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Subsidies to Technology Adoption when Firms' Information is Endogenous*

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Abstract

How should firms be incentivized to adopt new technologies when the technical merits and spillovers of such technologies are uncertain? We show that, when information is dispersed but exogenous, efficiency can be induced with simple (constant) subsidies. When, instead, firms must also be incentivized to collect information efficiently, subsidies must be conditioned on the ex-post profitability of the new technology and, when the cost of information acquisition is unknown to the planner, on the aggregate investment in the new technology. The optimal policy has a Pigou's flavor but accounts for the non-observability of firms' acquisition and usage of information.

Keywords: endogenous information, investment spillovers, optimal policy, welfare

JEL classification: D21, D62, D83

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1 Introduction

When deciding whether to adopt new technologies such as a new operating system or a new environment-friendly production process, firms face uncertainty about the profitability of their investments. Such an uncertainty may reflect limited familiarity with the new technology, but also the fact that its profitability may depend on whether it is adopted also by other firms. Importantly, this uncertainty is often endogenous, as firms can collect information about the new technology before investing.

In such contexts, how should the government incentivize firms to collect and use information in society's best interest? This question is at the center of an active policy debate as many countries are devoting significant resources to boost innovation and technology adoption in a number of fields, such as green technologies, the industrial internet of things, and fintech.[1]

We show that if the information the firms possess is dispersed but exogenous, efficiency can be induced by combining familiar subsidies correcting for firms' market power with additional state-invariant subsidies to innovating firms appropriately designed to make them use the available information efficiently. When, instead, firms must also be incentivized to collect information efficiently prior to investing, it becomes necessary to resort to more sophisticated policies that condition the subsidies to the innovating firms on the profitability of the new technologies and, when the cost of information is unknown to the policy maker, on the aggregate investment in the new technology. Such richer policies operate as a Pigouvian correction realigning the private value of information to its social counterpart by inducing firms to internalize the externality that their decisions impose on others. However, they account for the fact that neither the acquisition nor the use of information is verifiable. That Pigouvian taxes/subsidies can correct externalities when information is complete and firms' activities are verifiable is known. The paper's contribution is in showing that a specific version of such policies also creates the right incentives for information acquisition and its subsequent utilization.

In our model, the key externality originates in investment spillovers. Policies similar to those characterized in this paper can also be used to correct for other externalities. One example is the adoption of "greener" technologies that reduce pollution, where firms face uncertainty both about the technical merits of the new technologies and whether they will be used by a large enough number of firms to make them not only environment-friendly but also economically viable.

The paper is related to the literature investigating the interaction between investment under uncertainty, innovation, and the corrective role of taxation in the presence of externalities (see, e.g., Akcigit et al. (2018), (2022a), and the references therein). In particular, our work is related to Akcigit et al. (2022b), who investigate how to use policy to stimulate R&D investments in the presence of technology spillovers between firms that are heterogeneous and privately informed about

[1]See, for example, the European Commission policy briefs on advanced technologies for industry – https://ati.ec.europa.eu/reports/Policy-Briefs

their research productivity.[2] Our model abstracts from many effects considered in that paper. Our contribution is in endogenizing information about both the technical merits of new technologies and the spillovers associated with them and showing how appropriate subsidies can correct inefficiencies in both the acquisition and usage of information. Alvarez et al. (2022) study how to stimulate the adoption of new technologies in the fintech industry. This paper focuses on dynamic spillovers, but it does not investigate how to correct inefficiencies in the acquisition and usage of information. The latter topic is investigated in Pavan et al. (2022) who, however, focuses on information aggregation in financial markets, and does not consider spillovers in investment decisions or other direct payoff interdependencies among the relevant actors.

Our paper is also related to a broad literature in both micro- and macro-economics investigating incentives for information acquisition and efficient information usage. See, among others, Bergemann and Välimäki (2002) for how to use Vickrey-Clarke-Groves (VCG) transfers to incentivize agents to acquire information prior to participating in a mechanism, and Angeletos and La'o (2020) for optimal monetary policy over the business cycle with dispersed information. The contribution of our paper vis-a-vis this literature is in showing how to correct for externalities originating in investment spillovers and establishing that policies resembling Pigouvian corrections induce efficiency in both information acquisition and usage.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 contains all the key results. Section 4 concludes. All proofs omitted in the main text are in the Appendix at the end of the document.

2 The Model

The economy is populated by (i) a measure-1 continuum of firms each producing a differentiated intermediate good, (ii) a competitive retail sector producing a final good using the intermediate goods as inputs, (iii) a measure-1 continuum of homogenous workers, and (iv) a benevolent planner.

Each firm is run by a single entrepreneur who must decide whether to operate under an existing technology or adopt a new one. Indexing firms by $i \in [0,1]$, we denote by $n_i = 1$ (alternatively, $n_i = 0$) the decision by firm i to adopt the new technology (alternatively, retain the old one). Let

$$N = \int n_i di$$

denote the aggregate investment in the new technology, and $l_i \in \mathbb{R}_+$ the amount of labor employed by firm i. The amount of the intermediate good produced by firm i is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 1 \\ \Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 0 \end{cases}, \tag{1}$$

[2] See also Bloom et al. (2002) for the effects of R&D tax credits on innovation.

with $\gamma > 1$, $\beta \geq 0$, $\alpha \geq 0$, and $\psi \leq 1$. The variable $\Theta > 0$ proxies for the uncertainty that firms face at the time they make the relevant production decisions. The parameter γ scales the return differential between the two technologies, whereas the parameters α and β control for the returns to scale and the intensity of the investment spillovers, respectively. Finally, the parameter ψ captures the marginal productivity of labor. The variable Θ thus contributes both to the output differential between the two technologies and to the magnitude of the investment spillover, that is, the effect of aggregate investment N on individual output.

Whereas the new technology is more efficient than the old one, its differential is unknown at the time firms decide whether to adopt the new technology. After choosing which technology to use, each entrepreneur learns Θ and N, and then chooses the price p_i for the intermediate good it produces.[3] Finally, given Θ, N, and the realized demand for its intermediate good, firm i employs labor l_i on a competitive market to meet its demand. Labor is supplied by the continuum of measure-one workers.

Adopting the new technology costs $k > 0$. Such a cost can be interpreted as the disutility the entrepreneur incurs to familiarize with the new technology. What matters for the results is that such a cost is not mediated by a market that fully aggregates the entrepreneurs' dispersed information. The dependence of the production function on the aggregate investment N captures the idea that each entrepreneur benefits from the adoption of the new technology by the other entrepreneurs. That such spillovers affect both the entrepreneurs adopting the new technology and those retaining the old one is not essential for the results. What matters is that the output differential

$$(\gamma - 1)\Theta \left(1 + \beta N\right)^{\alpha} l^{\psi}$$

between the two technologies is increasing in both N and Θ.

The final good is produced by a competitive retail sector using the familiar CES technology

$$Y = \left(\int y_i^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}, \tag{2}$$

with $v > 1$ denoting the elasticity of substitution between goods. The price of the final good is P and the profits of the competitive retail sector are given by

$$\Pi = PY - \int p_i y_i di,$$

where p_i is the price of the intermediate good paid to firm i.

Let $\theta \equiv \log \Theta$. It is commonly believed that θ is drawn from a Normal distribution with mean 0 and precision π_θ. The realization of θ is unobserved by the entrepreneurs. Each entrepreneur i chooses the precision π_i^x of an additive private signal

$$x_i = \theta + \xi_i$$

[3] See the supplement for an extension in which firms set prices under dispersed information.

about θ, with ξ_i drawn from a Normal distribution with mean zero and precision π_i^x, independently from θ, and independently across i. The cost of information of precision π_i^x is equal to $\mathcal{I}(\pi_i^x)$, with $\mathcal{I}$ continuously differentiable and such that $\mathcal{I}'(0) = 0$, $\mathcal{I}^{'}(\pi_i^x) > 0$ and $\mathcal{I}^{''}(\pi_i^x) \geq 0$ for all $\pi_i^x > 0$. Such a cost can be interpreted as disutility of effort. The results extend to general/flexible information technologies but are best illustrated with the Gaussian structure described above.

Each entrepreneur maximizes her firm's profits, which are then used to finance the purchase of the final consumption good. Accordingly, each entrepreneur's objective function is given by

$$\Pi_i = \frac{p_i y_i - W l_i}{P} + T - k n_i - \mathcal{I}(\pi_i^x),$$

where W is the nominal wage rate, and T is a transfer to the firm in terms of the final consumption good.

Each worker uses his labor income to purchase the final consumption good by maximizing

$$U = \frac{W}{P} l - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \Upsilon,$$

where $l^{1+\varepsilon}/(1+\varepsilon)$ denotes the disutility of labor, with $\varepsilon > 0$, and Υ is a tax collected by the government. Because labor is undifferentiated, in equilibrium, each worker provides the same amount of labor. The government's budget is balanced implying that $\int T_i di = \Upsilon$.

A benevolent planner maximizes the ex-ante sum of the firms' profits and of all workers' utilities

$$\mathcal{W} = \mathbb{E}\left[\int \Pi_i di + U\right].$$

Using that (a) the total labor demand must equal the total labor supply, (b) the government's budget is balanced, (c) all entrepreneurs choose the same precision of private information in equilibrium, (d) firms' total revenues coincide with the total expenditure on the final good, and (e) the total consumption of the final good C coincides with its production Y, we have that the government's objective can be expressed as

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x).$$

The planner thus maximizes aggregate consumption, net of the costs to upgrade the technology, the labor costs, and the information-acquisition costs.

The timing of events is the following.

1. Nature draws θ.
2. Each entrepreneur i chooses the precision π_i^x of her private information.
3. Each entrepreneur i receives a private signal x_i about θ.
4. Entrepreneurs simultaneously choose n_i.

5. After θ and N are publicly revealed, entrepreneurs simultaneously set prices p_i.

6. The competitive retail sector chooses how much of each intermediate good to purchase taking the prices of the intermediate goods and the price P of the final good as given.

7. Given the demand y_i for her intermediate good, entrepreneur i hires l_i units of labor to meet her demand, taking N and θ as given.

8. A representative household comprising all workers and entrepreneurs chooses how much of the final good to buy taking the price of the final good P as given.

Because money in this economy has only a nominal effect on prices and plays no other role, we omit it.

The economy described above has two distinctive features: (a) the endogeneity of the firms' private information and (b) the investment spillovers.

3 Efficiency and Optimal Policy

3.1 Efficient Technology Adoption

The efficient allocation has three parts: the precision of private information, π^{x*}, a rule specifying how firms should choose between the two technologies based on their dispersed information x, and a rule describing how much labor each firm should employ as a function of θ and x (equivalently, θ and the technology adopted). These three parts are chosen jointly to maximize ex-ante welfare.[4]

Lemma 1 below focuses on technology adoption. The rule describing the efficient employment of labor is in the proof of Lemma 1, whereas the formula for the efficient precision of private information π^{x*} is in the proof of Lemma 3 in the Appendix.[5]

Lemma 1. *Let $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ and suppose that $\gamma^{\varphi} \geq 1+\beta$ and $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$. For any precision of private information π^x, there exists a threshold $\hat{x}(\pi^x)$ such that efficiency in technology adoption requires that each firm with signal $x > \hat{x}(\pi^x)$ adopts the new technology, whereas each firm with signal $x < \hat{x}(\pi^x)$ retains the old technology.*

Proof. See the Appendix.

The parameters' restrictions in the lemma guarantee that the social value of upgrading the technology (net of its disutility cost) is increasing in the fundamental and in the mass of firms adopting the new technology. Our key results below extend to economies in which the above restrictions are dispensed with. The characterization of the efficient allocation in such economies (in the Appendix) is, however, more convoluted.

[4] The notion of (decentralized) efficiency is standard and is the same as in Vives (1988), Angeletos and Pavan (2007), and Colombo, Femminis and Pavan (2014), among others.

[5] The reason for relegating these parts to the Appendix is that they are useful for comparative statics but not essential to follow the key arguments below.

3.2 Equilibrium Conditions

We start by characterizing the equilibrium allocations for given technology choice. The assumption that the retail sector is competitive implies that, in equilibrium, $\Pi = 0$ and that the price of the final good is equal to

$$P = \left(\int p_i^{1-v} di \right)^{\frac{1}{1-v}}, \tag{3}$$

with the demand for each intermediate good given by

$$y_i = C \left(\frac{P}{p_i} \right)^v, \tag{4}$$

where C is the consumption of the final good and is determined by the interaction between the representative consumer and the competitive retail sector. Furthermore, because labor is undifferentiated and the labor market is competitive, the supply of labor is given by

$$\frac{W}{P} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor. The demand for labor by each entrepreneur i is then given by

$$l_{1i} = \left(\frac{y_i}{\gamma\Theta \left(1 + \beta N\right)^{\alpha}} \right)^{1/\psi}, \tag{5}$$

if entrepreneur i adopted the new technology, and by

$$l_{0i} = \left(\frac{y_i}{\Theta \left(1 + \beta N\right)^{\alpha}} \right)^{1/\psi}, \tag{6}$$

otherwise. In both cases, the entrepreneur takes both N and Θ as given. Market clearing then implies that

$$\frac{W}{P} = \left(\int l_i di \right)^{\varepsilon}.$$

Let $p_1(\theta; \pi^x)$ and $l_1(\theta; \pi^x)$ (alternatively, $p_0(\theta; \pi^x)$ and $l_0(\theta; \pi^x)$) denote the equilibrium price and labor demand of each firm adopting the new technology (alternatively, retaining the old one). Note that the dependence of these functions on π^x comes from the fact that, in equilibrium, the fraction of firms adopting the new technology in state θ depends on π^x.

3.3 Optimal Policy

Given the above equilibrium conditions, we first characterize a simple policy implementing the efficient usage of information when the precision of private information π^x is exogenous. Next, we show that such a simple policy fails to induce the entrepreneurs to collect information efficiently, but a certain amendment guarantees efficiency in both the collection and the usage of information.

3.3.1 Exogenous Information

Suppose that the precision of private information is exogenous and equal to π^x. Let $r = py/P$ denote a representative firm's revenues in terms of the consumption of the final good. Next, let $\hat{C}(\theta; \pi^x)$ and $\hat{N}(\theta; \pi^x)$ denote, respectively, the amount of the final good consumed and the measure of firms adopting the new technology in state θ when the precision of private information is π^x, and all firms make all decisions efficiently.

Lemma 2. *Suppose that the conditions in Lemma 1 hold, and that the precision of private information is exogenous and equal to π^x. The following policy implements the efficient allocation. Each firm adopting the new technology receives a total transfer equal to*

$$\bar{T}_1(r) = \bar{s}_{\pi^x} + \frac{1}{v-1} r,$$

where

$$\bar{s}_{\pi^x} = \mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}(\theta;\pi^x)} \middle| \hat{x}(\pi^x), \pi^x \right],$$

with $\hat{x}(\pi^x)$ as defined in Lemma 1. Each firm retaining the old technology receives a total transfer equal to

$$\bar{T}_0(r) = \frac{1}{v-1} r.$$

Proof. See the Appendix.

When information is exogenous, efficiency in both technology adoption and in the subsequent employment of labor can be induced with a simple policy that combines the familiar revenue subsidy $r/(v-1)$, designed to offset firms' market power, with an additional (constant) subsidy $\bar{s}_{\pi^x}$ to the innovating firms. Naturally, those firms adopting the new technology expect higher revenues and hence a higher subsidy $r/(v-1)$. However, this subsidy alone is not enough to guarantee that firms adopt the new technology efficiently. This is because firms do not internalize that, by adopting the new technology, they increase other firms' output. The additional subsidy $\bar{s}_{\pi^x}$ to the innovating firms corrects for such an externality by guaranteeing that each firm with signal $x < \hat{x}(\pi^x)$ finds it optimal to retain the old technology, whereas each firm with signal $x > \hat{x}(\pi^x)$ finds it optimal to adopt the new one.

The term

$$\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}(\theta;\pi^x)}$$

in the formula for $\bar{s}_{\pi^x}$ represents the marginal externality created by the investment spillover. It coincides with the increase in the production of the final good that obtains if one increases N by a small amount $\varepsilon > 0$ around the efficient level $\hat{N}(\theta; \pi^x)$, holding firms' technology and employment decisions fixed. The subsidy $\bar{s}_{\pi^x}$ to the innovating firms is thus the externality expected by the "marginal innovator" with signal equal to the efficient threshold $\hat{x}(\pi^x)$.

3.3.2 Endogenous Information

We now turn to the case where firms' information is endogenous. Let π^{x*} denote the precision of the firms' information that maximizes welfare (its characterization is in the proof of Lemma 3 below).

Let $\partial\hat{N}(\theta;\pi^{x*})/\partial\pi^x$ denote the marginal variation in the measure of firms adopting the new technology at θ that obtains when one varies π^x infinitesimally at $\pi^x = \pi^{x*}$, holding the rule for technology adoption fixed at $\hat{n}(x;\pi^{x*})$.

Lemma 3. *Suppose that information is endogenous and that the economy satisfies the conditions in Lemma 1. Consider a policy that pays the firms retaining the old technology a total transfer equal to*

$$T_0^*(r) = \frac{1}{v-1}r,$$

and the firms adopting the new technology a total transfer equal to

$$T_1^*(\theta, r) = s(\theta) + \frac{1}{v-1}r,$$

where the additional subsidy $s(\theta)$ to the innovating firms is determined ex-post, after θ is revealed. Such a policy induces firms to acquire and use information efficiently only if $s(\theta)$ is non-decreasing and satisfies the following two conditions

$$\mathbb{E}\left[s(\theta)\,|\hat{x}(\pi^{x*}), \pi^{x*}\right] = \mathbb{E}\left[\left.\frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}\right|\hat{x}(\pi^{x*}), \pi^{x*}\right] \tag{7}$$

and

$$\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] = \mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]. \tag{8}$$

Proof. See the Appendix.

Condition (7) is a restriction on the expected value of the subsidy $s(\theta)$, whereas Condition (8) is a restriction on the covariance between the subsidy $s(\theta)$ and the marginal effect of more precise information on the aggregate investment $\hat{N}(\theta;\pi^{x*})$ in the new technology under the efficient allocation. Together with the condition that $s(\theta)$ is non-decreasing, the above two conditions guarantee that, in equilibrium, firms acquire information of precision π^{x*} and then use it efficiently. Note that the simple policy of Lemma 2, specialized to $\pi^x = \pi^{x*}$, satisfies Condition (7) but not Condition (8), and hence fails to induce efficiency in information acquisition. This is because a constant subsidy equal to the externality expected by the marginal investor with signal $\hat{x}(\pi^{x*})$ fails to induce the right covariance between the subsidy $s(\theta)$ and the (state-dependent) marginal effect of more precise information on aggregate investment $\partial\hat{N}(\theta;\pi^{x*})/\partial\pi^x$ necessary to realign the private benefit to information to its social counterpart.

To induce efficiency in both information acquisition and information usage it is necessary to let the subsidy $s(\theta)$ to the innovating firms vary with the profitability θ of the new technology, which can be inferred ex-post from the observation of the firms' output, using information about the inputs used (here labor) and the form of the production function.

Building on the previous results, the following proposition identifies a policy inducing efficiency in both information acquisition and technology adoption.

Proposition 1. *Irrespective of whether the economy satisfies the conditions in Lemma 1, the policy of Lemma 3 with a subsidy to the innovating firms equal to*

$$s(\theta) = \frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)} \tag{9}$$

induces all firms to acquire and use information efficiently.

Proof. Suppose that all other firms (a) acquire information of precision π^{x*}, (b) adopt the new technology when, and only when, it is socially efficient to do so, and (c) set the prices $\hat{p}_0(\theta;\pi^{x*})$ and $\hat{p}_1(\theta;\pi^{x*})$ that induce the efficient employment decisions. Then, in each state θ, irrespective of the precision π^x of its private information, each firm finds it optimal to set a price equal to $\hat{p}_0(\theta;\pi^{x*})$ when it retains the old technology, and equal to $\hat{p}_1(\theta;\pi^{x*})$ when it adopts the new technology. Furthermore, each firm assigns a private value to upgrading its technology that coincides with the planner's value (see the proof of Lemma 2 in the Appendix for the formal arguments). These properties hold irrespective of whether the economy satisfies the conditions in Lemma 1. The same properties also imply that the value that the firm assigns to acquire information coincides with the planner's value. Because the private cost of information also coincides with the social one, the above results imply that acquiring information of precision π^{x*} and then using the information efficiently (both when it comes to choosing the technology and setting the prices) is individually optimal for each firm expecting all other firms to do the same. Q.E.D.

As anticipated above, the state-contingent subsidy in (9) operates as a Pigouvian correction that induces each firm to internalize the effect of its technology choice on the production of the final consumption good when all other firms acquire and use information efficiently. To see this, let Λ denote the cross-sectional distribution of firms' technology and employment decisions (n_i, l_i). Let $C_N\left(\theta,\Lambda\right)$ denote the marginal change in the production of the final good that obtains when, holding θ and Λ fixed, one changes N in all firms' production functions by a small $\varepsilon > 0$, starting from $N = N_\Lambda$ where N_Λ is the aggregate investment in the new technology under the distribution Λ. Next, let $\hat{\Lambda}(\theta,\pi^{x*})$ denote the cross-sectional distribution of firms' technology and employment decisions (n_i, l_i) under the efficient allocation. Then

$$C_N\left(\theta,\hat{\Lambda}(\theta,\pi^{x*})\right) = \frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}.$$

That is, the state-dependent subsidy in (9) coincides with the marginal change in the production of the final good that obtains as a result of a marginal change in N, evaluated at $N = \hat{N}\left(\theta;\pi^{x*}\right)$, holding all firms' technology and employment decisions fixed at the efficient level. Such a policy is thus reminiscent of familiar Pigouvian corrections for complete-information economies. These corrections also induce firms to collect and use information efficiently when information is dispersed and endogenous.

The Pigouvian policy of Proposition 1 is not the unique one implementing the efficient allocation. Other state-contingent policies do the job. Furthermore, when information acquisition is verifiable, the planner can control separately the firms' incentives to acquire information, for example by taxing those firms that fail to acquire information of precision π^{x*}. However, one of the limitations of the above policies (including the one in Proposition 1) is that they require the planner to know the firms' information acquisition technology (formally, the type of signals that firms can acquire and their costs).[6] Such a knowledge may not be available in many markets of interest.

As the next proposition shows, this knowledge, however, is not essential: efficiency in both information acquisition and usage can be induced by conditioning the transfer to the innovating firms directly on the cross-sectional distribution of firms' technology and employment decisions.

Proposition 2. *Suppose that the planner does not know what type of information the firms can collect (equivalently, the cost of different information structures). Efficiency in both information acquisition and usage can be induced through a policy that pays to the non-innovating firms a transfer equal to*

$$T_0^{\#}(r) = \frac{1}{v-1} r,$$

and to the innovating firms a transfer equal to

$$T_1^{\#}(\theta, r, \Lambda) = C_N(\theta, \Lambda) + \frac{1}{v-1} r,$$

where Λ is the ex-post cross-sectional distribution of firms' technology and employment decisions (n_i, l_i), and where $C_N(\theta, \Lambda)$ is the marginal change in the production of the final good that obtains as a result of a marginal change in N holding all firms' technology and employment decisions fixed at the level specified by Λ.

Proof. Suppose that all other firms (a) acquire information efficiently (with information acquisition taking the form of a private signal mapping θ into a distribution over posterior beliefs over θ), (b) use information efficiently to make their technology choice, and (c) set prices in each state θ so as to induce the efficient employment (and hence production) choices. Then each firm has enough knowledge about the economy to compute the efficient allocation and has incentives to follow the same efficient policies as any of the other firms. In fact, the revenue subsidy $r/(v-1)$ guarantees that each firm, no matter its technology, after learning θ, has the right incentives to set the price for its intermediate good at a level that induces the efficient demand for its product, and hence the efficient employment decisions (see the proof of Lemma 2 where the result is established without using the

[6] In the context of Proposition 1, this knowledge is used to compute $\hat{C}(\theta; \pi^{x*})$ and $\hat{N}(\theta; \pi^{x*})$.

specific properties of the firms' information structure). Furthermore, when, in each state θ, the extra subsidy to the innovating firms takes the form of the marginal externality $C_N(\theta, \Lambda)$ exerted by N on the production of the final good (holding all firms' information, technology, and pricing rules fixed), the marginal value that each firm assigns to upgrading its technology coincides with the planner's value in each state (see the proof of Lemma 2). The above properties imply that the private value to information coincides with the social one and hence that all firms have the right incentives to acquire and then use information efficiently. Q.E.D.

The result in Proposition 2 illustrates the power of the Pigouvian logic. When the planner announces that innovating firms will receive a subsidy equal to the *ex-post* (marginal) externality $C_N(\theta, \Lambda)$ that each firm's technology choice exerts on the production of the final good, it re-aligns firms' objective with total welfare, non just at the interim stage but ex-post. The planner can then delegate to firms the computation of the efficient allocation while guaranteeing that, in equilibrium, they acquire and use information efficiently.

The policies of Propositions 1 and 2 also resemble VCG transfers but with the correction operating on the margin instead of the levels.[7] While the VCG transfers eliminate the wedge between the private and the social objectives by making firms' profits (net of the transfers) proportional to their contribution to total welfare, the policies in the above two propositions eliminate the wedge between the *marginal* private and social benefit of varying the firms' decisions.[8]

4 Conclusions

We investigate firms' incentives to learn about the profitability of new technologies when such technologies are affected by investment spillovers. We show that firms can be induced to acquire information about the new technologies efficiency and then use such information in society's best interest through a policy that, in addition to correcting for firms' market power, provides those firms adopting the new technology with a subsidy that makes them internalize the effects of their investments on other firms' production of intermediate and final goods.

The paper shows that the power of Pigouvian corrections extends to economies in which neither the collection nor the usage of information is observable. Similar results obtain in markets in which externalities originate in pollution and/or spillovers from investments in human capital.

In future work, it would be interesting to extend the analysis to economies in which firms, in addition to acquiring information about the profitability of new technologies, expand the set of available products over time and strategically choose when to replace existing products with new ones, thus contributing to the understanding of the innovation diffusion process.

[7]See Bergemann and Välimäki (2002) for the role of VCG payments in mechanism design with endogenous information acquisition.

[8]In our economy with a continuum of infinitesimal firms, VCG payments do not work, as the contribution of each firm to total welfare is zero.

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Appendix

Proof of Lemma 1. Fix π^x and drop it from all expressions to ease the notation. Efficiency requires that any two firms with the same technology employ the same amount of labor. Letting $n(x)$ denote the probability that a firm receiving signal x adopts the new technology, $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the firms adopting the new technology and by those retaining the old one, respectively, we have that the planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta C(\theta)d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta [l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))]^{1+\varepsilon} d\Omega(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N(\theta)-\int_x n(x)\Phi(x|\theta)\right)d\Omega(\theta),$$

where $\Omega(\theta)$ is the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}} \quad \text{(A.1)}$$

with

$$y_1(\theta) = \gamma\Theta(1+\beta N(\theta))^\alpha l_1(\theta)^\psi, \quad \text{(A.2)}$$

and

$$y_0(\theta) = \Theta(1+\beta N(\theta))^\alpha l_0(\theta)^\psi. \quad \text{(A.3)}$$

Using (A.1) and (A.2), the first-order condition with respect to $l_1(\theta)$ can be written as

$$\psi\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta(1+\beta N(\theta))^\alpha\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1} \\ -(l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta)))^\varepsilon = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \quad \text{(A.4)}$$

and using (A.1) and (A.2), we have that the above first order condition reduces to

$$\psi C(\theta)^{\frac{1}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^\varepsilon. \quad \text{(A.5)}$$

Following similar steps, the first order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^\varepsilon. \quad \text{(A.6)}$$

Using (A.2), (A.3), (A.5) and (A.6), we obtain that

$$l_1(\theta) = \gamma^\varphi l_0(\theta), \quad \text{(A.7)}$$

$$L(\theta) = l_0(\theta)\left[(\gamma^\varphi - 1)N(\theta) + 1\right]. \quad \text{(A.8)}$$

Hence, (A.1) becomes

$$C(\theta) = \Theta(1+\beta N(\theta))^\alpha l_0(\theta)^\psi \left((\gamma^\varphi - 1)N(\theta) + 1\right)^{\frac{v}{v-1}}. \quad \text{(A.9)}$$

Next, using (A.6), (A.8) and (A.9), we obtain that

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon-\psi}} \left(\Theta \left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1}{1+\varepsilon-\psi}} \left(\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}. \tag{A.10}$$

Note that $l_0(\theta) > 0$ for all θ. The above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1), for any θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0, 1]$, we have that

$$\mathcal{Q}(\theta) = C_N\left(\theta, \Lambda(\theta)\right) - k - L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right),$$

where $C_N\left(\theta, \Lambda(\theta)\right)$ is the marginal change in the production of the final good that obtains when one changes infinitesimally the proportion N of firms investing in the new technology, starting from $N(\theta)$ and holding $l_0(\theta)$ and $l_1(\theta)$ fixed. Observe that $\Lambda(\theta)$ is the cross-sectional distribution of (n_i, l_i) that obtains at θ when firms make decisions according to $(n_i(x), l_0(\theta), l_1(\theta))$.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right) = f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_{\theta} \mathcal{Q}(\theta) f\left(\theta|x\right)d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all firms receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ invest, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$ do not invest.

Next, use (A.1), (A.2) and (A.3) to obtain that

$$C_N\left(\theta, \Lambda(\theta)\right)) = \tfrac{v}{v-1} C(\theta)^{\frac{1}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + \tfrac{\alpha\beta}{1+\beta N(\theta)} C(\theta).$$

Finally, use (A.5) and (A.6) to observe that

$$L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right) = \psi C(\theta)^{\frac{1-v}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right).$$

We conclude that

$$\mathcal{Q}(\theta) = \left(\frac{v-\psi\left(v-1\right)}{v-1}\right) C(\theta)^{\frac{1}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + C(\theta)\frac{\alpha\beta}{1+\beta N\left(\theta\right)} - k. \tag{A.11}$$

Using (A.2), (A.3), (A.7), and (A.9), after some manipulations, we have that

$$C(\theta)^{\frac{1}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] =$$

$$= \left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1}{v-1}} \Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha} l_0(\theta)^{\psi}\left(\gamma^{\varphi}-1\right). \tag{A.12}$$

Using (A.9), we thus have that

$$\mathcal{Q}(\theta) = ((\gamma^{\varphi} - 1) N(\theta) + 1)^{\frac{v}{v-1}} \Theta (1 + \beta N(\theta))^{\alpha} l_0(\theta)^{\psi} \times \\ \times \left(\frac{\gamma^{\varphi} - 1}{\varphi[(\gamma^{\varphi} - 1) N(\theta) + 1]} + \frac{\alpha\beta}{1 + \beta N(\theta)} \right) - k.$$

Using (A.10) to substitute for the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we finally obtain that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi}{1+\varepsilon-\psi}} \Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}} ((\gamma^{\varphi} - 1) N(\theta) + 1)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)} - 1} (1 + \beta N(\theta))^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}} \times \\ \times \left(\frac{\gamma^{\varphi} - 1}{\varphi} + \frac{\alpha\beta ((\gamma^{\varphi} - 1) N(\theta) + 1)}{1 + \beta N(\theta)} \right) - k.$$

Note that, when the parameters satisfy the conditions in the lemma, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. Such a property implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ, which, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N, then implies that the first-best level of N is increasing in θ. This property in turn suggests that the efficient strategy $\hat{n}(x)$ is monotone. For any $\hat{x}$, then let

$$\bar{N}(\theta|\hat{x}) \equiv 1 - \Phi(\hat{x}|\theta)$$

and

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi}{1+\varepsilon-\psi}} \Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}} \left((\gamma^{\varphi} - 1) \bar{N}(\theta|\hat{x}) + 1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)} - 1} \left(1 + \beta \bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}} \times \\ \times \left(\frac{\gamma^{\varphi} - 1}{\varphi} + \frac{\alpha\beta \left((\gamma^{\varphi} - 1) \bar{N}(\theta|\hat{x}) + 1\right)}{1 + \beta \bar{N}(\theta|\hat{x})} \right) - k.$$

Observe that, under the parameters' restriction in the lemma, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous in $\hat{x}$, strictly increasing, and such that

$$\lim_{\hat{x} \to -\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x} \to +\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits one and only one solution. Denote such a solution by $\hat{x}$. Furthermore, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$ and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. Reintroducing the dependence on π^x, we conclude that, under the assumptions in the lemma, there exists a threshold $\hat{x}(\pi^x)$ satisfying

$$\mathbb{E}\left[\psi^{\frac{\psi}{1+\varepsilon-\psi}} \Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}} \left((\gamma^{\varphi} - 1) \hat{N}(\theta; \pi^x) + 1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)} - 1} \left(1 + \beta \hat{N}(\theta; \pi^x)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}} \times \right. \\ \left. \times \left(\frac{\gamma^{\varphi} - 1}{\varphi} + \frac{\alpha\beta \left((\gamma^{\varphi} - 1) \hat{N}(\theta; \pi^x) + 1\right)}{1 + \beta \hat{N}(\theta; \pi^x)} \right) \right| \hat{x}(\pi^x), \pi^x \right] = k,$$

with $\hat{N}(\theta;\pi^x) = 1 - \Phi(\hat{x}(\pi^x)|\theta;\pi^x)$, such that the investment strategy $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ satisfying the above first-order conditions constitute a solution to the planner's problem. Q.E.D.

Proof of Lemma 2. As in the proof of Lemma 1, we drop π^x from all formulas. We also drop θ when there is no risk of confusion.

Consider first the problem faced by a firm that has innovated. Each such firm chooses p_1 to maximize

$$\frac{p_1 y_1 - W l_1}{P} + T_1\left(\frac{p_1 y_1}{P}\right)$$

taking W and P as given, accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^v, \tag{A.13}$$

with C exogenous to the firm's problem, and accounting for the fact that, given y_1, the amount of labor that the firm needs to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta\left(1+\beta N\right)^\alpha}\right)^{\frac{1}{\psi}}. \tag{A.14}$$

The first-order condition with respect to p_1 is given by

$$(1-v)\,CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(p_1y_1/P\right)}{dr}\frac{d\left(p_1y_1\right)}{dp_1} = 0.$$

Combining (A.13) with (A.14), we have that

$$l_1 = \left(\frac{CP^v}{p_1^v\gamma\Theta\left(1+\beta N\right)^\alpha}\right)^{1/\psi}, \tag{A.15}$$

from which we obtain that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}.$$

Using (A.13), we also have that

$$\frac{d\left(p_1y_1\right)}{dp_1} = (1-v)\,CP^vp_1^{-v}.$$

Replacing these last formulas into the above first-order condition, and using (A.13) to express y_1 as $y_1 = CP^vp_1^{-v}$, we obtain that

$$(1-v)\,\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT\left(p_1y_1/P\right)}{dr}\frac{(1-v)\,y_1}{P} = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\frac{1-v}{v}\frac{y_1p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1\left(p_1y_1/P\right)}{dr}\frac{y_1p_1}{P} = 0. \tag{A.16}$$

Next use (1) and (4), along with (A.7) and (A.9), to observe that, in any equilibrium implementing the efficient use of information,

$$\hat{p}_1 = \left(\left(\gamma^{\varphi} - 1 \right) \hat{N} + 1 \right)^{\frac{1}{v-1}} \gamma^{\frac{\varphi}{1-v}} \hat{P} \left(\theta; \pi^x \right),$$

$$\hat{p}_0 = \left(\left(\gamma^{\varphi} - 1 \right) \hat{N} + 1 \right)^{\frac{1}{v-1}} \hat{P} \left(\theta; \pi^x \right),$$

and, using (3),

$$\hat{P} \left(\theta; \pi^x \right) = \left(\hat{p}_1^{1-v} \hat{N} + \hat{p}_0^{1-v} \left(1 - \hat{N} \right) \right)^{\frac{1}{1-v}}.$$

Now suppose that all other firms follow policies that induce the efficient allocations. Hereafter, we use "hats" to denote the efficient choices by such firms as well as the corresponding aggregate variables. Observe that market-clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon},$$

with $\hat{L}$ as defined in (A.8). Recall that, by virtue of Condition (A.5), efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon} \hat{l}_1 = 0.$$

Accordingly, from (A.16), we have that

$$\frac{1-v}{v} \frac{y_1 p_1}{\hat{P}} + \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1 \left(p_1 y_1 / P \right)}{dr} \frac{y_1 p_1}{\hat{P}} = 0. \tag{A.17}$$

From (A.13) we obtain that

$$\hat{y}_1^{-\frac{1}{v}} = \hat{C}^{-\frac{1}{v}} \frac{\hat{p}_1}{\hat{P}},$$

so that the first-order condition (A.17) becomes

$$\frac{1-v}{v} \frac{y_1 p_1}{\hat{P}} + \frac{\hat{y}_1 \hat{p}_1}{\hat{P}} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1 \left(p_1 y_1 / P \right)}{dr} \frac{y_1 p_1}{\hat{P}} = 0.$$

Multiplying all the addenda in the last condition by $\hat{P} / \left(y_1 p_1 \right)$, we obtain that

$$\frac{1-v}{v} + \frac{\hat{y}_1 \hat{p}_1}{y_1 p_1} \frac{l_1}{\hat{l}_1} + \frac{1-v}{v} \frac{dT_1 \left(p_1 y_1 / P \right)}{dr} = 0. \tag{A.18}$$

Condition (A.15) allows us to express the ratio between the amount of labor that the firm needs to hire and the efficient one in terms of the ratio between the firm's own price and the efficient one

$$\frac{l_1}{\hat{l}_1} = \left(\frac{\hat{p}_1}{p_1} \right)^{\frac{v}{\psi}}.$$

The ratio between the efficient revenue and the one obtained by the firm choosing p_1 can also be expressed in terms of the ratio between the firm's price and the efficient one. In fact, using (A.13), we have that

$$\frac{\hat{y}_1 \hat{p}_1}{y_1 p_1} = \left(\frac{\hat{p}_1}{p_1} \right)^{1-v}.$$

The first-order condition (A.18) thus becomes

$$\frac{1-v}{v}+\left(\frac{\hat{p}_1}{p_1}\right)^{1-v+\frac{v}{\psi}}+\frac{1-v}{v}\frac{dT_1\left(p_1y_1/P\right)}{dr}=0.$$

For the rule T to implement the efficient allocation, it must be that $p_1 = \hat{p}_1$ solves the above first-order condition. This is the case if and only if

$$\frac{1}{v}=\frac{v-1}{v}\frac{dT_1\left(\hat{p}_1\hat{y}_1/\hat{P}\right)}{dr}.$$

Because $\hat{p}_1\hat{y}_1/\hat{P}$ is state dependent, thus have that T_1 must be affine in r and satisfy

$$T_1\left(r\right)=s+\frac{1}{v-1}r, \tag{A.19}$$

with s invariant in r. Furthermore, one can show that, when all other firms follow policies that induce the efficient allocations, under the transfer rule (A.19), the payoff of each firm that adopted the new technology is quasi-concave in its own price, which implies that the above first-order condition is also sufficient for the firm to optimally choose $p_1 = \hat{p}_1$.

Applying similar arguments to those firms that retain the old technology, we have that a policy that provides a transfer equal to

$$T_0\left(r\right)=\frac{1}{v-1}r \tag{A.20}$$

to those firms retaining the old technology induces such firms to set a price equal to $\hat{p}_0$ in each state θ (equivalently, to hire the efficient amount of labor $\hat{l}_0$).

Next, consider the firms' technology adoption. Since firms do not know θ when they choose their technology, we reintroduce θ in the notation to highlight the uncertainty that they face. When the transfer rule T satisfies the conditions above, each firm anticipates that, if it innovates, in each state θ it will then set a price $\hat{p}_1(\theta)$, hire $\hat{l}_1(\theta)$ and produce $\hat{y}_1(\theta)$, whereas, if it retains the old technology, it will then set a price $\hat{p}_0(\theta)$, hire $\hat{l}_0(\theta)$ and produce $\hat{y}_0(\theta)$. As a result of these observations, each firm receiving a signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\mathcal{R}(\theta)|x\right]>0,$$

and retain the old one if the above inequality is reversed, where

$$\mathcal{R}(\theta)\equiv\hat{r}_1\left(\theta\right)-\hat{r}_0\left(\theta\right)-\frac{\hat{W}(\theta)}{\hat{P}\left(\theta\right)}\left(\hat{l}_1(\theta)-\hat{l}_0(\theta)\right)+T_1\left(\theta,\hat{r}_1(\theta)\right)-T_0\left(\hat{r}_0(\theta)\right)-k$$

is the extra profit (net of the subsidy) from adopting the new technology relative to retaining the old one, with

$$\hat{r}_f(\theta)\equiv\frac{\hat{p}_f\left(\theta\right)\hat{y}_f\left(\theta\right)}{\hat{P}\left(\theta\right)}$$

denoting the (real) revenue the firm generates in state θ by following the efficient policies, with $f=0$ in case the firm retains the old technology and $f=1$ if it innovates.

Recall that the Dixit and Stiglitz demand system (A.13) implies that

$$\hat{p}_f(\theta) = \hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$$

so that

$$\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}} \text{ for } f = 0, 1. \tag{A.21}$$

Also recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\mathcal{R}(\theta)$ can be rewritten as follows

$$\mathcal{R}(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) - k,$$

which, using (A.5) and (A.6), becomes

$$\mathcal{R}(\theta) = (1-\psi)\,\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_1(\theta)) - k.$$

Therefore, when the policy takes the form in (A.19) and (A.20), with $s(\theta)$ possibly depending on θ, using (A.21), we have that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\mathcal{R}(\theta)$ can be written as

$$\mathcal{R}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s(\theta) - k. \tag{A.22}$$

Now recall that efficiency requires that each entrepreneur invests if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$, and does not invest if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$, where, as shown in (A.11),

$$\mathcal{Q}(\theta) = \left(\tfrac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)\tfrac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

We conclude that, for the proposed policy to induce efficiency in information usage, it suffices that $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ whenever $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$ whenever $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$. When the economy satisfies the properties of Lemma 1, $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ has the single-crossing property, turning from negative to positive at $x = \hat{x}$. In this case, it suffices that $\mathbb{E}[\mathcal{R}(\theta)|\hat{x}] = 0$, and that $\mathbb{E}[\mathcal{R}(\theta)|x]$ has the single-crossing property, turning from negative to positive at $x = \hat{x}$. When the policy takes the form in (A.19) and (A.20), the above last two properties hold if $s(\theta)$ is non-decreasing in θ and satisfies

$$\mathbb{E}[s(\theta)|\,\hat{x}] = \mathbb{E}\left[\left.\frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)}\right|\hat{x}\right]. \tag{A.23}$$

To see this, use (A.10) and (A.12) to rewrite the first term in (A.22) as

$$\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)=$$

$$=\psi^{\frac{\psi}{1+\varepsilon-\psi}}\Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right),$$

and note that this expression is increasing in N (for given θ) and increasing in θ (for given N). Hence, when the second term in (A.22), which is equal to $s\left(\theta\right)$, is also non-decreasing in θ, $\mathcal{R}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]$ is non-decreasing in x. Because Condition (A.23) implies that $\mathbb{E}\left[\mathcal{R}(\theta)|\hat{x}\right]=0$, we then have that $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]>0$ for $x>\hat{x}$ and $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]<0$ for $x<\hat{x}$. The simple policy of Lemma 2 clearly satisfies all the above conditions and hence implements the efficient allocation. Q.E.D.

Proof of Lemma 3. The proof is in two parts. Part 1 characterizes the efficient precision of information, π^{x*}. Part 2 uses the characterization in part 1 to establish the claim in the lemma.

Part 1. Observe that, for any precision of private information π^x, irrespective of whether the economy satisfies the parameter restrictions in Lemma 1, Conditions (A.8) and (A.9) imply that ex-ante welfare can be expressed as

$$\mathbb{E}\left[\mathcal{W}|\pi^x\right]=$$

$$=\int_\theta\Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha}\hat{l}_0\left(\theta;\pi^x\right)^{\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}}d\Omega\left(\theta\right)+$$

$$-k\int_\theta\hat{N}\left(\theta;\pi^x\right)d\Omega\left(\theta\right)-\int_\theta\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right]^{1+\varepsilon}d\Omega\left(\theta\right)-\mathcal{I}(\pi^x).$$

Using the envelope theorem, we then have that, no matter whether the parameter restriction in the lemma hold, the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[C^*\left(\theta\right)\left(\frac{\alpha\beta}{1+\beta\hat{N}^*\left(\theta\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*\left(\theta\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]$$
$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]+\mathbb{E}\left[\hat{l}_0^*(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*\left(\theta\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x},\quad\text{(A.24)}$$

where

$$\hat{N}^*(\theta)\equiv\hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta)\equiv\hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta)\equiv\hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta)\equiv\gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},\quad\text{(A.25)}$$

$$\hat{y}_0^*(\theta)\equiv\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},\quad\text{(A.26)}$$

and

$$\hat{C}^*(\theta) = \hat{Y}^*(\theta) \equiv \left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta) + \hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}}.$$

Part 2. We show that, for the firms to acquire information of precision π^{x*} (and then use it efficiently), in addition to the conditions of Lemma 2 (specialized to $\pi^x = \pi^{x*}$), $s(\theta)$ must satisfy Condition (8).

To see this, suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy in the lemma, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π_i^x. This price is equal to $\hat{p}_1^*$ if the firm adopted the new technology and $\hat{p}_0^*$ if the firm retained the old technology, where $\hat{p}_1^*(\theta) \equiv \hat{p}_1(\theta;\pi^{x*})$ and $\hat{p}_0^*(\theta) \equiv \hat{p}_0(\theta;\pi^{x*})$.

Now let $\hat{W}^*(\theta) \equiv \hat{W}(\theta;\pi^{x*})$ and

$$\hat{P}^*(\theta) \equiv \left(\hat{p}_1^*(\theta)^{1-v}\hat{N}^*(\theta) + \hat{p}_0^*(\theta)^{1-v}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the transfer rule, so as to ease the exposition, we have that firm i's value function, for any choice π_i^x of its private information, is equal to

$$\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] + \\ + \mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i adopts the new technology when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = (\hat{p}_1^*\hat{y}_1^*)/\hat{P}^*$ and $\hat{r}_0^* = (\hat{p}_0^*\hat{y}_0^*)/\hat{P}^*$ under the new and the old technology, respectively.

From (A.13), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f = 0,1$. Substituting $\hat{r}_f^*$ into the above expression for $\Pi_i(\varsigma;\pi_i^x)$ and using (A.7), (A.25), and (A.26), we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^\alpha\right)^{\frac{v-1}{v}}\left(\left(\gamma^\varphi-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ &- \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^\varphi-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] + \\ &+ \mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$

$$-\,\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right]+$$

$$+\,\mathbb{E}\left[\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}. \quad \text{(A.27)}$$

where

$$\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}$$

is the marginal effect of varying π_i^x on the probability of adopting the new technology at θ, holding the rule ς fixed.

Using again the fact that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$, $f=0,1$, along with (A.25) and (A.26), we obtain that

$$\hat{r}_1^*-\hat{r}_0^* = \hat{C}^{*\frac{1}{v}}\Theta^{\frac{v-1}{v}}\left(1+\beta\hat{N}^*\right)^{\alpha\frac{v-1}{v}}\left(\gamma^{\frac{v-1}{v}}\hat{l}_1^{*\psi\frac{v-1}{v}}-\hat{l}_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (A.7) and the structure of the proposed transfer policy, we have that

$$\hat{T}_1^*-\hat{T}_0^* = s+\frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (A.27), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}l_0^{*\psi\frac{v-1}{v}}\right]+$$

$$-\,\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Now, recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$ where $\hat{n}^*(x) \equiv \hat{n}(x;\pi^{x*})$ is the efficient technology choice for a firm receiving signal x after acquiring information of precision π^{x*}.

Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$

$$-\,\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}$$

where $\partial \hat{N}^*/\partial \pi^x$ is the marginal change in the measure of firms adopting the new technology that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding $\hat{n}^*$ fixed. Note that in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^*(\theta)^{\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\hat{l}_0^*(\theta)^{\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\frac{\hat{W}^*(\theta)}{\hat{P}^*(\theta)}\left((\gamma^{\varphi}-1)\hat{l}_0^*(\theta)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right)\right]+$$
$$+\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x},\quad\text{(A.28)}$$

where we reintroduced θ in the arguments of the various functions.
Next, use the fact that the equilibrium wage satisfies

$$\frac{\hat{W}^*(\theta)}{\hat{P}^*(\theta)}=\hat{L}^*(\theta)^{\varepsilon}$$

and (A.7) to note that

$$\frac{\hat{W}^*(\theta)}{\hat{P}^*(\theta)}=\left(\hat{l}_1^*(\theta)\hat{N}^*(\theta)+\hat{l}_0^*(\theta)\left(1-\hat{N}^*(\theta)\right)\right)^{\varepsilon}=\hat{l}_0^*(\theta)^{\varepsilon}\left((\gamma^{\varphi}-1)\hat{N}^*(\theta)+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^*(\theta)^{\frac{1}{v}}=\hat{C}^*(\theta)\hat{C}^*(\theta)^{\frac{1-v}{v}}$, along with Condition (A.9) (computed at π^{x*}), we have that

$$\hat{C}^*(\theta)^{\frac{1}{v}}=\hat{C}^*(\theta)\left(\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\right)^{\frac{1-v}{v}}\hat{l}_0^*(\theta)^{\psi\frac{1-v}{v}}\frac{1}{(\gamma^{\varphi}-1)\hat{N}^*(\theta)+1}.$$

It follows that (A.28) is equivalent to

$$\mathbb{E}\left[\hat{C}^*(\theta)\frac{v(\gamma^{\varphi}-1)}{(v-1)\left((\gamma^{\varphi}-1)\hat{N}^*(\theta)+1\right)}\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]+$$
$$-\mathbb{E}\left[\hat{l}_0^*(\theta)^{1+\varepsilon}\left((\gamma^{\varphi}-1)\hat{N}^*(\theta)+1\right)^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}.\quad\text{(A.29)}$$

In part 1, we showed that π^{x*} is given by the solution to (A.24). Comparing (A.29) with (A.24), we thus have that the policy in Lemma 3 induces the firms to acquire the efficient precision of private information only if, in addition to $s(\theta)$ being non-decreasing and satisfying Condition (7) in Lemma 2, it also satisfies the following condition

$$\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right]=\mathbb{E}\left[\hat{C}^*(\theta)\left(\frac{\alpha\beta}{1+\beta\hat{N}^*(\theta)}\right)\frac{\partial\hat{N}^*(\theta)}{\partial\pi^x}\right],$$

which is equivalent to Condition (8) in the lemma. Q.E.D.

Subsidies to Technology Adoption when Firms' Information is Endogenous

Supplement

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Abstract

This document contains an extension to a richer family of economies in which the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

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S. 1 Richer Economies

Consider the following economy in which the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good. This last assumption is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property in turn isolates the frictions (and associated inefficiencies) that originate in the endogenous dispersion of information at the time technology choices are made from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's *market valuation* taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - W l_i}{P} + T\right)\middle| x_i, \pi_i^x\right] - kn_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open. The household faces a *"cash-in-advance"* constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is

$$PY \leq M.$$

The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The benevolent planner maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary rule $M(\cdot)$ and a transfer rule $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in the baseline model, with the exception that prices are set under dispersed information about θ (that is, with each p_i based on x_i instead of θ) and that the supply of money is state-dependent and governed by the monetary rule $M(\cdot)$. This richer economy is consistent with most of the assumptions typically in the pertinent literature.

S. 1 Efficient Allocation

The following proposition characterizes the efficient allocation in this richer economy.

Proposition S.1. *(1) Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$. *Suppose that* $\gamma^{\varphi} \geq 1+\beta$, $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$, *and* $0 \leq R \leq \bar{R}$, *with* $\bar{R} \equiv 1 - \frac{(v-1)(1+\varepsilon)}{(1+\varepsilon)v+\varepsilon\psi(1-v)}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency requires that* $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$
$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)}+\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\right)\middle|\hat{x}(\pi^x),\pi^x\right]=k,$$

$$\hat{N}(\theta;\pi^x) = 1-\Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^{x*})+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad \text{(S.1)}$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x), \quad \text{(S.2)}$$

where $\Theta \equiv \exp(\theta)$.
(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}+\frac{v}{v-1}\frac{(\gamma^{\varphi}-1)}{\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^{x*})+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^{x*})+1\right]^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^x}.$$

The restriction $0 \leq R \leq \bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease "too quickly" with C. Along with the other restrictions in the proposition, which are the same as in the baseline model, this property implies that the efficient investment strategy is monotone. When, instead, $R > \bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing in the new technology. As we clarify below, our key results extend to this case but the exposition is less transparent.

S. 2 Equilibrium Allocation

Firms choose both their technology and the price for their intermediate goods under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and the total investment N in the new technology. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units starting from a level of consumption equal to C.

Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each firm that invests in the new technology. The corresponding functions for the firms that retain the old technology are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[1]

Definition S.1. Given the monetary rule $M(\cdot)$ and the transfer policy $T(\cdot)$, an **equilibrium** is a precision π^x along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.2. The monetary rule $M^*(\cdot)$ along with the transfer rule $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as an equilibrium. That is, if they induce all firms to choose the efficient precision of information π^{x*}, follow the efficient rule $\hat{n}(x;\pi^{x*})$ to determine whether or not to upgrade their technology, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$ and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, in each state θ, induce demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), let $\hat{M}(\theta;\pi^x)$ denote the optimal monetary rule. Such a rule specifies the amount of money supplied to the representative

[1] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of firms N adopting the new technology depends on the precision π^x of the firms' information.

household in each state θ. It is designed so that, when all firms choose their technology according to $\hat{n}(x;\pi^x)$ and set prices according to $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$, the resulting employment decisions coincide with the efficient ones $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$.

The following lemma characterizes the monetary policy $\hat{M}(\theta;\pi^x)$.

Lemma S.1. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\theta;\pi^x)$ that, together with some transfer policy $\hat{T}(\cdot)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

where m is an arbitrary positive constant. The monetary rule $\hat{M}(\theta;\pi^x)$ induces all firms with the same technology to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\theta;\pi^x)$ implements the efficient allocation by inducing firms to disregard their private information about the fundamentals and set prices based only on the adopted technology. That prices do not respond to the firms' information about θ is necessary to avoid allocative distortions in the induced employment and productions decisions. Relative prices must not vary with the firms' signals about θ when the latter signals are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed rule, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the amount of money supplied to the realization of θ in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma S.1 in turn permits us to establish the following result:

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the transfer policy*

$$T_0^*\left(r\right) = \frac{1}{v-1}r,$$

and

$$T_1^*\left(\theta,r\right) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{1}{v-1}r.$$

along with the monetary policy

$$M^*(\theta) = m\hat{l}_0(\theta;\pi^{x*})^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}}$$

are optimal (i.e., implement the efficient acquisition and usage of information as an equilibrium).

The monetary policy in the proposition (which belongs to the family in Lemma S.1, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price rigidity by replicating the same allocations as under flexible prices. When paired with the transfer policy in the proposition, it guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x;\pi^{x*})$ to choose which technology to operate and then set prices $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$ that induce the efficient labor demands and hence the efficient production of the intermediate and final goods. This is accomplished through a transfer policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, it realigns the private value of upgrading the technology with the social value through an additional subsidy to the innovating firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

makes each firm internalize the marginal effect of the investment in the new technology on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

S. 2 Proofs.

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part (1). We drop π^x from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x adopts the new technology, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the firms adopting the new technology and by those retaining the old one, respectively. The planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k\int_\theta N(\theta) d\Omega(\theta) + \\ - \frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ - \int_\theta \mathcal{Q}(\theta)\left(N(\theta) - \int_x n(x)\Phi(x|\theta)\right) d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}} \tag{S.3}$$

with

$$y_1(\theta) = \gamma\Theta\left(1+\beta N(\theta)\right)^\alpha l_1(\theta)^\psi, \tag{S.4}$$

and

$$y_0(\theta) = \Theta\left(1+\beta N(\theta)\right)^\alpha l_0(\theta)^\psi. \tag{S.5}$$

Using (S.3) and (S.4), the first-order condition with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R}\left(y_1(\theta)^{\frac{v-1}{v}}N(\theta)+y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}}\left(\gamma\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}l_1(\theta)^{\psi\frac{v-1}{v}-1} - \left(l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right)^{\varepsilon}=0.$$

Letting

$$L(\theta)\equiv l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta)), \tag{S.6}$$

and using (S.3) and (S.4), we have that the first order condition above reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}}y_1(\theta)^{\frac{v-1}{v}}=l_1(\theta)L(\theta)^{\varepsilon}. \tag{S.7}$$

Following similar steps, the first order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1-vR}{v}}y_0(\theta)^{\frac{v-1}{v}}=l_0(\theta)L(\theta)^{\varepsilon}. \tag{S.8}$$

Using (S.4) and (S.5), the ratio between (S.7) and (S.8) can be written as

$$\gamma^{\frac{v-1}{v}}\left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi\frac{v-1}{v}}=\frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta)=\gamma^{\varphi}l_0(\theta). \tag{S.9}$$

Notice that (S.9) implies that, at the efficient allocation, the total labor demand, as defined in (S.6), is equal to

$$L(\theta)=l_0(\theta)\left[\left(\gamma^{\varphi}-1\right)N(\theta)+1\right]. \tag{S.10}$$

Using (S.4) and (S.5), we can also write aggregate consumption as

$$C(\theta)=\Theta\left(1+\beta N(\theta)\right)^{\alpha}\left(\gamma^{\frac{v-1}{v}}l_1(\theta)^{\psi\frac{v-1}{v}}N(\theta)+l_0(\theta)^{\psi\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (S.9), we can rewrite the latter expression as

$$C(\theta)=\Theta\left(1+\beta N(\theta)\right)^{\alpha}l_0(\theta)^{\psi}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{v}{v-1}}. \tag{S.11}$$

Next, use (S.9) and (S.11) to rewrite (S.8) as

$$\psi\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{\frac{1-vR}{v}}l_0(\theta)^{\psi\frac{1-vR}{v}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1-vR}{v-1}}\times \times\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}l_0(\theta)^{\psi\frac{v-1}{v}}=l_0(\theta)L(\theta)^{\varepsilon},$$

which, using (S.10), we can express as

$$\psi\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1-vR}{v-1}} =l_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}} \left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{S.12}$$

and by (S.9).

Note that $l_0(\theta) > 0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0, 1]$, we have that

$$\mathcal{Q}(\theta) \equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)} - k - L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed and varying the proportion of firms investing into the new technology and the amounts that each firm produces for given technology choice when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right) = f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_{\theta} \mathcal{Q}(\theta)f\left(\theta|x\right)d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all managers receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ adopt the new technology, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$ retain the old one.

Next, use (S.3) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)} = \tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] +$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (S.4) and (S.5) to observe that

$$y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$= \tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N\left(\theta\right) + y_0(\theta)^{\frac{v-1}{v}}\left(1-N\left(\theta\right)\right)\right)$$

$$= \tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses again (S.3).

Finally, use (S.7) and (S.8) to observe that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta)=\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N\left(\theta\right)}-k.$$

Using (S.4), (S.5), (S.9), and (S.11), after some manipulations, we have that

$$\begin{aligned}C(\theta)^{\frac{1-vR}{v}}&\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=\\&=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right).\end{aligned}\quad\text{(S.13)}$$

Using (S.11), we also have that

$$C(\theta)^{1-R}=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\begin{aligned}\mathcal{Q}(\theta)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}&\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\times\\&\times\left(\frac{\gamma^{\varphi}-1}{\varphi[\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1]}+\frac{\alpha\beta}{1+\beta N\left(\theta\right)}\right)-k.\end{aligned}$$

Next, recall that the optimal labor demand for the firms retaining the old technology is given by (S.12). Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\begin{aligned}\mathcal{Q}(\theta)=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}&\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\\&\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k.\end{aligned}$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties in turn suggest that the optimal investment policy is monotone. For any $\hat{x}$, then let

$$\bar{N}(\theta|\hat{x})\equiv 1-\Phi(\hat{x}|\theta)$$

denote the measure of firms investing in the new technology at θ when firms follow the monotone rule $n(x)=\mathbb{I}(x>\hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) \bar{N}(\theta|\hat{x})+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta \bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right) \bar{N}(\theta|\hat{x})+1\right)}{1+\beta \bar{N}(\theta|\hat{x})}\right)-k$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta)=\bar{N}(\theta|\hat{x})$.
Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous and strictly increasing in $\hat{x}$, and is such that

$$\lim_{\hat{x}\to-\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]<0<\lim_{\hat{x}\to+\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]=0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we then have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]<0$ for $x<\hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]>0$ for $x>\hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(x\geq\hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) \hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta \hat{N}\left(\theta;\pi^x\right)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times\right.$$
$$\left.\times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right) \hat{N}\left(\theta;\pi^x\right)+1\right)}{1+\beta \hat{N}\left(\theta;\pi^x\right)}\right)\right| \hat{x}(\pi^x),\pi^x\Bigg]=k,$$

with $\hat{N}\left(\theta;\pi^x\right)=1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.
Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}\left(x;\pi^x\right)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (S.1) and (S.2) in the proposition and $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x]>0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right) \hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta \hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right) \hat{N}(\theta;\pi^x)+1\right)}{1+\beta \hat{N}(\theta;\pi^x)}\right)-k$$

with $\hat{N}(\theta;\pi^x)=\int_{\theta} \hat{n}\left(x;\pi^x\right) d\Phi\left(x|\theta,\pi^x\right)$.

Part (2). For any precision of private information π^x, use Conditions (S.10) and (S.11) in Part (1) to write ex-ante welfare as

$$
\begin{aligned}
&\mathbb{E}\left[\mathcal{W}|\pi^x\right] = \\
&= \frac{1}{1-R}\int_\theta \Theta^{1-R}\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha(1-R)}\hat{l}_0\left(\theta;\pi^x\right)^{\psi(1-R)}\left(\left(\gamma^\varphi-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}(1-R)}d\Omega\left(\theta\right)+ \\
&\qquad -k\int_\theta \hat{N}\left(\theta;\pi^x\right)d\Omega\left(\theta\right)-\int_\theta\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[\left(\gamma^\varphi-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right]^{1+\varepsilon}d\Omega\left(\theta\right)-\mathcal{I}(\pi^x).
\end{aligned}
$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$
\begin{aligned}
&\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x} = \\
&= \mathbb{E}\left[\hat{C}\left(\theta;\pi^x\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}+\frac{v\left(\gamma^\varphi-1\right)}{\left(v-1\right)\left(\left(\gamma^\varphi-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+ \\
&-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+\mathbb{E}\left[\hat{l}_0\left(\theta;\pi^x\right)^{1+\varepsilon}\left(\left(\gamma^\varphi-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^\varepsilon\left(\gamma^\varphi-1\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]-\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.
\end{aligned}
$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Using (S.7) and (S.8), we have that

$$
\begin{aligned}
\hat{l}_1(\theta)\hat{L}\left(\theta\right)^\varepsilon &= \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}, \\
\hat{l}_0(\theta)\hat{L}\left(\theta\right)^\varepsilon &= \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.
\end{aligned}
$$

The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, the prices set by any two firms adopting the same technology coincide, so that they are independent of the signal x. Let $\hat{p}_1$ be the (state-invariant) price set by the firms investing in the new technology and $\hat{p}_0$ that set by firms retaining the old technology. Let $\hat{P}(\theta)$ denote the price of the final good when all firms follow the efficient policies. Efficiency requires that such prices satisfy

$$
\hat{l}_1(\theta)\hat{L}\left(\theta\right)^\varepsilon = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_1\right)^{v-1}, \tag{S.14}
$$

$$
\hat{l}_0(\theta)\hat{L}\left(\theta\right)^\varepsilon = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_0\right)^{v-1}, \tag{S.15}
$$

from which we obtain that

$$
\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},
$$

which, using (S.9), implies that

$$
\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0.
$$

The price of the final good is then equal to

$$\hat{P}(\theta) = \left((\gamma^{\varphi} - 1)\, \hat{N}(\theta) + 1 \right)^{\frac{1}{1-v}} \hat{p}_0. \tag{S.16}$$

Combining the cash-in-advance constraint $M = PC$ with (S.15), we then have that

$$\hat{l}_0(\theta) \hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)^{1-R} \hat{P}(\theta)^{v+R-2}\, \hat{p}_0^{1-v}$$

and therefore

$$\hat{l}_0(\theta) \hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)^{1-R} \left((\gamma^{\varphi} - 1)\, \hat{N}(\theta) + 1 \right)^{\frac{v+R-2}{1-v}} \hat{p}_0^{R-1},$$

where we also used (S.16). Finally, using Condition (S.10), we obtain that

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi} \hat{l}_0(\theta)^{1+\varepsilon} \left((\gamma^{\varphi} - 1)\, \hat{N}(\theta) + 1 \right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}} \hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.14). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m^{1-R} = \frac{1}{\psi} \hat{p}_0^{1-R}$. Q.E.D.

Proof of Proposition S.2. The proof is in two steps and establishes a more general result than the one in the proposition. Step 1 fixes the precision of information and identifies a condition on the transfer policy $T(\cdot)$ that guarantees that, when $T(\cdot)$ is paired with the monetary policy of Lemma S.1, and the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Step 2 identifies an additional restriction on the transfer policy that, when combined with the condition in Step 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in Steps 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $M(\cdot)$ and $T(\cdot)$ are the specific policies of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Step 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when this is no risk of confusion.

Consider first the pricing decision of a firm that adopts the new technology. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R} \left(\frac{p_1 y_1 - W l_1}{P} + T_1(r_1) \right) \middle| x \right], \tag{S.17}$$

where $r_1 = p_1 y_1 / P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C \left(\frac{P}{p_1} \right)^{v} \tag{S.18}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma \Theta (1 + \beta N)^{\alpha}} \right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.17) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R}\left((1-v)\,CP^{v-1}p_1^{-v}-\frac{W}{P}\frac{dl_1}{dp_1}+\frac{1}{P}\frac{dT_1\,(r_1)}{dr}\frac{d(p_1y_1)}{dp_1}\right)\middle|\,x\right]=0. \tag{S.19}$$

Using the fact that

$$\frac{dl_1}{dp_1}=-\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S.20}$$

$$\frac{d\,(p_1y_1)}{dp_1}=(1-v)\,CP^{v}p_1^{-v},$$

and (S.18), we have that (S.19) can be rewritten as

$$\mathbb{E}\left[C^{-R}\left((1-v)\,\frac{y_1}{P}+\frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1}+\frac{dT_1\,(r_1)}{dr}\frac{(1-v)\,y_1}{P}\right)\middle|\,x\right]=0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}C^{-R}\frac{y_1p_1}{P}+\frac{1}{\psi}C^{-R}\frac{W}{P}l_1+\frac{1-v}{v}C^{-R}\frac{dT_1\,(r_1)}{dr}\frac{y_1p_1}{P}\middle|\,x\right]=0. \tag{S.21}$$

Suppose that all other firms follow policies that induce the efficient allocations (meaning that they follow the rule $\hat{n}(x)$ to determine which technology to use and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the technology they adopted but not on the signal x, as in the proof of Lemma S.1). Hereafter, we add "hats" to all relevant variables to highlight that these are computed under the efficient policies.

Observe that market clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}}=\hat{L}^{\varepsilon}, \tag{S.22}$$

and recall that, as established in the Proof of Proposition S.1,

$$\hat{L}=\hat{l}_0\left[(\gamma^{\varphi}-1)\,\hat{N}+1\right].$$

Also consider that efficiency requires that

$$-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0.$$

Accordingly, using Condition (S.21), we have that, each firm adopting the new technology finds it optimal to set the price $\hat{p}_1$ only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}C^{-R}\frac{dT_1\,(\hat{r}_1)}{dr}\hat{r}_1\middle|\,x\right]=0, \tag{S.23}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.18), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (S.23) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\,(\hat{r}_1)}{dr}\hat{r}_1\middle|\,x\right]=0,$$

or, equivalently,

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\Bigg|\,x\right]=0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr=1/(v-1)$, the firm's first-order condition is satisfied. Furthermore, one can verify that, under the proposed transfer rule, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1=\hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r)=\frac{r}{v-1}+s=\frac{1}{v-1}\left(\frac{p_1y_1}{P}\right)+s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.17) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}l_1+s\left(\theta\right)\right)\Bigg|\,x\right].$$

Using (S.18) and (S.20), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\Bigg|\,x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\Bigg|\,x\right].$$

From the analysis above, we have that, when $p_1=\hat{p}_1$, $y_1=\hat{y}_1$ and $l_1=\hat{l}_1$ in each state θ. Furthermore, no matter x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1=\hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\Bigg|\,x\right]=0. \tag{S.24}$$

Using (S.24), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1=\hat{p}_1$, is negative. Because the firm's objective function has a unique stationary point at $p_1=\hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the firms retaining the old technology, we have that a transfer policy that pays each firm retaining the old technology a transfer equal to $T_0(r)=r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' technology choice. Hereafter, we reintroduce θ in the notation. When

$$T_0\left(r\right)=\frac{1}{v-1}r \tag{S.25}$$

and

$$T_1\left(\theta,r\right)=s(\theta)+\frac{1}{v-1}r, \tag{S.26}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by innovating, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by retaining the old technology, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta) \equiv \hat{C}(\theta)^{-R}\left(\hat{r}_1(\theta) - \hat{r}_0(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, after adopting the new technology and retaining the old one, respectively. Each firm receiving signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0,$$

and retain the old technology if the above inequality is reversed.

Recall from (S.18) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}(\theta)\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}}$ for $f = 0, 1$. Also recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1)

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon},$$

and

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon},$$

we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) = (1-\psi)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Next, use (S.18) to note that

$$\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}},$$

$f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v - \psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}s(\theta) - k. \quad \text{(S.27)}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm adopts the new technology if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and retains the old one if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient technology adoption rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.13) and (S.12), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \left((\gamma^{\varphi}-1)\hat{N}(\theta)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta\hat{N}(\theta)\right)^{\alpha}\right)^{1-R}\hat{l}_0(\theta)^{\psi(1-R)}(\gamma^{\varphi}-1),$$

and

$$\hat{l}_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left((\gamma^{\varphi}-1)\hat{N}(\theta)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, the first addendum in (S.27) can be rewritten as

$$\left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}(1+\beta N(\theta))^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}(\theta)^{-R}s(\theta)$ in (S.27) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the innovating firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a) $\hat{C}(\theta)^{-R}s(\theta)$ non-decreasing in θ;

(b)

$$\mathbb{E}\left[\hat{C}(\theta)^{-R}s(\theta)\middle|\hat{x}\right] = \mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta)^{1-R}}{1+\beta\hat{N}(\theta)}\middle|\hat{x}\right].$$

The analysis above also reveals that, when the transfer policy takes the form in (S.25) and (S.26) with

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)}$$

for all θ, and the monetary policy takes the form in Lemma S.1, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient technology rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Step 2. We now show that, when the economy satisfies the conditions in Proposition S.1, for the transfer policy in (S.25) and (S.26) to implement the efficient acquisition and usage of information when paired with the monetary policy

$$M^*(\theta) = m\hat{l}_0\left(\theta;\pi^{x*}\right)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}}$$

the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in Step 1, must also be such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient technology and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed transfer and monetary rules, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after adopting the new technology and equal to $\hat{p}_0^*$ after retaining the old one, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta)\equiv\hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta)\equiv\hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta)\equiv\hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta)\equiv\gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta)\equiv\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta)=\hat{Y}^*(\theta)\equiv\left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta)\equiv\hat{W}(\theta;\pi^{x*})$$

and

$$\hat{P}^*\left(\theta\right)\equiv\left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the transfer rule, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to

$$\bar{\Pi}_i(\pi_i^x)\equiv\sup_{\varsigma:\mathbb{R}\to[0,1]}\Pi_i(\varsigma;\pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i adopts the new technology when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^*=\hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^*=\hat{p}_0^*\hat{y}_0^*/\hat{P}^*$ under the new and the old technology, respectively. Using (S.18), we have that $\hat{r}_f^*=\hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f=0,1$. Hence,

$$\Pi_i(\varsigma;\pi_i^x) =\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma)+\hat{y}_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x).$$

Using

$$\hat{y}_1^*=\gamma\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_1^{*\psi}, \tag{S.28}$$

$$\hat{y}_0^*=\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}, \tag{S.29}$$

and

$$\hat{l}_1^*=\gamma^{\varphi}\hat{l}_0^*, \tag{S.30}$$

we have that

$$\Pi_i(\varsigma;\pi_i^x) =\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x).$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} = \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}, \tag{S.31}$$

where

$$\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}$$

is the marginal effect of varying π_i^x on the probability that the firm adopts the new technology at θ, holding fixed the rule ς.

Next, recall again that, for $f = 0, 1$,

$$\hat{r}_f^* \equiv \frac{\hat{p}_f^* \hat{y}_f^*}{\hat{P}^*} = \hat{C}^{*\frac{1}{v}} \hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.28) and (S.29), we have that

$$\hat{r}_1^* - \hat{r}_0^* = \hat{C}^{*\frac{1}{v}} \Theta^{\frac{v-1}{v}} \left(1 + \beta \hat{N}^*\right)^{\alpha \frac{v-1}{v}} \left(\gamma^{\frac{v-1}{v}} \hat{l}_1^{*\psi \frac{v-1}{v}} - \hat{l}_0^{*\psi \frac{v-1}{v}}\right).$$

Therefore, using (S.30) and the structure of the proposed transfer policy, we have that

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1} \hat{C}^{*\frac{1}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \hat{l}_0^{*\psi \frac{v-1}{v}}.$$

Substituting this expression in (S.31), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1} \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x} \hat{l}_0^{*\psi \frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R} \frac{\hat{W}^*}{\hat{P}^*} \left(\left(\gamma^{\varphi} - 1\right) \hat{l}_0^* \frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R} s \frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Next recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$, where $\hat{n}^*(x) \equiv \hat{n}(x;\pi^{x*})$ is the efficient technology choice for a firm receiving signal x after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1} \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \frac{\partial \hat{N}^*}{\partial \pi^x} \hat{l}_0^{*\psi \frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R} \frac{\hat{W}^*}{\hat{P}^*} \left(\left(\gamma^{\varphi} - 1\right) \hat{l}_0^* \frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R} s \frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^* / \partial \pi^x$ is the marginal change in the measure of firms adopting the new technology that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the transfer rule to induce efficiency in information acquisition (when paired with the monetary rule in the proposition), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$

$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\hat{N}^*}{\partial\pi^x}\right)\right]+$$

$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}. \quad \text{(S.32)}$$

Next, use (S.22) and (S.30) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}=\left(\hat{l}_1^*\hat{N}^*+\hat{l}_0^*\left(1-\hat{N}^*\right)\right)^{\varepsilon}=\hat{l}_0^{*\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\hat{C}^{*\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition S.1,

$$\hat{C}^*=\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\frac{v}{v-1}},$$

we have that

$$\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{1-v}{v}}\hat{l}_0^{*\psi\frac{1-v}{v}}\frac{1}{\left(\gamma^{\varphi}-1\right)\hat{N}^*+1}.$$

It follows that (S.32) is equivalent to

$$\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}^{*1-R}}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$

$$-\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$

$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}. \quad \text{(S.33)}$$

Recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[\hat{C}^{*1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}^*}+\frac{v\left(\gamma^{\varphi}-1\right)}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]$$

$$+\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \quad \text{(S.34)}$$

Comparing (S.33) with (S.34), we thus have that, for the rule T to implement the efficient acquisition and usage of information (when paired with the monetary rule in the proposition, which, by virtue of Lemma S.1, is the only monetary rule that can induce efficiency in both information usage and information acquisition), the subsidy s to the innovating firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right],$$

where we reintroduced the arguments of all functions.

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the innovating firms is equal to

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta; \pi^{x*})}{1 + \beta\hat{N}(\theta; \pi^{x*})}$$

in each state, then, as shown in Step 1, the private value $\mathcal{R}$ that each firm assigns to adopting the new technology coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition S.2. Q.E.D.

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