Part II

SECESSION THREATS & LOGNORMAL INCOME DISTRIBUTIONS[1]

Abstract: Secession threats arise worldwide for different reasons: ethnic divisions, religion, or economic issues. This paper studies how wealth and income distribution affect preferences for separation within a democratic country. Incomes are represented through two-parameters lognormal density functions. Given secession rule, we assume indifference between unification and separation in one of the regions at the beginning. If the parameters of the distribution vary in that region or in the rest of the country, would the region secede? We would expect that the region secedes if its wealth has increased and does not secede if wealth in the rest of the country has increased. Our paper shows that there are cases where the region secedes even if wealth in the rest of the country has increased, and cases where the region does not secede even if its wealth has increased. Such results depend upon the skewness of income distributions and the levels of taxation that follow.

Key Words: Secession, Income Distribution, Wealth, Taxation

JEL Code: H7, D3, H2

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1   Introduction

At the end of World War II, there were less than 80 independent countries worldwide; now the number of nations is around 200. China, the most populated one, has more than 1.2 billion inhabitants, but almost half of them has less than 5 millions inhabitants.

The breakup of colonial empires in the sixties and the collapse of USSR in the nineties are phenomena which can partially explain the increase in the number of nations. Many other secessions has happened worldwide; furthermore, there are centrifugal forces asking for decentralization and/or separation in many other countries. Secession threats arise worldwide for different reasons: ethnic divisions, religion, or economic issues. The processes leading to regionalism, separation and independence are sometimes violent like Chechnya versus Russia and sometimes non-violent like Scotland versus the United Kingdom.

Our starting point is the same of the model developed by Bolton and Roland (1997): separation is always inefficient from the economic point of view. Defense spending, for example, is more efficient in an unified country; furthermore, free trade among regions can be guaranteed much more easily in an unified country. On the other hand, we have to consider how the benefits from unification cannot be distributed among all citizens.

We will not focus on the ways to prevent secessions: other papers have already analyzed this topic in order to find the tax rate, the compensation scheme and the secession rule to prevent (inefficient) breakup of existing countries.[2] Our paper focuses on the effects of income distribution on secession threats within a country.

We assume the existence of a democratic country composed of several regions. Given secession rule, one of its regions, call it region A, is assumed to be indifferent between unification and separation at the beginning. Our analysis will show how the preferences for separation in region A would be affected by changes in wealth and distribution of incomes in the seceding region and also in the rest of the country, given the presence of efficiency losses from separation. Our model describes a decisional process where region A chooses between separation and unification on the basis of its own preferences; it does not display a strategic interaction between region A and the rest of the country.

Formally, our benchmark is the model by Bolton and Roland (1997) as

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[2] See, for example, Bolton and Roland (1996, 1997), Bordignon and Brusco (2001), Etro and Giarda (2002), Le Breton and Weber (2003).

interpreted by Alesina and Spolaore (2003):[3] individuals vote on taxation levels and their preferences depend upon their own income.

In our model incomes are distributed following a two-parameters lognormal density functions. We will develop our analysis focusing on wealth (median and average income levels) and income skewness (mean/median ratio[4]) through the parameters of the distribution functions. Bolton and Roland (1997) focused on differences in terms of income between median voters across regions; our paper focus on the effects of income skewness within regions.

Given indifference at the beginning for region A, we would expect that, if the rest of the country becomes richer, A does not secede (*ceteris paribus*); on the other hand, we would expect that, if region A becomes richer, A secedes (*ceteris paribus*). The results of our model are partially different: if the rest of the country becomes richer and its mean/median ratio increases, there are cases where A secedes (*ceteris paribus*); furthermore, if region A becomes richer and also its mean/median ratio increases, we can reasonably suppose that there are cases where region A does not secede (*ceteris paribus*).

This paper is organized as follows: Section 2 presents the model and discusses the assumptions; Section 3 is devoted to the algebraic analysis and Section 4 concludes.

2   The model

2.1   General assumptions

We assume the existence of a democratic country composed of several regions whose boundaries are exogenously given and immutable.

Our country can be represented through a spatial model where population has mass equal to 1 and it is continuously and uniformly distributed on the segment $[0, 1]$; furthermore, we assume that individuals are not mobile. Size (population) of region A equals $s_A \in (0, 1)$; size (population) of the rest of the country, call it R, equals $s_R \in (0, 1)$ and:

$$s_A + s_R = 1$$

holds.

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[3] The utility function used by Alesina and Spolaore in "The Size of Nations" (2003) to discuss the model by Bolton and Roland (1997) derives from the utility function they used in "On the Number and the Size of Nations" (1997).

[4] Graphically, mean/median income ratio is the inverse of the slope of the tangent to the Lorenz curve at the 50th percentile; details on this point can be found in Section 2.5.

Incomes are not uniformly distributed on the geographical space $[0, 1]$. The density functions of income distributions are given by $\phi_A(y_i)$ in region A and $\phi_R(y_i)$ in the rest of the country. We have:

$$\left.\begin{array}{c} y_{Am} < \bar{y}_A \\ y_{Rm} < \bar{y}_R \end{array}\right\} \Rightarrow y_m < \bar{y}$$

where y_m is median income and $\bar{y}$ is average income.[5]

Given a proportional taxation scheme, individuals vote on the tax rate within the jurisdiction where they live and Median Voter Theorem holds: the preferences of individuals over public spending are single peaked and depend upon their own income.[6]

There is perfect substitutability between public and private goods and tax revenues are assumed to finance public good provision and lump-sum redistribution.[7]

As we will show later on, we assume that public good provision is exogenous and independent from size in order to show in a simple way that it is not possible to have efficiency gains from separation. Under this assumption, any individuals of the seceding region will have to pay more taxes in order to finance the same level of public good, given that the size of the seceding region is strictly smaller than the one of the unified country and given that taxes are proportional to income.

2.2   Utility of individual i

Following these simplifying assumptions, the utility of individual i living in jurisdiction j (unified country or the seceding region A) is given by:

$$u_{ij} = g + (1 - \tau_j)y_i + T_j \tag{1}$$

where: g is exogenous public good provision, τ_j is the tax rate in jurisdiction j, y_i represents the income of individual i and T_j is the transfer each individual will get from the government in the jurisdiction where he lives.

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[5] Notice that in the paper we have subscript A for region A, subscript R for the rest of the country and no subscript for the unified country.

[6] In this model we abstract from heterogeneity of preferences over types of public goods, and focus on hereogeneity of incomes as the determinant of different preferences between individuals. This is the main difference with respect to the model of geopolitical organization developed by Alesina and Spolaore in 1997, where heterogeneity of preferences over public good is given by the distance from the point where the public good is located.

[7] As in the model by Meltzer and Richard (1981) we consider a tax-transfer scheme in which the revenues of a proportional income tax are redistributed lump-sum.

2.3 What changes in case of separation?

Scale economies are modeled in a very simple way: both public good provision (g) and cost of public goods (k) are exogenous and independent from size and $g = k$ holds. Furthermore, there are deadweight losses from taxation: in particular, 1 dollar of taxes provides $1 - \tau_j/2$ dollars for transfers and public goods.

Lump-sum redistribution, net of the costs of public good provision, is financed through a proportional taxation scheme and tax rate is chosen by the individuals of each jurisdiction; as a consequence, transfer from the government would be equal to:

$$T_A = \left(\tau_A - \frac{\tau_A^2}{2}\right) \bar{y}_A - \frac{k}{s_A}$$

in independent region A, whereas it is equal to:

$$T = \left(\tau - \frac{\tau^2}{2}\right) \bar{y} - k$$

in the unified country.

Notice that Median Voter Theorem holds, therefore the tax rate would be given by:

$$\frac{\partial u_A(y_{Am})}{\partial \tau_A} = 0 \Rightarrow \tau_A = \frac{\bar{y}_A - y_{Am}}{\bar{y}_A} \tag{2}$$

in independent region A; on the other hand, it is given by:

$$\frac{\partial u(y_m)}{\partial \tau} = 0 \Rightarrow \tau = \frac{\bar{y} - y_m}{\bar{y}} \tag{3}$$

in the unified country.

The utility of an individual living in independent region A would be:

$$u_A(y_i) = g + y_i + \frac{\bar{y}_A - y_{Am}}{2\bar{y}_A} \left[(\bar{y}_A - y_i) + (y_{Am} - y_i)\right] - \frac{k}{s_A} \tag{4}$$

The utility of the same individual living in the unified country is:

$$u(y_i) = g + y_i + \frac{\bar{y} - y_m}{2\bar{y}} \left[(\bar{y} - y_i) + (y_m - y_i)\right] - k \tag{5}$$

2.4 Secession rule and indifference condition

Secession Rule Region A secedes from the unified country when a majority of voters in region A is in favor of separation.[8]

Under the assumption that Median Voter Theorem holds and given secession rule, region A would secede if the utility of the median individual in A is higher under separation rather than under unification.

In our model we assume that the median individual in region A is indifferent between separation and unification at the beginning.

Indifference Condition $SU = u_A(y_{Am}) - u(y_{Am}) = 0$

Notice that if $SU > 0$ region A would secede; on the other hand, if $SU < 0$ region A would not secede.

If we substitute y_{Am} to y_i in (4) and (5), we obtain:

$$SU = \left\{ \frac{(\bar{y}_A - y_{Am})^2}{2\bar{y}_A} - \frac{k}{s_A} \right\} - \left\{ \frac{\bar{y} - y_m}{2\bar{y}} \left[(\bar{y} - y_{Am}) + (y_m - y_{Am}) \right] - k \right\} = 0$$

and after algebraic manipulations, we get:

$$SU = \frac{(y_m - y_{Am})^2}{2\bar{y}} + \frac{\bar{y}_A - \bar{y}}{2} + \frac{y_{Am}^2}{2\bar{y}_A} - \frac{y_{Am}^2}{2\bar{y}} - \frac{1 - s_A}{s_A} k = 0 \qquad (6)$$

2.5 Skewness index

It is possible to refer to different concepts in order to rank income distributions.[9] The ratio of mean to median income is mathematically simpler and easier to introduce in the model with respect to inequality-related indices and polarization-related indices.[10] Mean/median ratio refers to the skewness of the distribution and, graphically, it represents the inverse of the

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[8] We consider an "extremely weak" secession rule; this assumption seems reasonable when the central government is too weak to prevent a secession through military means.

[9] There are inequality related indices and polarization related indices; inequality and polarization focus on different aspects of a distribution. See, for example, Esteban and Ray (1994) and Wolfson (1994). For a complete discussion on this issue, see Cowell (1995) and Esteban and Ray (2005) .

[10] Mean/median ratio has been used as a proxy for both income inequality and income polarization in theoretical and empirical papers: see, for example, Meltzer and Richard (1981), Persson and Tabellini (1994, 2000), Wolfson (1994) and Alesina, Baqir and Easterly (1999).

slope of the tangent of the Lorenz curve at the $50th$ percentile. Given that average income is higher than median income; i.e., the income distributions are right-skewed, median/mean ratio equals 1 in case of egalitarian distribution of incomes and increases together with income skewness: the higher is the ratio, the higher is income skewness.

Skewness indices are:

$$SK_A = \frac{\bar{y}_A}{y_{Am}}$$

in region A, and:

$$SK = \frac{\bar{y}}{y_m}$$

in the unified country.

Let us go back to the tax rate chosen by individuals in region A and in the unified country. Taking into account (2) and (3) it is immediate to notice that within our framework the tax rate increases together with income skewness.

2.6   Two-parameters lognormal distribution function

We use two-parameters lognormal density functions to describe the distribution of incomes in our model.

The two parameters are the mean (μ) and the variance (σ^2) of the Normal density function:

$\mu = \int_{-\infty}^{+\infty} y f(y) dy$

$\sigma^2 = \int_{-\infty}^{+\infty} (y - \mu)^2 f(y) dy$

There are several reasons in order to justify the choice of such functions; lognormal distribution has convenient properties:[11]

It has a simple relationship with the normal distribution.

The interpretation of its parameters it is easy.

It generates symmetrical and non-intersecting Lorenz curves.

It provides a reasonable sort of fit to many actual data sets.

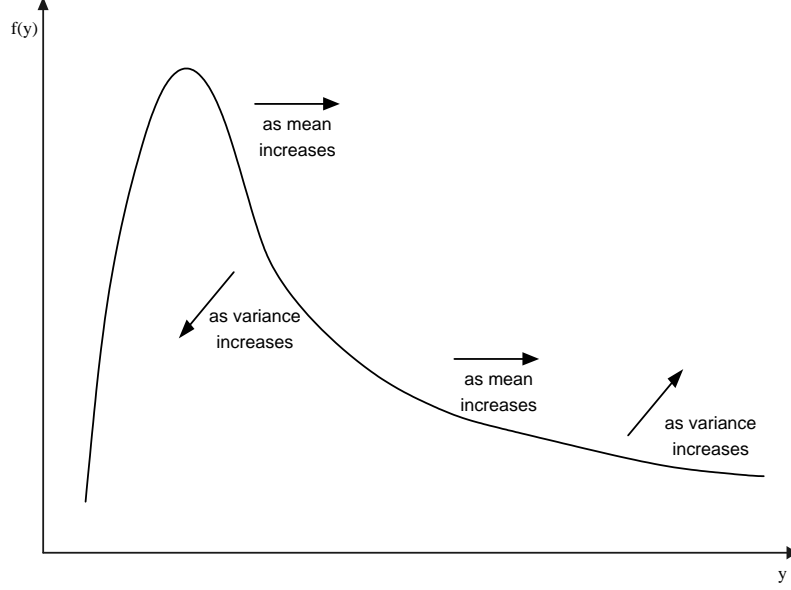[11] For a detailed discussion on this point, see Aitchinson and Brown (1957) and Cowell (1995).

Figure 1: Two-parameters Lognormal Distribution Function

If incomes are distributed through a two-parameters lognormal function, we have:

Region A	Unified country $(A + R)$
$\phi_A(y_i) = \frac{1}{y_i \sigma_A \sqrt{2\pi}} e^{-\frac{1}{2\sigma_A^2}(\ln y_i - \mu_A)^2}$	$\phi(y_i) = \frac{1}{y_i \sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\ln y_i - \mu)^2}$
$\bar{y}_A = e^{\mu_A + \frac{1}{2}\sigma_A^2}$	$\bar{y} = e^{\mu + \frac{1}{2}\sigma^2}$
$y_{Am} = e^{\mu_A}$	$y_m = e^{\mu}$
$SK_A = e^{\frac{1}{2}\sigma_A^2}$	$SK = e^{\frac{1}{2}\sigma^2}$

If we substitute the values of median and average income depending on mean and variance in (6), we obtain:

$$SU = \frac{(e^{\mu} - e^{\mu_A})^2}{2e^{\mu + \frac{1}{2}\sigma^2}} + \frac{e^{\mu_A + \frac{1}{2}\sigma_A^2} - e^{\mu + \frac{1}{2}\sigma^2}}{2} + \frac{e^{\mu_A^2}}{2e^{\mu_A + \frac{1}{2}\sigma_A^2}} - \frac{e^{\mu_A^2}}{2e^{\mu + \frac{1}{2}\sigma^2}} - \frac{1 - s_A}{s_A} k = 0$$

and, after algebraic manipulations, we get:

$$SU = \frac{1}{2}\left[e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right] - \frac{1-s_A}{s_A}k = 0 \tag{7}$$

Notice that in case of indifference at the beginning for R instead of A, an analogous equation holds.[12]

3 The analysis

If $SU = 0$, median individual in region A is indifferent between separation and unification. Our purpose is to check if changes in wealth and distribution of incomes lead to separation or not. We consider changes in the region involved in the break-up process, and also changes in the rest of the country.

3.1 Changes in the rest of the country

We consider variations in the parameters of the distribution function of the rest of the country having effects in the whole country, whereas the distribution function of region A remains unchanged. In order to simplify the derivation of the results, we consider variations in μ and σ^2 instead of variations in μ_R and σ_R^2, given (7).

3.1.1 μ increases

If the mean in the rest of the country increases, we have no effect on the skewness index as SK does not depend on μ:

The effect on SU is given by:

$$\frac{\partial SU}{\partial \mu} = \frac{1}{2}e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) < 0 \tag{8}$$

The second order derivative of SU with respect to μ gives us:

$$\frac{\partial^2 SU}{\partial \mu \partial \mu} = \frac{1}{2}e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) < 0$$

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[12]In particular, if R is indifferent between separation and unification at the beginning instead of A, expession (7) becomes:
$SU = \frac{1}{2}\left[e^{\mu}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_R}\left(e^{\frac{1}{2}\sigma_R^2} + e^{-\frac{1}{2}\sigma_R^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right] - \frac{1-s_R}{s_R}k = 0$

We have $SU > 0$ if the rest of the country is "poor enough" and poorer than region A; on the other hand, we have $SU < 0$ if the rest of the country is "rich enough" and richer than region A.
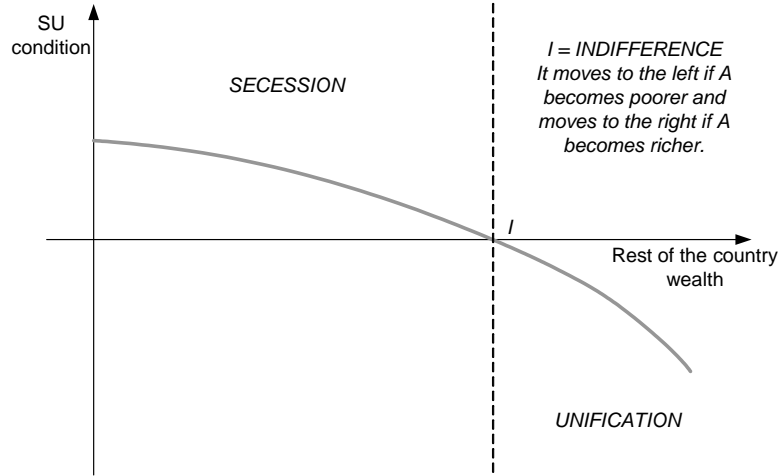

Figure 2: SU condition if R becomes richer

PROPOSITION 1   **Given $SU = 0$ and the following wealth increase in the rest of the country (with income skewness unchanged), A does not secede (*ceteris paribus*).**

Proof. If μ increases, both the median income and the average one increase in the rest of the country and in the whole country; on the other hand neither the skewness index nor the tax rate vary as a consequence.

The effects on the utility of median individual in A are the following:

$$\Delta u_{(y_{Am})}(\Delta T) > 0$$

under unification, and:

$$\Delta u_{(y_{Am})A}(\Delta T) < 0$$

under separation, due to efficiency losses.

Therefore, if μ increases, region A does not secede. ∎

In general, incentives to secede for region A decrease as wealth in the rest of the country increases.

3.1.2 σ increases

If σ increases, the effect on the skewness index is given by:

$$\frac{\partial SK}{\partial \sigma} = \sigma e^{\frac{1}{2}\sigma^2} > 0$$

On the other hand, the effect on SU condition is given by:

$$\frac{\partial SU}{\partial \sigma} = \frac{1}{2}\sigma \left[-e^{\mu} \left(e^{-\frac{1}{2}\sigma^2} + e^{\frac{1}{2}\sigma^2} \right) + 2e^{\mu_A - \frac{1}{2}\sigma^2} \right] \gtreqless 0 \qquad (9)$$

Notice that:

$$\frac{\partial SU}{\partial \sigma} = 0 \iff \sigma = \sqrt{\ln \left[2 \left(e^{\mu_A - \mu} - 1 \right) \right]}$$

It follows that the derivative of SU with respect to σ equals zero only if $\mu_A > \mu$, given $\sigma > 0$. As a consequence, we have to distinguish between two different cases: $\mu_A \leq \mu$ and $\mu_A > \mu$.

The second order derivative of SU with respect to σ gives us:

$$\frac{\partial^2 SU}{\partial \sigma \partial \sigma} = \frac{1}{2} \left[\left(\sigma^2 - 1 \right) \left(e^{\mu - \frac{1}{2}\sigma^2} - 2e^{\mu_A - \frac{1}{2}\sigma^2} \right) - \left(\sigma^2 + 1 \right) e^{\mu + \frac{1}{2}\sigma^2} \right] \gtreqless 0$$

Notice that the second order derivative is positive only if σ tends to 0^+; as a consequence, we assume:

$$\frac{\partial^2 SU}{\partial \sigma \partial \sigma} < 0$$

If $\mu_A \leq \mu$, SU is always negative.

If $\mu_A > \mu$, we have: $SU \gtreqless 0$ if the rest of the country is "not skewed" and less skewed than region A; $SU < 0$ if the rest of the country is "skewed enough" and more skewed than region A.
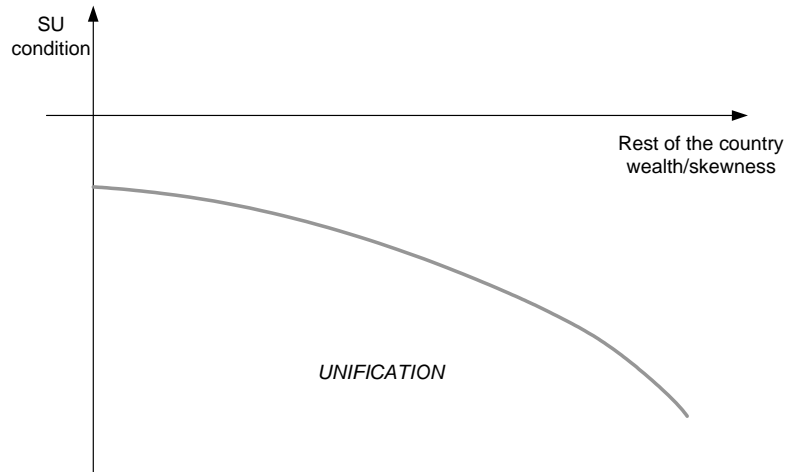
SU
condition

Rest of the country
wealth/skewness

UNIFICATION

Figure 3: SU if w/s in R increases $(\mu_A \leq \mu)$

SU
condition

?

(depending
on size and
cost of public
good)

SECESSION

I = INDIFFERENCE
It moves to the left if A
becomes poorer and
moves to the right if A
becomes richer.

I

Rest of the country
wealth/skewness

If the function cross the horizontal axis
twice, there is a case where sigma of
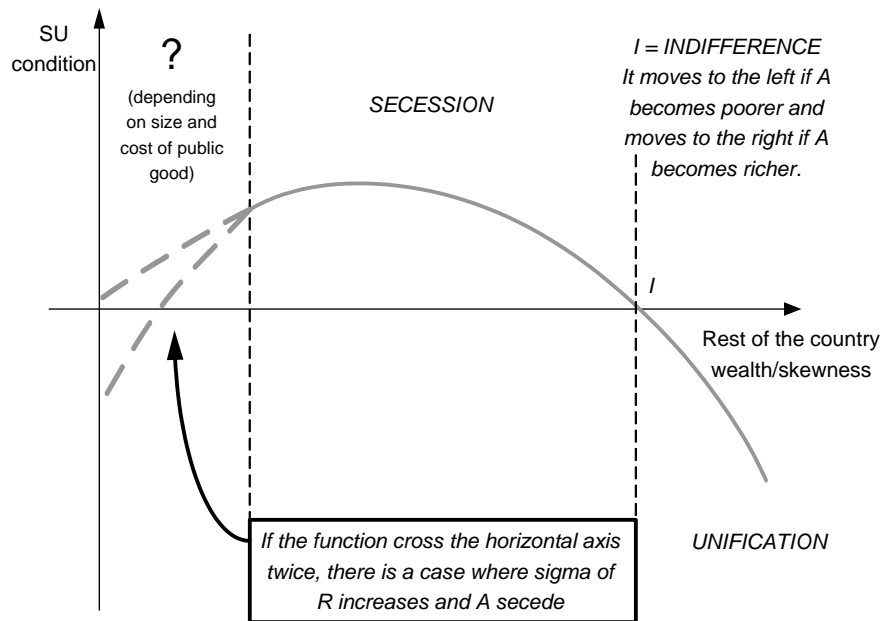*R increases and A secede*

UNIFICATION

Figure 4: SU if w/s in R increases $(\mu_A > \mu)$

PROPOSITION 2   **Given $SU = 0$ and the following wealth and income skewness increase in the rest of the country, there are cases where A secedes (*ceteris paribus*).**

Proof. If σ increases, the average income increases in the rest of the country and in the whole country, whereas the median income remains unchanged: the median individual becomes relatively poorer with respect to the individual with average income. As a consequence, income skewness and tax rate increase in the rest of the country and in the whole country.

The effects on the utility of the median individual in A are the following:

(i) If the median in region A is poorer than the median in the whole country we have that unification is Pareto superior to separation:

$$\Delta u_{(y_{Am})}(\Delta\tau, \Delta T) > \Delta u_{(y_{Am})A}(\Delta T)$$

Region A does not secede.

(ii) If the median in region A is richer than the median in the whole country we have that separation can be Pareto superior to unification as:

$$\Delta u_{(y_{Am})}(\Delta\tau, \Delta T) \gtreqless \Delta u_{(y_{Am})A}(\Delta T)$$

It depends upon the skewness of income distribution SK, the size of the seceding region s_A and the costs of public good k. ∎

In general, we would expect that incentives to secede for region A decrease as wealth and skewness in the rest of the country increases. Our analysis show that there are cases where such incentives increase; they increase if the median in region A is richer than the median in the rest of the country and σ is "low enough".

3.2   Changes in region A

We consider now variations in the parameters of the distribution function of region A having effects in the whole country, whereas the distribution function of the rest of the country remains unchanged. In particular, we consider variations in μ_A and σ_A^2 and also the variations in μ and σ^2 that follow.

3.2.1 μ_A increases

The mean is a linear operator, then:

$$\mu = s_A \mu_A + s_R \mu_R$$

If mean in region A increases, we have no effects on skewness indices as neither SK nor SK_A depend upon μ_A:

The effect on SU condition is given by:

$$\frac{\partial SU}{\partial \mu_A} = \frac{1}{2}\left[s_A e^{s_A \mu_A + s_R \mu_R}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right]$$

$$(10)$$

The analysis of this derivative is not straightforward; $\partial SU / \partial \mu_A$ is supposed to be negative if region A is "poor enough" and poorer than the rest of the country; on the other hand, $\partial SU / \partial \mu_A$ is supposed to be positive if region A is "rich enough" and richer than the rest of the country.

Let's consider now the second order derivative of SU condition with respect to μ_A:

$$\frac{\partial^2 SU}{\partial \mu_A \partial \mu_A} = \frac{1}{2}\left[s_A^2 e^{s_A \mu_A + s_R \mu_R}\left(e^{-\frac{1}{2}\sigma^2} - e^{\frac{1}{2}\sigma^2}\right) + e^{\mu_A}\left(e^{\frac{1}{2}\sigma_A^2} + e^{-\frac{1}{2}\sigma_A^2} - 2e^{-\frac{1}{2}\sigma^2}\right)\right]$$

The second order derivative is supposed to be negative only if μ_A tends to 0^+; it is positive otherwise.

We have $SU < 0$ if region A is "poor enough" and poorer than the rest of the country; on the other hand, we have $SU > 0$ if region A is "rich enough" and richer than the rest of the country.
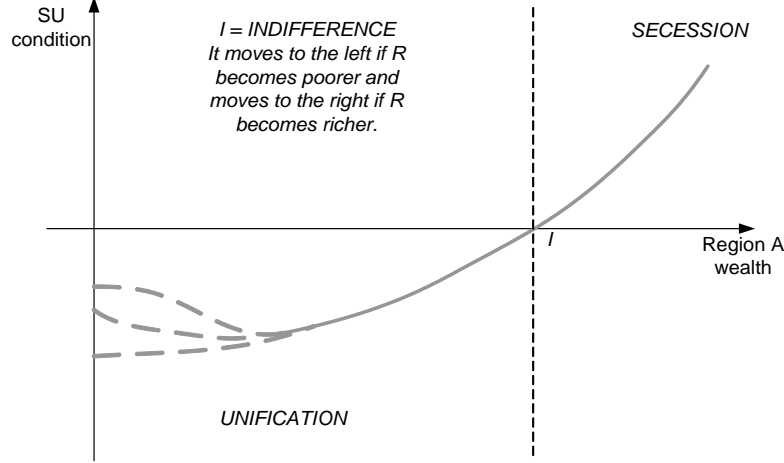
Figure 5: SU condition if Region A becomes richer

PROPOSITION 3   Given $SU = 0$ and the following wealth increase in region A (with income skewness unchanged), region A secedes (*ceteris paribus*).

Proof. If μ_A increases, both the median income and the average one increase in region A and in the whole country; on the other hand, neither the skewness index nor the tax rate vary as a consequence.

The effects on the utility of the median individual in A are the following:

$$\Delta u_{(y_{Am})}\left(\Delta y_{Am}, \Delta T\right) > 0$$

under unification (the effect is independent of the differential between y_{Am} and y_m), and:

$$\Delta u_{(y_{Am})A}\left(\Delta y_{Am}, \Delta T\right) > 0$$

under separation (the variation depends upon the differential between y_{Am} and y_m: positively if $y_{Am} > y_m$, negatively if $y_{Am} < y_m$).

As a consequence, if μ_A increases, the richer the median in region A with respect to the median individual in the whole country, the greater is supposed to be the positive effect under separation with respect to the positive effect under unification. If region A is poorer and poorer than the rest of the country, it could be the case that even if μ_A increases the median individual in A would prefer unification, but, if the median in A is indifferent

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between separation and unification at the beginning, region A secedes as a consequence of an increase in μ_A ∎

In order to analyze what happens following a variation in μ_A given every possible value of SU, we have to take care of different variables, but in general incentives to secede for region A increase together with the wealth of region A.

3.2.2 σ_A increases

The variance is not a linear operator; we have:

$$\sigma^2 = s_A^2 \sigma_A^2 + s_R^2 \sigma_R^2 + 2 s_A s_R \sigma_{AR}$$

where σ_{AR} is covariance.
Using Pearson Correlation Coefficient:

$$\rho_{AR} = \frac{\sigma_{AR}}{\sigma_A \sigma_R}$$

where $\rho_{AR} \in (-1 \ , \ 1)$, we can rewrite the variance in the whole country in terms of the variance in region A and in the rest of the country:

$$\sigma^2 = s_A^2 \sigma_A^2 + s_R^2 \sigma_R^2 + 2 s_A s_R \rho_{AR} \sigma_A \sigma_R$$

We can reasonably assume that the correlation between the variance in region A and the variance in the rest of the country is non-negative or, at least, "not too much negative".

Formally, we assume that the derivative of income variance in the whole country with respect to income variance in region A cannot be negative:

$$\frac{\partial \sigma^2}{\partial \sigma_A^2} \geq 0 \iff \rho_{AR} \geq -\frac{s_A \sigma_A}{s_R \sigma_R}$$

If σ_A increases, we have the following effects on skewness indices:

$$\frac{\partial SK}{\partial \sigma_A} = \left(s_A^2 \sigma_A + s_A s_R \rho_{AR} \sigma_R \right) e^{\frac{1}{2}(s_A^2 \sigma_A^2 + s_R^2 \sigma_R^2 + 2 s_A s_R \rho_{AR} \sigma_A \sigma_R)} > 0$$

$$\frac{\partial SK_A}{\partial \sigma_A} = \sigma_A e^{\frac{1}{2}\sigma_A^2} > 0$$

On the other hand, the effect on SU condition is given by:

$$\frac{\partial SU}{\partial \sigma_A} = -\frac{e^\mu}{2}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right)e^{-\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)} +$$

$$-\frac{e^\mu}{2}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right)e^{\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)} +$$

$$+\frac{e^{\mu_A}}{2}\sigma_A\left(e^{\frac{1}{2}\sigma_A^2} - e^{-\frac{1}{2}\sigma_A^2}\right) +$$

$$+e^{\mu_A}\left(s_A^2\sigma_A + s_A s_R \rho_{AR}\sigma_R\right)e^{-\frac{1}{2}\left(s_A^2\sigma_A^2 + s_R^2\sigma_R^2 + 2s_A s_R \rho_{AR}\sigma_A\sigma_R\right)}(11)$$

The analysis of this derivative is not straightforward; exactly as in case of an increase in σ, we consider the existence of two different cases: $\mu_A \geq \mu$ and $\mu_A < \mu$

Let's solve them graphically.

If $\mu_A \geq \mu$, SU is always positive.

Let us focus now on the case where $\mu_A < \mu$. $\partial SU/\partial \sigma_A$ is supposed to be negative if region A is "not skewed" and/or less skewed than the rest of the country; on the other hand, $\partial SU/\partial \sigma_A$ is supposed to be positive if region A is "skewed enough" and/or more skewed than the rest of the country.

The second order derivative of SU condition with respect to the skewness of region A is extremely complex: given our results in case of an increase in wealth and skewness in the rest of the country, the second order derivative can reasonably supposed to be positive (and negative only if σ_A tends to 0^+).

In general, SU is supposed to be positive if region A is "skewed enough" and more skewed than the rest of the country.
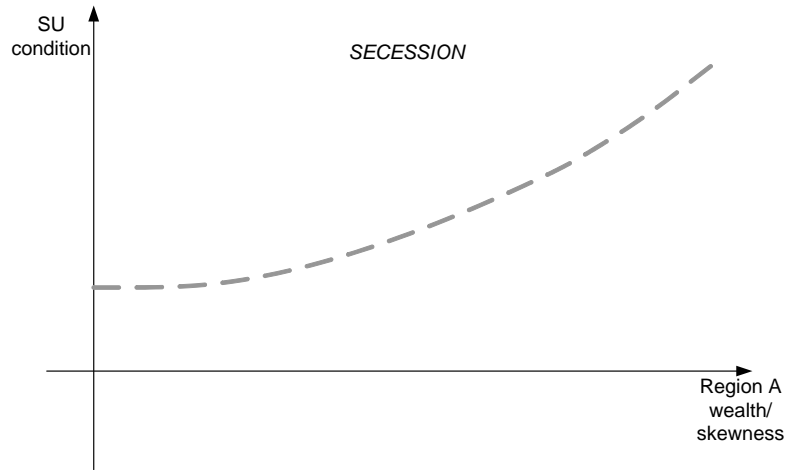
SU
condition

SECESSION

Region A
wealth/
skewness

Figure 6: SU if w/s in A increases $(\mu_A \geq \mu)$

SU
condition

If the function cross the horizontal axis twice, there is a case where sigma of A increases and A does not secede

SECESSION

I

Region A
wealth/
skewness

I = INDIFFERENCE
It moves to the left if R becomes poorer and moves to the right if R becomes richer.

?

(depending
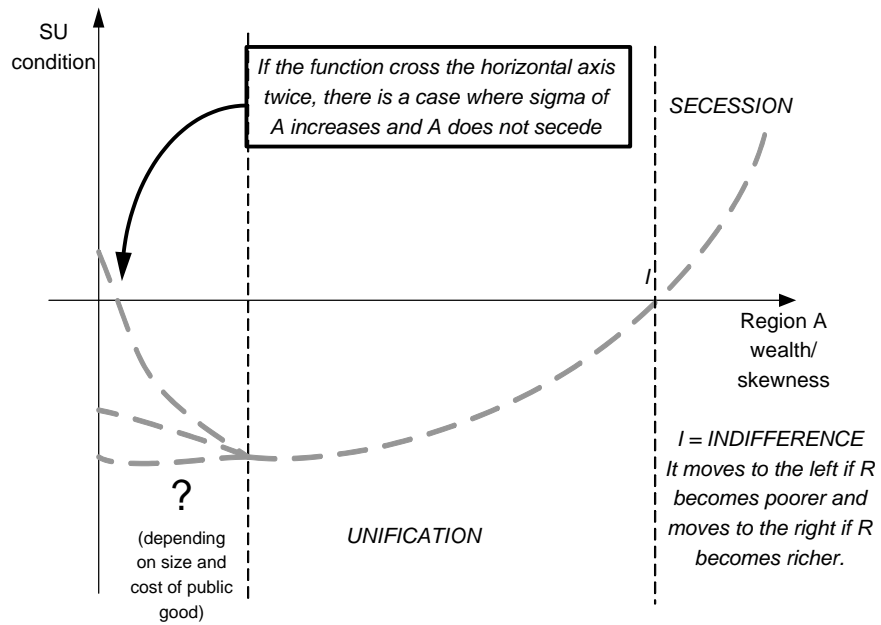on size and
cost of public
good)

UNIFICATION

Figure 7: SU if w/s in A increases $(\mu_A < \mu)$

PROPOSITION 4   **Given $SU = 0$ and the following wealth and income skewness increase in region A, there are cases where A does not secede (*ceteris paribus*).**

Proof. If σ_A increases, the average income increases in region A and in the country, whereas the median income remains unchanged: the median individual becomes relatively poorer with respect to the individual with the average income. As a consequence, the income skewness and the tax rate increase in region A and in the whole country.

The effects on the utility of the median individual in A are the following:

(i) If the median individual in region A is richer than the median in the whole country we have that separation is Pareto superior to unification as:

$$\Delta u_{(y_{Am})}(\Delta\tau, \Delta T) < \Delta u_{(y_{Am})A}(\Delta\tau, \Delta T)$$

(ii) If the median individual in region A is poorer than the median in the whole country we have ambiguous effects: it could be the case that an indifferent (at the beginning) median individual of region A would not secede as a consequence of an increase in σ_A. ∎

If we focus on variations in σ_A, the effects on SU are not easy to interpret. We made some simulations focusing on the effects of wealth, income skewness, size of the seceding region and cost of public good. If the median individual in A is "not too much poorer" than the median in the whole country, incentives to secede are supposed to increase for region A; on the other hand, the poorer the median in A with respect to the median in the country, the more ambiguous the effects of a variation in σ_A are supposed to be.

4   Conclusion

The model by Bolton and Roland (1997) showed that an increase in across regions inequality make separation more likely to occur. We develop an analysis of the effects of wealth and income distribution on the preferences of the seceding region. Our model shows a strong link between wealth and political separatism, but it also shows that there are cases where distribution effects overcompensate wealth effect. In particular, we find that an increase in the skewness of income distribution in the rest of the country can make separation more likely to occur, due to the different levels of taxation

chosen by the median voter. Such result could give an interesting hint in order to study the cases of regions asking for regionalism, separation and/or independence within developed countries: not only the already cited case of Scotland, but also Lombardia (Italy), Catalunya and Pais Vasco (Spain), Flanders (Belgium)...

We find several analyses whose results can be partially useful to check the goodness of fit of our theoretical model, even if they didn't deal explicitly with wealth, income skewness and secession threats; most of them deal with the issue of government decentralization.

Several empirical works found a negative correlation between average income and centralization: Wallis and Oates (1988) on the trends in fiscal centralization during the 20th century in state and local sector in the United States; Panizza (1999) on revenues and expenditure centralization ratios in a large sample of countries; on the other hand, the empirical analysis by Cerniglia (2003) on OECD countries shows that the correlation between income inequality and centralization seems to be positive.

Other works investigated the reasons of the collapse of Soviet Union and Russian Federation from economic perspectives. The theoretical model by Berkowitz (1997) on peripheral Russian regions shows how the impact of wealth increase on secession threats depends upon the efficiency gains from separation (in our model there are efficiency losses from separation), the substitutability between public and private goods (not considered in our model) and whether or not the demand for public good is stronger in center or periphery. Giuliano (2006) analyzed the arising of secessionism in former Soviet Union concluding that, through the framing on issues of ethnic economic inequality, nationalist leaders were able to politicize the ethnic issue by persuading people to view their personal life chances as dependent on the political fate of their ethnic community; economy becomes an instrument for politician to create secession wishes.

From a theoretical perspective, the model by Jaramillo, Kempf and Moizeau (2003) on the link between inequality and club formation gives us an interesting hint. The model shows that inequality leads to segmentation, therefore, given two distributions of endowments, the more inegalitarian generates more clubs; furthermore, a club becoming more and more inegalitarian is expected to break-up.

The results of Jaramillo, Kempf and Moizeau seems coherent with ours, in the sense that different levels of income inequality within regions make separation more likely to occur, but a clarification is in order; our model

shows that, due to different levels of taxation,[13] preferences for separation in the seceding region can increase together with income skewness in the rest of the country but they can decrease as income skewness increases in the seceding region.

In the very end of the paper, we need to note that almost nothing can be said on the effects of income skewness on secession threats in real world. An empirical analysis on this issue remains an unanswered question.

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[13]Let us recall that in our model the tax rate increases together with income skewness.

Glossary

s	size of region(s)
y_{ij}	income of individual i in region(s) j
$\bar{y}$	average income
y_m	median income
$u(.)$	utility function
g	public spending
τ	tax rate
T	transfer from the government
k	cost of public goods ($k = g$)
SU	Indifference condition
SK	Skewness Index (mean/median ratio)
$\phi_j(y_i)$	income distribution
μ_j	mean of the distribution
σ_j	standard deviation of the distribution
σ_j^2	variance of the distribution
$\sigma_{jj'}$	covariance
$\rho_{jj'}$	Pearson Correlation Coefficient
	SUPERSCRIPTS
$-$	average
	SUBSCRIPTS
i, j	individual, region(s)
m	median
A, R	region A, rest of the country

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