Journal of Economic Behavior and Organization 220 (2024) 279–304

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Research Paper

Conformism, social pressure, and the dynamics of integration ☆

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ARTICLE INFO

JEL classification:
D83
D85
J15
Z13

Keywords:
Integration
Networks
Peer pressure
Policies

ABSTRACT

We consider a model in which each individual belonging to an ethnic minority group is embedded in a network of relationships and decides whether she wants to be integrated in the society. Each individual wants her behavior to agree with her personal ideal preference for integration but also wants her behavior to be as close as possible to the average integration behavior of her peers. We show that there is always convergence to a steady-state and characterize it. We also show that different preferences for integration may emerge in steady state depending on the structure of the network. Then, we consider an optimal tax/subsidy policy which aim is to reach a certain level of integration in the population.

1. Introduction

In his book, *Assimilation, American Style*, Salins (1997) argues that an implicit contract has historically defined integration in America. As he puts it: "Immigrants would be welcome as full members in the American family if they agreed to abide by three simple precepts: First, they had to accept English as the national language. Second, they were expected to live by what is commonly referred to as the Protestant work ethic (to be self-reliant, hardworking, and morally upright). Third, they were expected to take pride in their American identity and believe in America's liberal democratic and egalitarian principles." Though hardly exhaustive, these three criteria certainly get at what most Americans consider essential to successful integration.

The same issues have been discussed and debated in Europe, especially over recent decades. According to the 2016 Eurostat statistics, 20.7 million people with non-EU citizenship are residing in the European Union. Additionally, 16 million EU citizens live outside their country of origin in a different Member State. Migration movements are on the rise both within and from outside the European Union.

The key to ensuring the best possible outcomes for both the migrants and the host countries (both in the European Union and the United States) is their successful integration into the host country.[1] However, integration is often fraught with tension, competition,

☆ We are grateful to the editor Friederike Mengel and three anonymous referees for very helpful comments. We also thank Sergio Currarini, Pietro Dindo, Carlos Bethencourt Marrero, Benjamin Golub, Theodoros Rapanos, Sebastiano Della Lena, and Thierry Verdier for their comments. The financial support from the Australian Research Council Discovery Project (DP200102547) is gratefully acknowledged by Yves Zenou.

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[1] Different countries have different views on the integration of immigrants. Certain countries, such as France, consider it to be a successful integration policy when immigrants leave their cultural background and are "assimilated" into the new culture. Other countries, such that the United Kingdom, consider that a successful

https://doi.org/10.1016/j.jebo.2024.02.021
Received 2 October 2022; Received in revised form 6 February 2024; Accepted 12 February 2024

Available online 22 February 2024

and conflict. There is strong evidence showing that family, peers, and communities shape the individual integration preferences and, therefore, affect integration decisions (see e.g., Bisin et al., 2016). In particular, there may be a conflict between an individual's integration choice and that of her peers and between an individual's integration choice and that of her family and community. For example, an ethnic minority may be torn between speaking one language at home and another at work.

In this paper, we study how these conflicting choices affect the long-run integration behaviors of ethnic minorities and how policies and economic incentives can affect these integration decisions.

We consider a model where each individual belonging to an ethnic minority group is embedded in a *directed* network of relationships. Each agent decides how much she wants to integrate to the majority group. At time t, this decision, denoted by action x_i^t for individual i, is continuous and is between 0 (no integration at all) and 1 (total integration). Each individual i wants to minimize the distance between x_i^t and the average integration choice of individual i's direct peers (that is, i's social pressure) but also between x_i^t and i's preference for integration at time t, denoted by s_i^t. At time $t+1$, the latter is determined for each individual i by a convex combination of x_i^t and s_i^t. In this framework, we study the dynamics of both integration choice x_i^t and preference for integration s_i^t and their steady-state values. Basically, this is a coordination game with myopic best-reply dynamics where, in each period, the agents choose their best responses to last-period actions.

The utility function of each individual captures both their desire to conform to the average action of their peers (that is, *social pressure*) and to their own *preference for integration*. To study the dynamics of s_i^t, we show that it suffices to look at the (row-normalized) adjacency matrix of the network even though the process of integration is described by a more general matrix. We also show that, independently of the network structure, convergence always occurs. We characterize the steady-state preferences for integration and show that they depend on the initial preference (at time $t = 0$) and on the individual position in the network. In particular, we demonstrate that, if an individual belongs to a closed communication class,[2] then her steady-state integration preference will be a weighted combination of the initial preferences of all agents belonging to the same communication class, where the weights are determined by the eigenvector centrality of each agent. If an agent does not belong to any closed communication class, her steady-state integration preference will be determined by a weighted combination of the steady-state integration preferences of the agents in the closed communication classes for which she has links or paths to.

We can explain why individuals from the same ethnic group can choose to adopt oppositional identities, that is, some integrate to the host country while others do not.[3] Indeed, some individuals may not be integrated in the host country because they live in a closed community formed of other people that interact just within the community itself—thus, not favoring integration—and because some of its members (for example, cultural leaders) have a strong influence on the group (Verdier and Zenou, 2018). In the terminology of our model, these are individuals who have a key position in the network—that is, those who have a high eigenvector centrality. On the contrary, other individuals from the same ethnic group may want to be integrated because their communities are not isolated or, if they are isolated, the social pressure is in favor of integration. Our model shows that these two types of behaviors can arise endogeneously in the steady-equilibrium, even for ex-ante identical individuals—that is, individuals with exactly the same characteristics. The key determinant of these integration choices is their position in the network and the initial preferences of the persons they are connected to.

We show that the steady-state integration actions of individuals are equal to their steady-state social pressure, so that for each agent, the distance between her action and that of her peers is equal to zero and the distance between her action and her own preference for integration is also equal to zero. This implies that total welfare is maximized in the steady-state equilibrium. This is not true at any period t outside of the steady state because individuals choose actions different from that of their peers.

Then, we generalize our utility function by introducing material (economic) incentives for integration, which are defined as the marginal benefits of exerting action x_i^t. We propose a new way of calculating the steady-state social pressure and actions and show that they do not depend on initial preferences but on the ex-ante heterogeneity of each agent, their taste for conformity, and their position in the network.

Finally, we derive some policy implications of our model with this generalized utility function. We determine the optimal tax/subsidy that needs to be given to each agent in order to reach a certain degree of long-run integration. For example, if the objective is that all ethnic minorities much reach an integration level of at least 50 percent, then we are able to calculate the level of tax/subsidy given to each agent; they depend on the marginal benefits of integration (material incentives) and the individual position in the network.

1.1. Related literature

We now relate our paper to the literature on the role of networks in the integration of ethnic minorities.[4]

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integration policy is that immigrants can keep their original culture while also accepting the new culture (or at least not rejecting it). In this paper, we will focus on the role of "integration" in the "integration" of ethnic minorities. However, integration can be defined as in Salins (1997) or in a broader way such as, for example, by the economic success of the individuals. For example, some groups such as the Chinese in the United States or in Europe, because of their economic success, can be considered as "assimilated" even if they do not interact too much with people from the majority culture.

2 For a definition of closed communication class and for other definitions and results about network and linear algebra, see Appendices B and C.

3 Studies in the United States have found that African American students in poor areas may be ambivalent about learning standard English and performing well at school because this may be regarded as "acting white" and adopting mainstream identities (Fordham and Ogbu, 1986; Wilson, 1987; Delpit, 1995; Ogbu, 1997; Ainsworth-Darnell and Downey, 1998; Fryer and Torelli, 2010; Patacchini and Zenou, 2016).

4 For overviews on the economics of networks, see Jackson (2008), Benhabib et al. (2011), Ioannides (2012), Jackson (2014), Jackson and Zenou (2015), Bramoullé et al. (2016) and Jackson et al. (2017).

The integration of minorities in a given country is an important research topic in social sciences, in general, and in economics, in particular (see e.g. Brubaker, 2001; Schalk-Soekar et al., 2004; Kahanec and Zimmermann, 2011; Algan et al., 2012). The standard explanations of whether immigrants integrate to the host country are because of parents' preferences for cultural traits (Bisin and Verdier, 2000), ethnic and cultural distance to the host country (Alba and Nee, 1997; Bisin et al., 2008), previous educational background (Borjas, 1985), country of origin (Beenstock et al., 2010; Borjas, 1987; Chiswick and Miller, 2011), and discrimination against immigrants (Alba and Nee, 1997).

However, despite strong empirical evidence, very few papers have studied the explicit impact of social networks and social pressure on the integration of ethnic minorities.

From an *empirical* viewpoint, there is a literature that looks at the impact of social networks on the integration choices of immigrants and ethnic minorities. To capture network effects, most economic studies have adopted ethnic concentration/enclave as the proxy for networks of immigrants in the host country (e.g., Damm, 2009; Edin et al., 2003). Other studies have used language group or language proficiency (Bertrand et al., 2000; Chiswick and Miller, 2002). The effects on integration are mixed. For example, Bertrand et al. (2000) and Chiswick and Miller (2002) showed that linguistic concentration negatively influenced immigrants' labor market performance in the US. In contrast, Edin et al. (2003) find that by correcting for the endogeneity of ethnic concentration, immigrants' earnings in Sweden were positively correlated with the size of ethnic concentration. Similar results were obtained by Damm (2009) for Denmark and Wang et al. (2021) for Australia.

From a *theoretical* viewpoint, there is a growing literature that shows that the integration of immigrants has been impacted by their cultural identity and cultural transmission (Bisin and Verdier, 2000, 2001; Akerlof and Kranton, 2010; Benabou and Tirole, 2011; Panebianco, 2014; Panebianco and Verdier, 2017; Bisin et al., 2016; Carvalho, 2016; Kim and Loury, 2019; Della Lena and Panebianco, 2021; Munoz-Herrera, 2021), their community cohesion, social interactions and social norms, and social networks (Mengel, 2008; Buechel et al., 2014, 2015; Moizeau, 2015; Bezin and Moizeau, 2017; Verdier and Zenou, 2017; Stark et al., 2018; Sato and Zenou, 2020; Anufriev et al., 2023; Boucher et al., 2021; Bhowmik and Sen, 2022; Itoh et al., 2022).

Our model is quite different to these papers because we show how social pressure (via network), initial inclinations (preferences, personal patterns), and material incentives are key determinants of the integration process of ethnic minorities. In particular, individuals with strong integration preferences at home may end up not being integrated because of their "isolated" position in the network while other minorities, belonging to closed-knit networks with social pressure in favor of integration, may end up being integrated even though their initial preferences were not in favor of integration. More generally, our model highlights the role of communities, ex ante heterogeneity, and parental influence (through their impact on children's early preferences) in the long-run integration of ethnic minorities.

The rest of the paper unfolds as follows. In the next section, we describe the benchmark model. In Section 3, we analyze the model when there are no material incentives for integration while, in Section 4, we study the model where agents are ex ante heterogeneous. In Section 5, we examine the policy implications of the model. Finally, Section 6 concludes. We provide all the proofs of the results in the main text in Appendix A. Appendices B and C provide definitions and results about linear algebra, convergence, and network topology.

We have created a not-for-publication Online Appendix. In the Online Appendix A, we provide some examples illustrating our main results while the Online Appendix B gives a detailed analysis of the speed of convergence in the benchmark model.

2. The integration choice and the dynamics of preferences

Consider a set I composed of n agents belonging to an ethnic minority, i.e., $I = \{1, ..., n\}$. Each agent $i \in I$ is born with an initial individual preference for integration, denoted as $s_i^0 \in [0, 1]$. This preference is influenced by the agent's idiosyncratic characteristics as well as the cultural and ethnic values passed down by their family, such as the original language and customs. Throughout the paper, we will illustrate our model and results with the language example in which the ethnic minority's mother tongue is different from the language spoken in the host country and integration is measured by the child's fluency in the host country's language.[5] While the language choice can serve as a measure of integration, the parents can transfer to the child a given practice of the language that shapes the initial preferences s_i^0. For example, the parents may choose to speak the language of the majority at home to foster an initial integration preference for their child or may choose to speak their native language, which may lead to a lower level of initial integration s_i^0.

The initial individual preference s_i^0 evolves over time through a process that we describe below and that will be endogenous with the social interactions and past choices and preferences. We refer to the personal preference of agent i at time t as $s_i^t \in [0, 1]$. Each agent $i \in I$ also has some idiosyncratic material incentives to make a specific integration choice. Using the language example, this illustrates how much it pays to be more or less proficient in the language of the majority. Subsequently, the agent is exposed to a group of peers who face the same decision.

At each time $t \in \mathbb{N}$, each individual $i \in I$ simultaneously chooses an action $x_i^t \in [0, 1]$, referred to as the *integration choice*.[6] This action represents the degree of integration of the agent into the majority culture of the host country. If $x_i^t = 0$, the individual chooses

[5] Language proficiency is a key driver of immigrant integration as it increases job opportunities and facilitates social and political participation. See e.g., Pont-Grau et al. (2015) and Foged and van der Werf (2023).

[6] Our framework can be extended without any change to $s_i^t \in X \subseteq \mathbb{R}$ with X being a compact interval, and $x_i^t \in X$. Notice that actual actions and individual preferences share the same support.

not to be integrated at all—for example, they only speak their native language. Conversely, if $x_i^t = 1$, the agent chooses complete integration—for example, they only speak the majority language. Thus, any choice $x_i^t \in (0, 1)$ corresponds to different integration levels—this may represent the fraction of time spent speaking the majority language.

Thus, the integration choice x_i^t of individual i is influenced by three competing motives. First, she wants to align her behavior with her individual preference for integration at time t, denoted by s_i^t, thereby seeking *consistency* between her own preferences s_i^t and her behavior x_i^t. Second, she also aims to align her choice with that of *social pressure*, defined as the average integration choices of her peers,[7] thereby demonstrating a *conformist* behavior. Last, she is driven by idiosyncratic *material incentives* for integration, which are influenced by her characteristics. At each time $t \in \mathbb{N}$, let $\mathbf{x}^t := (x_i^t)_{i\in I}$ and $\mathbf{s}^t := (s_i^t)_{i\in I}$. The timing of the model is as follows:

1. At the beginning of each period $t \in \mathbb{N}$, the vector of agents' personal preferences for integration is given by $\mathbf{s}^t$;
2. Agents make their integration choices $\mathbf{x}^t$. This choice is a response to simultaneous observations of the (weighted) actions of all other network members with whom individual i has a connection to;
3. At the end of period t, each agent $i \in I$ updates her individual preferences for integration, which leads to $\mathbf{s}^{t+1}$;
4. The process starts again.

Each agent $i \in I$ is embedded in a social network $\mathbf{g}$, which is characterized by a row-normalized or stochastic adjacency matrix $\mathbf{G}$, where $g_{ij} > 0$ if and only if i assigns a positive weight to j, and $g_{ij} = 0$ otherwise; moreover, the row-normalization of the matrix implies that the sum of each row is equal to 1. In other words, we consider a *directed* weighted network where links are weighted *outdegrees*. There are no self-loops, i.e., $g_{ii} = 0$. Because the network is *directed*, we do not impose any symmetry on the links in the network so that we allow for $g_{ij} \neq g_{ji}$. This simply means that any two agents can give different weights to each other. This allows for $g_{ij} > 0$ and $g_{ji} = 0$. For the rest of the paper, we assume $\mathbf{G}$ to be diagonalizable. We call N_i the set of out-neighbors of i—that is $N_i := \{j \in I \mid g_{ij} > 0\}$. The average action taken by i's neighbors (or peers) at time t is given by $\sum_{j\in N_i} g_{ij} x_j^t$.

To produce results comparable to the ones obtained in the standard literature in network games (Jackson and Zenou, 2015), we assume that agents observe the network and the activity in the network when they choose their actions.[8]

Consider a vector $\boldsymbol{\alpha} \in \mathbb{R}_+^I$. Then, each individual i at time t chooses x_i^t that maximizes the following utility function:

$$u_i^t(\mathbf{x}^t, s_i^t, \mathbf{G}) = \underbrace{\mathbb{1} \cdot \left(2\alpha_i x_i^t - \left(x_i^t\right)^2\right)}_{\text{Material incentives}} \underbrace{- \omega(x_i^t - \sum_j g_{ij} x_j^t)^2}_{\text{Conformism}} \underbrace{- (x_i^t - s_i^t)^2}_{\text{Consistency}}, \tag{1}$$

where $\omega > 0$.[9] The first term in the utility function represents agent i's *material incentives*. Let $\mathbb{1}$ be an indicator function that takes a value of 1 when agents have material incentives and 0 otherwise. Material incentives are generally crucial for integration choices as we will show in Section 4. However, we find it useful to first analyze the individual integration choice without material incentives (Section 3) as it allows for cleaner characterization of some key results. Observe that α_i combines two influences of individual i: personal characteristics (height, race, etc.) and material incentives (e.g., wages) that apply to such characteristics. To simplify the presentation, we refer to the first term in the utility function as material incentives for agent i. In particular, a higher value of $\alpha_i > 0$ indicates a greater individual marginal utility for taking action x_i.[10] Thus, the coefficient α_i represents the *economic incentives* for integration and can be influenced by targeted policies aiming to make integration more or less advantageous for ethnic minorities (as we will show in Section 5). In the language example, α_i could represent the wage premium of being fluent in the language of the majority group.[11] We assume the vector $\boldsymbol{\alpha}$ to be time-independent.

The second term on the right-hand side of equation (1) represents the social interaction or conformism part. It reflects the desire of each individual i to minimize the discrepancy between her integration choice x_i^t and the average integration choice of their peers $\sum_{j\in N_i} g_{ij} x_j^t$; the latter is referred to as *social pressure*. The utility loss incurred by not conforming to the average choice of her peers is given by $-\omega\left(x_i^t - \sum_{j\in N_i} g_{ij} x_j^t\right)^2$, where ω is her taste for *conformity*. This formulation is consistent with the standard approach used by economists to model conformity behaviors in social networks (Patacchini and Zenou, 2012; Liu et al., 2014; Blume et al., 2015; Boucher and Fortin, 2016; Boucher, 2016; Calabuig et al., 2018; Ushchev and Zenou, 2020; Boucher et al., 2024).

7 In terms of network, peers are defined as the outdegrees of each individual. This is without loss of generality. All results go through if peers are defined as indegrees or the network is undirected.

8 This assumption is commonly made in models using Nash equilibrium as an equilibrium concept. In our case, it can be viewed as a convenient way of capturing the convergence of behavior under less restrictive conditions than full observability of agents' actions. Specifically, we can consider a scenario where agents only observe the actions of their neighbors or just have some conjecture about them, and update these conjectures according to some feedback they receive from the model—for example, the past neighbors' actions. With strategic complementarities, if the Nash equilibrium of the game is unique (as it is usually the case in network games with local complementarities under some eigenvalue condition; see e.g., Jackson and Zenou, 2015), any adaptive learning process based on such conjectures converges to the unique Nash equilibrium of the game with full information. See, in particular, Battigalli et al. (2023), which study self-confirming equilibrium in network games.

9 Without loss of generality, we assume that material incentives and consistency have the same weight in the utility function (1).

10 Note that we use $2\alpha_i$ instead of α_i for computational convenience, which does not affect the generality of our analysis.

11 See e.g., Li (2013) who shows that speaking fluently the majority language leads to higher wages. See also Chiswick and Miller (2002), Dustmann and Fabbri (2003), and more recently, Foged et al. (2024) who show that language skills positively affect workers' productivity and wages.

The last term of equation (1) captures the agent's desire for *consistency* with their preferences. This implies that each agent cares about identity-driven ideal.[12] Specifically, each agent i derives a personal utility, which is a decreasing function of the distance between her chosen action x_i^t and her preference s_i^t. Observe that the weight each agent places on this part of the payoff is normalized to unity.

More generally, the utility function (1) captures the fact that a child of an immigrant makes integration choices in response to the incentives provided by the host society (captured by the material incentives α_i) and by social pressure (represented by the average choice of their direct friends). However, this choice may conflict with that of their parents, who may influence their initial preference s_0^i. In other words, there is a tension between economic incentives, personal preferences, and coordination with peers.

In terms of our language example, ethnic minorities are born with an ethnic language that is different from that of the majority. s_i^t is the initial preference for the majority language (for example, whether the parents speak themselves the majority language) and α_i is the marginal benefits in terms of wage of improving the fluency of the majority language. Social pressure $\sum_{j\in N_i} g_{ij} x_j^t$ is defined as the average fluency level in the majority language by i's peers. Then, individuals have to choose x_i^t their level of fluency in the majority language. If $x_i^t = 0$, then they don't speak at all the majority's language while if $x_i^t = 1$ they speak it fluently (that is, they are as fluent as the majority people). Clearly, an ethnic minority individual may face a dilemma between speaking one language at home and another at work.

We delve into the dynamics of individual preferences and analyze how they are influenced by individuals' integration choices and the network structure. Two crucial assumptions are made at this point. First, the network is assumed to be fixed whereas the profiles of integration choices $\mathbf{x}^t$ and individual preferences $\mathbf{s}_i^t$ evolve over time. Second, all individuals are myopic and naive, thereby maximizing their instantaneous utility at time t as described in equation (1).[13]

At each time $t \in \mathbb{N}$, once individual i has chosen own integration level x_i^t, she can update her own preference s_i^t depending on the previous action profile. We have the following dynamic equation:

$$\mathbf{s}^{t+1} = \gamma \underbrace{\mathbf{x}^t}_{\text{Time consistency}} + (1-\gamma) \underbrace{\mathbf{s}^t}_{\text{Anchoring}}, \tag{2}$$

where $\gamma \in (0,1)$ is a parameter γ that quantifies the level of consistency among all agents. This source of preference change is grounded in solid psychological principles and has found extensive application in economics (see, for instance, Akerlof and Dickens, 1982; Kuran and Sandholm, 2008). The dynamic equation (2) states that the evolution of individual preference is a linear convex combination. The first term captures the extent to which each individual is consistent with her own past integration choice while the second term indicates the degree to which she remains anchored to her past preferences.[14]

In terms of language, equation (2) implies that the preference for the majority language in the next period is a convex combination of the current preference for the majority language and the current chosen level of language fluency.

We want to emphasize that our underlying assumption is that the network remains the same during the dynamic process. We are aware that networks naturally evolve over time, especially when considering a long time horizon. However, in our analysis, we do not impose any specific duration for each period. We can envision a scenario where a fixed group of individuals interact multiple times every day. In such cases, each period can be interpreted as a short-term or even instantaneous interaction. Consequently, convergence can occur much more rapidly than any change in the network, thereby minimizing the potential issues associated with our assumption of time-homogeneous network structure. The same reasoning applies when considering any other fixed parameter of our model.

3. No material incentives for integration

In this section, we consider the case in which there are no material incentives, that is $\mathbb{1} = 0$. The utility function in equation (1) is now given by:

$$u_i^t(\mathbf{x}^t, s_i^t, \mathbf{G}) = -\omega \left(x_i^t - \sum_{j\in N_i} g_{ij} x_j^t \right)^2 - \left(x_i^t - s_i^t \right)^2. \tag{3}$$

[12] That individuals derive utility from following their personal ideals has been recognized in other analyzes of social interactions. See, for example, Akerlof (1997), Kuran and Sandholm (2008) and Della Lena and Dindo (2019). Even if these papers bear some similarities with our model, they do not explicitly incorporate a social network.

[13] This assumption is made for tractability purposes and is standard in the DeGroot type of models. Interestingly, there have been empirical studies (field and lab experiments) examining whether agents behave according to the DeGroot model or exhibit more Bayesian behavior. Corazzini et al. (2012), Chandrasekhar et al. (2020), and Grimm and Mengel (2020) demonstrate that individuals tend to behave in accordance with the DeGroot model, thereby suggesting that agents are myopic and possess limited cognitive abilities.

[14] It is worth noting that this concept relates to the literature on self-signaling (Bem, 1972; Benabou and Tirole, 2004, 2006), which posits that agents progressively discover their true norms by observing their own behavior, leading to updates in the direction of past behavior, resulting in the convergence of norms and behavior over time due to the behavior-to-norm force.

3.1. The integration choice

At each period of time t, each agent i chooses x_i^t that maximizes equation (3). By solving for the first-order condition, we obtain:

$$x_i^t = \left(\frac{1}{1+\omega}\right) s_i^t + \left(\frac{\omega}{1+\omega}\right) \sum_{j\in N_i} g_{ij} x_j^t. \tag{4}$$

In vector-matrix form, we have:

$$\mathbf{x}^t = \left(\frac{1}{1+\omega}\right) \mathbf{s}^t + \left(\frac{\omega}{1+\omega}\right) \mathbf{G}\mathbf{x}^t.$$

By setting $\theta := \omega/(1+\omega)$ and solving this equation, we obtain:

$$\mathbf{x}^t = (1-\theta)\,[\mathbf{I}-\theta\,\mathbf{G}]^{-1}\,\mathbf{s}^t =: \mathbf{b}_{\mathbf{s}^t}(\mathbf{g},\theta), \tag{5}$$

where $\mathbf{b}_{\mathbf{s}^t}(\mathbf{g},\theta)$ is the weighted Katz-Bonacich centrality (Bonacich, 1987; Katz, 1953; Ballester et al., 2006).[15] Given that $\mathbf{G}$ is a stochastic matrix, the largest eigenvalue of $\mathbf{G}$ is 1, then $[\mathbf{I}-\theta\,\mathbf{G}]$ is invertible and with non-negative entries if and only if $\theta < 1$, which is always true. Thus, there is a unique and interior solution to equation (5).

Equation (5) shows that x_i^t, the integration choice of each agent i at time t, depends on ω, the taste for conformity, her preference at time t, s_i^t, and her position in the network, as captured by her weighted Katz-Bonacich centrality.

3.2. The dynamics of individual preferences

Consider now the dynamics of preferences. Define $\mathbf{M}(\theta,\mathbf{G}) := [\mathbf{I}-\theta\mathbf{G}]^{-1}$. Then, by substituting (5) into (2), we obtain:

$$\mathbf{s}^{t+1} = [\gamma(1-\theta)\mathbf{M}(\theta,\mathbf{G}) + (1-\gamma)\mathbf{I}]\,\mathbf{s}^t. \tag{6}$$

Define

$$\mathbf{T} := \gamma(1-\theta)\mathbf{M}(\theta,\mathbf{G}) + (1-\gamma)\mathbf{I}. \tag{7}$$

Then, the dynamic equation (6) is a *time-homogeneous Markov process* where

$$\mathbf{s}^{t+1} = \mathbf{T}\,\mathbf{s}^t. \tag{8}$$

This transition mechanism takes into account each agent's preference and own equilibrium actions. The latter, as shown by (5), are dependent on the network structure and on the position of each agent in the network (captured by their Katz-Bonacich centrality). The *limiting preferences* can be calculated as a function of the initial preferences and of the weights derived using matrix $\mathbf{T}$ as follows:

$$\mathbf{s}^\infty = \lim_{t\to\infty} \mathbf{T}^t\,\mathbf{s}^0, \tag{9}$$

where $\mathbf{T}^t$ is the matrix of cumulative influences in period t.

Definition 1. A matrix $\mathbf{T}$ is convergent if $\lim_{t\to\infty} \mathbf{T}^t\,\mathbf{s}$ exists for all vectors $\mathbf{s} \in [0,1]^n$.

This definition imposes the convergence of preference profiles for all initial preferences vectors. If convergence fails to occur for a specific initial vector, it results in oscillations or cycles during the preferences' updating process. Note that the convergence relies on the characteristics of the matrix $\mathbf{T}$, which represents a non-trivial transformation of the network adjacency matrix $\mathbf{G}$. Indeed, $\mathbf{T}$ encompasses information regarding the equilibrium actions, the updating process, and the preference parameters of all agents involved in the network of interactions.

We will now present conditions for the convergence of $\mathbf{T}$ that solely rely on the topological properties of $\mathbf{G}$, *independently* of the preference parameters. This is particularly relevant because $\mathbf{G}$ is an observable network whereas $\mathbf{T}$ is not. First, notice that $(1-\theta)\mathbf{M}(\theta,\mathbf{G})$ is row-normalized.[16] It follows that $\mathbf{T}$ is also row-normalized since it is a convex combination of two row-normalized matrices. For any $\epsilon \in (0,1)$, define

$$\mathbf{G}_\epsilon := \epsilon\mathbf{I} + (1-\epsilon)\mathbf{G}. \tag{10}$$

$\mathbf{G}_\epsilon$ is the matrix of social interactions in which we consider weights *as if* all agents actually put some (homogenous) weight on themselves. Then, we can provide the following steady-state characterization.[17]

[15] The Katz-Bonacich centrality measure, which has proven to be extremely useful in game theoretic applications (Ballester et al., 2006), presumes that the power or prestige of a node is simply a weighted sum of the walks that emanate from it.

[16] Notice that, since $\mathbf{G}$ is row-normalized, then $\sum_{t=0}^{\infty}\theta^t\mathbf{G}^t\cdot\mathbf{1}$ is a vector with all entries equal to $\frac{1}{1-\theta}$. Since $\theta = \frac{\omega}{1+\omega}$, then $\frac{1}{1-\theta} = 1+\omega$. Then the sum of the entries of each row of the matrix $\mathbf{M}(\theta,\mathbf{G})$ is $1+w$. It is then immediate to see that $\frac{1}{(1+\omega)}\mathbf{M}(\theta,\mathbf{G}) =: (1-\theta)\mathbf{M}(\theta,\mathbf{G})$ is row-normalized.

[17] In Appendix B, we provide some standard linear algebra results about irreducibility and aperiodicity of matrices.

Proposition 1. *Consider a set of agents of size n. Each agent maximizes utility (3), with $\mathbf{s}^0 \in [0,1]^n$, and the updating rule is given by (8). Then,*

(*i*) *if* $\mathbf{G}$ *is an aperiodic matrix,*

$$\mathbf{s}^\infty = \lim_{t\to\infty} \mathbf{T}^t\,\mathbf{s}^{(0)} = \lim_{t\to\infty} \mathbf{G}^t\,\mathbf{s}^{(0)}; \tag{11}$$

(*ii*) *if* $\mathbf{G}$ *is a periodic matrix, for any* $\epsilon \in (0,1)$,

$$\mathbf{s}^\infty = \lim_{t\to\infty} \mathbf{T}^t\,\mathbf{s}^{(0)} = \lim_{t\to\infty} \mathbf{G}_\epsilon^t\,\mathbf{s}^{(0)}. \tag{12}$$

This result is of significant importance as it demonstrates that the steady-state vector of preferences, defined by the matrix $\mathbf{T}$, can be analyzed solely by examining the adjacency matrix $\mathbf{G}$ of the network, which is more likely to be observable by a policymaker. Furthermore, the convergence is guaranteed regardless of whether $\mathbf{G}$ is periodic or not.[18] The convergence is also independent of the preference parameters ω and γ. This result stems from the fact that the anchoring element in the preference dynamics makes each preference dependent on its past value, thereby breaking any potential cycles. Consequently, our model ensures convergence of integration preferences for any network, without imposing restrictions. Furthermore, we offer a straightforward method to determine the steady state of the dynamics even when $\mathbf{G}$ exhibits cycles. Proposition 1(*ii*) characterizes the steady state by considering a small perturbation of the original network.

Observe that, at any finite time t, $\mathbf{T}^t \neq \mathbf{G}^t$; they only coincide when $t \to \infty$. The result about the common asymptotic properties of matrices $\mathbf{T}$ and $\mathbf{G}$ given in Proposition 1 is based on the fact that these two matrices commute, i.e., $\mathbf{TG} = \mathbf{GT}$ (see the proof of Proposition 1 in Appendix A). Indeed, when $\mathbf{T}$ and $\mathbf{G}$ commute (which is based on the fact that $\mathbf{G}$ and $\mathbf{M}$ commute and are diagonalizable), then they have the same eigenvector $\mathbf{e}$ associated with the maximum eigenvalue, which is 1 here.[19] This implies that $\mathbf{e}^{\mathrm{T}}\,\mathbf{T} = \mathbf{e}^{\mathrm{T}}\,\mathbf{G} = \mathbf{1}\,\mathbf{e}^{\mathrm{T}}$, which proves the convergence result.

In Section A1 of the Online Appendix A, we provide a simple example that shows how the asymptotic properties of $\mathbf{T}$ and $\mathbf{G}$ are the same but, at the same time, how the two matrices differ during the convergence process.

3.3. Steady-state and welfare

Given our previous results on convergence of preferences, we are now able to characterize the steady-state integration choices of all individuals belonging to network $\mathbf{G}$. Recall first that

$$\mathbf{x}^t = (1-\theta)\,[\mathbf{I}-\theta\,\mathbf{G}]^{-1}\,\mathbf{s}^t = (1-\theta)\,[\mathbf{I}-\theta\,\mathbf{G}]^{-1}\,\mathbf{T}^t\mathbf{s}^{(\mathbf{0})}. \tag{13}$$

It should be clear from equation (3) that the welfare of the ethnic minorities is maximized when the utility of each agent is equal to zero because, in this case, there are no losses. This is when the individual preference and the integration effort of each individual are the same and are also equal to those of her neighbors in the network. We now analyze what happens in steady state.

Proposition 2. *In steady state, the equilibrium integration choices are given by:*

$$\mathbf{x}^\infty = (1-\theta)\,[\mathbf{I}-\theta\,\mathbf{G}]^{-1}\lim_{t\to\infty}\mathbf{G}^{\mathbf{t}}\mathbf{s}^{\mathbf{0}} = \mathbf{G}\mathbf{x}^\infty, \tag{14}$$

where $\mathbf{x}^\infty = \mathbf{s}^\infty$. *Moreover, for any network* $\mathbf{G}$, *the total welfare is maximized in steady state. If* $\mathbf{G}$ *is periodic, the same result holds by replacing* $\mathbf{G}$ *with* $\mathbf{G}_\epsilon$.

This proposition characterizes the equilibrium integration choices in steady state. Consider first equation (14). The first equality establishes a relationship between steady-state equilibrium actions and steady-state preferences for integration. These preferences are derived from the limit of equation (13) as t approaches infinity. However, this alone does not provide much information regarding how steady-state actions are influenced by the network. The second equality of (14) offers this characterization. Specifically, we have $\mathbf{x}^\infty = \mathbf{G}\mathbf{x}^\infty$, which indicates that the steady-state action vector is such that each agent chooses an action that corresponds to the average of their neighbors' actions. In other words, individuals conform to the average integration effort of their peers. As a result, given that $\mathbf{x}^\infty = \mathbf{s}^\infty$, all agents achieve a utility of zero in steady state, thereby maximizing total welfare.

It is worth noting that the specific levels to which individual preferences converge do not have an impact on welfare. This is because individual utility is solely determined by conformity and adherence to one's own preference. The chosen action itself does not have any other inherent consequences, although it does play a role in the context of material incentives for integration, which will be discussing in Section 4.

Observe that the results in Proposition 2 are only true in steady state and partly due to specific assumptions such as the time homogeneity of the network. As mentioned earlier, the time required to reach a steady state may vary depending on the specific

[18] To clarify, we use the term "aperiodic" for $\mathbf{G}$ to indicate that the submatrix associated with each closed communication class is aperiodic. Closed-communication classes are defined in Section C.1 of Appendix C.

[19] Since both $\mathbf{T}$ and $\mathbf{G}$ are row-normalized they both have the same largest eigenvalue equal to 1.

problem being analyzed. Given that we assume that the network does not change over time, we believe that our results correspond to situations in which the convergence to a steady state is quick and faster than any network update. Furthermore, our focus is solely on steady-state welfare levels, thereby disregarding the cumulative welfare levels throughout the convergence process.

Let us calculate the integration effort of each individual i at any moment in time t. It can be shown that[20]:

$$x_i^t - s_i^t = \sum_{j \neq i} m_{ij}(s_j^t - s_i^t), \tag{15}$$

where $\mathbf{M} = [\mathbf{I} - \theta \mathbf{G}]^{-1}$ and m_{ij} is its (i,j) entry. This implies that, at time t, whenever the difference between agent i's integration effort x_i^t and her individual preference for integration s_i^t is not equal to zero, there is some "cognitive dissonance" for individual i. Equation (15) also shows that this difference is equal to the one between the preferences of her path-connected peers and her own preference. Therefore, the agents who choose an integration level different from their integration preference are more likely to be strongly connected in the network to agents whose preferences differ from theirs. Proposition 2 demonstrates that, for any network $\mathbf{G}$, only in the steady state, integration choices are efficient and aligned with integration preferences, since welfare is not maximized during the dynamic convergence process.

Thus, if we consider cases of rapid convergence, Proposition 2 can be seen as a representation of the overall welfare that arises throughout the process. Conversely, in situations where convergence takes longer, our steady-state results can be viewed as a target for the society.

3.4. Equilibrium characterization

In this section, we delve into the characterization of long-run preferences and integration choices in relation to network structure. Given the equivalence between matrices $\mathbf{G}$ and $\mathbf{T}$ in the long run, we can study the dynamics by focusing on the shape of the long-run preferences, which are determined by the properties of the powers of matrix $\mathbf{G}$. Thus, we can leverage existing results and adapt them to our framework. Detailed results and examples are provided in Appendix C, while in this section, we discuss the implications of these findings.

We define a subset of agents $J \subset I$ as a *closed communication class* if the agents in J directly or indirectly influence each other, are not influenced by agents outside of J, but may influence agents outside of J. More precise definitions of communication classes can be found in Section C.1 of Appendix C. In essence, within a closed communication class, agents only have paths connecting them to other agents within the same class, and no paths into the class originate from agents outside the class. These sets of agents drive the entire dynamics of preferences. Specifically, all agents belonging to the same closed communication class converge to the same preference (and subsequently, the same action), while agents outside of any closed communication class converge to a convex combination of the limiting preferences observed within the closed communication classes. Consequently, in the steady state, the number of integration preferences will be equal to the number of closed communication classes, plus the number of agents outside of any closed communication class who are influenced by more than one communication class (since agents who are influenced by just one communication class have their preferences converging to the limit preference of the communication class they are influenced by). In sum, the convergence in terms of integration choices x_i^t and preference for integration s_i^t is not universal but operates differently within communication closed classes and among those with connections to different closed classes.

By examining the structure of the network represented by matrix $\mathbf{G}$, we can readily determine the number of steady-state preferences and the specific individual preference each agent converges to, as dictated by the powers of matrix $\mathbf{T}$. Proposition 8 in Appendix C provides a general result of convergence of the individual preference for integration $\mathbf{s}^{\infty}$ for any network.

To illustrate this result, consider the three networks displayed in Fig. 1: network 1 (left panel), network 2 (right panel) and network 3 (bottom panel). In these networks, there are always three groups of agents distinguished by their intra-group links. It can be observed that $\mathbf{G_2}$ (the adjacency matrix of network 2) can be obtained from $\mathbf{G_1}$ (network 1) by removing the link g_{15} in $\mathbf{G_1}$. Similarly, $\mathbf{G_3}$ (network 3) can be derived from $\mathbf{G_2}$ by removing the link g_{59} and adding the link g_{86}.

Network 1 is strongly connected, which implies that all individuals belong to the same communication class. Thus, all agents in this network will eventually converge to the same steady-state preference for integration. In contrast, network 2 exhibits one closed communication class—that is, $\{1,2,3\}$. As a result, all agents within this set will converge to the same long-run preference, while agents 4 to 9 will each converge to a different preference for integration. The preferences of these agents will be a convex combination of the preferences of agents in the closed communication class $\{1,2,3\}$.

Lastly, network 3 consists of two closed communication classes: $\{1,2,3\}$ and $\{4,5,6\}$. Thus, agents belonging to each closed communication class will converge to a unique preference for integration, while agents 7, 8, and 9 will converge to different preferences. These preferences will be determined by a convex combination of agents' preferences in their respective closed communication classes.[21]

In Online Appendix B, we study the speed of convergence of the individual preferences in our model and show that it does not only depend on the second largest eigenvalue of the adjacency matrix (as in Golub and Jackson, 2010, 2012), but also on the taste for

[20] See Appendix A for the derivation of (15).

[21] In Appendix C, we provide the exact calculation of each long-term preference for network 1 (see Section C.2.2), network 2 (see Section C.3.2) and network 3 (see Section C.4.2).

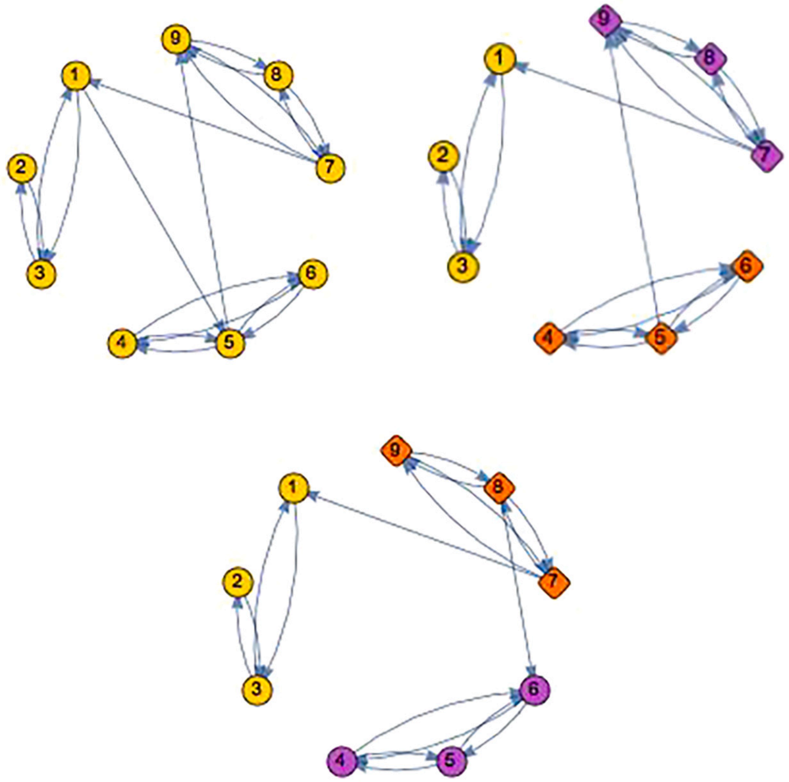

Fig. 1. Three different networks.

conformity and on the weight put on the impact of past integration decisions on current individual preferences during the preference updating process.

4. Material incentives for integration

Now, we introduce material incentives in the integration choices to see how the results change with respect to the previous case. In terms of utility function, we go back to equation (1) with $\mathbb{1} = 1$.

4.1. The integration choice

By solving for the first-order conditions of (1), we obtain:

$$x_i^t = \left(\frac{1}{2+\omega}\right)\alpha_i + \left(\frac{1}{2+\omega}\right)s_i^t + \left(\frac{\omega}{2+\omega}\right)\sum_j g_{ij}x_j^t. \tag{16}$$

The equilibrium actions are a convex combination of the individual i's material marginal incentives, α_i, her own preference for integrations, s_i^t, and the average actions of her neighbors, $\sum_j g_{ij}x_j^t$. Denote by $\theta' := \omega/(2+\omega)$ and by $\boldsymbol{\alpha} := (\alpha_i)_{i\in I}$. Using (16), we obtain the equilibrium actions in matricial form:

$$\mathbf{x}^t = \left(\frac{1}{2+\omega}\right)\left[\mathbf{I}-\theta'\,\mathbf{G}\right]^{-1}(\mathbf{s}^t+\boldsymbol{\alpha}) =: \mathbf{b}_{\mathbf{s}^t+\boldsymbol{\alpha}}(\mathbf{g},\theta'), \tag{17}$$

where $\mathbf{b}_{\mathbf{s}^t}(\mathbf{g},\theta')$ is the weighted Katz-Bonacich centrality. Since the largest eigenvalue of $\mathbf{G}$ is 1, $\left[\mathbf{I}-\theta'\,\mathbf{G}\right]$ is invertible and with non–negative entries if and only if $\theta' < 1$, which is always true. Thus, the equilibrium action x_i^t for each agent i at time t in (17) is unique and interior.

4.2. The dynamics of individual preferences

Define $\mathbf{M}\left(\theta',\mathbf{G}\right) := [\mathbf{I}-\theta'\mathbf{G}]^{-1}$. Then, by substituting (17) into (2), we obtain:

$$\mathbf{s}^{t+1} = \left[\frac{\gamma}{(2+\omega)}\mathbf{M}\left(\theta',\mathbf{G}\right)+(1-\gamma)\mathbf{I}\right]\mathbf{s}^t + \frac{\gamma}{(2+\omega)}\mathbf{M}\left(\theta',\mathbf{G}\right)\boldsymbol{\alpha}. \tag{18}$$

While this formulation is relatively simple and involves already-known elements, we cannot go further in the characterization of the dynamics of preferences and equilibrium behavior in terms of the network $\mathbf{G}$. This limitation arises from the impossibility to express the dynamics as a linear system $\mathbf{s}^{t+1} = \mathbf{A}\mathbf{s}^t$, which would enable a time-homogeneous Markov process analysis. Moreover it does not enable us to readily understand the relationship between the dynamics and the topological properties of the network $\mathbf{G}$, as for the case without material incentives.

To solve this issue, define by $\hat{\mathbf{G}}$ a *modified* network $\mathbf{G}$ matrix, in which we consider a fictitious directed network of $2n$ agents in which, on top of network $\mathbf{G}$, for each agent i, we create a *fictitious* agent f_i. Each agent i has a link towards f_i, no other agent in the network is linked to f_i, and f_i has no other link than herself (self-loop). Call F the set of fictitious agents. Formally, $\hat{g}_{i,f_i} > 0$, $\hat{g}_{f_i,f_i} = 1$, $\hat{g}_{f_i,j} = 0$ for every $j \in I$, and $\hat{g}_{f_i,f_j} = 0$ for every $f_j \neq f_i$. In particular, the first n rows of $\hat{\mathbf{G}}$ represent the links of fictitious agents while the last n rows represent the links of real agents.[22] This augmented weighted matrix $\hat{\mathbf{G}}$ helps us write the FOC in (16) in a matricial form and derive a time-homogenous Markov process for the individual preferences.

First, we check that $\hat{\mathbf{G}}$ is a stochastic matrix itself. Since $\theta = \omega/(1+\omega)$, we can define the $\hat{\mathbf{G}}$ matrix in its canonical form as follows:

$$\hat{\mathbf{G}} \equiv \left[\begin{array}{c|c} \mathbf{I} & \mathbf{O} \\ \hline \mathbf{D} & \theta\mathbf{G} \end{array}\right], \tag{19}$$

in which each block is of dimension $n \times n$, $\mathbf{D}$ is a diagonal matrix with entries $1/(1+\omega)$ on the main diagonal and 0 in all the other entries, and $\mathbf{O}$ is a matrix of zeros. Thus, $\hat{\mathbf{G}}$ is a stochastic matrix.

Second, we study the optimal action profile $\hat{\mathbf{x}}^t$. We are only interested in the integration actions of real agents and not of the fictitious ones; thus, we assume that each fictitious agent f_i always make an effort equal to α_i. Then, the first-order condition (16) can be written as follows:

$$\hat{\mathbf{x}}^t = \left(\frac{1}{2+\omega}\right)\hat{\mathbf{s}}^t + \left(\frac{1+\omega}{2+\omega}\right)\hat{\mathbf{G}}\hat{\mathbf{x}}^t, \tag{20}$$

where

$$\hat{\mathbf{x}}^t := \left[\begin{array}{c} \boldsymbol{\alpha} \\ \hline \mathbf{x}^t \end{array}\right], \qquad \hat{\mathbf{s}}^t := \left[\begin{array}{c} \boldsymbol{\alpha} \\ \hline \mathbf{s}^t \end{array}\right]. \tag{21}$$

This implies that (17) can now be written as:

$$\hat{\mathbf{x}}^t = \left(\frac{1}{2+\omega}\right)\left[\mathbf{I} - \left(\frac{1+\omega}{2+\omega}\right)\hat{\mathbf{G}}\right]^{-1}\hat{\mathbf{s}}^t = \left(\frac{1}{2+\omega}\right)\left[\mathbf{I} - \frac{\theta'}{\theta}\hat{\mathbf{G}}\right]^{-1}\hat{\mathbf{s}}^t. \tag{22}$$

Instead of working with $\mathbf{G}$, we will now work with $\hat{\mathbf{G}}$ while being aware that the first n rows of all matrices and vectors are about fictitious agents and, thus, without strict economic meaning.

4.3. Steady-state equilibrium

We have the following preference updating rule:

$$\hat{\mathbf{s}}^{t+1} = \gamma\hat{\mathbf{x}}^t + (1-\gamma)\hat{\mathbf{s}}^t. \tag{23}$$

By plugging the value of the equilibrium action given in (22) into this equation and denoting $\mathbf{M}(\frac{\theta'}{\theta}, \hat{\mathbf{G}}) := \left[\mathbf{I} - \frac{\theta'}{\theta}\hat{\mathbf{G}}\right]^{-1}$, we obtain:

$$\hat{\mathbf{s}}^{t+1} = \left[\frac{\gamma}{(2+\omega)}\mathbf{M}\left(\frac{\theta'}{\theta}, \hat{\mathbf{G}}\right) + (1-\gamma)\mathbf{I}\right]\hat{\mathbf{s}}^t. \tag{24}$$

Denote

$$\hat{\mathbf{T}} := \frac{\gamma}{(2+\omega)}\mathbf{M}\left(\frac{\theta'}{\theta}, \hat{\mathbf{G}}\right) + (1-\gamma)\mathbf{I},$$

which is the augmented $\mathbf{T}$ matrix, obtained by adding the n fictitious players. If we reshape the weights accordingly, we can rewrite (23) as follows

$$\hat{\mathbf{s}}^{t+1} = \hat{\mathbf{T}}\,\hat{\mathbf{s}}^t. \tag{25}$$

By doing so, we have written (18) as an augmented problem that follows the rule of a time non-homogenous Markov process. Thus, as we did for the case without material incentives, we can derive the following result:

[22] These fictitious agents do not have a real counterpart. They are just a technical device that transforms a complex problem into a simpler one. We can interpret each fictitious agent f_i as a stubborn alter-ego of i, who always plays α_i, and who influences i by trying to convince her to play α_i.

Proposition 3. *For any* $\mathbf{G}$*, we have:*

$$\hat{\mathbf{s}}^{\infty} = \lim_{t\to\infty} \hat{\mathbf{T}}^t \hat{\mathbf{s}}^{(0)} = \lim_{t\to\infty} \hat{\mathbf{G}}^t \hat{\mathbf{s}}^{(0)}. \tag{26}$$

Observe that, in network $\hat{\mathbf{G}}$, each fictitious agent f_i forms a closed communication class and there are no other closed communication classes, since each agent i is linked to at least, agent f_i whereas f_i is not linked to anyone. Therefore, the dynamics of the preferences (and action) is now represented by a matrix with n closed communication classes and n agents belonging to other non-closed communication classes. We can thus use the results of Proposition 8 of Appendix C.4 to obtain:

Proposition 4. *If the utility of each agent* i *is given by (1), then the steady-state preferences and actions of all* n *agents in the original network are equal to:*

$$\mathbf{s}^{\infty} = \left(\frac{1}{1+\omega}\right)[\mathbf{I}-\theta\mathbf{G}]^{-1}\boldsymbol{\alpha}, \tag{27}$$

$$\mathbf{x}^{\infty} = \left(\frac{1}{2+\omega}\right)\left[\mathbf{I}-\theta'\,\mathbf{G}\right]^{-1}\left[\mathbf{I}+\frac{1}{(1+\omega)}(\mathbf{I}-\theta\mathbf{G})^{-1}\right]\boldsymbol{\alpha}. \tag{28}$$

Independently of the initial preferences for integration, in the long run, integration preferences and choices are uniquely determined by the ex-ante heterogeneities (material incentives) of agents in terms of α_i and their position in the network. Since $\boldsymbol{\alpha}$ represents a vector of heterogenous economic incentives for integration, this result implies that economic incentives can drive integration preferences very far from their initial values. Indeed, suppose, for example, that the initial preferences are such that $\mathbf{s}^{(\mathbf{0})} \in [0, \frac{1}{2}]^n$, but that $\boldsymbol{\alpha} \in [\frac{2}{3}, 1]^n$. Then, in steady state, we have: $\mathbf{s}^{(\infty)} \in [\frac{2}{3}, 1]^n$. Observe that the weights of each preference are proportional to the Katz-Bonacich centrality of the agents in the $\mathbf{G}$ network; thus, the steady-state distribution of preferences will depend on the position of each agent in the network.

In Section A2 of the Online Appendix A, we provide an example that illustrates these results.

5. Policy implications

When material incentives matter, the long-term preferences and actions of agents are influenced by both the material incentives and the network topology, regardless of the initial preference values. Let us now examine how our model can inform a planner whose targeted policy incentives can assist in generating desirable outcomes such as a desired profile of long-term integration preferences or some specific long-run integration choices.

As previously discussed, since we mainly focus on scenarios for which convergence occurs rapidly, we consider policy makers who solely focus on the long-term (steady-state) outcomes and thus, disregard the convergence process.

5.1. Policy analysis

Consider a policy for which the policy-maker's objective function is to maximize the welfare of the minority group.[23] To maximize this welfare, the policymaker can optimally decide the level of material incentives that leads to a desired long-run integration choices profile $\mathbf{x}^*$, given that the long-run preferences $\mathbf{s}^*$ and choices $\mathbf{x}^*$ will be given by (27) and (28). By substituting (27) into (28), the optimal choices are given by.

$$\mathbf{x}^* = \left(\frac{1}{2+\omega}\right)\left[\mathbf{I}-\theta'\,\mathbf{G}\right]^{-1}\left(\boldsymbol{\alpha}+\mathbf{s}^*\right). \tag{29}$$

Denote by $\boldsymbol{\alpha}^*$ the target vector of individual material incentives that leads to the desired integration actions given network $\mathbf{g}$. Since material incentives capture wages, we study policies in which the policymaker changes wages to improve the integration of ethnic minorities. Indeed, many ethnic minorities are paid at the minimum wage; thus, a feasible policy for the policy-maker is to whether it wants to increase the minimum wage to raise the integration of ethnic minorities. In our model, we capture this policy by a positive or negative subsidy on wages. Such policies have been implemented in France, for example.[24]

Let $\boldsymbol{\sigma}^* := \boldsymbol{\alpha}^* - \boldsymbol{\alpha}$, so that $\boldsymbol{\sigma}^*$ is the vector of additional positive (subsidies) or negative (taxes) incentives that the planner should provide to each agent, on top of the existing material incentives $\boldsymbol{\alpha}$, in order to reach $\mathbf{x}^*$. Indeed, by knowing how agents choose their actions, how they are influenced by their peers, and how they update their preferences, the planner can determine $\boldsymbol{\sigma}^*$. That is, given initial preferences $\mathbf{s}^{\mathbf{0}}$, by changing the material incentives using $\boldsymbol{\sigma}^*$, the policymaker induces a change in the dynamics of preferences, which, in the long run, reaches $\mathbf{s}^*$, and, thus the desired action levels $\mathbf{x}^*$ chosen by all agents. In terms of timing, we just add one stage at time $t = 0$ where the planner sets the incentives $\boldsymbol{\sigma}^*$.

Proposition 5. *To reach the long-term choices* $\mathbf{x}^*$ *and preferences* $\mathbf{s}^*$*, the planner must give the following incentives to each individual:*

[23] We do not consider a welfare function that encompasses the population at large because we do not model the welfare of the majority group and the positive or negative externalities that the minority group may exert on the majority group.

[24] In our language example, this would mean a policy that increases the wage of ethnic minorities to improve their fluency in the majority language.

1. *If the objective of the policymaker is to reach homogenous long-term choices (i.e., $x_i^* = x_j^*$ for all i, j), and thus, homogenous long-term integration preferences, then $\boldsymbol{\sigma}^* = \boldsymbol{x}^* - \boldsymbol{\alpha} = \boldsymbol{s}^* - \boldsymbol{\alpha}$.*
2. *If the objective of the policymaker is to reach heterogeneous long-term preferences $\mathbf{s}^*$, then $\boldsymbol{\sigma}^* = \mathbf{s}^* - \boldsymbol{\alpha} + \omega(\mathbf{I} - \mathbf{G})\mathbf{s}^*$.*
3. *If the objective of the policymaker is to reach heterogeneous long-term actions $\mathbf{x}^*$, then $\boldsymbol{\sigma}^* = (2+\omega)\left[\mathbf{I} + \frac{1}{(1+\omega)}(\mathbf{I} - \theta\mathbf{G})^{-1}\right]^{-1}\left[\mathbf{I} - \theta'\mathbf{G}\right]\mathbf{x}^* - \boldsymbol{\alpha}$.*

This proposition characterizes the optimal intervention depending on whether long-run integration is homogeneous or heterogeneous. If the policymaker wants an homogeneous society in which all individuals reach the same integration level and have the same integration preference, then the marginal subsidy must be the difference between the individual desired choice (that also coincides with the desired preference for integration) and the agent's actual material incentive, independently of the network structure. Intuitively, the vector $\boldsymbol{\sigma}^*$ makes all agents ex-ante equal in terms of material incentives. Since long-run preferences and actions for integration are a convex combination of the $\boldsymbol{\alpha}^*$, it is straightforward to see that the actions and preferences will converge to the same value. Given that the network determines the weights of this convex combination and since all agents, after the policy intervention, are equal, then the network becomes irrelevant. Thus, the policymaker can mostly ignore the network and just introduce corrections to material incentives to make all agents equal. In this respect, if the policymaker aims at having total integration, the best she can do is to ignore the role of social interactions among agents and make material incentives the same for all agents.

The situation is different if the policymaker wants an heterogenous society in terms of integration. Whether she focuses on preferences or actions, the policymaker has to take into account the direct effect of subsidies on each individual's action but also on their neighbors; thus, the knowledge of the network is of paramount importance. In part 2 of Proposition 5, we show that the per-person subsidy depends on the taste for conformity, the network structure, and the ex ante heterogeneity in terms of αs. Indeed, by simply inverting (27), we have

$$\boldsymbol{\alpha}^* = (1+\omega)(\mathbf{I} - \theta\mathbf{G})\mathbf{s}^*. \tag{30}$$

In this equation, $(\mathbf{I} - \theta\mathbf{G})\mathbf{s}^*$ tells us how much each individual i's s_i^* differs from the average preference for integration of her direct neighbors. Moreover, in the dynamics of preferences, each agent is affected by her neighbors, which depends on the conformism parameter ω. Thus, the incentives $\boldsymbol{\alpha}^*$ should be equal to the vector of desired individual preferences $\mathbf{s}^*$, corrected by how much each individual i's desired preference s_i^* is larger or smaller than the average desired preference of her neighbors. The resulting actions are then given by (29).

The result becomes more complex if the policy-maker wants to reach a specific vector of heterogeneous actions. Indeed, by changing the actions' incentives, the policymaker should take into account the fact that these incentives are going to impact the preferences through the updating rule. This, in turn, affects the actions through conformism. Thus, the knowledge of the network is again important for a policymaker to implement a policy aiming at producing heterogenous levels of integration.

5.2. Example

To get an intuition about how this policy is shaped by the network structure, consider the networks displayed in Fig. 1 with nine agents. We assume the following vector of material incentives:

$$(\boldsymbol{\alpha})^T = \begin{pmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{pmatrix}. \tag{31}$$

First, consider a policy for which the planner wants an homogenous society with a maximal level of integration (part 1 of Proposition 5), that is, $s_i^* = 1$ for all i. Then, $\boldsymbol{\sigma}^* = \boldsymbol{s}^* - \boldsymbol{\alpha}$. Thus,

$$(\boldsymbol{\sigma})^T = \begin{pmatrix} 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \end{pmatrix}. \tag{32}$$

In steady state, everyone will have a preference for integration equal to $s_i^* = 1$ and will make an integration effort given by $x_i^* = 1$. Thus, there is total integration. This is because, for every individual, $\alpha_i^* = \alpha_i + \sigma_i^* = 1$, and actions are a convex combinations of all α^*s. The overall cost of the subsidy is given by $\sum_i \sigma_i^* x_i^* = \sum_i \sigma_i^*$, independently of the network structure. Thus, if the desired preference for integration is the same for everyone, the determination of the subsidy and its cost will be independent of the network structure.

Second, consider the case when the planner has different targets in terms of integration (parts 2 and 3 of Proposition 5). For example, let us study a policy in which the policymaker does not want any agent to have a long-term preference for integration below 0.5. Consider the first two networks in Fig. 1 and assume that the material incentives are given by (31). If there no intervention, the long-run preferences for integration in each network are given by[25]:

$$\mathbf{s}_{\mathbf{G}_1}^{\infty} = \begin{pmatrix} 0.201 & 0.223 & 0.270 & 0.450 & 0.539 & 0.565 & 0.669 & 0.785 & 0.842 \end{pmatrix}, \tag{33}$$

$$\mathbf{s}_{\mathbf{G}_2}^{\infty} = \begin{pmatrix} 0.154 & 0.220 & 0.262 & 0.450 & 0.539 & 0.565 & 0.664 & 0.784 & 0.841 \end{pmatrix}. \tag{34}$$

[25] Even if network 1 is strongly connected, the individuals will not have the same long-term preferences for integration because they have different αs. The same reasoning applies tor network 2, which has one closed communication class.

If the policy is such that everybody needs to have a level of long-term integration of at least 0.5, then all individuals who have an integration value below 0.5 need to increase it to 0.5 while those with values above or equal to 0.5 keep this same level. For the first two networks in Fig. 1, the objective of the policy should be

$$\mathbf{s}^*_{\mathbf{G}_1} = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.539 & 0.565 & 0.669 & 0.785 & 0.842 \end{pmatrix}, \tag{35}$$

$$\mathbf{s}^*_{\mathbf{G}_2} = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.539 & 0.565 & 0.664 & 0.784 & 0.841 \end{pmatrix}. \tag{36}$$

Using (30), we can compute the subsidy given to each agent in order to reach these steady-state preferences for integration. They are equal to

$$\boldsymbol{\alpha}^*_{\mathbf{G}_1} = \begin{pmatrix} 0.490 & 0.5 & 0.5 & 0.473 & 0.491 & 0.587 & 0.650 & 0.8 & 0.9 \end{pmatrix}, \tag{37}$$

$$\boldsymbol{\alpha}^*_{\mathbf{G}_2} = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.473 & 0.491 & 0.587 & 0.642 & 0.8 & 0.9 \end{pmatrix}. \tag{38}$$

Note that, in the second network, the first three agents need an incentive equal to the targeted preference for integration. This is because they form a closed communication class and thus, if the policymaker wants them to reach at least 0.5, she has to induce this behavior through incentives. Interestingly, agents 4 and 5 need incentives lower than 0.5 since conformism leads them to have higher long-run preferences for integration. Therefore, the final marginal subsidy or tax for each individual should be

$$\boldsymbol{\sigma}^*_{\mathbf{G}_1} = \begin{pmatrix} 0.390 & 0.3 & 0.2 & 0.073 & -0.008 & -0.012 & -0.049 & -1.110\times 10^{-16} & -1.1102\times 10^{-16} \end{pmatrix},$$

$$\boldsymbol{\sigma}^*_{\mathbf{G}_2} = \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.073 & -0.008 & -0.012 & -0.057 & 1.110\times 10^{-16} & -1.110\times 10^{-16} \end{pmatrix}.$$

In both networks, the policymaker gives a marginal subsidy to the first four agents and imposes a tax to all the other agents. Note, however, that we did not impose any budget constraint so that the cost of the policy can be positive or negative depending on the parameters. Indeed, the cost of the policy is given by $(\boldsymbol{\sigma}^*)^T\mathbf{x}$ and, in this example, is equal to 0.85 (network 1) and 0.91 (network 2).

Given that the objective is to have all agents converging to a steady-state preference for integration of at least 0.5, the cost would only be reduced by lowering the actions and preferences for integration for agents above 0.5. In our example, the cost would be zero once we choose an homogeneous steady-state preference vector of 0.5. In this case, everyone would choose an action equal to 0.5. Then, the agent with the initial preference of 0.9 will pay for the subsidy of the agent with an initial preference of 0.1. The agent with initial preference of 0.8 would pay for the agent with an initial preference of 0.2, and so forth. In this case, the overall cost of this policy would be zero; thus, this policy would be self-financed.

6. Conclusion

We consider a model where each individual (or ethnic minority) is embedded in a network of relationships and decides whether or not she wants to be integrated to the host country. First, each individual wants her behavior to agree with her personal preference for integration, which implies that there is *consistency* between her own preference and her integration behavior. Second, she also wants her behavior to be as close as possible to the average integration behavior of her peers (social pressure), which implies that she is a *conformist*. We show that there is always convergence to a steady-state and characterize it. We also derive some implications in terms of integration policies. More generally, our model highlights the tension that may exist for an ethnic minority deciding on her long-run integration between her initial preferences for integration, social pressure from her community, and her own marginal benefits of integration.

We view our model as a first stab at the complex issue of integration of ethnic minorities. Future research should incorporate an empirical analysis that examines the relative importance of individual preference for integration, social pressure, and material incentives (personal characteristics and/or wages) in the integration of minorities. This will allow to disentangle these different effects and address the relevant integration policy implications.

Declaration of competing interest

The authors have no conflicts of interest to report.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs of the results in the main text

Proof of Proposition 1. *First step:* Let us show first that $\mathbf{GM} = \mathbf{MG}$. To do that, we first prove the following lemma

Lemma 1. *If two matrices* $\mathbf{A}$ *and* $\mathbf{B}$ *commute and* $\mathbf{B}$ *is nonsingular, i.e.* $\mathbf{AB} = \mathbf{BA}$, *then* $\mathbf{AB}^{-1} = \mathbf{B}^{-1}\mathbf{A}$.

Proof. We have $\mathbf{AB} = \mathbf{BA}$. This implies that $\mathbf{B}^{-1}\mathbf{AB} = \mathbf{B}^{-1}\mathbf{BA}$, which is equivalent to:

$$\mathbf{B}^{-1}\mathbf{AB} = \mathbf{A}$$

This implies that

$$\mathbf{B}^{-1}\mathbf{ABB}^{-1} = \mathbf{AB}^{-1}$$

which is equivalent to

$$\mathbf{B}^{-1}\mathbf{A} = \mathbf{AB}^{-1}$$

This proves the lemma. □

Let us now show that $\mathbf{M}^{-1}$ and $\mathbf{G}$ commute, i.e. $\mathbf{M}^{-1}\mathbf{G} = \mathbf{GM}^{-1}$. We have $\mathbf{M} = (\mathbf{I} - \theta\,\mathbf{G})^{-1}$. Thus, $\mathbf{M}^{-1} = (\mathbf{I} - \theta\,\mathbf{G})$. As a result,

$$\mathbf{M}^{-1}\mathbf{G} = (\mathbf{I} - \theta\,\mathbf{G})\,\mathbf{G} = \left(\mathbf{G} - \theta\,\mathbf{G}^2\right) = \mathbf{G}\,(\mathbf{I} - \theta\,\mathbf{G}) = \mathbf{GM}^{-1}$$

Denote $\mathbf{A} = \mathbf{G}$ and $\mathbf{B} = \mathbf{M}^{-1}$. Since $\mathbf{G}$ and $\mathbf{M}^{-1}$ commute, then Lemma 1 shows that $\mathbf{GM} = \mathbf{MG}$.

Second step: Let us show that $\mathbf{TG} = \mathbf{GT}$. We have that

$$\mathbf{T} = \frac{\gamma}{(1+w)}\mathbf{M} + (1-\gamma)\mathbf{I}$$

This implies that

$$\begin{aligned}\mathbf{TG} &= \left[\frac{\gamma}{(1+w)}\mathbf{M} + (1-\gamma)\mathbf{I}\right]\mathbf{G} \\ &= \frac{\gamma}{(1+w)}\mathbf{MG} + (1-\gamma)\mathbf{G}\end{aligned}$$

Since $\mathbf{MG} = \mathbf{GM}$, this can be written as:

$$\begin{aligned}\mathbf{TG} &= \frac{\gamma}{(1+w)}\mathbf{GM} + (1-\gamma)\mathbf{G} \\ &= \mathbf{G}\left[\frac{\gamma}{(1+w)}\mathbf{M} + (1-\gamma)\mathbf{I}\right] \\ &= \mathbf{GT}\end{aligned}$$

Third step: Let us show that $\lim_{t\to\infty}\mathbf{T}^t = \lim_{t\to\infty}\mathbf{G}^t$. Assume that $\mathbf{G}$ is diagonalizable and aperiodic.[26] This implies that $\mathbf{T}$ is diagonalizable. Indeed, if $\mathbf{G}$ is diagonalizable, $\mathbf{M}$ is diagonalizable being a polynomial of $\mathbf{G}$. Since $\mathbf{T}$ is a linear convex combination of $\mathbf{I}$ and $\mathbf{M}$, two diagonalizable matrices, it is also diagonalizable.

Then, since we have seen that $\mathbf{G}$ and $\mathbf{T}$ commute, then they have the same eigenvectors (this is a standard result in linear algebra; see e.g., Strang, 2023). We know that $\mathbf{G}$ converges because $\mathbf{G}$ is aperiodic. Then, $\mathbf{G}$ and $\mathbf{T}$ have the same eigenvector associated with the maximum eigenvalue (which is 1 here), which we denote by $\mathbf{e}$. Since both $\mathbf{T}$ and $\mathbf{G}$ are row-normalized, they both have the same largest eigenvalue equal to 1. This implies that:

$$\mathbf{e}^{\mathrm{T}}\,\mathbf{T} = \mathbf{e}^{\mathrm{T}}\,\mathbf{G} = \mathbf{1}\,\mathbf{e}^{\mathrm{T}}$$

This proves part (i) of the proposition.

Assume now that $\mathbf{G}$ is periodic. Observe that $\mathbf{G}$ and $\mathbf{G}_\epsilon$ commute. Indeed

$$\mathbf{GG}_\epsilon = \mathbf{G}[\epsilon\mathbf{I} + (1-\epsilon)\mathbf{G}] = \epsilon\mathbf{G} + (1-\epsilon)\mathbf{G}^2 = [\epsilon\mathbf{I} + (1-\epsilon)\mathbf{G}]\mathbf{G} = \mathbf{G}_\epsilon\mathbf{G}$$

Then, by substituting $\mathbf{G}_\epsilon$ to $\mathbf{G}$ and using exactly the same proof as for the case when $\mathbf{G}$ was aperiodic, we obtain the proof of part (ii) of the proposition. □

Proof of Proposition 2. Consider, first, the dynamics of norms in (2). It is straightforward to see that, in steady state, $\mathbf{s}^\infty = \mathbf{x}^\infty$ when $\mathbf{s}^{\mathbf{t+1}} = \mathbf{s}^{\mathbf{t}} = \mathbf{s}^\infty$ and $\mathbf{x}^{\mathbf{t+1}} = \mathbf{x}^{\mathbf{t}} = \mathbf{x}^\infty$. Consider equation (5) that we report here:

$$\mathbf{x}^t = \left(\frac{1}{1+w}\right)[\mathbf{I} - \theta\,\mathbf{G}]^{-1}\,\mathbf{s}^t \tag{39}$$

Then, in steady state,

$$\mathbf{x}^\infty = \left(\frac{1}{1+w}\right)[\mathbf{I} - \theta\,\mathbf{G}]^{-1}\,\mathbf{x}^\infty \tag{40}$$

[26] With some abuse of notation, we say that $\mathbf{G}$ is aperiodic if the submatrix associated to each closed communication class is aperiodic.

We can write is as:

$$(1+w)\mathbf{x}^{\infty}+\omega\mathbf{G}\mathbf{x}^{\infty}=\mathbf{x}^{\infty} \tag{41}$$

With some algebra, it is immediate to have $\mathbf{x}^{\infty}=\mathbf{G}\mathbf{x}^{\infty}$. This means that $\mathbf{x}^{\infty}$ is the right eigenvector associated to the unit eigenvalue since $\mathbf{G}$ is row-stochastic.

Consider now the utility function in steady state. Since $\mathbf{x}^{\infty}=\mathbf{G}\mathbf{x}^{\infty}$, then $(x_i^{\infty}-\sum_j g_{ij}x_j^{\infty})^2$. Moreover, in steady state $x_i^{\infty}=s_i^{\infty}$. Then the utility is null for all agents, and the welfare is maximal. □

Proof of equation (15). Recall that

$$\mathbf{x}^t=\frac{1}{1+w}[\mathbf{I}-\theta\mathbf{G}]^{-1}\mathbf{s}^t$$

Then

$$\mathbf{x}^t-\mathbf{s}^t=\Big\{\frac{1}{1+w}[\mathbf{I}-\theta\mathbf{G}]^{-1}-\mathbf{I}\Big\}\mathbf{s}^t$$

Notice that the matrix $\mathbf{M}$ is a stochastic matrix. Denote $\mathbf{B}=\left\{\frac{1}{1+w}[\mathbf{I}-\theta\mathbf{G}]^{-1}-\mathbf{I}\right\}$. Then, $b_{ii}=m_{ii}-1<0$, $b_{ij}=m_{ij}>0$, for all $i\neq i$, and $\sum_{j\neq i}b_{ij}<1$. Moreover, $b_{ii}=-\sum_{j\neq i}b_{ij}$

Looking at the ith row of $\mathbf{x}^t-\mathbf{s}^t$, we get $x_i^t-s_i^t=\sum_{j\neq i}m_{ij}s_j^t-\sum_{j\neq i}m_{ij}s_i^t=\sum_{j\neq i}m_{ij}(s_j^t-s_i^t)$. □

Proof of Proposition 4. Using Proposition 8 in Appendix C.4, we can write the Markov process associated with $\hat{\mathbf{G}}$ by reconsidering equation (54) for our case. We obtain:

$$\hat{\mathbf{s}}^{\infty}=\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&\cdots&0&0\\0&0&0&1&0\\\mathbf{g}^{\infty}_{f_1}&\mathbf{g}^{\infty}_{f_2}&\cdots&\mathbf{g}^{\infty}_{f_n}&\mathbf{O}\end{pmatrix}\hat{\mathbf{s}}^{(0)} \tag{42}$$

where $\mathbf{g}^{\infty}_{f_i}$ is the vector of weights assigned by each agent in N to the specific fictitious agent f_i. Let us now consider how each of these vectors looks like. Using the same results as in Proposition 8, we have:

$$\mathbf{g}^{\infty}_{f_i}=[\mathbf{I}-\theta\mathbf{G}]^{-1}\mathbf{g}_{f_i} \tag{43}$$

where $\mathbf{g}_{f_i}$ is the column vector of weights that agents in N assign to the fictitious f_i. We now recall that only agent i assigns a positive weight to f_i and that this is equal to $\frac{1}{1+w}$. Then, by denoting by m_{ij} the generic entry of matrix $[\mathbf{I}-\theta\mathbf{G}]^{-1}$, and by $\mathbf{m}_{*\mathbf{i}}$, its i^{th} column, we obtain:

$$\mathbf{g}^{\infty}_{f_i}=[\mathbf{I}-\theta\mathbf{G}]^{-1}\mathbf{g}_{f_i}=\frac{1}{1+w}\mathbf{m}_{*\mathbf{i}} \tag{44}$$

We then can derive the following:

$$[\mathbf{g}^{\infty}_{1_\alpha}|\mathbf{g}^{\infty}_{2_\alpha}|\dots|\mathbf{g}^{\infty}_{n_\alpha}]=\frac{1}{1+w}[\mathbf{I}-\theta\mathbf{G}]^{-1} \tag{45}$$

which is row-normalized. This proves the first part of the proposition.

To find the equilibrium actions just plug $\mathbf{s}^{\infty}$ into (17). This completes the proof. □

Proof of Proposition 5. Consider equation (27). By simple algebra, we obtain:

$$\boldsymbol{\alpha}=(1+w)[\mathbf{I}-\theta\mathbf{G}]\mathbf{s}^{\infty} \tag{46}$$

Since $\theta=w/(1+w)$, we have:

$$\boldsymbol{\alpha}=[(1+w)\mathbf{I}-w\mathbf{G}]\mathbf{s}^{\infty}=\mathbf{s}^{\infty}+w(\mathbf{I}-\mathbf{G})\mathbf{s}^{\infty} \tag{47}$$

Given a target $\mathbf{s}^*$, we immediately have that the optimal $\boldsymbol{\alpha}^*$ is $\boldsymbol{\alpha}^*=\mathbf{s}^*+w(\mathbf{I}-\mathbf{G})\mathbf{s}^*$. Then

$$\boldsymbol{\sigma}^*=\mathbf{s}^*-\boldsymbol{\alpha}+w(\mathbf{I}-\mathbf{G})\mathbf{s}^* \tag{48}$$

Notice however that, if for all $i,j\in I$, $s_i^*=s_j^*$, then $(\mathbf{I}-\mathbf{G})\mathbf{s}^*=0$ since every agent has a target preference equal to the average of her neighbors' targets. Then $\boldsymbol{\sigma}^*=\boldsymbol{s}^*-\boldsymbol{\alpha}$. This proves the results 1 and 2. Result 3 is obtained by inverting (28). □

Appendix B. Some results in linear algebra

We state here (without proving them) some standard results in linear algebra.

Definition 2 *(Irreducible matrix).* $\mathbf{G}$ is said to be irreducible if all agents form a unique communication class. Otherwise $\mathbf{G}$ is reducible. In terms of network, $\mathbf{G}$ is irreducible if it is a strongly connected network.

Lemma 2. *Consider* $\mathbf{G}$ *to be an irreducible matrix. Then its largest eigenvalue* $\lambda_1 = 1$ *and there is a positive eigenvector associated to it.*

Definition 3 *(Periodicity).* A state in a Markov chain is periodic if the chain can return to the state only at multiples of some integer larger than 1. Formally, the period $d(x)$ of state $x \in S$ is:

$$d(x) = gcd\left\{n \in \mathbb{N}_+ : P^n(x,x) > 0\right\} \tag{49}$$

where S is the state space, P the transition probability matrix and gcd stands for greater common divisor. State x is aperiodic if $d(x) = 1$ and periodic if $d(x) > 1$

Lemma 3. *All agents belonging to the same communication class have the same period.*
If a matrix is periodic then $\lim_{t\to\infty} \mathbf{G}^t$ *does not exist and there are precisely* d *complex eigenvectors of length one.*

Definition 4 *(Primitive Matrix).* If $\mathbf{G}$ is aperiodic and irreducible, it is called *primitive*

Definition 5. Consider the eigenvector associated to the unit eigenvalue of a stochastic matrix $\mathbf{G}$. We refer to it as the Perron-Froebenius eigenvector, and we call it $\mathbf{e}(\mathbf{G})$.

Consider $\mathbf{Q}$ defined in equation (51).

Lemma 4. *Consider a matrix* $\mathbf{G}$ *in its canonical form. Then,* $\lim_{t\to\infty} \mathbf{Q}^t = 0$ *entry wise. This convergence happens geometrically fast.*

Appendix C. Convergence and network topology

C.1. Basic definitions

Let us provide some results of Markov processes that we write in terms of our model, where we do not have transition matrices that determine the probability of switching from one state to another, but, instead, we have the influence of the network.

We now propose some useful definitions.

Definition 6 *(Influenced Agents).* Agent i is influenced by agent j if there exists a sequence of nodes (agents) such that $g_{ik}g_{kw}\cdots g_{zj} > 0$. We denote this as $i \to j$. If j is also influenced by i then $i \leftrightarrow j$.

Notice that $i \to j$ does not mean that g_{ij} are linked, but that there is a path from i to j. Considering, for example, the first network $\mathbf{G_1}$ in Fig. 1. Then, we can see that $8 \to 6$ since $g_{87}g_{71}g_{15}g_{56} > 0$.

Definition 7 *(Communication Class).* A set $C \subset N$ is called a communication class if: (a) $i, j \in C \Rightarrow i \leftrightarrow j$; (b) $i \in C$ and $i \leftrightarrow j \Rightarrow j \in C$.

In words, a communication class is a subset of agents who are all (directly or indirectly) influenced by each other. In the networks in Fig. 1, the agents belonging to the same communication class have the same color. This means that the first network is composed of agents that all belong to the same communication class while, in the two other networks, each has three different communication classes.

Definition 8 *(Closed Communication Class).* A set $C \subset N$ is a closed communication class if: $i \in C$ and $j \notin C \Rightarrow i$ is not influenced by j.

Given that $g_{ii} = 0$, closed communication classes cannot be formed by singletons. Consider the networks given in Fig. 1. In network 2, there is one closed communication class composed of agents 1, 2 and 3 while, in network 3, there are two closed communication classes composed of agents 1, 2, 3 and 4, 5, 6, respectively. In these networks, closed communication classes are represented by circles while other communication classes (so called transient communication classes) by diamonds. Observe that the definition of closed communication classes implies that there are no links between agents belonging to two closed communication classes.

C.2. Convergence for strongly connected networks

C.2.1. General results

We first analyze the case of a *strongly connected* network $\mathbf{G}$. Recall that a network is strongly connected if all nodes belong to a unique closed communication class. There are, however, two main differences with the standard case. First, in the standard case, the steady-state preferences are given by the network matrix $\mathbf{G}$. In our case, we have seen that the dynamics of preferences is governed by $\mathbf{T}$ instead of $\mathbf{G}$. Second, the standard results only hold for aperiodic $\mathbf{G}$ while we will prove convergence of preferences also for periodic $\mathbf{G}$.

Proposition 6. *Assume that the network is strongly connected. If* $\mathbf{G}$ *is aperiodic, then,*

$$\lim_{t\to+\infty} \mathbf{T}^t\,\mathbf{s} = \begin{pmatrix} \mathbf{e}(\mathbf{G}) \\ \cdots \\ \cdots \\ \mathbf{e}(\mathbf{G}) \end{pmatrix} \mathbf{s} \quad \text{for every } \mathbf{s} \in [0,1]^n$$

If $\mathbf{G}$ *is periodic, the same result holds by considering the matrix* $\mathbf{G}_\epsilon$ *instead of* $\mathbf{G}$.

Proof of Proposition 6. Consider the case in which $\mathbf{G}$ is aperiodic. Then, as shown in Proposition 1, $\lim_{t\to+\infty}\mathbf{T}^t = \lim_{t\to+\infty}\mathbf{G}^t$. A strongly connected network implies $\mathbf{G}$ to be irreducible. As shown, for example, in Golub and Jackson (2010), for an irreducible and aperiodic network,

$$\lim_{t\to+\infty} \mathbf{G}^t = \begin{pmatrix} \mathbf{e}(\mathbf{G}) \\ \cdots \\ \cdots \\ \mathbf{e}(\mathbf{G}) \end{pmatrix}$$

Consider now the case in which $\mathbf{G}$ is periodic. Proposition 1 states that $\lim_{t\to+\infty}\mathbf{T}^t = \lim_{t\to+\infty}\mathbf{G}_\epsilon^t$. Then $\mathbf{G}_\epsilon$ is aperiodic. From the same reasoning as before we get

$$\lim_{t\to+\infty} \mathbf{G}_\epsilon^t = \begin{pmatrix} \mathbf{e}\mathbf{G}_\epsilon \\ \cdots \\ \cdots \\ \mathbf{e}\mathbf{G}_\epsilon \end{pmatrix}$$

This proves the result. □

The convergence results come from Markov chain theory together with our finding that the asymptotic properties of $\mathbf{T}$ and $\mathbf{G}$ are the same. While it is well known that, for a strongly connected $\mathbf{G}$, its limit is given by its Perron-Froebenius eigenvector, the way in which the two matrices $\mathbf{G}$ and $\mathbf{T}$ relate to each other is not straightforward, and depends on the way material incentives and preferences updating process interact. More precisely, the matrix $\mathbf{T}$ is *irreducible* because the associated network $\mathbf{G}$ is assumed to be strongly connected. Moreover, the matrix $\mathbf{T}$ is *aperiodic* because every agent assigns a positive weight to herself in the preference updating process ($\gamma > 0$) and, since $\mathbf{T}$ is a function of the network $\mathbf{G}$, which is strongly connected, there are always self loops. This implies that $t_{ii} > 0$, $\forall i$. Since $\mathbf{T}$ is an irreducible and aperiodic non-negative matrix, it is a *primitive* matrix. This guarantees that the process converges to a unique steady state. Finally, since the matrix $\mathbf{T}$ is *row stochastic*, its largest eigenvalue is 1, and therefore, there is a unique left eigenvector $\mathbf{e}(\mathbf{T})$ with positive components such that $\mathbf{e}(\mathbf{T}) = \mathbf{e}(\mathbf{T})\,\mathbf{T}$. The eigenvector property is just saying that $e_i = \sum_{j\in N} t_{ji} e_j$ for all i, so that the agents with greater influence have a greater weight in the final convergence levels.

Proposition 6 also derives a similar result when the matrix $\mathbf{G}$ is periodic. When this is the case, it is impossible to find $\lim_{t\to\infty}\mathbf{G}^t$. However, for the very same reasons described above, the matrix $\mathbf{T}$ is primitive. Then it is enough to study the perturbed matrix $\mathbf{G}_\epsilon$ instead of $\mathbf{G}$ to obtain the convergence result.

C.2.2. Example

Let us illustrate our convergence result for strongly connected networks stated in Proposition 6. In this proposition, we characterize the unique steady-state preference s_i^∞ for each individual i, which is the same for all individuals in the network, and show that it is equal to a weighted average of the initial preferences of all agents, where the weights are each agent's eigenvector centrality. This means that the more central an individual is in the network, the higher is her weight in the determination of the common steady-state preference. Let us illustrate this result for network 1 in the left panel of Fig. 1, which is a strongly connected network with one communication class (i.e. there is no agent outside the communication class) and understand the implications in terms of integration preferences.

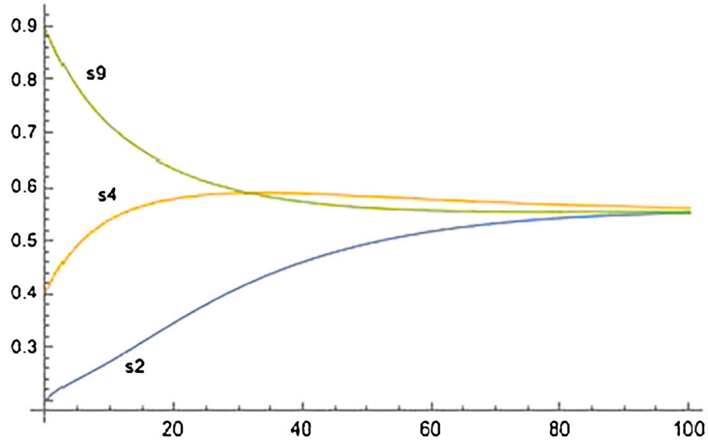

Fig. 2. Norm convergence in a strongly connected network.

The adjacency matrix of network 1 in Fig. 1 is given by:

$$\mathbf{G_1} = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Assume the following (transpose) vector of initial preferences:

$$[\mathbf{s^{(0)}}]^T = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{bmatrix} \tag{50}$$

We expect the preferences to converge to the same value, which is given by the weighted average of the initial preferences, where the weights are given by the eigenvector centralities of the agents. Assume that $w = \gamma = 0.5$. We easily obtain:

$$\mathbf{T_1^\infty} = \begin{pmatrix} \mathbf{e}(\mathbf{G}) \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \mathbf{e}(\mathbf{G}) \end{pmatrix} = \begin{bmatrix} 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.0952 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \\ 0.095 & 0.048 & 0.095 & 0.095 & 0.143 & 0.095 & 0.143 & 0.127 & 0.159 \end{bmatrix}$$

We can see that the most influencial individual is 9 because she is the more central agent (in terms of eigenvector centrality) in the network. The limit preference for integration will then be the weighted average of all initial preference, where the weights are given by each row of this matrix. It is easily verified that it is given by: $s_i^\infty = 0.559$ for all $i = 1, \dots, 9$. In other words, even though each agent has a very different initial integration preference, they all end up with the same preference equal to 0.559, which is slightly favorable to integration.

Let us now look at the dynamics of preferences and how they converge in steady state. For that, we examine the convergence of the integration preferences of individuals 2, 4, and 9, who belong to the same communication class but to different groups and have very different initial integration preferences. Fig. 2 illustrates this dynamics.

We can see that these individuals start with very different initial preference (at time $t = 0$) and then, as time passes by, they converge to the same weighted average preference $s^\infty = 0.559$. Individual 9 starts with a very high integration preference ($s_9^{(0)} = 0.9$) while individual 2 starts with $s_2^{(0)} = 0.2$, a preference not in favor of integration (for example, because her parents want to keep their original culture and language and feel threatened by the majority culture). Still, because of peer effects, social interactions and the structure of the network, these two agents end up with the same steady-state preference equal to 0.559, which implies a relatively favorable level of integration preference.

C.3. Convergence for networks with only one closed communication class

C.3.1. General results

Denote by $\mathbf{C}$ the $k \times k$ (transition) matrix associated to the closed communication class with k agents. Then, for this subset of agents only, the previous result about strongly connected networks holds. We need to understand, however, the convergence preferences of all the other agents in the network who do not belong to the closed communication class. Since their preferences depend directly or indirectly on that of the agents in the closed communication class, they also need to converge to the preference of the agents in the closed communication class. Denote by $\tilde{\mathbf{e}}(\mathbf{C}) = [\mathbf{e}(\mathbf{C})|0, \cdots, 0]$ the $1 \times n$ vector composed of $\mathbf{e}(\mathbf{C})$ augmented by $n-k$ entries equal to 0.

Proposition 7. *Assume that* $\mathbf{G}$ *only has one closed communication class of* k *agents. Then, in steady state, the preference of all agents in the network will converge to the same value. If* $\mathbf{C}$ *is aperiodic, then*

$$\mathbf{s}^{\infty} = \begin{pmatrix} \tilde{\mathbf{e}}(\mathbf{C}) \\ \cdots \\ \cdots \\ \tilde{\mathbf{e}}(\mathbf{C}) \end{pmatrix} \mathbf{s}^{(0)}$$

If $\mathbf{C}$ *is periodic, then the same result holds by considering the matrix* $\mathbf{C}_{\epsilon}$ *instead of* $\mathbf{C}$.

Proof of Proposition 7. Consider first the agents belonging to the closed communication class. The dynamics of their preferences is represented by the matrix $\mathbf{C}$. Previous proposition characterizes the convergence for the preferences of these agents. Consider now also agents not belonging to the closed communication class. The entire process is a uni-reducible Markov process and the steady state for all agents is the steady state of agents belonging to the closed communication class. □

As above, this result relies on the fact that $\mathbf{G}$ and $\mathbf{T}$ have the same asymptotic properties and that there exists only one closed communication class. Observe that, since the communication class is composed of more than one agent, then the convergence value is given by the Perron-Froebenius eigenvector centrality in the sub-matrix representing the communication class.

C.3.2. Example

Let us now study networks with only one closed communication class. In Proposition 7, we characterize the unique steady-state preference s_i^{∞} of each individual i belonging to the closed communication class and shows that it is equal to a weighted average of the initial preferences of the individuals belonging to the closed communication class, where the weights are determined by each agent's eigenvector centrality. Importantly, all individuals who do not belong to the closed communication class will be assigned a weight of zero when determining the common steady-state preference. In other words, in steady-state, all individuals will adhere to the same common integration preference, which is calculated by taking the weighted average of all initial preferences where the weights are equal to the eigenvector centrality of each individual who belongs to the closed communication class and zero for all the other individuals who do *not* belong to the closed communication class.

Consider network 2 in Fig. 1, which has only one closed communication class composed of agents 1, 2, 3. Thus, we expect the norm of all agents outside of the closed communication class to converge to the common steady-state integration preference of agents 1, 2, 3. The adjacency matrix of network 2 is given by:

$$\mathbf{G_2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

The matrix of the closed communication class $C = \{1, 2, 3\}$ is given by:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

and is periodic (period 2). To show the convergence, we need to determine $\mathbf{C}_{\epsilon}$ (see equation (10)), with, for example, $\epsilon = 0.1$. Then,

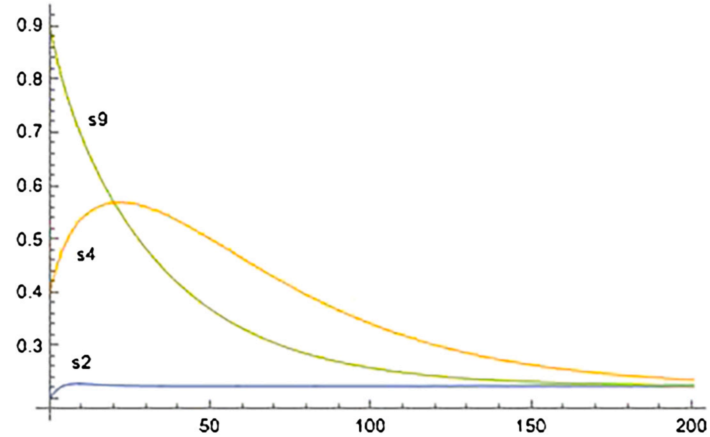

Fig. 3. Preference convergence with one communication class.

$$\mathbf{C}_\epsilon = \begin{bmatrix} \frac{1}{10} & 0 & \frac{9}{10} \\ 0 & \frac{1}{10} & \frac{9}{10} \\ \frac{9}{20} & \frac{9}{20} & \frac{1}{10} \end{bmatrix}$$

From $\mathbf{C}_\epsilon$ we can calculate $\mathbf{C}_\epsilon$. For that, in $\mathbf{G_2}$, we replace $\mathbf{C}$ by $\mathbf{C}_\epsilon$. In that case, the convergence matrix is given by:

$$\mathbf{T}_\mathbf{2}^\infty = \begin{pmatrix} \tilde{\mathbf{e}}(\mathbf{G}) \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \tilde{\mathbf{e}}(\mathbf{G}) \end{pmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Individual 3, who is the most central agent in the closed communication class, has the highest weight (i.e. 0.5). If we consider the same vector of initial preferences as in (50), then, the limit integration preference will be equal to:

$$s_i^\infty = 0.25 s_1^{(0)} + 0.25 s_2^{(0)} + 0.5 s_3^{(0)} = 0.225, \quad \text{for all } i = 1, \dots, 9$$

Clearly, all individuals who are not in the closed communication class (i.e. agents 4,..., 9) have no influence on this steady-state norm but will still adopt it. Indeed, as stated above, if there is only one closed communication class, then this class produces a system whose dynamics is independent of the dynamics of the other individuals in the network not belonging to this closed communication class.

Let us now examine the dynamics of preferences and how they converge in steady state. For that, we look at the convergence of the preferences of individuals 2, 4, and 9, who belong to different groups (only 2 belongs to the closed communication class). Fig. 3 displays these dynamics.

We can see that individual 4, who is the furthest away from the closed communication class and who starts with an initial integration preference of 0.4 follows first the preferences of individual 9. Then, eventually, she converges to the preference of the closed communication class. On the contrary, individual 9 is not influenced by individual 4 because no agent in 9's communication class is influenced by any agent from 4's communication class. What is interesting here is that, in terms of integration preferences, agents 4,..., 9, who have inherited a relatively high integration preference (especially agents 7, 8, 9) will end up having a steady-state preference of 0.225, which is not at all favorable to integration. This is due to their positions in the network, which are peripheral, and the fact that the agents who matter (those belonging to the closed communication class) have low initial preference for integration.

C.4. Convergence for any network

C.4.1. General results

Let us now state our result on convergence of social norms for any possible network. Given any network $\mathbf{G}$, we can partition the set of agents such that $N = C_1 \cup C_2 \cup \cdots \cup C_m \cup Q$ where $C_1 \cdots C_m$ are closed communication classes while Q is formed by all the remaining agents. Moreover we call C_p a generic communication class, $n_p := |C_p|$ and $n_Q := |Q| = n - \sum_{p=1}^m n_p$.

Consider now a network $\mathbf{G}$. By columns and rows transposition, we can write the matrix $\mathbf{G}$ as an upper triangular block matrix, in its *Canonical Form*, as follows:

$$\widetilde{\mathbf{G}} = \begin{pmatrix} \mathbf{C_1} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{C_2} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C_m} & 0 \\ \mathbf{Q}_{C_1} & \mathbf{Q}_{C_2} & \mathbf{Q}_{C_{\dots}} & \mathbf{Q}_{C_m} & \mathbf{Q} \end{pmatrix} \tag{51}$$

where $\mathbf{C_p}$ is the transition matrix associated to the C_p closed communication class, while the last rows report the matrices representing how much agents not belonging to any C_p are influenced by those agents in a generic C_p, for $i = 1, \dots, m$. Notice that the generic $\mathbf{Q}_{C_p}$ has dimension $n_Q \times n_p$, that every $\mathbf{C}_p$ is stochastic, and that $\mathbf{Q}$ is substochastic.

To illustrate this, consider network 3 in Fig. 1. Its adjacency matrix is given by:

$$\mathbf{G_3} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \tag{52}$$

where this matrix keeps track of *outdegrees* only and where the sum of each row is equal to one (row-normalized matrix). Consider, for example, the first row corresponding to agent 1. Since we only consider outdegrees, the link between 1 and 7 does not appear here and it only exists in row 7.

It is easily verified that this network has two closed communication classes, $C_1 = \{1,2,3\}$ and $C_2 = \{4,5,6\}$ and thus $Q = \{7,8,9\}$, which means that agents 7, 8, 9 do not belong to any closed communication class. Then, the canonical form of $\mathbf{G_3}$ is given by $\widetilde{\mathbf{G}}_3$, where

$$\mathbf{C_1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}; \mathbf{C_2} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}; \mathbf{Q} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}; \mathbf{Q}_{C_1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \mathbf{Q}_{C_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}. \tag{53}$$

In particular, $\mathbf{Q}_{C_1}$, which keeps track of the links between individuals who do not belong to any closed communication class, i.e. individuals 7, 8, 9, and those who belong to $C_1 = \{1,2,3\}$, shows that there is only one direct link between individuals 7 and 1 with weight 1/3. Similarly, $\mathbf{Q}_{C_2}$, which keeps track of the links between individuals who do not belong to any closed communication class, i.e. individuals 7, 8, 9, and those who belong to $C_2 = \{4,5,6\}$, shows that there is only one direct link between individuals 8 and 6 with weight 1/3.

Definition 9. Consider the left eigenvector associated to the unit eigenvalue of a stochastic matrix $\mathbf{G}$. This is the Perron-Froebenius eigenvector, and we call it $\mathbf{e}(\mathbf{G})$, which is defined as: $\mathbf{e}^T(\mathbf{G}) = \mathbf{e}^T(\mathbf{G})\mathbf{G}$.

Recall that $\mathbf{e}^T(\mathbf{G})$ is a row vector, and, given that $\mathbf{G}$ is a stochastic matrix, has positive entries whose sum is normalized to one. We are now ready to characterize the steady-state preferences in terms of the structure of the network $\mathbf{G}$. We first recall some well-known results stating that, if the network is strongly connected, then in steady state preferences converge to the same value (see Appendix C.2). This result easily extends to the case of network with just one closed communication class (see Appendix C.3). Consider now the case of a generic network $\mathbf{G}$.

While the convergence level of preferences of agents belonging to a closed communication class can be derived directly from previous results on strongly connected networks, the characterization of preferences of agents not belonging to any closed communication class is more challenging. Define $\mathbf{O}$ as a square matrix of zeros of dimension n_q. Denote by $\mathbf{e}(\mathbf{C_p})$ the Perron-Frobenius eigenvector for the communication class C_p, and denote by $e_i(\mathbf{C_p})$ its i^{th} entry.

Proposition 8. *Consider any network* $\mathbf{G}$. *Each agent* i *maximizes utility (3) and her preference follows the updating process described in (6). Assume* $\mathbf{G}$ *to be aperiodic. Then, in steady state,*

$$\mathbf{s}^{\infty} = \begin{pmatrix} \mathbf{C_1}^{\infty} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{C_2}^{\infty} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C_m}^{\infty} & 0 \\ \mathbf{Q}^{\infty}_{C_1} & \mathbf{Q}^{\infty}_{C_2} & \mathbf{Q}^{\infty}_{C_{\dots}} & \mathbf{Q}^{\infty}_{C_m} & \mathbf{O} \end{pmatrix} \mathbf{s}^{(0)} \tag{54}$$

where:

$$\mathbf{C_p^\infty} = \begin{pmatrix} \mathbf{e(C_p)} \\ \dots \\ \dots \\ \mathbf{e(C_p)} \end{pmatrix}^{n_p \times n_p}, \ \forall p = 1, \dots, m. \tag{55}$$

$$\mathbf{Q}_{C_p}^\infty = [\mathbf{q}_{1_p}^\infty | \mathbf{q}_{2_p}^\infty | \dots | \mathbf{q}_{n_{p_p}}^\infty]^{n_q \times n_p}, \ \forall p = 1, \dots, m, \tag{56}$$

and each column is given by the vector

$$\mathbf{q}_{i_p}^\infty = e_i(\mathbf{C_p})[\mathbf{I} - \mathbf{Q}]^{-1} \cdot \mathbf{Q}_{C_p} \cdot \mathbf{1}, \ \forall i \in C_p, \ \forall p = 1, \dots, m \tag{57}$$

If any $\mathbf{C_p}$ is periodic, then the same result holds by considering the corresponding perturbed matrix $\mathbf{C_{p\epsilon}}$.

Proof of Proposition 8. Consider first agents belonging to closed communication class. Then, by previous analysis it is proved that (55) holds.

Consider now the weight assigned by each agent in Q to any other in Q. It is a well-known result that the matrix of these weights is null.[27] This immediately comes from the fact that, taking the power matrix $\mathbf{G}^t$, the block corresponding to $\mathbf{Q}$ is given by $\mathbf{Q}^t$. Since $\mathbf{Q}$ is a substochastic matrix, then $\mathbf{Q}^\infty = \mathbf{O}$.

Consider now how each $\mathbf{g}_{i_p}^\infty$ is shaped. Recall, first, that all agents belonging to a communication class C_p have preferences converging to the same value. Call x_{kp} the weight that agent $k \in Q$ assigns in steady state to the communication class C_p. Then $\mathbf{x_p}$ is the vector of dimension n_q assigning the steady state weights of all agents in Q to the communication class C_p. As proved in Theorem 4.4 of Senata (1981), we have:

$$\mathbf{x_p} = [\mathbf{I} - \mathbf{Q}]^{-1} \mathbf{Q_{C_p}} \mathbf{1} \tag{58}$$

Then, if we want to determine the weight that agent $i \in C_p$ contributes to $\mathbf{x_p}$, it is enough to multiply $\mathbf{x_p}$ by $e_i(\mathbf{C_p})$. Indeed, we have:

$$\mathbf{g}_{i_p}^\infty = e_i(\mathbf{C_p})[\mathbf{I} - \mathbf{Q}]^{-1} \mathbf{Q}_{C_p} \mathbf{1}, \ \forall i \in C_p, \ \forall p = 1, \dots, m \tag{59}$$

This completes the proof. □

This result shows that the preferences of agents not belonging to any closed communication class converge to some average of the others' preferences, which is given by (54). Notably this result holds independently of $\mathbf{G}$ being periodic or not. Let us now show how the limit matrix is calculated.

Consider, first, the agents belonging to any of the closed communication classes C_p, identified by the matrix $\mathbf{C_p}$ of dimension n_p. Then, the long-run preferfences of agents belonging to C_p are given by $\mathbf{e(C_p)}$. Thus, we can characterize the block-diagonal matrix $\mathbf{C_p^\infty}$ as in (55).

Next, consider the agents who do not belong to any closed communication class, characterized by the matrix $\mathbf{Q}$. Notice that $\mathbf{Q}$ characterizes the network existing only among these agents, disregarding the links from and towards any other agent belonging to any closed communication class. In other words, $\mathbf{Q}$ is the adjacency matrix of the network $\mathbf{G}$ restricted to agents in Q (i.e. agents not belonging to any closed communication class). Consider now agents $h \in Q$, and $i \in C_p$. We would like to determine the weight that h assigns to i. In (56), $\mathbf{Q}_{C_p}^\infty$ gives the matrix of weights in steady state that all agents in Q assign to agents in C_p. The ith column of this matrix, denoted by $\mathbf{q}_{i_p}^\infty$ and reported in (57), represents the weights that agents in Q assign to agent $i \in C_p$. Their steady-state preference value depends on the weights directly or indirectly assigned to other agents in $\mathbf{Q}$ times how much each of them is directly linked to agents in any C_p times $e_i(\mathbf{C_p})$, that is how much $i \in C_p$ weights in the final preferences of agents in C_p.

Proposition 8 describes this result. Indeed, consider equation (57) and focus on how $\mathbf{q}_{i_p}^\infty$ is constructed. The matrix $[\mathbf{I} - \mathbf{Q}]^{-1}$, which is convergent given that $\mathbf{Q}$ is a substochastic matrix, is a Neumann series representing how much any two agents in Q directly or indirectly assign weight to each other. Now, select another agent $k \in \mathbf{Q}$ and call m_{hk} the generic entry of $[\mathbf{I} - \mathbf{Q}]^{-1}$. Then, m_{hk} gives how many direct or indirect walks h has towards k. Then, fixing agent $i \in C_p$, we have:

$$e_i(\mathbf{C_p})[\mathbf{I} - \mathbf{Q}]^{-1} \mathbf{Q}_{C_p} \mathbf{1} = e_i(\mathbf{C_p}) \sum_{k \in Q} m_{hk} \sum_{j \in C_p} g_{kj}, \ \forall h \in Q .$$

The weight $h \in Q$ assigns to $i \in C_p$ depends on how many paths $h \in Q$ has towards any other $k \in Q$, times how much weight any $k \in Q$ assigns to agents in C_p times how much weight $i \in C_p$ has in the final norm of C_p. $[\mathbf{I} - \mathbf{Q}]^{-1}$ is usually called the *Fundamental Matrix of Absorbing Chains*. By doing so, in terms of our model, we can fully characterize the steady-state level of preferences for any network.

Let us summarize how we calculate the steady-state preferences in any network. Consider a network where there are m closed communication classes, where, in each closed communication classes C_p, there are n_p individuals, and n_Q individuals who do not belong to any closed communication class. If the total number of individuals in the network is n, then $n = \sum_{p=1}^{p=m} n_p + n_Q$. First, we

[27] See, for example, Theorem 4.3 in Senata (1981).

need to write the adjacency matrix $\mathbf{G}$ in its canonical form $\tilde{\mathbf{G}}$ as in equation (51). In $\tilde{\mathbf{G}}$, there are three types of matrices that keep track of the weighted links between all agents in the network. First, there are the squared matrices $\mathbf{C}_p$, for $p = 1, ..., m$ (each has a dimension $n_p \times n_p$) corresponding to the closed communication classes. Second, there is the squared matrix $\mathbf{Q}$ (which is of dimension $n_Q \times n_Q$) of all agents who do not belong to any closed communication class. Third, there are the matrices $\mathbf{Q}_{\mathbf{C}_p}$, for $p = 1, ..., m$ (each has a dimension $n_Q \times n_p$), which keep track of the links between individuals who do not belong to any closed communication class and those who belong to the communication class C_p.

Given the matrix $\widetilde{\mathbf{G}}$, Proposition 8 characterizes the preferences of all individuals in steady state. First, for all individuals belonging to the same closed communication class C_p, they all converge to the same steady state preference, which is a weighted average of their initial preferences and where each weight is the eigenvector centrality of each individuals belonging to C_p. Second, for all the other individuals who do not belong to any closed communication class, their limiting preferences are calculated as in (56) and (57), which mean that the preferences of these individuals converge to some average of the preferences of agents belonging to the closed communication classes. However, this convergence depends on their position in the network and if there is a path between a given individual not belonging to any closed communication class and someone else in a closed communication class.

C.4.2. Example

Let us provide an example to illustrate the results stated in Proposition 8.[28] In this proposition, we show that different preferences will emerge in steady state. Indeed, if there are m closed communication classes and n_Q individuals who do not belong to any closed communication class, then there will be $m + n_Q$ steady-state preferences.

Consider network 3 in Fig. 1. Its adjacency matrix is given by (52). Let us now determine the steady state preferences of all individuals in the network. As stated above, since there are two closed communication classes and three individuals who do not belong to any closed communication class, there will be five preferences in steady state. It is easily verified that the communication class C_1 is periodic (period 2) so that we will use $C_{1\epsilon}$ with $\epsilon = 0.10$. Using (10), we have:

$$\mathbf{C}_{1\epsilon} = \begin{bmatrix} 0.1 & 0 & 0.9 \\ 0 & 0.1 & 0.9 \\ 0.45 & 0.45 & 0.1 \end{bmatrix}$$

We obtain:

$$\mathbf{T}_3^\infty = \begin{bmatrix} 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0. & 0. & 0. & 0.333 & 0.333 & 0.333 & 0 & 0 & 0 \\ 0. & 0. & 0. & 0.333 & 0.333 & 0.333 & 0 & 0 & 0 \\ 0. & 0. & 0. & 0.333 & 0.333 & 0.333 & 0 & 0 & 0 \\ 0.156 & 0.156 & 0.313 & 0.125 & 0.125 & 0.125 & 0 & 0 & 0 \\ 0.0938 & 0.0938 & 0.188 & 0.208 & 0.208 & 0.208 & 0 & 0 & 0 \\ 0.125 & 0.125 & 0.25 & 0.167 & 0.167 & 0.167 & 0 & 0 & 0 \end{bmatrix} \tag{60}$$

We see that the limit preference of the closed communication class $C_1 = \{1, 2, 3\}$ will only take into account the weights of individuals $1, 2, 3$ while that of the closed communication class $C_2 = \{4, 5, 6\}$ will only take into account the weights of individuals $4, 5, 6$. We also see that, to calculate the limit preference of each individual who do not belong to any closed communication class, i.e. individuals $7, 8, 9$, one puts a weight of zero for each of them and puts a positive weight for each other individual belonging to either C_1 or C_2. Interestingly, the weights depend on the position in the network and the distance between the individual who does not belong to any closed communication class and those who belong to a closed communication class. Take, for example, individual 7 (row 7 in $\mathbf{T}_3^\infty$). In the determination of her limit preference, the highest weight has been put on individual 3 because there is a direct link between individual 7 and someone from C_1 (i.e. individual 1), and individual 3 has the highest eigenvector centrality (0.5) of all individuals in C_1. The lowest weight has been put on individual 4, 5 or 6 because there is no direct link between individual 7 and anybody from C_2 and all individuals in C_2 have the same eigenvector centrality equal to 0.333.

To better understand this result, let us see how the matrix $\mathbf{T}_3^\infty$ is constructed. First, we calculate the matrix that keeps track of the walks of different lengths in the network only for individuals who do not belong to any closed communication class (i.e. individuals $7, 8, 9$):

$$[\mathbf{I} - \mathbf{Q}]^{-1} = \begin{bmatrix} \frac{15}{8} & \frac{9}{8} & 1 \\ \frac{9}{8} & \frac{15}{8} & \frac{8}{3} \\ \frac{3}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

Consider, for example, the weight each agent in Q assigns to agent 1 (who belongs to C_1) in steady state. This is given by $\mathbf{q}_{i_1}^\infty$, for $i = 7, 8, 9$. In Proposition 8, we show that: $\mathbf{q}_{i_1}^\infty = e_1(\mathbf{C_1})[\mathbf{I} - \mathbf{Q}]^{-1} \cdot \mathbf{Q}_{C_1} \cdot \mathbf{1}$. We have: $e_1(\mathbf{C_1}) = 0.25$. Then, we easily obtain:

[28] In Appendices C.2 and C.3, we provide examples where we calculate the convergence of preferences for strongly connected networks and networks with one closed communication class, respectively.

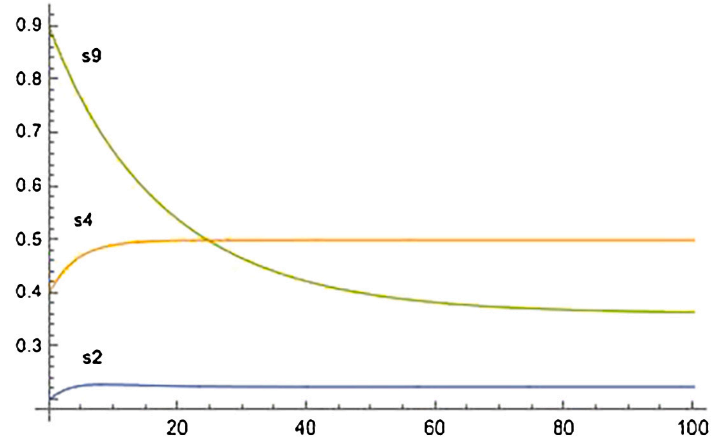

Fig. 4. Norm convergence with two communication classes.

$$\mathbf{q}_{i_1}^{\infty} = 0.25 \begin{bmatrix} \frac{15}{8} & \frac{9}{8} & 1 \\ \frac{9}{8} & \frac{15}{8} & \frac{8}{3} \\ \frac{3}{2} & \frac{3}{2} & 2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.156 \\ 0.0938 \\ 0.125 \end{bmatrix}$$

This means that, when calculating the steady-state preference, the weight individual 7 assigns to individual 1 is equal to 0.156. Let us see how it is calculated. We have:

$$q_{71}^{\infty} = \underbrace{0.25}_{e_1(C_1)} (\underbrace{\frac{15}{8}}_{m_{77}} \times \underbrace{\frac{1}{3}}_{g_{71}} + \underbrace{\frac{9}{8}}_{m_{78}} \times \underbrace{0}_{g_{81}} + \underbrace{1}_{m_{79}} \times \underbrace{0}_{g_{91}}) = 0.156$$

Thus, the positive weight of 0.156 that individual 7 assigns to 1 is due to the fact that there is a direct link between 7 and 1. Observe that the other individuals 8 and 9 also assign a positive weight to individual 1, albeit smaller, because of the direct link between 7 and 1. We can then perform the same exercise to determine how the weights are determined between individuals $7, 8, 9$ and individuals in C_2. Clearly, it will be through the link between 8 and 6. So, it is clear here how the position in the network of each individual has an impact on the weight that will be assigned in the steady-state preference.

To see that, let us now determine how preferences converge in steady state. Assume the following (transpose) vector of initial preferences:

$$[\mathbf{s}^{(\mathbf{0})}]^T = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{bmatrix} \tag{61}$$

It is then easily verified that the steady-state (transpose) vector of preferences is given by:

$$[\mathbf{s}^{(\infty)}]^T = \begin{bmatrix} 0.225 & 0.225 & 0.225 & 0.5 & 0.5 & 0.5 & 0.328 & 0.397 & 0.363 \end{bmatrix} \tag{62}$$

Indeed, all agents belonging to the first closed communication class $C_1 = \{1, 2, 3\}$ converge to the same preference equal to 0.225, all agents belonging to the second closed communication class $C_2 = \{4, 5, 6\}$ converge to the same preference equal to 0.5, and, finally, all the other agents $7, 8, 9$ converge to different steady-state preferences given by 0.398, 0.397 and 0.363, respectively. Even though individuals $7, 8, 9$ have high initial integration preferences, they end up converging to a much lower level of integration preference than individuals belonging to C_2 because their weights are zero and their steady state preference is a convex combination of the preferences of the two closed communication classes, which have preferences between 0.225 and 0.5.

Let us now study the dynamics of the integration preferences of individuals 2, 4, and 9, who belong to different groups. It is depicted in Fig. 4.

The integration preference of individual 2, who belongs to the first communication class C_1, converges to a low-value of integration (0.225) because all agents in this communication class have a low initial preference in terms of integration. The same intuition applies for the second closed communication class C_2 but with a higher value of the integration preference because of higher initial norms. Finally, for individuals 7, 8 and 9, who do not belong to any closed communication class, their steady-state preference is totally independent of their initial preferences, which have high values. Their steady-state preferences only depend on their position in the network, the distance between their location, and that of the two closed communication classes. We can see from network 3 in Fig. 1 that there is always a path between any agent 7, 8, 9 and the two communication classes. As a result, the limit preference will be a combination of the steady-state preferences of the two closed communication classes.

Appendix D. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jebo.2024.02.021.

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