UNIVERSITA' CATTOLICA DEL SACRO CUORE DI MILANO

UNIVERSITA' DEGLI STUDI DI MILANO

UNIVERSITA' DEGLI STUDI DI MILANO - BICOCCA

**GRADUATE SCHOOL IN THE ECONOMICS AND FINANCE OF PUBLIC ADMINISTRATION - DEFAP**
**ciclo XIX°**
**S.S.D. SECS-P/01**

# GEOPOLITICAL EFFECTS OF INCOME DISTRIBUTION

Tesi di Dottorato di: **Filippo Gregorini**
Matricola: **3303592**

**Anno Accademico 2007/2008**

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Coordinatore: **Ch.mo Prof. Massimo Bordignon**

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This dissertation is composed of three papers and discusses different aspects of the geopolitical organization of countries from a theoretical point of view.

In the first part, we develop an analysis on the effects of the distribution of incomes on size and public good provision in an international context. Heterogeneity concerns both the geographical space and the dimension of incomes. The utility of individuals depends upon their own location in the geographical space and their own income. In this chapter we refer to "median individual" in terms of "geographical location"; that is, the "median individual" is the individual living at the median distance between the center of the country and country borders. Both normative and positive solutions are considered.

The second paper studies the effects of wealth and income distribution on the willing to secede of regions within a democratic country. Heterogeneity concerns the dimension of incomes and individuals vote on the level of public spending. The utility of individuals depends upon their own income and it does not depend upon their own geographical location. Therefore, unlike the previous chapter, here we refer to "median individual" in terms of "income"; that is, the "median individual" is the individual with the median income.

In the last part of the dissertation, we focus on different indices used in the literature in order to rank income distributions. Our purpose is to analyze the relationships between the indices and we will also show the divergences in terms of policy implications on the basis of the measures used.

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## Part I

# POLITICAL GEOGRAPHY & INCOME INEQUALITIES[1]

**Abstract:** This paper displays an analysis of geopolitical organizations within the framework proposed by Alesina and Spolaore (1997), where heterogeneity concerns the geographical space. This model adds heterogeneity in the dimension of incomes, hence population is described by a double heterogeneity. In the normative equilibrium (social planner solution) the size of nations monotonically increases as income inequality increases, whereas the relationship between income inequality and public good provision within each nation can be strictly non-monotone. In the positive equilibrium (equilibrium geography) we find that in some cases there are no equilibria and it depends upon income inequality.

**Key Words:** Country Size, Public Good, Income Inequality, Tax Distortion

**JEL Code:** D6, H4, D3, H2

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# 1 Introduction

This paper studies a model of geopolitical organization where the size of nations and the level of public good provision are endogenous variables. Population is described by a double heterogeneity: individuals are located in a segment representing the world and there are different income levels. The introduction of income heterogeneity is the original contribution of our paper, whose benchmark models are Alesina and Spolaore (1997) and Etro (2006). Our purpose is to check the robustness of their results after the introduction of income inequalities as suggested in 1997 by Alesina and Spolaore. In the end of the paper, they highlighted five possible hints for future researches. Our analysis focus on their hint number four: "differences in income...may be crucial determinants...of the equilibrium size and number of countries"(page 1046).

Beyond this point, this paper is intended to discuss the effects of income inequality on public spending and political instability from a theoretical point of view.

Political geography have been already explored under many perspectives: the first works are Friedman (1977) and Buchanan and Faith (1987) on country formation and secessions. They can be considered pioneers of this discipline, whose diffusion increased together with the number of nations in the nineties, when country borders have been redrawn to an extent that is absolutely exceptional for a peacetime period.

In the model by Alesina and Spolaore (1997) the size of nations is endogenously determined through the trade-off between scale economies and heterogeneity; in their work population is uniformly distributed, geographical and preference dimensions coincide and public spending is exogenous and independent from size. In Etro (2006) public spending is endogenous and it depends upon size through a budget constraint. Etro considers also the elasticity of marginal utility from public good as a variable of his model.

Our analysis focuses on the effects of the introduction of income heterogeneity in the model of Alesina and Spolaore modified *à la* Etro; in particular, we will show how income inequality affects size and public good provision.

The effects of income heterogeneity have been already explored in similar contexts by Bolton and Roland (1997) and Haimanko, Le Breton and Weber (2005). Bolton and Roland analyzed how income differences between regions can influence the break-up or unification of countries. They are not interested in the determination of the size of nations; their model emphasizes political conflicts over redistribution policies in jurisdictions where the decision to separate or to unify is taken by majority voting. A trade-off between efficiency gains of unification and costs in terms of loss of control on political decisions is highlighted. Haimanko, Le Breton and Weber focused on threats of secession in a model where population is not uniformly distributed. They underline how efficiency implies stability only if the differences in citizens' preferences due to the geographical distribution of population are sufficiently small. If such differences are great enough efficient countries are not stable and redistribution schemes are needed in order to prevent secessions. Notice that both Bolton and Roland (1997) and Haimanko, Le Breton and Weber (2005) focus on threats of secession within a single country.

Our model considers a plurality of countries. Heterogeneity is given by individuals' location and income distribution. Furthermore, population is continuously and uniformly distributed and individuals are not mobile in contrast with the literature that follows Tiebout (1956). The issue of multi-dimensional heterogeneity in a context with a large number of jurisdictions has been already analyzed within the framework of Tiebout by Perroni and Scharf (2001).

Our analysis focuses on normative equilibrium[2] through a two stage process: in the first stage, an utilitarian social planner chooses the size of jurisdictions and the amount of public good within each jurisdiction;[3] in the second stage, the social planner chooses the location of public good in order to minimize the "costs of distance" from it within each jurisdiction. Our results can be summarized as follows: the size of nations depends upon income distribution; there is an inverse relationship between public good provision and income inequality but in a particular case global public good provision increases together with income inequality. We also check the stability of equilibria under rules for border redrawing; that is, the positive equilibria[4] of our model: we show that there are cases where positive equilibria do not exist and it depends upon the distribution of incomes.

This paper is organized as follows: Section 2 presents the model; Section 3 derives the normative equilibrium (social planner solution); Section 4 defines and characterizes the positive equilibrium (equilibrium geography);

[2] In this paper, the social planner solution can be considered as a "constrained optimum" or "second best solution", given that we assume the presence of a distortionary taxation scheme.

[3] Beyond different assumptions on individuals' mobility, the model by Perroni and Scharf does not consider the social planner solution and focus on a locational model of local fiscal choices where jurisdictions consist of open-membership coalitions of individuals and the levels of local public good provision are selected by majority voting.

[4] In this paper, we refer to positive equilibrium following the notions of stability discussed in Alesina and Spolaore (1997) and Etro (2006). Details are in Section 4.

Section 5 concludes. At the end of the paper, the Appendix contains some clarifications and proofs.

# 2 The model

## 2.1 General assumptions

World population has mass equal to 1 and it is continuously and uniformly distributed on the segment $[0, 1]$. We assume that individuals are not mobile.

Individuals are divided in two groups, call them "poor" and "rich". There is no income heterogeneity within groups. $y_P$ is the income of poor individuals, $\bar{y}$ represents average income, $y_R$ is the income of rich individuals and

$$y_R > \bar{y} > y_P > 0$$

holds.

For simplicity, from now on we will assume $y_P = y$ and $y_R = ky$, where $k > 1$ measures income differential between groups.

The parameter $\alpha$ represents the share of poor individuals and $1 - \alpha$ is the share of rich individuals. We assume that $\alpha$ is greater than 0.5 in order to guarantee the skewness to the right of income distribution; under such assumption we have that median income is strictly lower than average income as it is empirically observed.

In every point of the segment $[0, 1]$ there are $\alpha$ poor individuals and $1 - \alpha$ rich individuals.

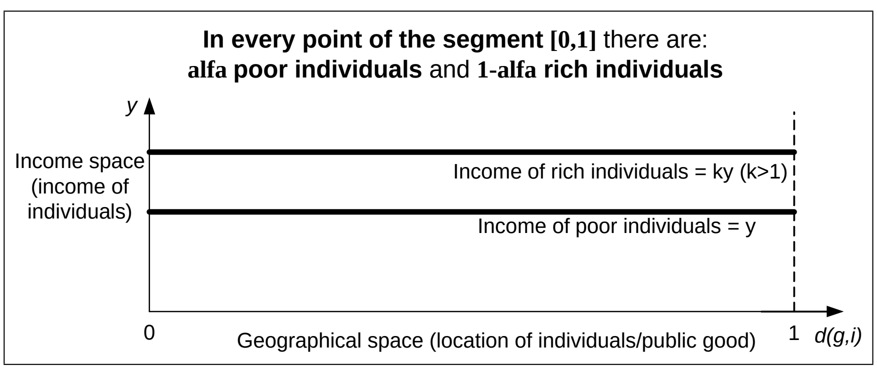


Figure 1: The dimensions of the model

## 2.2 Utility of individual $i$

Utility of individual $i$ in country $j$ depends upon public spending and private consumption:

$$U_{ij} = f(i)H(g_j) + u(c_i)$$

$H(\cdot)$ is the utility from public spending and $u(\cdot)$ is utility from private consumption. Utility from public spending depends upon the location of the individual $i$ through the function $f(i)$.

The utility function we use to test the effects of the introduction of income inequalities derives from our benchmark models Alesina and Spolaore (1997) and Etro (2006). In particular, the utility function of Etro, given the introduction of income differences, becomes:

$$U_{ij} = \frac{g_j^{1-\theta}}{1-\theta}[\lambda - a\ d(i, l_{gj})] + y_i - \frac{t_i^2}{2} \tag{1}$$

Utility from public spending $g$ is assumed to be isolelastic. $\theta \in (0, 1)$ represents the elasticity of marginal utility of public expenditure (the lower it is, the more public and private consumption are substitutable).

The term in parenthesis $[\lambda - a\ d(i, l_{gj})]$ concerns heterogeneity of preferences between individuals depending on their own distance from the point where public good is located: $\lambda > 0$ represents the maximum utility from public good, $a \geq 4\lambda$ reflects the costs of heterogeneity and $d(i, l_{gj}) = |i - l_{gj}|$ is the distance between the location of individual $i$ and the location of public good $g$ in jurisdiction $j$. Within our framework individuals' utility from public spending must depend upon their own location in the geographical space. In particular, we need that utility of individuals decreases together with the distance from the point where public good is located, *ceteris paribus.*

We also assume that utility is linear in private consumption.

There are specific assumptions on the technology of production of public goods; in particular, we assume that the cost function of taxation is a quadratic one. Such formalization is the same as the one of Etro; it entails the presence of diminishing marginal returns in the production process of public goods with a distortion of taxes increasing and convex in the taxation level. It is useful because of the mathematical tractability of the First Order Conditions. Alesina and Spolaore do not need to make any assumption on the technology of production of public goods because their utility from the exogenous public spending is a given parameter and the cost of production and tax distortions is another unrelated and exogenous parameter.

| | AS97 | E06 | G08 |
|---|---|---|---|
| Utility from public spending | $g$ | $\frac{g_j^{1-\theta}}{1-\theta}$ | $\frac{g_j^{1-\theta}}{1-\theta}$ |
| Heterogeneity of preferences | $1-al_i$ | $\lambda - al_i$ | $\lambda - a\,\lvert i - l_{gj} \rvert$ |
| "Costs" of taxation/public good | / | $-\frac{t_j^2}{2}$ | $-\frac{t_{ij}^2}{2}$ |

Given that in our model individuals differ about location and income, the utility of a poor individual $i$ in country $j$ is given by:

$$U_{i_P j} = \frac{g_j^{1-\theta}}{1-\theta}\left(\lambda - a\ d(i, l_{gj})\right) + y_j - \frac{t_{jP}^2}{2}$$

if $i \in P \subseteq [0, 1]$.

On the other hand, the utility of a rich individual $i$ in country $j$ is given by:

$$U_{i_R j} = \frac{g^{1-\theta}}{1-\theta}\left(\lambda - a\ d(i, l_{gj})\right) + ky_j - \frac{t_{jR}^2}{2}$$

if $i \in R \subseteq [0, 1]$.

### 2.3 Taxation scheme and budget constraint

Each individual pays taxes and enjoys benefits from public good in the country where he lives; taxes are assumed to be proportional with respect to income, therefore the tax rate is given by $\tau_j \in (0, 1)$ and we have:

$$t_i = \tau_j y_i$$

The budget constraint of our model derives from the assumptions on the distributions of population and incomes. Public spending equals tax revenue multiplied by country size. Notice that $s_j$, given uniform distribution of individuals, represents not only the size of the country but also its population.

Under the assumption that taxes are proportional with respect to income, the budget constraint for country $j$ is given by:

$$g_j = s_j \tau_j \left[\alpha y + (1-\alpha)\, ky\right] \tag{2}$$

## 3 Normative equilibrium (social planner solution)

We derive the normative equilibrium through a two-stage process. In the first stage, an utilitarian social planner chooses the number of nations and the level of public spending within each country; in the second stage, he locates the public good within each jurisdiction.

In the first stage, the utilitarian social planner maximizes:

$$W(g,s,t)=\sum_{j=1}^{N}\int_{s_j}U_{ij}di$$

$$s.t.:\ g_j=s_j t_j$$

Notice that the social planner observes the location of the individuals and also the distribution of incomes.

Our paper focus on symmetric partitions of the world, given the distribution of individuals and incomes. If countries are equal-sized, we have:

$$s_j=s \qquad \forall j\in(1,N)$$

As a consequence, from now on subscript $j$ will be omitted.

In the second stage, the social planner chooses the location of the public good within each jurisdiction in order to minimize the "costs of distance" from public good. As we have already pointed out in Section 2.2, the distance of each individual from public good is given by:

$$d(i,l_g)=|i-l_g|$$

The total cost of distance from $g$ within each country is given by:

$$L(i)=\int_s d(i,l_g)f(i)di$$

Under the assumption of uniform distribution, the previous integral reduces to:

$$L(i)=\int_s d(i,l_g)di$$

And the utilitarian social planner locates public good solving:

$$\min_{g\in j}L(i)=\min_{g\in j}\int_s d(i,l_g)di$$

It follows that public good is located in the middle of each jurisdiction.

As a consequence of the location of public good, the utilitarian social planner maximizes:

$$W(g,s,t) = \alpha\left[\frac{g^{1-\theta}}{1-\theta}\left(\lambda - a\frac{s}{4}\right) + y - \frac{t_P^2}{2}\right] + \\ +(1-\alpha)\left[\frac{g^{1-\theta}}{1-\theta}\left(\lambda - a\frac{s}{4}\right) + ky - \frac{t_R^2}{2}\right]$$

Under the budget constraint $g = st$. Notice that $s/4$ is the median distance between the center of the country and country borders.

Rearranging the terms in the previous equation we obtain:

$$W = \frac{g^{1-\theta}}{1-\theta}\left(\lambda - a\frac{s}{4}\right) + [\alpha y + (1-\alpha)\, ky] - \left[\alpha\frac{t_P^2}{2} + (1-\alpha)\,\frac{t_R^2}{2}\right]$$

The first term in square brackets equals average income; the second one equals the costs of taxation given the distribution of incomes.

Let's focus on the costs of taxation given income distribution. Given a proportional taxation scheme and the budget constraint $g = st$ we derive:

$$\alpha\frac{t_P^2}{2} + (1-\alpha)\,\frac{t_R^2}{2} = \frac{1}{2}\left(\frac{g}{s}\right)^2 \frac{\alpha + (1-\alpha)\,k^2}{[\alpha + (1-\alpha)\,k]^2} = \frac{1}{2}\left(\frac{g}{s}\right)^2 \psi$$

In order to get the normative equilibrium, the utilitarian social planner maximizes with respect to size and public good provision the following equation:

$$W\,(g,s) = \frac{g^{1-\theta}}{1-\theta}\left(\lambda - a\frac{s}{4}\right) + \bar{y} - \frac{1}{2}\left(\frac{g}{s}\right)^2 \psi \tag{3}$$

where:

$$\psi := \frac{\alpha + (1-\alpha)\,k^2}{[\alpha + (1-\alpha)\,k]^2} > 1 \tag{4}$$

is a linear transformation of the Generalized Entropy Index with parameter equal to 2.

Generalized Entropy Index, if the parameter equals 2, is given by:

$$E_2 = \frac{1}{2}\left[\int\left(\frac{y_i}{\bar{y}}\right)^2 dF - 1\right]$$

Where $F$ represents income distribution function.

Given the distribution of incomes in our model, we have:

$$E_2 = \frac{1}{2}\left\{\alpha\left[\frac{y}{\alpha y + (1-\alpha) ky}\right]^2 + (1-\alpha)\left[\frac{ky}{\alpha y + (1-\alpha) ky}\right]^2 - 1\right\}$$

It reduces to:

$$E_2 = \frac{1}{2}\left\{\frac{\alpha + (1-\alpha) k^2}{[\alpha + (1-\alpha) k]^2} - 1\right\}$$

Equivalently, we have:

$$\psi = 2E_2 + 1$$

Generalized Entropy Index is a convenient measure of income inequality as it satisfies important properties.[5]

Let's focus now on the economic interpretation of the Index. In our model, the index derives from the component of $W$ concerning the technology of production of public goods; *de facto*, it shows us the variation in the average costs of taxation in our model that follows the introduction of income inequalities. The numerator approximates the average costs of taxation we observe, given income distribution. The denominator approximates the costs of taxation we would have in case of uniform income; in such a case the costs of taxation would be minimized.

### 3.1 Derivation of the normative equilibrium

Let us consider the First Order Condition[6] of (3) with respect to size:[7]

$$\frac{\partial W}{\partial s} = -g^{1-\theta}\frac{a}{4(1-\theta)} + \frac{g^2}{s^3}\psi = 0$$

It follows that:

$$s = g^{\frac{1+\theta}{3}}\left[\frac{4(1-\theta)\psi}{a}\right]^{\frac{1}{3}} \tag{5}$$

[5] First of all, Generalized Entropy Index satisfies the Strong Principle of Transfers, so that any transfer of income from a rich person to a poor one reduces measured inequality proportionally to the distance in terms of income between the two individuals. Furthermore, the Index is income scale independent, so the measured inequality of the slices of the cake do not depend on the size of the cake. It is also population independent, so the measured inequality does not depend on the number of individuals we consider.

[6] Second Order Conditions are satisfied. See the Appendix for details.

[7] Notice that in the mathematical derivation of the equilibrium we do not take into account the constraint $s \in 1/N$ where $N$ is the number of nations; that is, a natural number greater than 1. Obviously, the constraint $s \in 1/N$ is taken into account in the results of the model.

The size of nation chosen by the social planner is an increasing and convex function of the provision of public goods. It increases together with the provision of public good in order to properly exploit the economies of scale. On the other hand, there is an inverse relationship between the costs of heterogeneity and size.

Merging (5) and the budget constraint (2), we obtain:

$$t^{*} = s^{\frac{2-\theta}{1+\theta}} \left[ \frac{a}{4(1-\theta)\psi} \right]^{\frac{1}{1+\theta}} = \Psi(s) \tag{5A}$$

($5A$) suggests a positive correlation between country size and average public spending per capita. Such correlation contrasts the empirical results of the paper by Alesina and Wacziarg (1998), who have showed a robust negative correlation between the two variables. ($5A$) also shows an inverse relationship between tax rate and income inequality; both in theoretical and empirical analyses there are controversies on this issue. Beyond this point, the economic intuition suggests that if income inequality increases it would be more difficult to target public good on the preferences of citizens.

The First Order Condition[6] of (3) with respect to public good provision gives us:

$$g = s^{\frac{2}{1+\theta}} \left( \lambda - a\frac{s}{4} \right)^{\frac{1}{1+\theta}} \left( \frac{1}{\psi} \right)^{\frac{1}{1+\theta}} \tag{6}$$

The public good provision chosen by the social planner is an inverted U function of the size of nations. Merging (6) and the budget constraint (2), we obtain:

$$t^{*} = s^{\frac{1-\theta}{1+\theta}} \left( \lambda - a\frac{s}{4} \right)^{\frac{1}{1+\theta}} \left( \frac{1}{\psi} \right)^{\frac{1}{1+\theta}} = \Phi(s) \tag{6A}$$

($6A$) suggests that there is a trade off between heterogeneity costs and benefits from scale economies in the provision of public goods. Both these effects are increasing in the size of nations, therefore the net benefits from public good provision are maximized at an "intermediate" size. The effect of income inequality is the same already discussed for ($5A$).

We derive the size of nations $s^{*} = 1/N^{*}$ solving:

$$\Psi(s) = \Phi(s)$$

It follows that:

$$s = \frac{4\lambda(1-\theta)}{a(2-\theta)} \tag{7}$$

Merging (6) and (7), we obtain:

$$g = \left(\frac{\lambda}{2-\theta}\right)^{\frac{3}{1+\theta}} \left[\frac{4\,(1-\theta)}{a}\right]^{\frac{2}{1+\theta}} \left(\frac{1}{\psi}\right)^{\frac{1}{1+\theta}} \tag{8}$$

**PROPOSITION 1** **The number of nation $N^* = 1/s^*$ is given by:**

$$\left\lfloor \frac{a\,(2-\theta)}{4\lambda\,(1-\theta)} \right\rfloor \textbf{ if } \begin{cases} \psi^* \le 1 \\ \psi^* > 1 \quad \& \quad \psi \ge \psi^* \end{cases}$$

$$\left\lceil \frac{a\,(2-\theta)}{4\lambda\,(1-\theta)} \right\rceil \textbf{ if } \psi^* > 1 \quad \& \quad \psi \le \psi^*$$

Where:

$$\lfloor \mathbb{R} \rfloor = \max \{ n \in \mathbb{N} \mid n \le \mathbb{R} \}$$

$$\lceil \mathbb{R} \rceil = \min \{ n \in \mathbb{N} \mid n \ge \mathbb{R} \}$$

$$\psi^* = \frac{1}{\lfloor N^* \rfloor \lceil N^* \rceil \left( \lfloor N^* \rfloor + \lceil N^* \rceil \right)} \frac{a g^{-(1+\theta)}}{2(1-\theta)}$$

The Appendix contains a detailed discussion on how the number of nations in the normative equilibrium depends upon income inequality.

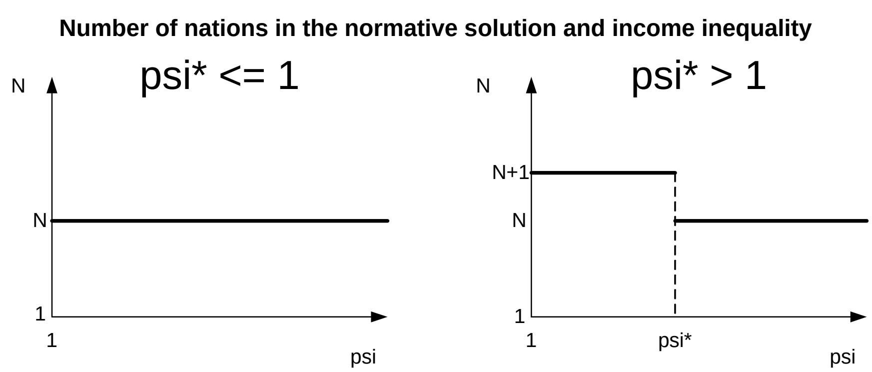


Figure 2: N* and income inequality

**PROPOSITION 2** **The provision of public good within each country is given by:**

$$g^* = \left(\frac{\lambda}{2-\theta}\right)^{\frac{3}{1+\theta}} \left[\frac{4(1-\theta)}{a}\right]^{\frac{2}{1+\theta}} \left(\frac{1}{\psi}\right)^{\frac{1}{1+\theta}}$$

Both size and public spending increase together with absolute utility from public good: the higher is the value of $\lambda$, the higher is the utility from $g$, *ceteris paribus*. Furthermore, optimal size and optimal public good provision decrease as the costs of heterogeneity increase.

Our analysis shows that, if income inequality increases, it would be optimal to lower the tax rate. Notice that the heterogeneity of preferences on public good increases together with income inequality; as a consequence, if income inequality increases, it would be harder to target the public good on the preferences of individuals. The economic intuition for our results follows: if income inequality increases, individuals prefer less public good because its "average distance" from the preferences of individuals increases together with the measured income inequality.

In our model the social planner can lower taxation increasing size and/or lowering the provision of public good, given the budget constraint $g = st$. On one hand, if the size of countries increases, the "geographical" heterogeneity of preferences on public good increases; on the other hand, if public good within each jurisdiction lowers, the "geographical" heterogeneity of preferences on public good does not increase. As a consequence, the social planner lowers the provision of public good if income inequality increases.

### 3.1.1 A particular case

If income inequality increases from $\psi 1$ to $\psi 2$ and $\psi 2 > \psi^* > \psi 1$ does not hold, the number of nations does not change and the provision of public good within each jurisdiction decreases; that is, the tax rate decreases (see Figure 3).

If income inequality shifts from $\psi 1$ to $\psi 2$ and $\psi 2 > \psi^* > \psi 1$ holds, the number of nations decreases from $\lceil N^* \rceil$ to $\lfloor N^* \rfloor$ and the provision of public good within each jurisdiction increases; that is, the tax rate increases (see Figure 4).

**PROPOSITION 3** **If income inequality increases from $\psi 1$ to $\psi 2$ and $\psi 2 > \psi^* > \psi 1$ holds, the provision of public good within each country increases.**

We prove such result focusing on the global provision of public good;[8] if it increases, tax rate has increased also within each jurisdiction.

**Proof.** The global provision of public good increases if:

$$\lfloor N^* \rfloor \;\; g^*(\lfloor N^* \rfloor) > \lceil N^* \rceil \;\; g^*(\lceil N^* \rceil)$$

If we rewrite (6) in terms of $N = 1/s$, the previous equation becomes:

$$\lambda \left( \frac{\lceil N^* \rceil^{1-\theta}}{\psi 1} - \frac{\lfloor N^* \rfloor^{1-\theta}}{\psi 2} \right) < \frac{a}{4} \left( \frac{\lceil N^* \rceil^{2-\theta}}{\psi 1} - \frac{\lfloor N^* \rfloor^{2-\theta}}{\psi 2} \right)$$

It holds, therefore the global provision of public good $Ng$ increases.

Let us consider now the budget constraint (2); the global budget constraint is given by:

$$Ng = Ns\tau\bar{y}$$

Given $Ns = 1$ and under the assumption that $\bar{y}$ does not change, $\tau$ increases within each jurisdiction if $Ng$ increases. ■

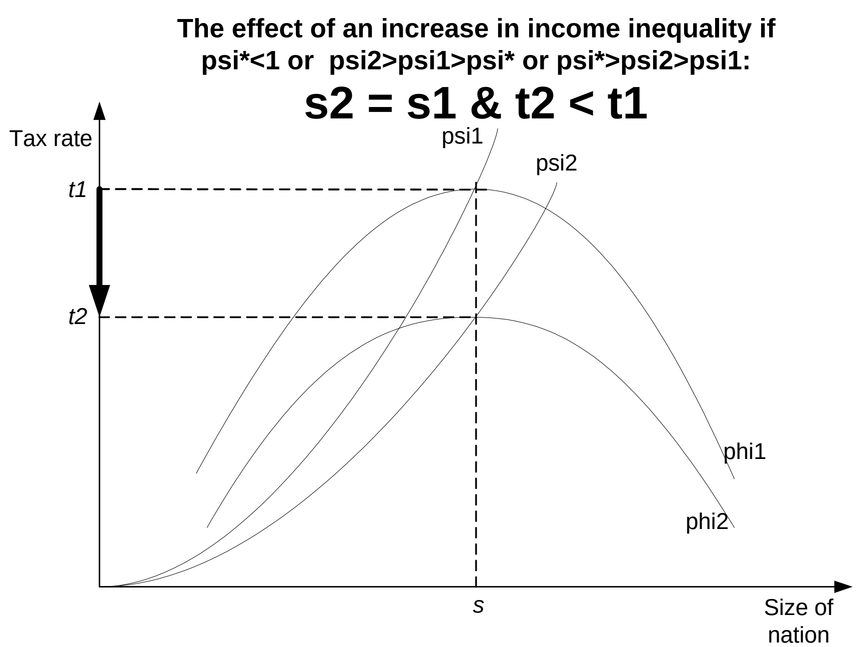


Figure 3: N* does not change

[8] In our model countries are equal-sized, hence the global provision of public good is given by public good provision within each country multiplied by the number of nation; that is, $Ng$.

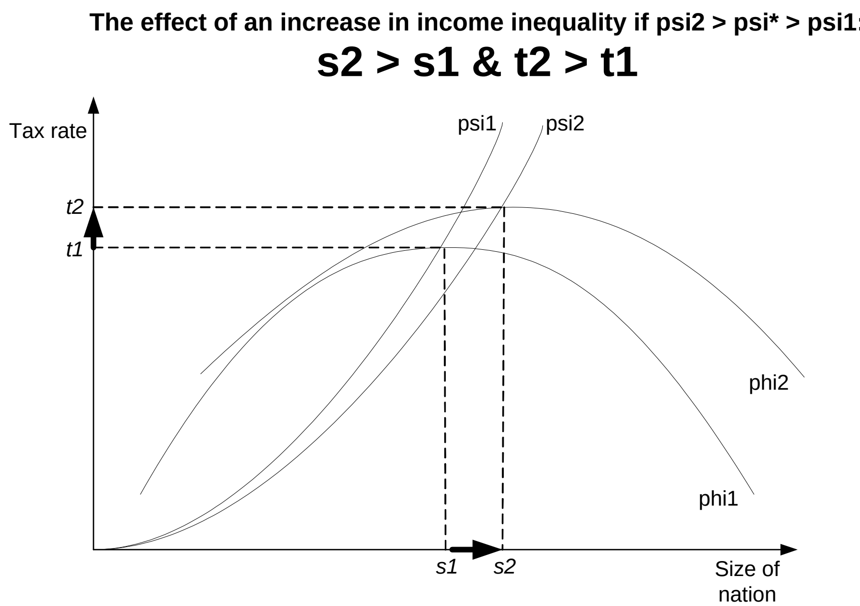


Figure 4: N* changes

### 3.2 Theoretical analysis and empirical analysis

After the introduction of income inequalities through a two-spike distribution, we show that in general there is an inverse relationship between income inequality and public spending. Such inverse relationship contrasts the results obtained by Persson and Tabellini (2000); in their theoretical model, optimal public good provision under proportional taxation scheme rises as median income drops relative to average income.[9] There are important differences between our model and the one of Persson and Tabellini. We derive the normative solution through the vision of an utilitarian social planner that maximizes the utility of the median individual in terms of geographical location; Persson and Tabellini maximize the utility of the individual with median income through a voting model where Median Voter Theorem holds. Beyond this point, both in Persson and Tabellini and in our model an increase in income skewness and/or income inequality leads to a smaller

[9] Persson and Tabellini consider income skewness (mean/median ratio); we consider income inequality (Generalized Entropy Index). Given our two-spike distributions, an increase in income differential between rich and poor increases both income skewness and income inequality.

redistribution in equilibrium.

The theoretical work by Lind (2007) confirms the inverse relationship between inequality and public spending. There are fundamental differences between our work and the one of Lind in terms of assumptions on income distribution: we consider a "spiked" distribution with perfect homogeneity within each group; Lind considers a distribution of incomes where within groups heterogeneity exists. The result of his model depends upon the differences in densities within different groups; he shows that a mean-preserving increase in between-groups inequality decreases the politically chosen tax rate. Given different assumptions on income distribution, we observe similar results within different frameworks.

The results of empirical analyses on the effects of income inequality on public expenditure seems to confirm our results. In the econometric analysis by Alesina, Baqir and Easterly (1999),[10] income inequality[11] has negative effect on per capita education spending. Also the work by Lindert (1996)[12] shows that an increase in income inequality lowers total public expenditure as share of GDP.

In general, it is possible to note that in most countries transfers rose more quickly during the 1960s and the 1970s, when income inequality was generally declining; in contrast, during the 1980s and the 1990s, inequality started to increase and government transfers rose less quickly with respect to the previous period.

In order to check the correlation between income inequality and public expenditure nowadays, we have built up a data set on population, Gini Index, total public expenditure as share of GDP and priorities (education, health and defense) in public expenditure as share of GDP worldwide.[13] An inverse correlation between income inequality and public expenditure exists and it is coherent with our results. Notice that we have focused our preliminary empirical analysis on public expenditure on education, health and defense (where direct transfers and subsidies should not be included) in order to limit the endogeneity between the two variables. To control the

[10] This paper is an econometric analysis of public spending at local level within the U.S..

[11] Alesina, Baqir and Easterly measure income inequality through mean/median ratio; *de facto*, they consider income skewness as a proxy for income inequality.

[12] This paper is an econometric analysis of the determinants of public spending in 19 OECD countries from 1960 to 1992.

[13] The sources of our data set are: Human Development Report of the United Nations (2005) for population; World Bank (2004, 2005) for Gini Inequality Index, total public expenditure and public expenditure on health; Unesco (2005) for public expenditure on education and Stockholm International Peace Research Institute (2005) for public expenditure on defense.

robustness of the correlation in the whole sample consisting of 87 countries we have also tested the correlation on different subsets: OECD countries, members of the European Union (EU25) and countries with at least 5 millions of inhabitants; in all the subsets a negative correlation between Gini Index and public expenditure on priorities exists.

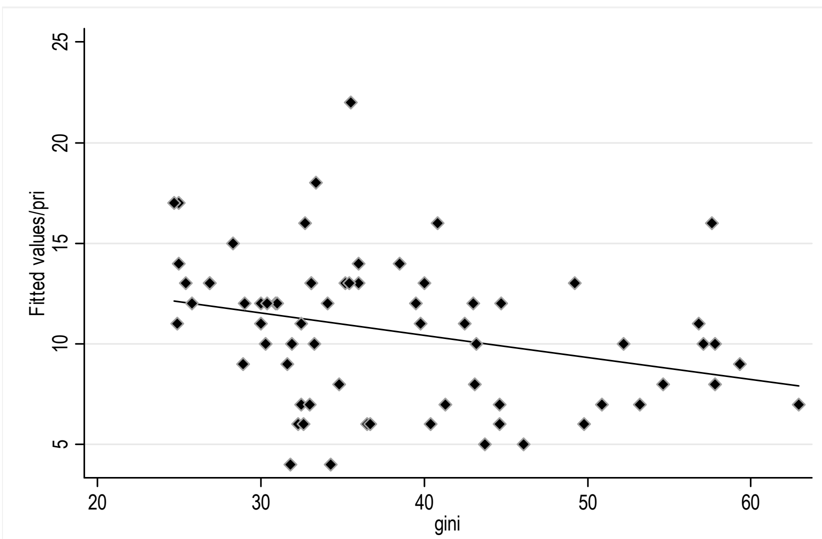


Figure 5: Gini Index & Public Expenditure (>5mlns.)

# 4 Positive equilibrium (equilibrium geography)

In this section, we check how different preferences on country size affects the equilibrium geography of the model, given the amount of public good chosen by the social planner.[14]

In order to study the equilibrium number of countries, we need to define rules for border redrawing. Under **Rule i**, we require that each individual can choose whether to live in its country or in autarchy; that is, without public good provision and taxation. Under **Rule ii**, we require that nobody

[14] We assume that also in the derivation of equilibrium geography the public good is located in the middle of each jurisdiction. Suppose that (i) the social planner minimizes the costs of distance from each place where public good is located or (ii) within each country the location of public good is decided by majority rule. In both cases public good is located in the middle of each jurisdiction. Alesina and Spolaore (1997), for example, use method (i) in the derivation of the social planner solution and method (ii) in the derivation of the equilibrium solution.

living at the border between two countries can be forced to belong to a country if he prefers to join the other one.

Rules for border redrawing can be summarized as follows:

**Rule i** *Each individual can choose between status quo and autarchy.*

**Rule ii** *Each individual at the border between two countries can choose which country to join.*

A configuration of $N$ countries is:

An i/ii-equilibrium if the borders of the $N$ nations are not subject to change under Rule i and Rule ii.

i/ii-stable if it is an i/ii-equilibrium and it is stable under Rule i and Rule ii.

Our notion of i/ii-stability implies that if an i/ii-equilibrium is subject to a "small" perturbation, the system returns to the original position. A "small" perturbation occurs when some individuals live in autarchy and/or some individuals change their citizenship.

Formally, a configuration of $N$ countries is i/ii-stable if and only if the following conditions hold:

$$V_P(s/2) \geq y \quad \text{(iP)}$$

$$\partial V_P(s/2)/\partial s \leq 0 \quad \text{(iiP)}$$

$$V_R(s/2) \geq ky \quad \text{(iR)}$$

$$\partial V_R(s/2)/\partial s \leq 0 \quad \text{(iiR)}$$

Where $V_i(s/2)$ is the expected utility of the individual $i$ living at country borders.

Under Rule i we require that for each individual the loss of utility deriving from taxation cannot be superior to the increase in utility deriving from public good provision.[15]

[15] Alesina and Spolaore (1997) do not need to explicitly consider **Rule i** given their assumptions on the parameters of the utility function. **Rule i** is equivalent to Condition 1 in Etro (2006). Notice that if **Rule i** holds for citizens living at country border *a fortiori* it holds for any other individual, given that in our model the utility of individuals decreases together with the distance from the middle of each country where $g$ is located, *ceteris paribus*.

Under Rule ii we require that each individual living at the border between two countries of different size will prefer to join the smallest one.[16]

From the previous section, the provision of public good chosen by the social planner (in terms of $s$) is given by:

$$g^* = \left[\frac{s^2}{\psi}\left(\lambda - a\frac{s}{4}\right)\right]^{\frac{1}{1+\theta}}$$

In this section, we assume that $g^*$ is the exogenous public good provision for every country size.

## 4.1 Derivation of the positive equilibrium

In order to check i/ii-stability, we have to consider the expected utilities of poor and rich individuals living at country borders given $g^*$:

$$V_P(s/2) = \left[\frac{s^2}{\psi}\left(\lambda - a\frac{s}{4}\right)\right]^{\frac{1-\theta}{1+\theta}} \left[\frac{\lambda - a\frac{s}{2}}{1-\theta} - \frac{1}{2\left[\alpha + (1-\alpha)k^2\right]}\left(\lambda - a\frac{s}{4}\right)\right] + y \qquad \text{(9P)}$$

$$V_R(s/2) = \left[\frac{s^2}{\psi}\left(\lambda - a\frac{s}{4}\right)\right]^{\frac{1-\theta}{1+\theta}} \left[\frac{\lambda - a\frac{s}{2}}{1-\theta} - \frac{k^2}{2\left[\alpha + (1-\alpha)k^2\right]}\left(\lambda - a\frac{s}{4}\right)\right] + ky \qquad \text{(9R)}$$

With respect to poor individuals, we have:

$$\frac{\left[-4\lambda(2\phi+\theta-1) + as(4\phi+\theta-1)\right]\left[\frac{s^2}{\psi}\left(\lambda - a\frac{s}{4}\right)\right]^{\frac{1-\theta}{1+\theta}}}{8\phi(\theta-1)} \geq 0 \qquad \text{(iP)}$$

$$\frac{\left\{16\lambda^2(\theta-1)(2\phi+\theta-1) + a^2s^2(\theta-2)(4\phi+\theta-1) - 4as\lambda[3-9\phi+\theta(5\phi+2\theta-5)]\right\}\psi\left[\frac{s^2}{\psi}\left(\lambda - a\frac{s}{4}\right)\right]^{\frac{2}{1+\theta}}}{s^3\phi(\theta^2-1)(as-4\lambda)^2} \leq 0 \qquad \text{(iiP)}$$

[16] Let us recall once again that in our model the utility of individuals decreases together with the distance from $g$, *ceteris paribus.* **Rule ii** is equivalent to Rule A in Alesina and Spolaore (1997) and Condition 2 in Etro (2006).

where:

$$\phi = \alpha + (1-\alpha)\,k^2$$

As a consequence, a configuration of $N$ countries is i/ii-stable for poor individuals only if it belongs to the interval:

$$\tilde{s}_P \in \left[\frac{2\lambda}{a}\frac{3-9\phi+\theta(5\phi+2\theta-5)+\sqrt{(\theta-1)^2-2\phi[3+\theta^2(2\theta-5)]-\phi^2[\theta(7\theta-6)-17]}}{(\theta-2)(4\phi+\theta-1)}, \frac{4\lambda}{a}\frac{2\phi+\theta-1}{4\phi+\theta-1}\right] \tag{10P}$$

Let us express the interval in $(10P)$ as:

$$I_P\,(\lambda,\alpha,k,\theta)$$

With respect to rich individuals, we have:

$$\frac{[-4\lambda(2\beta+\theta-1)+as(4\beta+\theta-1)]\left[\frac{s^2}{\psi}\left(\lambda-a\frac{s}{4}\right)\right]^{\frac{1-\theta}{1+\theta}}}{8\beta(\theta-1)} \geq 0 \tag{iR}$$

$$\frac{\left\{16\lambda^2(\theta-1)(2\beta+\theta-1)+a^2s^2(\theta-2)(4\beta+\theta-1)-4as\lambda[3-9\beta+\theta(5\beta+2\theta-5)]\right\}\psi\left[\frac{s^2}{\psi}\left(\lambda-a\frac{s}{4}\right)\right]^{\frac{2}{1+\theta}}}{s^3\beta(\theta^2-1)(as-4\lambda)^2} \leq 0 \tag{iiR}$$

where:

$$\beta = \frac{\alpha+(1-\alpha)\,k^2}{k^2} = \frac{\phi}{k^2}$$

As a consequence, a configuration of $N$ countries is i/ii-stable for rich individuals only if it belongs to the interval:

$$\tilde{s}_R \in \left[\frac{2\lambda}{a}\frac{3-9\beta+\theta(5\beta+2\theta-5)+\sqrt{(\theta-1)^2-2\beta[3+\theta^2(2\theta-5)]-\beta^2[\theta(7\theta-6)-17]}}{(\theta-2)(4\beta+\theta-1)}, \frac{4\lambda}{a}\frac{2\beta+\theta-1}{4\beta+\theta-1}\right] \tag{10R}$$

Let us express the interval in $(10R)$ as:

$$I_R\,(\lambda,\alpha,k,\theta)$$

In particular, we have:

The stable interval for poor individuals $I_P$ is always non-empty.

The stable interval for rich individuals $I_R$ can be non-empty or empty, depending on income distribution.

i/ii-stable equilibria exist only if $I_P \cap I_R$ is non-empty. Our analysis shows that $I_P \cap I_R$ can be non-empty only if $2\beta + \theta > 1$ holds.[17]

Assuming that $2\beta + \theta > 1$ holds, we have $\min I_P > \min I_R$ and $\max I_P > \max I_R$, therefore $I_P \cap I_R$ is non-empty only if:

$$\max I_R(\lambda, \alpha, k, \theta) > \min I_P(\lambda, \alpha, k, \theta)$$

or, equivalently:

$$\frac{2(2\beta+\theta-1)}{4\beta+\theta-1} - \frac{3-9\phi+\theta(5\phi+2\theta-5)+\sqrt{(\theta-1)^2-2\phi[3+\theta^2(2\theta-5)]-\phi^2[\theta(7\theta-6)-17]}}{(\theta-2)(4\phi+\theta-1)} > 0 \quad (11)$$

**PROPOSITION 4** **If $2\beta + \theta > 1$ the existence of i/ii-stable equilibria depends upon the values of $\alpha$, $k$, $\theta$. i/ii-stable equilibria do not exist otherwise.**

For poor individuals i/ii-stable size increases as income differential increases. In such a case poor could have more pro-capita public good in a greater country because of a multiplicative effect: if income differential increases and the size of country doubles, it follows that the provision of public good is more than doubled and their favorite size increases. The effect of an increase in the percentage of poor is opposite. In such a case, poor would have to pay a larger share of the tax burden in order to get the same provision of public good; as a consequence, they would prefer less public good provision and less distance from the government in a smaller country.

For rich individuals, the effects of an increase in income differential or in the percentage of poor increase are the same. In both cases, with taxes proportional to income, they pay a larger share of the tax burden. If they pay (relatively) more taxes they would prefer a smaller country to join more benefits from the public goods they have paid for. In extreme cases, if taxes and/or income differential are "too high", autarchy is preferred. Notice that an increase in income differential between rich and poor ($k$) lowers $\beta$; it follows that, in case of income differential "high enough", autarchy is preferred by rich individuals.[18]

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[17] $I_R$ is non empty if $2\beta + \theta > 1$ or $4\beta + \theta < 1$. If $4\beta + \theta < 1$, we have $I_P \cap I_R = \emptyset$.

[18] Given $\beta = \left[\alpha + (1-\alpha)k^2\right]/k^2$ and the necessary condition $2\beta + \theta > 1$, the higher $k$, the smaller the range of $\alpha$ and $\theta$ that satisfies $I_P \cap I_R \neq 0$.

Our analysis have showed that the results of Alesina and Spolaore and Etro for equilibrium geography seem to be not robust to the introduction of income inequalities in the sense that a "sufficiently high" income inequality implies no positive equilibria within the framework of Alesina and Spolaore.[18]

## 4.2 Income inequalities and instability

There are cases where an equilibrium geography does not exist. The higher is income inequality, the more the preferences of individuals on size diverge; if income differential is "high enough"[18] or $2\beta+\theta \leq 1$ an i/ii-stable equilibrium does not exist. A strong link between inequality and instability emerges.[19]

Let us compare such result with the ones of Haimanko, Le Breton and Weber (2005), who develop a model where heterogeneity is given by the distribution of individuals in the geographical space and incomes are not considered. They study how governments can prevent secession threats through redistribution schemes, given the distribution of individuals. In both the models geographical and preference dimensions coincide but we focus on income differences within an uniformly distributed population. In spite of these differences, their degree of polarization in the geographical distribution of individuals can be considered as a counterpart of the Generalized Entropy Index $\psi$. Following this argument, a comparison of the results is possible. Haimanko, Le Breton and Weber show that in case of an highly polarized population efficiency does not imply stability without redistribution; that is, the efficient size is greater than the stable one. Within our framework redistribution schemes cannot be implemented[20] and we show that in case of high income inequality no equilibrium geography is possible.

There are also empirical works on the link between income distribution and political instability. The econometric analysis by Alesina and Perotti (1996) on 71 countries between 1960 and 1985 shows that political stability

[19]Notice that in our model we consider also the effect of the substitutability between public and private goods on i/ii-stability. Given that $2\beta + \theta > 1$ is a necessary condition for the existence of an i/ii-stability, the higher the substitutability between public and private goods (the lower $\theta$), the more the model is expected to be i/ii-unstable, given income distribution.

[20]Alesina and Spolaore (1997) proved that in their model a redistribution scheme cannot be implemented (page 1054-1055). Given uniformly distributed population and pairwise majority voting on redistribution schemes, for every country size $s_j$ there will always be a majority against redistribution schemes formed by individuals living at a distance from the middle of each jurisdiction (where public good is located) that is not superior to the median one; that is, a majority formed by each $i$ living in $j$ such that $d(i_j, g_j) \leq s_j/4$.

is enhanced by the presence of a wealthy middle class. Alesina and Perotti focuses on causal relationship, but, as noted by Acemoglu and Robinson (2006), in many cases the existing literature on this topic is contradictory and focuses on correlations instead of causal relationships, therefore it is not useful for scientific purposes.

## 5 Conclusion

In this paper we have discussed the effects of the introduction of income inequality in well-known models on geopolitical organizations.

We find that in the normative solution there is in most cases an inverse relationship between income inequality and public spending, but our paper also shows that the size of jurisdictions depend upon income distribution and in a particular case public good provision increases together with income inequality. Our results shows that, after the introduction of income heterogeneity, the relationships between jurisdiction size, public spending and income inequality are non monotone.

Our main finding on equilibrium geography concerns the existence of equilibria. In our benchmark model stable equilibria exist, but after the introduction of income heterogeneity we show that there are cases where stable equilibria do not exist depending on income inequality; in particular, there is a direct relationship between income differential between rich and poor and instability.

The model of Alesina and Spolaore modified *à la* Etro seems not to be robust to the introduction of income heterogeneity. This result should not be interpreted as a negative one. Let us focus, for example, on the comparison between Haimanko, Le Breton and Weber (2005) and our paper: given the different assumptions of the models, our non-existence of equilibria is the counterpart of their need for redistribution schemes in order to prevent secessions. An important result in the theoretical literature is confirmed within our multidimensional framework.

As we have already pointed out at the very beginning of the paper, in 1997 Alesina and Spolaore highlighted five possible hints for future researches. As far as we can see, some of their “questions left open” are still open nowadays. In particular, it would be interesting to relax some of the assumptions on the distribution of individuals. Another interesting extension of the original model could concerns the mobility of individuals, so that Alesina and Spolaore (1997) could meet the framework proposed by Tiebout (1956).

# Appendix

## The normative number of nations as an integer number

The number of nation in the normative equilibrium is $\lfloor N^* \rfloor$ if and only if:

$$W(\lfloor N^* \rfloor) \geq W(\lceil N^* \rceil)$$

that is, if and only if:

$$\tfrac{ag^{1-\theta}}{4\lfloor N^* \rfloor(1-\theta)} + \frac{\psi}{2}\left(g\lfloor N^* \rfloor\right)^2 \leq \tfrac{ag^{1-\theta}}{4\lceil N^* \rceil(1-\theta)} + \frac{\psi}{2}\left(g\lceil N^* \rceil\right)^2$$

Rearranging the terms we obtain:

$$\left(\frac{1}{\lfloor N^* \rfloor} - \frac{1}{\lceil N^* \rceil}\right)\left(\frac{ag^{1-\theta}}{4(1-\theta)}\right) \leq \left(\lceil N^* \rceil^2 - \lfloor N^* \rfloor^2\right)\left(\frac{g^2}{2}\right)\psi$$

Notice that if $\psi \to +\infty$, the right-side is strictly greater than the left-side, therefore $\lfloor N^* \rfloor$ is the unique number of nations in the normative equilibrium.

If the right-side is strictly greater than the left-side, $\lfloor N^* \rfloor$ is the unique number of nations in the normative equilibrium.

The two sides equal if and only if $\psi = \psi^*$,[21] therefore both $\lfloor N^* \rfloor$ and $\lceil N^* \rceil$ are equilibrium number of nations

If the right-side is strictly smaller than the left-side, $\lceil N^* \rceil$ is the unique number of nations in the normative equilibrium.

[21] Let us recall (page 19):

$$\psi^* = \frac{1}{\lfloor N^* \rfloor \lceil N^* \rceil \left(\lfloor N^* \rfloor + \lceil N^* \rceil\right)} \frac{ag^{-(1+\theta)}}{2(1-\theta)}$$

**Second Order Conditions (Proposition 1/Proposition 2)**

The Hessian matrix of $W(s,g)$:

$$D^2W(s^*,g^*) = \begin{bmatrix} \frac{\partial^2 W}{\partial s \partial s} & \frac{\partial^2 W}{\partial g \partial s} \\ \frac{\partial^2 W}{\partial s \partial g} & \frac{\partial^2 W}{\partial g \partial g} \end{bmatrix}$$

$(s^*, g^*)$ are strict local maximizers of $W(s,g)$ if and only if:

$$\det D^2W(s^*,g^*) = \frac{\partial^2 W}{\partial s \partial s}\frac{\partial^2 W}{\partial g \partial g} - \left(\frac{\partial^2 W}{\partial s \partial g}\right)^2 > 0$$

Given the First Order Condition of (3) with respect to size:

$$\frac{\partial W}{\partial s} = -g^{1-\theta}\frac{a}{4\,(1-\theta)} + \frac{g^2}{s^3}\psi = 0$$

It follows that:

$$\frac{\partial^2 W}{\partial s \partial s} = -3\frac{g^2}{s^4}\psi < 0$$

Given the First Order Condition of (3) with respect to public good provision:

$$\frac{\partial W}{\partial g} = g^{-\theta}\left(\lambda - a\frac{s}{4}\right) - \frac{g}{s^2}\psi = 0$$

It follows that:

$$\frac{\partial^2 W}{\partial g \partial g} = -\theta g^{-(1+\theta)}\left(\lambda - a\frac{s}{4}\right) - \frac{1}{s^2}\psi < 0$$

Furthermore, we have:

$$\frac{\partial^2 W}{\partial s \partial g} = \frac{\partial^2 W}{\partial g \partial s} = -\frac{ag^{-\theta}}{4} + \frac{2g}{s^3}\psi$$

$(s^*, g^*)$ are strict local maximizers of $W(s,g)$ as long as:

$$\det D^2W(s^*,g^*) = \frac{g^2}{s^6}\,(2-\theta)\,(1+\theta)\psi^2 > 0$$

Second order conditions are satisfied.

## Glossary

| | |
|---|---|
| $y_{ij}$ | income of individual $i$ in country/jurisdiction $j$ |
| $\bar{y}$ | average income |
| $\alpha \in (0.5, 1)$ | share of poor individuals |
| $k \in (1, +\infty)$ | income differential between rich and poor individuals |
| $y_P = y$ | income of poor individuals |
| $y_R = ky$ | income of rich individuals |
| $U(.)$ | utility function |
| $f(.)$ | utility depending upon location |
| $H(.)$ | utility from public spending |
| $g$ | public spending / public good provision |
| $u(.)$ | utility from private consumption |
| $c$ | private consumption |
| $\theta \in (0, 1)$ | elasticity of marginal utility from public spending |
| $\lambda$ | maximum utility from public spending |
| $a \in [4\lambda, +\infty)$ | costs of heterogeneity |
| $i$ | location of individual $i$ |
| $l_g$ | location of public good $g$ |
| $d(.)$ | distance between $i$ and $l$ |
| $t$ | tax revenue |
| $\tau$ | tax rate |
| $s$ | size of country/jurisdiction |
| $W(.)$ | social welfare function |
| $L(.)$ | Total costs of distance from $g$ |
| $\psi$ | Inequality Index |
| $E_2$ | Generalized Entropy Index with parameter = 2 |
| $N$ | Number of nations |
| $\phi(.),\ \beta(.)$ | Variables depending upon $\alpha, k$ |
| $V(.)$ | Expected utility |
| $I(.)$ | Interval where stable size exists |
| | **SUPERSCRIPTS** |
| $-$ | average |
| $*$ | equilibrium value / threshold value |
| | **SUBSCRIPTS** |
| $i, j$ | individual, country/jurisdiction |
| $P, R$ | poor individual, rich individual |

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**Part II**

# SECESSION THREATS & LOGNORMAL INCOME DISTRIBUTIONS[1]

**Abstract:** Secession threats arise worldwide for different reasons: ethnic divisions, religion, or economic issues. This paper studies how wealth and income distribution affect preferences for separation within a democratic country. Incomes are represented through two-parameters lognormal density functions. Given secession rule, we assume indifference between unification and separation in one of the regions at the beginning. If the parameters of the distribution vary in that region or in the rest of the country, would the region secede? We would expect that the region secedes if its wealth has increased and does not secede if wealth in the rest of the country has increased. Our paper shows that there are cases where the region secedes even if wealth in the rest of the country has increased, and cases where the region does not secede even if its wealth has increased. Such results depend upon the skewness of income distributions and the levels of taxation that follow.

**Key Words:** Secession, Income Distribution, Wealth, Taxation

**JEL Code:** H7, D3, H2

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## 1 Introduction

At the end of World War II, there were less than 80 independent countries worldwide; now the number of nations is around 200. China, the most populated one, has more than 1.2 billion inhabitants, but almost half of them has less than 5 millions inhabitants.

The breakup of colonial empires in the sixties and the collapse of USSR in the nineties are phenomena which can partially explain the increase in the number of nations. Many other secessions has happened worldwide; furthermore, there are centrifugal forces asking for decentralization and/or separation in many other countries. Secession threats arise worldwide for different reasons: ethnic divisions, religion, or economic issues. The processes leading to regionalism, separation and independence are sometimes violent like Chechnya versus Russia and sometimes non-violent like Scotland versus the United Kingdom.

Our starting point is the same of the model developed by Bolton and Roland (1997): separation is always inefficient from the economic point of view. Defense spending, for example, is more efficient in an unified country; furthermore, free trade among regions can be guaranteed much more easily in an unified country. On the other hand, we have to consider how the benefits from unification cannot be distributed among all citizens.

We will not focus on the ways to prevent secessions: other papers have already analyzed this topic in order to find the tax rate, the compensation scheme and the secession rule to prevent (inefficient) breakup of existing countries.[2] Our paper focuses on the effects of income distribution on secession threats within a country.

We assume the existence of a democratic country composed of several regions. Given secession rule, one of its regions, call it region $A$, is assumed to be indifferent between unification and separation at the beginning. Our analysis will show how the preferences for separation in region $A$ would be affected by changes in wealth and distribution of incomes in the seceding region and also in the rest of the country, given the presence of efficiency losses from separation. Our model describes a decisional process where region $A$ chooses between separation and unification on the basis of its own preferences; it does not display a strategic interaction between region $A$ and the rest of the country.

Formally, our benchmark is the model by Bolton and Roland (1997) as

[2] See, for example, Bolton and Roland (1996, 1997), Bordignon and Brusco (2001), Etro and Giarda (2002), Le Breton and Weber (2003).

interpreted by Alesina and Spolaore (2003):[3] individuals vote on taxation levels and their preferences depend upon their own income.

In our model incomes are distributed following a two-parameters log-normal density functions. We will develop our analysis focusing on wealth (median and average income levels) and income skewness (mean/median ratio[4]) through the parameters of the distribution functions. Bolton and Roland (1997) focused on differences in terms of income between median voters across regions; our paper focus on the effects of income skewness within regions.

Given indifference at the beginning for region $A$, we would expect that, if the rest of the country becomes richer, $A$ does not secede (*ceteris paribus*); on the other hand, we would expect that, if region $A$ becomes richer, $A$ secedes (*ceteris paribus*). The results of our model are partially different: if the rest of the country becomes richer and its mean/median ratio increases, there are cases where $A$ secedes (*ceteris paribus*); furthermore, if region $A$ becomes richer and also its mean/median ratio increases, we can reasonably suppose that there are cases where region $A$ does not secede (*ceteris paribus*).

This paper is organized as follows: Section 2 presents the model and discusses the assumptions; Section 3 is devoted to the algebraic analysis and Section 4 concludes.

# 2 The model

## 2.1 General assumptions

We assume the existence of a democratic country composed of several regions whose boundaries are exogenously given and immutable.

Our country can be represented through a spatial model where population has mass equal to 1 and it is continuously and uniformly distributed on the segment $[0, 1]$; furthermore, we assume that individuals are not mobile. Size (population) of region $A$ equals $s_A \in (0, 1)$; size (population) of the rest of the country, call it $R$, equals $s_R \in (0, 1)$ and:

$$s_A + s_R = 1$$

holds.

[3] The utility function used by Alesina and Spolaore in "The Size of Nations" (2003) to discuss the model by Bolton and Roland (1997) derives from the utility function they used in "On the Number and the Size of Nations" (1997).

[4] Graphically, mean/median income ratio is the inverse of the slope of the tangent to the Lorenz curve at the 50th percentile; details on this point can be found in Section 2.5.

Incomes are not uniformly distributed on the geographical space $[0, 1]$. The density functions of income distributions are given by $\phi_A(y_i)$ in region $A$ and $\phi_R(y_i)$ in the rest of the country. We have:

$$\left.\begin{array}{l} y_{Am} < \bar{y}_A \\ y_{Rm} < \bar{y}_R \end{array}\right\} \Rightarrow y_m < \bar{y}$$

where $y_m$ is median income and $\bar{y}$ is average income.[5]

Given a proportional taxation scheme, individuals vote on the tax rate within the jurisdiction where they live and Median Voter Theorem holds: the preferences of individuals over public spending are single peaked and depend upon their own income.[6]

There is perfect substitutability between public and private goods and tax revenues are assumed to finance public good provision and lump-sum redistribution.[7]

As we will show later on, we assume that public good provision is exogenous and independent from size in order to show in a simple way that it is not possible to have efficiency gains from separation. Under this assumption, any individuals of the seceding region will have to pay more taxes in order to finance the same level of public good, given that the size of the seceding region is strictly smaller than the one of the unified country and given that taxes are proportional to income.

## 2.2 Utility of individual $i$

Following these simplifying assumptions, the utility of individual $i$ living in jurisdiction $j$ (unified country or the seceding region $A$) is given by:

$$u_{ij} = g + (1 - \tau_j)y_i + T_j \tag{1}$$

where: $g$ is exogenous public good provision, $\tau_j$ is the tax rate in jurisdiction $j$, $y_i$ represents the income of individual $i$ and $T_j$ is the transfer each individual will get from the government in the jurisdiction where he lives.

[5] Notice that in the paper we have subscript $A$ for region $A$, subscript $R$ for the rest of the country and no subscript for the unified country.

[6] In this model we abstract from heterogeneity of preferences over types of public goods, and focus on hereogeneity of incomes as the determinant of different preferences between individuals. This is the main difference with respect to the model of geopolitical organization developed by Alesina and Spolaore in 1997, where heterogeneity of preferences over public good is given by the distance from the point where the public good is located.

[7] As in the model by Meltzer and Richard (1981) we consider a tax-transfer scheme in which the revenues of a proportional income tax are redistributed lump-sum.

### 2.3 What changes in case of separation?

Scale economies are modeled in a very simple way: both public good provision ($g$) and cost of public goods ($k$) are exogenous and independent from size and $g = k$ holds. Furthermore, there are deadweight losses from taxation: in particular, 1 dollar of taxes provides $1 - \tau_j/2$ dollars for transfers and public goods.

Lump-sum redistribution, net of the costs of public good provision, is financed through a proportional taxation scheme and tax rate is chosen by the individuals of each jurisdiction; as a consequence, transfer from the government would be equal to:

$$T_A = \left(\tau_A - \frac{\tau_A^2}{2}\right)\bar{y}_A - \frac{k}{s_A}$$

in independent region $A$, whereas it is equal to:

$$T = \left(\tau - \frac{\tau^2}{2}\right)\bar{y} - k$$

in the unified country.

Notice that Median Voter Theorem holds, therefore the tax rate would be given by:

$$\frac{\partial u_A(y_{Am})}{\partial \tau_A} = 0 \Rightarrow \tau_A = \frac{\bar{y}_A - y_{Am}}{\bar{y}_A} \tag{2}$$

in independent region $A$; on the other hand, it is given by:

$$\frac{\partial u(y_m)}{\partial \tau} = 0 \Rightarrow \tau = \frac{\bar{y} - y_m}{\bar{y}} \tag{3}$$

in the unified country.

The utility of an individual living in independent region $A$ would be:

$$u_A(y_i) = g + y_i + \frac{\bar{y}_A - y_{Am}}{2\bar{y}_A}\left[(\bar{y}_A - y_i) + (y_{Am} - y_i)\right] - \frac{k}{s_A} \tag{4}$$

The utility of the same individual living in the unified country is:

$$u(y_i) = g + y_i + \frac{\bar{y} - y_m}{2\bar{y}}\left[(\bar{y} - y_i) + (y_m - y_i)\right] - k \tag{5}$$

### 2.4 Secession rule and indifference condition

**Secession Rule** Region $A$ secedes from the unified country when a majority of voters in region $A$ is in favor of separation.[8]

Under the assumption that Median Voter Theorem holds and given secession rule, region $A$ would secede if the utility of the median individual in $A$ is higher under separation rather than under unification.

In our model we assume that the median individual in region $A$ is indifferent between separation and unification at the beginning.

**Indifference Condition** $SU = u_A(y_{Am}) - u(y_{Am}) = 0$

Notice that if $SU > 0$ region $A$ would secede; on the other hand, if $SU < 0$ region $A$ would not secede.

If we substitute $y_{Am}$ to $y_i$ in (4) and (5), we obtain:

$$SU = \left\{ \frac{(\bar{y}_A - y_{Am})^2}{2\bar{y}_A} - \frac{k}{s_A} \right\} - \left\{ \frac{\bar{y} - y_m}{2\bar{y}} \left[ (\bar{y} - y_{Am}) + (y_m - y_{Am}) \right] - k \right\} = 0$$

and after algebraic manipulations, we get:

$$SU = \frac{(y_m - y_{Am})^2}{2\bar{y}} + \frac{\bar{y}_A - \bar{y}}{2} + \frac{y_{Am}^2}{2\bar{y}_A} - \frac{y_{Am}^2}{2\bar{y}} - \frac{1 - s_A}{s_A} k = 0 \qquad (6)$$

### 2.5 Skewness index

It is possible to refer to different concepts in order to rank income distributions.[9] The ratio of mean to median income is mathematically simpler and easier to introduce in the model with respect to inequality-related indices and polarization-related indices.[10] Mean/median ratio refers to the skewness of the distribution and, graphically, it represents the inverse of the

[8] We consider an "extremely weak" secession rule; this assumption seems reasonable when the central government is too weak to prevent a secession through military means.

[9] There are inequality related indices and polarization related indices; inequality and polarization focus on different aspects of a distribution. See, for example, Esteban and Ray (1994) and Wolfson (1994). For a complete discussion on this issue, see Cowell (1995) and Esteban and Ray (2005) .

[10] Mean/median ratio has been used as a proxy for both income inequality and income polarization in theoretical and empirical papers: see, for example, Meltzer and Richard (1981), Persson and Tabellini (1994, 2000), Wolfson (1994) and Alesina, Baqir and Easterly (1999).

slope of the tangent of the Lorenz curve at the $50th$ percentile. Given that average income is higher than median income; i.e., the income distributions are right-skewed, median/mean ratio equals 1 in case of egalitarian distribution of incomes and increases together with income skewness: the higher is the ratio, the higher is income skewness.

Skewness indices are:

$$SK_A = \frac{\bar{y}_A}{y_{Am}}$$

in region $A$, and:

$$SK = \frac{\bar{y}}{y_m}$$

in the unified country.

Let us go back to the tax rate chosen by individuals in region $A$ and in the unified country. Taking into account (2) and (3) it is immediate to notice that within our framework the tax rate increases together with income skewness.

## 2.6 Two-parameters lognormal distribution function

We use two-parameters lognormal density functions to describe the distribution of incomes in our model.

The two parameters are the mean ($\mu$) and the variance ($\sigma^2$) of the Normal density function:

$\mu = \int_{-\infty}^{+\infty} y f(y) dy$

$\sigma^2 = \int_{-\infty}^{+\infty} (y - \mu)^2 f(y) dy$

There are several reasons in order to justify the choice of such functions; lognormal distribution has convenient properties:[11]

It has a simple relationship with the normal distribution.

The interpretation of its parameters it is easy.

It generates symmetrical and non-intersecting Lorenz curves.

It provides a reasonable sort of fit to many actual data sets.

[11] For a detailed discussion on this point, see Aitchinson and Brown (1957) and Cowell (1995).

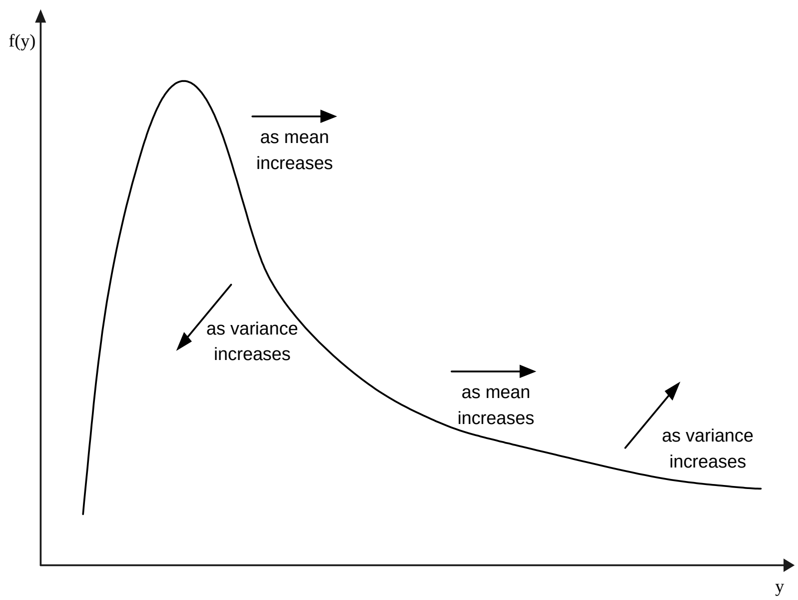


Figure 1: Two-parameters Lognormal Distribution Function

If incomes are distributed through a two-parameters lognormal function, we have:

| Region $A$ | Unified country $(A+R)$ |
|---|---|
| $\phi_A(y_i) = \frac{1}{y_i \sigma_A \sqrt{2\pi}} e^{-\frac{1}{2\sigma_A^2}(\ln y_i - \mu_A)^2}$ | $\phi(y_i) = \frac{1}{y_i \sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\ln y_i - \mu)^2}$ |
| $\bar{y}_A = e^{\mu_A + \frac{1}{2}\sigma_A^2}$ | $\bar{y} = e^{\mu + \frac{1}{2}\sigma^2}$ |
| $y_{Am} = e^{\mu_A}$ | $y_m = e^{\mu}$ |
| $SK_A = e^{\frac{1}{2}\sigma_A^2}$ | $SK = e^{\frac{1}{2}\sigma^2}$ |

If we substitute the values of median and average income depending on mean and variance in (6), we obtain:

$$SU = \frac{(e^{\mu} - e^{\mu_A})^2}{2e^{\mu + \frac{1}{2}\sigma^2}} + \frac{e^{\mu_A + \frac{1}{2}\sigma_A^2} - e^{\mu + \frac{1}{2}\sigma^2}}{2} + \frac{e^{\mu_A^2}}{2e^{\mu_A + \frac{1}{2}\sigma_A^2}} - \frac{e^{\mu_A^2}}{2e^{\mu + \frac{1}{2}\sigma^2}} - \frac{1 - s_A}{s_A} k = 0$$

and, after algebraic manipulations, we get:

$$SU = \frac{1}{2}\left[e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right] - \frac{1-s_A}{s_A}k = 0 \tag{7}$$

Notice that in case of indifference at the beginning for $R$ instead of $A$, an analogous equation holds.[12]

# 3 The analysis

If $SU = 0$, median individual in region $A$ is indifferent between separation and unification. Our purpose is to check if changes in wealth and distribution of incomes lead to separation or not. We consider changes in the region involved in the break-up process, and also changes in the rest of the country.

## 3.1 Changes in the rest of the country

We consider variations in the parameters of the distribution function of the rest of the country having effects in the whole country, whereas the distribution function of region $A$ remains unchanged. In order to simplify the derivation of the results, we consider variations in $\mu$ and $\sigma^2$ instead of variations in $\mu_R$ and $\sigma_R^2$, given (7).

### 3.1.1 $\mu$ increases

If the mean in the rest of the country increases, we have no effect on the skewness index as $SK$ does not depend on $\mu$:

The effect on $SU$ is given by:

$$\frac{\partial SU}{\partial \mu} = \frac{1}{2}e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) < 0 \tag{8}$$

The second order derivative of $SU$ with respect to $\mu$ gives us:

$$\frac{\partial^2 SU}{\partial \mu \partial \mu} = \frac{1}{2}e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) < 0$$

[12] In particular, if $R$ is indifferent between separation and unification at the beginning instead of $A$, expession (7) becomes:
$SU = \frac{1}{2}\left[e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_R}\left(e^{\frac{1}{2}\sigma_R^2} + e^{-\frac{1}{2}\sigma_R^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right] - \frac{1-s_R}{s_R}k = 0$

We have $SU > 0$ if the rest of the country is "poor enough" and poorer than region $A$; on the other hand, we have $SU < 0$ if the rest of the country is "rich enough" and richer than region $A$.

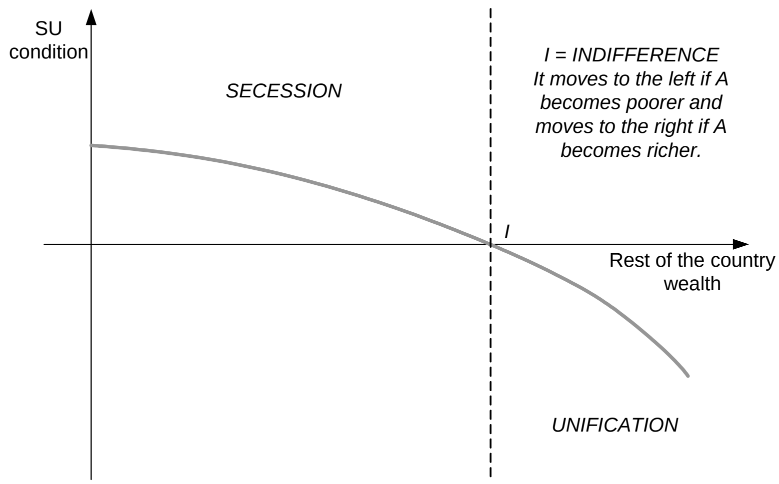


Figure 2: SU condition if R becomes richer

**PROPOSITION 1 Given $SU = 0$ and the following wealth increase in the rest of the country (with income skewness unchanged), $A$ does not secede (*ceteris paribus*).**

**Proof.** If $\mu$ increases, both the median income and the average one increase in the rest of the country and in the whole country; on the other hand neither the skewness index nor the tax rate vary as a consequence.

The effects on the utility of median individual in $A$ are the following:

$$\Delta u_{(y_{Am})} (\Delta T) > 0$$

under unification, and:

$$\Delta u_{(y_{Am})A} (\Delta T) < 0$$

under separation, due to efficiency losses.

Therefore, if $\mu$ increases, region $A$ does not secede. ■

In general, incentives to secede for region $A$ decrease as wealth in the rest of the country increases.

### 3.1.2 $\sigma$ increases

If $\sigma$ increases, the effect on the skewness index is given by:

$$\frac{\partial SK}{\partial \sigma} = \sigma e^{\frac{1}{2}\sigma^2} > 0$$

On the other hand, the effect on $SU$ condition is given by:

$$\frac{\partial SU}{\partial \sigma} = \frac{1}{2}\sigma \left[ -e^{\mu} \left( e^{-\frac{1}{2}\sigma^2} + e^{\frac{1}{2}\sigma^2} \right) + 2e^{\mu_A - \frac{1}{2}\sigma^2} \right] \gtreqless 0 \tag{9}$$

Notice that:

$$\frac{\partial SU}{\partial \sigma} = 0 \iff \sigma = \sqrt{\ln \left[ 2 \left( e^{\mu_A - \mu} - 1 \right) \right]}$$

It follows that the derivative of $SU$ with respect to $\sigma$ equals zero only if $\mu_A > \mu$, given $\sigma > 0$. As a consequence, we have to distinguish between two different cases: $\mu_A \leq \mu$ and $\mu_A > \mu$.

The second order derivative of $SU$ with respect to $\sigma$ gives us:

$$\frac{\partial^2 SU}{\partial \sigma \partial \sigma} = \frac{1}{2} \left[ \left( \sigma^2 - 1 \right) \left( e^{\mu - \frac{1}{2}\sigma^2} - 2e^{\mu_A - \frac{1}{2}\sigma^2} \right) - \left( \sigma^2 + 1 \right) e^{\mu + \frac{1}{2}\sigma^2} \right] \gtreqless 0$$

Notice that the second order derivative is positive only if $\sigma$ tends to $0^+$; as a consequence, we assume:

$$\frac{\partial^2 SU}{\partial \sigma \partial \sigma} < 0$$

If $\mu_A \leq \mu$, $SU$ is always negative.

If $\mu_A > \mu$, we have: $SU \gtreqless 0$ if the rest of the country is "not skewed" and less skewed than region $A$; $SU < 0$ if the rest of the country is "skewed enough" and more skewed than region $A$.

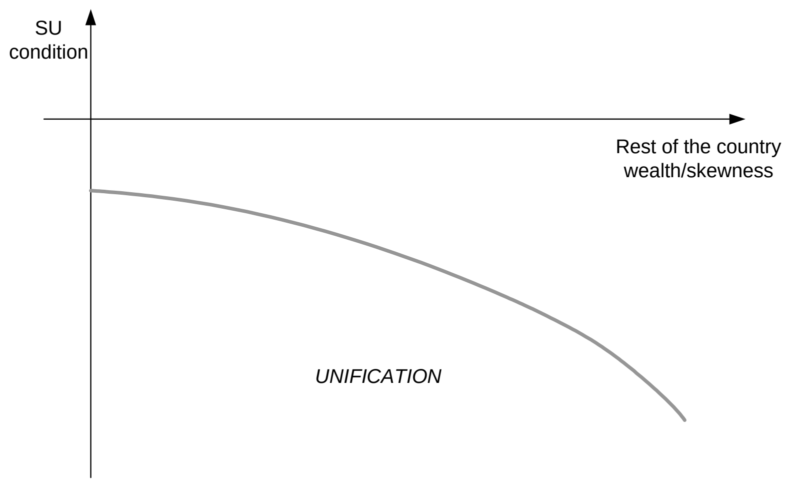


Figure 3: SU if w/s in $R$ increases ($\mu_A \leq \mu$)

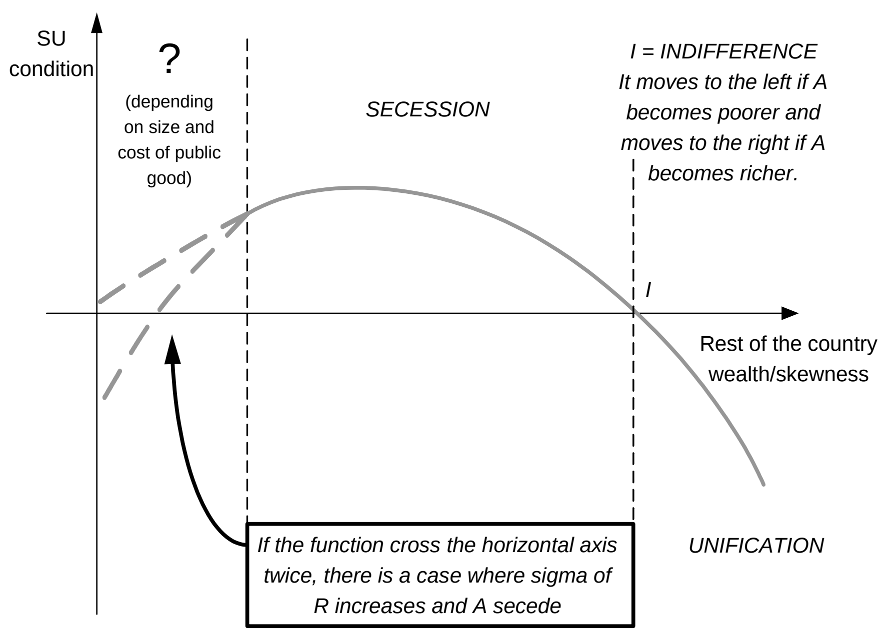


Figure 4: $SU$ if w/s in $R$ increases ($\mu_A > \mu$)

**PROPOSITION 2** **Given $SU = 0$ and the following wealth and income skewness increase in the rest of the country, there are cases where $A$ secedes (*ceteris paribus*).**

**Proof.** If $\sigma$ increases, the average income increases in the rest of the country and in the whole country, whereas the median income remains unchanged: the median individual becomes relatively poorer with respect to the individual with average income. As a consequence, income skewness and tax rate increase in the rest of the country and in the whole country.

The effects on the utility of the median individual in $A$ are the following:

(i) If the median in region $A$ is poorer than the median in the whole country we have that unification is Pareto superior to separation:

$$\Delta u_{(y_{Am})} (\Delta\tau, \Delta T) > \Delta u_{(y_{Am})A} (\Delta T)$$

Region $A$ does not secede.

(ii) If the median in region $A$ is richer than the median in the whole country we have that separation can be Pareto superior to unification as:

$$\Delta u_{(y_{Am})} (\Delta\tau, \Delta T) \gtreqless \Delta u_{(y_{Am})A} (\Delta T)$$

It depends upon the skewness of income distribution $SK$, the size of the seceding region $s_A$ and the costs of public good $k$. ■

In general, we would expect that incentives to secede for region $A$ decrease as wealth and skewness in the rest of the country increases. Our analysis show that there are cases where such incentives increase; they increase if the median in region $A$ is richer than the median in the rest of the country and $\sigma$ is "low enough".

## 3.2 Changes in region $A$

We consider now variations in the parameters of the distribution function of region $A$ having effects in the whole country, whereas the distribution function of the rest of the country remains unchanged. In particular, we consider variations in $\mu_A$ and $\sigma_A^2$ and also the variations in $\mu$ and $\sigma^2$ that follow.

### 3.2.1 $\mu_A$ increases

The mean is a linear operator, then:

$$\mu = s_A\mu_A + s_R\mu_R$$

If mean in region $A$ increases, we have no effects on skewness indices as neither $SK$ nor $SK_A$ depend upon $\mu_A$:

The effect on $SU$ condition is given by:

$$\frac{\partial SU}{\partial \mu_A} = \frac{1}{2}\left[s_A e^{s_A\mu_A+s_R\mu_R}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right] \tag{10}$$

The analysis of this derivative is not straightforward; $\partial SU/\partial\mu_A$ is supposed to be negative if region $A$ is "poor enough" and poorer than the rest of the country; on the other hand, $\partial SU/\partial\mu_A$ is supposed to be positive if region $A$ is "rich enough" and richer than the rest of the country.

Let's consider now the second order derivative of $SU$ condition with respect to $\mu_A$:

$$\frac{\partial^2 SU}{\partial \mu_A \partial \mu_A} = \frac{1}{2}\left[s_A^2 e^{s_A\mu_A+s_R\mu_R}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right]$$

The second order derivative is supposed to be negative only if $\mu_A$ tends to $0^+$; it is positive otherwise.

We have $SU < 0$ if region $A$ is "poor enough" and poorer than the rest of the country; on the other hand, we have $SU > 0$ if region $A$ is "rich enough" and richer than the rest of the country.

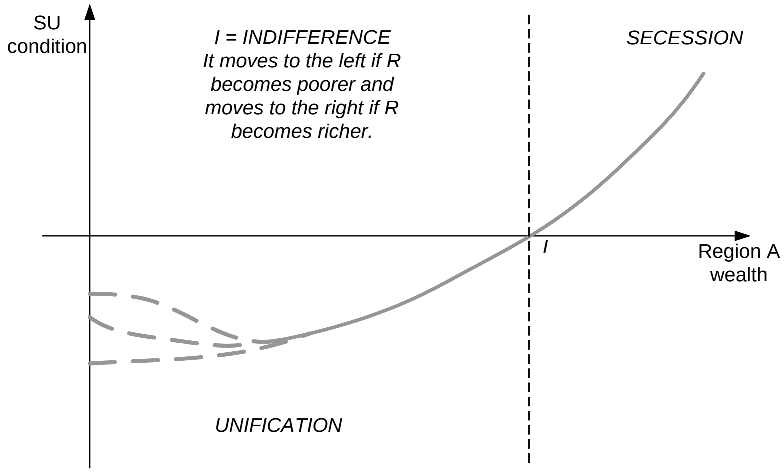


Figure 5: SU condition if Region A becomes richer

**PROPOSITION 3** **Given $SU = 0$ and the following wealth increase in region $A$ (with income skewness unchanged), region $A$ secedes (*ceteris paribus*).**

**Proof.** If $\mu_A$ increases, both the median income and the average one increase in region $A$ and in the whole country; on the other hand, neither the skewness index nor the tax rate vary as a consequence.

The effects on the utility of the median individual in $A$ are the following:

$$\Delta u_{(y_{Am})} (\Delta y_{Am}, \Delta T) > 0$$

under unification (the effect is independent of the differential between $y_{Am}$ and $y_m$), and:

$$\Delta u_{(y_{Am})A} (\Delta y_{Am}, \Delta T) > 0$$

under separation (the variation depends upon the differential between $y_{Am}$ and $y_m$: positively if $y_{Am} > y_m$, negatively if $y_{Am} < y_m$).

As a consequence, if $\mu_A$ increases, the richer the median in region $A$ with respect to the median individual in the whole country, the greater is supposed to be the positive effect under separation with respect to the positive effect under unification. If region $A$ is poorer and poorer than the rest of the country, it could be the case that even if $\mu_A$ increases the median individual in $A$ would prefer unification, but, if the median in $A$ is indifferent between separation and unification at the beginning, region $A$ secedes as a consequence of an increase in $\mu_A$ ■

In order to analyze what happens following a variation in $\mu_A$ given every possible value of $SU$, we have to take care of different variables, but in general incentives to secede for region $A$ increase together with the wealth of region $A$.

### 3.2.2 $\sigma_A$ increases

The variance is not a linear operator; we have:

$$\sigma^2 = s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \sigma_{AR}$$

where $\sigma_{AR}$ is covariance.
Using Pearson Correlation Coefficient:

$$\rho_{AR} = \frac{\sigma_{AR}}{\sigma_A \sigma_R}$$

where $\rho_{AR} \in (-1\ ,\ 1)$, we can rewrite the variance in the whole country in terms of the variance in region $A$ and in the rest of the country:

$$\sigma^2 = s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R$$

We can reasonably assume that the correlation between the variance in region $A$ and the variance in the rest of the country is non-negative or, at least, “not too much negative”.

Formally, we assume that the derivative of income variance in the whole country with respect to income variance in region $A$ cannot be negative:

$$\frac{\partial\sigma^2}{\partial\sigma_A^2} \geq 0 \iff \rho_{AR} \geq -\frac{s_A\sigma_A}{s_R\sigma_R}$$

If $\sigma_A$ increases, we have the following effects on skewness indices:

$$\frac{\partial SK}{\partial\sigma_A} = \left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right) e^{\frac{1}{2}(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R)} > 0$$

$$\frac{\partial SK_A}{\partial\sigma_A} = \sigma_A e^{\frac{1}{2}\sigma_A^2} > 0$$

On the other hand, the effect on $SU$ condition is given by:

$$
\begin{aligned}
\frac{\partial SU}{\partial \sigma_A} &= -\frac{e^{\mu}}{2}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right) e^{-\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)} + \\
&\quad -\frac{e^{\mu}}{2}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right) e^{\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)} + \\
&\quad +\frac{e^{\mu_A}}{2}\sigma_A\left(e^{\frac{1}{2}\sigma_A^2} - e^{-\frac{1}{2}\sigma_A^2}\right) + \\
&\quad +e^{\mu_A}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right) e^{-\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)}
\end{aligned} \tag{11}
$$

The analysis of this derivative is not straightforward; exactly as in case of an increase in $\sigma$, we consider the existence of two different cases: $\mu_A \geq \mu$ and $\mu_A < \mu$

Let's solve them graphically.

If $\mu_A \geq \mu$, $SU$ is always positive.

Let us focus now on the case where $\mu_A < \mu$. $\partial SU/\partial\sigma_A$ is supposed to be negative if region $A$ is "not skewed" and/or less skewed than the rest of the country; on the other hand, $\partial SU/\partial\sigma_A$ is supposed to be positive if region $A$ is "skewed enough" and/or more skewed than the rest of the country.

The second order derivative of $SU$ condition with respect to the skewness of region $A$ is extremely complex: given our results in case of an increase in wealth and skewness in the rest of the country, the second order derivative can reasonably supposed to be positive (and negative only if $\sigma_A$ tends to $0^+$).

In general, $SU$ is supposed to be positive if region $A$ is "skewed enough" and more skewed than the rest of the country.

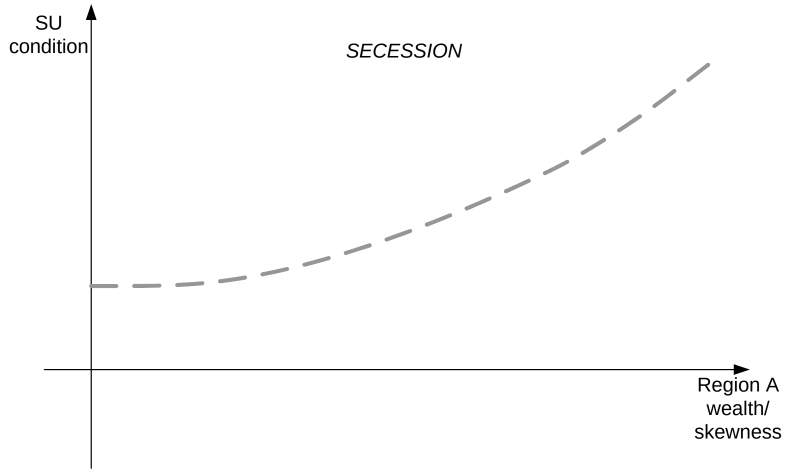


Figure 6: SU if w/s in A increases ($\mu_A \geq \mu$)

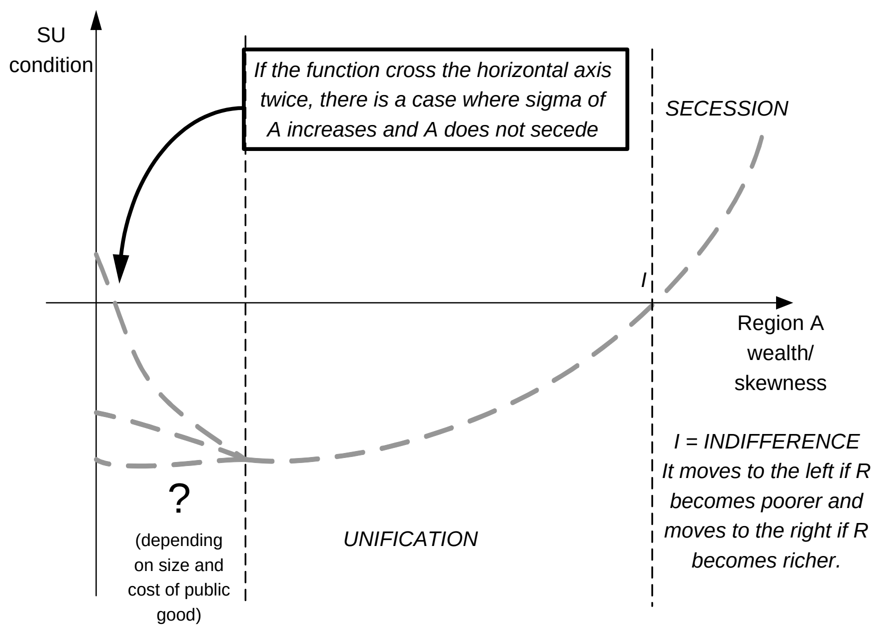


Figure 7: SU if w/s in $A$ increases ($\mu_A < \mu$)

**PROPOSITION 4** **Given $SU = 0$ and the following wealth and income skewness increase in region $A$, there are cases where $A$ does not secede (*ceteris paribus*).**

**Proof.** If $\sigma_A$ increases, the average income increases in region $A$ and in the country, whereas the median income remains unchanged: the median individual becomes relatively poorer with respect to the individual with the average income. As a consequence, the income skewness and the tax rate increase in region $A$ and in the whole country.

The effects on the utility of the median individual in $A$ are the following:

(i) If the median individual in region $A$ is richer than the median in the whole country we have that separation is Pareto superior to unification as:

$$\Delta u_{(y_{Am})}(\Delta\tau, \Delta T) < \Delta u_{(y_{Am})A}(\Delta\tau, \Delta T)$$

(ii) If the median individual in region $A$ is poorer than the median in the whole country we have ambiguous effects: it could be the case that an indifferent (at the beginning) median individual of region $A$ would not secede as a consequence of an increase in $\sigma_A$. ■

If we focus on variations in $\sigma_A$, the effects on $SU$ are not easy to interpret. We made some simulations focusing on the effects of wealth, income skewness, size of the seceding region and cost of public good. If the median individual in $A$ is "not too much poorer" than the median in the whole country, incentives to secede are supposed to increase for region $A$; on the other hand, the poorer the median in $A$ with respect to the median in the country, the more ambiguous the effects of a variation in $\sigma_A$ are supposed to be.

## 4 Conclusion

The model by Bolton and Roland (1997) showed that an increase in across regions inequality make separation more likely to occur. We develop an analysis of the effects of wealth and income distribution on the preferences of the seceding region. Our model shows a strong link between wealth and political separatism, but it also shows that there are cases where distribution effects overcompensate wealth effect. In particular, we find that an increase in the skewness of income distribution in the rest of the country can make separation more likely to occur, due to the different levels of taxation chosen by the median voter. Such result could give an interesting hint in order to study the cases of regions asking for regionalism, separation and/or independence within developed countries: not only the already cited case of Scotland, but also Lombardia (Italy), Catalunya and Pais Vasco (Spain), Flanders (Belgium)...

We find several analyses whose results can be partially useful to check the goodness of fit of our theoretical model, even if they didn't deal explicitly with wealth, income skewness and secession threats; most of them deal with the issue of government decentralization.

Several empirical works found a negative correlation between average income and centralization: Wallis and Oates (1988) on the trends in fiscal centralization during the $20^{th}$ century in state and local sector in the United States; Panizza (1999) on revenues and expenditure centralization ratios in a large sample of countries; on the other hand, the empirical analysis by Cerniglia (2003) on OECD countries shows that the correlation between income inequality and centralization seems to be positive.

Other works investigated the reasons of the collapse of Soviet Union and Russian Federation from economic perspectives. The theoretical model by Berkowitz (1997) on peripheral Russian regions shows how the impact of wealth increase on secession threats depends upon the efficiency gains from separation (in our model there are efficiency losses from separation), the substitutability between public and private goods (not considered in our model) and whether or not the demand for public good is stronger in center or periphery. Giuliano (2006) analyzed the arising of secessionism in former Soviet Union concluding that, through the framing on issues of ethnic economic inequality, nationalist leaders were able to politicize the ethnic issue by persuading people to view their personal life chances as dependent on the political fate of their ethnic community; economy becomes an instrument for politician to create secession wishes.

From a theoretical perspective, the model by Jaramillo, Kempf and Moizeau (2003) on the link between inequality and club formation gives us an interesting hint. The model shows that inequality leads to segmentation, therefore, given two distributions of endowments, the more inegalitarian generates more clubs; furthermore, a club becoming more and more inegalitarian is expected to break-up.

The results of Jaramillo, Kempf and Moizeau seems coherent with ours, in the sense that different levels of income inequality within regions make separation more likely to occur, but a clarification is in order; our model shows that, due to different levels of taxation,[13] preferences for separation in the seceding region can increase together with income skewness in the rest of the country but they can decrease as income skewness increases in the seceding region.

In the very end of the paper, we need to note that almost nothing can be said on the effects of income skewness on secession threats in real world. An empirical analysis on this issue remains an unanswered question.

[13] Let us recall that in our model the tax rate increases together with income skewness.

## Glossary

| | |
|---|---|
| $s$ | size of region(s) |
| $y_{ij}$ | income of individual $i$ in region(s) $j$ |
| $\bar{y}$ | average income |
| $y_m$ | median income |
| $u(.)$ | utility function |
| $g$ | public spending |
| $\tau$ | tax rate |
| $T$ | transfer from the government |
| $k$ | cost of public goods ($k = g$) |
| $SU$ | Indifference condition |
| $SK$ | Skewness Index (mean/median ratio) |
| $\phi_j(y_i)$ | income distribution |
| $\mu_j$ | mean of the distribution |
| $\sigma_j$ | standard deviation of the distribution |
| $\sigma_j^2$ | variance of the distribution |
| $\sigma_{jj'}$ | covariance |
| $\rho_{jj'}$ | Pearson Correlation Coefficient |
| | **SUPERSCRIPTS** |
| $-$ | average |
| | **SUBSCRIPTS** |
| $i, j$ | individual, region(s) |
| $m$ | median |
| $A, R$ | region $A$, rest of the country |

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# Part III
# DIVERGENCES IN INCOME DISTRIBUTION RANKING & POLICY IMPLICATIONS[1]

**Abstract:** This paper focuses on the differences between inequality and polarization measures in order to rank income distributions. Using two-spike distributions, our analysis shows the behaviour of inequality measures and polarization measures; in particular, we focus on monotonicity or non-monotonicity with respect to variations in the parameters of the distribution on the basis of the characteristics of each index. This paper also shows that the policy implications of theoretical and empirical models could diverge depending on the chosen measure.

**Key Words:** Economic Methodologies, Income Distribution, Public Good Provision

**JEL Code:** B4, D3, H4

[1] I am grateful to Mario Gilli, Conchita d'Ambrosio, Chiara Gigliarano and Frank Cowell for their useful comments.

# 1 Introduction

There are many empirical papers and books on the policy effects of the distribution of incomes and there are many ways to rank distributions. Several indices referring to different concepts are used in the literature. Our purpose is to explore the relationships between the indices and we will also show that the policy implications of different ways to rank distributions could diverge, even if the measures we analyze refer to the same theoretical concept.

First of all, we explain the differences between the concepts of inequality and polarization given a generic income distribution,[2] then we will explore the different characteristics of the indices.

Second, we consider the case of the simplest possible non-uniform distribution of incomes, a two-spike distribution, calculating how variations in one of its parameters affects each index,[3] then we focus on similarities between measures referring to different concepts and differences between measures referring to the same concept.

In the end, we discuss how the policy implications of theoretical and empirical models could diverge depending upon the chosen measure.

We focus on different literatures. Theoretical works, like Gini (1939), Theil (1967), Atkinson (1970), Lam (1986), Wolfson (1994), Esteban and Ray (1994), Wang and Tsui (2000) and the handbook by Lambert (1993). Empirical works on the analysis of trends in polarization and inequality, like Esteban, Gradìn and Ray (2007), Wolfson (1997) and Zhang and Kanbur (2001). Empirical works on the policy implications of income distribution, like Persson and Tabellini (1994) on the link between income inequality and growth rate of the economy and Alesina and Perotti (1996) on the effects of income distribution on political stability of countries. We also focus on analyses on the effect of income inequality on public expenditure like Lindert (1996) and Milanovic (1999). Finally, the works by Corneo and Grüner (2002) and Ravallion and Lokshin (2000) give us interesting hints in order to study the policy implications of the indices.

This paper is organized as follows: Section 2 presents the indices; Section 3 describes the behaviour of the indices in case of two-spike distributions; Section 4 discusses the policy implications and Section 5 briefly concludes.

[2] We consider right-skewed income distributions; i.e., distributions where median income is lower than average income. Notice that there are no empirically observed income distributions worldwide where median income is higher than average income.

[3] In this paper we consider the case of non mean preserving spreads, in order to be able to study the effects of variations in one parameter of the distribution that is independent of variations in the other parameters.

## 2 Income distributions ranking

### 2.1 Inequality indices

*A standard measure of inequality is a scalar representation of the interpersonal difference in income within a given population.*

Frank A. Cowell

**Gini Inequality Index**

Gini Inequality Index is based on Lorenz curves method. In the sense of Gini (1939), inequality is the "difference" between Lorenz curve and equality line. The Index is defined as a ratio where the numerator is the area between equality line and Lorenz curve and the denominator is the area under uniform distribution line.

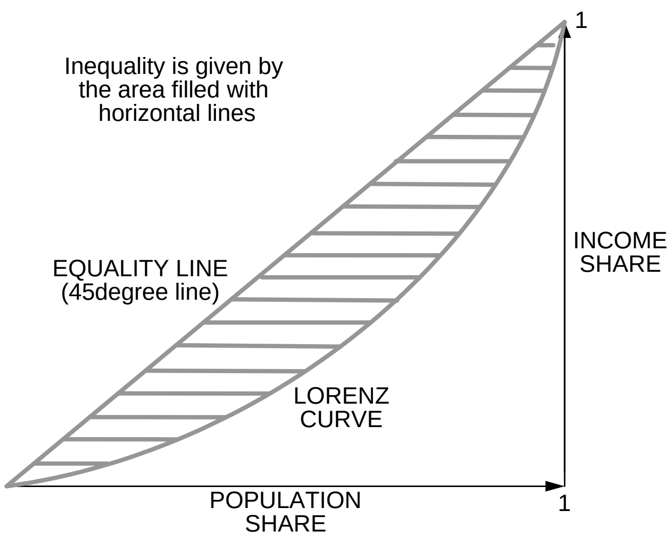


Figure 1: Gini Index

If Lorenz curve given distribution $X$ can be represented by:

$$Lorenz\ curve = L_X(q)$$

where $q$ represents income percentile(s), Gini Index is given by:

$$G = 1 - 2\int_0^1 L_X(q)dq$$

where $\int_0^1 L_X(q)dq$ is the area under the Lorenz curve.

Gini Coefficient takes values between 0 (perfect income equality) and 1 (perfect inequality; i.e., only one person holds richness).

Gini Coefficient gives more "weight" to the incomes around the mode and less "weight" to the ends of the distribution; it satisfies the weak principle of transfers, it is not decomposable and it is independent of population scale and income scale.

### Theil's Entropy Inequality Index

Theil's definition of his own Index is: "[The Theil Index can be interpreted] as the expected information content of the indirect message which transforms the population shares as prior probability into the income shares as posterior probabilities" (1967). This index does not deal with the Lorenz curves method; it deals with the concept of entropy, which can be considered the "degree of disorder" of a system, as stated by Cowell (1995): in particular, if we refer to inequality measurement, entropy can be expressed as:

$$entropy = \sum_{i=1}^{n} p_i h\left(p_i\right) = -\sum_{i=1}^{n} p_i \ln\left(p_i\right)$$

where: $n$ is the number of individuals and $p_i$ is the share of person $i$ in total income.

If we use Theil Index, overall inequality can be expressed through a weighted sum of the inequality values for every income subgroups:

$$T = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\bar{y}} \ln \frac{y_i}{\bar{y}}$$

where: $n$ is the number of individuals; $y_i$ is the income of individual $i$ and $\bar{y}$ is average income.

The Theil's Entropy Inequality Index takes value 0 in case of perfect income equality and increases together with income inequality.

Theil's Entropy Index gives the same "weight" to every income group; it satisfies the strong principle of transfers, it is decomposable and it is independent of population and income scale.

**Atkinson Inequality Index**

Gini Coefficient and Theil's Entropy Index do not give more "weight" to the bottom end of the distribution. Atkinson Index, *de facto,* attaches different "weights" to different income levels depending on the parameter $e \in (0, +\infty)$ which represents inequality aversion; that is, the higher $e$, the higher the "weight" given to poor. If $e \longrightarrow 0^+$, inequality aversion is minimized, and Atkinson Index approaches zero for every income distribution; on the other hand, for every possible unequal distribution, the Atkinson Index increases together with $e$ (*ceteris paribus*):

$$A(e) = 1 - \left[ \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{y_i}{\bar{y}} \right]^{1-e} \right]^{\frac{1}{1-e}}$$

where: $n$ is the number of individuals, $y_i$ is the income of individual $i$ and $\bar{y}$ is average income.

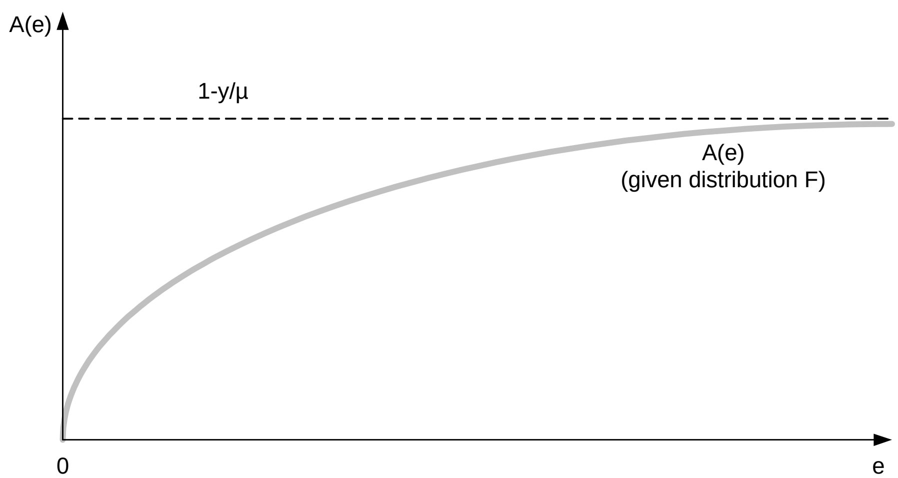


Figure 2: Atkinson Index given income distribution F

In case of extreme inequality aversion we have $e \longrightarrow +\infty$ and Atkinson Index reduces to:

$$A(e \longrightarrow +\infty) = 1 - \frac{y_L}{\bar{y}}$$

where: $y_L$ is the lowest income within the jurisdiction and $\bar{y}$ is average income.

If inequality aversion is maximized all the "weight" is given to the poorest class and Atkinson Index is given by the distance between lowest and average income.

The Atkinson Inequality Index takes values between 0 (perfect income equality) and 1 (perfect income inequality); this range is valid for every $e > 0$.

As we've already pointed out, Atkinson Index gives different "weights" to different income groups depending on $e$; furthermore, it satisfies the weak principle of transfers, it is decomposable (if $e \neq +\infty$) and it is independent of population and income scale.

## 2.2 Polarization indices

> *Polarization places more emphasis on "clustering". Many phenomena, such as "the disappearing middle class", can be described as "polarization".*
>
> Xiaobo Zhang and Ravi Kanbur

### Wolfson Polarization Index

Wolfson Index is derived from the method of the Lorenz curves.

Let's consider Figure 3: polarization is given by the area filled with vertical lines; this area is delimited at the bottom by the tangent to the Lorenz curve at the $50th$ percentile. From that area it is possible to derive the polarization curve: the higher is the curve, the more the distribution is spread away from the median value, the weaker is the middle class, the higher is income polarization.

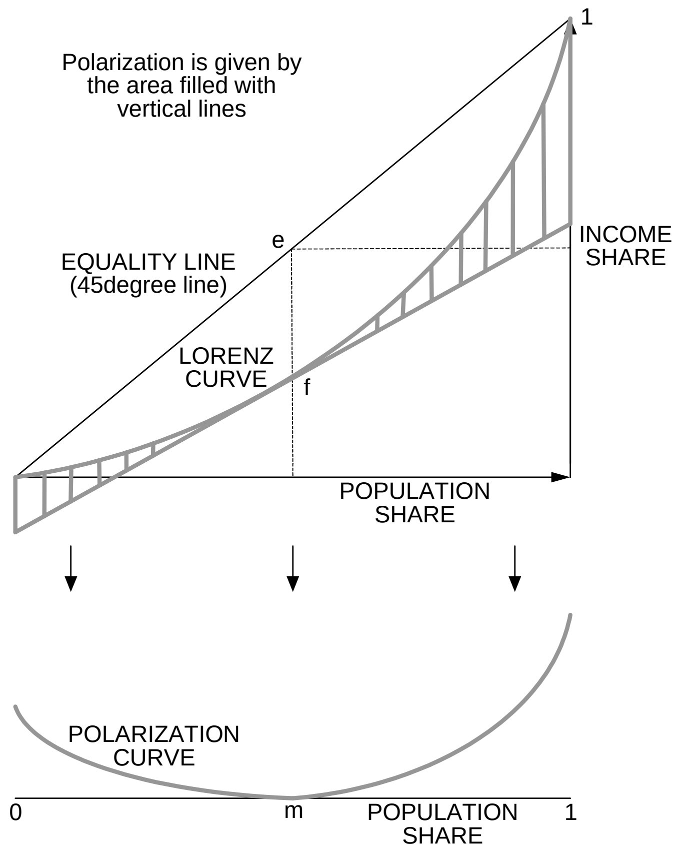


Figure 3: Wolfson Index

In order to show the difference between inequality and polarization, let us consider the case of a Pigou-Dalton Transfer. If the Transfer is from an individual above the median to an individual below the median (and nobody crosses the median because of the Transfer) both inequality and polarization decline: in such a case both the Lorenz curve and the tangent line at the $50th$ percentile move closer to uniform distribution line. On the other hand, if a Pigou-Dalton Transfer occurs between individuals on the same side with respect to the median, we observe that the Lorenz curve moves closer to uniform distribution line, whereas the tangent line at the $50th$ percentile is unaffected by the Transfer; in such a case inequality decreases as Lorenz curve moves closer to uniform distribution line and polarization increases as polarization curve goes up.

Formally, Wolfson Polarization Index is given by:

$$W = 2\frac{\bar{y}}{y_m}\left[2\left(0.5 - L(0.5)\right) - G\right]$$

where: $\bar{y}$ is average income; $y_m$ is median income; $L(0.5)$ is the income share of the bottom half of the population and $G$ is Gini Coefficient.

*De facto*, Wolfson Index measures the distance of a given distribution with respect to the one where all the population is concentrated at the median value.

The Wolfson Polarization Index take values between 0 (minimum income polarization) and 1 (maximum income polarization). It gives the same "weight" to every income group..

### Esteban and Ray Polarization Index

The purpose of Esteban and Ray (1994) is to distinguish between inequality and polarization through examples from discrete distributions: in some cases, given a variation in the number of groups and/or in the distance between different groups, inequality goes up and polarization goes down, or vice versa. They also impose "reasonable" axioms to allowable measures of polarization; a distribution of individual attributes (natural logarithm of income) is polarized if: (i) there is a high degree of homogeneity within each group, (ii) there is a high degree of heterogeneity across groups, and/or (iii) there is a small number of significantly sized groups, given that small groups carry little "weight" in order to measure polarization.

Esteban and Ray introduce a continuous Identification Function:

$$I(\pi_{ci}) = \pi_{ci}^{s}$$

where: $\pi_{ci}$ is the population share belonging to income class $c$ of individual $i$ and $s \in (0\ ,\ 1.6]$ is the polarization sensitivity parameter (sensitivity increases together with $s$).

The Identification Function is increasing in the population share $\pi_i$ belonging to the same income class of individual $i$. For every individual, his sense of identification is increasing in the number of individuals with the same income level as him.

Furthermore, Esteban and Ray introduce a continuous Alienation Function:

$$a\left(\delta\left(\ln y_i, \ln y_j\right)\right) = \left|\ln y_i - \ln y_j\right|$$

which is non decreasing in the income distance between individual $i$ and individual $j$. The Alienation Function characterizes the antagonism between individuals caused by income differences

Summarizing, the effective antagonism felt by $y_i$ towards $y_j$ is given by:

$$F(I, a)$$

and polarization in the sense of Esteban and Ray is given by the sum of all the antagonisms within population:

$$ER = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{ci} \pi_{cj} F\left(I(\pi_{ci}), a\left(\delta\left(\ln y_i, \ln y_j\right)\right)\right)$$

The Esteban and Ray Polarization Index is positive and increases together with income polarization. It gives the same "weight" to every income group; that is, it is "symmetric".[4]

### "Asymmetric" Esteban and Ray Polarization Index

In the last part of their paper, Esteban and Ray (1994) discuss on the "symmetry" of their polarization measure: they argue that the alienation felt by poor with respect to rich is not the same of the one felt by rich with respect to poor. As a consequence, they consider the case where the Alienation Function registers positive values only for income values greater than that of the individual considered; that is, a case where different "weights" are given to different income groups.

### Wang and Tsui Polarization Index

Following Wolfson (1994) and partially Esteban and Ray (1994), Wang and Tsui (2000) create a new class of polarization indices where the approach is "symmetric" and the focus is explicitly on the median income of the distribution. Accordingly with this Index, polarization is given by the average of a concave transformation of the distance with respect to median income.

Formally, Wang and Tsui Polarization Index is given by:

$$WT = \frac{\theta}{n} \sum_{i=1}^{n} \left( \left| \frac{y_i - y_m}{y_m} \right| \right)^r$$

[4] Notice that, even if the two indices refer to different theoretical concepts, Esteban and Ray (1994) compare their own index with Gini Coefficient. In the words of Esteban and Ray (1994, page 834): "Indeed barring the fact that we are using the logarithm of incomes, our measure would be the Gini if polarization sensitivity were equal to zero".

where: $\theta$ is a positive scalar; $n$ is the number of individuals; $y_i$ is the income of person $i$; $y_m$ is median income and $r$ is a coefficient between 0 and 1.

The Index is positive and increases together with income polarization.

Wang and Tsui Polarization Index gives the same "weight" to every income group.

## 3 Two-spike distributions

Most of the authors dealing with the theoretical definitions of the measures underlines the basic concepts of the indices in an informal way, through the examples of multiple spike or multiple densities. In particular, Wolfson (1994) and Esteban and Ray (1994, 2005) focused on the differences between inequality and polarization through the description of the effects of shifts of population mass or squeezes of densities. They found that sometimes the variations in the measured inequality and in the measured polarization diverge.

Divergences between inequality and polarization emerge also in empirical works. Wolfson (1997) calculated Gini Index and Wolfson Polarization Index using data on the distribution of incomes in Canada from the sixties to the nineties; he found that inequality diverges with respect to polarization in 20% of observations. Zhang and Kanbur (2001) calculated polarization indices (Wolfson, Esteban and Ray, Wang and Tsui) in 28 Chinese provinces from 1983 to 1995; their analysis showed that in general polarization grows up but at different rates depending on the chosen measure.

In order to check if such results are given to data and/or to the differences between the measures, we use two-spike income distributions. Such distributions have been already used in the literature on income distribution; see, for example, the works on income inequality by Lam (1986) and Fields (1993) on income inequality in dual economy models and the paper by Burger (2001) on the effects of inequality aversion in the Atkinson Index. The basic features of the two-spike distributions are similar to the ones of the multiple-spike distributions we find in Esteban and Ray (1994) on income polarization; "spiked" distributions show perfect homogeneity within each group.[5] We find support for the choice of two-spike distributions also from the empirical analysis by Esteban, Gradìn and Ray (2007) on five OECD countries: they showed that the results for different polarization measures

[5] As we have already pointed out, perfect homogeneity within groups is one of the basic features of polarization (Esteban and Ray 1994, page 824).

are similar if population is divided in two, three or four groups; in particular, for higher values of polarization sensitivity parameter, two-groups representation turns out to yield higher levels of polarization.

We assume that population has mass equal to 1. Given a two-spike income distribution, individuals are divided in two groups, call them "poor" and "rich", and there is no income heterogeneity within groups. $y_P = y$ is the income of poor individuals and $y_R = ky$ is the income of rich individuals, where $k > 1$ measures income differential between income groups. The income distribution is right-skewed, then the share of poor individuals $\alpha$ belongs to $(0.5, 1)$ and $1 - \alpha$ is the share of rich individuals.

In order to summarize, we have:

$$y_P = y_m = y$$

$$y_R = ky$$

$$\bar{y} = \alpha y + (1 - \alpha)ky$$

If we use two-spike distributions and such distributions are assumed to be right skewed, we are not able to distinguish between low and middle class as a consequence: the median income equals the lowest one. This is the main problem with two-spike distributions, given that in particular the concept of polarization is strongly linked with the "weight" of the middle with respect to the ends of the distribution of incomes.

As we have already pointed out (footnote 3, page 62), in our paper we consider the effects of variations in $\alpha$ or in $k$. These variations are "non mean-preserving". In such a case we are not able to distinguish between inequality and income effects but we can consider variations in $\alpha$ that does not affect $k$ and variations in $k$ that does not affect $\alpha$. Also Burger (2001), for example, explicitly focused on non mean-preserving variations in income distribution in his analysis on the Atkinson Index.

## 3.1 Inequality indices

### Gini Inequality Index

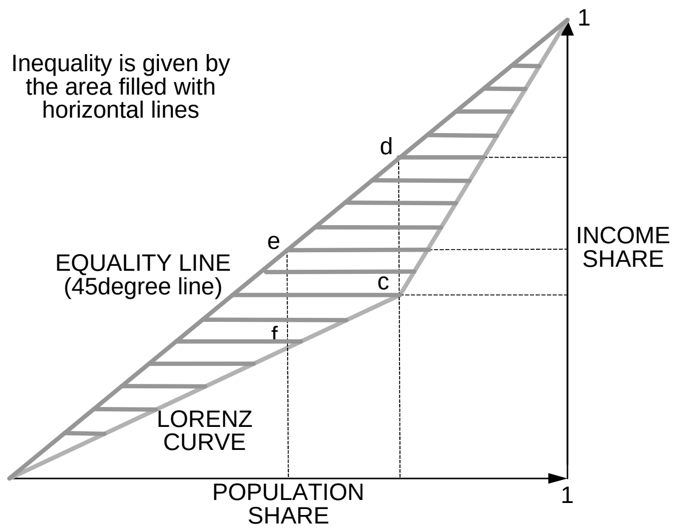


Figure 4: Gini Index (two-spike distributions)

Gini Index is given by the area between equality line and Lorenz curve, therefore we have:[6]

$$G = 2\left[\frac{\alpha\left(\alpha - \frac{\alpha}{\alpha+(1-\alpha)k}\right)}{2} + \frac{(1-\alpha)\left(\alpha - \frac{\alpha}{\alpha+(1-\alpha)k}\right)}{2}\right]$$

After algebraic manipulation, we obtain:

$$G = \alpha\left[1 - \frac{1}{\alpha+(1-\alpha)k}\right]$$

The effects of income differential and percentage of poor on Gini Index are the following:

[6]In Figure 4 we have:
$c = \left(\alpha\ ,\ \frac{\alpha}{\alpha+(1-\alpha)k}\right)$
$d = (\alpha\ ,\ \alpha)$
$e = (0.5\ ,\ 0.5)$
$f = \left(0.5\ ,\ \frac{0.5}{\alpha+(1-\alpha)k}\right)$

$$\frac{\partial G}{\partial k} = \frac{\alpha(1-\alpha)}{[\alpha + (1-\alpha)k]^2} > 0 \tag{1}$$

$$\frac{\partial G}{\partial \alpha} = 1 - \frac{k}{[\alpha + (1-\alpha)k]^2} \gtreqless 0 \tag{2}$$

*Gini Index is (strictly) monotonically increasing in the income differential between rich and poor and non monotone in the percentage of poor.*

In Figure 5, we show the behaviour of the derivative of Gini Index (and Theil Index) with respect to the percentage of poor.

**Theil's Entropy Inequality Index**

Theil Inequality Index is given by:

$$T = \frac{\alpha}{\alpha + (1-\alpha)k} \ln \frac{1}{\alpha + (1-\alpha)k} + \frac{(1-\alpha)k}{\alpha + (1-\alpha)k} \ln \frac{k}{\alpha + (1-\alpha)k}$$

After algebraic manipulations, we obtain:

$$T = \frac{(1-\alpha)k}{\alpha + (1-\alpha)k} \ln k - \ln [\alpha + (1-\alpha)k]$$

The effects of income differential and percentage of poor within population are the following:

$$\frac{\partial T}{\partial k} = \frac{\alpha(1-\alpha)}{[\alpha + (1-\alpha)k]^2} \ln k > 0 \tag{3}$$

$$\frac{\partial T}{\partial \alpha} = \frac{k-1}{\alpha + (1-\alpha)k} - \frac{k}{[\alpha + (1-\alpha)k]^2} \ln k \gtreqless 0 \tag{4}$$

In Figure 5, we show the behaviour of the derivative of Theil Index (and Gini Index) with respect to the percentage of poor.

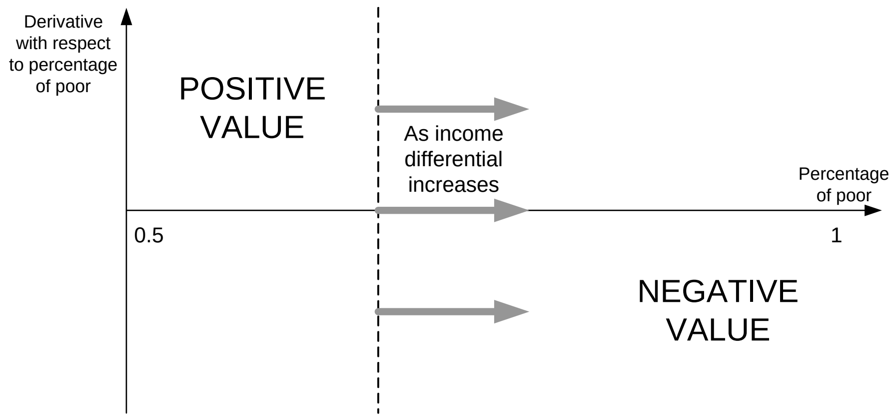


Figure 5: Derivative of Gini and Theil w.r.t. poor

*Theil Index is (strictly) monotonically increasing in the income differential between rich and poor and non monotone in the percentage of poor.*

**Atkinson Inequality Index** $(e \longrightarrow +\infty)$

Atkinson Inequality Index $(e \longrightarrow +\infty)$ is given by:

$$A = 1 - \frac{1}{\alpha + (1-\alpha)k}$$

7

The effects of income differential and percentage of poor on Atkinson Index $(e \longrightarrow +\infty)$ are the following:

$$\frac{\partial A}{\partial k} = \frac{1-\alpha}{[\alpha + (1-\alpha)k]^2} > 0 \tag{5}$$

$$\frac{\partial A}{\partial \alpha} = \frac{1-k}{[\alpha + (1-\alpha)k]^2} < 0 \tag{6}$$

*Atkinson Index $(e \longrightarrow +\infty)$ is (strictly) monotonically increasing in the income differential between rich and poor and (strictly) monotonically decreasing in the percentage of poor.*

[7] Notice that if we compare Atkinson Index $(e \longrightarrow +\infty)$ with Gini Index in case of two-spike distributions we have: $G = \alpha A$.

## 3.2 Polarization indices

### Wolfson Polarization Index

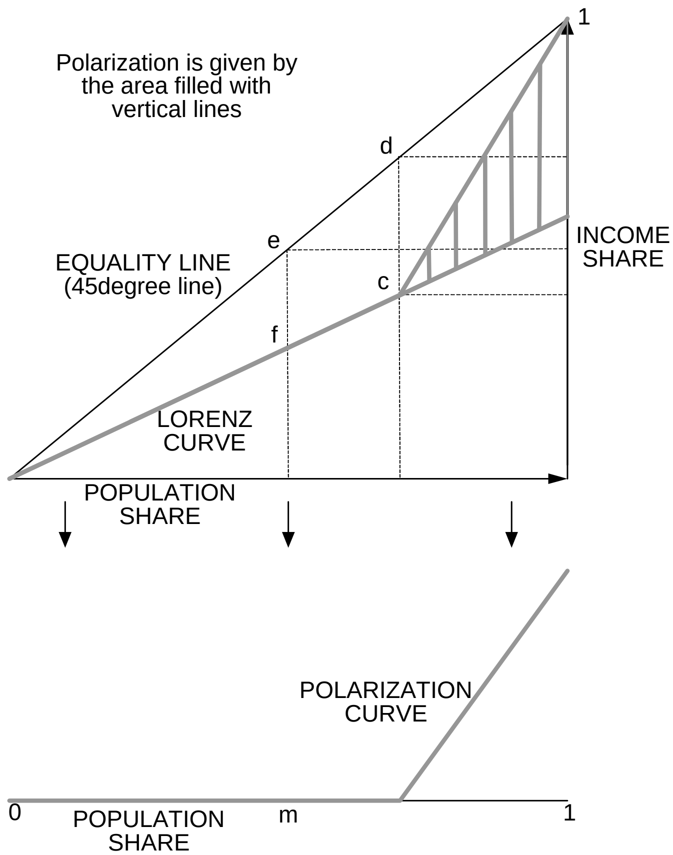


Figure 6: Wolfson Index (two-spike distributions)

Wolfson Index in case of two-spike distribution is given by:

$$W = 2\left[\alpha + (1-\alpha)k\right]\left[1 - \frac{1}{\alpha + (1-\alpha)k} - \left(\alpha - \frac{\alpha}{\alpha + (1-\alpha)k}\right)\right]$$

After algebraic manipulation, we obtain:

$$W = 2(1-\alpha)^2(k-1)$$

The effects of income differential and percentage of poor on Wolfson Index are the following:

$$\frac{\partial W}{\partial k} = 2(1-\alpha)^2 > 0 \tag{7}$$

$$\frac{\partial W}{\partial \alpha} = -(1-\alpha)(k-1) < 0 \tag{8}$$

*Wolfson Index is (strictly) monotonically increasing in the income differential between rich and poor and (strictly) monotonically decreasing in the percentage of poor.*

### Esteban and Ray Polarization Index

Esteban and Ray Polarization Index is given by:

$$ER = \left[\alpha^{1+s}(1-\alpha) + (1-\alpha)^{1+s}\alpha\right] \ln k$$

where $s \in (0\ ,\ 1.6]$ is the polarization sensitivity parameter and alienation felt by poor individuals with respect to rich ones equals alienation felt by rich individuals with respect to poor ones.

The effects of income differential and percentage of poor on Esteban and Ray Polarization Index are the following:

$$\frac{\partial ER}{\partial k} = \frac{\alpha^{1+s}(1-\alpha) + (1-\alpha)^{1+s}\alpha}{k} > 0 \tag{9}$$

$$\frac{\partial ER}{\partial \alpha} = \left\{(1+s)\left[\alpha^s(1-\alpha) - (1-\alpha)^s\alpha\right] + (1-\alpha)^{1+s} - \alpha^{1+s}\right\} \ln k < 0 \tag{10}$$

*Esteban/Ray Polarization Index is (strictly) monotonically increasing in the income differential between rich and poor and (strictly) monotonically decreasing in the percentage of poor.*

### "Asymmetric" Esteban and Ray Polarization Index

Given that poor individuals feel alienation with respect to rich ones, it may be argued that such alienation is greater than the one felt by rich individuals with respect to poor ones (*ceteris paribus*). As we have already pointed out before, this argument has been discussed by Esteban and Ray (1994, 2005); they consider the extreme case in which individuals simply do not feel alienation with respect to poorer ones.

If alienation is felt only by poor individuals with respect to rich ones, Esteban and Ray "Asymmetric"Index is given by:

$$ER(A) = \left[\alpha^{1+s}(1-\alpha)\right]\ln k$$

The effects of income differential and percentage of poor on the Index are the following:

$$\frac{\partial ER(A)}{\partial k} = \frac{\alpha^{1+s}(1-\alpha)}{k} > 0 \tag{11}$$

$$\frac{\partial ER(A)}{\partial \alpha} = \left[(1+s)\alpha^{s}(1-\alpha) - \alpha^{1+s}\right]\ln k \gtreqless 0 \tag{12}$$

In Figure 7, we show the behaviour of the derivative of Asymmetric Esteban and Ray Index with respect to the percentage of poor.

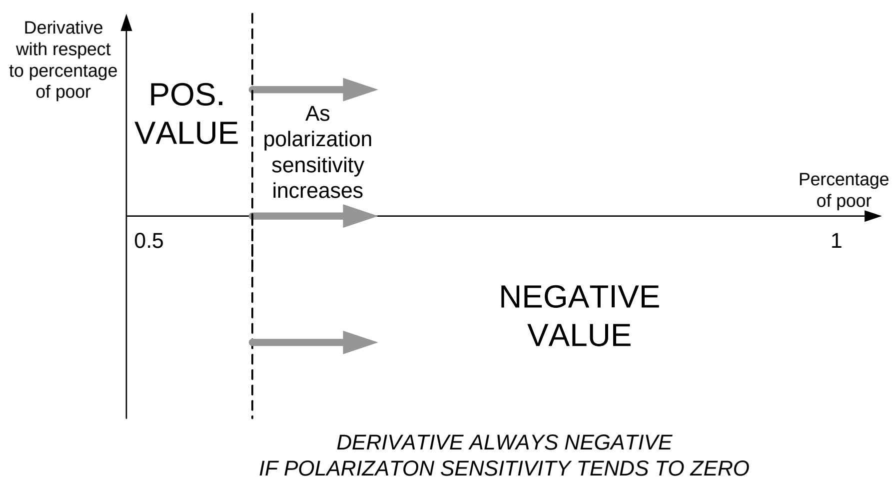


Figure 7: Derivative of ER(A) w.r.t. poor

*Esteban / Ray Asymmetric Polarization Index is (strictly) monotonically increasing in the income differential between rich and poor and non monotone in the percentage of poor.*

**Wang and Tsui Polarization Index**

Wang and Tsui Index is given by:

$$WT = \theta(1-\alpha)(k-1)^{r}$$

The effects of the parameters of the distributions on the Index are the following:

$$\frac{\partial WT}{\partial k} = \theta(1-\alpha)r(k-1)^{r-1} > 0 \qquad (13)$$

$$\frac{\partial WT}{\partial \alpha} = -\theta(k-1)^{r} < 0 \qquad (14)$$

*Wang/Tsui Polarization Index is (strictly) monotonically increasing in the income differential between rich and poor and (strictly) monotonically decreasing in the percentage of poor*

### 3.3 Income inequality and income polarization

| | Inequality | | | Polarization | | | |
|---|---|---|---|---|---|---|---|
| | $G$ | $T$ | $A_p$ | $W$ | $ER$ | $ER(A)_p$ | $WT$ |
| $k$ increases | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $\alpha$ increases | $+/-$ | $+/-$ | $-$ | $-$ | $-$ | $+/-$ | $-$ |

($p$ means "more weight" to the bottom end of the distribution)

All the indices are (strictly) monotonically increasing in the income differential between rich and poor: the more the incomes of rich and poor differ, the more there is inequality and polarization.

Different is the case of the percentage of poor: $G$, $T$ and $ER(A)$ are non monotone in the percentage of poor; $A$, $W$, $ER$ and $WT$ are (strictly) monotonically decreasing in the percentage of poor.

Let us focus on inequality indices. The index $A$ gives all the "weight" to the lowest income class and inequality is strictly monotonically decreasing in the percentage of poor. On the other hand, inequality indices that does not "overweight" the bottom end of the distribution, $G$ and $T$, are non-monotone in the percentage of poor.

Let us focus now on polarization indices. The indices $W$, $WT$ and $ER$ are "symmetric" and they all are strictly monotonically decreasing in the percentage of poor. The index $ER(A)$ is non-monotone with respect to the percentage of poor and it gives different "weights" to different income groups; that is, it is an "asymmetric" index.[8]

[8] The question of the symmetry/asymmetry of the alienation between rich and poor is for sure an interesting topic, but the concepts of polarization is by definition symmetric: following the paper by Esteban and Ray (1994), the less the size of the groups differs, the higher polarization (ceteris paribus). As a consequence $ER(A)$, in our opinion, cannot be properly considered as a measure of polarization.

**PROPOSITION 1a** **Given a right-skewed two-spike income distribution, inequality indices are (strictly) monotone in the percentage of poor only if they assign all the "weight" to the poorest income class. They are non-monotone otherwise.**

**PROPOSITION 1b** **Given a right-skewed two-spike income distribution, polarization indices are non-monotone in the percentage of poor only if they give asymmetric "weights" to income groups. They are strictly monotone otherwise.**

Our analysis confirms the results of the empirical works by Wolfson (1997) and Zhang and Kanbur (2001): polarization and inequality sometimes diverge and there are differences between different inequality measures and between different polarization measures.

# 4 Policy implications

## 4.1 What does this Index measure?

Given that inequality and polarization refer to different aspects of a distribution, in the literature there are cases where the same measure is linked with different concepts and cases where different measures are linked with the same concept.

Mean/Median Ratio, in many writing on "inequality and growth", like Persson and Tabellini (1994), is used as an approximation of income inequality; on the other hand, Wolfson (1994) refers to Mean/Median Ratio as a polarization-related statistic, even if he calls it "a measure of income skewness".[9]

The econometric analysis by Alesina and Perotti (1996) showed that inequality increases instability. They measured income inequality through the income share of the third and the fourth quintile of the population: other

[9]Income skewness is given by the distance between average and median income: the wider is the distance, the higher is income skewness. The Index simply compares this two incomes and can be calculated even if do not observe the whole distribution. Graphically, the Index represent the inverse of the slope of the tangent of the Lorenz curve at the $50th$ percentile. Formally, Mean/Median Skewness Index is given by:

$$S = \frac{\bar{y}}{y_m}$$

Where $\bar{y}$ is average income and $y_m$ is median income. The Index equals 1 in case of egalitarian distribution of incomes and increases together with income skewness.

authors would consider it as a polarization-related measure, given that the weakening of the middle class is at the basis of the concept of polarization.

Lindert (1996) analyzed the determinants of public spending in 19 OECD countries from 1960 to 1992: following his definitions of the variables income inequality and income skewness,[10] Lindert shows contrasting results. An increase in income skewness raises social public expenditure and lowers non-social public expenditure; on the other hand, an increase in income inequality lowers total public expenditure as share of GDP. The anti-spending effect of greater income inequality is in contrast with theories predicting that greater income inequality raises public expenditure, like Meltzer and Richard (1981): they considered Mean/Median Ratio as the determinant of the spending effect, but it refers to income skewness, not to income inequality. Milanovic (1999) analyzed public spending in 24 countries from the 1970s to the 1990s; he found support on the fact that higher income inequality, measured through Gini Index, raises redistribution.

## 4.2 Two-spike distributions & the real world

### 4.2.1 Social Rivalry Effect

In the econometric analysis by Corneo and Grüner (2002) on International Social Survey Programme data (1992), it is shown that an increase in Social Rivalry Effect ($SRE$) makes the individuals less likely to support redistribution. If we associate a social value to each income class, where $v_c$ is the social value associated to income class $c$, $SRE_c$ is given by downward value differential minus upward value differential with respect to the two neighboring classes of $c$:

$$SRE_c = v_c - v_{c-1} - (v_{c+1} - v_c)$$

where:

$$SRE = \sum_{c=1}^{C} |SRE_c|$$

$SRE$ does not depend on group size and increases as income differential between neighboring classes increases: we can consider either income inequality or income polarization: both of them go up as $SRE$ increases.

[10] Lindert (1996) considers: (i) natural logarithm of the ratio between first and third income quintile ("upper income gap") and (ii) natural logarithm of the ratio between third and fifth income quintile, named ("lower income gap"). His inequality index is given by (i) plus (ii); his skewness index is given by (i) minus (ii).

Given that for Corneo and Grüner (2002) an increase in Social Rivalry Effect makes the individuals less likely to support redistribution, it follows that an increase in inequality or polarization due to an increase in income differential should lower public expenditure, given our two-spike distributions.

### 4.2.2 Income trajectories

It is possible to "test" another result of the econometric analysis by Corneo and Grüner (2002); they found that a rising-income trajectory inhibits demand for redistribution. Such finding is confirmed by the empirical analysis on the "tunnel-effect" in Russia by Ravallion and Lokshin (2000).

Given a two-spike distribution of incomes, income-trajectories go up if: (i) the income differential increases (income of poor individuals and percentage of poor unchanged), (ii) the income of poor individuals increases (income differential and percentage of poor unchanged) and/or (iii) the percentage of poor decreases (income of poor individuals and income differential unchanged).

The comparison between these works and our analysis of two-spike distributions shows that there are no contradictions if we focus on changes in income differential.

If we focus on changes in the income of poor individuals, we see that our distributions are neutral to changes in wealth affecting the whole population given that such changes do not affect inequality nor polarization.

If we focus on a decrease in the percentage of poor, we have already analyzed the monotonicity or non-monotonicity of the indices; in particular, we observe that for two-spike distributions the effects on public expenditure depend upon the indices we use to rank distributions. If we consider measures of inequality, $A$ is monotone in the percentage of poor, then public expenditure should decrease; on the other hand, $G$ and $T$ are non-monotone, then policy implications in terms of public spending could diverge. If we consider measures of polarization, $W$, $ER$ and $WT$ are monotone in the percentage of poor, then public expenditure should decrease; on the other hand, $ER(A)$ is non monotone, then policy implications in terms of public spending could diverge.

A rising-income trajectory that follows a decrease in the percentage of poor can make $G$, $T$ and $ER(A)$ whether increase or decrease, depending on the percentage of poor within population and other variables.[11] It follows

[11] In Figure 5 and Figure 7, we see that the sign of the derivative with respect to percentage of poor depends upon income differential for Gini Index and Theil Index; on

that the policy implications could diverge, even if in the empirical works by Corneo and Grüner (2002) and Ravallion and Lokshin (2000) it is shown that a rising-income trajectory should mean less redistribution.

### 4.3 Furthermore...

If we refer to two-spike (and right-skewed) distributions there is no differences between the lowest income and the median one, as we've already pointed out in Section 3.

If lowest income equals median income, that's a problem for measures depending on the difference between the given distribution and the one where the income of all the individuals equals median income, for example $WT$. Furthermore, Lindert (1996) calculates his indices assuming that a wider "lower income gap"[10] implies less social spending: using two-spike distributions, lower income gap could equal zero.[12] Other works refer to redistribution and public spending processes as a battle between the ends and the middle of the distribution; in a two-spike distribution we have a "great middle" which also includes the bottom end.

An analysis on the divergences in policy implications in case of multiple-spike or more complex distributions is the main question left open.

## 5 Conclusion

There are different ways to rank distributions; they refer to different concepts: inequality, polarization (or skewness); they can diverge even if they refer to the same concept; they can have similar behaviour even if they refer to different concepts.

The choice of one measure instead of another one is not neutral: each measure refers to particular aspect of the distributions; each measure has its own characteristics: it gives, for example, different "weights" to different income groups or the same "weight" to every income group. We have shown how policy implications could diverge depending on the chosen measure in case of two-spike income distributions. Our results hold for two-spike distributions, but they can reasonably be supposed to hold also for more complex ones.

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the other hand, it depends upon polarization sensitivity for Esteban and Ray Asymmetric Index.

[12] In particular, "lower income gap" in the sense of Lindert equals zero if poor individuals are more than 60% of the population, that is, if $\alpha > 0.6$.

## Glossary

| | |
|---|---|
| $y_i$ | income of individual $i$ |
| $\bar{y}$ | average income |
| $y_m$ | median income |
| $\alpha \in (0.5, 1)$ | share of poor individuals |
| $k \in (1, +\infty)$ | income differential between rich and poor individuals |
| $y_P = y$ | income of poor individuals |
| $y_R = ky$ | income of rich individuals |
| $n$ | number of individuals |
| $L_X(q)$ | Lorenz curve given distribution $X$ |
| $q$ | income percentile(s) |
| $h(.)$ | function of income shares |
| $p_i$ | share of individual $i$ in total income of country/region |
| $e \in (0, +\infty)$ | inequality aversion |
| $z \in (1, +\infty)$ | coefficient |
| $L(0.5)$ | income share of the bottom half of the population |
| $ID(.)$ | identification function |
| $AL(.)$ | alienation function |
| $\delta(.)$ | function of income differential |
| $F(.)$ | function of antagonism between individuals |
| $\pi_c$ | population share (belonging to income class $c$) |
| $ps \in (0, 1.6]$ | polarization sensitivity parameter |
| $\omega \in (0, +\infty)$ | coefficient |
| $\theta \in (0, +\infty)$ | positive scalar |
| $r \in (0, 1)$ | coefficient |
| $v_c$ | social value (associated with income class $c$) |
| $SK$ | Skewness Index (mean/median ratio) |
| | **SUPERSCRIPTS** |
| $-$ | average |
| | **SUBSCRIPTS** |
| $i, j, c$ | individual, individual, individual in an income class |
| $m$ | median |
| $C$ | income class |
| $P, R, L$ | poor individual, rich individual, lowest income |
| $p$ | "more weight" to the bottom end of the distribution |

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