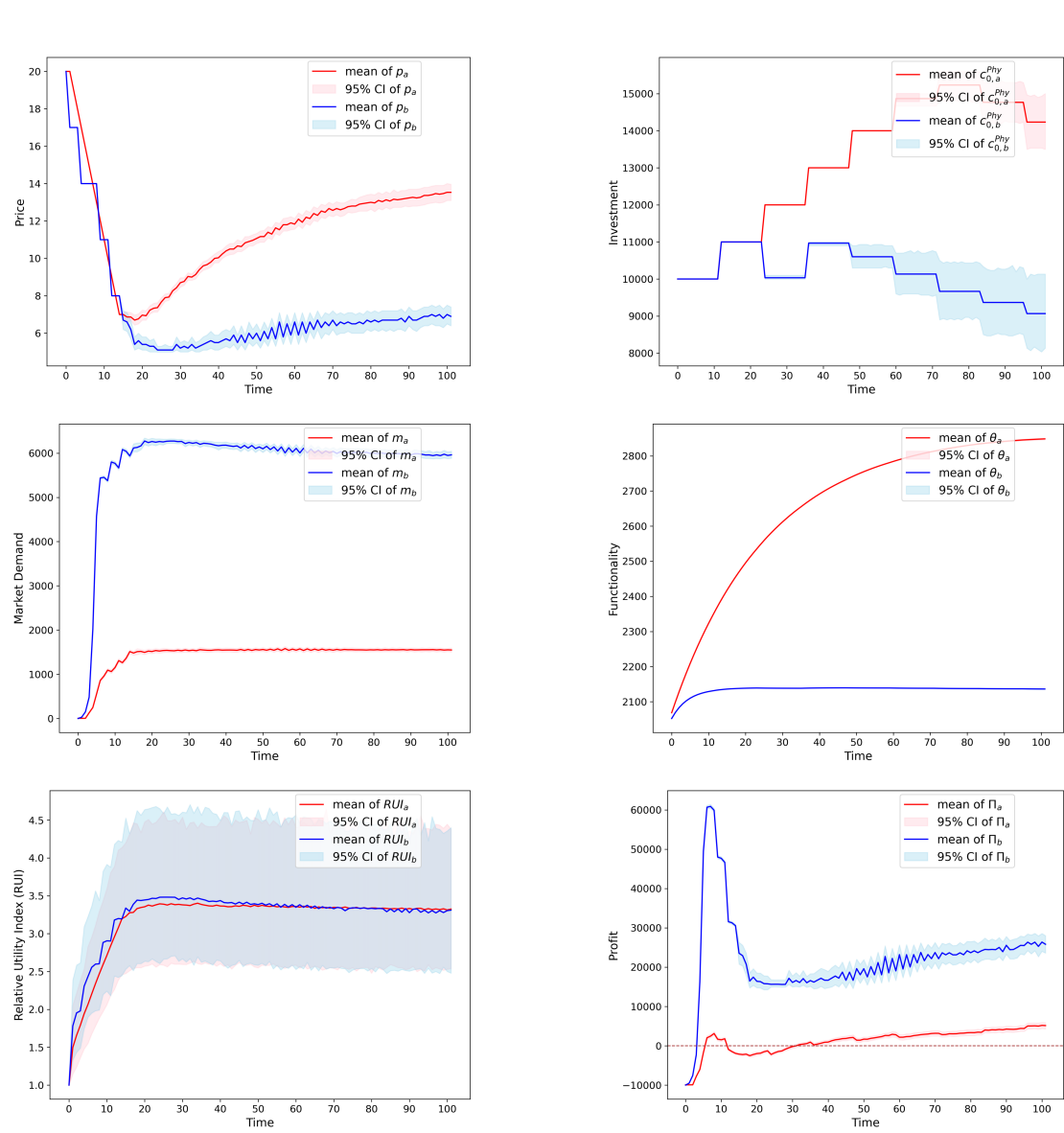


Figure 2.104: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (1, 3)$.

With congestion and network effects ($\beta_0 = 0.1, \beta_4 = 50$)

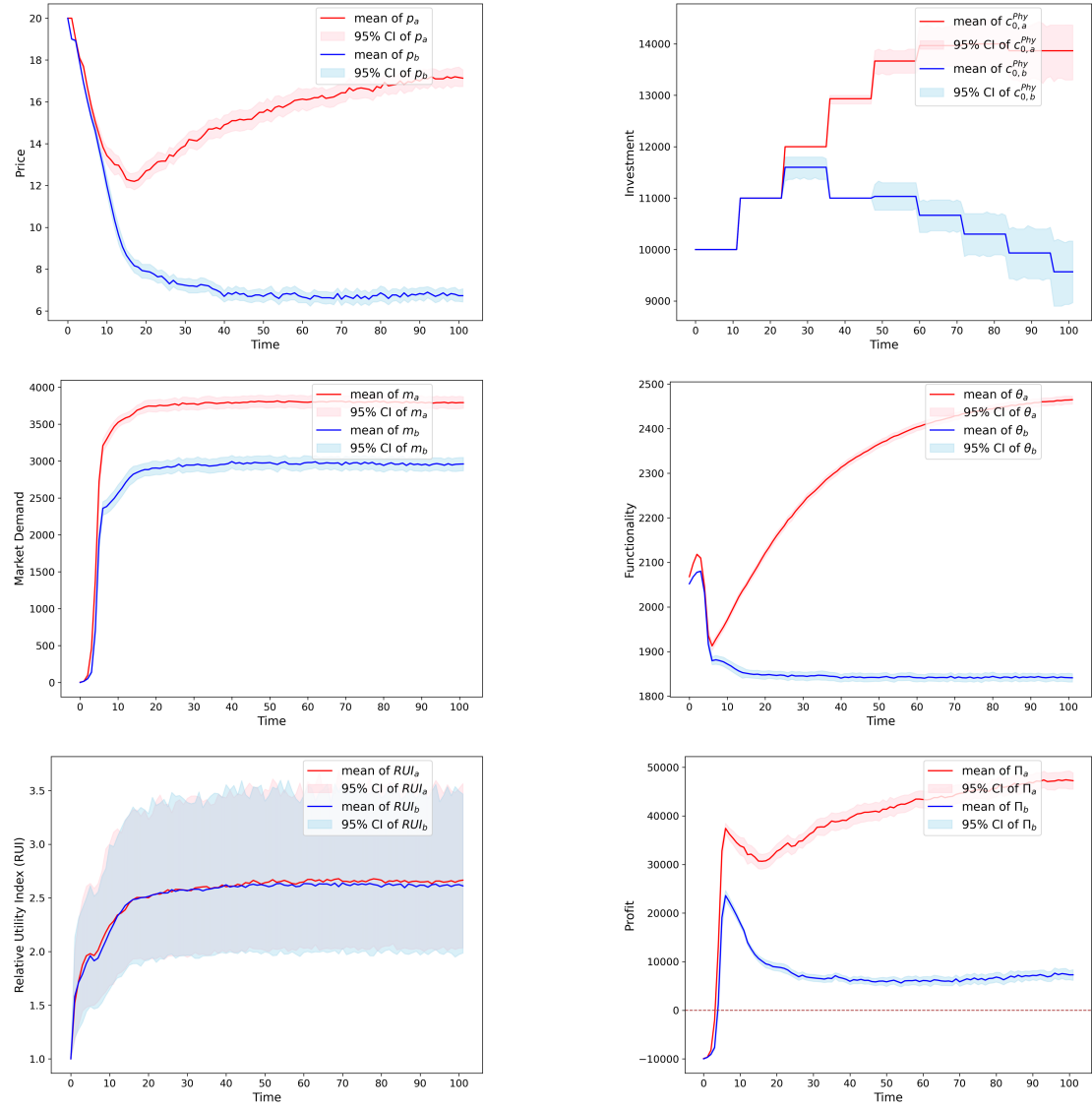


Figure 2.105: Mean and confidence interval of simulations. Benchmark case.

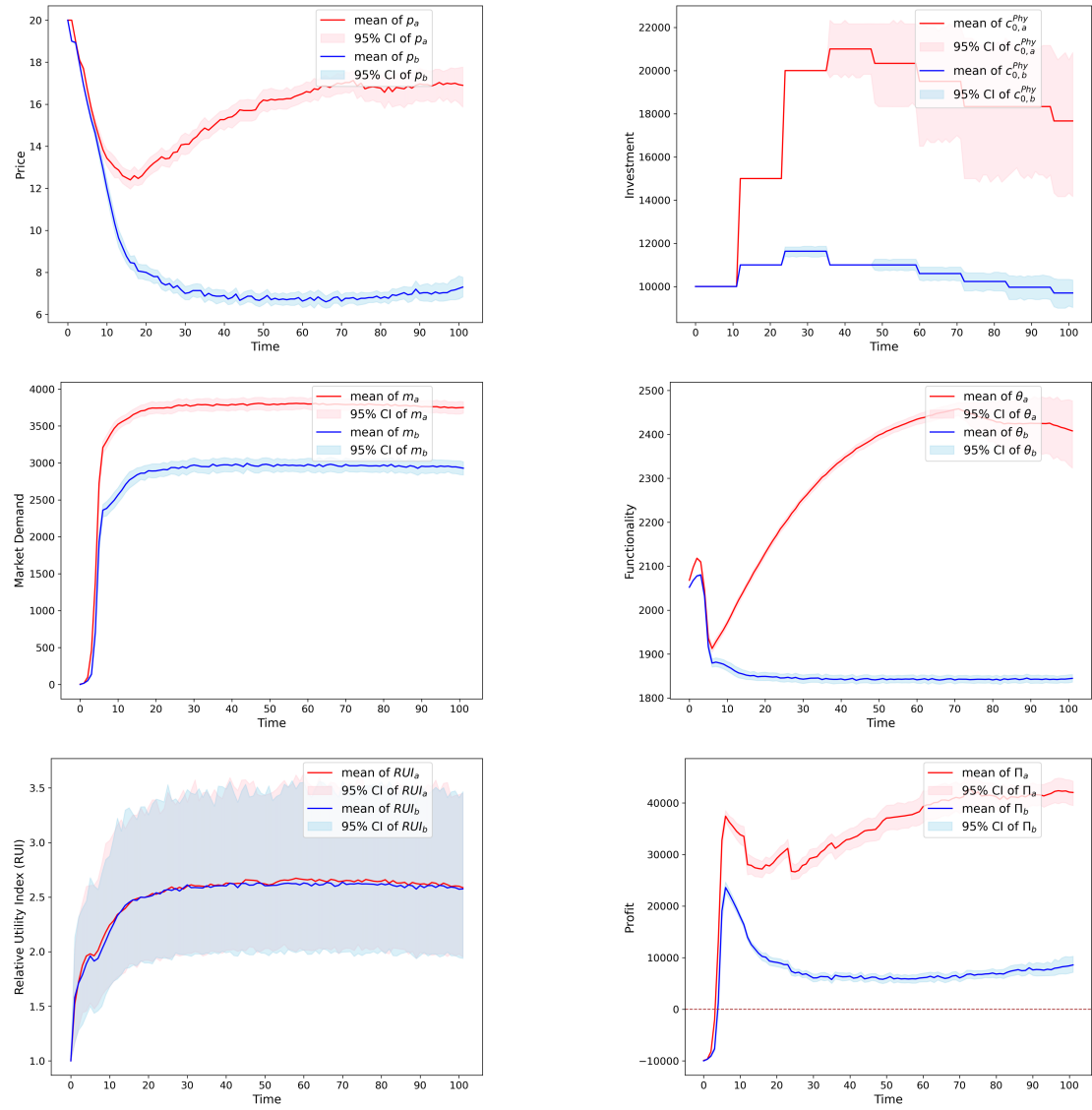


Figure 2.106: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (5000, 1000)$.

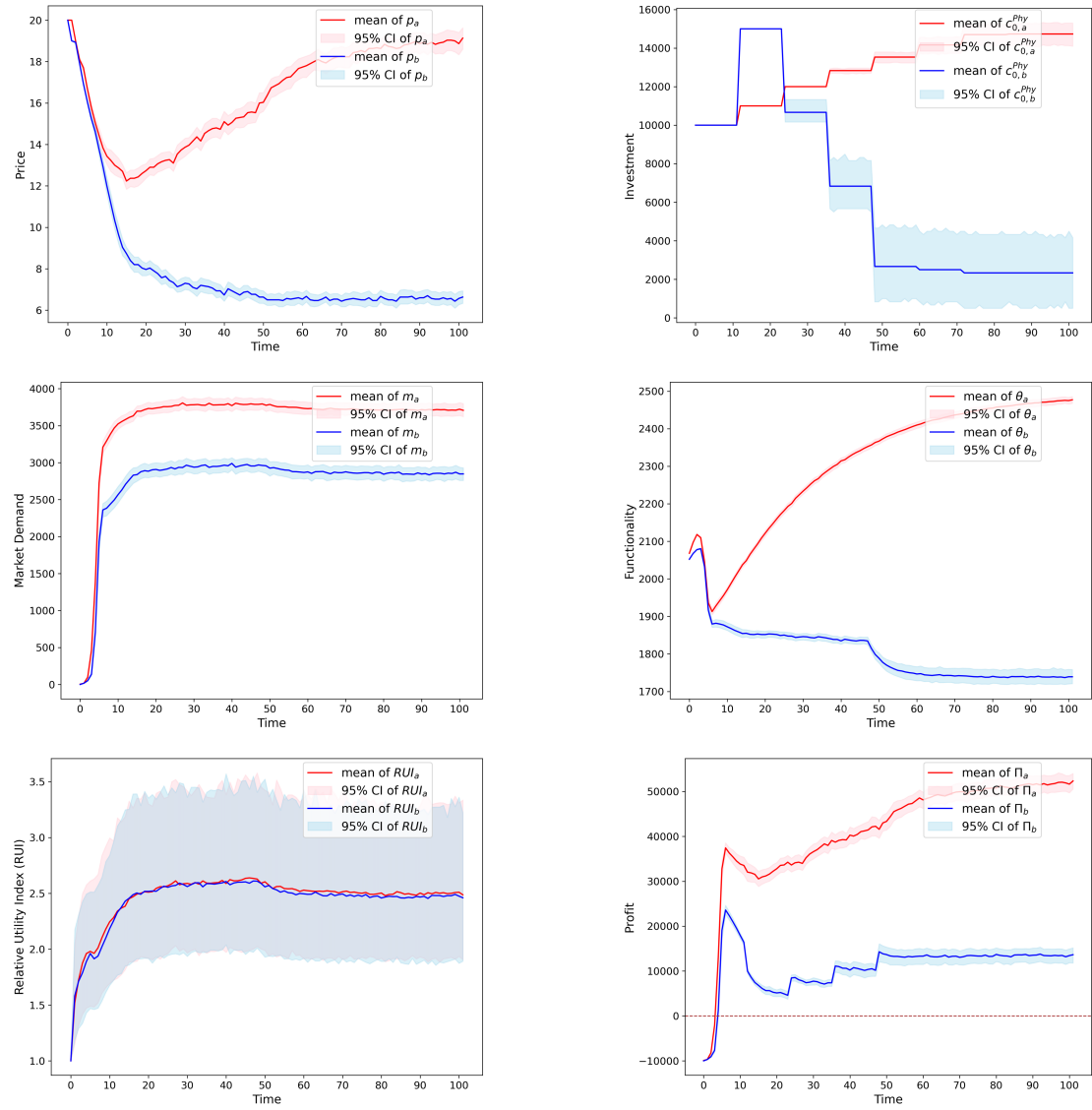


Figure 2.107: Mean and confidence interval of simulations. Policy: $(\eta_a, \eta_b) = (1000, 5000)$.

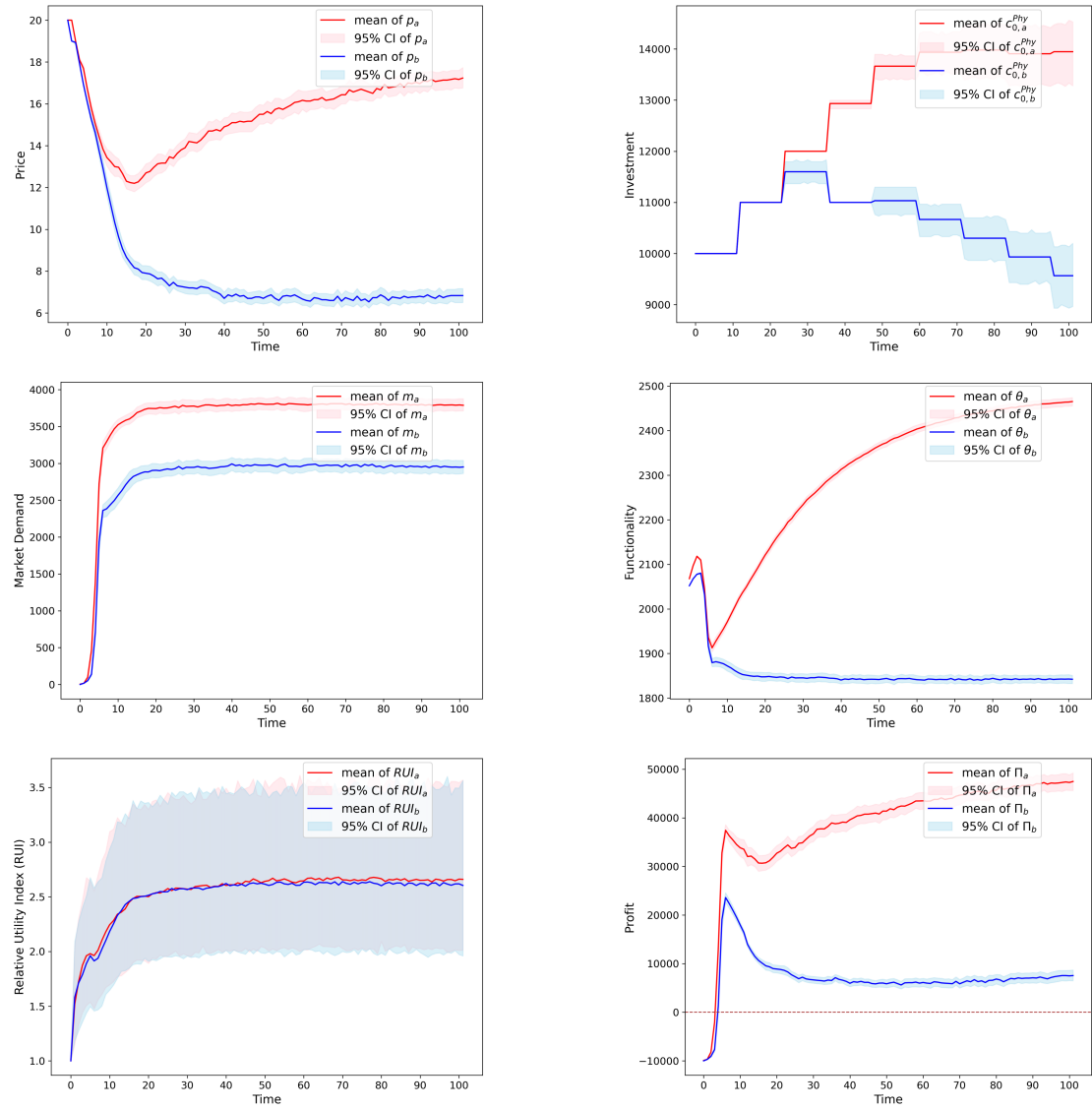


Figure 2.108: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (100, 0)$.

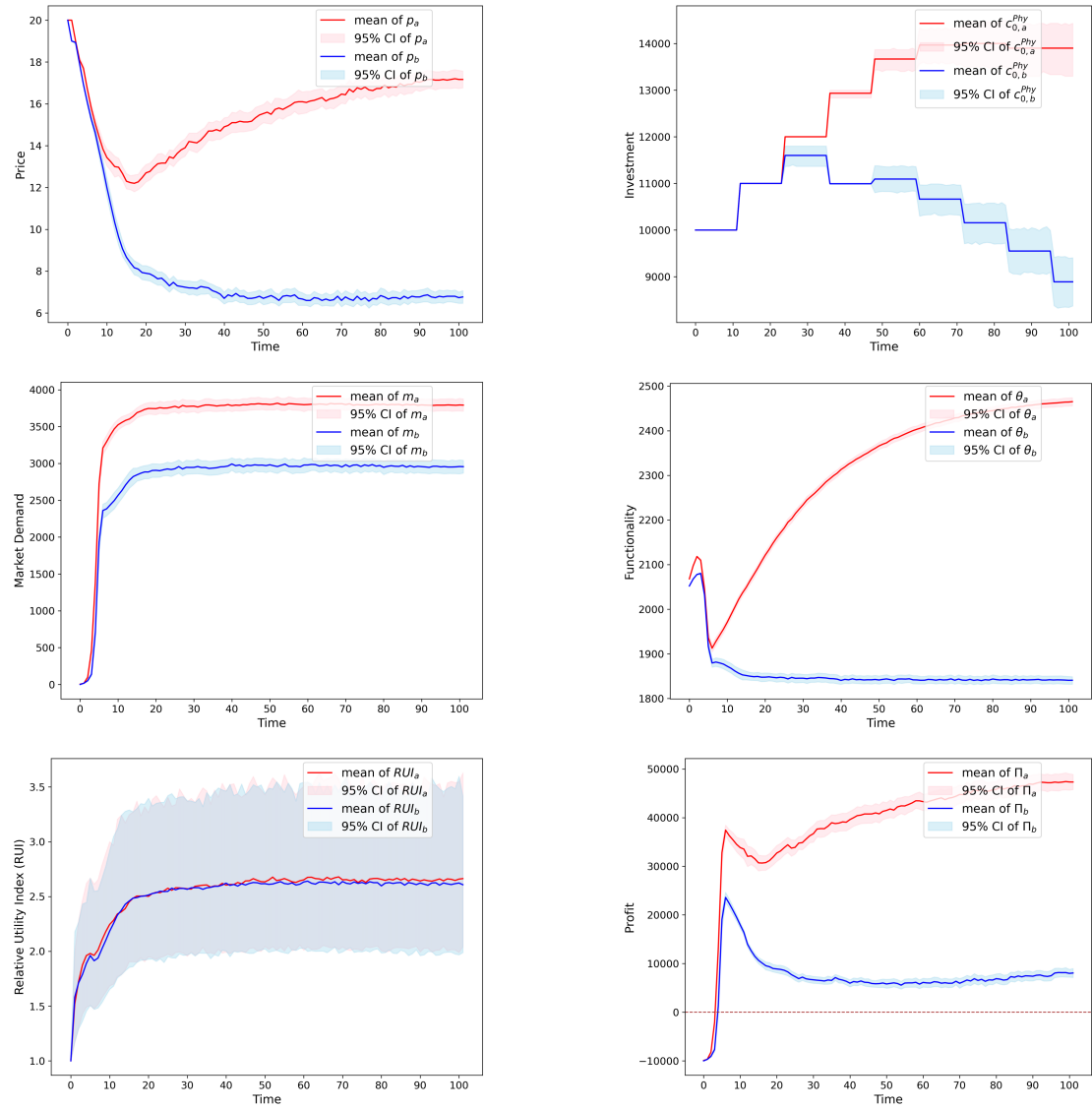


Figure 2.109: Mean and confidence interval of simulations. Policy: $(\psi_a, \psi_b) = (0, 100)$.

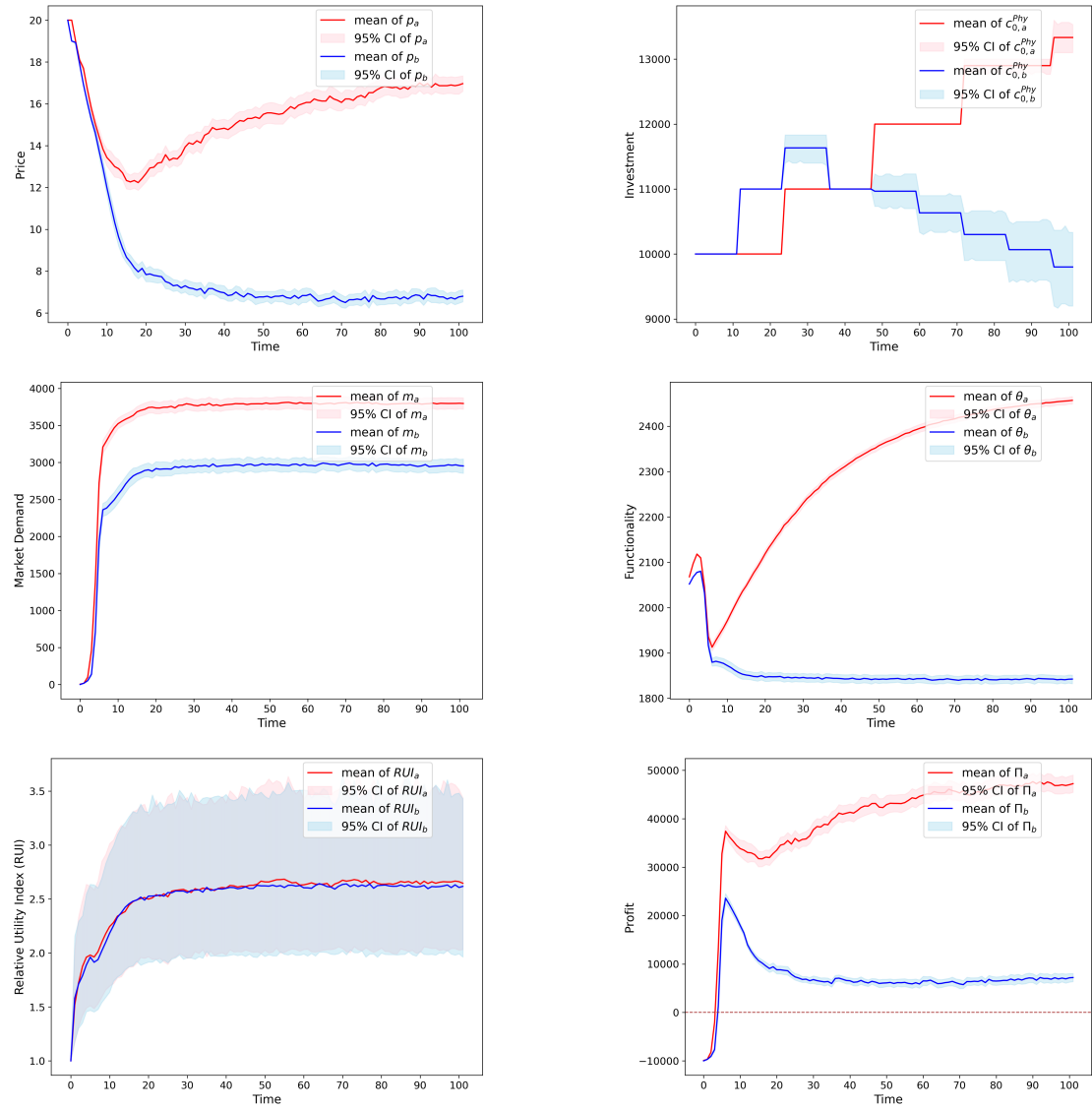


Figure 2.110: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (24, 12)$.

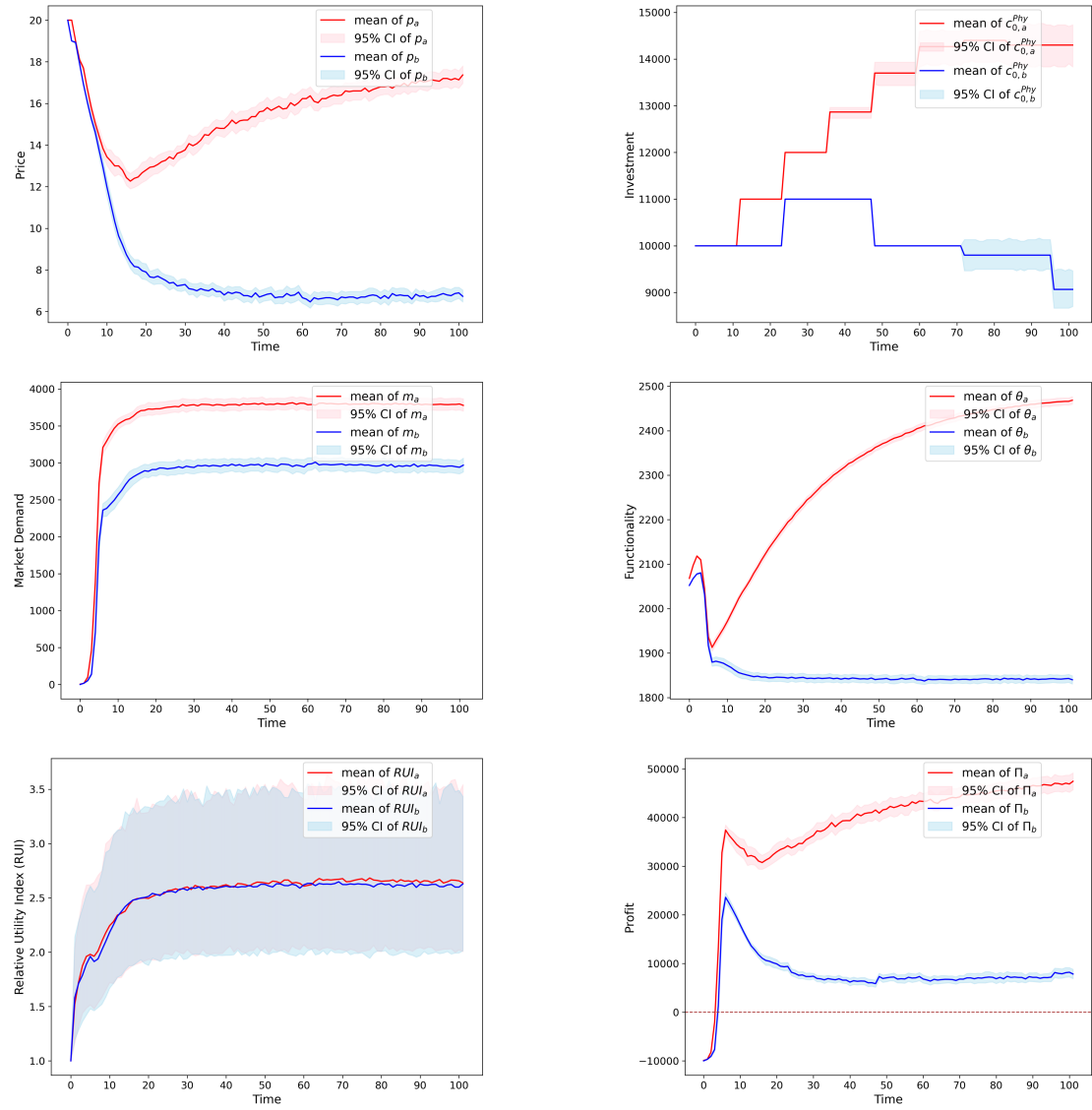


Figure 2.111: Mean and confidence interval of simulations. Policy: $(\tau_a, \tau_b) = (12, 24)$.

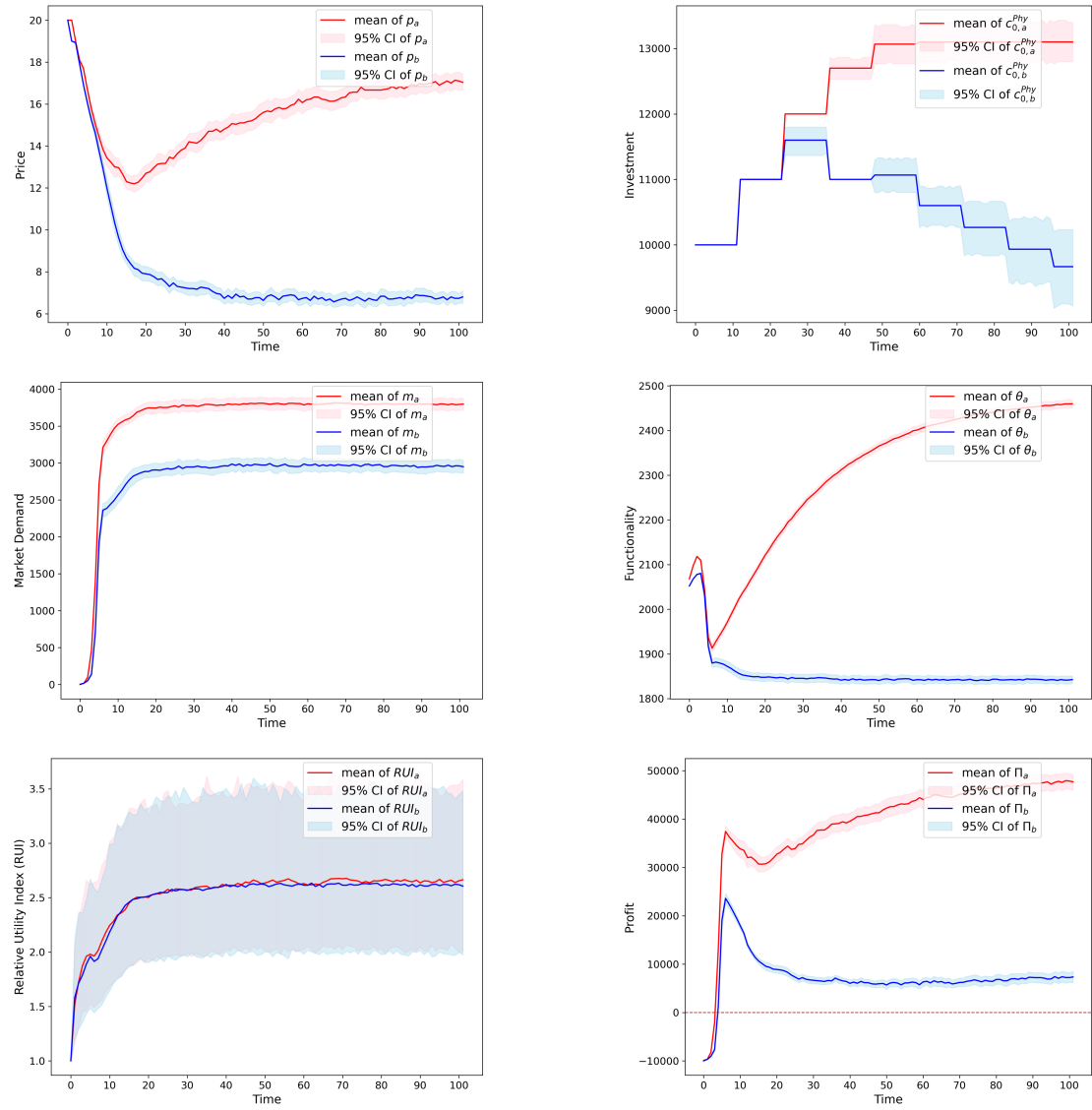


Figure 2.112: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.1, 0.05)$.

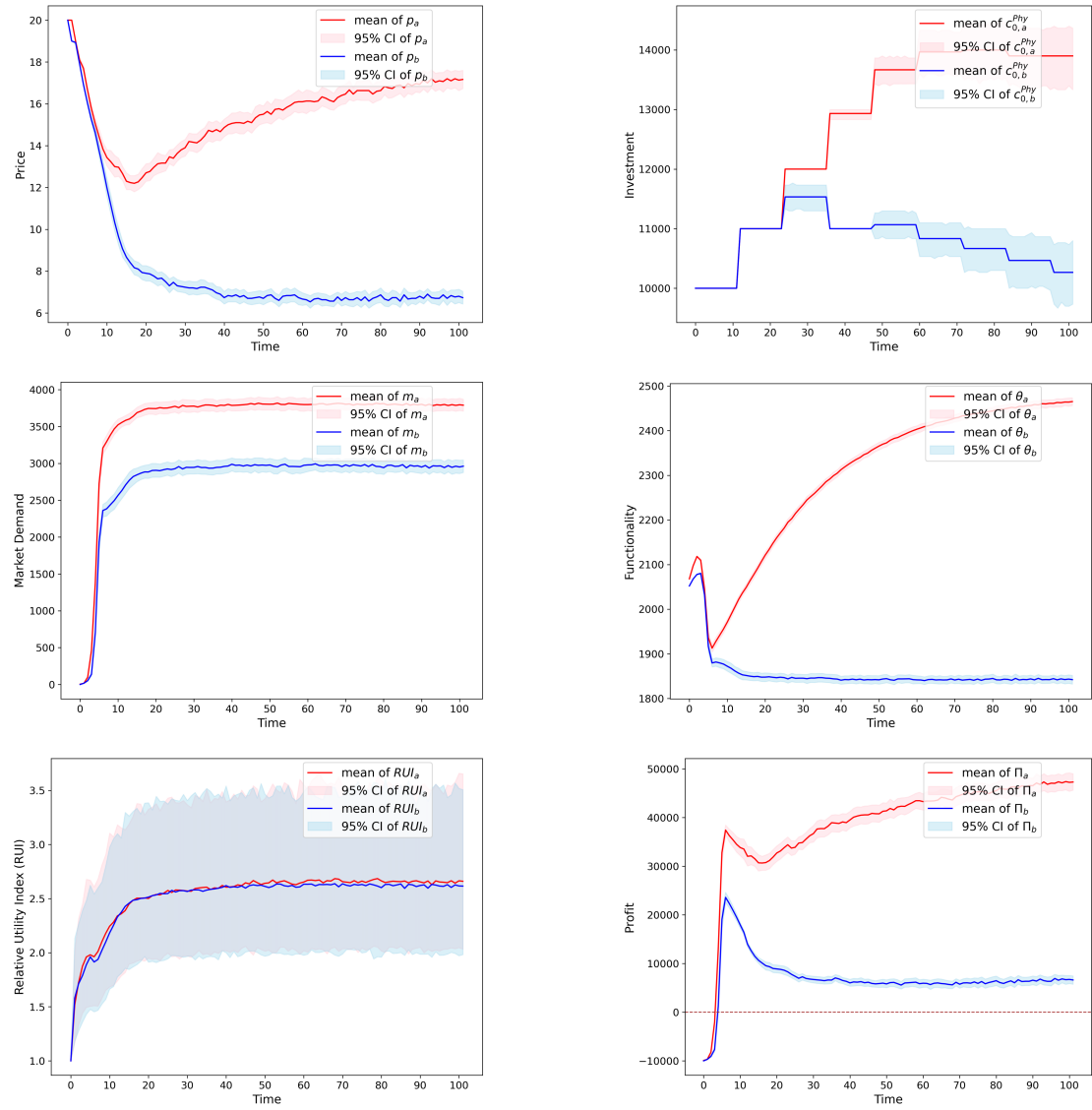


Figure 2.113: Mean and confidence interval of simulations. Policy: $(\vartheta_a, \vartheta_b) = (0.05, 0.1)$.

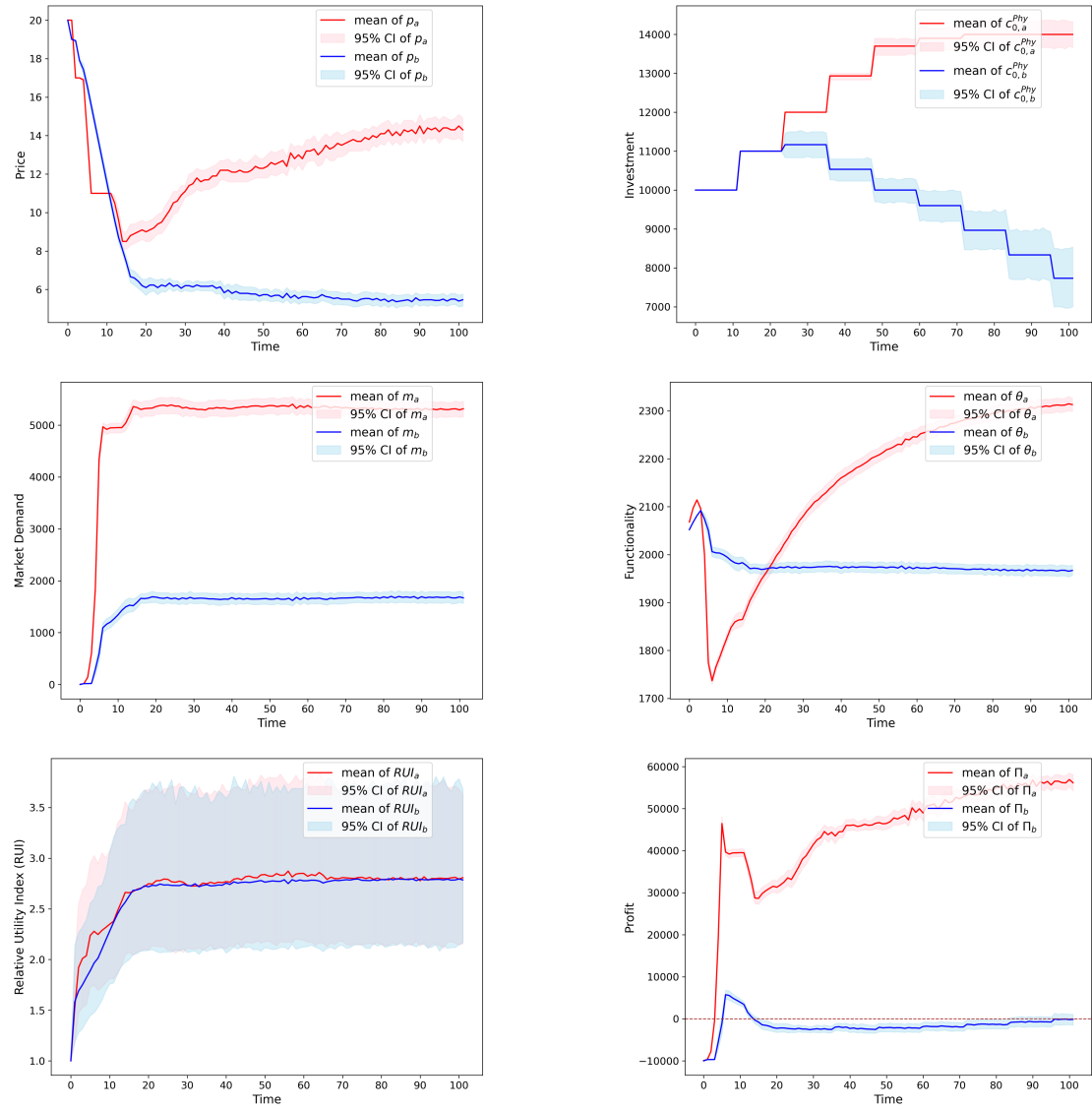


Figure 2.114: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (3, 1)$.

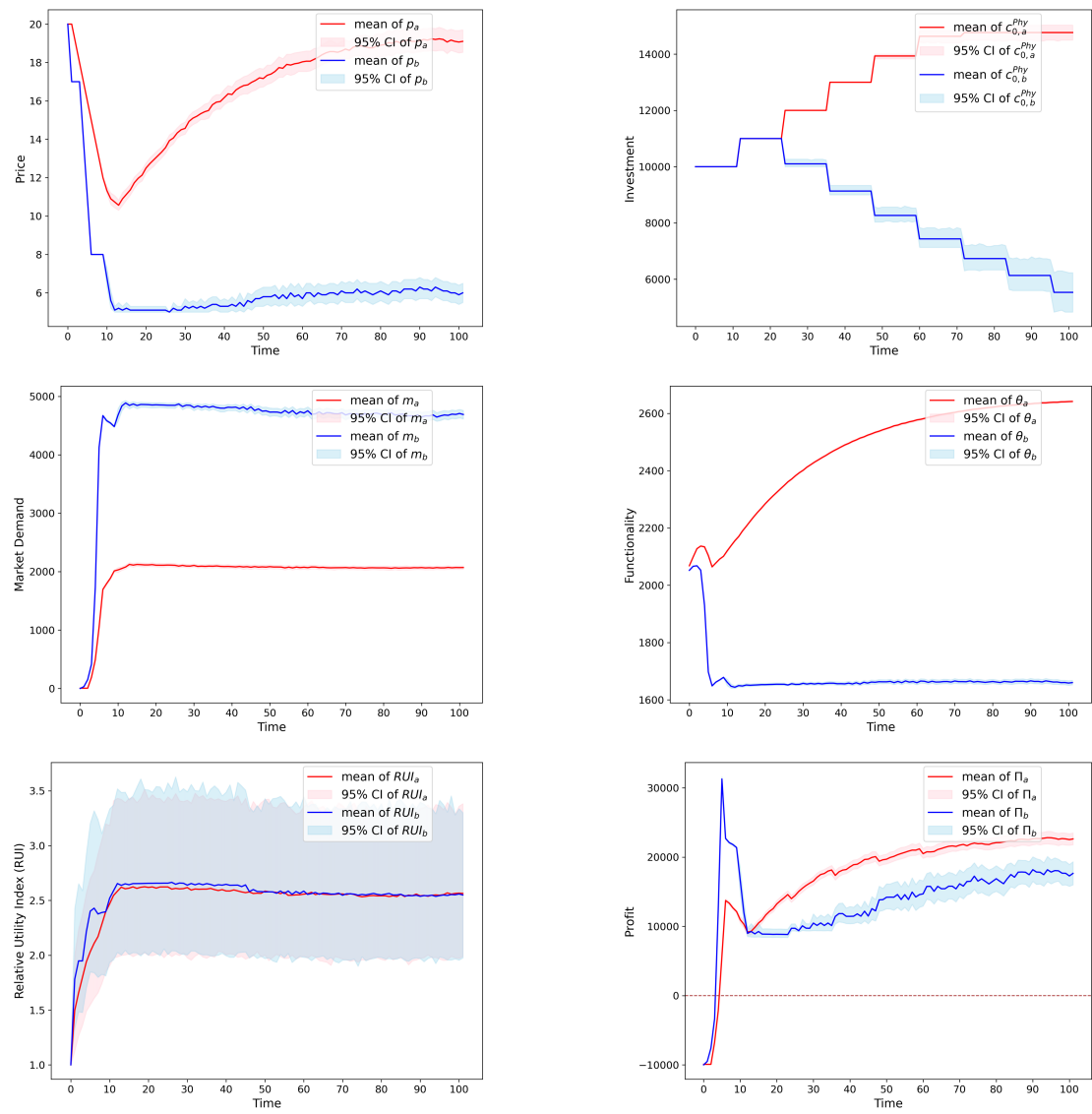


Figure 2.115: Mean and confidence interval of simulations. Policy: $(\xi_a, \xi_b) = (1, 3)$.

Chapter 3

Parameter Estimation Methods in an Agent-based Computational Model: Insights on ICT Industrial Dynamics and Policies

Despite the potential of the agent-based modeling (ABM) to uncover emergent phenomena and provide policy insights, the estimation and calibration of model parameters remains a significant challenge. This paper addresses this challenge by proposing a computational model tailored to the ICT industry, where strategic firm behavior and network effects are central. We apply recent advances in parameter estimation—specifically the simulated method of moments and neural network-based surrogate models—using financial data from U.S. telecommunication firms (AT&T and T-Mobile) to calibrate and validate the model, and to identify parameter values and distributions. By systematically comparing these calibration methods, we assess their suitability and practical implications for agent-based economic modeling. We aim to advance the methodological rigor in ABM parameterization and contribute to the broader discussion on improving the validity and applicability of computational economic models. Our findings provide insights into overcoming industry-specific challenges, enhancing the reliability of ABMs, and bridging the gap between theoretical modeling and real-world applications.

Keywords: Parameter Estimation; Agent-based Modeling; ICT Industry; Industrial Dynamics

3.1 Introduction

The complexity of economic systems calls into question the traditional rationality assumption often employed in economic research. As noted by Dawid and Delli Gatti (Dawid & Delli Gatti 2018), macroeconomic dynamics emerge from the interactions of a large number of heterogeneous agents making diverse decisions and engaging in the production and exchange of goods and information. In fact, this complexity is not limited to macroeconomic settings; even in models of firm competition, incorporating factors such as heterogeneous consumer preferences, information diffusion through social networks, and asymmetric firm technologies significantly increases system complexity. In response to these challenges, researchers following the behavioral tradition have critiqued top-down modeling approaches and advanced the use of agent-based computational models (ABMs). These models specify behavioral rules for various types of agents, such as firms, consumers, households, and governments, and explore the implications of different policy interventions through simulation. While ABMs offer valuable insights into emergent dynamics, they also pose methodological challenges, particularly concerning the estimation and calibration of model parameters.

Aligning model-generated data with empirical observations is crucial for interpreting the essential mechanisms embedded in a computational model. As Beck & Arnold (1977) note, parameter estimation provides a systematic framework for effectively using data to estimate constants in mathematical models and to support the modeling of complex phenomena. In recent years, both frequentist and Bayesian statistical approaches have been employed for parameter estimation and calibration in economic models. Notable examples include the Brock and Hommes (1998) asset pricing model and several medium-scale macroeconomic agent-based models, such as CATS, Eurace@Unibi, and the Caiani et al. (2016) stock-flow consistent (SFC) model (Brock & Hommes 1998, Dawid & Delli Gatti 2018, Caiani et al. 2016). In this study, we propose a computational model to capture the strategic selection of firm service capacity and the influence of network effects in the ICT industry. We apply several recent estimation techniques to calibrate model parameters using firm-level empirical data. This allows us to assess model validity, recover plausible parameter values and distributions, and compare the relative performance and appropriateness of different calibration methods.

Building on the existing literature, our study explores the specific challenges involved in estimating and calibrating parameters within industry-specific agent-based models, with particular attention to the ICT sector. We investigate how parameters can be effectively estimated and calibrated in such settings, and examine the extent to which incorporating real-world data enhances model validity and empirical relevance. In doing so, we also compare different calibration methods to assess their suitability for economic agent-based modeling.

Specifically, we focus on two representative firms in the U.S. telecommunications sector, AT&T and T-Mobile, given their substantial market shares and direct competitive interactions. While other firms such as Verizon and US Cellular are also present in the market, our analysis centers on AT&T and T-Mobile to capture the dynamics of a duopolistic competition. This focus is motivated by two main considerations. First, these firms have historically engaged in more direct rivalry, as evidenced by two high-profile merger proposals. In 2011, AT&T announced its intention to acquire T-Mobile USA for \$39 billion from its parent company, Deutsche Telekom. However, the proposed merger was ultimately blocked, with critics arguing that it would have reduced competition and failed to meet the standards of the U.S. Department of Justice and Federal Trade Commission’s Horizontal Merger Guidelines (Besen et al. 2013). More recently, the 2020 Sprint–T-Mobile merger reshaped the competitive landscape, effectively establishing AT&T and T-Mobile as the primary competitors in the post-merger market. Although a coalition of U.S. states challenged the merger on antitrust grounds, it was eventually approved (Williams 2020). Second, the U.S. telecom market is better characterized as an asymmetric “2+2” structure, where Verizon and AT&T constitute the larger players, and T-Mobile and Sprint represent the smaller ones, rather than a symmetric four-player configuration (Williams 2020). For analytical clarity, we model this market as a duopoly, with firm a representing a larger incumbent (e.g., AT&T) and firm b a smaller challenger (e.g., T-Mobile). This abstraction preserves the core competitive dynamics without much loss of essential information.

The structure of our research proceeds as follows. We begin by presenting the theoretical model together with the corresponding real-world data, including necessary preprocessing procedures such as data cleansing and rescaling. Next, we evaluate the feasibility of parameter estimation by applying several established approaches to a simplified version of the model. Based on their comparative performance, we identify the SMM as a suitable framework for calibrating the full model. To improve computational efficiency and to cover the model calibration quality, a neural-network surrogate is introduced to approximate the agent-based model’s mapping between parameters and simulated moments. In the final stage, we analyze the estimation outcomes and conduct statistical assessments to evaluate key properties of both the empirical data and the model-generated results.

3.2 Literature review

Agent heterogeneity and boundedly rational decision rules have long been central to micro-founded economic modeling. Within this tradition, agent-based models (ABMs) provide a natural framework for studying aggregate (macroeconomic) dynamics as an emergent outcome of interacting heterogeneous agents and heuristic

behavioral rules.

Regarding calibration and estimation, the ABM literature is commonly organized around two broad paradigms: *frequentist* and *Bayesian* approaches. On the frequentist side, Grazzini & Richiardi (2015) develop and apply simulated minimum distance (SMD) methods, including the simulated method of moments (SMM).¹ While the resulting estimators can be consistent and asymptotically normal under standard conditions, a persistent practical challenge is that the choice of moments (or other summary statistics) is not uniquely determined. Consequently, the quality of parameter estimates depends critically on selecting moments that are sufficiently informative about the underlying mechanisms. Platt (2022) review a broader set of recent developments, including information-theoretic criteria, simulation-based approaches to maximum likelihood estimation, and computationally efficient surrogate modeling techniques. Complementing these contributions, Vandin et al. (2022) propose automated, model-independent, and tool-supported procedures for inspecting simulations and conducting counterfactual analysis; importantly, their framework distinguishes transient dynamics from steady-state behavior using statistical model checking (SMC) and related toolboxes. Finally, Kukacka & Sacht (2023) emphasize empirical validation challenges in macroeconomic models with behavioral heuristics and nonlinear switching, showing that simulated maximum likelihood methods can identify behavioral effects that may remain hidden under standard econometric approaches.

Bayesian approaches to calibration and estimation have also received substantial attention in the ABM literature. Grazzini et al. (2017) develop a Bayesian estimation framework for ABMs based on likelihood construction and sampling, showing that a kernel density estimation (KDE)–based likelihood can outperform several prominent SMD procedures in systematic comparisons. In a related spirit, Lamperti (2018) emphasize information-theoretic criteria for selecting parameterizations that replicate empirical time-series properties, proposing a divergence-based metric (GSL-div) to discipline model fit. Moving toward applied macroeconomic contexts, Delli Gatti & Grazzini (2020) study Bayesian estimation and forecasting in an ABM setting and complement the estimation exercise with post-simulation diagnostics, including ergodicity checks. Platt (2020) provide a comparative assessment of calibration strategies by contrasting time-series-model-based approaches across two benchmark models—the Brock & Hommes (1998) asset pricing model and the INET Oxford housing market model. Further contributions extend Bayesian calibration to settings with time variation and heterogeneity: Kim et al. (2022) highlight the importance of dynamic and heterogeneous calibration in ABMs, while Akbar et al. (2022) compare Bayesian inference to classical econometric inference in the context of human capital and economic growth. Beyond

¹Also known as the method of simulated moments (MsM).

macro-growth applications, Gao et al. (2022) employ likelihood-based Bayesian methods to quantify behavioral primitives such as risk preferences. During the COVID-19 period, Bayesian calibration was also used to discipline policy-oriented simulation models: Delli Gatti & Reissl (2022) combine an epidemiological block with a macroeconomic block to analyze health–economy trade-offs, and Hadley et al. (2022) apply Bayesian estimation to improve hospitalization predictions. More recently, researchers have increasingly combined simulation-based estimation with machine learning to improve computational efficiency and calibration performance. For instance, Fen (2022) use normalizing flows to accelerate Bayesian estimation in settings with heterogeneous and representative agents, and Dyer et al. (2022) apply graph neural networks for simulation-based inference. Related work introduces learning-based updates in financial-market ABMs (Wang (2022)) and investigates heterogeneous expectations and adaptive learning within New Keynesian environments (Elias (2022)). In addition, Platt (2022) demonstrate how neural networks can be integrated into Bayesian estimation workflows for economic simulation models, while Yu (2022) explore reinforcement learning and modified activation functions to represent heterogeneous behavioral adaptation. Barde (2025) establishes the feasibility and relevance of macroeconomic empirical parameter estimation for ABM developers, emphasizing how Bayesian calibration can improve the empirical fit and credibility of scenario-based policy analysis. From our perspective, this study extends the existing literature in three main respects. First, it integrates several parameter estimation strategies into a micro-founded model of industrial dynamics (especially in the ICT market), thereby enabling a direct comparison of alternative calibration approaches within an empirically oriented agent-based framework. Second, it moves beyond a purely methodological assessment by using these estimation strategies to identify relatively best-fit parameter settings and to evaluate the extent to which the simulated data are able to reproduce salient stylized facts of observed market dynamics. Third, it situates this calibration exercise within a duopolistic environment characterized by heterogeneous consumer preferences, congestion effects, and network interactions, thus linking parameter estimation to substantive questions of firm competition, strategic behavior, and welfare-relevant outcomes. In particular, we employ the SMM to obtain a data-consistent calibration based on economically meaningful summary statistics. To alleviate the computational burden associated with repeated ABM evaluation, we further introduce a neural-network surrogate that approximates the mapping from parameters to simulated moments, thereby enabling a faster and more systematic exploration of the parameter space. The combined framework contributes to the empirical grounding of simulation-based industrial organization models and supports counterfactual as well as policy analysis in a setting capable of capturing the strategic logic of duopolistic competition.

3.3 Model design

This section describes the computational model developed in a related study by the author (Wang 2025). The model framework remains unchanged, and the following description is mainly based on the prior work. The key distinction in the calibration framework is the relaxation of assumptions on consumer-utility parameters, which allows for greater heterogeneity.

3.3.1 General model structure

- **Firms:** The firms that hold the copyright and license of the product. In our basic duopolistic competition model, there are two firms, referred to as firms a and b .
- **Consumers:** The users who are potential buyers of the product. In the model, consumers are numerically identified, ranging from user 1 to user n ($n \geq 1$).

The architecture of our model is outlined below, providing a clear representation of the interactions and relationships among these diverse agents.

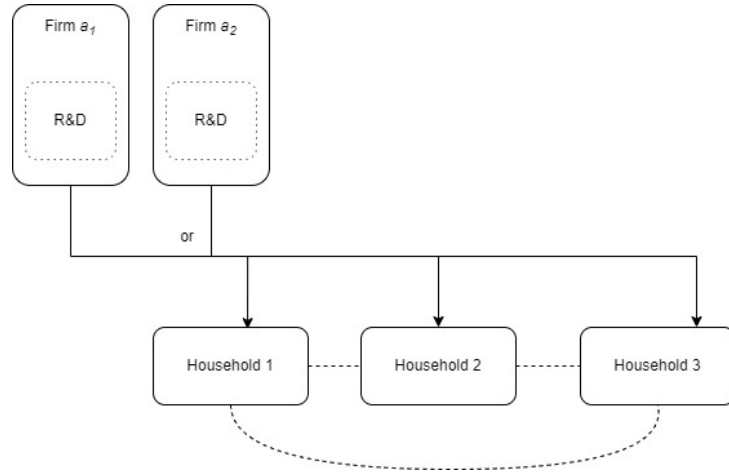


Figure 3.1: Model architecture

Sequence of events in each period:

1. **Contract Negotiation:** Firms engage with R&D teams to outline contracts and determine the investment in capacity.

2. **Product Development:** During this phase, R&D teams concentrate on product innovation, involving algorithm design and the application of intellectual effort to finalize the product's features.
3. **Payments and Sales:** Firms remunerate R&D teams and cover additional expenses, such as regulatory compliance, maintenance, and marketing. Subsequently, firms set product prices, enhance the physical R&D investment, and market their offerings on online platforms, thereby reaching a global consumer base.
4. **Consumer Decision-Making:** Consumers, who have been informed of the products, assess the available products based on detailed descriptions, user reviews, and prior experiences.² They may also explore free-trial versions. This decision-making process entails two steps: firstly, determining whether to make a purchase, and secondly, choosing which product to buy. Notably, consumers face no transaction costs.
5. **Transaction Completion and Profit Calculation:** Following the completion of transactions, each Firm calculates its profit, which is contingent on the choices made by Consumers.
6. **Product Consumption:** Consumers begin to use the products they have purchased and share their experiences within their social networks, shaping their overall user experience.
7. **Information Sharing:** Consumers share the information of the products with their connections.

Firms

In a duopolistic setting, firms produce products which are heterogeneous on product functionality, i.e. the products are vertically differentiated.³ The service capacity is crucial for the product functionality, while there may also be bias when consumers judge the products because of consumer congestion. This is because specifically in the ICT market, the consumer congestion (the situation when a large number of consumers use the product simultaneously) can negatively affect the “observed functionality”, which is linked with consumers’ utility level (user experience). Since the focus in the model is to capture the consumer congestion effect, we assume that the real product functionality is the same as the service

²It is assumed that in the initial period a small proportion of consumers receives the market information.

³Firm i produces a single type of product which is defined as product i .

capacity of the firm. We define the service capacity of product i in period t as $\chi_{i,t}$ and have the following equations for describing the product features across periods.

The service capacity in the initial period should be

$$\chi_{i,0} = S_{i,0}A_{i,0}$$

where $S_{i,0}$ represents the physical component and $A_{i,0}$ represents the technological component. The Physical component in period $t + 1$ ($t \geq 0$) $S_{i,t+1}$ can be formulated as

$$S_{i,t+1} = (1 - \beta^D)S_{i,t} + \beta_i^C \ln(c_{0,i,t}^{Phy} + 1)$$

where β^D represents the depreciation rate, β_i^C represents the update effectiveness, $c_{0,i,t}^{Phy}$ represents the physical investment in the product. Therefore, we have

$$\chi_{i,t+1} = S_{i,t+1}A_{i,t+1} = ((1 - \beta^D)S_{i,t} + \beta_i^C \ln(c_{0,i,t}^{Phy} + 1))A_{i,t+1}$$

The initial observed functionality of product i can be formulated as

$$\theta_{i,0} = \beta_2\chi_{i,0} - \beta_0\bar{m} + \beta_3$$

where $\bar{m}$ represents the number of initial users, parameters $\beta_2, \beta_0, \beta_3 \geq 0$. The dynamics of the updated observed functionality can be therefore formulated as

$$\theta_{i,t+1} = \beta_2\chi_{i,t+1} - \beta_0m_{i,t} + \beta_3$$

Consumers

Consumers are assumed to have heterogeneous preferences for product functionality. In practice, some individuals purchase network services to meet basic needs, while others, who spend more time online, may demand products with higher functionality. To capture this diversity, we assume that consumer j has a preference for product functionality represented by x_j , where $x_j \sim U[0, 1]$. A higher value of x_j indicates that the consumer places greater importance on product functionality. We define the utility level of consumer j considering product i in period t as $u_{j,i,t}$ and we have

$$u_{j,i,t} = \bar{u} + x_j\theta_{i,t} + \beta_4\mu_{i,j,t-1} - \beta_5p_{i,t}$$

where $\mu_{i,j,t-1}$ represents the number of consumer j 's connections buying product i in period $t - 1$. Parameters $\bar{u}, \beta_4, \beta_5 \geq 0$.

To allow more heterogeneity in consumers, we extend the equation above to

$$u_{j,i,t} = \bar{u}_{j,i} + x_j\theta_{i,t} + \beta_{4,j}\eta_{j,i,t-1} - \beta_{5,j}p_{i,t}$$

$\bar{u}_{j,i}$ captures consumer-specific baseline utility, accounting for unobserved factors such as brand perception that influence purchasing behavior.⁴ The parameter $\beta_{4,j}$ reflects heterogeneous responses to network effects, recognizing that some consumers are peer-followers, while others may be indifferent or even contrarian. Similarly, $\beta_{5,j}$ allows for variation in price sensitivity, with some consumers more willing to pay for higher-priced products. Such heterogeneity in consumer behavior is well-documented in the literature (Leibenstein 1950, Milgrom & Roberts 1986, Calzada et al. 2023).⁵

The total number of consumers buying product i in period t should be

$$m_{i,t} = \sum_j^{J_{t-1}} \mathbb{1}_{u_{j,i,t} > u_{j,-i,t}} + \sum_j^{J_{t-1}} \beta_{j,i,t} \mathbb{1}_{u_{j,i,t} = u_{j,-i,t}}$$

where J_{t-1} represents the set of consumers who have participated in the market. Firms calculate their profits at the end of period t :

$$\Pi_{i,t} = m_{i,t}(p_{i,t} - c_{res,i,t}) - c_{0,i,t}^{Phy} = m_{i,t}p_{i,t}^C - c_{0,i,t}^{Phy}$$

After the initial two periods, firms start adjusting product prices based on “market analyses” of a sampled group of consumers and adjust their investment levels according to historical profitability. Both adjustment heuristics are incorporated into the model (Wang 2025). The primary purpose of the model is to replicate symmetric and asymmetric oligopolistic competition and to analyze the resulting market outcomes under different firm policies. This framework facilitates a deeper understanding of firms’ decision-making processes and the implications of alternative policy designs.

3.3.2 Data collection

Raw data description

We collect all available data from the official websites of AT&T⁶ and T-Mobile⁷, Statista⁸, and Refinitiv⁹. Among all the data collected, we select the same time range of both firms (2013Q4–2023Q4). Data for ten years could provide a thorough overview of the development of the two main players in the market. We focus on

⁴In the calibration $\bar{u}_i \sim U[\bar{u}_{i,low}, \bar{u}_{i,high}]$ and the values are assigned to each consumer.

⁵In the calibration $\beta_{4,j}$ and $\beta_{5,j}$ are assumed to follow truncated normal distributions and the related parameters are included in the objective parameters.

⁶<https://investors.att.com/financial-reports/quarterly-earnings/2025>

⁷<https://investor.t-mobile.com/financials/annual-reports/default.aspx>

⁸<https://www.statista.com/>

⁹<https://www.lseg.com/en>

the real time series of gross revenue, market demand (number of subscriptions), operation cost, and operation profit. We explain the rule of selection as:

- **Revenue:** “Operating Revenue” from Income Statement. Quarterly basis.
- **Cost:** “Operating Cost” from Income Statement. Quarterly basis.
- **Profit:** “Operating Profit”. Equal to the subtraction of **Revenue** and **Cost**. Quarterly basis.
- **Market demand:** “Number of subscriptions” from Operating Metrics. Annual basis.
- **Subscription price:** Calculated from the existing data following the formula $p = R/m$ (**Revenue** divided by **Market demand**).

Since the firms offer different product and services to consumers and the exact subscription prices are not explicit in the firms’ reports, we calculate **Subscription price** $p_{i,t}$ by dividing **Revenue** with **Number of Consumers** to get the quarterly basis data.¹⁰

The market demand of the two firms can be approximated by the number of subscriptions in every period. In the financial statements and quarterly reports of AT&T, the annual wireless connections and broadband connections are given, but since some consumers are using them both, thus, the market demand may not be a simple sum of these numbers. To solve this problem, we propose a parameter ν representing the proportion of consumers using both services among total consumers. According to our calculations, with the information of “single-users”, $m_{wireless}$ and $m_{broadband}$, the total number of consumers should be $\frac{1}{1+\nu}(m_{wireless} + m_{broadband})$. Under different values of ν , we approximate the market demand for AT&T from 2013-Q1 to 2016-Q4. Referring to the data provided by *Statista*, we show the approximated market demand with different ν ’s and select the appropriate value (0.15) for the further steps.¹¹

¹⁰For the annual basis data, we calculate the time series by dividing **Sales** with **Number of Consumers** and show the difference. We assume the marginal cost $c_{res,i,t}$ to be 0, so that $p_{i,t} = p_{i,t}^C$.

¹¹*Statista* provides quarterly market demand data for AT&T starting from 2017-Q1, whereas for T-Mobile, data are available from as early as 2010-Q1. Therefore, no further approximation is needed for T-Mobile’s data prior to 2017-Q1.

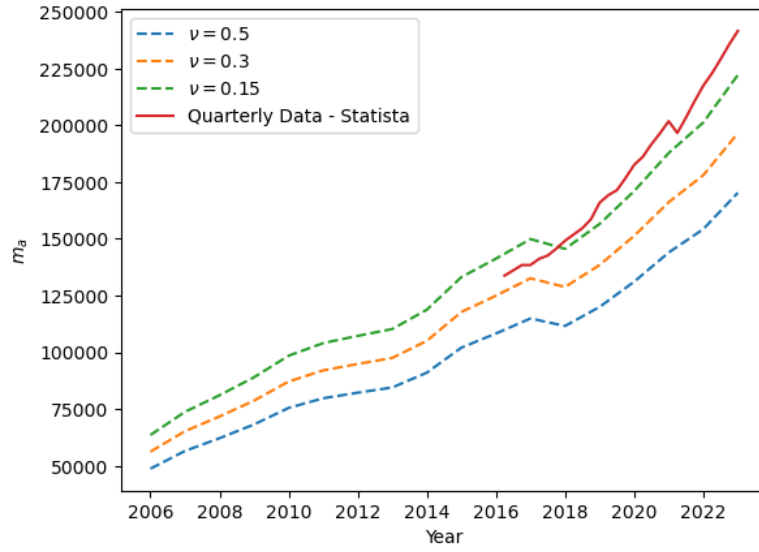


Figure 3.2: AT&T Market Demand. Unit: Thousand.

Other available data related to the two firms are shown in Figure 3.3.

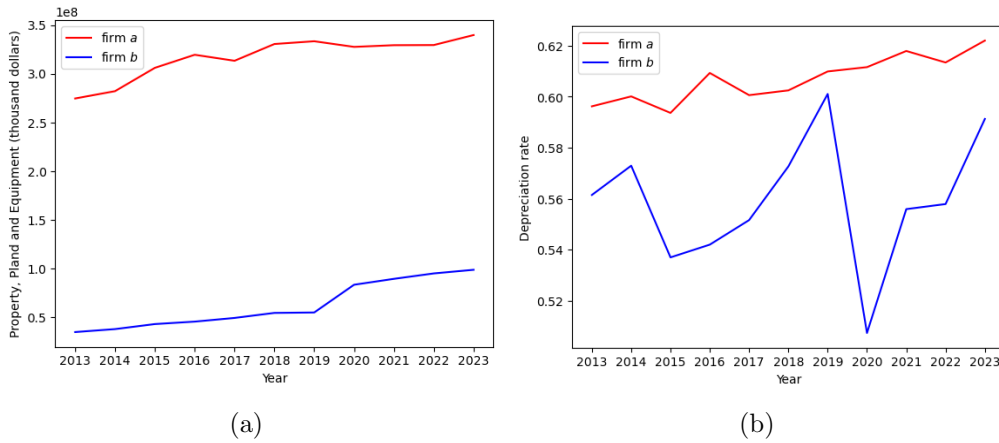


Figure 3.3: Property, plant & equipment and Depreciation Rate of both firms. We denote AT&T and T-Mobile US with firm a and b respectively.

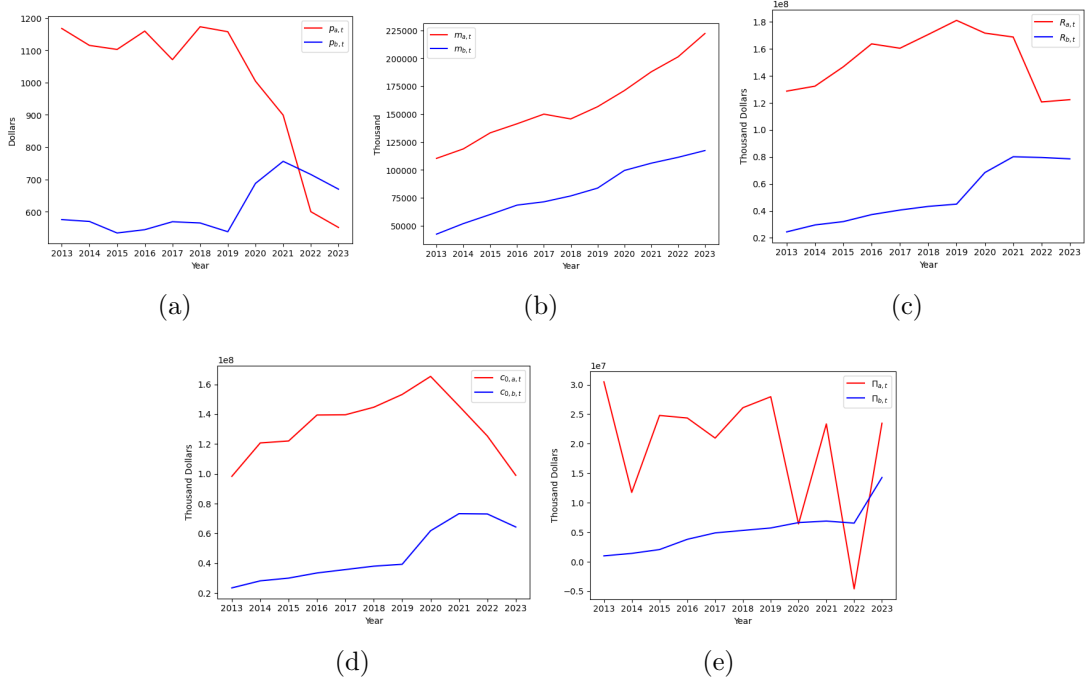


Figure 3.4: Operating status of both firms.

From the operating status, we observe that compared with firm b , firm a has a much larger PPE, higher market demand, higher revenue, and larger operating costs. Moreover, firm a experiences more fluctuations in profit. Firm b shows a much lower PPE level but experiences gradual growth, with a more noticeable increase after 2018. Both firms have a large depreciation rate on the infrastructure (> 0.5). Besides, the price of firm a shows a general decreasing trend, while the price of firm b experiences some increases. Moreover, since 2019, the revenue gap and the expense gap become smaller. Overall, firm a leads in most financial metrics (price, demand, revenue, and profit) but experiences more volatility. Firm b starts with lower values but exhibits consistent growth across all indicators, suggesting an improving competitive position.

Measuring Capacity Expansion Costs

A key assumption in our model is that all investment costs are allocated to service capacity expansion. However, translating this theoretical concept into empirical measures requires careful consideration of available financial data. We discuss three alternative approaches to operationalize capacity expansion costs from the firms' financial statements:

A01: Extracting capacity-related items from operating costs. Among the

items reported under operating costs, there exists a “cost of equipment” category. However, this item is more closely related to the cost of goods sold rather than service capacity-related investment, making it unsuitable for our purposes.

A02: Using R&D intensity as a proxy. The financial statements do not explicitly report R&D-related costs. Moreover, even if such data were available, R&D expenditures do not necessarily correspond to service capacity-related investment, as they may include activities unrelated to infrastructure expansion.

A03: Defining costs as a proportion of total revenue. We define the capacity expansion cost as:

$$c_{0,i} = \frac{PP\&E_i}{TotalAsset_i} \cdot R_i,$$

where $PP\&E_i$ denotes property, plant, and equipment, $TotalAsset_i$ represents total assets, and R_i is the total revenue of firm i . This formulation captures the intuition that a proportion of revenue, weighted by the firm’s fixed asset intensity, reflects capacity-related investment. Note that the fixed asset rate varies annually according to changes in the balance sheet.

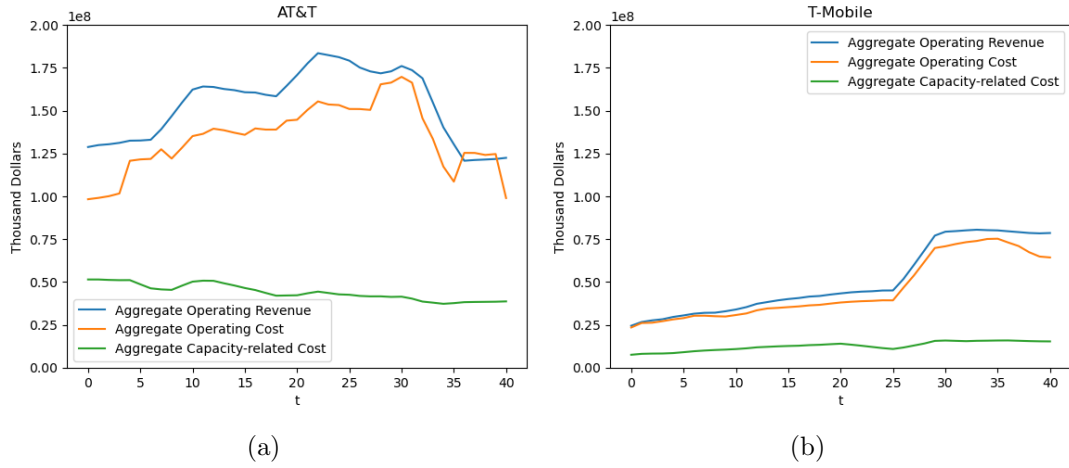


Figure 3.5: Operating cost and capacity-related cost.

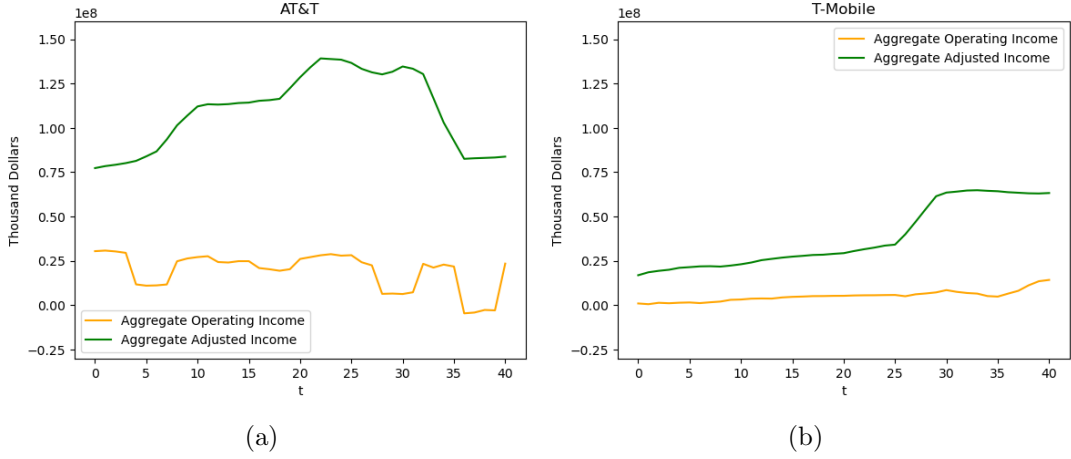


Figure 3.6: Operating income and adjusted income.

For the subsequent calibration steps, we adopt **A03**, as it best aligns with our assumption that investment costs are specifically directed toward service capacity expansion.

Data Transformation

Analyzing competitive dynamics in the presence of complex economic effects, such as consumer congestion and network effects, is essential for understanding market behavior. Parameter estimation enables us to quantify these effects and assess their impact on firm competition. To improve calibration quality, researchers have explored various data transformation techniques. For example, Grazzini & Richiardi (2015) rescaled and detrended their model to achieve stationarity around a balanced growth path in DSGE estimation. Within our framework, we apply temporal disaggregation and linear interpolation to convert the data to a monthly frequency, facilitating more precise calibration.

Since the raw market demand data (number of customers) is reported annually while cost and profit data are available quarterly, frequency alignment is necessary for accurate model fitting. Sax & Steiner (2013) and Selvam (2020) introduced methods for temporal disaggregation of time series data, enabling the conversion of low-frequency data into higher-frequency series. The primary aim is to estimate an unknown high-frequency series y such that its aggregated values (sums, averages, or endpoints) align with a known low-frequency series y_t . This approach is particularly relevant in our context, where we must reconcile annual consumer counts with quarterly financial metrics.

Given the frequency of price updates and the structure of the simulation units, it is beneficial to introduce a finer division of the time scale. Linear interpolation

is a common method for fitting data to a smaller scale while maintaining the consistency of the data's trend. Although Friedman (1962) argues that relying solely on the time series itself is insufficient and that incorporating related time series would improve estimation, obtaining suitable related monthly time series for the firms we investigate is not straightforward. Thus, we use a simple method to transform the quarterly data into monthly data and summarize ten years' data (2013-Q4 to 2023-Q4, 121 data points) into the final dataset.

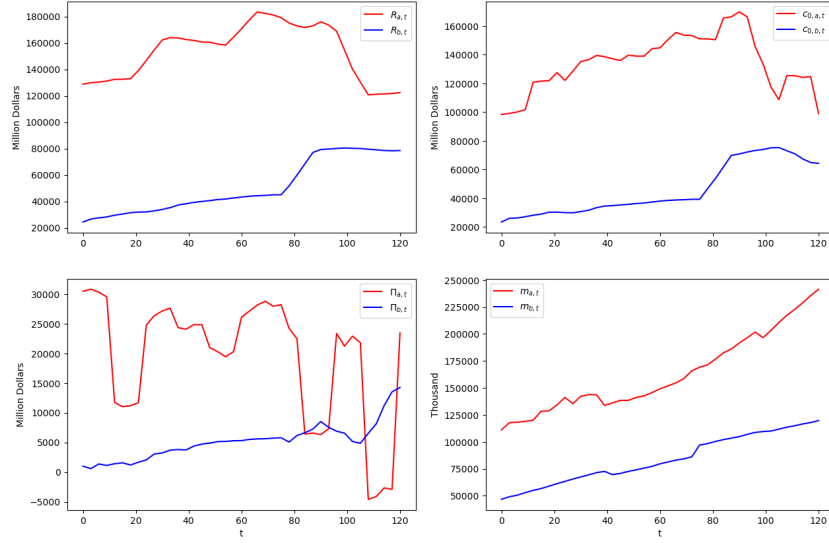


Figure 3.7: Monthly data of AT&T (firm *a*) and T-Mobile (firm *b*).

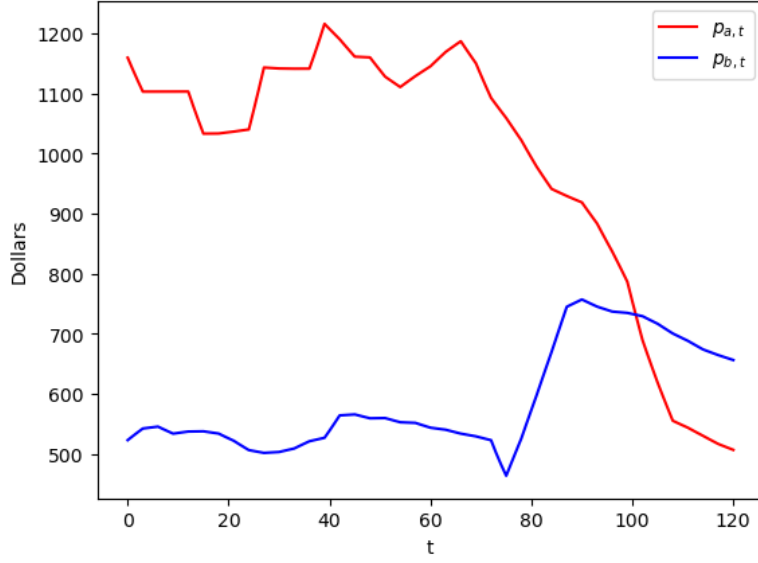


Figure 3.8: Monthly price data of AT&T (firm a) and T-Mobile (firm b).

3.4 Parameter estimation for the simplified model

3.4.1 Model calibration

Since the original model includes complexity in comparing the utility and aggregating the demand from the individual consumer level, we apply a simplified version among the initial examples as follows:

$$\begin{aligned}
 \chi_{i,0} &= S_{i,0} \\
 \chi_{i,t+1} &= S_{i,t+1} = \beta^D S_{i,t} + \ln(c_{0,i,t}^{Phy} + 1) + \nu_{i,t+1} \\
 \theta_{i,0} &= \beta_2 \chi_{i,0} - \beta_0 m_{i,ini} + \beta_3 + \varepsilon_{i,0} \\
 \theta_{i,t+1} &= \beta_2 \chi_{i,t+1} - \beta_0 m_{i,t} + \beta_3 + \varepsilon_{i,t+1} \\
 m_{i,t+1} &= \lambda_1 \theta_{i,t+1} + \lambda_2 p_{i,t+1} + \lambda_3 \\
 \Pi_{i,t} &= m_{i,t} p_{i,t} - c_{0,i,t}^{Phy}
 \end{aligned}$$

In this model, the market demand is defined as a linear function of observed functionality and price. Keep the newly introduced parameters fixed ($\lambda_1 = 0.05$, $\lambda_2 = -10$, $\lambda_3 = 5,000$), we explain our initial estimation examples based on a model without incorporating the individual consumer utility or consumer network effect. In this simplified specification, most coefficients are treated as fixed benchmark values and kept constant throughout all calibration runs.

To allow the space for parameter estimation, we introduce the error terms in the product functionality ($\chi_{i,t}$) and the observed service capacity ($\theta_{i,t}$), which improves the model at the basis of the deterministic features. For simplicity, we assume the process noise $\nu_{i,t} \sim N(0, \sigma_1^2)$ and the observation noise $\varepsilon_{i,t} \sim N(0, \sigma_2^2)$.

We start our approach by estimating a single parameter β_0 , which represents the effect of consumer congestion, while holding all other parameters as constants. Then, we show the possibility of including more parameters in the estimation procedure. Following the literature, we select four typical statistical methods to estimate the model: the Kalman Filter (KF), the Method of Moments (MM), the Simulated Method of Moments (SMM), and the Bayesian Markov Chain Monte Carlo (MCMC). For the optimization steps in SMM, we consider the Nelder-Mead method, since it is commonly used as a simplex algorithm under the structure of bound-constrained minimization (Nelder & Mead 1965).¹²

Kalman Filter

Kalman Filter is widely used for estimating time series data. As Haykin et al. (2001) describes, it provides a recursive solution in that each updated estimate of the state is computed from the previous estimate and the new input data. In our application, the procedure is given as follows:

1. For $i \in \{a, b\}$, generate the state vector and system dynamics as

$$\begin{aligned} x_{i,t} &= \begin{pmatrix} \chi_{i,t} \\ \beta_{0,i} \end{pmatrix} \\ \chi_{i,t+1} &= \beta^D S_{i,t} + \ln(c_{0,i,t}^{Phy} + 1) + \nu_{i,t+1} \\ \beta_{0,i,t+1} &= \beta_{0,i,t} + \nu_{\beta_{0,i,t+1}} \end{aligned}$$

The state transition matrix is given as

$$F_{i,t} = \begin{pmatrix} \beta^D & 0 \\ 0 & 1 \end{pmatrix}$$

2. Set up the observation model for each firm and the measurement matrix $H_{i,t}$.

$$\begin{aligned} \theta_{i,t+1} &= \beta_2 \chi_{i,t+1} - \beta_{0,i} m_{i,t} + \beta_3 + \varepsilon_{i,t+1} \\ H_{i,t} &= [\beta_2, -m_{i,t}] \end{aligned}$$

¹²In the codes, we apply the *Python* package *SciPy*. While the Nelder-Mead method is suitable for low-dimensional optimization in the simplified model, we switch to Latin Hypercube Sampling (LHS) in the complete model. LHS is more appropriate for exploring high-dimensional parameter spaces efficiently and is better suited to the simulation-based structure of the full model.

3. Initialize the Kalman Filter. Initial state estimator:

$$x_{i,0} = \begin{pmatrix} \hat{\chi}_{i,0} \\ \hat{\beta}_{0,i} \end{pmatrix}$$

Initial covariance matrix:

$$P_{i,0} = \begin{pmatrix} Var(\hat{\chi}_{i,0}) & 0 \\ 0 & Var(\hat{\beta}_{0,i}) \end{pmatrix}$$

Process noise covariance matrix:

$$Q_i = \begin{pmatrix} Var(\nu_{i,t}) & 0 \\ 0 & Var(\nu_{\beta_{0,i,t}}) \end{pmatrix}$$

Measurement noise covariance matrix:

$$R_i = Var(\varepsilon_{i,t+1})$$

4. Update steps for each firm.

$$\begin{aligned} x_{i,t+1|t} &= F_{i,t}x_{i,t|t} \\ P_{i,t+1|t} &= F_{i,t}P_{i,t|t}F_{i,t}^T + Q_i \\ y_{i,t+1} &= \theta_{i,t+1} - H_{i,t}x_{i,t+1|t} \\ K_{i,t} &= P_{i,t+1|t}H_{i,t}^T(H_{i,t}P_{i,t+1|t}H_{i,t}^T + R_i)^{-1} \\ x_{i,t+1|t+1} &= x_{i,t+1|t} + K_{i,t}y_{i,t+1} \\ P_{i,t+1|t+1} &= (I - K_{i,t}H_{i,t})P_{i,t+1|t} \end{aligned}$$

where $y_{i,t+1}$ denotes the measurement residual and $K_{i,t}$ denotes the Kalman gain.

5. Repeat Steps 1.-4. for each period using observed data $\theta_{i,t}$, $m_{i,t}$, $c_{0,i,t}^{Phy}$ for each firm. At each step, the Kalman filter produces an updated estimate of the state vector $x_{i,t}$ and the associated uncertainty $P_{i,t}$.

After iterating through the full time horizon T , the final estimated state vector $x_{i,T|T}$ provides the filtered estimates of the service capacity $\chi_{i,T}$ and the consumer congestion parameter $\beta_{0,i}$ at the end of the sample.¹³

¹³While extensions such as the Extended Kalman Filter or Particle Filter could be applied to the simplified model to accommodate greater nonlinearity, we do not pursue them here. Our primary focus is to identify the most efficient estimation strategies for the complete model, for which simulation-based approaches are more appropriate.

Method of Moments

The Method of Moments is a classical statistical approach for parameter estimation. Given that most equations in our simplified model are linear, this method can be applied efficiently. The key step is to construct a moment condition for the parameter β_0 .

The population moment condition is given by

$$\mathbb{E}[g(Z_t, \beta_0)] = 0$$

where $Z_t = (\theta_{i,t+1}, \chi_{i,t+1}, m_{i,t})$, $g(Z_t, \beta_0) = \theta_{i,t+1} - \beta_2 \chi_{i,t+1} + \beta_0 m_{i,t} - \beta_3$. We assume exact identification, meaning that the number of moment conditions equals the number of parameters to be estimated.¹⁴

Identification requires that $m_{i,t}$ exhibits non-zero variation, and that all components of Z_t are jointly observable and uncorrelated with the error term $\varepsilon_{i,t+1}$. These assumptions ensure that the moment condition uniquely pins down the value of β_0 .

Based on the moment condition, the parameter β_0 can be estimated as

$$\beta_0 = \frac{\beta_2 \mathbb{E}[\chi_{a,t+1}] + \beta_3 - \mathbb{E}[\theta_{i,t+1}]}{\mathbb{E}[m_{i,t}]}$$

This analytical expression provides a transparent and interpretable starting point for parameter recovery under the linear structure of the simplified model. However, in more complex settings that involve latent states, simulation-driven randomness, or nonlinear dynamics, closed-form estimation becomes infeasible. In such cases, simulation-based methods such as the Simulated Method of Moments (SMM) or Bayesian MCMC offer more flexible and robust alternatives.

Simulated Method of Moments

The Simulated Method of Moments is an extension of the Method of Moments. This method allows the estimation when the moments of the data cannot be calculated explicitly from the model. Following Hansen's insight that properly chosen moment conditions can serve as sufficient statistics for structural identification, our work explores how well a computational model calibrated via SMM can reproduce key economic moments observed in real-world time series (Hansen 1982). We describe the procedure as follows:

¹⁴In this case, only β_0 is estimated. A potential extension to multiple parameters would involve the Generalized Method of Moments (GMM). However, we focus on the Simulated Method of Moments (SMM), which shares GMM's core principles but is better suited to simulation-based models.

1. Generate a simulated time series with the model for profits, costs, and prices given initial values of the parameters to be estimated.
2. Calculate the statistical moments of the simulated data (mean, variance, etc.) that correspond to the empirical moments (calculated from real data).¹⁵
3. Compare the moments of the simulated data to the empirical moments from the real data. The goal is to find the value of β_0 that minimizes the difference between the empirical moments and the simulated moments.
4. Use least squares optimization to adjust β_0 and minimize the difference between the moments.

Although SMM is often applied to complex simulation-based models, it remains well-suited for linear systems like our simplified setup, providing a flexible and consistent approach to parameter estimation.

Bayesian Markov Chain Monte Carlo

The Bayesian Markov Chain Monte Carlo requires the prior belief or knowledge of the parameter distribution. This method allows for the incorporation of prior beliefs and yields full posterior distributions over the parameters, providing not only point estimates but also measures of uncertainty, which are valuable in simulation-based models with implicit randomness. The procedure is described as follows:

1. Specify a Likelihood Function related to the observed data to the model parameters. In this setting, we assume a normally distributed likelihood based on the residuals between observed and simulated time series.
2. Specify prior distributions for the parameters of interest. To remain consistent with other methods and to reflect limited prior knowledge, we assume independent normal priors with large variance.
3. Set Up the MCMC Sampler to sample from the posterior distribution of model parameters.¹⁶ The resulting posterior samples for the parameters are used for inference.

¹⁵We consider more sophisticated statistical moments as well as economic moments as we calibrate the complete model.

¹⁶We apply the Ensemble Sampler from *Python* package *emcee*, specifying 50 parallel walkers. The walkers are initialized by drawing from a normal distribution centered at zero with a standard deviation of 10. The sampler is run for 500 iterations per walker. To ensure robustness of the posterior inference, we discard the first 10 steps of each chain as burn-in and apply thinning (retaining every 10th sample).

4. Compute the posterior mean and credible intervals for each parameter based on the sampled chains. These estimates reflect the most plausible parameter values given both the observed data and prior beliefs.

This Bayesian MCMC framework allows us to quantify uncertainty in parameter estimates and compare parameter distributions across model variants. It is particularly well-suited to agent-based settings, where structural noise and simulation dependence complicate traditional estimation methods. At the same time, it remains applicable and effective even in linear systems, offering a consistent and probabilistically grounded approach to parameter inference.

3.4.2 Estimation results

We first select an example set of time series in our model for the initial estimation. The profit mean time series over random seeds of the Monte Carlo simulations are generated by a computational model in Wang (2025). Both firms are under symmetric setting. Among the generated data, we assume that: (a) time series for $p_{i,t}$, $c_{0,i,t}^{Phy}$, $m_{i,t}$ and $\Pi_{i,t}$ ($i \in \{a, b\}$) can be observed, (b) $\Pi_{a,t}$ and $\Pi_{b,t}$ are the “example time series” to calibrate, and (c) β_0 is the only parameter for estimation.¹⁷ Given limited prior information, we initialize β_0 at 0 when implementing the different estimation methods, consistent with the theoretical restriction $\beta_0 \geq 0$. The objective is to check the estimation performances by comparing the example time series $\Pi_{i,t}$ and the estimated time series using different methods.

For performance evaluation, we employ the commonly used Mean Squared Error (*MSE*) and its standardized forms: Root Mean Squared Error (*RMSE*) and Normalized Root Mean Squared Error (*NRMSE*). These metrics capture the same underlying concept and move in the same direction: lower values indicate better estimation accuracy. To reduce redundancy, we report only the *NRMSE* in the main analysis. The indices can be formulated as follows:¹⁸

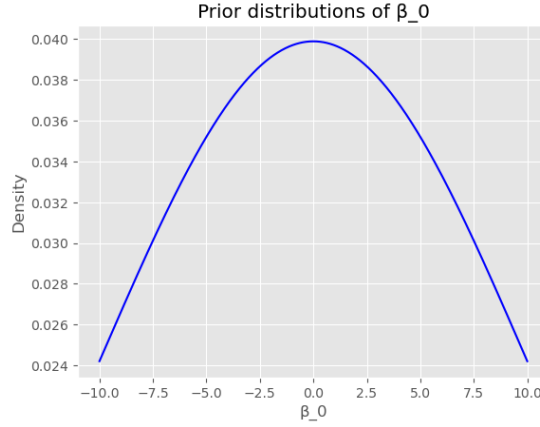
$$\begin{aligned}
 MSE &= \frac{1}{T} \sum_{t=1}^T (Y_{i,t} - \hat{Y}_{i,t})^2 \\
 RMSE &= \sqrt{MSE} \\
 NRMSE &= \frac{\sqrt{MSE}}{\bar{Y}_i}
 \end{aligned}$$

We observe that all of the methods we apply can calibrate the trend of the example time series, while SMM and MCMC can provide much closer estimation results

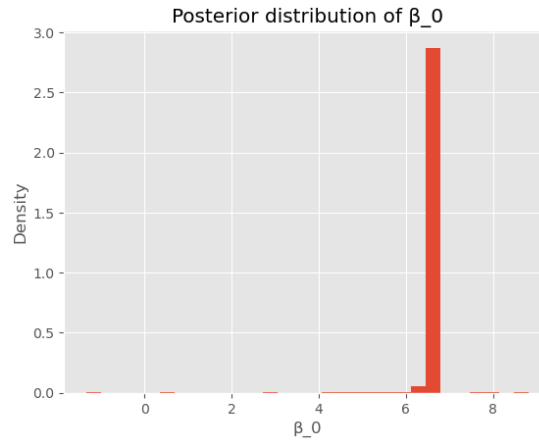
¹⁷Based on the symmetric assumption, the estimated parameter $\beta_0 = \beta_{0,a} = \beta_{0,b}$.

¹⁸Here our target is fitting the profit time series, therefore $Y_{i,t} = \Pi_{i,t}$, $\bar{Y}_i = \bar{\Pi}_i = \frac{1}{T} \sum_t \Pi_{i,t}$.

than the other two methods. The mean squared error of the two latter methods are also lower than that of the two former methods. We also observe that as the estimated parameter β_0 increases, the MSE is smaller, indicating that it is more likely that β_0 should be positive in the model. It implies that the congestion effect negatively impacts the observed functionality, which accords with our assumptions.



(a)



(b)

Figure 3.9: Prior and posterior distributions of estimated β_0 .

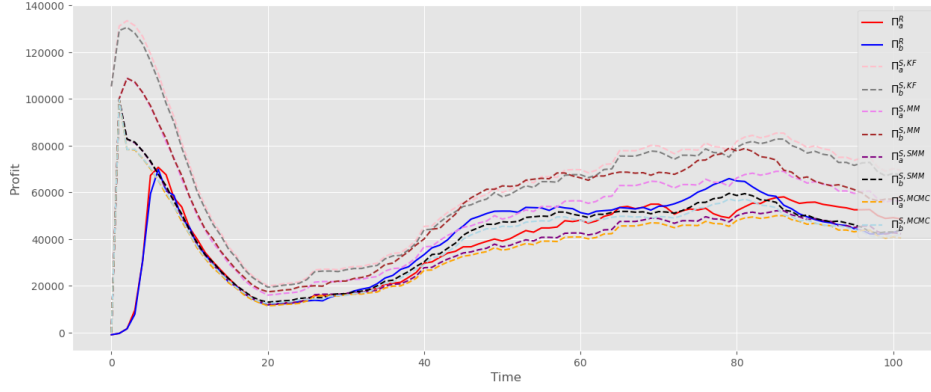


Figure 3.10: Comparison of the example time series and the time series with estimated parameter β_0 .

Method	$\hat{\beta}_0$	$NRMSE$
KF	-5.989	0.82
MM	0.069	0.55
SMM	5.513	0.48
MCMC	6.453	0.34

Table 3.1: Model performances with single parameter $\hat{\beta}_0$.

Method	$\hat{\beta}_0$	$\hat{\beta}_2$	$\hat{\beta}_3$	$NRMSE$
SMM	5.510	3	10,000	0.40
SMM	0.710	2.29	10,000	0.48
SMM	0.002	-5.94	23,054.66	0.41
MCMC	6.450	3	10,000	0.29
MCMC	2.480	-0.99	10,000	0.39
MCMC	-217.040	-202.64	-70,082.02	0.18

Table 3.2: Model performances with (a) $\hat{\beta}_0$, (b) $\hat{\beta}_0, \hat{\beta}_2$, and (c) $\hat{\beta}_0, \hat{\beta}_2, \hat{\beta}_3$. Numbers in **boldface** are assumed to be fixed during the parameter estimation.

Among the calibration methods, while Simulated Method of Moments (SMM) and Bayesian Markov Chain Monte Carlo (MCMC) techniques demonstrate superior performance in addressing the challenges related to complexities in the economy, we argue that although Bayesian MCMC methods can offer rich statistical information, including full posterior distributions and credibility intervals, they require substantial additional infrastructure, including prior specification, likelihood approximations, and convergence diagnostics, which are computationally intensive in

simulation-based models like ours. Given the implicit randomness and high dimensionality of our agent-based setting, we adopt the Simulated Method of Moments (SMM) as a more tractable and economically interpretable approach for parameter calibration.

In the complete model (with agent-based simulations), we select SMM for calibrating model parameters. Apart from the relatively better performance, there are other reasons to explain our selection: First of all, since the consumer behavior and market outcomes in the model depend critically on the initialization and propagation of stochastic interactions, governed by random seeds, the data-generating process is not explicitly tractable or analytically expressed, which violates the core assumption of the Kalman Filter (explicit linear-Gaussian state-space structure). Besides, the SMM can naturally accommodate randomness that only manifests through repeated simulations, without requiring knowledge of the functional form of the underlying stochastic process. Given the stochastic nature of agent-level decisions and network-induced feedback, the empirical moments are path-dependent and converge only over repeated simulations. This makes the classical Method of Moments less reliable unless simulation noise is controlled via extensive replications.

While the combination of the SMM and LHS provides a systematic way to explore the parameter space, it remains constrained by the finite number of simulation draws. The “best” parameter set identified through this procedure is therefore optimal only within the sampled grid, and potentially promising parameter combinations outside the selected hypercube points may remain unexplored. To enhance coverage of the parameter space and improve the efficiency of the search process, it is feasible to train a surrogate model, which is a machine learning approximation of our ABM, to predict simulated moment outcomes over a broader range of parameters. This surrogate-assisted calibration allows for more extensive experimentation and sensitivity analysis at a fraction of the computational cost. Recent studies highlight the increasing relevance of this approach in computational economics, where surrogate or emulator models are employed to approximate complex simulations and facilitate calibration (Lamperti et al. 2018, Dyer et al. 2022, Angione et al. 2022). Intuitively, the neural network can map structural parameters directly to the calibration bias, enabling fast evaluation of candidate parameter vectors without repeatedly simulating the full ABM. We therefore incorporate this method into our calibration framework.

3.5 Parameter estimation for the complete model

The complete model differs from the simplified version in several important respects. In the simplified model, uncertainty arises from process noise $\nu_{i,t}$ and

observation noise $\varepsilon_{i,t}$, implying that service capacity and observed functionality may vary stochastically within each period. By contrast, in the complete model, uncertainty primarily stems from consumer heterogeneity, as well as from firms' adaptive behaviors in market analysis and investment updating. Although the complete model does not explicitly incorporate process or observation noise, it captures a richer and more realistic form of uncertainty through its structural complexity. In particular, it allows for the modeling of individual consumer preferences, rather than assuming a representative or homogeneous consumer group. From this perspective, the complete model offers a more reasonable and behaviorally grounded framework for analyzing firm and consumer dynamics.

3.5.1 Model parameters

A crucial parameter related to the market demand is the population of consumers. Our consumer network follows a small-world structure, and we expect that a smaller representative sample can still capture the essential topology and dynamics of interactions. For the population size, we apply the logistic growth model, which is a common method for estimating the growth rate concerning the saturation (Oliver 1987, O'Kelly 2001, Aoki & Yoshikawa 2002). The equation has the form of

$$m_t = \frac{K}{1 + e^{-r(t-t_0)}}$$

where m_t represents the number of consumers at time t , K represents the maximum population size, r represents the growth rate, and t_0 represents the inflection point, at which there is maximum growth. Without loss of generality, we assume that each node in the consumer network represents 100,000 consumers.¹⁹ This abstraction serves to reduce computational complexity while also reflecting the real-world clustering of consumers who adopt and use products in similar ways. Although the demand in the real data keeps increasing, we believe that at some point, there will be a slower trend of increase because of market saturation. Based on the estimation of this model, we set the upper bound of 4698 as the total number of nodes in the consumer network.

¹⁹Here we apply a linear transformation. See **5.2 Estimation setup**.

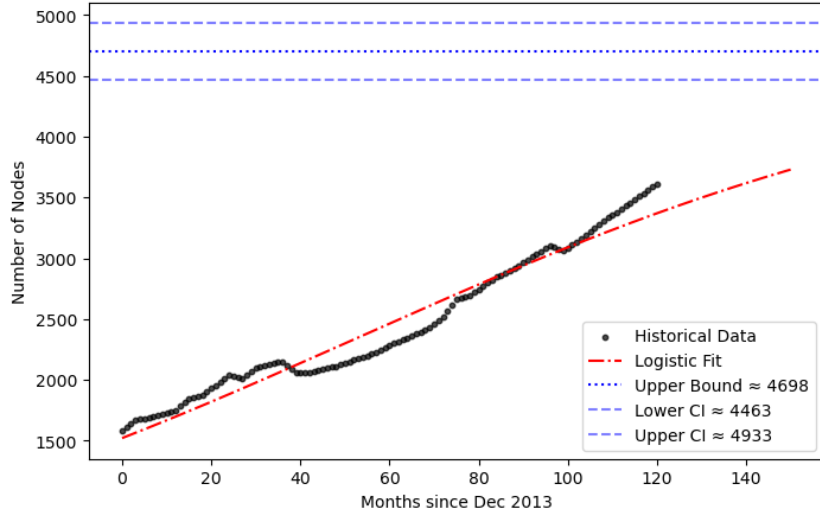


Figure 3.11: Logistic Growth Model Fit to Real Demand Data.

In our model, we also include the price and investment updates in the dynamics. For the discrete update steps, we calculate the mean and standard deviation of the differences in prices and investments.

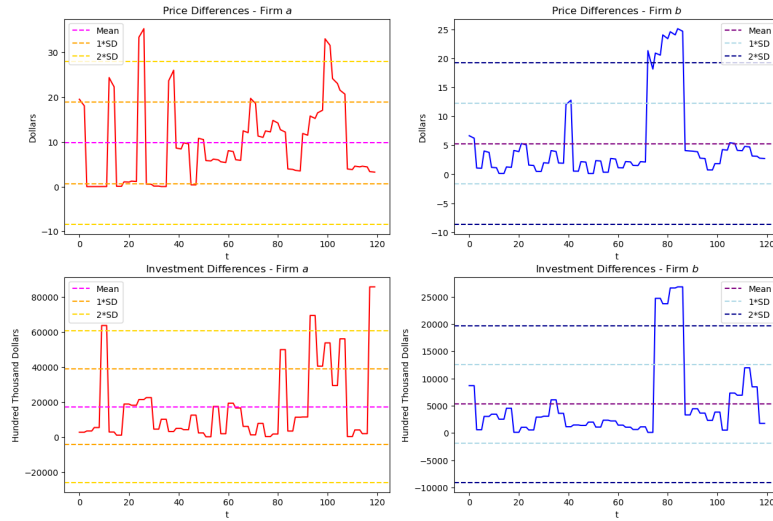


Figure 3.12: Differences in the price and cost time series.

From the real data, we find that among periods, the firms' adjustment steps for prices and investments vary. On average, the price adjustment of firm *a* (AT&T) is larger than that of firm *b* (T-Mobile), and at the same time, the investment

adjustment of firm a is also larger as well. Therefore, in the further calibration step, we refer to values which present this asymmetry as well.

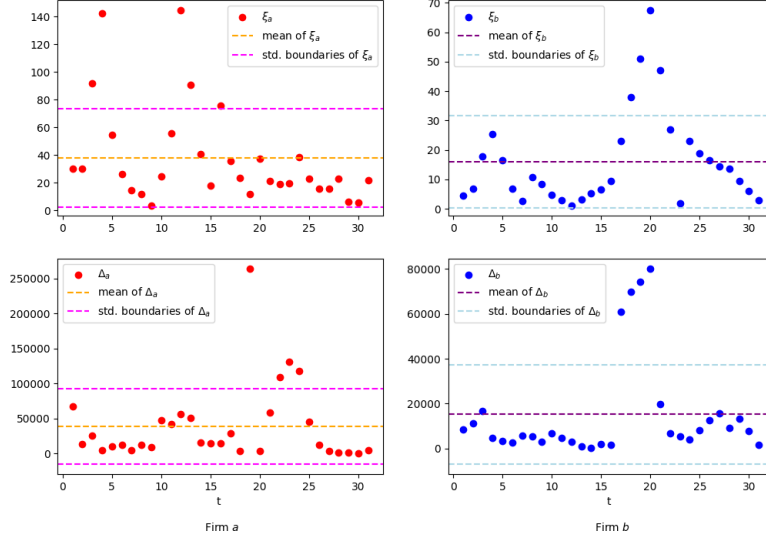


Figure 3.13: Updating steps estimation based on real data.

We consider the following parameters as the objective parameters:

- Parameters related to the observed functionality and congestion effect: $\beta_2, \beta_0, \beta_3, A_a$
- Consumer switching probability p_{sw}
- Parameters related to the consumer network G : k_G, p_G
- Parameters related to the price updates: ξ_a, ξ_b
- Parameters related to the investment updates: $\tau_a, \tau_b, \vartheta_a, \vartheta_b, \eta_a, \eta_b$

These parameters define the core mechanisms of the model, governing the formation of observed product functionality, the dispersion of information among consumers, and firms' decision-making in pricing and investment.

In total, we have 26 fixed parameters and 21 objective parameters for our ABM. As shown in Table 3.3 and Table 3.4, the fixed parameters are set based on real data or theoretical considerations, while the objective parameters are calibrated to match the observed market behavior.²⁰

²⁰Some parameters are set as “fixed”, because they do not have a direct empirical counterpart in the available data and are not separately identifiable in the present specification, for instance, β_a^C and β_b^C . These coefficients are not themselves parameters of substantive interest.

Parameter	Value	Description	Setting / Source
β^D	0.58	Depreciation rate (in firm capacity)	Real data
β_a^C	2	Investment effectiveness of firm a	Fixed
β_b^C	2	Investment effectiveness of firm b	Fixed
$\bar{u}_{a,low}$	700	Consumers' baseline utility for firm a (Lower bound)	Fixed
$\bar{u}_{a,high}$	1200	Consumers' baseline utility for firm a (Upper bound)	Fixed
$\bar{u}_{b,low}$	600	Consumers' baseline utility for firm b (Lower bound)	Fixed
$\bar{u}_{b,high}$	700	Consumers' baseline utility for firm b (Upper bound)	Fixed
ψ_a	656.144	Exploration amplitude for firm a	Estimated based on real data adjustments
ψ_b	656.144	Exploration amplitude for firm b	Estimated based on real data adjustments
A_b	1	Technological level of firm b	Fixed
w_R	600	Reservation utility threshold	Fixed
n_{ini}	46	Initially informed consumers	Fixed (1% of n)
n	4698	Total consumers (network nodes)	Estimated based on logistic growth
S_a	10	Initial physical capacity of firm a	Fixed
S_b	10	Initial physical capacity of firm b	Fixed
$c_{0,a}^{ini}$	19664.922	Initial investment of firm a	Real data
$c_{0,b}^{ini}$	2868.908	Initial investment of firm b	Real data
$c_{res,a}$	15.667	Marginal cost of firm a	Estimated based on real data
$c_{res,b}$	15.456	Marginal cost of firm b	Estimated based on real data
p_a^{ini}	44.315	Initial price of firm a 's product	Real data
p_b^{ini}	20	Initial price of firm b 's product	Real data
n_s	469	Sample size for market survey	Fixed (10% of n)
ι_a	0.01	Small price adjustment of firm a	Fixed
ι_b	0.01	Small price adjustment of firm b	Fixed
$a\beta_6$	-100	Lower truncation bound for consumer's network sensitivity	Fixed
$b\beta_6$	100	Upper truncation bound for consumer's network sensitivity	Fixed

Table 3.3: Fixed parameters and their descriptions.

Parameter	Range	Description
β_2	[1, 20]	Functionality sensitivity on service capacity
β_0	[0, 2]	Functionality sensitivity on consumer congestion
β_3	[1990, 2100]	Functionality sensitivity intercept
p_{sw}	[0.0001, 0.1]	Consumer switching probability
ϑ_a	[0.0001, 0.1]	Investment update threshold of firm a
ϑ_b	[0.0001, 0.1]	Investment update threshold of firm b
η_a	[0, 2,000]	Investment step of firm a
η_b	[0, 2,000]	Investment step of firm b
A_a	[1, 1.3]	Technological level of firm a
k	[2, 5]	Number of nearest neighbors each consumer is initially connected to on each side
p	[0.0001, 0.01]	Probability of rewiring each consumer connection
τ_a	[2, 36]	Number of periods between firm a 's two consecutive investment updates
τ_b	[2, 36]	Number of periods between firm b 's two consecutive investment updates
ξ_a	[0, 1]	Price adjustment step of firm a
ξ_b	[0, 1]	Price adjustment step of firm b
b_{β_5}	[30, 50]	Upper truncation bound of consumer's price sensitivity
μ_{β_5}	[0, 50]	Mean of distribution of consumer's price sensitivity
σ_{β_5}	[1, 150]	Standard deviation of distribution of consumer's price sensitivity
μ_{β_6}	[50, 100]	Mean of distribution of consumer's network sensitivity
σ_{β_6}	[1, 10]	Standard deviation of distribution of consumer's network sensitivity
p_{buy}	[0, 1]	Purchasing probability when utility is satisfied

Table 3.4: Objective parameters and descriptions

3.5.2 Estimation setup

Due to the large scale of the real-world data, we apply a linear transformation to mitigate potential measurement distortions arising from scale discrepancies. Each time series is rescaled by dividing it by an appropriate scaling factor. The specific values and justifications for these scaling factors are presented below.

To improve computational efficiency in parameter estimation, especially given the

Variable	Scaling Factor	Justification
Price	26.15	Ratio of the initial real-world price to the initial model-defined price.
Demand	100	Scaled to reduce the number of nodes in the consumer network while preserving generality.
Revenue	2.615	Scaled proportionally to ensure consistency with scaled price and demand.
Cost	2.615	Aligned with revenue scaling to maintain proportionality in cost structure.
Profit	2.615	Scaled in accordance with revenue and cost to preserve profit comparability.

Table 3.5: Scaling factors applied to key economic variables.

wide parameter space and high dimensionality, we combine parallelization with sampling and batching techniques. Using the Python package *ray*, we initiate a local cluster where parallelizable tasks (marked with `@ray.remote`) are distributed across multiple CPU cores, which can significantly accelerate simulation runs (Alsuwaiyel 2022). Moreover, we apply Latin Hypercube Sampling (LHS) and batch processing to efficiently explore the parameter space. These techniques rely on orthogonal arrays to ensure representative coverage for computer experiments and numerical integration (Tang 1993, Seaholm et al. 1988). In the SMM calibration step, we generate 100 parameter samples and run simulations using 30 random seeds per sample. Practically, this setup runs efficiently on platforms such as *Google Colab Pro*, where, for instance, estimating five samples with a batch size of 10 takes less than one minute using High RAM (v28).²¹

3.5.3 Methodology

SMM

For SMM, we redesign the algorithm to include more complexity (aggregate number of consumers, heuristics, etc.). The specific steps are shown as follows:

1. **Specification of the Simulation and Moment Structure**

Let $\theta \in \Theta \subset \mathbb{R}^d$ denote the vector of structural parameters to be estimated. Define the set of empirical target moments $m^{\text{real}} \in \mathbb{R}^k$, and corresponding simulated moments $m^{\text{sim}}(\theta)$ derived from model output. A diagonal weight

²¹For instance, estimating five samples with one random seed and a batch size of 10 takes less than one minute using High RAM, v28 on *Google Colab Pro*.

matrix

$$\Omega = \text{diag}(\omega_1, \dots, \omega_k)$$

is introduced to assign relative importance to each moment component.

2. Construction of Candidate Parameter Sets via Latin Hypercube Sampling (LHS)

A set of N candidate parameter vectors $\{\theta^{(1)}, \dots, \theta^{(N)}\}$ is drawn using Latin Hypercube Sampling over the admissible space Θ , ensuring stratified coverage of each dimension and improved space-filling efficiency relative to purely random sampling.

3. Monte Carlo Simulation of Model Moments

For each candidate vector $\theta^{(i)}$, the model is simulated S times using independent random seeds to account for stochasticity. Let $m_s^{\text{sim}}(\theta^{(i)})$ be the simulated moment vector for run s . The mean simulated moment is calculated as:

$$\bar{m}^{\text{sim}}(\theta^{(i)}) = \frac{1}{S} \sum_{s=1}^S m_s^{\text{sim}}(\theta^{(i)})$$

4. Computation of the Loss Function

For each candidate $\theta^{(i)}$, the discrepancy between simulated and empirical moments is evaluated via a weighted quadratic loss function:

$$L(\theta^{(i)}) = (\bar{m}^{\text{sim}}(\theta^{(i)}) - m^{\text{real}})^\top \Omega (\bar{m}^{\text{sim}}(\theta^{(i)}) - m^{\text{real}})$$

This procedure yields a scalar loss value summarizing the moment mismatch for each parameter set.

5. Selection of Optimal Parameter Sets

All candidate vectors are ranked according to their associated loss values $L(\theta^{(i)})$. The top-performing sets, i.e., those that minimize the loss, are retained and stored for further analysis and out-of-sample validation.

Since we have a large set of model parameters, we can classify all parameters into two categories: (a) predefined parameters based on justification from relevant literature, empirical data, public reports, or logical reasoning, and (b) objective parameters that are to be estimated.

To account for the initial discrepancy between the empirical time series and the model setup, particularly the fact that the model assumes zero initially informed consumers, whereas the empirical data indicates otherwise, we examine the evolution of the loss values across different burn-in periods. Based on the results illustrated in Figure 3.14, we select a burn-in period of 50. Given that the full

time series contains 121 observations, this choice ensures sufficient data remains for calibration without compromising estimation accuracy.

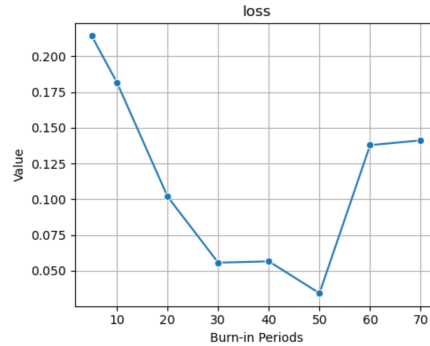


Figure 3.14: Number of burn-in periods and the minimum losses.

The model calibration workflow is shown in Figure 3.15.

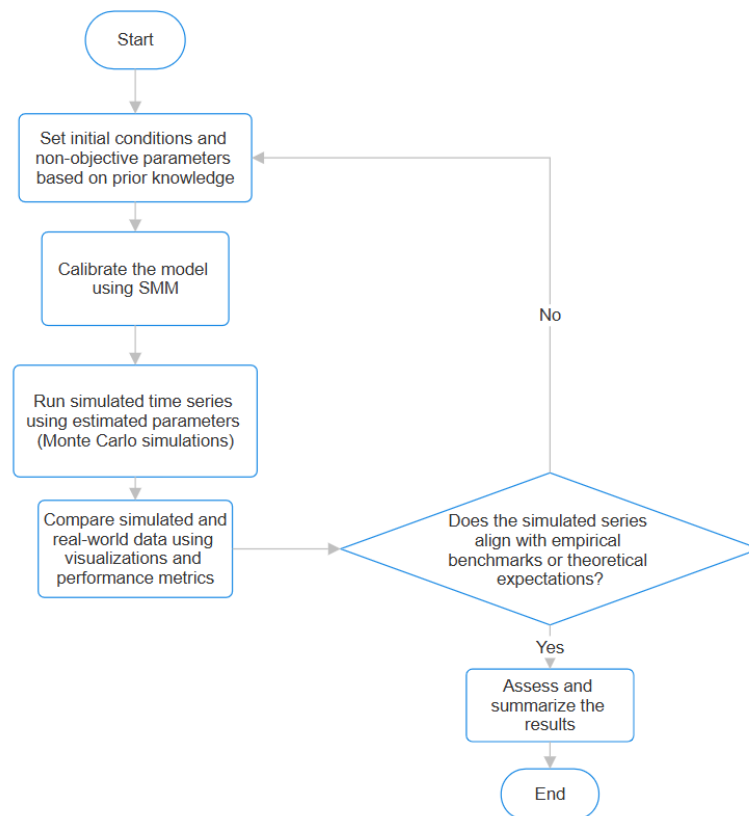


Figure 3.15: Model calibration workflow.

We select the statistical and economic moments from both the real-world data and the simulated data. The goal is to match the time series with minimal loss. From the statistical side, we consider the rolling mean, the rolling variance, and the trend of economic variables as the moments. From the economic side, research related to model calibration also introduces economic moments, focusing on economic interpretability (Keane 2010, Adam et al. 2016). Thus, we include the following economic moments:

- Arc price elasticity: $(\frac{m_{i,t+1}-m_{i,t}}{\frac{1}{2}(m_{i,t+1}+m_{i,t})})/(\frac{p_{i,t+1}-p_{i,t}}{\frac{1}{2}(p_{i,t+1}+p_{i,t})})$
- Arc cost elasticity: $(\frac{m_{i,t+1}-m_{i,t}}{\frac{1}{2}(m_{i,t+1}+m_{i,t})})/(\frac{c_{0,i,t+1}-c_{0,i,t}}{\frac{1}{2}(c_{0,i,t+1}+c_{0,i,t})})$
- Cost to revenue: $\frac{c_{0,i,t}}{R_{i,t}}$
- Market share: $\frac{R_{i,t}}{R_{i,t}+R_{-i,t}}$
- Correlations and autocorrelations

Following the traditional SMM approach, we take the mean and standard deviation of the statistical and economic moments that we have considered, and further select the moments which do not contain anomaly values (for example, infinity or not defined). In total we have 26 moments for our calibration process.

NN Surrogate

Throughout the SMM approach, we observe that the direct calibration remains challenging because of two reasons: (a) it is difficult to jointly match the trend and magnitude of all key economic variables, and (b) running the ABM repeatedly is computationally expensive. To overcome these issues, we also propose a neural network surrogate approach. This approach can approximate the ABM-generated realizations and allow efficient re-use of simulation data, and can enable gradient-based parameter search eg. Adam optimizer and out-of-sample validation.

In the surrogate-based calibration, a neural network (NN) is trained to approximate the mapping from model parameters to calibration quality. The input of the network is a 21-dimensional parameter vector representing the structural parameters of the agent-based model, and the output is a single scalar value, denoted as $\text{NRMSE}_{\text{avg}}$. In other words, the network learns a function

$$f : \mathbb{R}^{21} \rightarrow \mathbb{R},$$

which maps each parameter configuration to a score that measures the overall fit quality between simulated and empirical data. The target variable $\text{NRMSE}_{\text{avg}}$

is defined as the arithmetic mean of the Normalized Root Mean Squared Error (NRMSE) computed across four key economic variables: prices, investments, number of consumers, and profits.

Let Y_t^R denote the real (target) time series for a given variable. We define the “mean time series” across all random seeds as

$$Y_t^S = \frac{1}{\Omega} \sum_{\omega} Y_{\omega,t}^S$$

where ω is the random seed order and Ω is the number of random seeds. Thus, Y_t^S represents the corresponding simulated series generated by the model.²² Let T be the total length of the series and b a non-negative integer representing the burn-in period. The Normalized Root Mean Squared Error (NRMSE) is computed after discarding the first b observations as follows.

- **Slice after burn-in:**

$$t = b, b + 1, \dots, T - 1$$

- **Mean Squared Error (MSE):**

$$\text{MSE} = \frac{1}{T - b} \sum_{t=b}^{T-1} (Y_t^S - Y_t^R)^2$$

- **Root Mean Squared Error (RMSE):**

$$\text{RMSE} = \sqrt{\text{MSE}}$$

- **Normalization:** Define the range of the real series over the same time window as

$$\text{range}_R = \max_{t \in [b, T-1]} Y_t^R - \min_{t \in [b, T-1]} Y_t^R.$$

- **Normalized RMSE:**

$$\text{NRMSE} = \frac{\text{RMSE}}{\text{range}_R}.$$

Finally, the overall objective used for surrogate training is defined as the simple arithmetic mean of the NRMSE values across the four economic variables (prices, investments, number of consumers, and profits).²³

²²Another metric for the calibration label can be constructed also from the distribution of single-run outcomes. We show the results in the parameter optimization step.

²³The label $\text{NRMSE}_{\text{avg}}$ measures the discrepancy between the empirical data and the simulated mean time series. It differs from the internal training of the model or validation loss, which is defined as $\text{MSE}_{\text{scaled}} = \frac{1}{n} \sum_i (\hat{y}_i^{\text{scaled}} - y_i^{\text{scaled}})^2$ and computed on normalized targets during surrogate learning. A smaller $\text{NRMSE}_{\text{avg}}$ suggests a better fit to the target data across the economic variables.

$$\text{NRMSE}_{\text{avg}} = \frac{1}{8} \sum_i (\text{NRMSE}_{p_i} + \text{NRMSE}_{c_{0,i}} + \text{NRMSE}_{m_i} + \text{NRMSE}_{\Pi_i})$$

where $i \in \{a, b\}$.

The surrogate training and validation process was conducted using the high-performance computing (HPC) clusters at Paderborn University, where the ABM was executed for 40,000 single runs to generate a sufficiently rich dataset of parameter-output pairs. For numerical stability and comparability across variables, all inputs and outputs were normalized prior to training: model parameters were standardized by subtracting their empirical means and dividing by their standard deviations, while the target variable $\text{NRMSE}_{\text{avg}}$ was scaled to the range $[0, 1]$ using min-max normalization. The full pipeline proceeds as follows:

1. Generate 40,000 simulation samples.
2. Apply advanced oversampling strategies (binary, gradient, and exponential) to improve the density of low-error regions.
3. Define the parameter ranges.
4. Resample the data based on the distribution of the label.
5. Perform data preprocessing and normalization.
6. Train the improved surrogate model on the processed data.
7. Conduct local optimization within fixed parameter bounds to identify parameter configurations minimizing $\text{NRMSE}_{\text{avg}}$.
8. Extract the optimized parameter set from the surrogate predictions.
9. Validate the optimization results using the same surrogate model.
10. Perform detailed error analysis to assess prediction accuracy and calibration robustness.

Despite the extensive simulation runs, the distribution of the target variable remained highly skewed and heavy-tailed, indicating a persistent imbalance in the dataset. In particular, only a small subset of parameter configurations produced high-quality fits (low $\text{NRMSE}_{\text{avg}}$ values), while the majority corresponded to poor matches. This imbalance may bias the surrogate model toward over-represented regions of the parameter space and limit its predictive accuracy in low-error areas. To mitigate this issue, the dataset was oversampled with a focus on small-label observations using binary, gradient-based, and exponential oversampling strategies.

This procedure effectively increased the representation of high-quality samples and improved the surrogate’s ability to approximate the response surface around optimal regions.

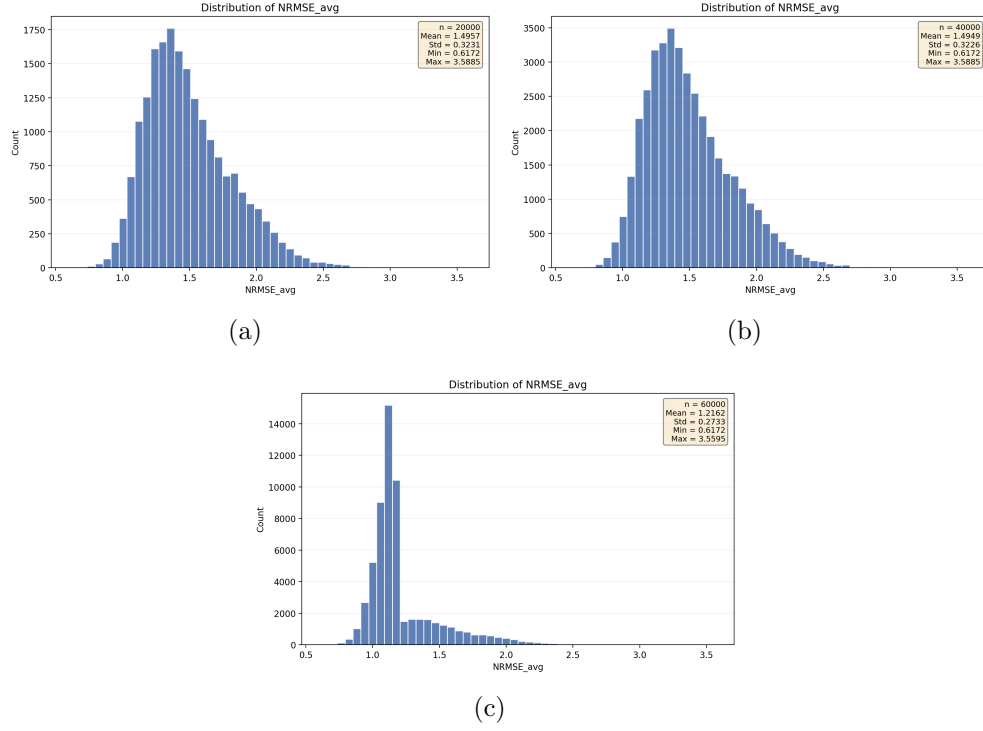


Figure 3.16: Histogram of label distribution based on (a) 20k samples, (b) 40k samples, and (c) 60k samples including oversampling.

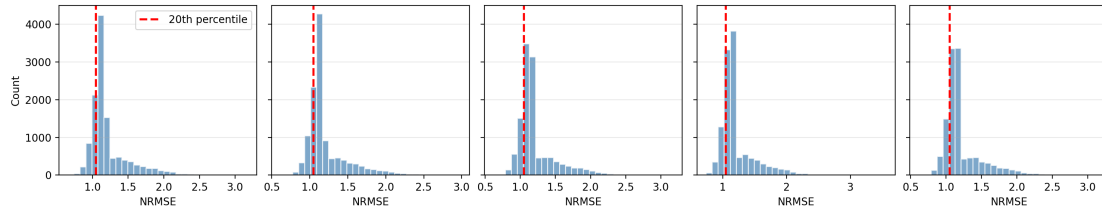


Figure 3.17: Label distribution within the five-fold cross validation.

The final stage of the calibration process involves a local optimization loop designed to identify the parameter vector β^* that minimizes the surrogate-predicted $\text{NRMSE}_{\text{avg}}$. The optimization is performed using the Adam algorithm, which iteratively updates parameter estimates based on adaptive learning rates and gradient

estimates. Model checkpoints are stored every 5,000 steps to monitor convergence and prevent overfitting. Each optimized candidate is subsequently validated through a separate evaluation of the original ABM, where the reported performance corresponds to the mean $\text{NRMSE}_{\text{avg}}$ over 30 random seeds.

To improve estimation quality, we define a benchmark configuration with simple baseline settings and evaluate multiple treatment configurations designed to enhance model fit. In the main experimental design, we generate simulation data from 1,000 parameter sets, each evaluated under 200 random seeds. Using these samples, we train multiple neural-network surrogate models and select the surrogate with the highest out-of-sample R^2 . The selected surrogate is then used to search for promising candidate parameter sets. We subsequently re-run the original ABM under these candidates, collect time series of key economic variables, and evaluate fit using the average normalized root mean squared error, $\text{NRMSE}_{\text{avg}}$. If a candidate yields a lower $\text{NRMSE}_{\text{avg}}$ than the current best parameterization, we update the benchmark to this new best candidate; otherwise, we retain the existing parameter setting.

3.5.4 Estimation and validation

Based on the transformed real data and the designed algorithm, we compare the real time series and the simulated mean time series of the key variables and show the parameter set that brings the minimum loss. The figures indicate that the parameter estimates obtained from the SMM-based calibration yield the weakest overall fit among the evaluated methods. In contrast, the parameter configurations corresponding to the 20k- and 50k-checkpoint surrogate models produce results that closely resemble the within-sample outcomes.²⁴ Although some discrepancies remain between the real and simulated price trajectories, the parameters derived from the within-sample calibration achieve a relatively good match in terms of simulated investments, market demand, and profits. Consequently, the subsequent economic interpretation and discussion are primarily based on the within-sample results.

²⁴20k- and 50k-checkpoints represent the final estimated parameters through 20,000 and 50,000 learning steps based on the NN-surrogate.

Parameter	SMM	Within-sample	20k-checkpoint	50k-checkpoint
β_2	12.690	6.393	1.000	1.000
β_0	1.494	0.366	0.334	0.334
β_3	2025.821	2020.98	2002.22	1990.85
A_a	1.211	1.022	1.000	1.000
p_{sw}	0.075	0.0815	0.0787	0.0807
b_{β_5}	39.638	32.01	36.87	36.10
μ_{β_5}	14.268	37.92	49.29	49.94
σ_{β_5}	139.299	139.18	134.07	129.44
μ_{β_4}	93.982	63.96	60.80	60.68
σ_{β_4}	4.945	3.207	1.536	1.338
ϑ_a	0.013	0.0839	0.0525	0.0555
ϑ_b	0.025	0.0936	0.1000	0.1000
p	0.0017	0.0076	0.0100	0.0100
ξ_a	0.8832	0.4090	0.5755	0.5583
ξ_b	0.5186	0.3983	0.2871	0.2806
η_a	330.24	1580.03	1594.95	1615.41
η_b	451.31	785.32	778.69	796.76
p_{buy}	0.8098	0.7366	0.7857	0.7807
k	2	4	5	5
τ_a	9	12	9	9
τ_b	2	16	15	14

Table 3.6: Comparison of estimated parameter vectors between best estimate of SMM, best within-sample fit and surrogate optimization checkpoints.

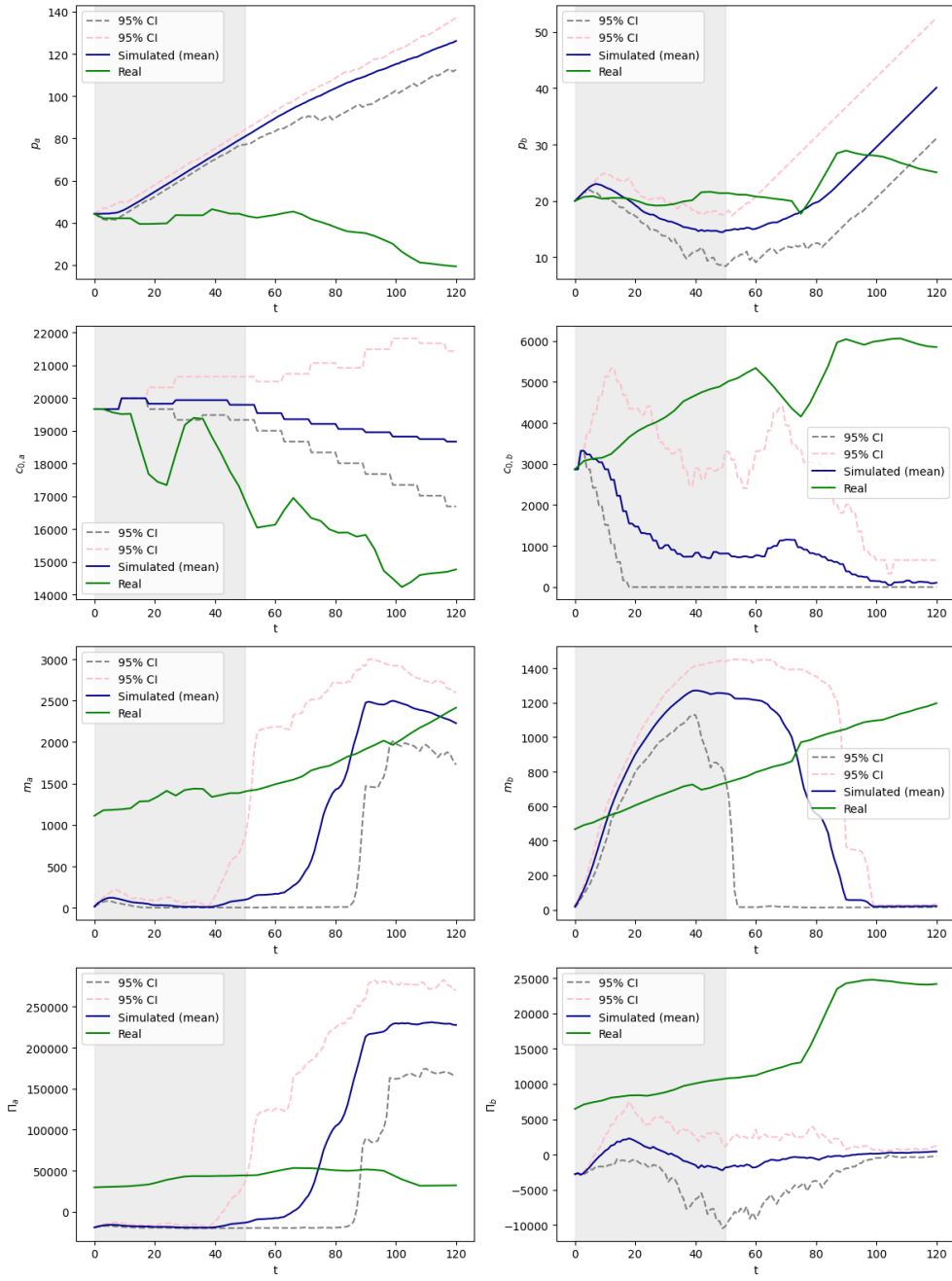


Figure 3.18: Real and simulated time series with best-fit parameters based on SMM.

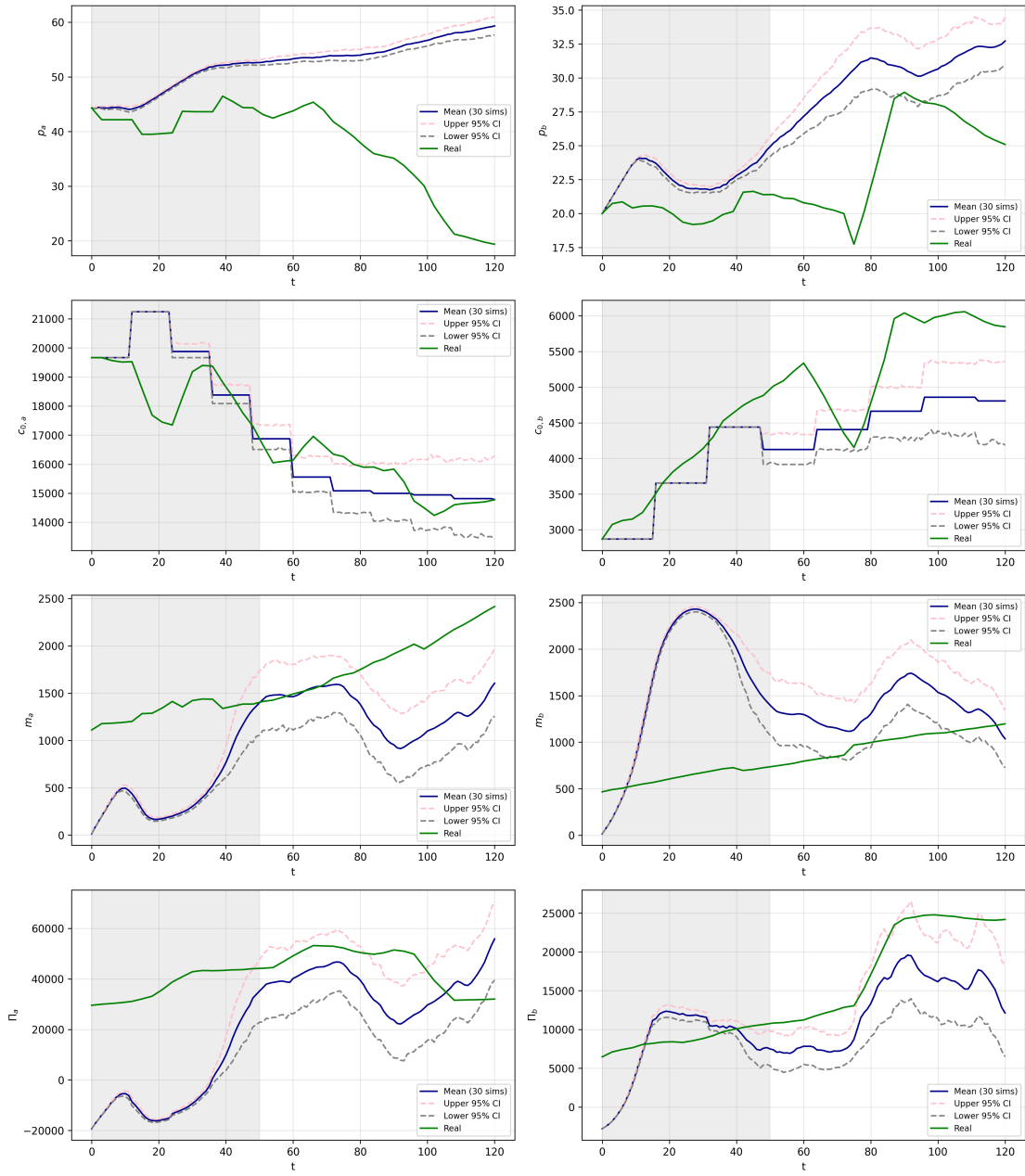


Figure 3.19: Real and simulated time series with best-fit parameters within the samples.



Figure 3.20: Real and simulated time series with parameters at the 20k-step checkpoint.

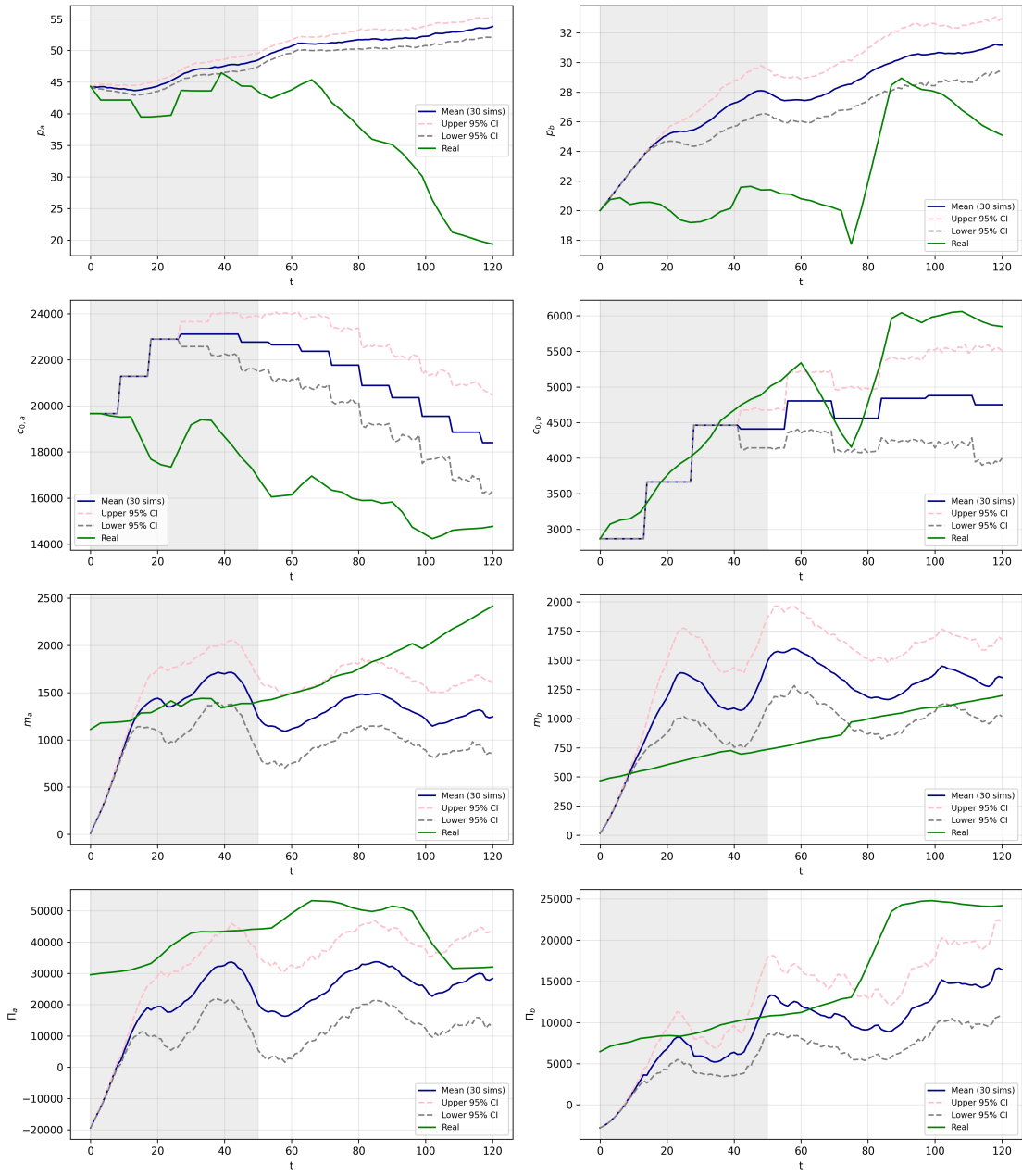


Figure 3.21: Real and simulated time series with parameters at the 50k-step checkpoint.

The estimated parameter values reveal several key insights into the market dynamics represented by the model. First, consumer congestion effects, captured by β_0 , are non-negligible, implying that excessive service usage negatively affects perceived functionality. Second, differences in technological levels (A_a) do not appear to dominate the competitive landscape, suggesting that both firms operate with comparable technological efficiency. Third, the consumer switching probability (p_{sw}) indicates a strong lock-in effect, meaning that consumers tend to remain with their current providers once network ties are established. Moreover, the large dispersion in β_5 reflects substantial heterogeneity in consumer price sensitivity, while the strongly positive network externality parameter (β_4) confirms that consumer utility increases with the adoption of others. The estimated connectivity parameters further suggest that the underlying consumer network is relatively sparse, limiting the speed of information diffusion. Finally, firm a , relative to firm b , is characterized by a lower investment update threshold, a larger price adjustment step, a higher learning rate in investment, and more frequent adaptation of its investment levels—indicating a more aggressive and responsive adjustment strategy.

The correlation analysis further indicates that the model behavior is not dominated by any single parameter; rather, it emerges from the joint influence of several key parameters. A closer interpretation of the results reveals that the contrasting market outcomes are not primarily driven by technological asymmetries between the competing firms. Instead, the observed differences originate from variations in strategic behavior and adjustment dynamics. Firm-specific decision rules, particularly those governing investment and pricing updates, play a more decisive role in shaping long-run performance than static technological advantages. In other words, even when firms possess comparable technological capabilities, they can exhibit markedly different trajectories depending on how they learn from and react to market feedback. Overall, the fit quality of the model appears most sensitive to behavioral and structural parameters, such as congestion and price adjustment mechanisms, rather than to technological disparities.

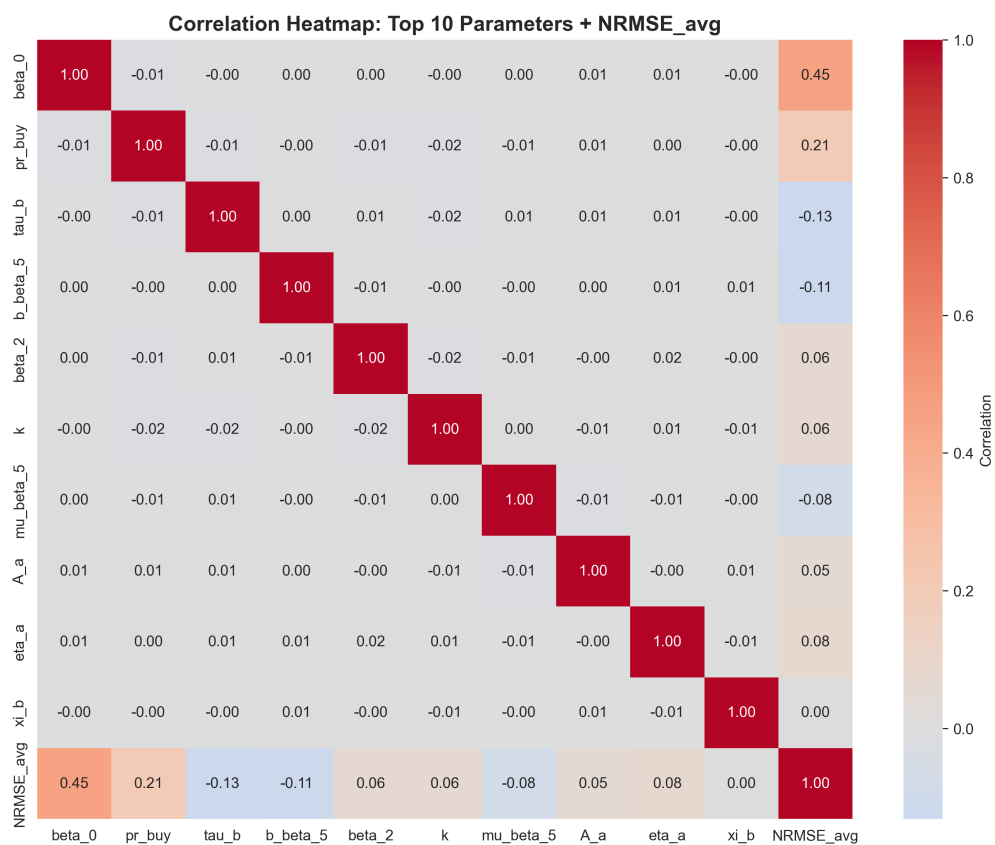


Figure 3.22: Correlations between the label and the features.

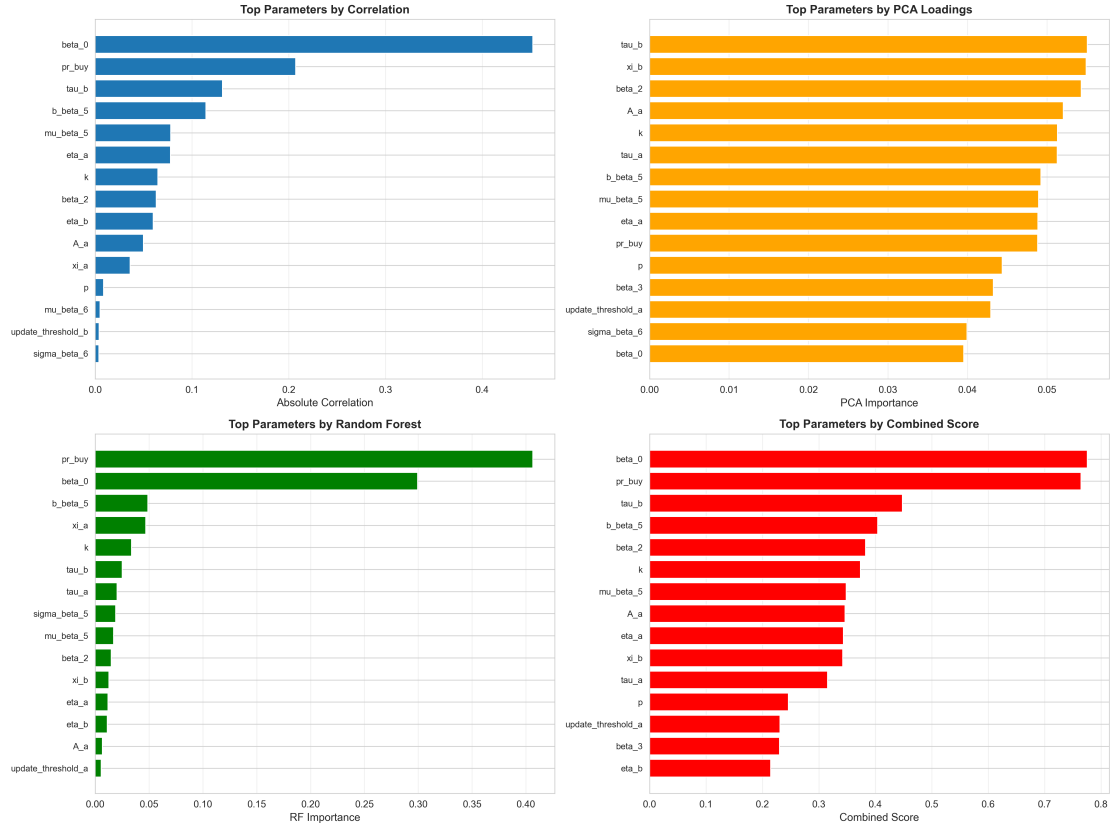


Figure 3.23: PCA feature importances.

3.5.5 Parameter optimization

To address the high-dimensionality challenge in surrogate-based optimization, we restrict attention to the ten most relevant parameters, identified by the combined feature-importance score reported in Figure 21, and hold the remaining eleven parameters fixed at their benchmark values. Using the Paderborn HPC infrastructure, we then collect more than 10,000 parameter samples, each evaluated under 200 random seeds to stabilize the stochastic simulation outcomes. Based on this expanded dataset, we specify a benchmark neural-network surrogate with parsimonious architecture and train multiple alternative (treatment) network configurations to improve predictive fit and overall calibration performance.

So far, the results based on the “mean time series” approach can mimic the trend of real profits, yet they do not fully capture the variance observed among individual simulation runs. To address this limitation, we define an alternative metric based on the normalized root mean squared error (NRMSE) between the real and

simulated time series. Here, we compute the loss for each simulation run and then average across runs. Specifically, we define the MSE of the simulated time series under random seed ω as:

$$\text{MSE}_\omega = \frac{1}{T-b} \sum_{t=b}^{T-1} (Y_{\omega,t}^S - Y_t^R)^2$$

Thus, we have:

$$\begin{aligned} \text{RMSE}_\omega &= \sqrt{\text{MSE}_\omega} \\ \text{range}_R &= \max_{t \in [b, T-1]} Y_t^R - \min_{t \in [b, T-1]} Y_t^R \\ \text{NRMSE}_\omega &= \frac{\text{RMSE}_\omega}{\text{range}_R} \\ \text{NRMSE}_{\text{avg}} &= \frac{1}{8} \sum_i (\text{NRMSE}_{p_i} + \text{NRMSE}_{c_{0,i}} + \text{NRMSE}_{m_i} + \text{NRMSE}_{\Pi_i}) \\ \overline{\text{NRMSE}_{\text{avg}}} &= \frac{1}{\Omega} \sum_{\omega} \text{NRMSE}_\omega \end{aligned}$$

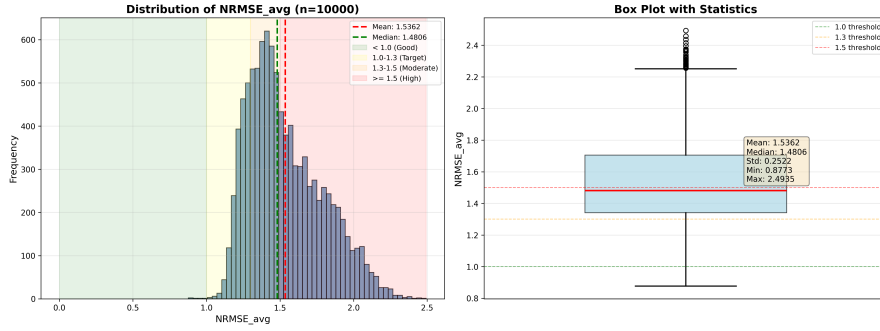


Figure 3.24: Distribution of the label (NRMSE) of the new collected samples.

Experiment	Key Modification	Total R^2	Low-region R^2	Total MAE	Low MAE	Time (s)
Benchmark	Weighted MSE ($p=3, w=10$)	0.85	0.49	0.0722	0.0532	120
Treatment1	Higher LR ($10^{-4} \rightarrow 5 \times 10^{-4}$)	0.94	0.77	0.0458	0.0350	133
Treatment2	Stronger base weight ($w=20$)	0.85	0.49	0.0722	0.0532	126
Treatment3	Unweighted MSE	0.85	0.28	0.0737	0.0639	80
Treatment4	LR= 5×10^{-4} , $w=20$	0.94	0.77	0.0459	0.0350	82
Treatment5	$p=5.0, w=20$	0.84	0.56	0.0736	0.0489	87

Table 3.7: Neural Network surrogates and the metrics.

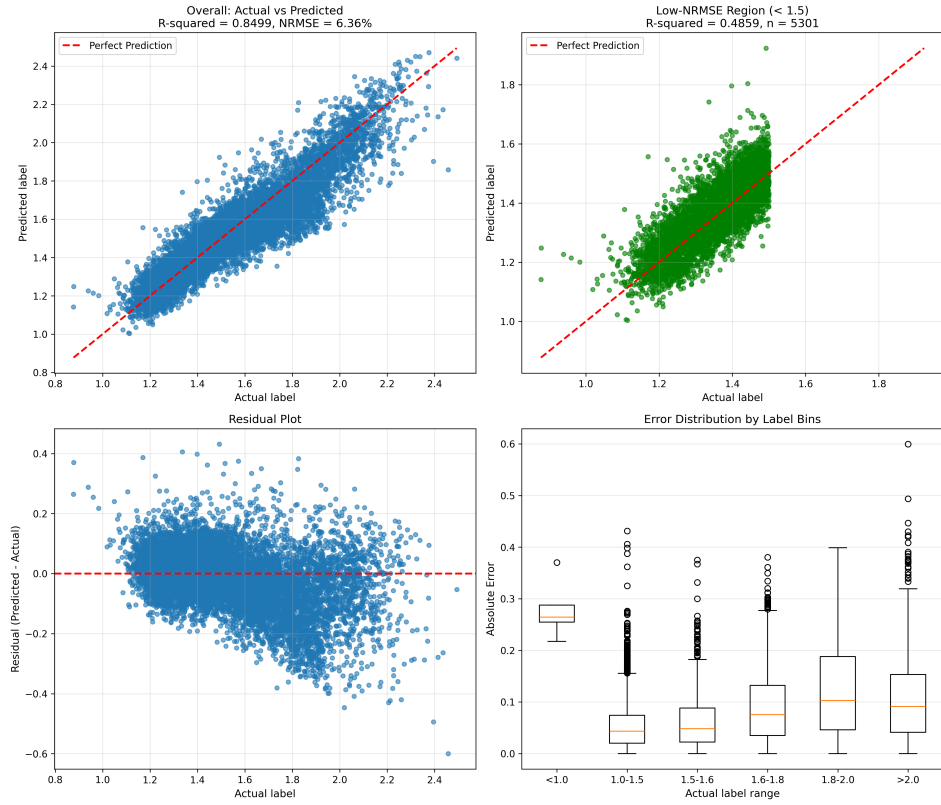


Figure 3.25: Performance of Benchmark NN Surrogate.

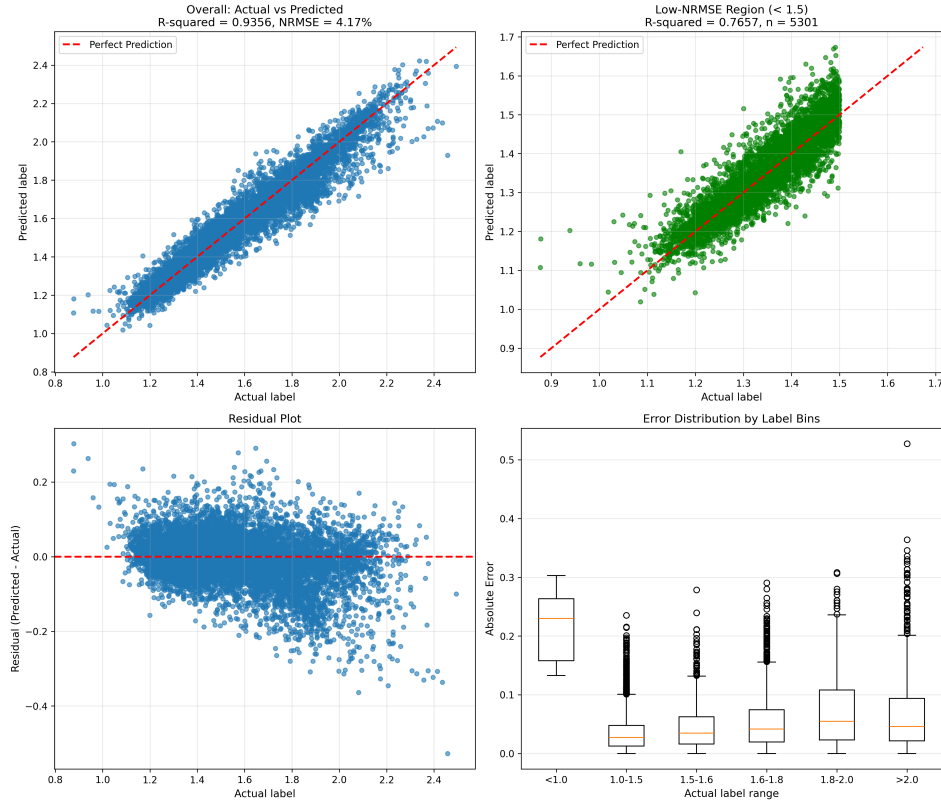


Figure 3.26: Performance of Treatment NN Surrogate (Treatment1).

Further experiments demonstrate that most selected calibrations can at least partially capture the dynamics of real duopolistic competition. We experimented with two NRMSE-based metrics and ultimately adopted the mean NRMSE across individual simulation runs ($\overline{\text{NRMSE}}_{\text{avg}}$), as this approach better reflects the typical behavior of the model at the level of individual realizations under stochastic dynamics. Among all tested configurations, the best-fit parameters identified within the sample (based on the mean NRMSE of single runs, using all 21 parameters) exhibit the most stable fit, as evidenced by the narrowest confidence intervals and the closest alignment with the real profit time series. Accordingly, we use these parameter values to interpret the results and conduct policy analysis in the following subsection.

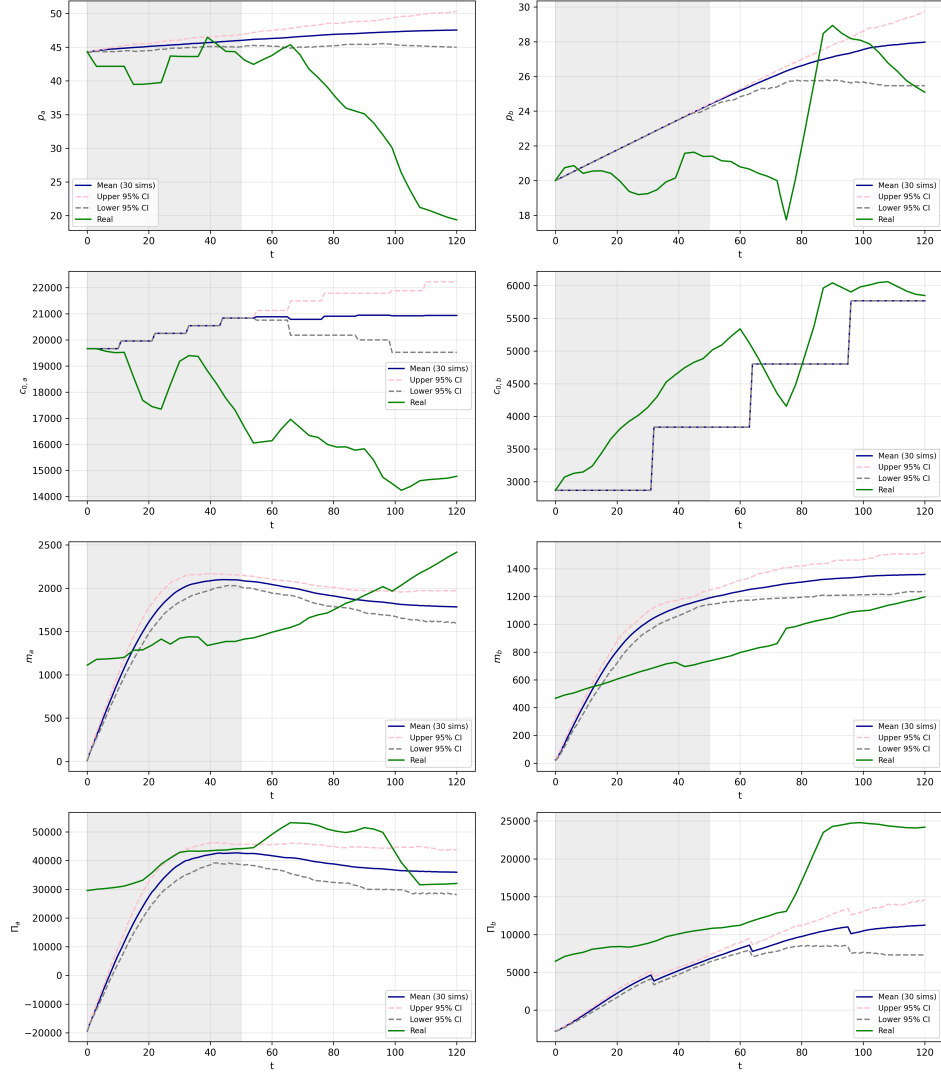


Figure 3.27: Results with best-fit parameters within the samples (based on the mean NRMSE of single runs, 21 parameters).

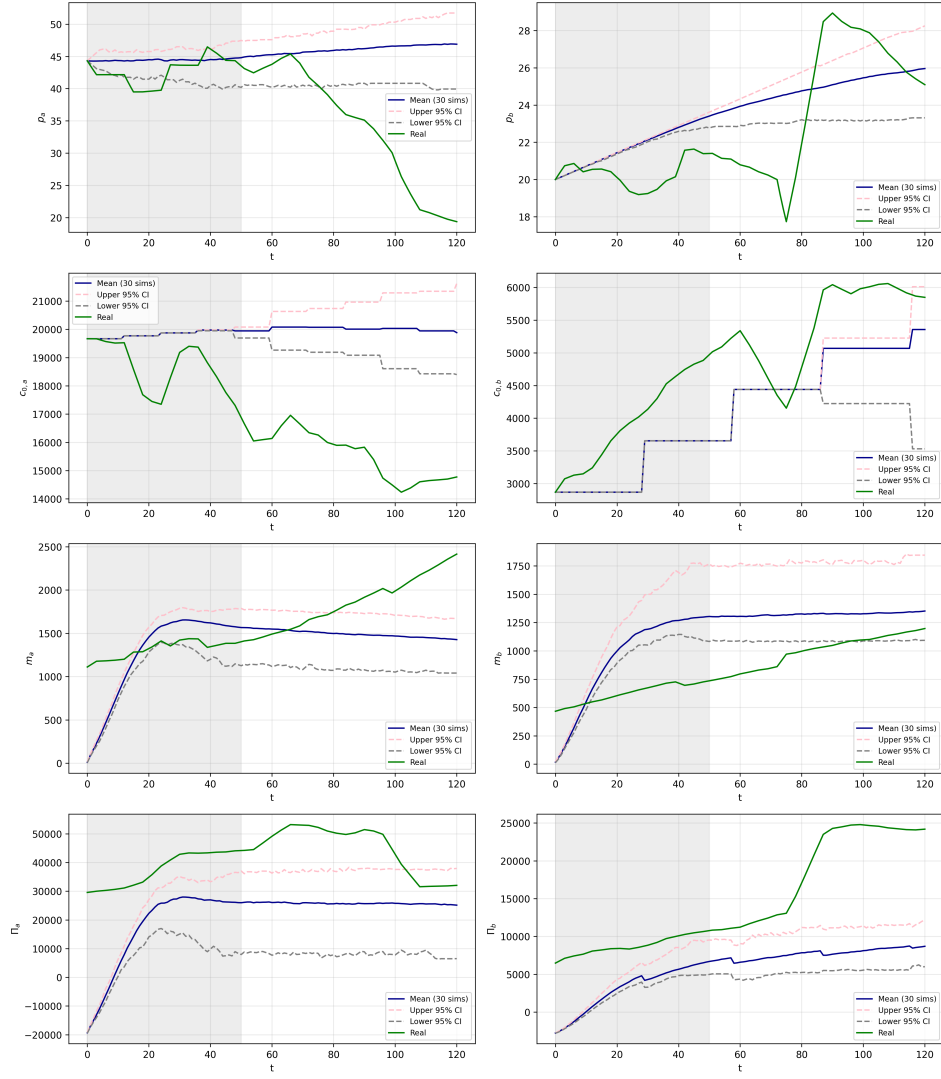


Figure 3.28: Results with best-fit parameters within the samples (based on the mean NRMSE of single runs, 10 parameters).

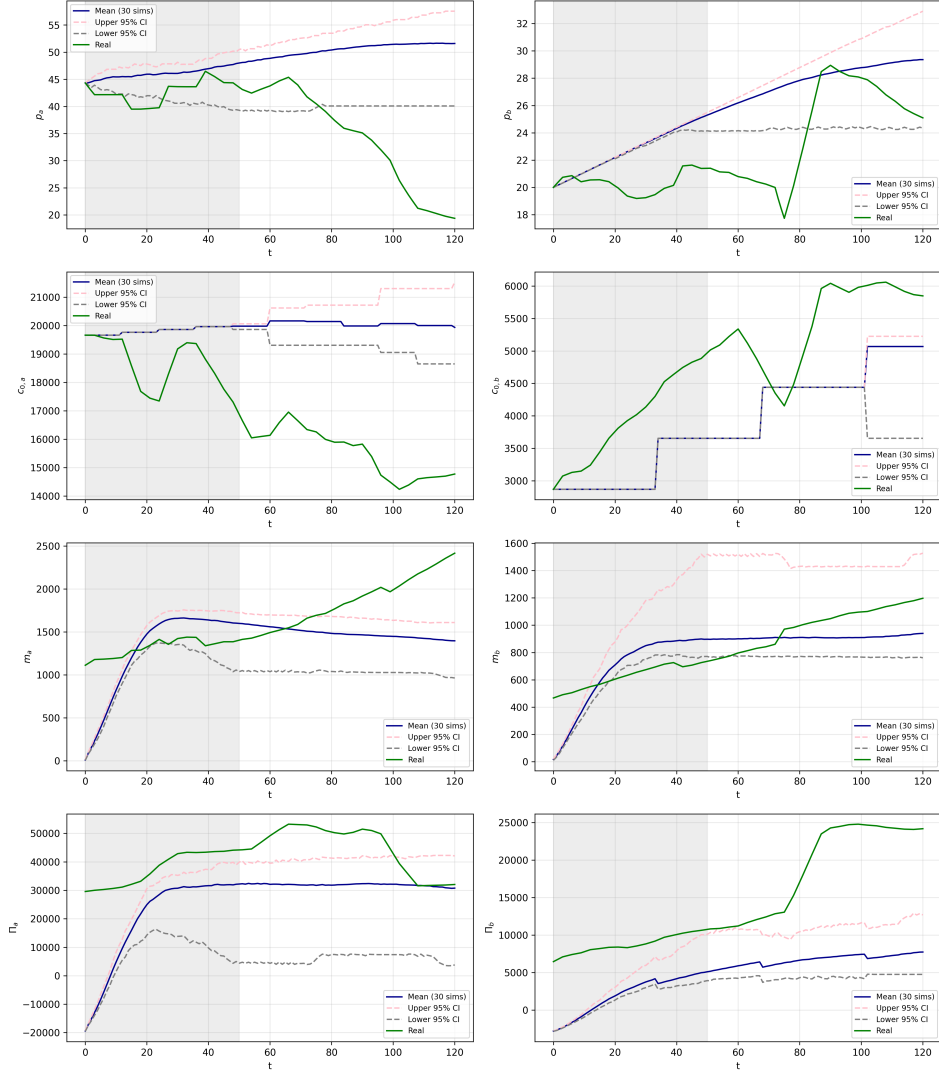


Figure 3.29: Results with best-fit parameters selected by the NN Surrogate (based on the mean NRMSE of single runs, 10 parameters).

Parameter	Within-sample (21 params)	Within-sample (21 params)	Within-sample (10 params)	NN Surrogate (10 params)
Metric	NRMSE _{avg}	NRMSE _{avg}	NRMSE _{avg}	NRMSE _{avg}
β_2	6.393	1.346	5.483	2.856
β_0	0.366	0.044	0.109	0.100
β_3	2020.98	2029.65	2020.98	2020.98
A_a	1.022	1.296	1.219	1.157
p_{sw}	0.0815	0.0391	0.0518	0.0720
b_{β_5}	32.01	49.31	44.26	48.00
μ_{β_5}	37.92	32.51	15.26	24.16
σ_{β_5}	139.18	141.09	139.18	139.18
μ_{β_6}	63.96	78.16	63.96	63.96
σ_{β_6}	3.207	9.179	3.207	3.207
ϑ_a	0.0839	0.0493	0.0839	0.0839
ϑ_b	0.0936	0.0739	0.0936	0.0936
p	0.0076	0.0034	0.0076	0.0076
ξ_a	0.4090	0.0674	0.4090	0.4090
ξ_b	0.3983	0.0783	0.0627	0.100
η_a	1580.03	293.05	103.31	100.00
η_b	785.32	965.85	785.32	785.32
p_{buy}	0.7366	0.9592	0.7739	0.7146
k	4	4	4	4
τ_a	12	11	12	12
τ_b	16	32	29	34

Table 3.8: Comparison of within-sample parameter estimates and NN surrogate optimization results.

3.5.6 Policy analysis

Policy analysis is conducted using the best-fit parameter configuration obtained from the calibration stage as the benchmark. Within this framework, we implement one-dimensional (and pairwise) policy deviations by varying the four strategic controls of a focal firm—price adjustment step size ξ_i , investment update threshold ϑ_i , investment learning step η_i , and investment adjustment window τ_i —while holding the rival firm’s controls fixed at their benchmark values. This design isolates the marginal impact of alternative policy settings and allows us to evaluate the implied payoff consequences across policy combinations. The assessment of policy effectiveness relies on paired Wilcoxon signed-rank tests following the procedure introduced in Wang (2025). For the surrogate-based parameter optimization, we generate 2,000 candidate samples (1,000 with respect to each firm) and validate each configuration using 30 random seeds. Firm performance is summarized using

two profit-based metrics: (i) the average profit over the final 30 periods,

$$\bar{\Pi} = \frac{1}{30} \sum_{t=T-29}^T \Pi_t,$$

and (ii) discounted profit over the full horizon,

$$\Pi^D = \sum_{t=0}^T \delta^t \Pi_t, \quad \delta = 0.80.$$

Together, we evaluate overall policy performance by aggregating metric-level evidence across random seeds. For each policy and each metric, we compute the seed-by-seed difference between the treatment and benchmark outcomes as

$$d_s = \text{metric}_s^{\text{treat}} - \text{metric}_s^{\text{bench}}, \quad s = 1, \dots, S,$$

and summarize directional changes via the counts

$$n_+ = \#\{s : d_s > 0\}, \quad n_- = \#\{s : d_s < 0\}, \quad n_0 = \#\{s : d_s = 0\}.$$

We regard a policy as practically improving a given metric if a strict majority of seeds exhibits a positive change,²⁵

$$P(d_s > 0) \approx \frac{n_+}{S} > 0.5.$$

To assess statistical significance, we apply the paired Wilcoxon signed-rank test to the difference sample $\{d_s\}_{s=1}^S$ under the null hypothesis that the median difference equals zero. Using a significance level of $\alpha = 0.05$, we deem an effect statistically valid when $p < 0.05$. Accordingly, a policy is classified as “effective” for a given metric only if it satisfies both criteria simultaneously: (i) practical improvement, $\frac{n_+}{S} > 0.5$, and (ii) statistical support from the Wilcoxon test, $p < 0.05$.

We evaluate multiple predictive models for mapping parameters to simulated outcomes, including linear baselines (OLS, Ridge, and Lasso), a neural-network benchmark (MLP regressor), and nonlinear ensemble methods such as Random Forests and Gradient Boosting. Model hyperparameters are selected via cross-validation, and performance is assessed using predictive accuracy measured by R^2 together with stability across folds. Overall, the Gradient Boosting w.r.t. firm *a* ($R^2 = 0.9655$) and the Random Forest Regressor w.r.t. firm *b* ($R^2 = 0.9317$) delivers the strongest and most robust performance among the candidate specifications.

²⁵We use this “majority” (median-oriented) criterion to reduce sensitivity to extreme outcomes.

Model	NRMSE	NMAE	R^2	Score
Gradient Boosting	0.0455	0.0339	0.9655	
Random Forest	0.0464	0.0350	0.9641	
Neural Network	0.0608	0.0473	0.9383	
Linear Regression	0.1155	0.0932	0.7779	
Lasso Regression	0.1155	0.0933	0.7779	
Ridge Regression	0.1155	0.0933	0.7778	

Table 3.9: Predictive performance of candidate models w.r.t. firm *a*. NRMSE and NMAE are normalized by the range of the target variable.

Model	NRMSE	NMAE	R^2	Score
Random Forest	0.0672	0.0515	0.9317	
Gradient Boosting	0.0699	0.0519	0.9261	
Neural Network	0.0957	0.0784	0.8613	
Linear Regression	0.1161	0.0987	0.7957	
Lasso Regression	0.1162	0.0988	0.7955	
Ridge Regression	0.1162	0.0988	0.7955	

Table 3.10: Predictive performance of candidate models w.r.t. firm *b*. NRMSE and NMAE are normalized by the range of the target variable.



Figure 3.30: Profit prediction of firm a based on sampled data.

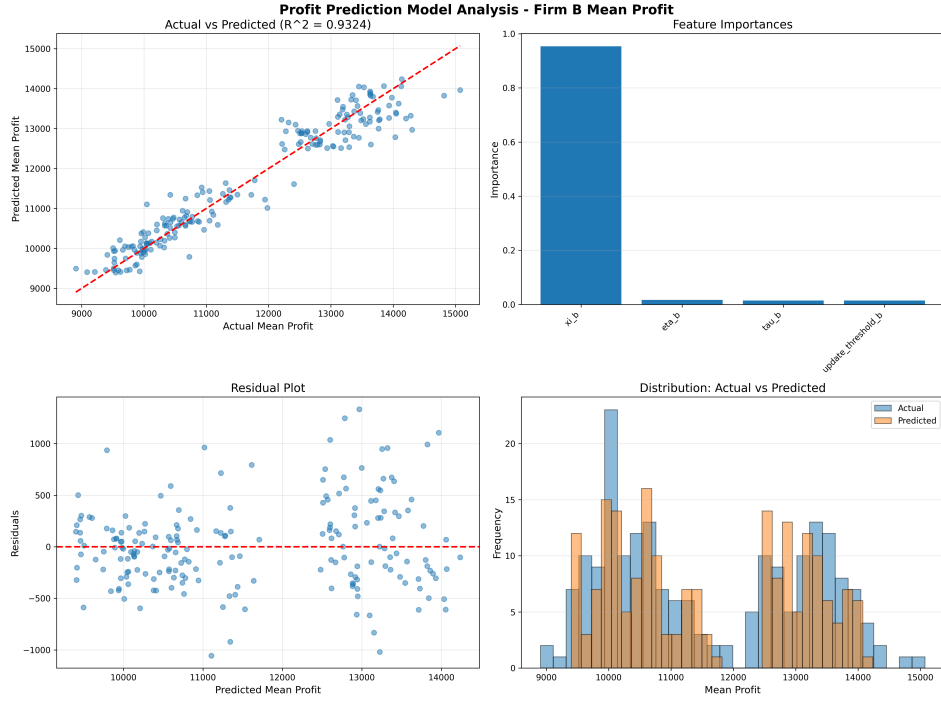


Figure 3.31: Profit prediction of firm b based on sampled data.

We set up surrogate models to map the control variables of one firm to the mean profit of last 30 periods. After training the surrogate, we perform parameter search directly on the approximated objective by solving $\max_{\theta \in \Theta} \hat{f}(\theta)$, where $\hat{f}(\theta)$ denotes the surrogate-predicted performance criterion over the feasible parameter set Θ . To reduce sensitivity to non-convexity and local minima, we apply a portfolio of optimization methods, including L-BFGS-B as a gradient-based local optimizer, SLSQP for constrained local search, and global exploration routines such as Differential Evolution and Basin Hopping, complemented by multi-start strategies. Candidate solutions obtained from these algorithms are then compared to assess robustness of the resulting parameter recommendations.

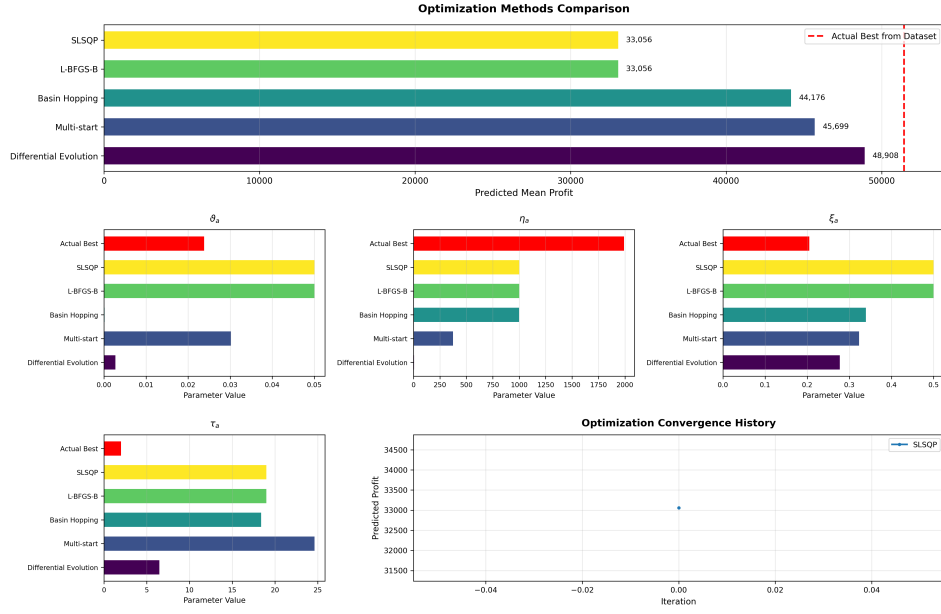


Figure 3.32: Surrogate optimization of firm a based on sampled data.

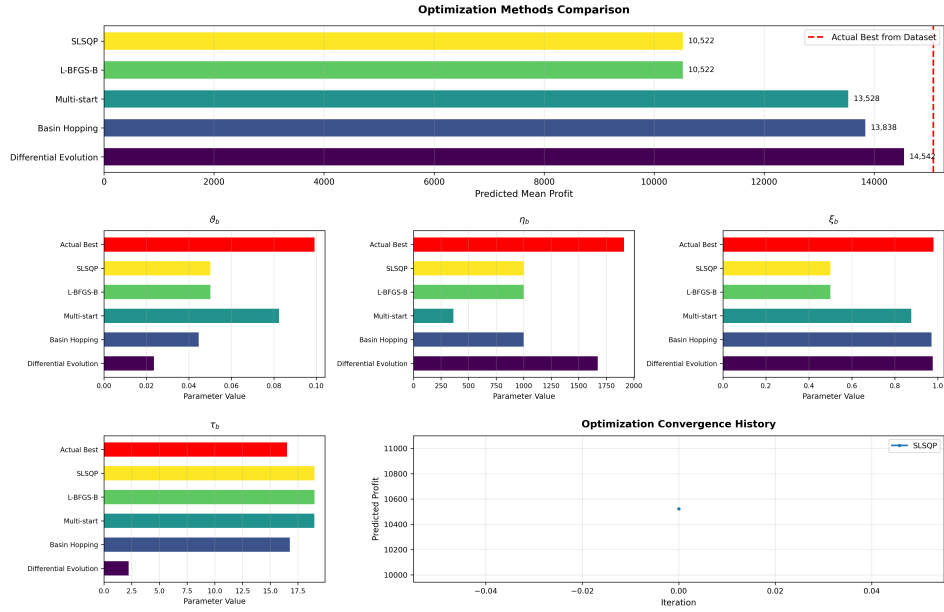


Figure 3.33: Surrogate optimization of firm b based on sampled data.

Based on the paired Wilcoxon signed-rank tests, a large share of the evaluated policy combinations are classified as effective. From the perspective of firm a ,

84.7% of policy combinations are effective when effectiveness is assessed using the mean profit over the final periods, and 73.3% are effective when assessed using discounted profit. From the perspective of firm b , the corresponding shares are 97.5% for the mean-profit metric and 97.5% for discounted profit. Overall, these results indicate that most policy combinations improve firm outcomes according to our effectiveness criteria. Consistent with the surrogate-based optimization results, the best-performing surrogate solution is obtained under Differential Evolution, and the price-adjustment control ξ_i emerges as the dominant strategic lever relative to the other three controls. In particular, the best-response outcomes imply $\xi_a = 0.2050$ when holding $\xi_b = 0.0783$ fixed, whereas $\xi_b = 0.9800$ is optimal when holding $\xi_a = 0.0674$ fixed. Taken together, the policy implications suggest that each firm benefits from increasing its own price-adjustment step size when the opponent's strategy is held constant, highlighting that asymmetric price-adjustment moves can be individually profitable.

3.6 Robustness check

3.6.1 Stationarity test

Stationarity refers to a crucial feature of a stochastic process that allows for consistent estimation of its statistical moments from observed time series data. In a strictly stationary process, the statistical properties—such as mean and variance—do not vary over time (Grazzini 2012). Ergodicity, on the other hand, is a foundational concept that ensures long-run averages computed from a single realization of the process are representative of the overall distribution. Specifically, an ergodic process exhibits asymptotic independence, meaning that observations far apart in time are effectively uncorrelated. When a process satisfies both stationarity and ergodicity, a single, sufficiently long time series can be used to reliably infer the characteristics of the underlying data-generating process, with sample moments converging almost surely to their population counterparts as the sample length grows (Hayashi 2000, Grazzini 2012). Previous studies have emphasized that conditions such as stationarity and ergodicity are essential for the reliable estimation of parameters in agent-based models (ABMs) (Grazzini 2012, Delli Gatti & Grazzini 2020). In order to ensure that the calibration process meaningfully reflects the properties of the actual data and the dynamics of the economic system, we conduct the following statistical tests.

We test the stationarity for both real and simulated time series. For the real data, testing stationarity helps validate whether it's reasonable to treat time averages (like means, variances, autocorrelations) as stable, while for the simulated data, it's important to ensure that the current model generates plausibly stationary

series if the real-world process is assumed stationary. For the simulated data, we select the model parameters that bring the best data fit to be the parameters in our model and run the computational model to obtain 30 profit time series based on different random seeds.

Real data

We apply the Augmented Dickey-Fuller (ADF) test to check stationarity. In addition, we decompose the time series to check properties. From the results, we show that all variables are likely non-stationary.

Economic Variable	ADF Statistic	p-value	Stationarity
R_a	-1.9769	0.2968	Non-stationary
R_b	-0.8251	0.8116	Non-stationary
$c_{0,a}$	-1.1193	0.7073	Non-stationary
$c_{0,b}$	-1.2928	0.6325	Non-stationary
m_a	1.2382	0.9962	Non-stationary
m_b	-0.3288	0.9213	Non-stationary
p_a	0.1868	0.9715	Non-stationary
p_b	-1.8636	0.3494	Non-stationary
Π_a	-1.0979	0.7159	Non-stationary
Π_b	0.0486	0.9624	Non-stationary

Table 3.11: ADF Test Results for Economic Variables

We analyze the data using autocorrelation functions (ACFs) and time series decompositions to investigate underlying dynamics. A comparison of the ACFs and decompositions of firm revenues reveals that both series display strong short-term persistence and signs of mean reversion. However, their temporal patterns differ significantly in terms of complexity and volatility. Firm a 's revenue displays clear cyclical behavior, with autocorrelations oscillating around zero and eventually returning to positive values. This reflects a dynamic adjustment process, likely driven by adaptive pricing, investment fluctuations, and consumer switching, which enables firm a to periodically recover and re-enter growth phases. The decomposition further reveals a non-monotonic trend and visibly volatile residuals, suggesting sensitivity to both internal strategy shifts and external shocks. In contrast, firm b 's revenue exhibits a smoother and more persistent trajectory. Its ACF shows a monotonic decay, and the decomposition indicates a steadily rising trend with lower-magnitude residuals. These patterns suggest that firm b follows a more passive or stabilization-oriented strategy, potentially with less aggressive exploration and lower exposure to demand-side volatility. Overall, the comparison

shows firm *a*'s more volatile but responsive position in the market, contrasted with firm *b*'s slower but more stable growth path.

The investment decisions of both firms are closely linked to their revenue dynamics. The decomposition of operating costs reveals that firm *a* exhibits a more volatile cost structure, with a non-monotonic trend and larger residual fluctuations. In contrast, firm *b*'s operating costs follow a smoother, steadily increasing trend, with less pronounced irregularities. Compared with firm *a*, firm *b*'s investment pattern shows a more gradual evolution and lacks signs of recovery or cyclical adjustment, implying more passive or rigid strategic behavior. This asymmetry reflects differing investment approaches and is consistent with the dominant and adaptive role played by firm *a* in the market.

From the demand side, the autocorrelation patterns for both firms reveal similar dynamics: both series exhibit strong short-term persistence followed by a gradual mean-reverting pattern. This suggests that consumer behavior is shaped by temporary loyalty and switching inertia, while market shares tend to rebalance over time. The seasonal-trend decompositions further support this view. Both firms experience steadily increasing trends, yet firm *a* exhibits greater irregularity in its residual component, indicating a higher degree of volatility and responsiveness to market fluctuations. In contrast, firm *b*'s demand follows a more stable path with smoother deviations around the trend. This comparison highlights that while overall demand dynamics are symmetric in structure, firm *a*'s customer base is more sensitive to short-term shocks or strategic interventions.

On the other hand, the ACFs of prices for both firms reveal consistent and symmetric dynamics, with strong short-term persistence and gradual mean reversion. This suggests that firms adjust prices cautiously over time, likely due to adjustment frictions or reliance on adaptive heuristics. The decomposition further supports this interpretation: both firms exhibit stable seasonal patterns and relatively smooth trends. However, firm *a* shows a declining trend in pricing, while firm *b* displays a more abrupt upward shift, possibly reflecting a late strategic correction. The residual component for firm *a* is more irregular, indicating greater short-term responsiveness or pricing volatility. The slight rebound in autocorrelations at longer lags points to cyclical pricing behavior, potentially driven by strategic interactions, delayed consumer responses, or feedback from profitability dynamics.

The autocorrelation function of profit for firm *a* reveals a complex and cyclical structure, indicative of internally generated economic oscillations. These may arise from dynamic interactions among pricing, investment, and consumer response mechanisms. The seasonal-trend decomposition further supports this interpretation: firm *a*'s profit series exhibits a non-monotonic trend, substantial seasonal fluctuations, and large, irregular residuals, highlighting the firm's sensitivity to both endogenous feedback and exogenous disturbances. In contrast, firm *b*'s profit

displays a smoother trajectory, with a steadily increasing trend and more regular seasonal and residual components. The ACF decays gradually without pronounced oscillations, suggesting more stable and persistent dynamics. This asymmetry highlights distinct strategic roles: while firm a appears to engage in active exploration and aggressive market positioning, firm b seems to follow a more passive, stabilization-oriented approach. The higher volatility in firm a 's profit may reflect both its willingness to take risks and its greater susceptibility to amplified feedback effects in a competitive environment.

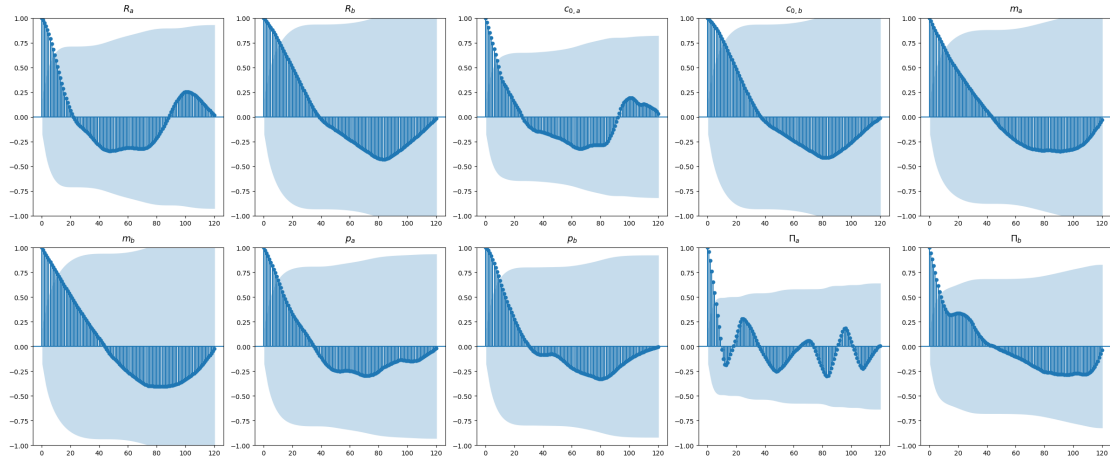


Figure 3.34: Autocorrelation function (ACF) of the real time series.

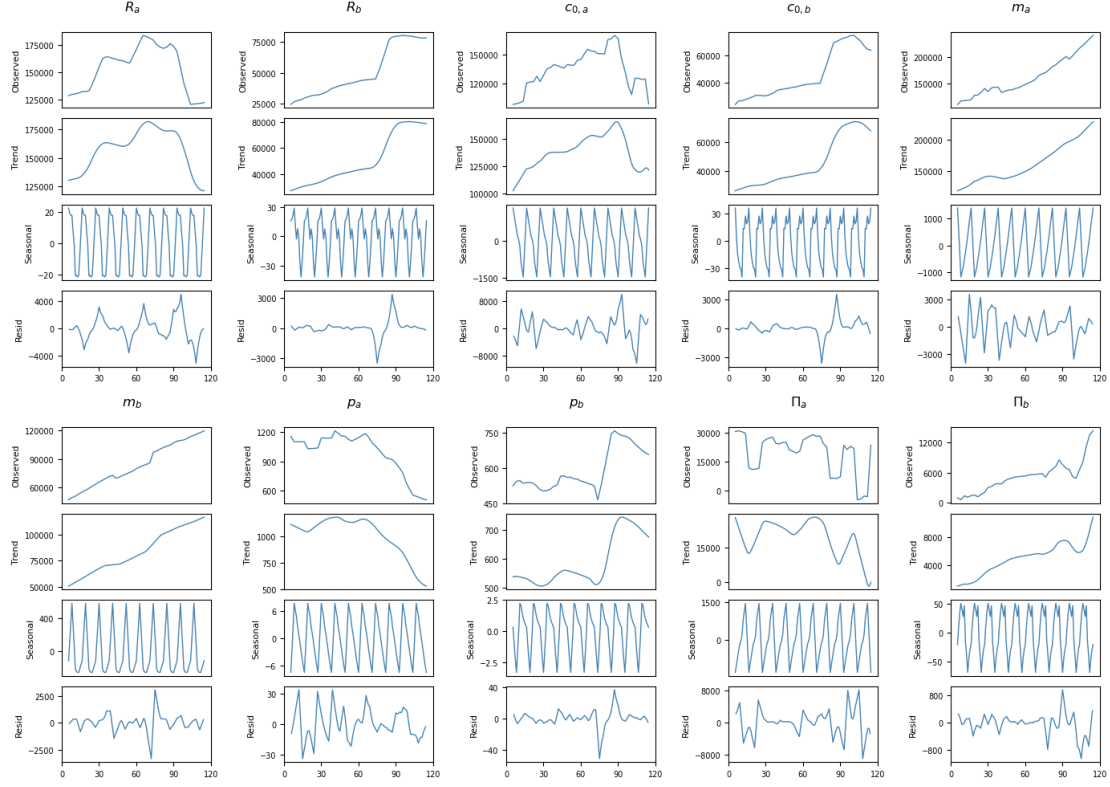


Figure 3.35: Decomposition of the real time series.

Simulated data

Different from the real data, since the simulated data generated from agent-based simulations are complex and may lack assumption over the data generator process, we apply the non-parametric test, namely the Wald–Wolfowitz test, for testing data stationarity. This test shows (non-)random patterns of the data. The null hypothesis states that there is no systematic pattern (non-randomness) in how observations fall above or below the chosen cutoff (Wald & Wolfowitz 1940, Grazzini 2012).

As we run the one-sample Wald–Wolfowitz Runs Test for the single round of simulated time series, we find that for firm a 's profit series, the fraction of rejections ($p < 0.05$) is 96.67%, and for firm b 's profit series, the fraction of rejections ($p < 0.05$) is 93.33%. This exhibits statistically significant non-random patterns around their means during the post-burn-in period, and suggests that the time series generated by the model display persistent or structured dynamics inconsistent with weak stationarity. Following Grazzini (2012), we interpret this as a violation

of the moment constancy assumption required for ergodicity and reliable moment-based calibration.

3.6.2 Ergodicity test

Since the ergodicity property refers to a set of time series, we conduct the test only for simulated time series. The two-sample Wald–Wolfowitz test suggests whether samples come from the same distribution. The null hypothesis states that the samples come from an identical distribution. If most of the samples satisfy the null hypothesis, then there is stronger evidence that the model is ergodic.

For the 30 simulated time series, we conduct pairwise comparisons using the two-sample Wald–Wolfowitz test, yielding $\binom{30}{2} = 435$ statistical tests in total. We record the frequency with which the null hypothesis (identical distribution, $p \geq 0.05$) and the alternative hypothesis (different distributions, $p < 0.05$) are supported. The results reveal that only 0.23% of all pairwise tests for firm *a* and 7.82% for firm *b* satisfy H_0 . These extremely high rejection rates indicate that the profit time series generated by the model are not statistically indistinguishable, even after the burn-in period. The near-complete rejection of the null hypothesis across pairwise comparisons provides strong empirical evidence against ergodicity, demonstrating that the ABM exhibits path-dependent dynamics in which different simulation runs do not converge to the same statistical distribution. This finding is consistent with the notion that agent-based models can display persistent heterogeneity and path-dependent behavior under the current parameter settings. In short, we find that both the real data and the simulated data are much likely to be non-stationary, and the simulated system is not ergodic. This mismatch undermines the validity of moment-based calibration and helps explain the discrepancy observed in the results.

3.7 Conclusion

Our research makes two main contributions. First, it incorporates established parameter estimation techniques into an IO-oriented agent-based modeling framework so as to link real-world data to a computationally designed market environment. This provides a structured setting for studying key questions in industrial organization, particularly duopolistic competition in the presence of heterogeneous consumer preferences, congestion effects, and network-based interactions. Second, it proposes and implements a systematic calibration procedure for identifying relatively best-fit parameter settings in this class of models. By combining the Simulated Method of Moments (SMM) with a neural-network surrogate framework, the analysis demonstrates how alternative estimation tools can be used in

a complementary manner to narrow the parameter space, strengthen the empirical relevance of the model, and recover parameter configurations under which the simulated market reproduces salient stylized facts of duopolistic competition.

The findings of this study also provide several important insights. A key achievement of the analysis lies in the development of a systematic calibration framework for an empirically grounded agent-based duopoly model. Within this framework, the Simulated Method of Moments (SMM) serves as a structured device for linking simulated outcomes to observed economic variables, while the neural-network surrogate complements this process by approximating ABM-generated realizations and enabling the efficient re-use of simulation data. In addition, the surrogate framework facilitates gradient-based parameter optimization through the Adam optimizer and allows for out-of-sample validation. Together, these elements enable a broader and more disciplined exploration of the parameter space, support the identification of relatively best-fit parameter settings, and clarify the conditions under which the simulated data are able to reproduce meaningful stylized facts. Overall, the results demonstrate the value of this integrated framework not only in computational terms, but also in its capacity to uncover richer economic insights into duopolistic competition, strategic interaction, and evolving market dynamics.

Both the policy analysis and the surrogate-based optimization identify the price-adjustment parameter ξ as the most influential firm control. Within the predictive models, ξ displays substantially greater feature importance than the other three strategic controls, underscoring its central role in shaping firm performance and market outcomes. The fact that this pattern emerges consistently across different components of the analysis strengthens the economic relevance of the result. At the same time, the surrogate-based optimization proves valuable by identifying economically plausible regions of the parameter space that had not been explored previously. These regions merit further targeted simulation and validation. Taken together, these findings show that the calibration framework serves a dual purpose. It helps identify relatively best-fit parameter settings and reveals which strategic variables exert the strongest influence on outcomes in a duopolistic competitive environment.

In addition, we find that the current model successfully reproduces several important stylized facts of the observed market and thus provides a meaningful representation of duopolistic competition in an empirically informed setting. The simulated data capture key patterns of firm interaction and market dynamics, demonstrating that the framework is able to generate economically interpretable regularities rather than merely isolated numerical correspondences. At the same time, the comparison between simulated and empirical time series suggests that some dynamic properties of the real market are only partially reflected in the present specifi-

cation. Although both the simulated and empirical data exhibit non-stationary behavior, the statistical results indicate differences in their underlying time-series properties. This opens a productive avenue for future work, particularly through the incorporation of richer dynamic mechanisms or additional exogenous influences, such as inflationary developments or the COVID-19 pandemic, which may have affected real-world price dynamics. Nevertheless, the present framework already offers a useful basis for policy analysis under a fixed parameter configuration, which remains a standard and practically relevant feature of agent-based models. Accordingly, the contribution of the model lies not only in the calibration exercise itself, but also in its capacity to depict the strategic logic of a duopolistic market in a tractable and empirically grounded manner.

While the current model does not explicitly account for all external shocks or structural shifts, it nonetheless provides a systematic and empirically grounded framework for studying how simulated and real-world economic time series may be brought into closer correspondence. The contribution of the study therefore lies not only in the identification of relatively best-fit parameter settings, but also in showing which aspects of market dynamics can already be reproduced within an agent-based duopoly framework and which aspects remain open for further development. Building on this foundation, future research may proceed in two complementary directions: first, by enriching the theoretical structure of the model to generate a wider range of realistic non-stationary dynamics; and second, by refining the empirical estimation strategy so that it more closely captures firms' investment behavior and strategic decision-making processes.

Code availability

To support transparency and reproducibility, the code underlying the results presented in this study is publicly accessible at https://github.com/ZHongli626/ICT_ABM_Parameter_Estimation.

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SHORT CURRICULUM VITAE

ZHONGLI WANG

ACADEMIC EDUCATION

since 2021 <i>Bielefeld, Germany</i> <i>Milan, Italy</i> (Sept. 2022 – Sept. 2023)	Bielefeld University and Università Cattolica del Sacro Cuore <i>PhD Candidate</i> EPOC Program (Marie Skłodowska-Curie Grant No. 956107); Research Interests: Computational Economics, Industrial Organization, Heterogeneous Agents; Thesis Topic: <i>Strategic Behavior and Economic Dynamics in ICT Markets: An Agent-Based Framework</i> .
2018 – 2021	University of Zurich , <i>Master of Arts in Business and Economics</i> (magna cum laude). Major: Banking and Finance; Minor: Data Science.
2015 – 2018	Nankai University , <i>Bachelor of Science in Finance</i> (GPA 4.39/5.0).
2014 – 2018	Nankai University , <i>Bachelor of Arts in German Language and Literature</i> (GPA 5.0/5.0).

RESEARCH

Publication	Polytope Fraud Theory , <i>International Review of Financial Analysis</i> , Volume 97, 2025. In collaboration with Dongshuai Zhao, Florian Schweizer, Didier Sornette. Introduces ten financial red flags based on activist short sellers and uses supervised learning (notably XGBoost) for fraud detection.
Prepublication	Agentic Workflows for Economic Research: Design and Implementation , arXiv. In collaboration with Hankui Wang, Jiachen Yi, Herbert Dawid, Philipp Harting. Proposes an AI-agent workflow for economic research with human-in-the-loop oversight across the research lifecycle.
Prepublication	Heterogeneous Consumers and the Data Value Chain: An Agent-Based Approach . Studies how data-restriction levels shape software-market dynamics using a game-theoretic and agent-based framework.

See Chapter 1.

- Prepublication **Strategic Firm Capacity Selection under the Network Effects: A Computational Model of Capacity Investment Dynamics in the ICT Industry.**
Develops an agent-based model of competing ICT firms that adjust investments in service capacity to manage congestion and demand under network effects.
See Chapter 2.
- Prepublication **Parameter Estimation Methods in an Agent-based Computational Model: Insights on ICT Industrial Dynamics and Policies.**
Compares simulated method of moments and neural-network surrogates using AT&T and T-Mobile data for ABM calibration.
See Chapter 3.

CONFERENCES, WORKSHOPS AND SEMINAR PRESENTATIONS

- 10/2025 Organizer and speaker, workshop on *AI Agents in Science*, Wissenswerkstadt Bielefeld.
- 09/2024 Speaker, 10th Meeting of the German Network for New Economic Dynamics (GENED 2024).
- 08/2024 Speaker, International Conference on Challenges in Managing Smart Products and Services.
- 07/2024 Speaker, 27th Annual Workshop on Economics with Heterogeneous Interacting Agents (WEHIA 2024).
Speaker, Economic Policy in Complex Environments Conference, Milan.
- 03/2024 Organization team member, International EPOC Doctoral Workshop, Ca' Foscari University of Venice.
- 01/2024 Speaker, 9th World Congress of the International Microsimulation Association.
Speaker, Graz Schumpeter Summer School 2024.
- 10/2023 Speaker, GENED 2023.
- 06/2023 Participant, ISEO Summer School.
- 12/2022 Organization team member and participant, 17th BiGSEM Workshop.