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ESSAYS ON FINANCIAL ECONOMICS AND COMPLEXITY

Tesi di dottorato di: Andrea Gurgone
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FOREWORD

The broad aim of this project is to build and study a macro-financial model that includes both real and financial aspects of the economy, ideally to obtain a comprehensive framework for the analysis of systemic risk and instabilities. In particular the inclusion of credit and interbank networks allows to explicitly model relations between firms and banks and within the banking sector, as well as to account their joint relevance for contagion and instabilities in a broad macroeconomic context.

Central to the motivation of the thesis there is need to include a financial sector into macroeconomic models and to study their interaction, an issue which raised a growing consensus among scholars after the financial crisis of 2008. For instance *Werner (2012)* claims that the separation of the disciplines of economics from banking and finance prevented economists from understanding the systemic effects of banks on the macro-economy, while finance researchers focused on the microeconomics of banks, thus ignoring potential feedback from the real to the financial sector. An obstacle to merge the two sides could be identified with the existing methods that are not suited to explicitly incorporate a network of banks into the macroeconomy, especially because in canonical models the dynamics of an entire sector is described by a single representative agent. Although many attempts have been conducted in the mainstream literature, for instance by means of New-Keynesian DSGE models with financial frictions, the reduction of the financial sector to a representative bank widely limits the possibility to gain insights about the systemic effects of financial institutions. From this viewpoint the descriptive power of macro-financial models would be enhanced if they are considered as complex systems, whose aggregate dynamics arise from the interaction of their many components. During the last years some researchers developed innovative methods to overcome these limitations, expanding to economics the study of complex systems and computational methods normally employed in physics and computer science. Such innovation gave rise to agent-based-computational economics *(see among the other Tesfatsion, 2003; Hommes, 2006)*, which proved to be a suitable instrument to link macro and financial aspects of the economy. Because of this, the methodology adopted for the project combines an agent-based-model (ABM) with financial networks, giving rise to a multi-agent macro-financial model with an interbank market.

The thesis presents a model which combines the financial sector with the real sector in a full-fledged macroeconomic framework, where real and financial sides are interdependent and endogenously determined. It could be considered part of the long-term project to build a comprehensive agent-based model of the macro-economy started in *Gabbi et al. (2015)*, whose objectives are in part shared and extended throughout the next chapters. Along the same lines as these scholars, the model attributes a remarkable relevance to the interbank market. It is widely accepted that the interbank market is a potential channel for contagion, as a matter of fact interlocked exposures of financial institutions might trigger systemic failures through knock-on effects. In addition the model does not only include direct exposures, but also indirect relations and feedback effects from the real to the financial sector and vice-versa. Each sector of the economy is in a debt-credit relationship with the others, besides banks trade in an intra-sectoral market. This complex network of connections might generate non trivial dynamics through direct and undirected links. The structure of the financial network is meant as a tool to analyse what are the effects of an interbank market when a full macroeconomic context is taken into account. This question has not been deeply explored so far, as most of the existing contributions focus on the financial aspects or consider the real economy as a black box, whose function is to demand credit and supply deposits to financial intermediaries. The preferred way to assess interbank effects consists in changing the degree of connectivity of the interbank network, varying from scarcely connected to very dense topologies. This approach has the advantage to isolate the contribution of connectivity *ceteris paribus*, so that all the patterns observed in the outcomes of simulations could be lead back to the degree of connectivity in a casual sense.

As described above, the financial network is composed by an interbank market together with a canonical credit market by which banks supply credit to the real sector. The activities of banks in these two networks are also related to two kinds of risk: credit and liquidity risk. The former arises in case of the insolvency of borrowers, the latter is determined by the inability of a financial institution to borrow or by interbank frictions that prevent the market from channelling funds where they are needed. Such risks can be faced with internal risk-management strategies, or with microprudential regulation. The goal of both measures is to ensure the financial robustness of individual institutions, but they fail to account for another concept of stability, that is systemic stability. The latter is not ensured despite banks are individually financially sound because the whole financial sector might be overexposed to the same risks, or because a systemic important financial institution declares bankrupt, thus negatively affecting the activity of the others. In these cases microprudential regulation is ineffective to guarantee stability as its underlying logic neglects the aggregate dimension. Rather it should be complementary to an holistic approach, *i.e.* macroprudential policy, which takes into account the welfare of the financial system as a whole. Macroprudential regulation should reduce or avoid systemic effects, addressing interconnections and common exposures of financial institutions. In principle macroprudential policies should be able to tackle any kind of systemic risk by choosing the proper policies. This is possible only if the risk is rightly assessed, in other words the effectiveness of regulations strongly depends on the reliability of the measure of systemic risk. In turn this poses the problem of choosing the right measure to evaluate systemic risk. Different indicators have been proposed so far, but none have turned out to be better than the others. It is worth to remark two distinctions which are employed in the second chapter of the dissertation: the first one is between market based and network based measures. Market-based indicators are built starting from cross correlations of the balance sheets of financial institutions or their market indexes, for instance return on equity, in order to determine the systemic importance of each bank, while network-based indicators exploit the exposures of financial institutions jointly with the structure of the financial network. The second distinction is between vulnerability and impact. Measures of systemic risk can be obtained from both, but their purposes are different. Vulnerability accounts for the susceptibility of banks to defaults. Those institutions that are more costly to recapitalize correspond to the most systemically important, as they entail the highest cost for the collectivity, for instance those associated to a bail-out. Conversely measures of impact account for the effect of the distress or default of a specific financial institution on the rest of the system, thus those banks that cause the greatest losses to the rest of the system are the most systemically important ones. In the end the development of effective macroprudential tools is still in progress and regulators should carefully take into account many heterogenous theoretical contributions to formulate policies. In this perspective the second chapter of the dissertation aims at clarifying the effectiveness of macroprudential rules derived from different kinds of systemic risk measures.

The thesis is divided in two chapters: the first one develops an agent based model and analyses its outcomes through numerical simulations. The second one exploits the ABM to focus on prudential policies oriented to the reduction of systemic risk. In Chapter 1 I develop an agent based model which incorporates real and financial features. In particular it is introduced a credit market where firms borrow from banks and an interbank market where liquidity is traded within the banking sector. The distinctive characteristic is the contemporaneous presence of goods, credit, labour and interbank markets. Existing contributions accounting for an interbank market often assume a simplified exogenous real side of the economy, thus ignoring households and/or firms. On the other hand there are many contributions that are keen to model real and financial aspects, but ignore interbank markets. Furthermore the model complies with stock-flow consistency, whose purpose is to control that all the resources in each sector are properly accounted and nothing else is created or destroyed. Such feature is needed *a-fortiori* in the agent-based model, as it is advisable to monitor what happens every time a string of code is added or changed. By far this is the longest part of the project, because agent-based-modelling requires a remarkable effort from a programming point of view. In addition the literature about this field is growing fast but not standardized. The model reproduces endogenous business cycles and it is able to replicate some stylized facts about business and credit cycles, while the interbank market has an important role for stability and efficiency. In particular prudential regulation, combined with adaptive expectations can exacerbate the precautionary behaviour of banks during a recession, inducing liquidity hoarding by sound banks. Furthermore connectivity of the interbank market has a twofold effect: on one side it supports credit to the real economy, but on the other it increases liquidity hoarding because credit exposure of banks is greater. The model can be employed as a workhorse for assessing risk and instability generated or propagated through the financial sector.

A set of macroprudential rules aimed at the reduction of systemic risk is presented in the second chapter of the thesis. Next such rules are compared through a policy experiment performed on the ABM previously developed. The proposed macroprudential regulation consists in setting minimum capital requirements for banks built on four systemic risk indicators. The first two measures are referred to as market-based measures, namely they are estimates derived from market indicators. The latter two are network-based measures, meaning that they are computed through an algorithm which explicitly accounts for the network topology of credit and interbank markets to assess potential losses and cascades of defaults of firms and banks. In turn each of the two set of indicator is divided in measures of vulnerability or measures of impact of financial institutions. The policy experiment reveals that measures based on vulnerability of banks perform better than those based on impact, reducing contagious defaults without worsening the macroeconomic performance. This might due to the fact that policies built on impact does not account for vulnerable financial institutions with low impact, that might be systemic as an herd.

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CONTENTS

1

LIQUIDITY HOARDING IN A MACROECONOMIC AGENT-BASED MODEL WITH AN INTERBANK MARKET

ABSTRACT We develop a macroeconomic agent-based model that consists of firms, banks and households who interact on labour, goods, credit and interbank markets. The model builds on existing literature by endogenising pricing decisions in each of the four markets as well as the choice of leverage by both banks and firms. These features lead the model to produce endogenous fluctuations that reproduce some of the essential facts of observed financial-economic cycles. In particular, liquidity hoarding by banks and sharp increases in interest rates play an essential role during financial downturns, as has been observed in actual data. Increasing the degree of connectivity on the interbank market improves the availability of credit to the real economy but intensifies liquidity hoarding.

1.1 INTRODUCTION

Following the 2007-2008 crisis, it came to be widely accepted that price flexibility and monetary stability are not enough to ensure macroeconomic stability, and that financial stability needs to be targeted both for its own sake and as a key component of any macroeconomic framework. Some of the lessons that have followed the crisis are that: (i) large financial institutions have the potential to generate systemic risk that is not confined just to the financial sector but also affects the real economy; (ii) large banks are especially prone to creating systemic risk, both for other banks via the interbank market and for real sector firms via credit markets; (iii) a key channel through which systemic risk operates is balance sheet contagion, *i.e.* losses arising in the balance sheet of one institution have the tendency to spread to other institutions.

An early contribution to the analysis of how banks affect macroeconomic stability was by Carl Chiarella (*Chiarella et al. (2012)*) who showed that the expansion of banking activities into non-traditional areas such as stock trading can expose both financial markets and the real economy to instability, even when the central bank undertakes appropriate monetary policy. By contrast a Fisherian system in which banks hold 100% reserves against demand deposits and are excluded from stock trading can support macroeconomic stability while guaranteeing a sufficient loan supply to businesses. While this paper did not employ an agent-based model (ABM), *Carl Chiarella* was among the first group of economists to work within that framework, *e.g.* *(Chiarella et al., 2009, 2002)*. In recent years these types of models have been widely employed to study the channels through which balance sheet contagion spreads through the economy.

Three channels for balance sheet contagion have been identified: (1) the direct 'knock-on' effect, as default by one bank creates losses on the balance sheets of its creditors; (2) an indirect 'fire sale' effect, as banks that suffer losses deleverage by selling off assets, leading to collapsing asset prices that undermine otherwise liquid and solvent banks; (3) another indirect 'liquidity hoarding' effect, as healthy institutions react to the accumulation of losses in the banking system by preferring the safety of cash and refusing to provide liquidity on the interbank market to their distressed counterparts. Of the three channels, the last is probably the most important one through which the interbank market contributes to the spread of financial crises.

Indeed one can argue that precisely because the interbank market is normally so important in redistributing liquidity within the banking sector in order to maximise its transmission to the real economy, when crisis hits it quickly becomes subject to the reversal of liquidity provision by the hoarding of liquidity by otherwise sound banks.

It is not surprising then that liquidity hoarding was identified by several authors as a key trigger of the 2007-8 financial and economic crisis (see, *e.g. (Allen and Carletti, 2008; Heider et al., 2009; Acharya and Merrouche, 2010)*) However in our opinion it has been one of the least studied, at least within the framework of the ABMs that we have mentioned above.

In this paper, we develop a macroeconomic ABM that incorporates households and firms who interact with each other on labour and goods markets; banks who take in deposits from other sectors, lend to firms on a credit market and to each other on an interbank market, a government which collects taxes, makes transfer payments and issues debt and a Central Bank that buys government debt and acts as lender of last resort to the banking system. The economy is closed. Credit flows constitute the only feasible mechanism for exchange, with all transactions concluded via transfers between the bank balances of agents. Firms and banks are assumed to be price setters. Imperfect competition arises by the assumption that each customer can visit only a subset of sellers and that each borrower can access only a limited number of lenders. Agents are boundedly rational in that they use simple rules-of-thumb to make decisions, but these rules are updated in light of experience. Another feature of the model is stock-flow consistency in the sense of *Godley (2007)*, ensuring that value is not accidentally created or destroyed. Disequilibrium is a possibility in the model: markets do not clear at every time step, thus rationing might occur and the *short side rule (Bénassy, 2002)* is applied.

The main contribution of the model is in the way it treats the banking sector. We endogenise banks' strategies as to how much lending they wish to undertake at any given time, who they lend to and at what interest rates. To be precise, banks determine a target leverage ratio on the basis of an expected shortfall measure which varies with financial market conditions, they prioritise counter-parties for lending to on the basis of their perceived risk of defaulting and charge interest rates to those whom they lend to on the basis of both that perceived risk and their own sense of economic vulnerability as proxied by their expected shortfall.

To our knowledge this is the first attempt to determine all three aspects of bank strategy in a single, unified framework. We find that the model is capable of generating endogenous business cycles without imposing any external disturbance. In line with the stylized facts, we find that there is a close link between financial and business cycles, with periods of credit expansion followed by recessions, whose severity could be predicted by credit growth during the previous expansionary phase. Moreover we find evidence that when financial downturns occur, banks contribute to them by withholding liquidity from the interbank and credit markets and seeking higher interest rates on the funds which they choose to make available. These effects occur via a decrease in the maximum leverage that banks are willing to undertake and an increase in the interest rates that they charge on the funds that they do offer for lending. Finally we find that an increase in interbank connectivity on one hand improves credit to the real economy but on the other exacerbate liquidity hoarding.

To our knowledge, the ABMs that have been developed thus far can be broadly categorised into three groups: those that are mainly concerned with the macroeconomic role of the banking sector as a whole and thus lack the interbank market as an additional channel for contagion; those that focus primarily on the interbank market and finally, those that combine elements of the two.

In the first group are the seminal papers of *Delli Gatti et al. (2009, 2011); De Masi and Gallegati (2012); Ashraf et al. (2017)*. Other papers in this vein include *Assenza et al. (2015); Caiani et al. (2015); van der Hoog and Dawid (2015); Dosi et al. (2010); Lilit et al. (2016)*. These models study contagion among firms that operate within the real sector as well as between the firms and banks but do not study the role of the interbank market in magnifying contagious effects.

By contrast, papers in the second group either ignore the real macroeconomy entirely or treat it as an exogenous source of shocks to the banking sector. For instance in *Gabbi et al. (2015)* the real sector is considered as a black-box that demands bank loans and creates exogenous shocks to banks' deposits, while the banking sector reacts to these external factors by its own optimal behaviour on the credit and interbank markets. A similar approach is adopted in *Iori et al. (2006)*, while *Georg*

(2013); Allen et al. (2009); Montagna and Kok (2013); Lux (2015) are pure models of interbank networks.

In the last group, *Tedeschi et al. (2012)* modelled a three sector economy with goods, credit and interbank markets in order to study the correlation between bankruptcy cascades and endogenous business cycles. Other contributions along these lines include *Grilli et al. (2014)*. However, while these papers do consider both credit and interbank markets they do not endogenise banks' strategies as we do, neither do they take up the issue of liquidity hoarding as a specific phenomenon that amplifies losses on the interbank market.

It should also be noted that while several of the mentioned above do assume that lenders follow strategies for screening potential borrowers on grounds of perceived risk and charging them interest rates accordingly, none of them relates these strategies to the lender's own perceived vulnerability. An exception is that of *(Delli Gatti et al., 2011)* who assume that, controlling for borrower risk, more financially sound lenders offer lower interest rates as a competitive strategy. Our own formulation makes this concept more precise, via linking financial soundness to the concept of expected shortfall which is well known as both a tool of risk management and as a benchmark for determining capital adequacy under Basel regulations.

The rest of the paper is structured as follows: section 1.2 contains the model and its underlying assumptions. Simulations are described in section 1.3, conclusions are in section 1.4.

1.2 THE MODEL

Our macroeconomic model represents a simplified version of the one used in *(Delli Gatti et al., 2011)* as far as the structure of final goods production is concerned, but extends the latter by introducing a detailed model of the interbank market in which loan supplies, selective rationing and interest rates are all determined endogenously. Additionally we build on *Godley (2007)* to model the sectoral structure of the economy.

1.2.1 *Overview*

The economy is composed of five types of agents: households, firms, banks, a government and a central bank (hereafter, CB). The (discrete) numbers of households, firms and banks are N^H, N^F, N^B respectively. Interactions take place on different markets: firms and households meet on markets for goods and for labour, while firms borrow from banks on the credit market and banks exchange liquidity on the interbank market.[1] The CB buys government-issued bills on the bond market.

The sole role of the government is to make transfer payments to the household sector, funding these by issuing bills and collecting taxes. The CB generates liquidity by buying government bills and providing advances to those banks that require them; it furthermore holds banks' reserve deposits in its reserve account.

Households work and spend their income, which is made up of wage income, asset income and transfers on buying consumption goods, adding to their assets and paying taxes. In the labour market, households are represented by unions in their wage negotiations with firms, while on the capital market, they own firms and banks, receiving a share of profits as part of their asset income.

Firms borrow from banks in order to pay their wage bills in advance, hire workers, produce and sell their output on the goods market.

The banking sector provides credit to firms, subject to regulatory constraints. In each period every bank tries to anticipate its liquidity needs and accesses the interbank market as a lender or a borrower, thus the interbank market works as a mechanism to ensure the proper flow of credit to the real economy. If a bank is short of liquidity, it seeks an advance from the CB.

1 There is no market for deposits. We assume that each bank has a fixed and equal number of depositors.

This section contains some general specifications of the model, *i.e.* the timing, matching mechanisms in the goods, credit and interbank markets, and the maturity structure of loans to the firm sector. Sec.s 1.2.2-1.2.7 describe in detail the behaviour of each class of agents and their respective balance sheets.

Timing

The sequence of events in the model is described below.

1. Banks compute the expected shortfall based on observed losses and choose their maximum credit supply. Firms decide their planned hiring and production levels, and use these to compute their credit demand.

2. The credit market opens: each bank computes the interest rate charged to each possible borrower. Firms enter the market in ascending order of default probability, so that less risky ones have greater priority than more risky ones. They seek out potential lenders, sorting banks in ascending order of the interest rates that they charge.

3. The labour market operates and production takes place. Firms compute their labour demand in line with their planned output levels. They hire workers on the basis of a frictional matching process and all employed workers are paid the same wage, which is set each period by a union.

4. Households spend their consumption budget, starting from sellers that charge lower prices.

5. Firms and banks that obtain positive profits pay taxes and distribute dividends. They update the dividend share.

6. The credit market closes. In case of default, firms cannot repay their loans and their creditors suffer losses. Bankrupt firms are replaced with newborn start-ups.

7. Banks form an expectation about the liquidity buffer they need in the next period.

8. The CB's standing facility provides advances to illiquid banks, while liquid one pay back previous advances (if any) taken from the CB are paid back.

9. The interbank market closes: if banks are insolvent, they cannot repay their interbank debts. A loop cycle accounts for potential cascades of bankruptcies on the interbank market. Banks are recapitalized by their creditors.

10. The interbank market opens: borrowers and lenders exchange liquidity. Demand and supply are respectively determined by the difference between a bank's expected liquidity buffer and its actual liquidity.

11. The government collects tax revenues and issues bills, which are bought by the Central Bank. Unions update their required wage rate following a Phillips rule.

It is worth stressing that the interbank market takes place in order to allow banks to collect the funds they need to keep a proper level of liquidity in each period. Should they be short of liquidity before the closing of the market, the discount window of the CB provides the shortfall. Without loss of generality, an alternative sequence would be to end each day with the closing of the interbank market in step 10, while moving steps 11 and 12 to the beginning of the day.

The matching mechanisms and network structure

Consumers and firms interact on the goods market, firms and banks on the credit market, while banks exchange liquidity with each other on the interbank market. A network structure of linkages between buyers and sellers determines the interaction in each market.

A simple matching process operates in the goods market, where each household observes a subset of firms in a random order, sorts them in ascending order and starts to spend its consumption budget, starting from the cheapest ones. The process ends once the budget is exhausted or the household has visited all the firms in its subset. The potential out-degree of the consumer nodes in the network is equal to the number of firms in the system, since each household is linked to all the firms, but can visit only a fraction of them at each round of interactions. This friction is introduced in order to model search costs.

The parameter $Fh \in (0, 1]$ determines the share of sellers that can be visited by each household at the opening of the market. If $Fh = 1$, meaning that each household can visit all the firms, they would spend their entire consumption budget at the cheapest sellers, while the most expensive firms would be unable to sell all their goods. On the other hand, for low values of Fh, when just a small sample of firms can be visited, the buyers are likely to end up not exhausting their budgets, although the most expensive sellers could sell more than in the previous case. Thus there is a tradeoff between demand rationing and unsold output, depending on the value of Fh.

Interbank and credit markets work with a simple matching mechanism, where each borrower observes a subset of potential partners, choosing the lender according to the interest rate asked. Borrowers in turn are sorted in ascending order with respect to their default probabilities, so that the more risky banks are the first to be rationed in case the loan supply is not enough to meet total demand. The rationale of this simple rule is to allow banks to limit their exposure by rationing those agents that are more likely to be insolvent, when there is contraction in credit supply. The number of potential partners is determined by the network topology, which is fixed during each simulation.

The financial topology of the system is represented by a non-bipartite network where N^F firms are randomly linked to N^B banks in the credit network and banks are coupled with other banks in the interbank network. The firm-bank credit network is generated starting from an *Erdös-Rényi* random graph, with $N = N^F + N^B$ total number of nodes. The adjacency matrix contains at most $N(N-1)/2$ edges between nodes, each one created with probability $p = 0.5$. Finally the credit network is composed of the first N^F rows and the last N^B columns of the adjacency matrix. The interbank structure is modelled using the *Barabási and Albert (1999)* (B-A) model, where starting from a subset of nodes, each new node connects to x existing nodes with a probability proportional to the degree of the latter. The generated topology is scale-free, as has been found for real world interbank network, see for instance *Iori et al. (2008)* for the Italian E-MID market. Moreover the B-A algorithm produces hubs, similar to those formed by the more interconnected banks in real world networks.

Although the overall structure is static, it reduces the complexity of the system for the purposes of this article; a more realistic set-up of the network structure and its implications for the economy will be examined in further research.

The lagged structure of maturities

The introduction of a heterogeneous lagged structure of maturities of firm:bank loans is a new feature in this family of ABM. It

Time	Group 1	Group 2	Group 3
1	close		
2	open	close	
3		open	close
4	close		open
5	open	close	
6		open	close
7	close		open
8	open	close	
9		open	close
10	close		open
11	open	close	
12		open	close

(a)

Time	Firm 1	Firm 2	Firm 3
1	close		
2	open	close	
3		open	close
4			open
5		close	
6		open	
7			close
8			open
9			
10	close	close	
11	open	open	
12			close

(b)

Table 1: (a) Lag structure. There are 3 groups of firms starting their cycles with a unit lag one from another. By assumption each group starts with an equal stock of loans (at time 0). At $t = 1$ the firms in group 1 repay the loan and enters the credit market at time 2. Firms in group 2 repay the loan at time 2 and demand credit at time 3 and so on.
(b) Random lag structure. With a random lag structure, firms have different maturities of loans. Each time the credit market opens a new maturity is negotiated.

assumes that firms have only intermittent access to the credit market at intervals of time that differ in length across firms. This leads to heterogeneity in bank portfolios, even though banks operate in the credit market at each time.

The simplest implementation of this structure is done by dividing firms in groups of the same size that have loans in their balance sheet at the beginning of the simulation. Then each group repays the loan according to a time schedule defined by a matrix S and enters the credit market in the next period. In case of default, when the firm is revived, it may change its starting group depending on the next opening of the credit market. A sketch of the matrix S is reported in *Table 1a*, which contains an example of a simplified lagged structure.

An alternative is to assume a random maturity length of loans for each firm (*see Table 1b*), ranging from one period to some maximum possible duration, which is determined. each time a firm enters the credit market to seek credit. Such a structure is aimed to model more realistically the duration mismatch across banks' portfolios. Henceforth, we assume this is how the structure of maturities is determined in the model.

1.2.2 *Households*

The household sector consists of N^H units indexed by i. Households work, buy consumption goods and save. All the households are assumed to be equal shareholders in banks and firms. Each household also has a deposit account at some bank, whereas each bank is a creditor of an equal number of households. In case of default, as shareholders, households use their deposits to recapitalize firms or banks, while as depositors they may lose a part of their savings after the default of their creditor bank.

The net worth of the i-th household is defined as the value of deposits kept in a bank account.[2]

$$nw^H_{i,t} = D^H_{i,t} \tag{1}$$

Households receive their income from wages, interest on deposits and dividends. The law of motion of deposits is given by the accounting equation (2), which states that the variation in the deposits from $t-1$ to t, defined $\Delta D_t \equiv D_t - D_{t-1}$, is given by the interest rate r^D on deposits at time $t-1$, plus worked hours N^H times the wage rate W, net of the tax rate θ, plus the constant dividend share δ^k on net profits of owned firms and banks $(1-\theta)\Pi^k$, with $k = f, b$, minus consumption. Moreover there is an exogenous fiscal component G, which consists of transfers to the household sector, such that everyone receives the same amount $\frac{G}{N^H}$, which adds to the disposable income.

$$\Delta D^H_{i,t} = D^H_{i,t-1} r^D + (1-\theta)\left(W_{t-1} N^H_{i,t-1} + \sum_{k=f,b} \delta^k \Pi^k_{i,t-1}\right) - C_{i,t-1} + \\ + \frac{G}{N^H} \tag{2}$$

2 Households' net wealth also includes the values of shares in firms and banks. However, these components of net wealth cannot be explicitly valued since there are no secondary stock markets in the present generation of this model, nor are credit markets open to borrowing by households. Shares in businesses therefore only carry implicit value and are excluded from the computation of net wealth.

The equation of consumption resembles a permanent income rule *(see Modigliani and Brumberg, 1954)* and states that households consume a fraction c_1 of their disposable income and a fraction c_2 of their wealth. In nominal terms:

$$C^d_{i,t} = c_1 \left[(1-\theta) W_t N^H_{i,t} + \frac{G}{N^H} \right] + c_2 D^H_{i,t} \tag{3}$$

It is worth noticing that (3) can be interpreted as a consumption budget that each agent wishes to spend during each period. If a household is rationed on the goods market, such involuntary saving increases its stock of deposits.

1.2.3 *The labour market*

Each household supplies one unit of labour inelastically, making the total labor force equal to N^H. However, due to hiring frictions that will be described later, there are at any time t only $\hat{N}_t \leq N^H$ *employable* workers. All employable workers are available to work at a wage determined by unions so that the actual number of employed will be determined by the demand for labour by firms, which in turn is in proportion to their available liquidity.

All workers are equally productive and all firms have access to the same production technology, so all employed workers are paid the same wage.

Unemployment arises as bankrupt firms fire their workers, and is not immediately eliminated despite the emergence of new firms, due to frictions in the search-matching process. Each job seeker can meet at most n prospective employers in a given period but a meeting need not result in a vacancy being filled, despite the fact that firms and workers are homogeneous in wages paid and productivity levels respectively. We assume that there is a subjective 'compatibility' factor that determines whether a given firm-worker pair are mutually suited to work with each other. Mutual compatibility is determined during an interview that takes place prior to employment. Assuming that the (simple) probability of "success" in any given interview is p, the probability of k successful interviews in n trials is given by

$$p(s) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k};$$

where s stands for success. Setting $n = 2$, $k = 1$ this boils down to

$$p(s) = 2p(1-p)$$

Furthermore we arbitrarily assume that $p = 0.6$ so the probability of "success" is $p(s) = 0.48$.

A candidate who is compatible might be hired depending on the firm's demand for labour. If aggregate labour demand is lower than the available number of workers $\hat{N}_t$, each firm can hire its desired labour demand. Otherwise it is assumed that firms hire in proportion to their demands with respect to the total labour demand, so that full employment cannot be exceeded.[3]

We assume that the union adjusts the wage rate sluggishly, based on an adaptive mechanism, in order to prevent the wage time series jumping up or down sharply. The adjustment takes into account a simple moving average of past realized values of inflation ($\hat{\pi}^p$) and unemployment($\hat{u}$) over the last τ^w periods.The wage inflation rate $\pi^w \equiv \frac{W_t}{W_{t-1}}$ is

$$\pi_t^w = \hat{\pi}_t^p - \sigma_1(\hat{u}_t - u^*) - \sigma_2(\hat{u}_t - \hat{u}_{t-1}) \tag{4}$$

with $\hat{\pi}^p \equiv \frac{P_t}{P_{t-1}}$ the price inflation rate computed on the average price level, $\hat{u}_t$ is the unemployment rate, u^* is the natural unemployment rate. The term $\sigma_2(\hat{u}_t - \hat{u}_{t-1})$ is the effect on wage inflation of a change in the unemployment rate, which can be thought of as mimicking a process of *hysteresis* in the evolution of wage inflation.

1.2.4 *Government and the central bank*

The roles of government and the CB are crucial to understand the logic of the model. Every agent tries to achieve a positive net worth, except the government and CB. The former is always in debt with the CB and the latter has zero net worth. According to the aggregate balance sheet identity for the whole economy,

3 For instance, if $\hat{N}_t = 100$ and $\sum_{j=1}^{N^F} N_j^d = 120$, with $N_1^d = 30$, then firm 1 can hire $\frac{N_1^d}{\sum_j N_j^d}\hat{N}_t = 25$ units of labour. Otherwise, if $\sum_j N_j^d < \hat{N}_t$, the constraint is not binding and firm 1 can hire all its labour demand.

the negative net worth of the government is balanced by the positive net worth of the private sectors so that aggregate net worth is zero (see sec. 1.A.2 in appendix for further details).

$$\sum_{i \in N^H} nw^H_{i,t} + \sum_{j \in N^F} nw^F_{j,t} + \sum_{h \in N^B} nw^B_{h,t} + nw^G_t = 0$$

Government bills are sold directly to the CB. They have one-period maturity and pay an interest rate r^B. The government's budget constraint can be expressed in the single equation

$$\Delta B_t = B_{t+1} - B_t = r^B B_t + G - T_t - \Pi^{CB}_t \tag{5}$$

where B_t is the outstanding stock of government bills at time t, G are (constant) transfers to households, T_t are tax revenues and Π^{CB}_t are the profits of the CB, repatriated to the government. The expenditure on transfers is assumed to be exogenous. In each period a part of the aggregate demand consists of G, which in turn is financed with the money created by the CB. This particular mechanism is known as a *pure auto-economy* in *Hicks et al. (1974, p. 51)* and is detailed in *Godley (2007, chap. 2)*.

The circuit starts when government issues bills that are bought by the CB. The funds raised from this sale are transferred to the household sector's bank accounts. The firm sector borrows funds from banks, pays workers to produce and then sells the goods to households. Firms then deposit revenues from sales in banks. Notice that banks are creditors of the CB, since the high powered money on their account can be asked by firms at any time and in turn can be claimed by them from the CB. The circuit closes once taxes are collected by the government and firms repay loans.

The governmental deficit is countercyclical, acting as a stabilizer. During bad times tax collections are low, so the amount of liquidity in the system increases by the budget deficit due to the constant level of public expenditure. In good times, when $T_t > G + r^B B_t$, the stock of bills is lowered and the excess liquidity is destroyed. In this simple setting we do not assume any limit to the stock of public debt or debt/GDP ratio. The event of a spiral driven by interest on outstanding debt is ruled-out by Eq.5, as profit of the CB are transferred to the government. Additionally the stock of bills cannot grow indefinitely because public transfers increase the budget of households, which in turn is spent in consumption goods, thus governmental deficit helps the economy to recover from the crisis. Finally when the crisis is beyond, money in the balance of agents collected with taxes and the stock of debt decreases.

The profits of the CB are given by the interest payment on advances A, bills B and reserves R:

$$\Pi_t^{CB} = A_{t-1} r^H + B_{t-1} r^B - R_{t-1} r^L \tag{6}$$

Interest rates on advances r^H and reserves r^L are fixed and form the corridor through which lending to firms and banks takes place.

Furthermore the CB acts as a lender of last resort, providing liquidity to the banking sector. Its behaviour is described with respect to bank h by

$$\Delta A_{h,t} = -\min(R_{h,t} + I_{h,t}^l - rrD_{h,t} - I_{h,t}^b, A_{h,t-1}) \tag{7}$$

When a solvent bank h has liquidity (taking into account interbank lending I^l and interbank borrowing I^b) that exceeds the advances due to the CB, i.e. $R_{h,t} + I_{h,t}^l - rrD_{h,t} - I_{h,t}^b \geq A_{h,t-1}$ it extinguishes the debt $A_{h,t-1}$. If instead $A_{t-1,h}$ is greater or equal than $R_{h,t} + I_{h,t}^l - rrD_{h,t} - I_{h,t}^b$ the bank either refunds a part of the debt ($R_{h,t} + I_{h,t}^l - rrD_{h,t} - I_{h,t}^b > 0$) or borrows ($R_{h,t} + I_{h,t}^l - rrD_{h,t} - I_{h,t}^b < 0$).

1.2.5 *The business sector*

There are N^F firms indexed by j that produce an homogeneous good using labour alone. In order to hire workers, firms need to pay the wage bill in advance. We assume that this cash-in-advance constraint always binds so that hiring decisions are constrained by credit conditions.[4]

4 We take this approach to modelling hiring decisions in order to focus on the role of credit in driving the business cycles. Note that available liquidity effectively binds how many workers a firm can employ, given the cash-in-advance constraint on paying workers. Absent this constraint, firms could have an indeterminate demand for labour because of (i) the linearity of the production technology, (ii) the exogeneity of the wage rate to their hiring decisions and (iii) their ability to manipulate the price of their own product.

The balance sheet of a firm is composed of bank deposits D^F on the asset side while liabilities consist of loans L^F.[5]

$$nw_t^F = D_t^F - L_t^F \tag{8}$$

The amount of credit demanded by firms depends on their preferred capital structure (in terms of loans to equity ratio), which is chosen in line with *dynamic trade-off theory*. The latter is a refined version of the standard trade-off theory, introduced in an ABM context by *Riccetti et al. (2013, 2015)*[6]. The main idea, pioneered by *Fischer et al. (1989)*, is that firms determine their capital structures via a long-run leverage target, but they cannot attain it immediately due to market frictions or adjustment costs. They try to converge to the long-run objective by partially adjusting their leverage in the short run. In our model, contrary to *(Riccetti et al., 2013, 2015)*, we assume that there are no frictions preventing firms form achieving their long-run leverage targets in the short run. Moreover the vector of long-run leverages is assumed to follow a log-normal distribution.

$$L_{j,t}^d = nw_{j,t}^F \ell_j \tag{9}$$

where ℓ_j is the given target ratio.

If a firm has obtained a loan, it uses it to hire as many workers as it can.[7] The labour demand is the ratio of the stock of loans of j to the wage rate.

$$N_{j,t}^d = \frac{L_{j,t}}{W_t} \tag{10}$$

By assumption, firms keep their internal resources as a buffer to protect themselves against unfavourable outcomes on the goods market and use credit to hire on the labour market. The effect of such behaviour is twofold: on one hand it provides a sort of collateral to loans, limiting the default risk faced by banks, on the other it allows us to model a credit economy, where production strictly depends on borrowing.

5 Inventories are not included as they are assumed to fully depreciate each period. These assumptions, which are in line with *Delli Gatti et al. (2011)*, rule out business cycle driven by the accumulation of unsold inventories. Rather they allow for purely credit-driven business cycles determined by financial factors.

6 See *Flannery and Rangan (2006)* for empirical evidence

7 The hired workforce might be different from the labour demand (see 1.2.3).

Each firm produces a homogeneous good using labour N alone. The output supplied by the j-th firm is

$$Y^s_{j,t} = \alpha N_{j,t} \tag{11}$$

where α is a constant productivity factor.

In our simple setting we assume that firms compete only on prices, thus ignoring quantity decisions. Our choice is driven by the objective to model a credit economy, where output is determined by the availability of credit and credit depends on net worth of firms. We follow a similar approach to *Delli Gatti et al. (2009, 2010)*, where production is a function of firms' net worth. This could be thought as a rule of thumb adopted from boundedly-rational firms rather than solving complicated optimization problem. Net worth also determines the amount of credit lent by banks because in presence of asymmetric information lenders assess the financial soundness of firms by observing a simple sign, *i.e.* net worth. The latter reflects the ability of firms to generate cash flows from profits, enhancing capital accumulation, hence the greater the net worth, the bigger the amount of external finance that banks are willing to lend.

Each firm has some monopoly power since search costs prevent all consumers to sample all firms each time. This implies that a firm can charge a price higher than its marginal costs. However, firms that charge lower prices increase their market shares. The pricing mechanism is a delicate job that may affect the outcome of the model: in principle a good pricing rule would clear the market at each time, but this ideal condition cannot be always satisfied, indeed shifts in the consumption budget of households, the cost structure and bounded rationality prevent firms from setting the market clearing price.

As in *Dosi et al. (2013)*, we assume that firms use their previous market shares to compute the price they charge by adding a mark-up on their costs of production. The mark-up charged by firm j at time t, $\mu_{j,t}$, follows the rule

$$\mu_{j,t} = \mu_{j,t-1}\left(1 + \Delta y_{j,t-1}\right) \tag{12}$$

where y_j represents firm $j's$ market share and $\Delta y_{j,t-1} = y_{j,t-1} - y_{j,t-2}$ is the change in market share between the previous two periods. According to this rule, each firm computes the most recently observed change in market share, then forms an expectation about the mark-up it can apply: if the difference is positive, the past price could be raised by an amount proportional to the size of the increase, otherwise price is too high with respect to competitors and should be reduced. Those firms that went bankrupt during the previous period start with a mark-up equal to the initial one and reset their memory with respect to past market shares.

Individual price is then determined as:

$$P_{j,t} = \max[uc_{j,t}, uc_{j,t}(1 + \mu_{j,t})] \tag{13}$$

The cost of producing one unit is $uc_{j,t}$, which is defined as the ratio of wage bill plus the debt service to j's output.

$$uc_{j,t} = \frac{W_t N_{j,t} + \sum_{h=1}^{N^b} r^f_{jh,t-k} L^F_{jh,t-k}}{Y^s_{j,t}} = \frac{W_t}{\alpha} + \frac{\sum_{h=1}^{N^b} r^f_{jh,t-k} L^F_{jh,t-k}}{Y^s_{j,t}} \tag{14}$$

The interest rate on outstanding loans to bank $h$is indicated by r^f_{jh}[8]. This formulation accounts for how the burden of debt service on firms is passed on to consumers via prices, representing a transmission channel from the financial to the real side of the economy.

Following the time-line of the model, after production and pricing took place, the goods market opens and consumers spend their consumption budget following the matching mechanism described in section 1.2.1. The output sold by firm j is denoted by Y_j. The firm's gross profits Π^F are given by

$$\Pi^F_{j,t} = P_{j,t} Y_{j,t} - W_t N_{j,t} + D^F_{j,t-1} r^D - \sum_{h=1}^{N^b} r^f_{jh,t-k} L^F_{jh,t-k}; \tag{15}$$

where $t-k$ refers to a loan taken k periods before. In words, gross profits equal sales revenues minus wage costs and interest charges. If $\Pi^F_{j,t} > 0$ the firm pays taxes and dividends, otherwise it absorbs the losses. Net profits equal gross profits minus taxes imposed at the rate θ. A fixed share δ^f of net profits is distributed as dividends, with each shareholder of firm j

8 $t-k$ indicates the time at which the loan was extended.

receiving the same amount. The remaining part represents retained profits. The net worth nw^F of firm j evolves as

$$nw^F_{j,t} = nw^F_{j,t-1} + (1-\theta)(1-\delta^f)\Pi^F_{j,t-1} \tag{16}$$

If $nw^F_{j,t} \geq 0$ the firm's debt can be serviced, otherwise the firm is insolvent at the end of the period and bankruptcy occurs.

1.2.6 *The banking sector*

There are N^B banks, indexed by h. They finance themselves with short-term liabilities and provide loans with longer maturities to the real sector. In case they have an excess or a shortage of liquidity, they exchange it on the interbank market or borrow from the CB in case of rationing. [9]

The asset side of their balance sheet includes outstanding loans to firms, indexed by j and banks, indexed by q, denoted respectively by L^F and I^l, plus liquidity R. Liabilities could include banks' own funds and external liabilities, such as interbank borrowing I^b towards creditors, indexed by z, deposits D^B and advances A from the CB.

Bank h's net worth is given by:

$$nw^B_{h,t} = R_{h,t} + \sum_{j=1}^{J} L^F_{hj,t-k_j} + \sum_{q=1}^{Q} I^l_{hq,t-1} - D^B_{h,t} - A_{h,t} - \sum_{z=1}^{Z} I^b_{zh,t-1} \tag{17}$$

where k_j is the maturity of outstanding loans held by firm j.

At the beginning of each period banks face credit requests from firms and try to serve these in full, while respecting the regulatory constraints. A bank h can supply up to the maximum

9 It is worth noticing that the banking sector cannot finance itself without limits just by creating new deposits by lending to firms. Rather money is controlled by the CB that finances government's expenditure. Banks can provisionally anticipate liquidity to firms, then they access the interbank market to retrieve funds and comply with prudential regulation. The overall liquidity of the banking system correspond to the money supply of CB, hence if the interbank market is frictionless, money is never created by banks but they lend the existing funds to firms. If there are frictions on the interbank market, an illiquid bank can obtain a loan from the CB, but it will pay it back as soon as it finds a cheaper source of liquidity, for instance substituting CB's funds with interbank loans.

amount allowed by the regulator net of the outstanding stock of loans.[10]

$$L^s_{h,t} = \lambda nw^B_{h,t} - \sum_{j=1}^{J} L^F_{hj,t-k_j} - \sum_{q=1}^{Q} I^l_{hq,t-1} \tag{18}$$

The credit market matches (*see sec. 1.2.1*) firms with a predetermined number of banks. A generic firm chooses to take the loan out from that bank in its own subset that offers the lowest interest rate. Each bank charges an interest rate, taking into account their counter-party risk and their own cost of funds, leading to heterogeneous interest rates. Furthermore, banks prefer to extend loans to borrowers in order of increasing default probabilities, so that risky firms are more likely to be rationed in case the supply is insufficient.

Eq. (19) describes the default risk $\rho^f_{t,hj}$ perceived by bank h for firm j:

$$\rho^f_{t,hj} = 1 - e^{-v(\ell_j ES_{h,t})} \tag{19}$$

The default probability is an increasing function of borrower $j's$ leverage rate ℓ_j, corrected for the financial vulnerability perceived by bank h at time t, in terms of its own expected shortfall, $ES_{h,t}$. The latter accounts for the fact that an increase in bank h's vulnerability due to greater expected losses induces it to place greater weight on the risk created by a given default. As will be clearer in the next paragraph, this mechanism amplifies negative sentiments regarding the economy by raising interest rates when non performing loans increase, therefore deteriorating credit conditions and lowering the net worth of firms, increasing the likelihood of further defaults. Although constructed differently, this mechanism resembles in a broad sense the *network-based financial accelerator* as a mechanism to amplify shocks *(Delli Gatti et al., 2010)*.

The interest rate at which bank h offers to lend funds to firm j is denoted by $r^f_{t,hj}$. It is a function of the cost of funds, j's

10 The prudential constraint states that the stock of loans to net worth ratio cannot exceed a parameter λ:

$$\frac{L_t}{nw^B_t} \leq \lambda$$

Define the loan supply as the change in the stock of loans

$$L_t = L_{t-1} + L^s_t$$

then by substitution: $L^s_t = \lambda nw^B_t - L_{t-1}$.

specific probability of default and the recovery rate φ_j in case of default. The latter is defined as $\varphi_j = \frac{1}{\ell_j}$ and represents a gross measure of how much a bank can recover from the original loan in case of default.[11].

$$r^f_{t,hj} = \frac{1 + cf_{h,t} - \rho^f_{j,t}\varphi_j}{1 - \rho^f_{j,t}} - 1 \tag{20}$$

where $cf_{h,t}$ is bank $h's$ cost of funds, given by

$$cf_{h,t} = \omega^D_{h,t} r^D + \omega^A_{h,t} r^H + \omega^I_{h,t} r^b_{t-k,h} \tag{21}$$

The cost of funds depends on the composition of the bank's liabilities, with $\omega^i_{h,t}$ representing the share of each source of liquidity (deposits, advances, interbank borrowing) over liabilities.

$$\omega^i_{h,t} = \frac{i_{h,t}}{D^B_{h,t} + A_{h,t} + I^b_{h,t}} \quad i = D^B, A, I^b \tag{22}$$

Gross profits $\Pi^B_{h,t}$ are

$$\begin{aligned} \Pi^B_{h,t} = {} & R_{h,t-1} r^L + \sum_{j=1}^{J} L^F_{hj,t-k_j} r^f_{hj,t-k_j} + \sum_{q=1}^{Q} I^l_{hq,t-1} r^b_{hq,t-1} + \\ & - A_{h,t-1} r^H - D_{h,t-1} r^D \end{aligned} \tag{23}$$

If positive, these are subject to taxes at the rate θ, then the fixed share δ^b is distributed to shareholders.

The loss for a bank in case of a default by firm j at time t is equal to the stock of loans outstanding to the firm, minus the debt that can be serviced in case of default. In other words it is the difference between the stock of loans and the firm's own deposits, i.e. the (negative) net worth of the defaulted firm.

$$loss^F_{t,hj} = L^F_{j,t-k_j} - D^F_{j,t} = -nw^F_{j,t};$$

with $nw^F_{j,t} \leq 0$.

Things are slightly more complex in case of an interbank default, indeed a bank has more than one creditor, including depositors, the CB and other banks. Moreover it is assumed that the asset portfolios of defaulting banks remain intact, as

11 Remember that the net worth of a borrowing firm is used as collateral, hence in case of default the creditor has residual claims to it).

they are recapitalised after default. In other words, there are no fire sales of assets. Each creditor of the defaulting bank claims its share of residual assets in proportion to its claims on the defaulter's aggregate liabilities $\mathcal{L}_{\sqcup,\amalg}$, as will be detailed in sec. 1.2.7. A creditor bank h then loses a part of its loan to the failed bank q according to:

$$loss^B_{t,hq} = \frac{I^l_{hq}}{\mathcal{L}_{t,q}} nw^B_{t,q}$$

It is worth noting that contagion can arise. If a borrower defaults, the creditor bank may become insolvent and go into bankruptcy as well, triggering a series of bankruptcies or losses on the interbank and credit markets.

The net worth of bank h updates with the retained profits minus the losses:

$$\Delta nw^B_{h,t} = (1-\theta)(1-\delta^b)\Pi^B_{h,t} - \sum_{j=1}^{J} loss^F_{t,hj} - \sum_{q=1}^{Q} loss^B_{t,hq} \tag{24}$$

Finally, in order to understand the change in liquidity, the accounting equation describing the law of motion of R is reported below. Here $\Delta L_{h,t}$ indicates the change between $t-1$ and t of loans in both credit and interbank markets.

$$\begin{aligned}\Delta R_{h,t} &= \Delta D^B_{h,t} + \Delta A_{h,t} - \Delta L_{h,t} + (1-\theta)(1-\delta)\Pi^B_{h,t} + \\ &- \sum_{j=1}^{J} loss^F_{t,hj} - \sum_{q=1}^{Q} loss^B_{t,hq}\end{aligned} \tag{25}$$

Minimum capital requirement and financial leverage

We adopt a simplified version of Basel III regulatory constraints. In detail we suppose that banks must comply with a minimum capital requirement (solvency ratio) and a financial leverage ratio.

The minimum capital requirement (MCR) implies that the ratio of net worth:weighted assets must be greater than a regulatory parameter ϕ, where the weight is given by the *expected shortfall* (ES) computed by each bank depending upon past losses in line with the Basel framework. The ES is computed as the average of the value of losses on the overall portfolio of the bank, exceeding the historical VAR over the last n periods at 97.5% confidence level. In other words the ES represents the expected percentage loss on the portfolio in worst case scenarios (occurring with probability 2.5% or less), over the last n observed periods.[12] Thus

$$nw^B_{h,t} \geq \phi_t ES_{h,t} \left(\sum_{j=1}^{J} L^F_{hj,t-k_j} + \sum_{z=1}^{Z} I^l_{t-1,hz} \right)$$

If the net capital of a bank h is lower than the potential losses on the loan portfolio, h is in violation of its MCR. This means that its portfolio is too risky, or its financial leverage rate is too high. As a response, h attempts to move to a safer position by de-leveraging until its capital complies with the prudential rules. This is done by reducing its credit supply and by not renewing the outstanding loans.

The condition above can be rewritten in form of a leverage rate:

$$\frac{\sum_{j=1}^{J} L^F_{hj,t-k} + \sum_{z=1}^{Z} I^l_{t-1,hz}}{nw^B_{h,t}} \leq \frac{1}{\phi_t ES_{h,t}}$$

In parallel with minimum capital requirements, banks must comply with a maximum financial leverage, by which they cannot exceed a threshold λ, set by the regulator uniformly across all banks.

$$\frac{\sum_{j=1}^{J} L^F_{hj,t-k} + \sum_{z=1}^{Z} I^l_{t-1,hz}}{nw^B_{h,t}} \leq \lambda$$

It follows that a bank may increase its leverage up to a maximum threshold λ^{max}, depending on which constraint is stricter:

$$\lambda^{max}_{h,t} = \min \left(\frac{1}{\phi_t ES_{h,t}}, \lambda \right)$$

So long as the statutory constraint is not binding, the maximum leverage ratio will change in response to ES, meaning that when expected losses are high, λ^{max} will be low and so on.

12 The losses over loans ratio is preferred to the absolute losses approach, as the latter would be a wrong signal if the relative size of the actual portfolio respect to the one used in the computation of ES changes over time. If this is the case, the bank should deleverage even when its stock of loans is lower than ES.

The loan supply in eq. (18) can be rewritten with λ^{max} in place of λ

$$L^{s}_{h,t} = \lambda^{max}_{h,t} nw^{B}_{h,t} - \sum_{j=1}^{J} L^{F}_{hj,t-k_j} - \sum_{q=1}^{Q} I^{l}_{hq,t-1} \tag{26}$$

The interbank market

The interbank market is the place where banks exchange liquidity, mutually protecting themselves against the risk of shortages. At the opening of the market banks exchange funds in order to have a buffer of liquidity large enough to face unexpected outflows during the period. There are two main reasons why bank seek interbank funds: the first corresponds to endogenous changes in their balance sheets due to firm defaults, withdrawals etc., the other is to ensure that they have enough liquidity to serve those firms to whom loans are made during the credit market.[13]

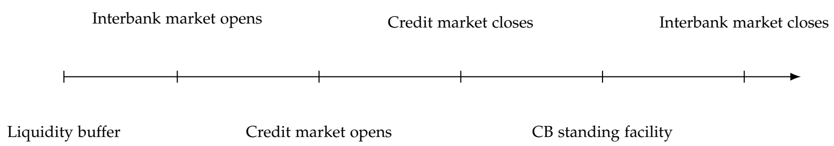


Figure 1: Timeline of the interbank market

Fig. 1 shows a stylized time-line of the interbank market. At the opening of an interbank session each bank tries to anticipate how much liquidity it needs to "survive" until the closing of the market. The *liquidity buffer* is determined on the basis of each bank's specific characteristics, like its structure of maturities. We assume that those banks unable to comply with it will not lend on the credit market, reducing their lending activity in order to lower their risk of illiquidity.

This in turn implies that rationing might arise on interbank market, causing a decrease of credit to the real economy, whenever banks cannot obtain the liquidity they need from the interbank market. After the closing of the credit market it might be the case that a bank has not enough liquidity to comply with the minimum reserve ratio or it cannot pay back interbank loans. In

13 In our approach banks form an internal liquidity coverage ratio based on their individual characteristics. As an alternative banks might enter the interbank market to comply with a prudential liquidity coverage ratio, as under Basel III *(see Lilit et al., 2016)*.

such circumstances it ask an advance at the CB (see eq. 7), which exposes it to the highest interest rate costs of all the possible sources of funds.

To assess the needed liquidity, a bank h computes a liquidity ratio (LR), defined as

$$LR_{h,t} = \frac{R_{h,t} - rrD^{B}_{h,t}}{\Omega_{h,t}} \geq 1$$

The numerator is the liquidity held at the CB net of the compulsory reserve ratio on deposits. In the denominator Ω represents the liquidity buffer, that is the difference between expected cash outflows out^E and inflows in^E of a bank during a single period. LR must be greater or equal than one.

The liquidity buffer of bank h is defined as

$$liq^{\Omega}_{h,t} = out^{E}_{t+1,h} - in^{E}_{t+1,h} \tag{27}$$

The expected cash outflows consist of the payment of interest rates on deposits plus a fixed share accounting for their run-off rate, that is assumed to be time varying with expected shortfall, plus the advances borrowed from the CB[14] and plus the amount of deposits originating from loans that will be repaid at the end of the period by the subset of firms J (last term on the right hand side).

$$out^{E}_{h,t+1} = (r^D + ES_{h,t})D^{B}_{h,t} + (1 + r^H)A_{h,t} + \sum_{j \in J} D^{F}_{hj,t}$$

The expected cash inflows are given by the sum of interest payments on loans by the subset of firms j, plus the principal of loans that will be paid back at the end of t by borrowers V corrected by the expected shortfall, plus the interest paid by the CB on reserves. It is worth noticing that banks form an expectation about their liquidity need, based on the state of the economy. When losses are large, their desired liquidity is larger than during periods of stability.

14 Given the sequence of events in the interbank market, a bank that borrowed from the CB in t prefers to pay back the advance in $t + 1$ and to resort to interbank liquidity, rather than to roll over the loan at unfavourable interest rate

$$in^E_{h,t+1} = \sum_{j \in J} L^F_{hj,t} r^f_{hj,t-k_j} + (1 - ES_{h,t}) \sum_{v \in V} L^F_{hv,t} + r^L R_{h,t}$$

At each session of the interbank market, if the liquidity is below the buffer, that is $R_{h,t} - rrD^B_{h,t} \leq \Omega_{h,t}$, bank h enters the interbank market as a borrower; otherwise it enters as a lender. Interbank demand and supply are described in eq.s (28)-(29).[15]

$$I^d_{h,t} = \Omega_{h,t} - (R_{h,t} - rrD^B_{h,t}) \tag{28}$$

$$I^s_{h,t} = \min\left(R_{h,t} - rrD^B_{h,t} - \Omega_{h,t}, \lambda^{max}_{h,t} nw^B_{h,t} - \sum_{j \in J} L^F_{hj,t-k} \right) \tag{29}$$

The interbank rate r^b is the minimum rate at which h is willing to lend interbank funds, if not met it keeps them at the CB, where they are remunerated at the set rate r^L. r^b is adjusted for the default probability of the counterparty, ρ^b and for a fixed recovery rate φ^b in case of default. For an hypothetical borrower z it is:

$$r^b_{hz,t} = \frac{1 + r^L - \varphi^b \rho^b_{hz,t}}{1 - \rho^b_{hz,t}} - 1 \tag{30}$$

The default probability computed by h for z is a function of the observed financial leverage of z. Moreover, as in eq. (19), it takes into account the vulnerability perceived by h via its own expected shortfall.

$$\rho^b_{hz,t} = 1 - e^{-s(lev^{obs}_{z,t} ES_{h,t})} \tag{31}$$

As discussed in sec. 1.2.1, riskier borrowers are more likely to be rationed in case liquidity is scarce. It is also worth stressing that the model allows for multiple lending, thus a bank can borrow from several lenders until its desired borrowing is satisfied, but a bank cannot be a borrower and lender at the same time.

15 I^s and I^d represent total loan supply and demand on the interbank market, which not equal actual borrowing or lending due to the failure of market clearing.

1.2.7 *Bankruptcies and new entrants*

If the net worth turns negative firms or banks go bankrupt. Their losses are absorbed by the balance sheets of their creditors, that could fail as well. Each defaulted agent is then recapitalized and enters the system again. Households are shareholders of firms and banks, so they participate to profits receiving dividends. For the sake of simplicity, each firm or bank is assumed to be owned by an equal number of households, which coincide with depositors for the bank.

Firms

When a firm defaults on loans, banks may not lose the entire amount of the loan because they can seize the defaulting firm's deposits. This results in a loss equal to the net worth of the failed firm. The credit market permits for multiple lenders, hence if a bankrupted firm has more than one creditor its default affects all outstanding loans, with each creditor suffering a loss proportional to the size of its loan with respect to the net worth of the borrower. After the default, firms leave the market and are replaced in the next period by new start-ups, which are initialized without liabilities and with positive deposits obtained from a random share of shareholders' wealth.

Banks

A bankrupt bank may also have multiple credit relationships, indeed the liabilities side of banks' balance sheet includes deposits from firms and households, interbank funds and advances from the CB. In case of default each creditor suffer a loss proportional to its credit with respect to the net worth of the bank. There is just one creditor that is always guaranteed, namely the CB. This assumption responds to the fact that in the real world advances or open market operations are fully collateralized, but since the model does not include collateral, it is assumed that the CB cannot make losses.

A bank in default does not leave the market, but it cannot operate in the financial markets until it is recapitalized with a *bail-in*. For conciseness we treat all banks in the same way, without considering the *too-big-to-fail* or *too-interconnected-to-fail issues*. A *bail-in* consists in the conversion of liabilities ($\mathcal{L}$) in assets ($\mathcal{A}$), in other words creditors turn a part of their deposits into bank capital, such that [16]:

$$\mathcal{A} \geq \frac{1}{1-0.03}\mathcal{L}$$

We assume a that the minimum time needed to complete the bankruptcy procedure and recapitalization cannot be lower than t^{recap} periods. In any case the bail-in is successful only if creditors have enough capital to reach the required asset/liabilities target, otherwise the bank remains in default until creditors can afford such operation.

Finally is worth noting that the default of a bank may trigger the defaults of its creditors, namely firms and banks. Households respond only with their deposits, so that they end up without net worth in the worst case, while if a firm loses a part of its deposits, it may not be able to repay loans and go bankrupt. A similar reasoning applies for banks, whose balance sheet includes interbank loans.

16 The assets-liabilities ratio is set above the the minimum leverage rate defined by Basel III: since $\frac{capital(tierI)}{\mathcal{A}} \geq 3\%$, then $\mathcal{A} \geq \frac{1}{1-3\%}\mathcal{L}$.

1.3 RESULTS

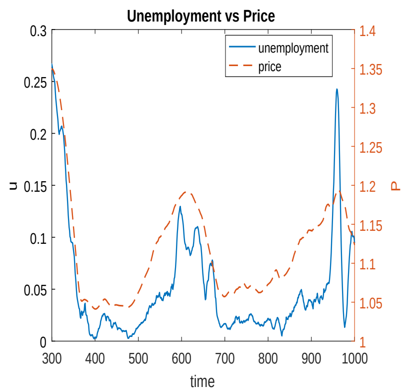
The dynamics of the model are explored through numerical simulations. Subsection 1.3.1 discusses the macro-dynamics of the model and investigates whether it can reproduce known empirical regularities. The calibration is done in 1.A.1.

1.3.1 *Baseline dynamics*

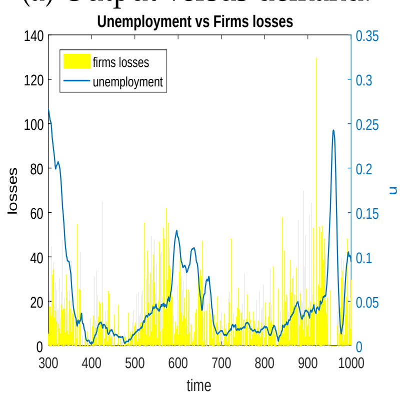
The complex dynamics generated by the model do not admit closed form solutions; rather they are analysed by means of numerical simulation. The following figures describe the baseline dynamics of the model and its ability to replicate empirical regularities.

Business cycles

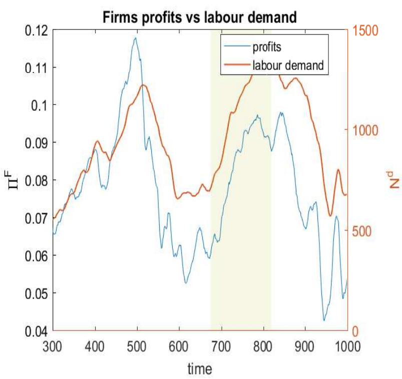
(a) Output versus demand.



(b) Unemployment versus price.



(c) Losses in the firms sector versus unemployment.



(d) Profits in the firms sector versus labour demand.

Figure 2: Business cycle dynamics

The model exhibits endogenous business cycles. Firms face competition on the goods market. They hire and pay wages to households, who in turn visit sellers in order to buy goods at the most favorable price. As long as a firm is profitable, its net worth increases and it demands more and more credit from banks, but a greater size implies an enhanced output, increasing the risk that it will be unable to sell its entire output. During good times, unemployment remains low and prices increase, driven by increasing mark-ups made possible by high demand (see fig. 2a and 2b).

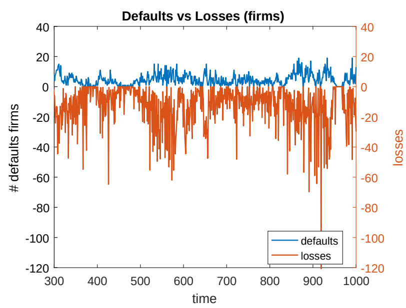
This induces pressure on nominal wages, as set by unions, resulting in rising costs of production. The last periods of a boom show high prices, given that firms have been increasing their mark-up while labour cost have risen along with nominal wages. At some point those who charge the highest prices cannot sell their entire output, suffering losses which in turn lead to declines in net worth or, in the worst case, bankruptcy of those firms that cannot repay their loans. Note that the price level keeps rising for some time following the onset of a recession. This phenomenon is in line with what is reported by *Claessens et al. (2009)*. In our model, it reflects the continuing effect of rising labour costs due to the sluggish adjustment of nominal wages.

The most important consequence of bankruptcies is the drop in aggregate demand: when a firm defaults, workers are fired and enter the pool of the unemployed. An increase in unemployment reduces aggregate demand for several periods. Note that labour incomes account for about 50% of the aggregate consumption budget of households. Moreover new entrants into the business sector are limited in borrowing by their size, while the surviving firms cannot change their labour demand immediately to absorb the unemployed workers, due to the matching friction discussed in section 1.2.3.

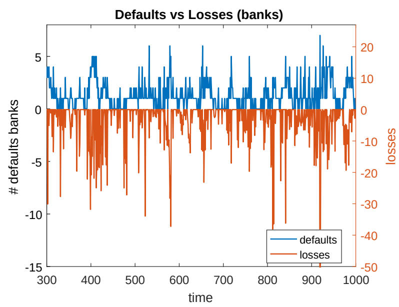
Declines in aggregate demand may trigger a cascade of defaults in the firm sector (see fig. 2c); in particular the default of bigger firms, in terms of net worth, may yield a drop in consumption so as to trigger a cascade of defaults. For instance, one can notice a sharp economic downturn at $t = 950$ in fig. 2a, with a sharp decline in output and aggregate demand, caused by a huge loss some periods earlier, at $t = 900$ as depicted in fig. 2c.

Turning to banks, the dynamic patters appear to coincide with those of firms. A peak in firm defaults is associated with a peak in bank defaults. Figs. 3a-3d offer a detailed description of what happens in both sectors. It is worth noting that a *self-reinforcing process* exists: defaults in the business sector weaken the balance sheets of banks, triggering defaults of the most fragile ones. In turn these bankruptcies affect the banks' creditors, and may trigger another cascade of defaults (see fig. 3c).

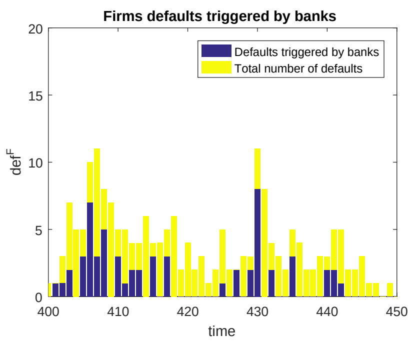
Recovery begins as the firm sector produces less and charges lower prices, both because mark-ups have shrunk during the recession and nominal wages have reduced due to lower inflation and high rates of unemployment. Under these conditions firms can make positive profits and begin to hire more workers again. As profits increase, labour demand moves upward (fig.2d). On the aggregate demand side, diminishing prices and lower unemployment raise real wages and yield a positive income effect, which enhances consumption demand, resulting in a recovery of the economy.



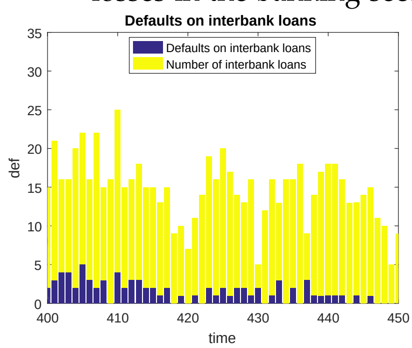
(a) Number of defaults versus losses in the business sector.



(b) Number of defaults versus losses in the banking sector.



(c) Number of firms bankruptcies triggered by banks defaults (blue) over total number of bankruptcies in the business sector (yellow).



(d) Number of interbank defaults (blue) over total number of interbank loans (yellow).

Figure 3: Business cycle dynamics

The financial side

We described above the dynamics of the real sector. We now consider the dynamics of the financial sector and how these interact with those of firms.

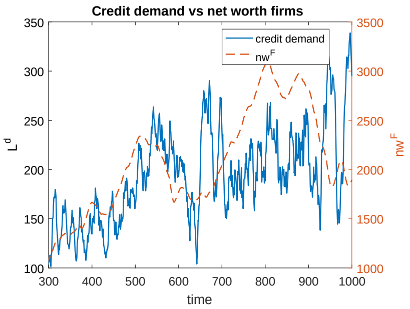
The financial accelerator is evident in our model, with variations in interest rates and the availability of funds on the credit market being the main transmission mechanisms. Cascades of default in the business sector inflict losses on creditors, leading in turn to an increase in their expected shortfall (ES). If losses are high enough and affect the ES of a critical mass of banks, there is a generalized increase in interest rates in credit and interbank markets, as suggested by eq.s (20) and (30). Fig. 4c shows that interest rates on both markets and the ES of banks all follow similar patterns.

In addition, banks try to limit their exposure to the risk of insolvency so they reduce credit supply in line with eq. (26). Credit availability is strictly related to bank capital due to regulatory constraints, hence if non performing loans lead to a reduction in the net worth of the banking sector, credit supply is reduced accordingly (see fig. 4b). At the same time, firms reduce their credit demand during recessionary periods, as in fig. 4a, given the reduction in their own net worth caused by losses and failures. One feature of the two figures is that the loan supply by banks has a higher order of magnitude than does loan demand by firms, at least when credit is abundant. This is the result of the feature of the model whereby firms enter the credit only intermittently, when outstanding loans mature, while banks can offer loans during each period.

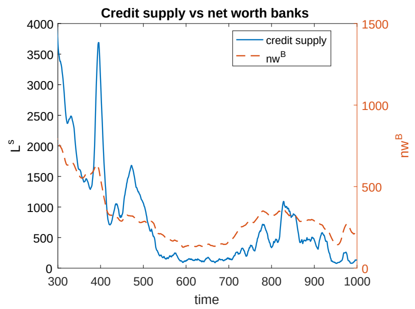
Below we examine the mechanism that gives rise to hoarding in greater detail.

Recall that a bank can supply credit up to λ^{max} times its net worth, minus its outstanding stock of loans, where λ^{max} is determined by the bank's expected shortfall (ES). The aggregate dynamics of λ^{max} lags behind that of the real economy with the lag depending on the value of the average expected shortfall across the banking system (see fig. 4d). During quiet periods there are no relevant losses for banks; hence, lending activity can be maintained despite relatively low values of leverage. When the system experiences substantial losses, the average expected shortfall increases and lending activity is tightened, but this happens with a delay: the maximum value of expected shortfall is reached some periods after the default cascade has occurred.

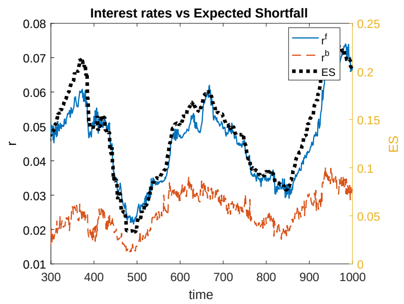
This reflects a pro-cyclical effect of prudential regulation and how it affects capital requirements. The danger is that tightening capital regulations can prolong the slump and delay the recovery, thereby imposing spill-over effects from the financial to the real side of the economy.



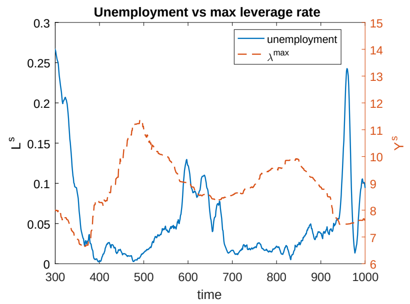
(a) Credit demand versus aggregate net worth of firms (dashed).



(b) Credit supply versus aggregate net worth of banks (dashed)



(c) Interbank (dashed) and credit interest rates versus expected shortfall (dotted).



(d) Unemployment and maximum allowed leverage rate (dashed).

Figure 4: Dynamics of the financial side.

Our model is able to replicate the stylized fact described in *Jordà et al. (2011)*, *i.e.* excess credit during a boom phase determines the amplitude and the severity of the subsequent recession. We can investigate this effect in our model by comparing the dynamics of an aggregate measure of leverage, *i.e.* bank loans/GDP, with those of unemployment. Fig. 5 displays the result: Humps in unemployment are preceded by peaks in credit:GDP, with the large expansion in credit:GDP between $t = 700$ and $t = 900$ followed by a large spike in unemployment at $t = 950$.

The *boom and bust* cycles observed during the simulations could be explained in terms of credit expansion and contraction: the boom phase ends when credit inverts its growing trend, giving way to increasing unemployment. Conversely the slump phase runs out when profits in the business sector start again to increase, so firms raise their labour demand and ask more credit

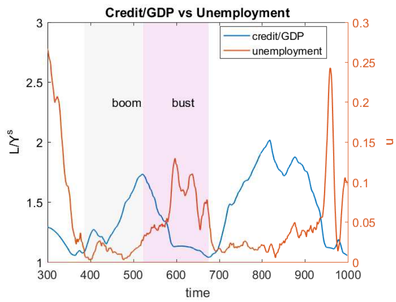
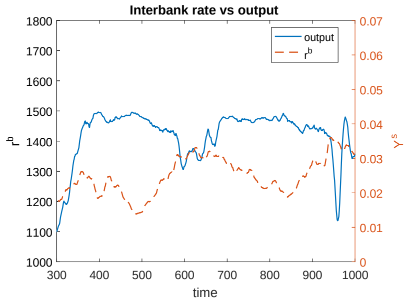


Figure 5: Credit to output ratio over unemployment rate (dashed).

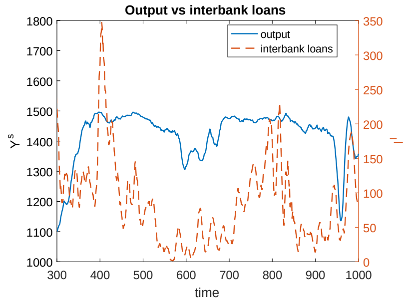
Interbank lending markets

The model also reproduces declines in interbank lending volumes and increased interest rates during periods of financial turmoil.

Interbank interest rates (eq. 30) depends upon the counterparty risk of default, represented by banks' financial leverage, and the vulnerability perceived by each bank given its expected shortfall. In general r^b follows a similar pattern as ES, recall fig. 4c, but its local dynamics is also determined by the default probability of borrowers, computed as a function of financial leverage. In accordance with stylized facts, during a downturn the interbank rate reaches a peak, as displayed in fig. 6a: some periods before it increases, supported by banks' financial leverage that tends to raise with credit expansion. At the same time there is a reduction in traded volumes, (fig.6b), mainly led by the supply side. Recall that banks are subject to regulatory constraints on their loanable funds, so they lend in proportion to their net worth (eq.s (28)-(29)). As a result, there are disruptions in interbank activity along with credit rationing, which in turn exacerbates reductions in loan supply to firms and give rises to rationing on the credit market.



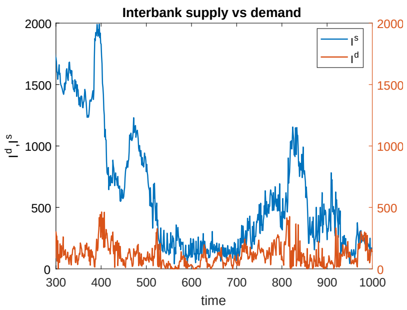
(a) Interbank rate versus output (dashed).



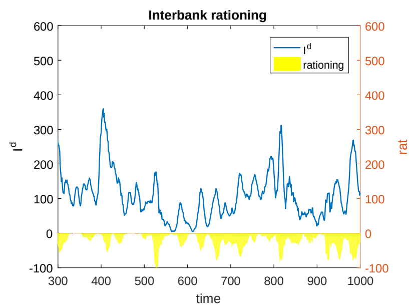
(b) Interbank loans versus output (dashed).

Figure 6: During a crisis the interbank rate increases, while the traded volume diminishes.

However emergence of interbank rationing is more complex than the simple supply-demand mismatches. Of course there is rationing when the supply is less than demand, but there are other channels as well. One is the structure of the interbank network that might prevent potential borrowers from connecting with lenders. The second is the rationing behaviour of banks; by choosing the order of borrowers on the basis of the counterparty's perceived default probability, the riskier ones are placed the last in queue even though they might be in most immediate need of funds. The interaction of the three mechanisms leads to rationing in the interbank market, which becomes more pronounced when there are peaks in demand, as it is clear from fig. 7b.



(a) Interbank supply versus interbank demand.



(b) Interbank demand versus credit rationing (yellow area).

Figure 7: Interbank credit rationing

1.3.2 *The role of the interbank market.*

In this section, we explore the role that the interbank market plays in generating negative externalities on the real sector when it is malfunctioning, as a result of banks' self-fulfilling expectations giving rise to liquidity hoarding.[17] The path of unemployment is represented for the benchmark case by the topmost in Fig 8, as can be seen there is a prolonged recession lasting from approximately t=650 till almost $t = 900$. This means that a considerable part of this run is characterised by recession, which enables us to study in greater detail the manner in which contagion arises within the interbank market and then spreads to the real economy.

Interbank spill-over effects

The interbank market produces a negative externality on the real sector as a consequence of rationing of interbank funds. The beginning of the process that generates spill over effects on the real sector can be attributed to an increase of expected shortfall due to a cascade of defaults in the firms sector. This effect is related to the endogenous business cycle dynamics, as described in sec. 1.3.1. The increase in expected shortfall leads to a decrease in the maximum leverage that banks can undertake and an increase in illiquid banks facing rationing on the interbank market. Rationed banks must borrow from the central banks at high rates, as a result the cost of funds soars. Along with this, those banks that can access loans on the interbank market are charged higher rates (see fig. 8).

The increased cost of funds spills over to the interest rate charged on new loans to firms, whose balance sheets are in turn weakened, thus generating instability. Moreover the cost of borrowing from the CB reduces profits in the banking sector and causes the default of banks in the lower part of the wealth distribution. Such losses in turn contribute to weaken the balance sheets of creditors, as the distress transfers to the business sector through a loss of the deposits that they held in failing banks.

Fig 9 depicts the size distribution of banks and firms before and after the economic downturn (upper panels) and the sources of firm and bank defaults (lower panels). As can be seen there

17 Note that the benchmark time series for macroeconomic variables underlying the results of this sub-section corresponds to a different run than the one underlying the results of the previous sub-sections.

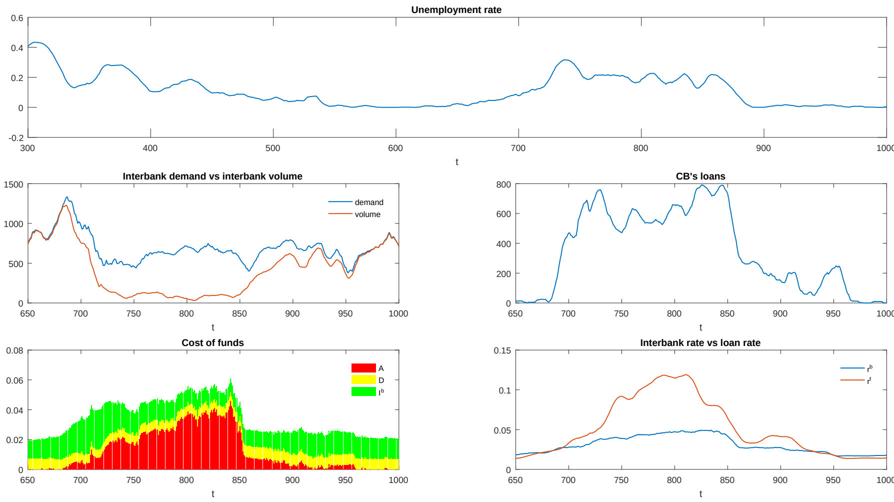


Figure 8: Top: unemployment rate. Top-left: aggregate interbank demand and total exchanged volume. *Top-right*: aggregate liquidity borrowed from CB. *Bottom-left*: time pattern of cost of funds. CB's loans (A, red), deposits (D, yellow), interbank funds (I^b, green). *Bottom-right*: time evolution of interbank interest rate (r^b, blue) and interest rate to firms (r^f, red).

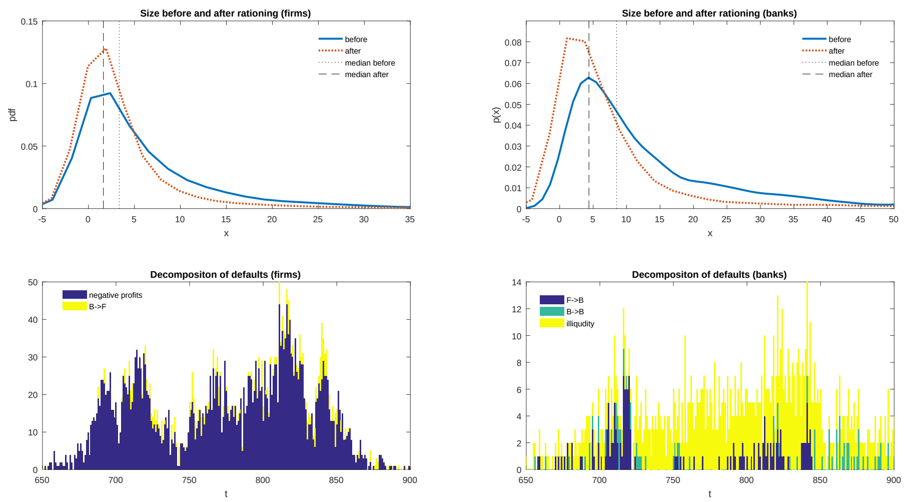


Figure 9: *Top-left*: estimated probability density function for sizes of banks (*top-left* panel for sizes of firms) measured during the 200 periods before after the slump at t=700.
Bottom-left: decomposition of total defaults in the firms sector over time. Firms might go into default because of negative profits from the goods market (blue) or because creditor banks are insolvent on their deposits (yellow). *Bottom-right*: decomposition of total defaults in the banking sector over time. Banks might default due to insolvency of firms (blue F-B), because of insolvency of interbank loans (green, B-B) or because they cannot bear the interest service to CB (yellow, liquidity).

is some shift to the left in the distribution of sizes, although this is relatively small given our assumption that resources for replacing bankrupt enterprises come from the household sector rather than from the corporate sector. The decomposition of defaults shows that while firms default mainly because of negative profits on the goods market, illiquidity plays a major role in bank defaults.

Interbank connectivity

In this section we present the results from an artificial experiment whose aim is to observe the effects of interbank connectivity on the economy. We simulate the model with different levels of interbank connectivity and check for network effects on the system.[18] As mentioned in sec. 1.2.1, interbank network is obtained by growing a Barabasi-Albert model with m initial nodes, adding one node with n edges at each iteration. The initial number of nodes represents the core of the network, where every node is connected with the other. As new nodes are added, they tend to form the peripheral part of the structure. In order to check the effects of increased interconnectivity, we generate nine networks by increasing both the size of the core and the number of edges for new nodes. The values of m and n are chosen to increase linearly the density of the network between 0.1 (least connected graph) and 0.9 (most connected graph). Values are reported in the bottom part of table 2[19].

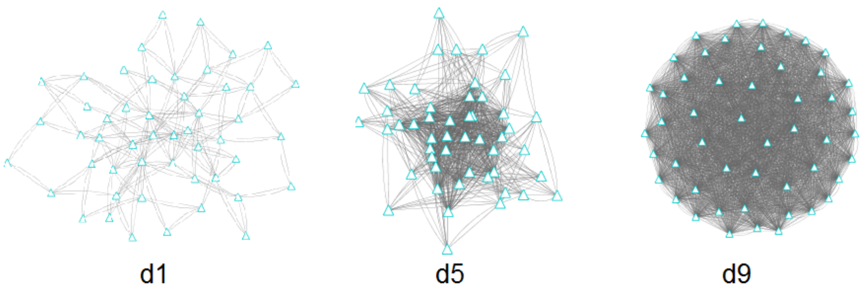


Figure 10: Structure of the interbank market at connectivity d1, d5, d9.

18 We conducted 100 Monte Carlo replications for each level of connectivity. Each simulation lasts for 2000 periods (where the first 300 periods are discarded to eliminate the transient).

19 The resulting structures define a set of fixed network topologies that constrain the maximum degree of each node. Realized degrees of nodes observed throughout the simulations can be at most equal to such determined potential degrees.

Network statistics	Connectivity								
	d1	d2	d3	d4	d5	d6	d7	d8	d9
Density	0.09	0.20	0.30	0.40	0.50	0.60	0.71	0.80	0.89
Avg degree	4.08	9.80	14.88	19.52	24.52	29.24	34.60	39.04	43.84
Max degree	13.00	24.00	30.00	34.00	37.00	39.00	43.00	46.00	47.00
Avg cluster coef	0.14	0.26	0.52	0.68	0.75	0.86	0.92	0.98	0.97
Avg path length	2.72	1.90	1.75	1.64	1.51	1.40	1.28	1.19	1.08
Initial nodes (m)	5	10	22	28	33	37	41	44	47
New edges (n)	2	5	5	5	5	5	5	5	1

Table 2: Network statistics for the interbank network

The main finding of the experiment is that augmenting interbank connectivity yields growing levels of liquidity hoarding. We measure liquidity hoarding as the share of disposable liquidity for lending, I^s (see eq.s (28)-(29)), that is retained by lenders over their total liquidity. Aggregate liquidity hoarding at each period of the simulation is defined as the sum of the liquidity targets of all lenders ($\mathcal{L}$) over the sum of their total liquidity ($R - rrD^B$)

$$hoard_t = \sum_{h \in \mathcal{L}} 1 - \frac{I^s_{h,t}}{liq_{h,t}}$$

The range is restricted to $[0, 1]$, zero is no hoarding at all, 1 is hoard all disposable liquidity.

In our model liquidity hoarding arises for precautionary reasons, as banks are concerned about their future level of liquidity but they fail to internalize the consequences of their behaviour, in line with the findings in *Gai and Kapadia (2010)* and in agreement with a part of the evidence provided in *Acharya and Merrouche (2010, p.4-5)*, who find that those banks that suffered greater losses during the crisis hoarded more liquidity, also in response to payment uncertainty. We present the results starting from the main finding, then we decompose it and explain what elements contribute to it and why.

Fig. 11 shows that liquidity hoarding is increasing with connectivity, together with Expected Shortfall (ES). Liquidity hoarding relates with losses, because banks adopt an internal risk mechanism based on historical losses, so that their expectations could be considered as adaptive. Perceived risk in turn is measured by Expected Shortfall (ES), as the average of losses over the loan portfolio exceeding the historical VaR during the last n periods at 97.5% confidence level. The more losses on loans affect their portfolio, the higher is banks' expected shortfall.

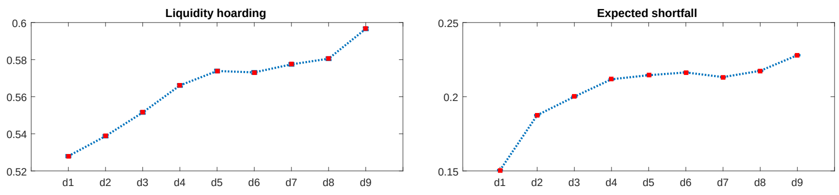


Figure 11: Average liquidity hoarding and expected shortfall of banks at different levels of connectivity. Each point represents the individual average per period computed on each set of Monte Carlo simulations. Error bars represent standard errors.

Fig. *12* displays losses over the entire portfolio for interbank and firms loans and shows the increasing incidence of insolvencies from the firms sector.

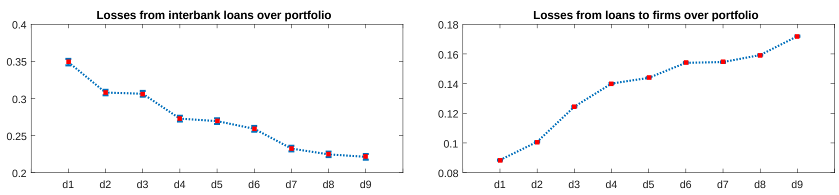


Figure 12: Incidence of losses on loans portfolios of banks at different levels of connectivity. Each point represents the individual average per period computed on each set of Monte Carlo simulations. Error bars represent standard errors.

Moving to a state with higher losses on loans makes bank more careful. This is reflected in their expected shortfalls which in turn determine the liquidity target and interbank supply (eq.s (27) and (28)-(29)). In the end growing connectivity leads to increasing levels of liquidity hoarding. This result is relevant because although during normal times the benefits of connectivity prevails (little rationing of interbank funds), during bad times liquidity hoarding makes it more difficult to access to interbank funds. For instance during a financial crisis the interbank market might be distressed, thus failing to provide the needed liquidity and exacerbating the downturn.

Having said that, we turn our attention to investigate the reasons that make ES growing with connectivity. As mentioned above, losses originating in the firm sector affect the portfolio of banks, determining an increase in ES. Such losses can be attributed to two reasons: first, greater connectivity implicates more defaults of firms on the goods market; second, banks are more exposed to firms, thus more fragile.

The first reason descends from the fact that improving interbank connectivity allows more firms to access to credit because banks can efficiently channel funds towards the real sector. This also entails that the goods market has an higher number of participants, so the market is more competitive and firms lose part of their market power. This result is displayed in fig. 13. But moving towards a competitive market yields lower profits in the firms sector. Average profits and Return on Equity (ROE) confirm this hypothesis, as shown in fig. 14. Finally competition results in lower losses (which could be considered the negative part of profits) and an higher number defaults (fig. 15).

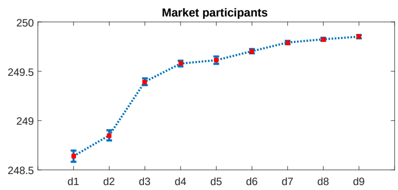


Figure 13: Average number of firms participating to the goods market per period at different levels of connectivity. Error bars represent standard errors.

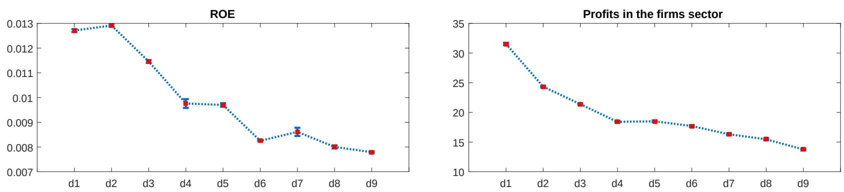


Figure 14: Average return on equity (ROE) and profits of firms per period at different levels of connectivity. Error bars represent standard errors.

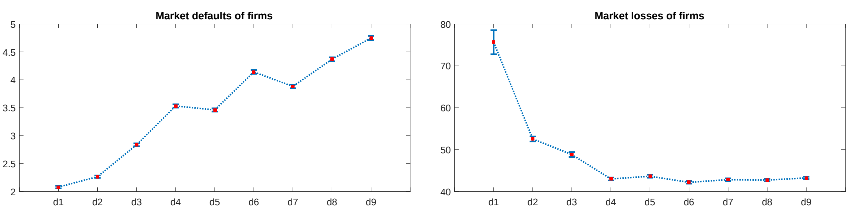


Figure 15: Average defaults and losses of firms from negative profits per period at different levels of connectivity. Error bars represent standard errors.

The second reason regards the increasing exposure of the banking sector to firms along with connectivity, which amplifies the pattern described above. When connectivity grows, the ratio of borrowers per lender increases, as a consequence of the rise of the number of firms that can access to credit and a reduction of lenders. The latter derives from the fact that if more banks supply loans, borrowers can choose the best banks in terms of interest rates, thus those banks with the highest rates are rationed (fig. 16 right). Moreover the better access to credit allows borrowers to satisfy their demand from fewer banks, that is multiple loans reduce with connectivity (fig. 16 left). As a result the credit market is characterized by an higher concentration of borrowers per bank and lower diversification when the degree of interbank connectivity grows.

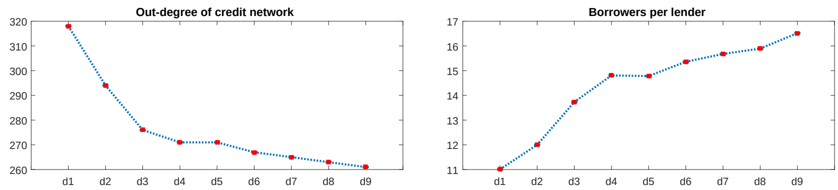


Figure 16: Average out-degree of credit network per period at different levels of connectivity. Average number of borrowers (firms) per bank per period at different levels of connectivity. Error bars represent standard errors.

Although the firms sector is characterized by a decreasing pattern of losses, the effect of such losses on banks is rather increasing with connectivity. The top-right panel of fig.s *17-18* displays an increasing pattern in the composition of defaults and losses originated from insolvencies of firms on loans. Banks suffer relatively more from exposures towards the firm sector, as described above, rather than interbank losses or negative profits. Fig.s *17-18* display the absolute and relative number of defaults and the amount of losses in the banking sector at different levels of connectivity. The top-left graph in both panels shows that a more interconnected interbank network is beneficial for banks as the incidence of negative profits is gradually reduced. Banks realize negative profits when their income from interest payments is lower than the cost of liquidity, namely interest service on deposits, interbank borrowing and CB's loans. In particular the likelihood to be rationed in interbank borrowing diminishes with the growth of network's density, hence banks do not need to rely on CB's finance at the penalty rate to meet their liquidity targets. The decreasing pattern of the cost of liquidity is displayed in fig. 19, together with interest rates and lending from the CB.

The bottom-left panels in fig.s *17-18* shows the impact of interbank defaults and losses at different levels of connectivity. In general interbank contagion is decreasing with connectivity, at odds to what one would expect since a more interconnected network should imply an enhanced risk of contagion. A possible explanation relates to defaults from negative profits, which trigger interbank losses. In general negative profits are triggered by interbank borrowing. Even those banks that borrowed from CB demand liquidity on the interbank market in order to substitute advances with cheaper liquidity, thus reducing their cost of funding. This implies that banks realizing negative profits have interbank liabilities in their balance sheet, hence the increase of connectivity reduces interbank losses. On closer inspection contagious defaults on the interbank market are emerging at low levels of connectivity. However the contrasting effects of contagion and risk diversification could be observed only from d1 to d3, where increasing connectivity leads to a rise of contagious defaults in d2. From d3 onwards contagion is dominated by risk diversification: increasing the number of links reduces the exposure of lenders to a specific borrower, thus enhancing robustness to interbank losses.

Last but not least, macroeconomic performance is improved by growing connectivity (fig. 20). This finding suggests that on average the benefits of connectivity are greater than its drawbacks, even if during downturn liquidity hoarding contributes more to worsen liquidity.

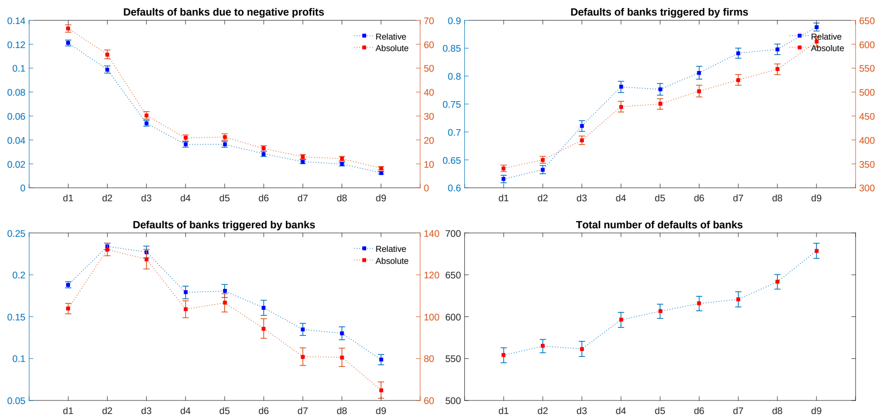


Figure 17: Relative and absolute contributions to defaults in the banking sector at different levels of connectivity. Each point is the average total defaults per period computed on each Monte Carlo simulations. Relative defaults reflect the ratio of each kind of default to total defaults. Error bars represent standard errors.

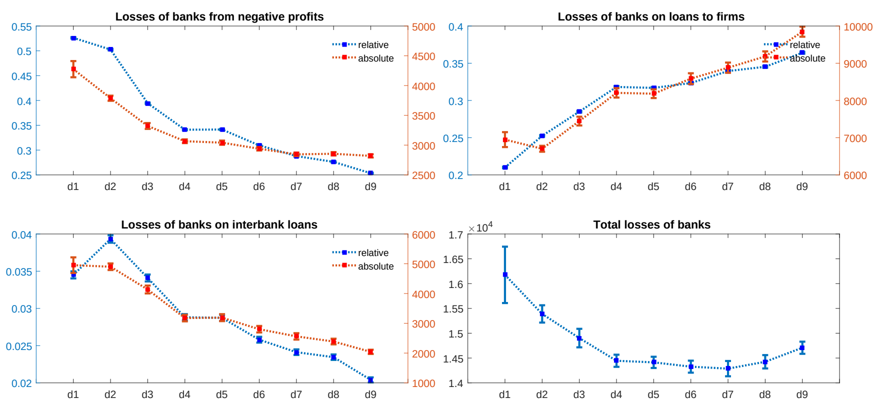


Figure 18: Relative and absolute contributions to losses in the banking sector at different levels of connectivity. Each point represents the average total losses per period computed on each set of Monte Carlo simulations. Relative losses reflect the ratio of losses of each kind to total losses. Error bars represent standard errors.

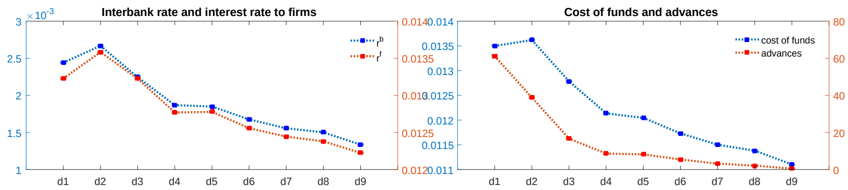


Figure 19: (Left) Interbank rate (r^b), interest rate to firms (r^f) (right) cost of funds and advances from the CB at different levels of connectivity. Each point represents the individual average total losses computed on each set of Monte Carlo simulations. Error bars represent standard errors.

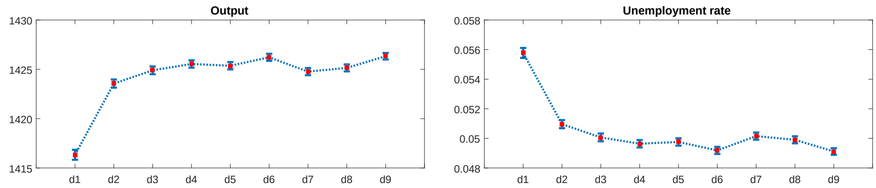


Figure 20: Average output and unemployment rate per period at different levels of connectivity. Error bars represent standard errors.

1.4 concluding remarks

We developed an agent-based model that incorporated both real and financial sectors, including an interbank market which complemented the credit market in facilitating the provision of liquidity by banks to the real economy. The set of potential interactions among lenders and borrowers was governed by static network structures in each market. A distinctive feature of the model consisted in the way in which the effects of prudential regulation were accounted for: in particular minimum capital requirements and the maximum allowed financial leverage depended upon the expected shortfall computed by each bank on the basis of the losses experienced in the recent past. Moreover we generated heterogeneity in banks' loan maturities by assuming a lagged structure of maturities of loans to the firm sector.

The model's dynamics were explored through simulations. We first showed that endogenous business cycles can occur, driven by price and wage dynamics that result from the behaviour of agents rather than through any external disturbance. In the boom phase, unemployment is low, demand is high, firms increase output by taking on more credit, while prices rise both because unions set higher wages and because firms increase mark-ups. The latter sows the seeds for a future downturn as at some point there is over-production of output and firms begin to experience balance sheet losses.

Second, we showed that the effects of firm closures in the real sector are amplified by the financial side of the economy via a financial accelerator. In particular, a recession creates spill-over effects in financial markets, such as the reduction of credit availability and higher interest rates. Our analysis shows that these are mainly driven by prudential regulations that lead to liquidity hoarding by financially sound banks, rather than by the disruptive effects of bank failures. In other words, bank failures disrupt the economy not by their direct effect, but by their indirect effect in inducing precautionary liquidity hoarding by sound institutions. We also showed that excessive credit growth during an expansionary phase is a signal of the severity of the subsequent recession, as the stock of loans in the balance sheets of agents is an indicator of the potential losses during a crisis.

Third, the financial amplification mechanism is strictly related to the pro-cyclical effects of prudential regulation. In order to comply with capital regulations, banks must hold enough capital to cover potential losses as estimated by applying the expected shortfall measure, which in turn is based on its VaR estimates over a fixed number of past periods. The key here is the possibility of risk misperception at different stages of the business cycle. In good times, observed losses are low so the VaR and expected shortfall measures are low *ex ante*. Capital constraints are therefore generous. As a slump begins, VaR and expected shortfall measures rise *ex post* and the constraints tighten, contributing to the credit crunch which aggravates the slump. While the model is not primarily aimed at policy analysis, these results support the Basel III aim of designing prudential regulation that focuses on limiting credit expansion during booms, in order to reduce the extent and severity of the downturn.

Lastly we find that increasing connectivity supports credit to the real economy and improves macroeconomic performance, but on the other hand it increases liquidity hoarding. A greater number of links in the interbank market reduces interbank rationing improving banks' liquidity. In turn it promotes credit to the real sector: on one hand more firms can access to credit and compete on the goods market, but at the cost of higher fragility. On the other banks are more exposed towards the firm sector, thus there are greater losses on loan portfolios as connectivity grows. As a results banks are more prudent and hoard more liquidity.

In future research, we plan to improve the performance of the model by introducing dynamic elements into the real economy, via capital accumulation or, as an alternative, endogenous growth in labour productivity, as in *Delli Gatti et al. (2011)*. In addition the timing of events can be changed to account for overnight exchanges on the interbank market, by assuming that the business sector operates at lower frequencies than the financial one. Trades on the interbank market could include other kinds of assets with different maturities. Another important issue that we assign to future research in an extensive analysis of the effects of interbank architecture on real economy.

BIBLIOGRAPHY

Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *The American Economic Review*, 102(3):59–64.

Acharya, V. V. and Merrouche, O. (2010). Precautionary hoarding of liquidity and inter-bank markets: Evidence from the sub-prime crisis. Technical report, National Bureau of Economic Research.

Adrian, T. and Brunnermeier, M. K. (2016). Covar. *The American Economic Review*, 106(7):1705–1741.

Aldasoro, I. and Alves, I. (2016). Multiplex interbank networks and systemic importance an application to european data. *Journal of Financial Stability*.

Allen, F. and Carletti, E. (2008). The role of liquidity in financial crises.

Allen, F., Carletti, E., and Gale, D. (2009). Interbank market liquidity and central bank intervention. *Journal of Monetary Economics*, 56(5):639–652.

Alter, A., Craig, B. R., and Raupach, P. (2014). Centrality-based capital allocations.

Ashraf, Q., Gershman, B., and Howitt, P. (2017). Banks, market organization, and macroeconomic performance: An agent-based computational analysis. *Journal of Economic Behavior & Organization*, 135:143–180.

Assenza, T., Delli Gatti, D., and Grazzini, J. (2015). Emergent dynamics of a macroeconomic agent based model with capital and credit. *Journal of Economic Dynamics and Control*, 50:5–28.

Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. *science*, 286(5439):509–512.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS one*, 10(6):e0130406.

Bardoscia, M., Caccioli, F., Perotti, J. I., Vivaldo, G., and Caldarelli, G. (2016). Distress propagation in complex networks: the case of non-linear debtrank. *PloS one*, 11(10):e0163825.

Battiston, S., Caldarelli, G., D'Errico, M., and Gurciullo, S. (2016). Leveraging the network: a stress-test framework based on debtrank. *Statistics & Risk Modeling*, 33(3-4):117–138.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? financial networks, the fed and systemic risk. *Scientific reports*, 2:541.

Bénassy, J.-P. (2002). *The macroeconomics of imperfect competition and nonclearing markets: a dynamic general equilibrium approach*.

Benoit, S., Colletaz, G., Hurlin, C., and Pérignon, C. (2013). A theoretical and empirical comparison of systemic risk measures.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Borio, C., Furfine, C., Lowe, P., et al. (2001). Procyclicality of the financial system and financial stability: issues and policy options. *BIS papers*, 1:1–57.

Brownlees, C. T. and Engle, R. (2012). Volatility, correlation and tails for systemic risk measurement. *Available at SSRN*, 1611229.

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *The American Economic Review*, 104(2):379–421.

Caiani, A., Godin, A., Caverzasi, E., Gallegati, M., Kinsella, S., and Stiglitz, J. E. (2015). Agent based-stock flow consistent macroeconomics: Towards a benchmark model. *Available at SSRN*.

Chiarella, C., Flaschel, P., Hartmann, F., and Proaño, C. R. (2012). Stock market booms, endogenous credit creation and the implications of broad and narrow banking for macroeconomic stability. *Journal of Economic Behavior & Organization*, 83(3):410–423.

Chiarella, C., Iori, G., et al. (2002). A simulation analysis of the microstructure of double auction markets*. *Quantitative finance*, 2(5):346–353.

Chiarella, C., Iori, G., and Perelló, J. (2009). The impact of heterogeneous trading rules on the limit order book and order flows. *Journal of Economic Dynamics and Control*, 33(3):525–537.

Claessens, S., Kose, M. A., and Terrones, M. E. (2009). What happens during recessions, crunches and busts? *Economic Policy*, 24(60):653–700.

Danielsson, J., Valenzuela, M., and Zer, I. (2016). Learning from history: volatility and financial crises.

De Masi, G. and Gallegati, M. (2012). Bank–firms topology in italy. *Empirical Economics*, 43(2):851–866.

Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P., and Gallegati, M. (2011). *Macroeconomics from the Bottom-up*. Milano: Springer Milan.

Delli Gatti, D., Gallegati, M., Greenwald, B., Russo, A., and Stiglitz, J. E. (2010). The financial accelerator in an evolving credit network. *Journal of Economic Dynamics and Control*, 34(9):1627–1650.

Delli Gatti, D., Gallegati, M., Greenwald, B. C., Russo, A., and Stiglitz, J. E. (2009). Business fluctuations and bankruptcy avalanches in an evolving network economy. *Journal of Economic Interaction and Coordination*, 4(2):195–212.

Dosi, G., Fagiolo, G., Napoletano, M., and Roventini, A. (2013). Income distribution, credit and fiscal policies in an agent-based keynesian model. *Journal of Economic Dynamics and Control*, 37(8):1598–1625.

Dosi, G., Fagiolo, G., and Roventini, A. (2010). Schumpeter meeting keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, 34(9):1748–1767.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Fischer, E. O., Heinkel, R., and Zechner, J. (1989). Dynamic capital structure choice: Theory and tests. *The Journal of Finance*, 44(1):19–40.

Flannery, M. J. and Rangan, K. P. (2006). Partial adjustment toward target capital structures. *Journal of Financial Economics*, 79(3):469–506.

Gabbi, G., Iori, G., Jafarey, S., and Porter, J. (2015). Financial regulations and bank credit to the real economy. *Journal of Economic Dynamics and Control*, 50:117–143.

Gai, P. and Kapadia, S. (2010). Liquidity hoarding, network externalities, and interbank market collapse. In *Proc. R. Soc. A*, volume 466, page 439.

Gauthier, C., Lehar, A., and Souissi, M. (2010). Macroprudential regulation and systemic capital requirements. Technical report, Bank of Canada Working Paper.

Georg, C.-P. (2013). The effect of the interbank network structure on contagion and common shocks. *Journal of Banking & Finance*, 37(7):2216–2228.

Godley, W., L. M. (2007). Monetary economics: an integrated approach to credit, money, income production and wealth.

Grilli, R., Tedeschi, G., and Gallegati, M. (2014). Bank interlinkages and macroeconomic stability. *International Review of Economics & Finance*, 34:72–88.

Heid, F. (2007). The cyclical effects of the basel ii capital requirements. *Journal of Banking & Finance*, 31(12):3885–3900.

Heider, F., Hoerova, M., and Holthausen, C. (2009). Liquidity hoarding and interbank market spreads: The role of counterparty risk.

Hicks, J. R. et al. (1974). Crisis in keynesian economics.

Hommes, C. H. (2006). Heterogeneous agent models in economics and finance. *Handbook of computational economics*, 2:1109–1186.

Iori, G., De Masi, G., Precup, O. V., Gabbi, G., and Caldarelli, G. (2008). A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278.

Iori, G., Jafarey, S., and Padilla, F. G. (2006). Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, 61(4):525–542.

Jordà, Ò., Schularick, M. H., and Taylor, A. M. (2011). When credit bites back: leverage, business cycles, and crises. Technical report, National Bureau of Economic Research.

Kaufman, G. G. and Scott, K. E. (2003). What is systemic risk, and do bank regulators retard or contribute to it? *The Independent Review*, 7(3):371–391.

Kleinow, J., Moreira, F., Strobl, S., and Vähämaa, S. (2017). Measuring systemic risk: A comparison of alternative market-based approaches. *Finance Research Letters*, 21:40–46.

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50.

Lilit, P., Napoletano, M., and Roventini, A. (2016). Taming macroeconomic instability: Monetary and macro prudential policy interactions in an agent-based model. *Available at SSRN 2710131*.

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52:A11–A23.

McGill, R., Tukey, J. W., and Larsen, W. A. (1978). Variations of box plots. *The American Statistician*, 32(1):12–16.

Modigliani, F. and Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data. *Franco Modigliani*, 1.

Montagna, M. and Kok, C. (2013). Multi-layered interbank model for assessing systemic risk. Technical report, Kiel Working Paper.

Pankoke, D. (2014). Sophisticated vs. Simple Systemic Risk Measures. Working Papers on Finance 1422, University of St. Gallen, School of Finance.

Poledna, S., Molina-Borboa, J. L., Martínez-Jaramillo, S., Van Der Leij, M., and Thurner, S. (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability*, 20:70–81.

Poledna, S. and Thurner, S. (2016). Elimination of systemic risk in financial networks by means of a systemic risk transaction tax. *Quantitative Finance*, 16(10):1599–1613.

Rabemananjara, R. and Zakoian, J.-M. (1993). Threshold arch models and asymmetries in volatility. *Journal of Applied Econometrics*, 8(1):31–49.

Riccetti, L., Russo, A., and Gallegati, M. (2013). Leveraged network-based financial accelerator. *Journal of Economic Dynamics and Control*, 37(8):1626–1640.

Riccetti, L., Russo, A., and Gallegati, M. (2015). An agent based decentralized matching macroeconomic model. *Journal of Economic Interaction and Coordination*, 10(2):305–332.

Rodríguez-Moreno, M. and Peña, J. I. (2013). Systemic risk measures: The simpler the better? *Journal of Banking & Finance*, 37(6):1817–1831.

Silva, T. C., da Silva, M. A., Tabak, B. M., et al. (2016). Modeling financial networks: a feedback approach. Technical report.

Tarashev, N. A., Borio, C. E., and Tsatsaronis, K. (2010). Attributing systemic risk to individual institutions.

Tedeschi, G., Mazloumian, A., Gallegati, M., and Helbing, D. (2012). Bankruptcy cascades in interbank markets. *PloS one*, 7(12):e52749.

Tesfatsion, L. (2003). Agent-based computational economics: Modeling economies as complex adaptive systems. *Information Sciences*, 149(4):263–269.

Thurner, S. and Poledna, S. (2013). Debtrank-transparency: Controlling systemic risk in financial networks. *arXiv preprint arXiv:1301.6115*.

van der Hoog, S. and Dawid, H. (2015). Bubbles, crashes and the financial cycle: Insights from a stock-flow consistent agent-based macroeconomic model.

Webber, L. and Willison, M. (2011). Systemic capital requirements.

Werner, R. A. (2012). Towards a new research programme on 'banking and the economy'—implications of the quantity theory of credit for the prevention and resolution of banking and debt crises. *International Review of Financial Analysis*, 25:1–17.

1.A APPENDIX

1.A.1 *Calibration and initial values*

The parameters of the model are reported in Table 3. The number of periods for a benchmark simulation are set to 1000. The number of agents is chosen to balance the trade-off between the duration of each run and similarity with empirical evidence, in particular to reproduce network effects. Notice that the same value is assigned to the interest rates r^L, r^D and r^B. The value of r^B is arbitrary, since it is not influential in the results of the simulations, indeed by construction the only buyer of bills is the CB, whose profits are transferred again to the government. The deposits market is not explicitly modelled here and the rate on deposits is constant and equal to the rate on reserves. It could be read as a zero profit condition for banks that are not lending, so that they can only realize profits or losses through loans. The parameter *Fh* represents the share of firms that are observed by the households, sorting prices in ascending order, buying starting from the cheaper firms. As *Fh* approaches 1, the market becomes more competitive, that is each household can screen all the firms before buying. It means that the firms with prices above the average do not sell their output and realize negative profits. If *Fh* is low, the market is not competitive and firms may sell their goods despite prices are not below the mean. Furthermore there is a trade-off in terms of rationed consumers and unsold output: low values of *Fh* corresponds to more rationed consumers (that cannot visit enough firms to spend entirely the budget), less unsold output and less firm defaults, as the volatility of profits is lower.

At the beginning of the simulations the most important initial values refer to the net worth of the agents. All the agents start with the same amount of resources that is going to change during the simulations due to the heterogeneity in their behaviour. Firms are endowed with an amount of deposits that let them borrow and produce the output of full employment, namely $D^F = \frac{Y^{full}}{1+lev^F}$. Households start without net worth, as they will receive their income in form of wages, interest, dividends and transfers. Banks have the deposits of the firm sector as liabilities, hence they need a positive capital to be able to lend on the credit and the interbank market. The initial liquidity of the banking sector is $R = \frac{1}{1-rev}D$, so that their equity-assets ratio equals *rev*. The transfers *G* are obtained from the steady state solution of a simplified model and correspond to the steady state level of full employment.

$$G = \frac{W^0 N^{full}\left[1 + \mu - (1-\theta)\left(c_1 + c_2 \frac{1-c_1}{c_2 - r^D}\right)\right]}{\left(c_1 + c_2 \frac{1-c_1}{c_2 - r^D}\right)}$$

Table 3: Calibration of the baseline model

Parameter	Description	Value
T	Length of the simulation	1000
N^F	Number of firms	250
N^H	Number of households	750
N^B	Number of banks	50
α	Labour productivity	2
W^0	Ideenitial wage rate	2
θ	Tax rate	0.4
δ	Dividend share	0.5
c_1	Marginal propensity to consume out of income	0.8
c_2	Marginal propensity to consume out of savings	0.2
r^L	Interest rate on reserves	0.01
r^D	Interest rate on deposits	0.01
r^B	Interest rate on bills	0.01
r^H	Interest rate on advances	0.05
rr	Reserve coefficient	0.03
v_f	Sensitivity of r^f to the default probability	0.14
v_b	Sensitivity of r^b to the default probability	0.02
λ	Maximum leverage rate banks	24
τ	Length of firms' and banks' memory	10
τ^w	Length of unions' memory	120
τ^{ES}	Length of losses memory (ES)	100
σ_1	Sensitivity of the wage rate to unemployment	0.05
σ_2	Sensitivity of the wage rate to hysteresis	0.15
Fh	Share of firms observed on the goods market	0.2
rev	Equity/assets ratio of the recapitalized banks	0.08
L^{max}	maximum duration of loans	30
L^{min}	minimum duration of loans	2
t^{recap}	minimum time between bankrupt and recapitalization of banks	5

1.A.2 *Accounting*

The equations of the model are divided in the behavioural and accounting kind. The model includes both *stock* and *flow* variables. A stock-flow consistent accounting system can verify consistency among them. It is composed by a *transactions flow matrix* and a *balance sheet matrix*. The former describes the changes in the stock variables between the beginning and the end of any time period while the latter describes the level of the stock variables at a given time. The result of these matrices is to provide a description of the model from an accountancy viewpoint.

Tab. 4-8 report the balance sheets of a representative agent from each sector, while tab. 9-10 allow to retrieve the stock and the flow variables and the accounting identities.

Balance sheets

The balance sheet of each sector is reported below:

Table 4: Balance sheet of firms

Assets	Liabilities
D^F liquidity deposited at the bank	L^F stock of loans
	nw^F net worth

Table 5: Balance sheet of banks

Assets	Liabilities
R reserves at the CB	A advances from the CB
$L = L^F + I^l$ outstanding loans (firms and interbank)	D^B outstanding deposits
	I^b interbank borrowing
	nw^B net worth

Table 6: Balance sheet of households

Assets	Liabilities
D^H liquidity in bank deposits	
	nw^H net worth

Table 7: Balance sheet of the Central Bank

Assets	Liabilities
A advances to the banks	R cash reserves
B treasury bills	

Table 8: Balance sheet of the government

Assets	Liabilities
	B treasury bills
	nw^G net worth

Aggregate balance sheet and transactions matrix

Tab. 9 represent the aggregate balance sheet of the economic system. The sum of each row and column is zero and each element for a class of agents balances with the corresponding one. Since it is assumed that there is no physical capital and that inventories are perishable the rows in the firms accounts sum to zero and the sum of the net worths is zero as well. Notice that this implies that the government has a negative net worth.[20]

$$\sum_{i \in N^H} nw_i^H + \sum_{j \in N^F} nw_j^F + \sum_{h \in N^B} nw_h^B + nw^G = 0$$

Tab. 10 represents the aggregate transactions taking place in the system. Each flow should move from a class of agents to another (the intra-class flows are not displayed at the aggregate level) as it reported on the rows. The aggregate flows occurring within a class of agents is represented on the columns and may be divided in current and capital accounts. The current account describes the current inflows and outflows due to payments or earnings, while the capital account describes the changes in the balance sheet of the agents, that is the change in assets or liabilities.

Table 9: Aggregate balance sheet

	HH	Firms	Banks	CB	G	$\sum$
Deposits	$+D^H$	$+D^F$	$-D^B$			0
Loans		$-L^F$	$+L^F$			0
Bills				$+B$	$-B$	0
Reserves			$+R$	$-R$		0
Advances			$-A$	$+A$		0
Firms capital						0
Banks capital						0
Net worth	$-nw^H$	$-nw^F$	$-nw^B$		$-nw^G$	0
$\sum$	0	0	0	0	0	0

Variables measured at current prices. Assets(+), liabilities(-).

20 In a model with physical capital and/or inventories their sum plus the sum of the net worths should be zero.

Table 10: Aggregate transactions flow matrix

	HH	Firms		Banks		CB		G	$\sum$
		CA	KA	CA	KA	CA	KA		
Consumption	$-C$	$+C$							0
Govern exp	$+G$							$-G$	0
Production		Y							0
Wages	$+WN$	$-WN$							0
Taxes	$-T^H$	$-T^F$				$-T^B$		$+T$	0
Profits Firms	$+\delta\Pi^F$	$-\Pi^F$	$+(1-\delta)\Pi^F$						0
Profits Banks	$+\delta\Pi^B$			$-\Pi^B$	$+(1-\delta)\Pi^B$				0
Profits CB						$-\Pi^{CB}$		$+\Pi^{CB}$	0
Deposits interest	$+r^D D^H$	$+r^D D^F$		$-r^D D$					0
Loans interest		$-r^f L^f$		$+r^f L^f$					0
Bills interests						$+r^B B$		$-r^B B$	0
Reserves interests				$+r^R R$		$-r^R R$			0
Advances interests				$-r^H A$		$+r^H A$			0
ΔLoans			$+\Delta L$		$-\Delta L$				0
ΔBills							$-\Delta B$	$+\Delta B$	0
ΔReserves					$-\Delta R$		$+\Delta R$		0
ΔDeposits	$-\Delta D^H$		$-\Delta D^F$		$+\Delta D$				0
Δ Advances					$+\Delta A$		$-\Delta A$		0
$\sum$	0	0	0	0	0	0	0	0	0

Variables measured at current prices. Sources of funds(+), uses of funds(-).

MACROECONOMIC AND FINANCIAL SERIES

Figure 21: Key macroeconomic variables.

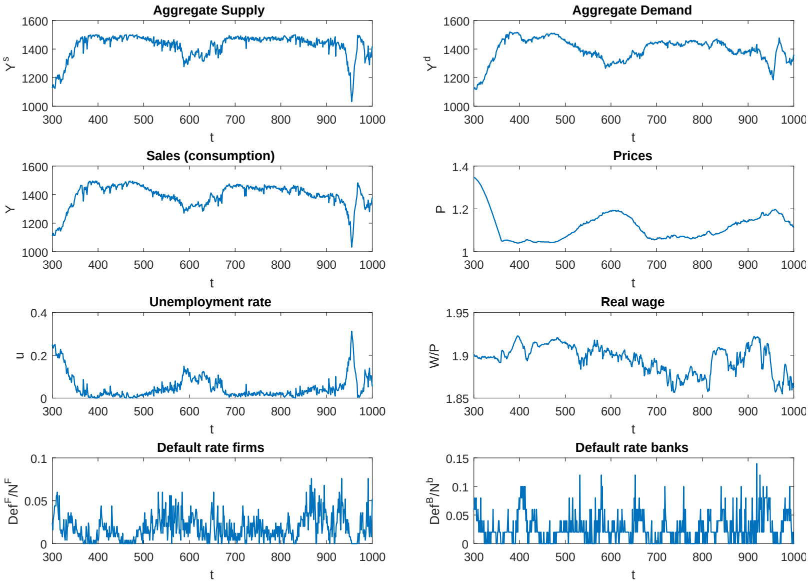
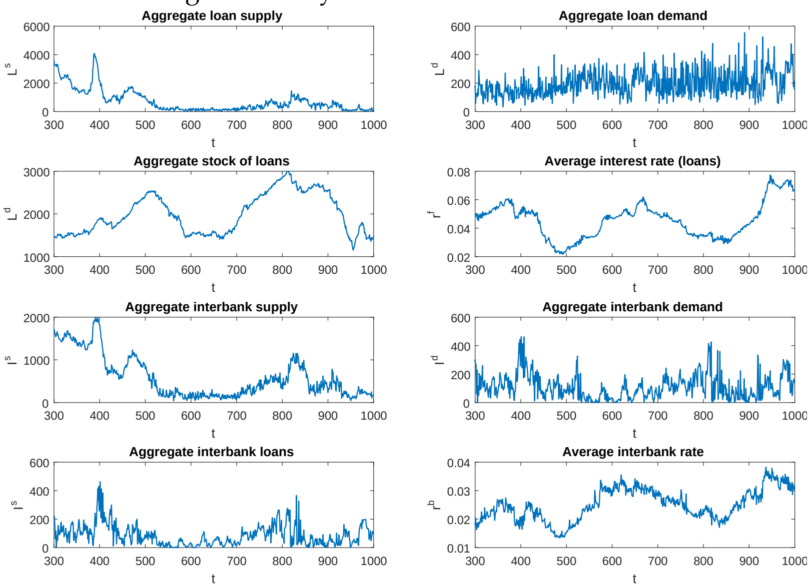


Figure 22: Key macroeconomic variables.



GINI INDEX

$$Gini_t = N + 1 - \frac{2}{N-1} \frac{\sum_{i=1}^{N} (N+1-i) y_{i,t}}{\sum_{i=1}^{N} y_i}$$

with N total sample size

y_i wealth of the poorest i individuals in the sample, where individuals are sorted in ascending order with respect to wealth y.

2

REDUCING SYSTEMIC RISK WITH MACRO-PRUDENTIAL CAPITAL REQUIREMENTS: COMPARATIVE POLICY EXPERIMENTS

ABSTRACT We exploit a multi-agent macroeconomic model to conduct policy experiments with several macroprudential rules derived from measures of systemic risk. Our objective is to obtain new insights about the effectiveness of systemic risk metrics when applied to minimum capital requirements of financial institutions. We find that requirements based on vulnerability of financial institutions are better than those based on impact.

2.1 INTRODUCTION

The concept of *systemic risk* (SR) is relatively recent in economic and financial literature. The first appearances of the term in scientific articles date back to the early '90s, even if citations reveal that most of these contributions have been revived after 2008, when the term regained strength with the crisis. A general definition is provided in *Kaufman and Scott (2003, p. 371)*:

"*Systemic risk refers to the risk or probability of breakdowns in an entire system, as opposed to breakdowns in individual parts or components, and is evidenced by comovements (correlation) among most or all the parts.*"

Systemic risk denotes a risk involving the entire economy, not just single financial institutions. Similarly policies aimed at the reduction of systemic risk should consider the macro environment, namely macro-prudential regulation, as it is explained in *Borio et al. (2001)*.

Macroprudential policies should be viewed as complementary to micro-prudential ones and should be designed to improve the resistance and the resilience of the financial system to unforeseen events. However there are several reasons that make such policies hard to implement, for instance they should be built upon a reliable measure of systemic risk. This is not an easy task, as it is not clear which measure out of the many performs better and under what circumstances given that systemic events can be observed infrequently, for instance *Danielsson et al. (2016)* reports a banking crisis on average every 35 years for OECD countries. Moreover incomplete information makes it difficult for banks to rightly predict their systemic importance, hence they might perceive as unfair the prudential requirements imposed by the central regulator. Conversely regulatory measures based on systemic risk could be adopted by each institution as an internal risk management model, but asymmetric information or adverse incentives might undermine their realization, leading to the failure to internalize the effects of individual contribution to systemic risk.

In this paper we abstract from the problem concerning how to credibly apply systemic risk regulation to banks, rather we focus on the comparison of capital surcharges derived from different systemic risk measures. We resort to an hypothetical reasoning, assuming that banks can successfully adopt capital rules aimed at the reduction of systemic risk, then we observe the outcomes of such policies produced by the multi-agent macro-economic model presented in Chapter 1. Results could shed some light on the effectiveness of systemic risk indicators.

Many methods to measure systemic risk have been proposed so far, but there is not a consensus among scholars. We consider two alternative approaches, namely *market-based* (SRISK and $\Delta CoVaR$) and *network-based* (DebtRank) measures. A further distinction between them consists in what they measure: vulnerability or impact. Measures of vulnerability focus on the effect of a systemic crisis on the capital of banks, in other words they account for the susceptibility of each institution conditional to a systemic event. Measures of impact capture the effect produced by a distressed bank on the entire financial system in terms of lost capital .

The first measure of vulnerability is based on *LRMES* (Long Run Marginal Expected Shortfall) *(Acharya et al., 2012)*, which contributes to compute the cost of re-capitalizing a bank in case of distress (SRISK). The second one is derived from a stress test based on *DebtRank*, in such a way to obtain an expected shortfall describing individual vulnerabilities. The chosen indicators of impact are $\Delta CoVaR$ *(Adrian and Brunnermeier, 2016)* and a *DebtRank* based measure of individual impact.[1]

Policy experiments consist in employing systemic risk measures to determine minimum capital requirements of banks, then to test their effectiveness by means of our multi-agent macro model. In the first set of experiments we assume that minimum capital requirements are set on the basis of vulnerability measures, so that more fragile agents are required to have a proper equity capital. However this might not be satisfactory, as it does not operate on the systemic impact of banks. The second set of experiments is similar, but capital requirements depend on the impact of banks on the system, or the extent of externalities they produce in case of default.

We find that policies based on vulnerability perform better than those based on impact, both from the viewpoint of reducing systemic risk and macroeconomic stability. Moreover network based measures are more accurate but more volatile than market based measures.

The paper is organized as follows. Sec. 2.2 presents the related literature, sec. 2.3 describes distress dynamics, systemic risk

1 The DebtRank framework provides both measures of vulnerability and impacts. However the term "*DebtRank*"specifically refers to the systemic impact of banks in its original formulation. For further details see *Battiston et al. (2016)*

measures and prudential policies. Sec. 2.4 goes through results of the policy experiments. Conclusions are in sec. 2.5.

2.2 RELATED LITERATURE

Several studies have been carried out so far aiming at the comparison of systemic risk measures. The novelty of this paper is the attempt to compare network based and market based measures of systemic risk, both from the perspective of vulnerability of single institutions and of individual impacts on the financial system. Another departure from the existing literature consists in the method of assessment, that is performed by means of a multi-agents model, rather than regressions on observed data of financial institutions. Our simulated economy produces data about returns on assets and at the same time includes a network structure, thus it allows for a comparison on these two dimensions.

Part of the literature aims at the comparison of different measures of systemic risk by means of econometric methods. *Benoit et al. (2013)* provide a theoretical and an empirical comparison of three market based measures of systemic risk, namely MES, SRISK and $\Delta CoVaR$. They resort to a common framework to show similarities and properties of these measures, then they turn to an empirical approach to compare the top US financial institutions and to determine whether different measures of SR leads to identify the same SIFI. The main finding is that the three measures cannot fully capture multiple aspects of SR (Systemic Risk), but their variability can be explained by a single market measure of firm characteristics. *Kleinow et al. (2017)* empirically compare four widespread measures of systemic risk, namely MES, Co-Risk, $\Delta CoVaR$ and LTD using data on US financial institutions. Their estimates point out that the four metrics are not consistent with each other over time, hence it is not possible to fully rely on a single measure. *Rodríguez-Moreno and Peña (2013)* consider six measures of systemic risk using data from stock, credit and derivative markets. They quantitatively evaluate such metrics through an "horse race", exploiting a sample composed of the biggest European and US banks. Their results favour SR measures based on simple indicators obtained from credit derivatives and interbank rates, rather than more complex metrics whose performance is not so satisfactory. Similarly *Pankoke (2014)* compares and contrasts sophisticated versus simple measures of systemic risk and conclude that simple measures have more explanatory power.

Another set of contributions draws from the econophysics literature. They focus on network based measures of systemic risk. *Battiston et al. (2016)* propose a network based stress test building the DebtRank algorithm. The framework is flexible enough to account for impact and vulnerability of banks, as well as to decompose the transmission of financial distress in various rounds of contagion and to estimate distribution of losses. Such methodology is exploited to perform a stress-test on a panel of European banks. The outcome indicates the need to include contagion effects (or indirect effects) in future stress-tests of the financial system, otherwise systemic risk might be underestimated. *Thurner and Poledna (2013)* suggest an agent-based model with an incentive scheme to reduce systemic risk on the interbank market, which consists in the disclosure of information about the systemic importance of banks computed through DebtRank and Katz centrality. Next they test the mechanism by means of an agent-based macroeconomic model with real and financial aspects. The incentives deter banks from borrowing from systemically important agents, thus reducing systemic risk without altering the efficiency of the financial network. A more sophisticated mechanism is studied in *Poledna and Thurner (2016)*, which involves a tax on systemic risk. The amount of the tax is determined by the marginal contribution of each transaction to systemic risk, quantified by the DebtRank methodology. The scheme is implemented in a macro-financial agent-based model, where the systemic risk tax is compared to a Tobin tax and to a model without any tax. The systemic risk tax turns out to be the best option, as links within the interbank network organizes in such a way to eliminate systemic risk when the tax is levied.

Some papers are relevant for our analysis because they translate systemic risk measures into capital surcharges, despite it is not their primary objective. For instance *Gauthier et al. (2010)* compare capital allocation rules derived from five different measures of systemic risk by means of a network based model of interbank relations applied to a dataset about the six greatest banks of Canada. They claim that requirements based on current capitalization of banks would not minimize total risk, hence they employ an iterative optimization process to solve for the fixed point, so that the optimal allocation minimizes total risk, keeping constant the total amount of capital. The adopted framework leads to a reduction of the probability of systemic crises of about 25%, however results are sensitive to the inclusion of derivatives and cross shareholdings in the data. *Alter et al. (2014)* study a reallocation mechanism of capital in a model of interbank contagion. They assume that the overall capital required for the banking system is constant. Then they consider reallocation rules based on network centrality metrics, where optimal shares of capital are those that minimize expected bankruptcy costs. Finally they compare total expected bankruptcy costs of defaulted banks with the benchmark scenario and show that allocation rules based on centrality measures outperform credit risk measures.

Tarashev et al. (2010) propose a methodology borrowed from game theory to attribute systemic risk to financial institutions, namely the Shapley value. They impose capital requirements in order to achieve a target for system-wide risk, from both a micro and a macro prudential perspective. They find that when banks differ only in their exposures to a common risk factor, macroprudential rules aimed at equalizing systemic risk importance of financial institutions perform better. In other words if capital surcharges are set in order to equalize individual contributions to systemic risk, then a lower level of aggregate capital is needed to reach the system-wide risk objective. *Webber and Willison (2011)* draw on a similar approach to define "*systemic capital requirements*". It is assumed that the regulator is willing to tolerate a systemic-wide risk level and aims at reaching the most parsimonious feasible capitalization at the aggregate level. Such dilemma is formally translated into a constrained optimization problem, whose solution includes both the unique level of capital in the banking system and its distribution across banks. Systemic capital requirements result increasing in balance sheet size and in the value of interbank obligations, however they are also found to be strongly pro-cyclical, then a correction would be needed to obtain unbiased requirements.

2.3 THE MODEL

2.3.1 *Distress dynamics*

We elaborate on the macroeconomic ABM built in Chapter 1. The financial system is populated by N^B banks and N^F firms which interact on the credit and interbank market. Banks and

$$\begin{tikzcd}
\text{Firms-Banks (loans)} \arrow[d] \arrow[dd, bend left=60] \\
\text{Banks-Banks (interbank loans)} \arrow[d] \\
\text{Banks-Firms (deposits)} \arrow[uu, bend left=60]
\end{tikzcd}$$

Figure 1: The diagram represents the way distress is transmitted across banks and firms.

firms default if their equity turns negative. Distress propagates through defaults in the credit and interbank market. We refer to the part of the model which accounts for distress propagation as "*default stage*".

During the simulations endogenous shocks hit the system. They originate from market dynamics, in other words some firms cannot re-pay loans due to a negative outcome in the goods market. Shocks propagate from firms to banks, within banks and from banks to firms. The process goes on until there are no new losses.

LIQUIDATION OF ASSETS In order to enhance contagion dynamics, we propose that during each "*defaults stage*" defaulted banks must sell their illiquid assets in order to repay creditors. An agency of the Central Bank acts as a liquidator, namely it buys all the assets (loans to firms or interbank loans) of defaulted banks. The sale price of an asset indexed by j is determined by a fixed haircut γ, times the net present value[2] of j, NPV_j:

$$p_{j,t}^{CB} = \gamma NPV_{j,t}$$

At the end of the default stage assets are sold to survived banks at the price

$$p_{j,t}^{B} = NPV_{j,t}$$

Banks that comply with regulatory capital requirements and are liquid enter the market as buyers, sorted by the distance between

2 We assume that the net present value is equal to the book value of loans. In other words present valuation is computed through a simple interest rate rather than a compound one, in order to be consistent with the simple accounting system of the model.

actual and target leverage (so that they act in a consistent way with their objective.). In case of sale the ownership of loans and its cash flows are transferred to buyers, while maturities and interest rates remains unchanged.

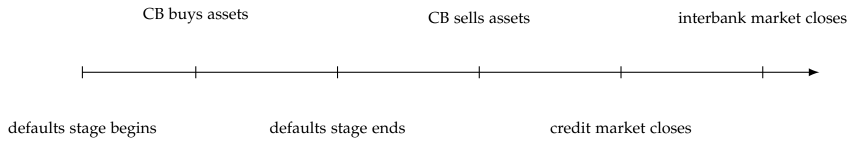


Figure 2: Timeline of liquidation

It is worth remarking that distress propagates only from defaults and not with relative equity losses. The process is different from the previous model because liquidation of assets decreases the amount that is recovered by the creditors, thus increasing systemic losses. The CB has a role similar to that of *Federal Deposit Insurance Corporation* (FDIC) for US. Loans are trasferred from defaulting banks to other banks, so that the total exposure of active banks increases since the stock of loans in the system is owned just by active banks.

RECOVERY RATES The effective loss on the value of each asset during the "default stage" is $A_{ij}(0)(1-\varphi_{ij}(t))$, where φ is the recovery rate and $A_{ij}(0)$ is the value of a generic asset owed by j to i at the beginning of the cycle. In general each creditor of j can recover the fraction of its assets given by the borrower's assets ($\mathcal{A}$) to liabilities ($\mathcal{L}$) ratio $\varphi_{ij}=\frac{\mathcal{A}_j}{\mathcal{L}_j}$. However the nominal value of assets is not immediately convertible in cash, indeed some assets are illiquid and must be sold to compensate creditors. We denote the liquidation value of assets of bank j with $\mathcal{A}^{liq}_{j,t}$, with $\mathcal{A}^{liq}_{j,t} \leq \mathcal{A}_{j,t}$. The actual recovery rate can be written as:

$$\varphi_{ij}=\frac{\mathcal{A}^{liq}_j}{\mathcal{L}_j}$$

Furthermore we assume that there is a pecking order of creditors, so that they are not equal from the viewpoint of bankruptcy law: the most guaranteed is CB, then depositors and finally banks with interbank loans.

For instance those creditors who claim interbank loans towards the defaulted bank j recover the part of j's assets left after the other classes of creditors have been compensated. For instance the recovery rate on an interbank loan can be expressed as

$$\varphi_{ij} = \max\left(0, \frac{\mathcal{A}_j^{liq} - A_j^{CB} - D_j}{\mathcal{L}_j - A_j^{CB} - D_j}\right)$$

where A^{CB} are CB's loans to j and D are j's deposits.

It is worth noticing that *loss given default* (LGD) is $LGD \equiv 1 - \varphi$, so that the net worth of creditor i updates as:

$$nw_{i,t}^B = nw_{i,t-1}^B - LGD_{ij,t} I_{i,t}^l$$

DEFAULT STAGE ALGORITHM The sequential structure of the algorithm employed in the ABM to model the "*default stage*" of banks and firms is described below.

while there are losses

1. FIRMS $\xrightarrow{\text{loans to firms}}$ BANKS

   Banks update their net worth by writing-off assets towards defaulted firms and adding an amount of liquidity equal to $R_i(\tau) = L_{ij}^F(0)\varphi_j^L(\tau)$.

   The same quantity is subtracted by the balance sheet of defaulted firms, loans are written-off borrowers' liabilities.

2. BANKS $\xrightarrow{\text{interbank loans}}$ BANKS

   Banks with negative equity declare default.

   Creditors (other banks) update their balance sheets by writing-off interbank assets towards defaulted banks and adding an amount of liquidity equal to $R_i(\tau) = I_{ik}^l(0)\varphi_k^{Ib}(\tau)$.

   The same quantity is subtracted by the balance sheet of defaulted banks, while interbank loans are written-off borrowers' liability side.

   Repeat until the default cascade on the interbank market ends (denote it with $t = \tau$).

3. BANKS $\xrightarrow{\text{deposits of firms}}$ FIRMS

   Banks with negative equity declare default.

Firms update their balance sheet accounting for losses on deposits: $D_{jk}(\tau) = D_{jk}(0)\varphi_k^D(\tau)$. At the same time banks reduce deposits at the same amount.

end

2.3.2 *Accounting for systemic risk*

In this section we define minimum capital requirements (MCR) based on SR measures. The first set of MCR is built on vulnerability of banks to a systemic crisis, while the second one is based on their systemic impact on the financial sector

In all cases minimum capital requirements are expressed in an intuitive way, as SR indexes range in the interval $[0,1]$.

Banks must hold a minimum net worth equal to a fraction of their risk weighted assets (RWA)[3], $nw_t^B \geq \psi RWA_t$, where ψ is a parameter determined by SR metrics.

If a systemic risk measure equals 0, $\psi = \frac{1}{\lambda}$, a bank must have a capital greater or equal than a threshold fixed by the regulator, namely $nw^B \geq \frac{1}{\lambda}RWA$, where $\frac{1}{\lambda} = 0.04$. The calibration of λ is chosen to enhance the model dynamics. When the systemic risk measure equals 1, then $\psi = 1$ and capital requirements are as strict as possible, so that equity should equal assets, $nw^B = RWA$.

Banks comply with capital requirements by setting loan supply L^s and interbank supply I^s in accordance with their maximum allowed stock of loans. If capital is lower than requirements, banks de-leverage by suspending their lending activity until they fulfil with minimum capital. Bank i can supply loans up to a multiple of its net worth (nw^B), net of outstanding loans (L) at the beginning of t. Its loan supply is

$$L_{i,t}^s \leq \frac{1}{\psi} nw_{i,t}^B - L_{i,t}$$

DebtRank

An advantage of the DebtRank algorithm is that it can provide both measures of vulnerability and impact of banks (see

3 For the sake of simplicity we assign a weight equal to 1 to loans to firms and to interbank lending, while liquidity is assumed to be riskless with a weight of 0. Risk weighted assets of bank i can be expressed as $RWA_{it} = 1 \times (L_{it}^F + I_{it}^l) + 0 \times R_{it} = L_{it}^F + I_{it}^l$. Total assets can be written as $\mathcal{A}_{i,t} = RWA_{i,t} + R_{i,t}$.

sec. 2.A.2 on page 110 for details). Vulnerabilities are obtained by imposing a shock on firms' deposits and repeating the DebtRank algorithm 250 times.[4] Individual vulnerability produced by the stress test is expressed in terms of the relative equity loss of each bank at the last step of the algorithm $h_T \equiv \frac{nw_T^B - nw_0^B}{nw_0^B}$. Next an *expected shortfall* (ES^{ind}) is computed at the α level of the relative equity loss h_T for each bank. We define the systemic risk index for vulnerability obtained from the DebtRank algorithm as $DR_{i,t}^{vul} = ES_{i,t}^{ind}$.

The impact of each bank on the rest of the system is accounted for by DR^{imp}, that is the overall loss in capital that is produced by the default of bank i. The values of DR^{imp} for each institution are obtained by imposing its default before running the algorithm.

$$DR_i^{imp} = \sum_{j=1}^{N^b} h_{j,T} \; nw_0^B$$

The final value of DR^{imp} is obtained by repeating the process 250 times and, imposing each time a different random draw from the distribution of recovery rates.[5] From this set of realizations we compute for each bank the average value of observations beyond the $95th$ percentile, *i.e.* an expected shortfall over the realized DR^{imp} of each bank.

Although our approach is similar to that adopted in *Battiston et al. (2016)*, there are some differences that it is worth to remark. We have adapted the algorithm to account for the structure of the underlying macro-model, as described in greater detail in sec. 2.A.2 on page 110. Given that the macro-environment includes firms, we first impose the shock on firms' assets, next the induced distress transmits linearly to the assets of creditors (i.e. banks). This allows to capture the specific dynamics of the distress process. In detail we impose a depreciation of firms' deposits such that the relative equity loss of firms corresponds to the realized value of the shock. ($D' = shock \times nw_0^F + D_0$). The amount of deposits lost by firms is uniformly transferred to the household sector, then banks update deposits and equity.

4 The shock is a $N^f \times 1$ vector of independent draws from a truncated log-normal distribution with parameters $\mu = -7.3$, $\sigma = 1.5$, ranging in the interval $[0.0001, 0.1]$. The distribution function has been chosen to fit the realized distribution of shocks in our benchmark model. Moreover parameters and truncation have been calibrated so that the aggregate capital shortage computed through DebtRank is similar to SRISK as much as possible.

5 Recovery rates on each kind of assets are randomly extracted from a vector of observations generated by the benchmark model, see sec. 2.A.2.

SRISK and ΔCoVaR

SRISK *(Brownlees and Engle, 2012)* is a widespread measure of systemic risk based on the idea that the latter arises when the financial system as a whole is under-capitalized, leading to externalities for the real sector. The SRISK of a financial firm i is defined as the quantity of capital needed to re-capitalize a bank conditional on a systemic crisis

$$SRISK_{i,t} = \max\left[0,\ \frac{1}{\lambda}\mathcal{L}_i - \left(1 - \frac{1}{\lambda}\right) nw^B_{i,t} MES^{sys}_{i,t+h|t}\right]$$

where $\mathcal{L}$ are liabilities and $MES^{Sys}_{i,t+h|t} = E\left(r_{i,t+h|t}|r < \Omega\right)$ is the tail expectation of the firm equity returns conditional on a systemic event, that happens when i's equity returns r are less than a threshold value Ω from $t - h$ to t.

At each period of the simulation, every bank computes its own SRISK with a rolling regression on a time window of 200 periods. Required information are market and individual returns. We compute returns on equity (ROE) of bank i as the relative change in its net worth from the beginning of t to t', where the latter indicates the time between the defaults stage and banks' recapitalization in the timing of the model.[6] Similarly market returns are calculated as the sum of relative changes in net worth weighted by the net worth of each bank.

$$ROE_{i,t} = \frac{nw^B_{i,t'} - nw^B_{i,t}}{nw^B_{i,t}} \quad \text{with } t' > t$$

$SRISK$ is built on the idea that capital shortages in case of a systemic crisis can be inferred from the tail of the distribution of negative equity returns during normal days, given that a crisis is a very rare event whose data are barely available. To ensure theoretical consistency with the idea of $SRISK$, we exclude from the sample those observations in periods when aggregate losses are greater than 0.4 times market capitalization, which corresponds to periods of downturn in our benchmark model. In such case, banks compute SRISK with the last available value of MES^{sys}.

Another widespread measure of systemic risk is ΔCoVaR, which quantifies the systemic distress conditional to the distress

6 Otherwise in case a bank goes into default its return would be upwards biased by shareholders' capital.

of a specific financial firm, therefore it assesses the impact of a bank on the financial system.

CoVaR is implicitly defined as the VaR of the financial system (*sys*) conditional on an event $C(r_{i,t})$ of institution i

$$Pr\left[r_{sys,t} \leq CoVaR^{sys|C(r_i)} \mid C(r_{i,t})\right] = \alpha$$

where r represents ROE and the conditioning event $C(r_i)$ corresponds to a loss of i equal or above its Var_α^i level.

$\Delta CoVaR$ is the part of systemic risk that can be attributed to i, or *"the change in the value at risk of the financial system conditional on an institution being under distress relative to its median state " (Adrian and Brunnermeier, 2016, p.1)*.

$$\Delta CoVaR_\alpha^{sys|i} = CoVaR_\alpha^{sys|r_i = VaR_{i,\alpha}} - CoVaR_\alpha^{sys|r_i = VaR_{i,0.5}}$$

$\Delta CoVaR$ is a statistical measure of tail-dependency between market returns and individual returns, which is able to capture co-movements of variables in the tails and account for both spillovers and common exposures.

A flaw of $\Delta CoVaR$ is its (at best) contemporaneity with systemic risk, thus it fails to capture the build-up of risk over time and suffers of procyclicality. Furthermore contemporaneous measures lead to the *"volatility paradox"(Brunnermeier and Sannikov, 2014)*, inducing banks to increase the leverage target when contemporaneous measured volatility is low. A workaround would be to substitute contemporaneous with a forward-looking version of $\Delta CoVaR$ *(Adrian and Brunnermeier, 2016, p.1725)*. The latter is obtained by projecting on the regressors of $\Delta CoVaR$ their estimated coefficients, where the independent variables include individual banks' characteristics and macro-state variables. Nevertheless our model lacks the wide range of variables that can be employed in empirical works, as a results our measure of forward $\Delta CoVaR$ turns out to be strongly proportional to the *VaR* of banks, thus failing to capture the build up of systemic risk. However if we consider minimum capital requirements, the absolute value of Delta CoVaR is not important, but what matters is its relative value. In other words, even if systemic risk is low we use the relative value of $\Delta CoVaR$ (see sec. 2.3.3) to establish capital requirements, assuming that the systemic rank of financial institutions does not change too quickly over time.

2.3.3 *Minimum Capital Requirements*

In order to conduct policy experiments with different SR indicators we need to define minimal capital requirements (MCR) in a comprehensive way such that the systemic risk metrics above could be compared.

Our unifying procedure builds on the approach of *Acharya et al. (2012)*, but we depart from it in the definition of expected capital shortfall. Expected capital shortfall (CS) is the difference between minimum regulatory capital expressed as a fraction $\frac{1}{\lambda}$ of risk weighted assets (RWA) and book value of equity in case of a crisis, in other words it is the capital needed to restore the capital adequacy ratio to the value set by regulator ($\frac{nw^B}{RWA} \geq \frac{1}{\lambda}$):

$$\begin{aligned} CS_{i,t+\tau|t} &= \max\left\{0,\ E_t\left[\frac{1}{\lambda}RWA_{i,t+\tau} - nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right]\right\} \\ &= \max\left\{0, E_t\left[\frac{1}{\lambda}\mathcal{L}_{i,t+\tau} - R_{i,t+\tau}|\ crisis_{t+\tau}\right] + \right. \\ &\left. - E_t\left[(1-\frac{1}{\lambda})nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right]\right\} \end{aligned}$$

By assumption debt and liquidity[7] are unchanged in case of crisis, hence

$$E_t\left[\mathcal{L}_{i,t+\tau} - R_{i,t}|\ crisis_{t+\tau}\right] = \mathcal{L}_{i,t} - R_{i,t+\tau}$$

It turns out that

$$\begin{aligned} CS_{i,t+\tau|t} &= \max\left\{0,\ \frac{1}{\lambda}(\mathcal{L}_{i,t} - R_{i,t}) - E_t\left[\left(1-\frac{1}{\lambda}\right)nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right]\right\} \\ &= \max\left\{0,\ \frac{1}{\lambda}\left(RWA_{i,t} - nw^B_{i,t}\right) - E_t\left[\left(1-\frac{1}{\lambda}\right)nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right]\right\} \end{aligned}$$

MCR based on vulnerability

CASE 1: SRISK If we are interested in computing SRISK, the conditional expectation becomes:

$$E_t\left[nw^B_{i,t+\tau}|\ crisis_{t+\tau}\right] = (1 - LRMES_{i,t})nw^B_{i,t}$$

7 In other words we assume that in case of crisis the major change in net worth is derived from losses on loans while cash flows and liabilities are constant. Moreover even if there were a change in liabilities, for instance in deposits, the difference $\mathcal{L} - R$ remains constant because deposits and R would be reduced by the same amount.

where $LRMES$ is the Long Run Marginal Expected Shortfall representing the vulnerability of banks in case of extreme events (see sec. 2.A.2 on page 112).

SRISK is:

$$SRISK_{i,t} = \frac{1}{\lambda} RWA_{i,t} - \left[1 - \left(1 - \frac{1}{\lambda}\right) LRMES_{i,t}\right] nw^B_{i,t}$$

Capital requirements for bank i are then obtained by imposing $SRISK = 0$, so that it should always maintain a capital buffer great enough to avoid recapitalization during periods of distress

$$nw^B_{i,t} \geq \frac{\frac{1}{\lambda}}{1 - (1 - \frac{1}{\lambda}) LRMES_{i,t}} RWA_{i,t} \tag{1}$$

CASE 2: DEBTRANK-VULNERABILITY We apply the vulnerability measure derived from DebtRank to capital requirements. In such case the conditional expectation is:

$$E_t \left[nw^B_{i,t+\tau} | \; crisis_{t+\tau} \right] = (1 - DR^{vul}_{i,t}) nw^B_{i,t}$$

Expected capital shortfall is

$$CS_{i,t+\tau|t} = \max \left\{ 0, \; \frac{1}{\lambda} RWA_{i,t} - \left[1 - \left(1 - \frac{1}{\lambda}\right) DR^{vul}_{i,t}\right] nw^B_{i,t} \right\}$$

As in the previous case, capital requirements derive from the condition $CS_{i,t+\tau|t} = 0$, hence:

$$nw^B_{i,t} \geq \frac{\frac{1}{\lambda}}{1 - (1 - \frac{1}{\lambda}) DR^{vul}_{i,t}} RWA_{i,t} \tag{2}$$

MCR based on impact

We adopt a top-down approach to ensure consistency between the previous rules and those based on impact. In other words minimum capital requirements are defined starting from the aggregate and then deriving the individual requirements that each bank must satisfy to reach the aggregate objective.

To this end, we rewrite the expected capital shortage in aggregate terms. The idea underlying the measures presented below is that banks contribute to reduce the aggregate expected capital shortage in proportion to their systemic impact.

$$\sum_{i=1}^{N^b} CS_{i,t+\tau|t} = \frac{1}{\lambda}\left(\sum_{i=1}^{N^b} RWA_{i,t} - \sum_{i=1}^{N^b} nw^B_{i,t}\right) + \\ - E_t\left[\left(1-\frac{1}{\lambda}\right)\sum_{i=1}^{N^b} nw^B_{i,t+\tau} | \; crisis_{t+\tau}\right]$$

CASE 3: ΔCOVAR We assume that each bank contributes to expected aggregate capital shortage proportionally to its impact, which is captured by $\Delta CoVaR^{sys|i}_{\alpha}$. It measures the change in value at risk of the financial system at α level when the institution i shifts from its normal state (measured with losses equal to its median Var) to a distressed state (losses greater than or equal to its Var).

To keep internal consistency and to avoid aggregation issues we assume that aggregate capital shortage is the sum of individual $SRISK$ values, computed with the same procedure of sec.2.3.3, so that $\sum_{i=1}^{N^b} CS_{i,t+\tau|t} = \sum_{i=1}^{N^b} SRISK_{i,t}$.

Each bank should contribute to expected capital shortage in proportion to its systemic importance. That means that the additional capital required for each bank is

$$nw^+_{i,t} = \frac{\Delta CoVaR^{sys|i}_t}{\sum_{i=1}^{N^b} \Delta CoVaR^{sys|i}_t} \sum_{i=1}^{N^b} CS_{i,t+\tau|t}$$

Hence the target level of capital for bank i is given by the minimum regulatory level of capital plus the additional capital:

$$nw^{tag}_{i,t} = \frac{1}{\lambda} RWA_{i,t} + nw^+_{i,t}$$

A rule for capital requirements which is consistent with those descending from vulnerability is

$$nw^{tag}_{i,t} \geq \frac{\frac{1}{\lambda}}{1-(1-\frac{1}{\lambda})\zeta_i} RWA_{i,t} \tag{3}$$

with[8] $\zeta_i = \frac{nw^{+}_{i,t}}{(1-\frac{1}{\lambda})(\frac{1}{\lambda}RWA_{i,t}+nw^{+}_{i,t})}$.

case 4: debtrank-impact As before, we choose a top-down approach and we assume that each bank contributes to aggregate expected capital shortage in proportion to its economic impact. In this case the aggregate capital shortage is the sum of individual CS computed with the DebtRank algorithm as described in sec. 2.3.3.

The contribution of each bank to aggregate CS depends on its impact measured by DR:

$$nw^{+}_{i,t} = \frac{DR^{imp}_{i,t}}{\sum_{i=1}^{N^b} DR^{imp}_{i,t}} \sum_{i=1}^{N^b} CS_{i,t+\tau|t}$$

The capital target of bank i is given by minimum regulatory capital plus an addition:

$$nw^{tag}_{i,t} = \frac{1}{\lambda}RWA_{i,t} + nw^{+}_{i,t}$$

Minimum capital requirements for i are given by:

$$nw^{tag}_{i,t} \geq \frac{\frac{1}{\lambda}}{1-(1-\frac{1}{\lambda})\chi_i}RWA_{i,t} \tag{4}$$

with $\chi_i = \frac{nw^{+}_{i,t}}{(1-\frac{1}{\lambda})(\frac{1}{\lambda}RWA_{i,t}+nw^{+}_{i,t})}$.

2.4 policy experiments: results

This section presents the results of the policy experiment, whose aim is to show what would happen if measures of systemic risk were employed to determine minimum capital requirements of banks. A snapshot of the behaviour of SR measures over a business cycle is shown in sec. 2.4.1. Autocorrelation is analysed in sec. 2.4.2, finally empirical distributions arising from the simulation with the ABM are presented in sec. 2.4.3.

8 If $nw^{+}_{i,t} = 0 \Rightarrow \zeta_{i,t} = 0$ and $nw^{tag}_{i,t} = \frac{1}{\lambda}RWA_{i,t}$. Moreover we assume that $\zeta \in [0,1]$, so that banks must hold at most an amount of capital equal to their assets.

2.4.1 *SR over the business cycle*

In this section we compare systemic risk metrics along both their dimensions during a downturn generated by model dynamics.

Fig. 3 displays unemployment and default rates during a crisis episode. The unemployment rate raises around $t = 540$ as a consequence of a cascade of defaults of firms and banks. When the cascade ends unemployment smoothly declines and returns to the pre-crisis value around $t = 650$.

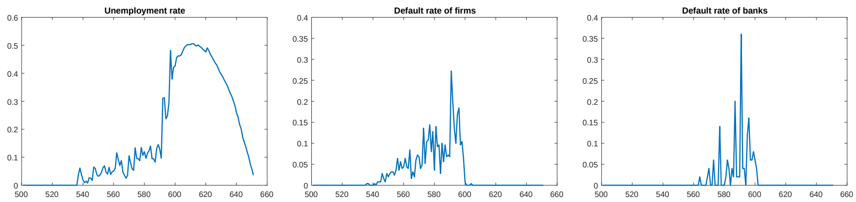


Figure 3: Unemployment rate, defaults of firms and banks during a crisis.

Time series of SR measures for bank i are displayed in fig. 4. It can be noticed that the values of market based measures tend to be more persistent, that is they display higher autocorrelation (see sec. 2.4.2 for correlation analysis). Furthermore network based measures exhibit higher values before the crisis, while the other take on greater values after the beginning of the downturn. In particular $\Delta CoVaR$ is procyclical, meaning that it accounts for the systemic impact after the financial institution is affected by the crisis, hence its value increases only around $t = 580$. At the opposite DR^{imp} records a greater impact of bank i before it suffers losses, while after $t = 580$ its impact is considerably diminished. The third graph of fig. 4 shows a comparison of expected capital shortages (ECS) computed through LRMES and DR^{vul}. After $t = 580$, the two series are almost identical, but before it DR-ECS produces higher and more volatile values, although both follow a similar pattern. This is due to different structural characteristics of the two measures resulting from their dissimilar construction.

Fig.s 5 and 6 depict the value of systemic risk at different periods of the crisis. Market based measures are represented in fig. 5, where the horizontal axis measures vulnerability with LRMES and the vertical axis reports impact calculated with $\Delta CoVaR$. The size of circles represents leverage rate of banks (loans to net worth), while colour reflects net worth. Notice first that banks tend to move towards the right part of the graph on the horizontal axis as time approaches to $t = 585$, but

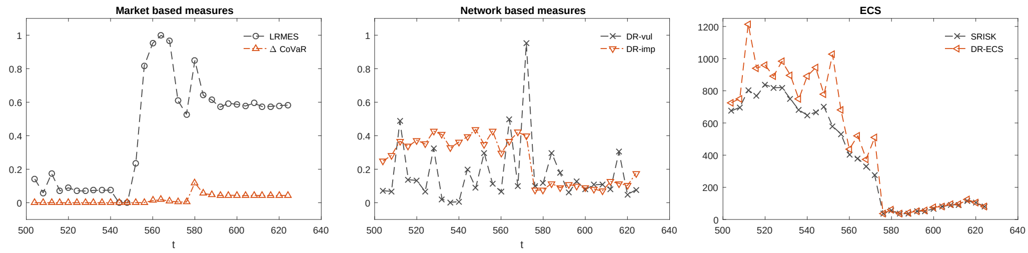


Figure 4: Market-based measured of a randomly chosen bank, network based measures of a randomly chosen bank, aggregate expected capital shortage (ECS).

observed impacts are low. $\Delta CoVaR$ remains near 0 until $t = 595$, when the cascade of defaults turns towards the end. During the next periods systemic risk metrics are more heterogeneous, but leverage and net worth are far lower in the aftermath of the crisis.

A different picture of systemic risk is provided in fig. 6, where the impact of bank measured by DR^{imp} stays constant until $t = 590$, while vulnerability increases. In this particular case[9] a large number of banks is systemic from the viewpoint of impact, but their potential could be realized only if they become vulnerable and suffer losses. After the most severe phase of the crisis, network based measures shift towards the left-lower corner, meaning that they are neither vulnerable or dangerous because there is no more capital to be lost.

We compared market and network based measures from a qualitative point of view. We can conclude that market based metrics are quite stable over time and show high persistence, although $\Delta CoVaR$ is pro-cyclical and turns out to be ineffective as an early warning signal to predict the crisis. On the other hand LRMES provides a good description of banks' vulnerabilities but its value does not decline after the crisis. At the opposite network based measures seems to be more precise but display more volatility, as they are sensitive to small changes in the network of exposures. In particular impact is quite steady during the phase of build-up of systemic risk preceding the crisis, while vulnerability increases as capital is lost and agents become more fragile.

9 All combinations with one or more systemically important financial institutions can be generated by the model.

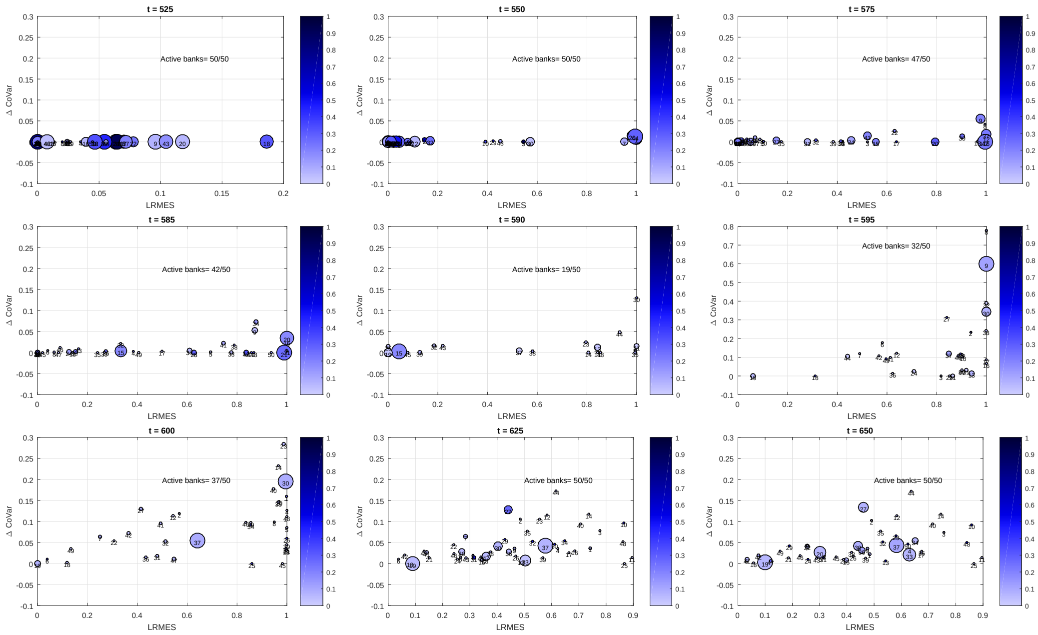


Figure 5: Market based SR risk measures over the business cycle. Circle size represents assets to equity ratio, colour is the normalized equity.

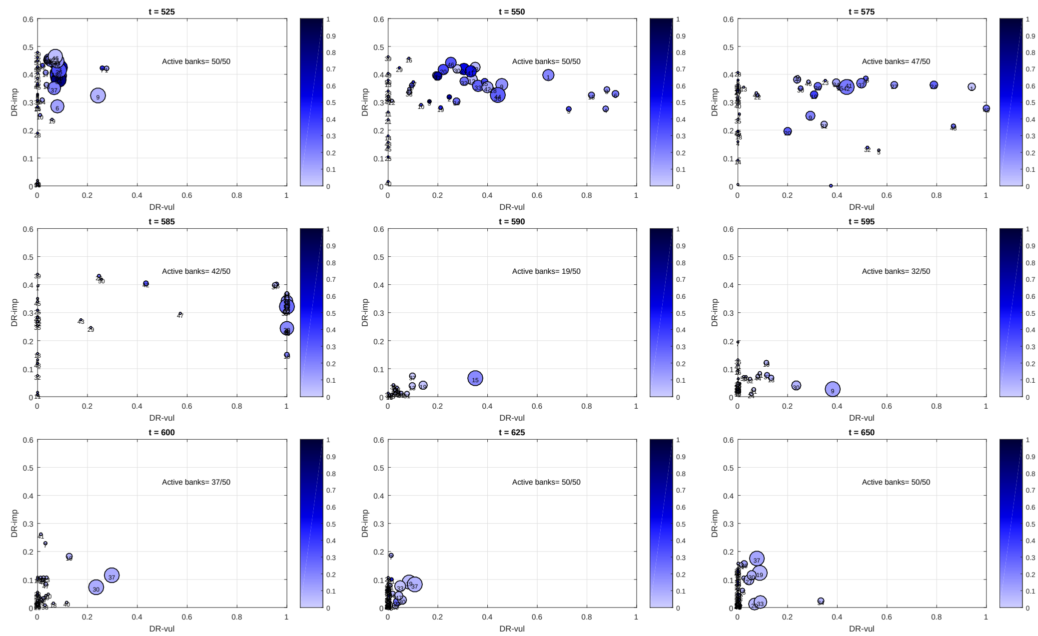


Figure 6: Network based SR risk measures over the business cycle. Circle size represents assets to equity ratio, colour is the normalized equity.

2.4.2 *Correlation analysis*

A desirable property of SR measures would be to be stable over time, at least when considering rank, so that the ranking of the more systemically important financial institutions has not high variability in a given time span and identifies the same set of banks. We study the auto-correlation of SR metrics to understand how stable they are. First we consider a measure of rank correlation (Kendall's tau) and then sample autocorrelation and partial sample autocorrelation.

Kendall's tau (τ^k) is a non-parametric measure of correlation between pairs of ranked variables. If two variables are perfectly correlated $\tau^k = 1$, otherwise if there is no correlation at all $\tau^k = 0$. If one variable is the reverse of the other $\tau^k = -1$.

$$\tau^k = \frac{C - D}{n(n-1)/2}$$

where C and D are the total number of concordant and discordant pairs and n is the sample size.

Moreover when two variables are statistical independent, a z statistics built on τ^k tends to be distributed as a standard normal, therefore it can be tested the null of no correlation versus the alternative of non-zero correlation.

We compute τ^k between the rank of SR measures of each bank and its lagged values over a representative simulation of 2000 periods. Results are reported in Tab. 1. When market-based measures are considered, the ranking has a high autocorrelation, whose persistence is still high after 15 lags, especially for $\Delta CoVaR$. On the other hand network based measures are autocorrelated but to a lower extent. The difference could be explained in terms of construction, as market-based measures are obtained from conditional variances (or conditional VaR), which in turn are estimated through a TGARCH model, where conditional variances are assumed to follow an autoregressive process (see sec. 2.A.2). Conversely network based measures do not assume any dependence on past values, rather they depend on the network structure and credit-debt relationships, so that the outcome of the DebtRank algorithm might change as a result of small variations in network configuration.

Kendall's tau

SR metric	Lags +1	+5	+10	+15
LRMES	0.869	0.676	0.566	0.496
	(0.000)	(0.000)	(0.000)	(0.000)
$\Delta CoVaR$	0.885	0.716	0.615	0.546
	(0.000)	(0.000)	(0.000)	(0.000)
DR-vul	0.664	0.513	0.444	0.384
	(0.000)	(0.000)	(0.000)	(0.000)
DR-imp	0.648	0.518	0.430	0.377
	(0.000)	(0.000)	(0.000)	(0.000)

Table 1: Kendall's correlation coefficients. Reported statistics refers to the average of τ^k computed for each bank. The share of p-values exceeding 0.01 are reported in parenthesis.

Fig.s 7-8 are obtained from the observed values of a randomly chosen bank, nevertheless the autocorrelation function (ACF) and the partial autocorrelation function (PACF) follow the same recurring pattern for all banks. A visual analysis of autocorrelation functions in fig. 7 shows that they are highly autocorrelated and decay at a low rate pointing out that the series are not stationary. The differentiated series displayed in fig. 8 are stationary, however it is not trivial to model their stochastic process from their pattern and estimating a model for the time series would be beyond the aim of the paper. An exception is the DR^{vul} index, whose ACF alternates positive and negative values that die out slowly, while the PACF cuts off after 4 lags. Following the Box-Jenkins method, DR^{vul} is best described by an AR(6) process.

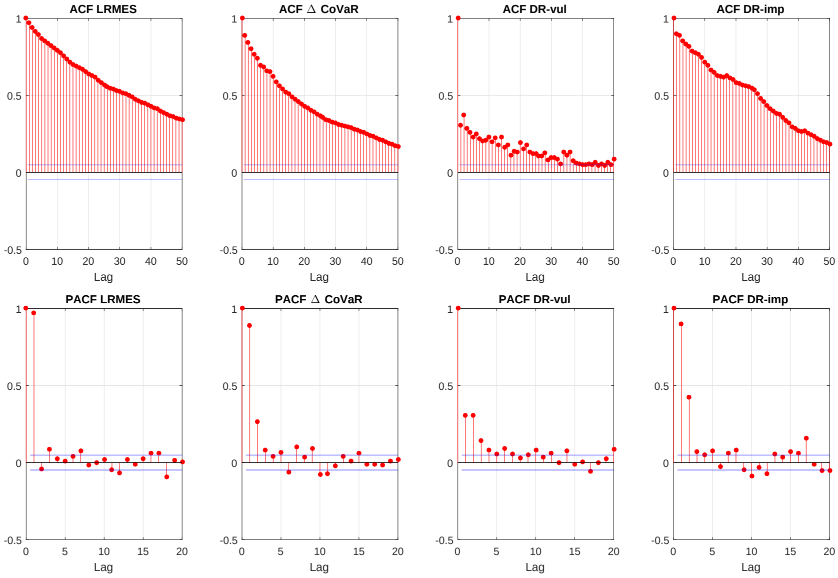


Figure 7: Sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of the time series of systemic risk measures for a randomly chosen bank.

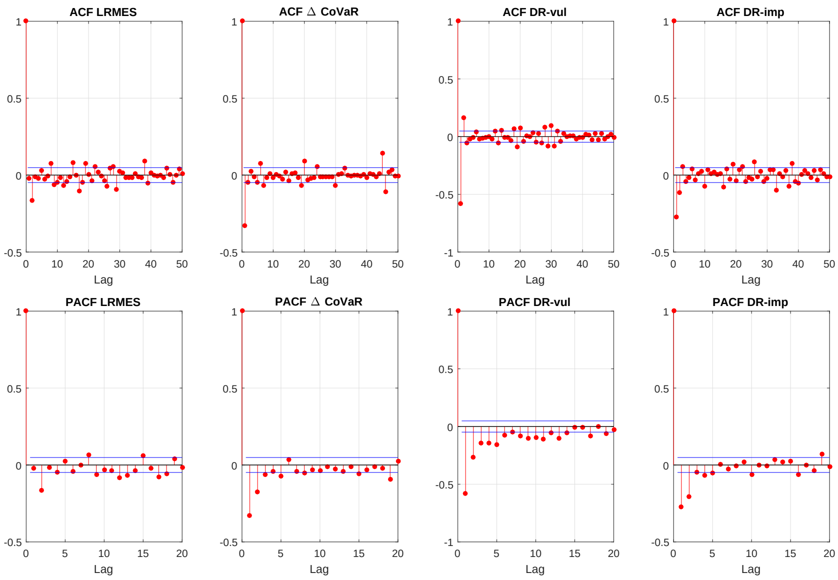


Figure 8: Sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of differenced series of systemic risk measures for a randomly chosen bank.

2.4.3 *Empirical distributions*

We present and compare the distributions of selected variables generated from our model under different settings of macroprudential regulation. A description of such variables can be found in sec. 2.A.3 on page 114. The distributions are obtained by running 50 Monte Carlo simulations of the agent-based-model under each rule, where each simulation net of the transient period has a length of 1000 periods.
Fig.s 9- 13 include a comparison of the effects of minimum capital requirements on selected variables by means of boxplots[10] (upper part) and decumulative distribution functions[11] (lower part). An analysis of periodicity of business cycles is presented in fig. 12.

We assess the effectiveness of macroprudential policies mainly analysing two variables: contagious defaults and unemployment. If systemic capital requirements work properly, contagion should be reduced with respect to the benchmark case. Fig. 9 displays total defaults due to contagion and contagious defaults in firms and banking sectors. We do not include those defaults

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10 The red line in each box represents the sample median, while the blue lines below and above the median are the first and third quartile of the sample. The black lines above and below the box are the whiskers, which extend from the nearest quartile to 1.5 times the interquantile range. Observations above the top whisker (or below the bottom whisker) are outliers, represented by a red cross. Notches display a confidence interval above and below the median

$$median \ \pm 1.57 \times \frac{Inter\ Quantile\ Range}{\sqrt{n}}$$

If the notches of a pair of boxplots do not overlap, we can reject the null that the medians come from the same population with 95% confidence, namely their difference is statistically significant *(McGill et al., 1978)*.

11 The decumulative distribution function (ddf), or survival function is defined as the probability that a variable X takes a value greater than or equal to x:

$$\widehat{F}(X) = prob(X \geq x)$$

By contrast the cumulative distribution function (cdf) indicates the probability that a a variable X takes a value lower or equal than x:

$$F(X) = prob(X \leq x)$$

Then ddf can be written as:

$$\widehat{F}(X) = 1 - F(X)$$

We favor ddf over cdf because it allows a better visualization of the right tail of distributions. To further improve presentation of data we take the log of both axis (Loglog).

induced by negative profits realized on goods market or interest payment in case of banks. Systemic capital requirements are unable to reduce the median number of contagious defaults, indeed notches of boxplots overlap, but rather they reduce the size of tails of distributions. This can be noticed from the lower part of fig. 9, which shows the tails by means of decumulative distribution functions. From this point of view macroprudential policies are successful in reducing systemic events because they limit the extent of default cascades as the right tails of the distributions reveal. This result extends to defaults rates of firms and bank as shown in fig. 10, so macroprudential policies reduce the more extremes values of default rates.

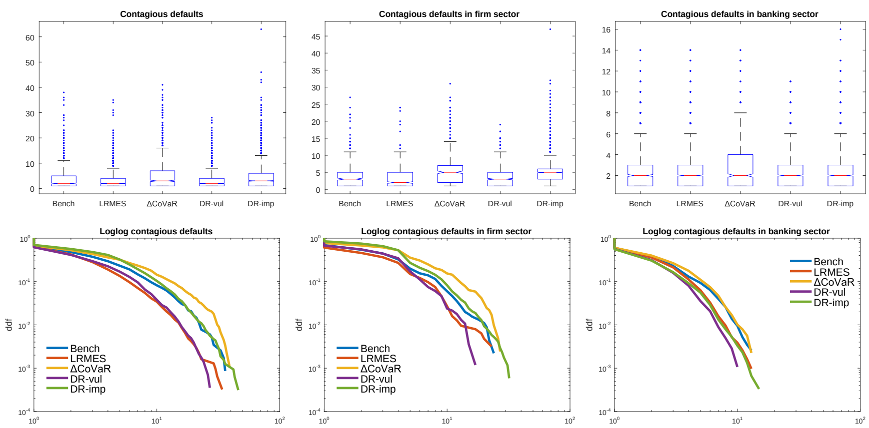

Figure 9: Distribution of contagious defaults of banks and firms for different SR measures.

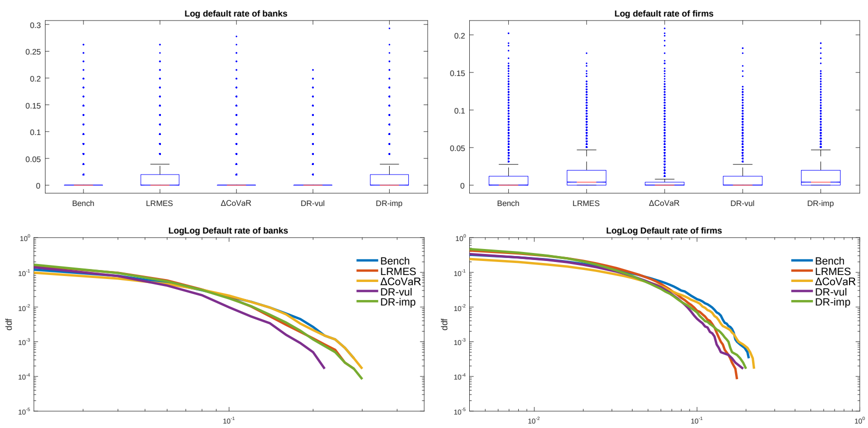

Figure 10: Distribution of default rate of banks and firms for different SR policies.

The analysis of unemployment reveals the effects of capital requirements on business cycles. In the setting of our ABM the production function has only labour input and inventories disappear at the end of each period, hence output corresponds to the number of worked hours. Fig. 11 includes the distribution of the unemployment rate and its duration, defined as the consecutive number of periods in which the unemployment rate exceeds 0.05. The distribution of the unemployment rate is similar under all rules, but if we look at the duration policies built on DR^{vul} and DR^{imp} outperform the others, while the rule based on $\Delta CoVaR$ creates longer periods of unemployment.

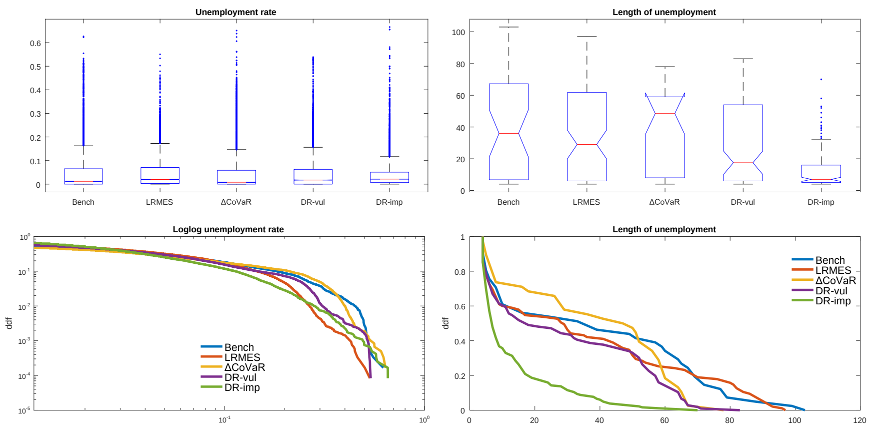

Figure 11: Distribution of unemployment rate, length of unemployment and inflation rate for different SR policies.

Furthermore we evaluate the periodicity of unemployment under all rules. To do that, we resort to autocorrelation of the time series, as the period of the time series of unemployment corresponds to the period of autocorrelation.[12] Qualitative results are presented in fig. 12. The average interval between the peaks of two crises for the first three macroprudential rules is similar to the benchmark case of 290 periods, with a maximum of 289 periods for $LRMES$ and a minimum of 286 for $\Delta CoVaR$. In contrast DR^{vul} has a period of 177 with many more peaks than other cases. Such pattern points out an irregular behaviour where frequency of crises is higher, but the unemployment rate is similar to the benchmark case while the length of each episode is shorter (see fig. 11). Even if absolute values of periods might

12 We consider five time series of 1500 periods, where each one is generated with a different policy rule. First we apply a moving average filter with a time window of 50 periods to smooth the signal and get rid of disturbances, then the autocorrelation function is computed with backward and forward lags. Next we identify peaks on the ACF and periods of occurrence, finally the period is computed as the average between time distances of consecutive pairs of peaks.

partially change for signals generated with different random seeds, this result is robust in terms of ranking.

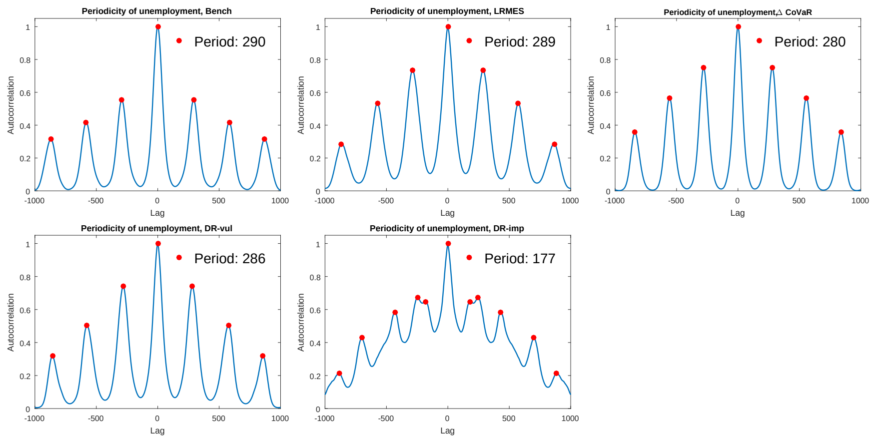

Figure 12: Autocorrelation analysis of periodicity of unemployment for different SR policies.

Fig. 13 displays the distribution of ψ, (see sec. 2.3.2), that is the parameter built with SR metrics which determine capital adequacy ratio of banks. Its distribution reveals how capital requirements are stringent or loose, with $1 \leq \psi \leq 24$, where $\psi = 1$ means that a bank must hold an amount of net worth equal to its risk weighted assets. However what matters for the effectiveness of macroprudential policies is not the value of ψ itself, rather how it is assigned to each bank. Low values of ψ corresponds to higher capital adequacy ratios, but do not automatically reduce systemic risk. In other words if ψ does not reflect the contribution of systemic risk of a bank, minimum capital requirements are not effective. From fig. 13 it turns out that the rule based on DR^{vul} has the highest median value of ψ and it is less stringent than the others.

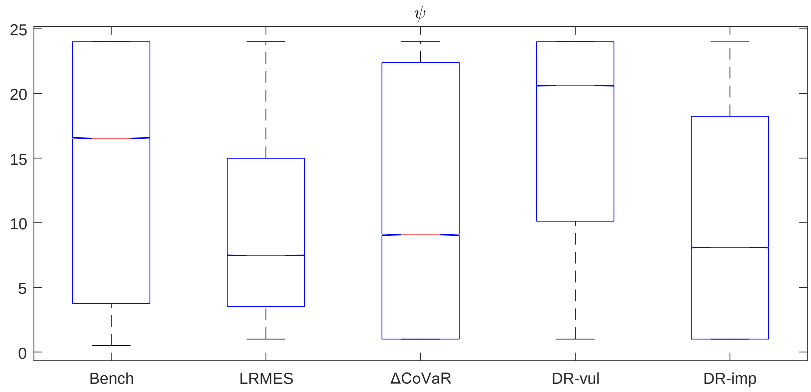

Figure 13: Distribution of maximum regulatory assets to net worth ratio of banks for different SR measures.

In conclusion the results of the comparison between macro-prudential rules point to the rule based on DR^{vul} as the best one for both contagion and unemployment dimensions. If we consider the graphs in this section we can conclude that it is the one which best maps systemic risk into capital requirement: on one hand it reduces contagious defaults and the duration of unemployment without increasing the frequency of crises, on the other it allows relatively loose capital adequacy ratios, thus avoiding excessive credit rationing.

2.5 CONCLUDING REMARKS

We designed policy experiments based on the agent-based model previously developed in order to understand what kind of macro-prudential rule is the most effective to contrast systemic risk and to stabilize the economy. In the present setting banks are subject to minimum capital requirements built on systemic risk measures. Four measures of systemic risk are considered: the first set is composed by two market based measures (*LRMES* and Δ*CoVaR*), while the second one includes network based indexes (*DR-vul* and *DR-imp*). In turn each set contains a measure of vulnerability, which states how much a financial institution is vulnerable to an adverse shock and one measure of impact, which accounts for the effects of the distress of a bank on the financial system. Capital surcharges are obtained with a simple rule by which required capital is proportional to the contribution of each bank to expected capital shortage.

In sec. 2.4 we analysed and compared macroprudential rules to detect those rules which are better at reducing systemic risk without worsening macroeconomic performance. We find that network based measures are more sensitive to the build-up of systemic risk, but they are much more volatile than market based measures. Moreover $\Delta CoVaR$ turns out to be pro-cyclical, that is it measures systemic importance of banks during or immediately after a crisis, hence it is unable to anticipate the extent to which a bank can be systemic. Correlation analysis reveals that all measures are highly time-persistent as they show autocorrelation for many lags. A similar result holds when we consider the autocorrelation of ranks by means of Kendall's tau. In that case ranking of financial institutions does not vanish in few periods but shows a lasting behaviour, meaning that ranks of systemic importance of banks do not reverse from one period to the other. Finally we compared the empirical distributions of selected variables generated by the ABM. In order to assess the performance of systemic capital requirements, we focus on two variables: contagious defaults and unemployment. The first reflects how macroprudential rules are good in avoiding cascades of defaults in a systemic perspective. The second one describes the macroeconomic performance of the economy under different capital requirements. We find that policies based on vulnerability perform better than the other set. In particular the rule derived from DR^{vul} beats that based on $LRMES$. It reduces both contagious defaults and the length of unemployment, without increasing the frequency of cycles. Moreover it is the rule that better maps systemic risk into capital requirements because it does not impose stringent capital adequacy ratios to banks and at the same time performs better than others with respect to cascades of defaults and unemployment.

We hypothesize two reasons to explain why systemic risk measures based on vulnerability are better than metrics accounting for impact. Firstly the policy rule derived from impact might be too naive. It assigns capital requirements based on relative values of impact, so even in tranquil periods the bank which has the highest value of impact, though a moderate one in absolute terms, is subject to the maximum capital adequacy ratio. This might lead to biased capital requirements, as a consequence the policy is ineffective in reducing systemic risk. A workaround would be to assign capital requirements respecting the ranking determined by SR measures but in a way that minimizes expected aggregate losses (or capital shortage). This implies the minimization of a multivariate loss function with respect to the share of contribution to systemic risk of each bank by means of computational techniques (for instance simulated annealing). However such approach combined with the calculation of DR^{imp} or $\Delta CoVaR$ in each period of a multi-agent model would be too much computationally expensive, especially because it would require to repeat the simulation until the algorithm converges to a global optimum. The second reason is that regulating financial institutions relying only on their impacts might be misleading: even if it avoids the default of systemic banks, it does not account for low-impact but vulnerable financial institutions that could be systemic as an herd, triggering the default of systemically important banks.

This paper could be further extended in several directions. One out of many would be to determine prudential policies based both on vulnerability and impact, with a holistic approach which looks beyond the interest of single financial institutions. Furthermore our concept of systemic risk is mainly related to banks. Although the network approach includes also firms in the financial network, measures of vulnerability and impact are only referred to banks. Firms might be systemically important as well, at least from the point of view of credit-debt relationships with banks. In addition we ignored some features that characterize the firm sector, such as trade-credit, stock-market or over-the-counter derivatives. The macro-prudential polices should keep into account the financial network as a whole without excluding any of the relevant agents.

BIBLIOGRAPHY

Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *The American Economic Review*, 102(3):59–64.

Acharya, V. V. and Merrouche, O. (2010). Precautionary hoarding of liquidity and inter-bank markets: Evidence from the sub-prime crisis. Technical report, National Bureau of Economic Research.

Adrian, T. and Brunnermeier, M. K. (2016). Covar. *The American Economic Review*, 106(7):1705–1741.

Aldasoro, I. and Alves, I. (2016). Multiplex interbank networks and systemic importance an application to european data. *Journal of Financial Stability*.

Allen, F. and Carletti, E. (2008). The role of liquidity in financial crises.

Allen, F., Carletti, E., and Gale, D. (2009). Interbank market liquidity and central bank intervention. *Journal of Monetary Economics*, 56(5):639–652.

Alter, A., Craig, B. R., and Raupach, P. (2014). Centrality-based capital allocations.

Ashraf, Q., Gershman, B., and Howitt, P. (2017). Banks, market organization, and macroeconomic performance: An agent-based computational analysis. *Journal of Economic Behavior & Organization*, 135:143–180.

Assenza, T., Delli Gatti, D., and Grazzini, J. (2015). Emergent dynamics of a macroeconomic agent based model with capital and credit. *Journal of Economic Dynamics and Control*, 50:5–28.

Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. *science*, 286(5439):509–512.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS one*, 10(6):e0130406.

Bardoscia, M., Caccioli, F., Perotti, J. I., Vivaldo, G., and Caldarelli, G. (2016). Distress propagation in complex networks: the case of non-linear debtrank. *PloS one*, 11(10):e0163825.

Battiston, S., Caldarelli, G., D'Errico, M., and Gurciullo, S. (2016). Leveraging the network: a stress-test framework based on debtrank. *Statistics & Risk Modeling*, 33(3-4):117–138.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? financial networks, the fed and systemic risk. *Scientific reports*, 2:541.

Bénassy, J.-P. (2002). *The macroeconomics of imperfect competition and nonclearing markets: a dynamic general equilibrium approach*.

Benoit, S., Colletaz, G., Hurlin, C., and Pérignon, C. (2013). A theoretical and empirical comparison of systemic risk measures.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Borio, C., Furfine, C., Lowe, P., et al. (2001). Procyclicality of the financial system and financial stability: issues and policy options. *BIS papers*, 1:1–57.

Brownlees, C. T. and Engle, R. (2012). Volatility, correlation and tails for systemic risk measurement. *Available at SSRN*, 1611229.

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *The American Economic Review*, 104(2):379–421.

Caiani, A., Godin, A., Caverzasi, E., Gallegati, M., Kinsella, S., and Stiglitz, J. E. (2015). Agent based-stock flow consistent macroeconomics: Towards a benchmark model. *Available at SSRN*.

Chiarella, C., Flaschel, P., Hartmann, F., and Proaño, C. R. (2012). Stock market booms, endogenous credit creation and the implications of broad and narrow banking for macroeconomic stability. *Journal of Economic Behavior & Organization*, 83(3):410–423.

Chiarella, C., Iori, G., et al. (2002). A simulation analysis of the microstructure of double auction markets*. *Quantitative finance*, 2(5):346–353.

Chiarella, C., Iori, G., and Perelló, J. (2009). The impact of heterogeneous trading rules on the limit order book and order flows. *Journal of Economic Dynamics and Control*, 33(3):525–537.

Claessens, S., Kose, M. A., and Terrones, M. E. (2009). What happens during recessions, crunches and busts? *Economic Policy*, 24(60):653–700.

Danielsson, J., Valenzuela, M., and Zer, I. (2016). Learning from history: volatility and financial crises.

De Masi, G. and Gallegati, M. (2012). Bank–firms topology in italy. *Empirical Economics*, 43(2):851–866.

Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P., and Gallegati, M. (2011). *Macroeconomics from the Bottom-up*. Milano: Springer Milan.

Delli Gatti, D., Gallegati, M., Greenwald, B., Russo, A., and Stiglitz, J. E. (2010). The financial accelerator in an evolving credit network. *Journal of Economic Dynamics and Control*, 34(9):1627–1650.

Delli Gatti, D., Gallegati, M., Greenwald, B. C., Russo, A., and Stiglitz, J. E. (2009). Business fluctuations and bankruptcy avalanches in an evolving network economy. *Journal of Economic Interaction and Coordination*, 4(2):195–212.

Dosi, G., Fagiolo, G., Napoletano, M., and Roventini, A. (2013). Income distribution, credit and fiscal policies in an agent-based keynesian model. *Journal of Economic Dynamics and Control*, 37(8):1598–1625.

Dosi, G., Fagiolo, G., and Roventini, A. (2010). Schumpeter meeting keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, 34(9):1748–1767.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Fischer, E. O., Heinkel, R., and Zechner, J. (1989). Dynamic capital structure choice: Theory and tests. *The Journal of Finance*, 44(1):19–40.

Flannery, M. J. and Rangan, K. P. (2006). Partial adjustment toward target capital structures. *Journal of Financial Economics*, 79(3):469–506.

Gabbi, G., Iori, G., Jafarey, S., and Porter, J. (2015). Financial regulations and bank credit to the real economy. *Journal of Economic Dynamics and Control*, 50:117–143.

Gai, P. and Kapadia, S. (2010). Liquidity hoarding, network externalities, and interbank market collapse. In *Proc. R. Soc. A*, volume 466, page 439.

Gauthier, C., Lehar, A., and Souissi, M. (2010). Macroprudential regulation and systemic capital requirements. Technical report, Bank of Canada Working Paper.

Georg, C.-P. (2013). The effect of the interbank network structure on contagion and common shocks. *Journal of Banking & Finance*, 37(7):2216–2228.

Godley, W., L. M. (2007). Monetary economics: an integrated approach to credit, money, income production and wealth.

Grilli, R., Tedeschi, G., and Gallegati, M. (2014). Bank interlinkages and macroeconomic stability. *International Review of Economics & Finance*, 34:72–88.

Heid, F. (2007). The cyclical effects of the basel ii capital requirements. *Journal of Banking & Finance*, 31(12):3885–3900.

Heider, F., Hoerova, M., and Holthausen, C. (2009). Liquidity hoarding and interbank market spreads: The role of counterparty risk.

Hicks, J. R. et al. (1974). Crisis in keynesian economics.

Hommes, C. H. (2006). Heterogeneous agent models in economics and finance. *Handbook of computational economics*, 2:1109–1186.

Iori, G., De Masi, G., Precup, O. V., Gabbi, G., and Caldarelli, G. (2008). A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278.

Iori, G., Jafarey, S., and Padilla, F. G. (2006). Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, 61(4):525–542.

Jordà, Ò., Schularick, M. H., and Taylor, A. M. (2011). When credit bites back: leverage, business cycles, and crises. Technical report, National Bureau of Economic Research.

Kaufman, G. G. and Scott, K. E. (2003). What is systemic risk, and do bank regulators retard or contribute to it? *The Independent Review*, 7(3):371–391.

Kleinow, J., Moreira, F., Strobl, S., and Vähämaa, S. (2017). Measuring systemic risk: A comparison of alternative market-based approaches. *Finance Research Letters*, 21:40–46.

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50.

Lilit, P., Napoletano, M., and Roventini, A. (2016). Taming macroeconomic instability: Monetary and macro prudential policy interactions in an agent-based model. *Available at SSRN 2710131*.

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52:A11–A23.

McGill, R., Tukey, J. W., and Larsen, W. A. (1978). Variations of box plots. *The American Statistician*, 32(1):12–16.

Modigliani, F. and Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data. *Franco Modigliani*, 1.

Montagna, M. and Kok, C. (2013). Multi-layered interbank model for assessing systemic risk. Technical report, Kiel Working Paper.

Pankoke, D. (2014). Sophisticated vs. Simple Systemic Risk Measures. Working Papers on Finance 1422, University of St. Gallen, School of Finance.

Poledna, S., Molina-Borboa, J. L., Martínez-Jaramillo, S., Van Der Leij, M., and Thurner, S. (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability*, 20:70–81.

Poledna, S. and Thurner, S. (2016). Elimination of systemic risk in financial networks by means of a systemic risk transaction tax. *Quantitative Finance*, 16(10):1599–1613.

Rabemananjara, R. and Zakoian, J.-M. (1993). Threshold arch models and asymmetries in volatility. *Journal of Applied Econometrics*, 8(1):31–49.

Riccetti, L., Russo, A., and Gallegati, M. (2013). Leveraged network-based financial accelerator. *Journal of Economic Dynamics and Control*, 37(8):1626–1640.

Riccetti, L., Russo, A., and Gallegati, M. (2015). An agent based decentralized matching macroeconomic model. *Journal of Economic Interaction and Coordination*, 10(2):305–332.

Rodríguez-Moreno, M. and Peña, J. I. (2013). Systemic risk measures: The simpler the better? *Journal of Banking & Finance*, 37(6):1817–1831.

Silva, T. C., da Silva, M. A., Tabak, B. M., et al. (2016). Modeling financial networks: a feedback approach. Technical report.

Tarashev, N. A., Borio, C. E., and Tsatsaronis, K. (2010). Attributing systemic risk to individual institutions.

Tedeschi, G., Mazloumian, A., Gallegati, M., and Helbing, D. (2012). Bankruptcy cascades in interbank markets. *PloS one*, 7(12):e52749.

Tesfatsion, L. (2003). Agent-based computational economics: Modeling economies as complex adaptive systems. *Information Sciences*, 149(4):263–269.

Thurner, S. and Poledna, S. (2013). Debtrank-transparency: Controlling systemic risk in financial networks. *arXiv preprint arXiv:1301.6115*.

van der Hoog, S. and Dawid, H. (2015). Bubbles, crashes and the financial cycle: Insights from a stock-flow consistent agent-based macroeconomic model.

Webber, L. and Willison, M. (2011). Systemic capital requirements.

Werner, R. A. (2012). Towards a new research programme on 'banking and the economy'—implications of the quantity theory of credit for the prevention and resolution of banking and debt crises. *International Review of Financial Analysis*, 25:1–17.

2.A APPENDIX

2.A.1 *Structure of the financial network*

Most of the existing network analyses consider financial firms operating in a *monoplex* network, for instance overnight transactions. However recent contributions *(Aldasoro and Alves, 2016; Poledna et al., 2015; Silva et al., 2016)* stress the importance of considering multilayer networks in order to assess the true magnitude of systemic risk. From this perspective banks operate in more than one market, and their relationships could be represented with two or more layers. Although our model is not characterized by a multi-layer network, banks operates in more than one dimension. Such structure can be represented by a (non) bi-partite network[13], that describes the relationships of banks with firms and other banks on the credit and interbank markets. In our simplified framework banks form a credit/debt relationships. In detail banks exchange funds in the interbank market, while they extend loans to firms in the credit market. Firms (and households) hold their deposits on banks accounts. A brief representation of the layered structure of the financial network is in fig. 14, whose links indicates the potential channels for contagion.

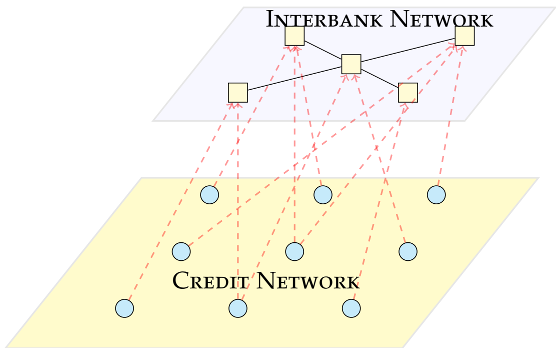

Figure 14: Structure of the financial network: in the top layer there are banks, whose links describe the interbank market. The nodes in the bottom layer represents firms, whose links towards banks depict the credit network. The deposits network is not displayed.

13 Technically there cannot be cycles in a bi-partite network, since no links are present within the same group of nodes. We refer to the graph as a non-bipartite.

2.A.2 Methods

DebtRank

We employ a differential version of the DebtRank algorithm in order to provide a network measure of systemic risk. Differential DebtRank *(Bardoscia et al., 2015)* is a generalization of the original DebtRank *(Battiston et al., 2012)* which improves the latter by allowing agents to transmit distress more than once. Moreover our formulation has similarities with *Battiston et al. (2016)*, where it is assumed a sequential process of distress propagation. In our case we first impose an external shock on firms' assets, then we sequentially account for the propagation to the banking sector through insolvencies on loans, to the interbank network and to firms' deposits.

The relative equity loss for banks (h) and firms (f) is defined as the change in their net worth (respectively nw^B, and nw^F) from $\tau = 0$ to τ with respect to their initial net worth. In particular the initial relative equity loss of firms happens at $\tau = 1$ due to an external shock on deposits:

$$h_i(\tau) = \min\left[\frac{nw_i^B(0) - nw_i^B(\tau)}{nw_i^B(0)}\right]$$
$$f_j(\tau) = \min\left[\frac{nw_j^F(0) - nw_j^F(\tau)}{nw_j^F(0)}\right]$$

The dynamics of the relative equity loss in firms and banks sectors is described by the sequence:

- Shock on deposits in the firms sector:

$$f_j(1) = \min\left[1,\ \frac{D_j^F(0) - D_j^F(1)}{nw_j^F(0)}\right] = \min\left[1,\ \frac{loss_j(1)}{nw_j^F(0)}\right]$$

- Banks' losses on firms' loans:

$$h_i(\tau+1) = \min\left[1,\ h_i(\tau) + \sum_{j \in J} \Lambda_{ij}^{fb}(1 - \varphi_j^{loan})(p_j(\tau) - p_j(\tau - 1))\right]$$

- Banks' losses on interbank loans:

$$h_i(\tau+1) = \min\left[1,\ h_i(\tau) + \sum_{k \in K} \Lambda_{ik}^{bb}(1 - \varphi_k^{ib})(p_k(\tau) - p_k(\tau - 1))\right]$$

- Firms' losses on deposits:

$$f_j(\tau+1) = \min\left[1, f_j(\tau) + \Lambda_{jk}^{fb}(1 - \varphi_k^{dep})(p_k(\tau) - p_k(\tau - 1))\right]$$

Where p_j is the default probability of debtor j and $\varphi^i,\ i = \{loan, ib, dep\}$ is the recovery rate on loans, interbank loans and deposits. Recovery rates on each kind of assets are randomly extracted from a vector of observations generated by the benchmark model.

For the sake of simplicity we can define it as linear in f_j (h_k for banks), so that $p_j(\tau) = h(\tau)$ [14]. Λ is the exposure matrix that represents credit/debt relationships in the firms-banks network. It is written as a block matrix, where Λ^{bb} refers to the interbank market, Λ^{bf} refers to deposits, Λ^{fb} refers to firm loans and Λ^{ff} is a matrix of zeros.

$$\Lambda = \begin{bmatrix} \Lambda^{bb} & \Lambda^{bf} \\ \Lambda^{fb} & \Lambda^{ff} \end{bmatrix}$$

The exposure matrix Λ represents potential losses over equity related to each asset at the beginning of the cycle, where each element has the value of assets at the numerator and the denominator is the net worth of the related creditor. in our specification firms have no intra-sector links, hence $\Lambda^{ff} = 0$. In case there are $N^b = 2$ banks and $N^f = 3$ firms, the matrix Λ looks like:

$$\Lambda = \begin{bmatrix} 0 & \frac{Ib_{12}}{nw_2^B} & \frac{D_{13}}{nw_1^F} & \frac{D_{12}}{nw_2^F} & \frac{D_{15}}{nw_3^F} \\ \frac{Ib_{21}}{nw_1^B} & 0 & \frac{D_{23}}{nw_1^F} & \frac{D_{24}}{nw_2^F} & \frac{D_{25}}{nw_3^F} \\ \frac{L^f_{31}}{nw_1^B} & \frac{L^f_{32}}{nw_2^B} & 0 & 0 & 0 \\ \frac{L^f_{41}}{nw_1^B} & \frac{L^f_{42}}{nw_2^B} & 0 & 0 & 0 \\ \frac{L^f_{51}}{nw_1^B} & \frac{L^f_{52}}{nw_2^B} & 0 & 0 & 0 \end{bmatrix}$$

In order to obtain VaR and ES we introduce an unbounded relative equity loss $\hat{h}$, whose upper bound at 1 has been removed to account for effective equity losses. This index runs in parallel with h, and it is determined by the same variables (except $\hat{h}_i(\tau)$).

$$\hat{h}_i(\tau+1) = \hat{h}_i(\tau) + \sum_{j \in J} \Lambda^{fb}_{ij} (p_j(\tau) - p_j(\tau-1))$$

Given the definition of $\hat{h}_{i,\tau} = \frac{nw^B_{i,\tau} - nw^B_{i,0}}{nw^B_{i,0}}$, we obtain the expected shortfall of relative equity loss (ES^{ind}) by means of the stress test described in sec. **??** on page ??. In turn ES^{ind} is defined as the arithmetic mean of all the observation exceeding the VaR at 95%.

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14 In a more realistic setting the default probability could be written as

$$p_j(\tau) = f_j(\tau) \exp(\alpha(h_j(\tau)) - 1)$$

where if $\alpha = 0$ it corresponds to the linear DebtRank, while if $\alpha \to \infty$ it is the Furfine algorithm *(Bardoscia et al., 2016)*. Moreover we can assume that deposits are not marked-to-market, but they respond to the Furfine algorithm, in other words the distress propagates only in case of default of the debtor. For deposits it might be reasonable to assume

$$p_j^D(\tau-1) = \begin{cases} 1 & if\ h_k(\tau-1) = 1 \\ 0 & otherwise \end{cases}$$

The change in equity of bank i is equivalent to the change in its assets, assuming that liabilities are constant.

$$\hat{h}^b_{i,\tau} \equiv \frac{nw^B_{i,0} - nw^B_{i,\tau}}{nw^B_{i,0}} = \frac{\mathcal{A}_{i,0} - \mathcal{A}_{i,\tau}}{nw^B_{i,0}}$$
$$\frac{\hat{h}^b_{i,\tau} nw^B_{i,0}}{\mathcal{A}_{i,0}} = \frac{\mathcal{A}_{i,0} - \mathcal{A}_{i,\tau}}{\mathcal{A}_{i,0}}$$

Given that $nw^B_{i,0}$ and $\mathcal{A}_{i,0}$ are initial values which does not change during the stress tests, we can define the expected shortfall in terms of relative asset loss as

$$ES^{\mathcal{A}}_{i,t} = \frac{ES^{ind}_{i,t} nw^B_{i,0}}{\mathcal{A}_{i,0}}$$

The two variables are similar, however $ES^{\mathcal{A}}$ has values in $[0, 1]$ because banks cannot loose more than their assets[15], while ES^{ind} is contained between 0 and the realized rate of leverage of banks.

SRISK

SRISK *(Brownlees and Engle, 2012)* is a widespread measure of systemic risk based on the idea that the latter arises when the financial system as a whole is under-capitalized, leading to externalities for the real sector. To apply the measure to our model we follow the approach of *Brownlees and Engle (2012)*. The SRISK of a financial firm i is defined as the quantity of capital needed to re-capitalize a bank conditional to a systemic crisis

$$SRISK_{i,t} = \min\left[0,\ \frac{1}{\lambda}\mathcal{L}_i - \left(1 - \frac{1}{\lambda}\right) nw^B_{i,t}(1 - MES^{Sys}_{i,t+h|t})\right]$$

where $MES^{Sys}_{i,t+h|t} = E\left(r_{i,t+h|t}|r < \Omega\right)$ is the tail expectation of the firm equity returns conditional on a systemic event, that happens when i's equity returns r from $t-h$ to t are less than a threshold value Ω.

Acharya et al. (2012) propose to approximate MES^{Sys} with its *Long Run Marginal Expected Shortfall* (LRMES), defined as a

$$LRMES_{i,t} = 1 - \exp\{-18MES^{2\%}_{i,t}\}$$

LRMES represents the expected loss on equity value in case the market return drops by 40% over the next six months. Such approximation is obtained through extreme value theory, by means of the value of MES that would be if the daily market return drops by -2%[16].

The bivariate process driving firms' (r_i) and market (r_m) returns is

15 Our version of DR does not include fire sales

16 As an alternative, V-lab computes it as $1 - exp(\log(1-d)\beta)$, where d is the six-month crisis threshold for the market index decline, its default value is 40% while and β is the firm's CAPM. Such specification allow to obtain LRMES for thresholds different from 40%.

$$
\begin{aligned}
r_{m,t} &= \sigma_{m,t}\epsilon_{m,t} \\
r_{m,t} &= \sigma_{i,t}\rho_{i,t}\epsilon_{m,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}\xi i,t \\
(\xi_{i,t}, \epsilon_{m,t}) &\sim F
\end{aligned}
$$

where $\sigma_{m,t}$ is the conditional standard deviation of market returns, $\sigma_{i,t}$ is the conditional standard deviation of firms' returns, $\rho_{i,t}$ is the conditional market/firm correlation and ϵ and ξ are i.i.d. shocks with unit variance and zero covariance ϵ and ξ are i.i.d. shocks with unit variance and zero covariance.

$MES^{2\%}$ is expressed setting $\Omega = -2\%$:

$$
MES_{i,t-1}^{\Omega} = \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\epsilon_{m,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right) + \sigma_{it}\sqrt{1-\rho_{i,t}^2}E_{t-1}\left(\xi_{i,t}|\epsilon_{m,t} < \frac{\Omega}{\sigma_{m,t}}\right)
$$

Conditional variances $\sigma_{m,t}^2$, $\sigma_{i,t}^2$ are modelled with a TGARCH model from the GARCH family *(Rabemananjara and Zakoian, 1993)*. Such specification captures the tendency of volatility to increase more when there are bad news:

$$
\begin{aligned}
\sigma_{m,t}^2 &= \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 I_{m,t-1}^- + \beta_m \sigma_{m,t-1}^2 \\
\sigma_{i,t}^2 &= \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 I_{i,t-1}^- + \beta_i \sigma_{i,t-1}^2
\end{aligned}
$$

$I_{m,t}^- = 1$ if $r_{m,t} < 0$ and $I_{i,t}^- = 1$ when $r_{i,t} < 0$, 0 otherwise.

Conditional correlation ρ is estimated by means of a symmetric DCC model *(Engle, 2002)*. Moreover to obtain the *MES* it is necessary to estimate tail expectations. This is performed with a non-parametric kernel estimation method *(see Brownlees and Engle, 2012)*.

Open-source Matlab code is available thanks to *Sylvain Benoit*, and *Gilbert Colletaz*, *Christophe Hurlin*, who developed it in *Benoit et al. (2013)*.

ΔCoVaR

Following *Adrian and Brunnermeier (2016)* $\Delta CoVaR$ is estimated through a quantile regression (Koenker and Bassett Jr (1978)) on the α^{th} quantile, where r_{sys} and r_i are respectively market-wide returns on equity and bank i's returns. Quantile regression estimates the α^{th} percentile of the distribution of the independent variable given the regressors, rather than the mean of the distribution of the dependent variable as in standard OLS regressions. This allows to compare how different quantiles of the dependent variables might affect the regressand, hence it is suitable to analyse tail events. While *Adrian and Brunnermeier (2016)* employ an estimator based on an augmented regression, we further simplify the estimation of $\Delta CoVaR$ following the approach in *Benoit et al. (2013)*, which is consistent with the original formulation.

First we regress individual returns on market returns:

$$
r_{sys,t} = \gamma_1 + \gamma_2 r_{i,t} + \varepsilon_{\alpha,t}^{sys|i}
$$

The estimated coefficients (denoted by $\widehat{\ }$) are employed to build CoVaR. The conditional VaR of bank i ($Var^i_{\alpha,t}$) is obtained from the quasi maximum likelihood estimates of conditional variance generated by the same TGARCH model described above *(see Benoit et al., 2013, p.38)*.

$$CoVar^{sys|i}_{\alpha,t} = \widehat{\gamma}_1 + \widehat{\gamma}_2 Var^i_{\alpha,t}$$

Finally $\Delta CoVar$ is obtained from the difference between the α^{th} and the median quantile of $CoVar$.

$$\begin{aligned}
\Delta CoVar^{sys|i}_{\alpha,t} &= CoVar^{sys|i}_{\alpha,t} - CoVar^{sys|i}_{0.5,t} \\
\Delta CoVar^{sys|i}_{\alpha,t} &= \hat{\gamma}_2 \left(VaR^i_{\alpha,t} - VaR^i_{0.5,t} \right)
\end{aligned}$$

2.A.3 *Description of variables*

A description of the variables represented in sec. 2.4 is provided below.

Default rates Number of defaults per period of firms (banks) over total number of firms (banks).

Credit rationing and interbank rationing Computed as the average per period of $\frac{loan\ demand\ -\ obtained\ loan}{loan\ demand}$ for each agent (firms or banks).

Credit volume and interbank volume Total outstanding stock of credit to firms or interbank funds per period.

Liquidity hoarding Share of disposable interbank liquidity (I^s) retained by lenders over their total liquidity:

$$1 - \frac{I^s}{liq}$$

Liquidity hoarding is obtained as the average of individual hoarding per period.

Leverage of banks Weighted average of $\frac{\text{stock of loans}}{\text{net worth}}$ of each bank in each period. Weights are given by individual stock of loans (both to firms and banks).

Unemployment rate Number of worked hours over maximum amount of hours.

Lenght of unemployment Length of periods of sustained unemployment. It is computed as the length of periods with unemployment greater than 5%, lasting at least three periods.

Contagious defaults Total number of defaults of firms and banks less those triggered by negative profits.

3 DISCUSSION

This final chapter of the dissertation provides a brief overview and a discussion of the most relevant results, so that the reader can better understand the contributions of this research work and its limitations.

Chapter 1 can be considered the most important part of the thesis, thus the discussion of its results absorbs the majority of this section. It required a great effort to build and code an ABM describing a credit economy, where several classes of agents interact with each other giving rise to a complex macro-dynamics. To our knowledge it is one of the first attempts to study liquidity hoarding of banks in a full-fledged agent-based macroeconomic model. The interbank market has an important role in redistributing liquidity in the banking sector, which in turn promotes the access to credit by non-financial firms. Although liquidity hoarding behaviour might not be relevant during good times, it turns out to be one of the key drivers of financial crises during turbulent times when the majority of banks is concerned about excessive liquidity outflows. We find that during recessions liquidity hoarding behaviour of financial sound banks works as an amplification channel of systemic risk. Those banks with disposable liquidity are both concerned about counterparty risk and excessive cash outflows, hence their supply of interbank funds is strongly revised downwards. This result relates to the literature on liquidity hoarding, for instance *Acharya and Merrouche (2010)* provide evidence that liquidity hoarding was precautionary in nature during the crisis in 2007. Although this finding is supported by empirical evidence, our formulation of interbank supply does not allow to decompose precautionary reasons between counterparty risk *(Heid, 2007)* and uncertainty about liquidity volatility *(Allen et al., 2009)*.

Failing to retrieve cheap funds on the interbank market, banks in need of liquidity are forced to borrow from the CB's standing facility at a penalty rate, which increases their probability of default, causes a rise of interest rates to firms through the cost of funds and limits the supply of credit to the real sector. This result is in line with our expectations and it is a consequence of the model assumptions. In particular we suppose that the cost of illiquidity is represented by the penalty rate at which rationed banks borrow from the CB, rather than imposing the bankrupt of illiquid institutions, as it is often assumed in other models. It turns out to be an effective choice, because it allows the transmission of interbank distress to the rest of the economy in several ways: the cost of funds soars, transmitting the distress to the real sector through interest rate; banks with a structural shortage of liquidity are forced to pay an high price for CB's advances, eventually causing their failure which contributes to worsen the crisis; the reduction in aggregate credit supply resulting from the impossibility to lend for banks that have been rationed on the interbank market affects the real sector, limiting firms to hire part of their labour demand. This intensifies unemployment and depresses aggregate demand. On the other hand our assumptions do not take into account that borrowing from CB's standing facility should be associated with the '*stigma*' of the peers, guided by the idea that the inability to obtain interbank liquidity is a signal that a bank is riskier than the other, hence it is not considered a reliable counterpart. Moreover the same '*stigma*' effect might lead a bank to build its own buffer of liquidity during periods of uncertainty, thus eliminating the event of borrowing from the CB, but also reducing the available interbank liquidity for other banks. Another drawback is that CB's standing facility assures any bank against the risk of being illiquid, irrespective to the behaviour of banks. This could give rise to a moral hazard in bank risk taking.

Broadening the picture, liquidity hoarding can be attributed to prudential regulation of banks, which turns out to be pro-cyclical. Imposing a counter-cyclical capital buffer could improve financial stability. In our model banks fail to internalize the consequences of their actions, in other words prudential measures of banks only consider them in isolation from the whole financial system, thus ignoring the systemic effects of their behaviour. Excess leverage and liquidity hoarding are the realization of such behaviour, by which the rational choices from a single-oriented perspective are at odds with aggregate welfare. However in our model the major reason for pro-cyclicality is given by banks' expectations, that are based on *Expected Shortfall* computed on historical losses. In this setting banks do not learn how to avoid negative outcomes, but just rely on past observations to take their choices for the future. Despite in the real world banks employ sophisticated models to protect against risk, *Borio et al. (2001)* claims that the financial system is pro-cyclical, namely it amplifies the swings of the real economy. In light of this, adaptive expectations are a consistent way to model pro-cyclical behaviour observed in the financial sector.

It is widely recognised that the transition phase between a boom and a recession along the business cycle is amplified by the financial accelerator *(Bernanke et al., 1999)*. An acceleration mechanism also operates in our setting: it contributes to deepen the crisis with higher interest rates and the reduction of credit to the real sector (and interbank funds). Firms become more fragile and output is reduced, thus cascades of defaults and rising unemployment contribute to lower aggregate demand, accelerating the dynamics of the recession. Our contribution is tightly related to the '*network based financial accelerator*' *(Delli Gatti et al., 2010)*, where an initial shock is amplified through the interest rate channel. When the net worth of lenders is reduced by insolvencies on loans, they charge higher interest rates increasing the financial fragility of borrowers. If the leverage rate of borrowers rise, lenders will charge even higher interest rates. The process might continue until borrowers are not able to service their debt, generating defaults that further worsen the fragility of agents and lead to default avalanches that involve the entire economy. In *(Delli Gatti et al., 2010)* the financial accelerator mechanism is implemented assuming that lenders charge a rate decreasing with their net worth and increasing with leverage rates of borrowers. Our framework is similar, but this mechanism is extended to the interbank market too and involves supply of credit and interbank funds. Rather than considering the net worth of lenders, we consider a measure of perceived risk based on historical losses, *i.e.* expected shortfall. Another important difference is that we assume that firms have heterogeneous but constant leverage rates, hence the change in interest rates to firms is only determined by lenders' conditions. As a result, the acceleration mechanism is slower on the credit network, nonetheless it is effective. In addition it should exhibit more persistence since the expected shortfall of banks changes slowly as *de-facto* it is a moving average of losses over the last n periods. Anyway our model could be improved by endogenising the leverage rate of firms, in order to obtain a financial accelerator both on credit and interbank networks.

In the last part of Chapter 1 we conduct an artificial experiment changing the connectivity on the interbank market, to understand what are the effects on liquidity hoarding. We find that more connectivity is beneficial for the circulation of liquidity in the banking sector and promotes access to credit by firms. Furthermore it improves the macroeconomic performance with lower unemployment (higher output). On the other hand it entails growing levels of liquidity hoarding, which are justified by higher perceived risk due to the greater fragility of the banking sector with respect to firms. This result is relevant since during a crisis the liquidity in the financial sector might not be distributed to banks in need, but rather it could be hoarded for precautionary reasons. This finding is a novelty in the literature, however we should be careful about its significance and support it with empirical research. Liquidity hoarding raises as a consequence of two factors: a more competitive goods market and an increase in the exposure of banks to firms, with reduced diversification of loans.

The first reasons seems fair, as enhancing access to credit allows firms to produce according to their plans. In the goods market there is a reduction of profit (and losses) margins arising from the need to compete with other firms. As a result the number of defaults increases, but losses are smaller. At the end banks suffer more defaults, even if total losses are lower. However the origin of losses shifts from illiquidity (negative profits) to contagion from firms, while the reduction of losses triggered on the interbank market is not relevant. In this case greater liquidity hoarding can be attributed to the way risk is perceived by banks, that is a function of losses on loans (to firms or banks) portfolio. If expected shortfall was also accounting for losses due to negative profits, results would change. Two modifications could improve our model: the first would be to separate risk measures derived from losses on loans to firms and banks, so that prudential measures of banks can vary depending on which market is considered. The second one is to introduce another measure accounting for the risk of illiquidity, which increases when a bank is rationed on the interbank market or with deposit volatility.

The second reason is a consequence of our starting assumptions. In particular the borrowers per lender ratio increases because firms can switch to a new lender if the interest rate is lower. If some frictions were introduced, the number of lender would not be subject to a significant change. For example a sticky partner selection mechanism or a low connected topology of the credit network would achieve this aim. Turning to the number of loans per period measured by the average out-degree, the increase in connectivity has a counter-intuitive result, that is to reduce diversification on the credit market. This largely descend by the reduction of firms with multiple loans, because more banks provide credit in each period not being rationed on the interbank market. Augmenting the density of the credit network would reduce multiple loans even at lower levels of interbank connectivity. At the opposite the introduction of a credit rationing mechanism aimed at limiting the amount of credit lent to each borrower can increase risk diversification on the credit network, so the raise of out-degree would not be robust to the aforementioned changes. Our conjecture is that the first of the two reasons has the stronger effect on liquidity hoarding, because losses are the main cause of increases in ES. A comparison between the two reasons above would reveal which one has the stronger effect on liquidity hoarding.

The last remark concerning Chapter 1 is about interbank contagion. According to the related literature, in general growth of connectivity yields to a greater number of defaults from direct exposures *(Iori et al., 2006; Tedeschi et al., 2012)*, since balance sheet contagion prevails on risk diversification. This also occurs in our model at the lower levels of connectivity but the effect of diversification prevails over contagion beyond a given degree, thus defaults and losses triggered by insolvencies on interbank loans decrease. The increasing pattern of defaults that can be observed growing the connectivity until a certain degree is a well known behaviour and it is in accordance with the literature. Moreover empirical studies report low levels of density in interbank networks, therefore our outcomes are consistent with the literature when the density of the network is low. On the other hand a further growth of connectivity leads to an opposite behaviour, that is a decrease of contagious defaults or losses. The overall dynamics suggests that the effects of connectivity are non-linear: at the starting degrees contagion prevails over diversification, next it occurs the opposite. This result can hardly be empirically validated, above all because high levels of connectivity are only observed in the core of interbank networks. Such theoretical finding entails some thoughts: first, it is confirmed the non-linear effect caused by the increase of the connectivity level, albeit in this case the pattern of defaults and losses looks like a curve with an inflection point. If the result was confirmed, policies aimed at a large increase of density on the interbank network would be beneficial by reducing the likelihood of default cascades. Second, the banks in the core and those in the periphery should be subject to different prudential policies, as in one case the transmission of the disease would be more dangerous. Moreover the interaction of these two groups of banks should be further investigated from a regulatory viewpoint.
When diversification prevails over contagion we attribute the pattern of defaults and losses to the limited overall exposures within the banking sector. The size of losses deriving from failures on the interbank market is not big enough to shock the capital of banks and give rise to systemic events. We identified several reasons for limited systemic effects: one is a matter of timing. A consistent part of interbank trades is overnight, but we assume that banks access to the interbank market just one time per period, thus reducing the exposures between banks. Moreover interbank loans last one period, hence banks cannot be borrowers and lenders at the same time. Another reason is that the number of banks is limited to 50 for computational reasons, while a larger number might enhance the magnitude of contagion. Banks trade only one kind of assets on the interbank market, whereas interbank networks are typically multi-layered. In addition there are not intermediaries or settlement banks that connect nodes without direct connections. Of course the presence of intermediary banks could broaden the extent of contagion.

The second chapter of this dissertation contains a comparison of two sets of macroprudential capital requirements by means of policy experiments. The first one is built on market-based measures, the other one on network based metrics. A further distinction regards what do measures account for: vulnerability or impact. The aim of policy experiments is to establish whether capital requirements derived from Systemic Risk (SR) measures are better than the benchmark specification of the model and to explain which one performs better and why.

Results reward the policy rule derived from the vulnerability network-based measure (DR^{vul}), that is the one which exploits the DebtRank algorithm to assess the vulnerability of banks. The second best solution is the rule based on $LRMES$. In short policies based on vulnerability perform better than those based on impact.

The best ranking of DR^{vul} could be firstly explained because it is a model-based measure. In other words the DebtRank algorithm takes advantage of the same model employed for simulations to determine vulnerability or impact. Of course this permits to represent the systemic importance of agents with an high degree of accuracy. In our case this seems to provide an advantage to network-based measures, at least if we compare each one with the corresponding market-based measure. However our insights should not be taken as literal, but should be tested on data. When the underlying model is simple, the default cascades can be well replicated by DebtRank, which provides the best description of systemic risk. At the opposite, when analysing complex models or data with multiple balance sheet exposures or non trivial correlation in asset prices, it might be convenient to adopt a statistical approach with market-based measures. If we reverse these considerations we could provide an explanation of why market-based measures are the second-best choice. The model only includes few assets which are not marked-to-market, so it appears too simple in order to be accurately described by market-based-measures. In conclusion the best performance of network-based policy rules could be explained by the structure of the model: when it is simple, DebtRank provides a very good description of systemic risk, but if it is intricate then policies built on market-based measure could better capture the variability and correlation of data.

Minimum capital requirements derived from measures of impact do not achieve the same performance of their alternatives. This result is puzzling because a capital regulation based on impact should reduce the systemic effect deriving from the defaults of those institutions that could generate the larger losses in the system. We hypothesized two reasons to explain it. The first one is strictly connected to the way we construct minimum capital requirements. We determine them depending on the relative values of impact. This poses an issue, because when the absolute value of impact is low, those banks with the higher relative impact are asked a disproportionate contribution. If expected capital shortfall is high but absolute impacts are low, the most systemic banks in relative terms must contribute much more than the other by raising their own capital. This would be serious issue when the system is very capitalized with banks showing high vulnerabilities and low impacts. The literature includes several approaches to derive systemic capital requirements. The most similar contribution are *Alter et al. (2014); Webber and Willison (2011)*. In *Alter et al. (2014)* for instance capital is reallocated within the banking sector in order to minimize contagion effects according to network based measures of systemic risk. These authors consider a banking system, abstracting from the rest of the economy, where capital is reallocated in order to minimize systemic losses. Despite this approach is effective in reducing systemic risk, it can unlikely be adopted by a central regulator because it is not acceptable to take capital away from a bank and to redistribute it to other institutions without violating property rights. This is the reason why we let banks adjust their capital rather than redistribute it. The approach in *Webber and Willison (2011)* is similar from a conceptual viewpoint, although reallocation is obtained by a more elaborate process. It involves a constrained minimization problem to determine the minimum level of capital reaching a given constraints in terms of VaR, where the shares of capital of banks are adjusted during the algorithm to reach the best allocation. This approach can certainly determine the best allocation of capital within the banking sector, however the same considerations about redistribution hold. Furthermore it has an high degree of computational intensity since the algorithm calculates the VaR of the system for each possible capital allocation. It appears difficult to embed it in our agent-based model, as the number of operations and consequently the elapsed time would increase exponentially.

In the second place capital requirements purely derived from impact measures should capture the risk of '*systemic as a herd*' institutions. Impact accounts for the effect of default (or distress) of a bank on the rest of the system, thus ignoring its own degree of vulnerability. Thus under impact-based capital requirements it might happen that a group of low-impact but high-vulnerable banks is subject to loose capital regulation. If such group of banks is exposed to the same risk factor, it might be '*systemic as a herd*', thus spreading the disease to the entire economy. Of course DR^{imp} does not account for this dimension of systemic risk, but $\Delta CoVaR$ of *Adrian and Brunnermeier (2016)* should in principle capture it, however it suffers of contemporaneity so that it should be substituted with forward $\Delta CoVaR$. The latter is obtained by projecting on the regressors of $\Delta CoVaR$ their estimated coefficients, where the independent variables include individual banks' characteristics and macro-state variables (see sec. 2.4.3). However such a predictive index cannot be effectively computed from our model, because of the strong dependency of forward $\Delta CoVaR$ on banks' VaR.

In conclusion macroprudential polices could be more effective if they rely on measures that include both impact and vulnerability. Financial institutions are characterized at the same time by two dimension of risk: impact and vulnerability. Regulation should account for both dimensions, otherwise banks would be regulated irrespective of one risk dimension. The very last thought of this chapter is dedicated to macroeconomics. We used to consider systemic risk as an issue regarding only the financial sector represented by financial institutions. However the complex structure of the economic system is not limited to them, but it includes households and firms among the other, besides the balance sheets of agents are highly interrelated. In such a macro-environment firms can be systemically important, or for instance households might behave as an herd. From this perspective it would be desirable for macroprudential regulation to adopt a holistic approach, in which the economic system is viewed as a complex organism whose parts cannot be fully understood in isolation from the rest.