2. Computable General Equilibrium Models: Macroeconomics and Closures

The debate on macroclosures became popular in the late 1970s and in the early 1980s because of two contemporary events. The first is that Amartya Sen published his famous paper discussing four fundamental closures for a simple CGE model in 1963 on the wave of the strong academic debate between Neoclassicals and Neokeynesians. The second is that the first large- scale applied models were constructed (Adelman and Robinson for South Korea (1977), and Taylor *et al.* for Brazil (1980)) and their results were surprising.

With the improvements in computer science and more powerful mainframes, large- scale applied models were built. First attempts were made to conduct these analyses with Walrasian models interpreting any solution's deviation as the measurement of imperfect competitive behaviour and market failures. However, each country was a different case. Each of them had a different structure and different relationships among macroeconomic aggregates. So, each modeller's aim was to construct a more country- specific model. To succeed, the closure problem was crucial. As Taylor (1990) said: "*a sense of institutions and history necessarily enters into any serious discussion of macro causality*".

The debate started when Sen (1963) analysed a simple version of a closed CGE model and stated that "*it is no longer possible* […] *to simultaneously maintain the value of public consumption expenditures at a predetermined level, to compensate the economic agents according to marginal productivity in terms of the value of the factors of production they hold and to satisfy the labour market equilibrium*" (Decaluwé, Martens, and Savard, 2000). From a mathematical point of view, the system was over-determined and this meant it had more than one solution. Practically speaking, the problem was to have a squared system with an equal number of endogenous variables and equations. In this specific case the modeller had to choose to drop a specific assumption.

Depending on which assumption is dropped, the model has a different closure[1]: Neoclassical, Keynesian, Johansen, or Kaldorian[2]. Thus, the problem, from Sen's point of view, was theoretical and was derived from an extensive debate after Kaldor's review on income distribution.

A further step in the closure debate was the 1979 paper of Taylor and Lisy. Their work was based on the intuition that the results of an applied CGE model are affected by an aspect which is not usually analysed. Based on their experience with an applied model for Brazil, they were particularly concerned with distributional changes. We may describe their aim using Llunch's (1979) words: "*they wanted to see why policy experiments with their Brazil model had a large impact on the price level, a minor one on the labour share and almost none on aggregate output. with the model stripped down to the bare essentials, they found that in the hurry to disaggregate over commodities and agents, a different dimension had been forgotten: the disaggregation over closing rules*". They compared a traditional neoclassical system with two other Keynesian closures to see how the same model works. Effectively, when this happens many changes take place. The Keynesian closures allow for changes in output through the multiplier when changes in wages, and consequently in prices, occur.

Moreover, the core version of Sen's model was extended to include government (Rattsø, 1982, Robinson, 2003), and the external sector (Taylor and Lisy, 1979; Decaluwé, Martens, and Monette, 1987; Dewatripont and Michel, 1983; Robinson, 2003)[3]. In this case, the closure problem still holds, but becomes more complex. When the modeller closes a model, it refers to *ex- ante* equilibriums in different markets. For instance it should determine how the savings-investments market works, which aggregate is predetermined and which one moves to reach the equilibrium. In a closed economy, the only *ex- ante* equilibrium conditions to specify are the labour and the saving- investments markets. In an open economy we have to introduce a new equilibrium condition in the foreign exchange rate and to count for new sources of savings in the savings- investments balance.

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[1] Llunch (1979) simply reduces the problem to the dichotomy between Neoclassical and Keynesian closures. He states that the closure problem may be solved dropping one equation. If the modeller chooses to drop the exogenous fixed investments' assumption he obtains a Neoclassical closure. If the full employment assumption is dropped he has the Keynesian closure.

[2] These labels do not strictly trace the original work of the corresponding authors, but each of these definitions has its own variants. What is defined as "Kaldorian" is not properly related to the work of Nicolas Kaldor but it contains many different approaches: Neo- Keynesian, Neo- Marxian, Structuralist and obviously Kaldorian in a strict sense.

[3] A concise summary of the state- of - the - art in the closure debate is presented in table 1.

The aim of this chapter is dual. First, we want to describe in a theoretical way the different macroclosures that may be applied in a CGE model, focusing on the adjusting mechanism at the base of each one and how the structure of the model itself changes as a response to a change in the closures. Second, we want to quantify the effects of a closure rule choice. Therefore, we develop three simplified models. Two are for a closed economy, both with and without Government, and one is for an open economy. We apply the different closures and we discuss the final results. We are particularly interested in describing how the closures affect the result of a model, and furthermore to understand the impact of opening the model while applying the same closure. In other words, we are interested in comparing the results of the closed and open economy model with the same closure.

In the following pages, a brief summary of the state- of- the- art in macroclosure debate is presented. Here, fundamental papers are cited and for each of them we highlight which kind of model is investigated (i.e. closed or open), the nature of the analysis (i.e. theoretical or empirical application), which closure rules are applied (according to our distinction into the four fundamental closures) and final results.

Table 1: The State- of- the- Art in the Macroclosure Debate

Author	Framework	Problem	Closure	Result
Sen (1963)	Closed Economy	He recognizes a theoretical problem in the mathematical structure of a closed CGE: the system is over-determined thus it has more than one solution. It is impossible while fixing investments to have marginal productivity remuneration and full-employment.	He recognizes four main closure rules: Neoclassical, Keynesian, Johansen, and Kaldorian. Each of them drops one specific assumption.	Applying each of these closures the system is now determined with a unique solution.
Taylor, and Lisy (1979)	Open Economy	Analysis of the impact of different closure rules in a CGE with a distributional focus, as the large- scale model for Brazil they have already developed.	Neoclassical vs Keynesian closures.	The closure choice matters. The results of the Neoclassical approach are very different from the ones of the Keynesian. Moreover, the effects of a Keynesian closure are mitigated when any macroeconomic aggregate is fixed in nominal terms.
Llunch (1979)	Open Economy (more precisely the same of Taylor and Lisy (1979))	Analysis of few alternative closures on a simplified version of the Taylor and Lisy (1979) model.	Neoclassical with full employment as the reference. Classical unemployment and Keynesian unemployment.	The closure rule matters. However the author reduces its role. He supposes as sufficient how the modeller closes the labour market. The labour market rules characterize the closure of the model.
Rattsø (1982)	Closed and Open Economy	Analysis of the different closures and application to the original Johansen model.	He applies the four closures Sen had already classified.	He quantitatively analysed the effects of a different closure choice.
Dewatripont, and Michel (1983)	Open Economy	Study the closure rule problem in different exchange rate regimes.		When there is fixed exchange rate, the model has already closed. So, the closure rule is crucial only in a case of floating exchange rate.
Decaluwé, Martens, and Monette (1987)	Open Economy	Study in an open economy framework, the possibility of different closure rules, and their effects respect to supply and demand shocks.	They apply the usual four closures in a floating exchange rate regime.	They derive different magnitudes in the effects of the closure choice if they suppose a supply disturbance (increase in the capital stock) or a demand disturbance (increase in exports).

(table 1 continues)

Author	Framework	Problem	Closure	Result
Taylor (1990, 1991)	Literature Survey	Presentation of the concept of the problem, what the closure choice means.	He concentrates on the Kaldorian closures in comparison with the Neoclassical. The Kaldorian closures contain the main element of the Keynesian one (the aggregate demand effect) so that it is a comparison among the three models. Moreover he describes closures for heterodox models (Loanable funds closure, and the Pigou or Real Balance effect closure).	Theoretical presentation and analysis of the macroeconomics behind an adopted closure rule.
Abdelkhalek, and Martens (1996)	Open Economy	How to choose the appropriate closure rule when there is no prior information.	Neoclassical, Keynesian, Johansen.	The solution of the problem is testing the significance of the simulation imposing upper and lower bounds for each closure.
Thissen (1998)	Literature Survey	Analysis of the likely closures for a generic CGE model	He describes the four closures but he splits the Kaldorian closure into four different closures: the Neo-Keynesian that is the Kaldorian in a strict sense, the Kaleckian or Structuralist, the Loanable funds closure, and the Pigou or Real Balance effect closure.	A taxonomy of the different closures and a classification of empirical CGE models.
Decaluwé, Martens, and Savard (2000)	Open Economy	Effects of the alternative closures of the Neoclassical approach.	Keynesian, Kaldorian, Johansen.	There are different relations at the basis of each assumption. Mainly, they recognize a different mechanism for income generation and distribution.
Robinson (2003)	Closed and Open Economy	Analysis of the different closure rules in a closed and an open economy.	The four closure of Sen both in the closed economy and the open economy version.	He stresses the role of foreign savings in closing the saving-investment gap.
Gibson (2008)	Closed Economy	The closure problem may be overcome.	Keynesian vs Neoclassical closures.	The need of a choice in the closure rule may be overcome when we introduce multi-agents and dynamic.

I. The original Sen's dilemma

As previously cited, the closure rule problem arises through two distinct avenues. In mathematical terms this choice has to solve the problem of a system where the number of equations is not equal to the number of endogenous variables. In practice, the modeller decides which variables are endogenous and which ones are exogenous. Furthermore, the modeller's decision is a personal belief about the economic structure when deciding a plausible adjustment process. This statement was formally carried out by Sen in his 1963 paper.

Here, he demonstrates the simplest case of a closed economy without Government where the closure choice still matters[4]. As Rattsø (1982) presented, the framework is composed of 7 equations. In this model one product is produced with constant returns to scale (CRTS) technology, and factors are paid according to the value of their marginal productivity (equations 1 and 2). Then, only capital and labour are employed and they are fixed in supply (equations 6 and 7). Because of the exhaustion theorem, the total income is divided between profits and a wage bill (equation 3). In the model, there are two classes of agents, wage earners and rentiers, and each of them has a specific saving propensity. Moreover, investments are fixed in real terms. To reach equilibrium in the system, savings and investments must be equal.

Table 2: The original Sen's Model

$X = f(N, K)$	(1)
$PF_N = w$	(2)
$PX = rK + wN$	(3)
$PI = s_R rK + s_W wN$	(4)
$I = \bar{I}$	(5)
$N = \overline{N}$	(6)
$K = \overline{K}$	(7)

Source: ***Rattsø (1982)***

However if we count for the endogenous variables, there are only six: X, N, K, I, w/P, r/P. This means the system is over-determined. In order to be solved, it must have as many equations as unknowns.

According to Sen we must drop one assumption, but this choice is not trivial. There are a minimum of four possible choices, although as Robinson (2003) stresses, "*the different macroclosure models range along a continuum*". However, in terms of reference we mainly focus on the *Neoclassical*, *Keynesian*, *Kaldorian* and *Johansen* model closures. In a concise form, this choice may be reduced to dropping one specific equation. In the *Neoclassical closure*

[4] To have a quantitative exposition of the Sen's model and an empirical application in an archetype economy see section II. For the simulation we have employed, see the MPSGE/GAMS software.

we drop equation 5 so investments are not exogenously determined but endogenous, and consequently their amount is equal to savings. The *Keynesian closure* allows for unemployment which eliminates equation 6. In this case labour supply is not fixed, but endogenized. The *Johansen closure* is a mid- point between the Neoclassical and the Keynesian. It maintains the neoclassical setup on the production side but there is also an exogenous level of investments (as in Keynes). In this case, the fundamental mechanism works through an endogenous fiscal policy instrument[5]. Finally, there is the *Neo-Keynesian closure* (in Sen's terminology, otherwise also defined *Kaldorian*), where an income distribution mechanism acts.

These four models may be classified on the basis of the factor market and the laws it follows. From this perspective, the *Neoclassical* and the *Johansen* closures may be compared. Both of them assume that the production side has full utilisation of available resources so that real wage and the rate of return to capital are determined[6]. Therefore, the production side is completely separated from the demand side where the two models differ. There is no room for an interaction between the two sides.

Neoclassicals suppose there is a level of investments that equals the total amount of savings that are fixed in the economy. The Johansen closure assumes exogenous investments and endogenous consumption, whose volume adjusts to liberate sufficient savings.

The other two options consider more complicated interactions. The *Keynesian* possibility supposes that a supply- demand interaction determines employment level, output, and relative prices. The *Kaldorian* closure supposes that employment and output are fixed but income redistribution takes place and frees the necessary savings.

In the table below, we present schematically how the different closures model the assumptions on the factor market, and the assumptions on the *ex- ante* identity between savings and investments.

In the summary below, we highlight which variables in the core model are fixed and which ones are not. Thus, the final step is to describe which adjusting mechanism acts and the interactions inside the model itself. As Taylor (1991) points out: "*prescribing closure boils down to stating which variables are endogenous or exogenous in an equation system largely based upon macroeconomic accounting identities, and figuring out how they influence one another. When one is setting up a model for any economy, the closure question becomes more*

[5] This means an endogenous consumption.

[6] Real wage is determined by the solution of the first- order condition in the maximization problem the producers face. And the return to capital is interpreted as the residual.

interesting, transforming itself to one of empirically plausible signs of "effects" and, more important, a perception of what are the driving macroeconomic forces in the system".

Table 3: A summary of the four macroclosures sssumptions

	Neoclassical	**Keynesian**	**Johansen**	**Kaldorian**
Equilibrium in the factor market	Full-Employment	Unemployment	Full- Employment	Full-employment
Ex- ante equilibrium in savings-investments	Saving- driven	Exogenous investment	Exogenous investment	Exogenous investment
Variables				
P	Numeraire	Numeraire	Numeraire	Numeraire
N	Fixed		Fixed	
K	Fixed	Fixed	Fixed	Fixed
I		Fixed	Fixed	Fixed
w		Fixed		Fixed
s_R, s_W	Fixed	Fixed		Fixed

II. The closure rule problem in a closed economy without Government

To follow with our simulations on detecting how the closures work and the peculiarities of each model, we use a numerical representation of an archetype economy. The numerical values are as follows: total output, $X = 100$, is divided among consumption out of wages, $C_w = 45$, consumption out of profits, $C_r = 20$, and investments, $I = 35$. All prices are set equal to one in the base level. Total output is produced employing labour, $L = 60$, and capital, $K = 40$.

The saving propensities are assumed to be $s_w = 0.25$ and $s_r = 0.5$ for workers and capitalists, respectively. For the sake of simplicity we assume that we have a Cobb- Douglas production function.

Then, to summarize the values, we adopt an MCM (Micro- Consistency Matrix) which is the starting point for the building of the MPSGE code.

Box 10: An illustrative MCM for a closed economy without government

	ACT	WORK	RENT	INV
PX	100	-45	-20	-35
w	-60	60		
r	-40		40	
SAV		-15	-20	35

Source: ***Author's own model***

a)The Neoclassical closure for a closed economy

In the *Neoclassical closure* there are no fixed investments (the real investment target is abandoned). This implies the existence of a mechanism that causes investments to be equal to savings at the full employment level. Simply, whatever is saved is invested. The adjusting mechanism, not explicitly modelled, is an interest rate effect like in the Solow growth model

(1956)[7]. The total effect on production is nil. There is no GDP effect. In this way the only effect is compositional on total demand. This means that when investments move to equal savings, there is a contemporary opposite movement in the other demand components (namely consumption). In order to increase the GDP level, we have to increase the available inputs so that firms may move towards a north-eastern isoquant[8].

To better explain these mechanisms we refer to box 11, where a simple closed economy model is presented in MCP format (Mixed Complementarity Format[9]). Then, we will assume two different shocks: a demand side shock with a 10% increase in real investments, and a supply side shock with a 10% increase in capital supply.

Box 11: The MCP format for a Neoclassical closed economy model without government

$$w^{\beta} \cdot r^{(1-\beta)} = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT) / PX) + INV \quad (2)$$

$$LS = G = GDP \cdot \beta \cdot \left(\frac{r}{w}\right)^{(1-\beta)} \quad (3)$$

$$KS = G = GDP \cdot (1-\beta) \cdot \left(\frac{w}{r}\right)^{\beta} \quad (4)$$

$$WORK = E = w \cdot LS - alphaz \cdot (PX \cdot INV) \quad (5)$$

$$RENT = E = r \cdot KS - (1 - alphaz) \cdot (PX \cdot INV) \quad (6)$$

$$WORK = L = (1 - s_w) \cdot w \cdot LS \quad (7)$$

$$RENT = L = (1 - s_r) \cdot r \cdot KS \quad (8)$$

GDP= total domestic production, *PX*= output price, w= wage rate, *r*= rental rate of capital, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers.
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: ***Authors' own model***

In the box above, we summarize the fundamental relations that describe the model. Equation (1) is the dual representation of the production function. Firms employ labour and capital (*LS* and *KS*) paid *w* and *r*, respectively. Theoretically speaking, this equation represents the "*zero profit condition*" for sector *X*: production costs are greater or equal to final sale prices when firms act in perfect competition. The production function is a CD function with an elasticity of substitution between inputs equal to β. Then, equations (3) and (4) follow

[7] This closure, although correct in macroeconomic terms, partly contradicts the macro nature of the CGE model where it is employed. In the CGE there is no money or financial market. However, the mechanism is based on a monetary variable (the interest rate) which is not directly described by the model. This issue is part of the debate on Neoclassical CGE models (see Robinson (2003)).

[8] For a diagrammatical representation of isoquants in the plane see Varian (1992).

[9] For a description of the MCP format in describing CGE models, see Rutherford T. F. (1987, 2005), Markusen J.R. (2002), Mathiesen L. (1985a, 1985b).

as the *Shepard's lemma*: the first derivative of the production function with respect to an input equals the ratio of the input itself with respect to total production[10]. Equation (2) represents a "*market clearing condition*". It simply states that in real terms production is fully exhausted by consumption (in this case of two classes, workers and rentiers, *WORK* and *RENT* respectively) and investments (*INV*). Equations (5) and (6) are the "*income balance*" equations: total income is devoted to consumption and savings. Since here we are in a Neoclassical context, savings are equal to investments. A difference from the original Sen's model is the utilization of parameter *alphaz*. It represents the share of workers' savings with respect to the total private savings.

This means each consumer participates in totalling investments according to this share. Finally, equations (7) and (8) are "*constraint conditions*" which define consumption as the residual income after decisions about saving.

If we count for the variables of the model, we have 4 parameters, β, *alphaz*, s_w, and s_r; and we have 9 variables, *GDP*, *LS*, *KS*, *INV*, *w*, *r, WORK, RENT, PX*. To solve the system we need an equal number of unknowns and relations so we have to fix one variable exogenously. Since we want to build a Neoclassical model, we suppose that *LS* is fixed and the identity between savings and investments holds.

Let us describe the first possible shock: a demand side shock due to a 10% increase in real investments[11]. As we have previously assumed, this kind of shock leads to a simple reallocation of the available output. Firms face the same production function since they have the same amount of input. If the input combination is the same, the firm is on the same isoquant so that total output doesn't change (from relations (1), (3) and (4)). However, investments increase by assumption and this means that private consumption (in this case a combination of workers' and rentiers' consumptions) has to decline (to satisfy relation (2)) .

From relations (5) and (6) we derive the negative relationship between private consumption and investments. From relations (7) and (8) we derive the consequence of a negative relationship between consumption and savings.

Quantitative results are presented in table 5. Real and nominal GDP are stable at the benchmark level, as are labour and capital employment. A change occurs in the private consumption levels. Workers diminish their consumption by more than 3% while rentiers diminish theirs by 10%. The increase in investments (by assumption, 10%) is satisfied by a

[10] For the mathematical proof, see Varian (1992).

[11] Formally, when we follow a Neoclassical model, we should use another expression to define this shock: a 10% increase in total savings. In this way we capture the causality inside the model: a change in savings stimulates a change in investments and not the other way round.

contemporaneous increase in workers' and rentiers' savings (both increased by 10%). It is valuable to highlight that the two social classes' free available savings depend upon the *ex-ante alphaz* share.

More properly, the change in available savings allows investments to increase. The causal chain goes from savings to investments as the fundamental element in the Neoclassical framework.

When we move to a supply side shock (namely a 10% increase in capital supply) a bit more complicated mechanism takes place. The production function does not change, and so the ratio r/w is stable. However, in the new situation labour is the scarce factor and its remuneration increases, and as a consequence the profit rate increases. Since both factor prices are raised, the final price PX increases as well according to relation (1). In real terms there is the same output level and redistribution is all that takes place between capitalists and workers. The former faces a higher income so that they allocate this increase between consumption and savings, while workers reduce their consumption in favour of savings.

This effect is a price effect: now good X is more expensive causing workers to decide to consume less because their real income is lower while capitalists increase their consumption because of the increase in their real income.

As before, numerical results of the simulation are presented in table 6. The supply side shock affects nominal variables, the general price level, and the profit rate-wage ratio. As a consequence, the changes in real variables are driven from a price effect. It is worth noting that real investments are not affected. Also in this case the *alphaz* parameter is fixed at its benchmark level as in the case of the demand side shock.

b) The Keynesian closure for a closed economy

In the *Keynesian closure* labour market equilibrium does not necessarily exist. Each activity employs labour according to an increasing function of production and decreasing in real wages. In this way, households' income is determined and savings are adjusted in order to bring savings and investments into equilibrium. This may be different from those at the full employment level. Here the multiplier effect takes action. When investments increase, there is a higher demand for production so that firms have to hire extra workers up to the full-employment level. With this kind of closure, this simple CGE model becomes a textbook case of a multiplier model with expansionary effects on output and employment as Keynes predicts.

As Robinson (2003) describes, we may have different models which satisfy Keynes' prescriptions. Specifically, he discusses two different Keynesian closures. Both of them are coherent with Keynesian macroeconomics although they suppose an economic system that

works rather differently. The fundamental assumptions adopted are both a multiplier mechanism and an exogenous investment level. But the labour demand may be modelled differently. In the first case (Robinson calls it the "*Keynesian 1 closure*"), labour supply is supposed to be endogenous so the adjusting mechanism works through adjustments in the real wage. But this model assumes firms are on they labour demand curve, so that wages decline to give firms an incentive to hire extra- workers.

A different story is for "*Keynesian 2 closure*". In this case wages are fixed and the labour supply is assumed to be free. Firms are not on their labour demand curve and there is a distortion between effective wages and the marginal productivity.

Although the original debate did not consider these peculiarities, in our work we want to apply what we call "*Bastard Keynesian closure*" (using the terminology of von Arnim and Taylor (2006, 2007a, 2007b)). It is nothing else than what Robinson defines as "*Keynes 1 closure*". The multiplier still works but the labour market is Neoclassical in fashion: firms are on their labour demand curve and pay labour according to its marginal productivity. It is likely to have unemployment but it could be eliminated through a reduction in wages.

The "Bastard Keynesian" closure is presented formally in box 12 in the MCP format.

Box 12: The MCP format for a "Bastard Keynesian" closed economy model without government

$$w^{\beta} \cdot r^{(1-\beta)} = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT) / PX) + INV \quad (2)$$

$$m \cdot LS = G = GDP \cdot \beta \cdot \left(\frac{r}{w}\right)^{(1-\beta)} \quad (3)$$

$$KS = G = GDP \cdot (1-\beta) \cdot \left(\frac{w}{r}\right)^{\beta} \quad (4)$$

$$WORK = E = w \cdot m \cdot LS - alphaz \cdot (PX \cdot INV) \quad (5)$$

$$RENT = E = r \cdot KS - (1 - alphaz) \cdot (PX \cdot INV) \quad (6)$$

$$WORK = L = (1 - s_W) \cdot w \cdot m \cdot LS \quad (7)$$

$$RENT = L = (1 - s_r) \cdot r \cdot KS \quad (8)$$

GDP= total domestic production, *PX*= output price, *w*= wage rate, *r*= rental rate of capital, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers, *m*= endogenous labour supply multiplier.
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: *Authors' own model*

Essentially, the model is similar to the Neoclassical version. The main difference is the introduction of *m*, the endogenous labour supply multiplier. It answers the question of how many workers want to be employed. This is a way to model unemployment or under-employment. In this way any change in *m* has to be interpreted as a change in labour supply. Fundamentally, the model works like the previous one. In this case, however, there are 8 equations in the model, 4 parameters, β, *alphaz*, s_w, and s_r, and 10 unknowns, *w*, *r*, *PX*, *GDP*, *WORK*, *RENT*, *INV*, *m*, *LS*, and *KS*. So, we have to fix 2 variables: the first one is *INV*,

according to Keynes' ideas on exogenous investment level, and the second is the choice of w as the numeraire of the model.

Also in this case, we suppose that in our economy the two shocks occur. The interesting aspect is to compare the results with the ones of the Neoclassical closure.

Firstly we suppose a 10% increase in real investments occurs. The mechanism is the one described above, that is, a textbook case of multiplier effect. An increase in investments is an increase in a final demand component. To satisfy it, firms have to hire extra workers at the full employment level. This choice affects the level of m, which increases. Labour becomes the abundant factor so that profit rate increases as well.

Both social classes face higher income and they allocate a higher portion to consumption. Savings also increase in order to balance the higher investments.

Numerically, it is interesting to note that a 10% increase in investments stimulates a more than proportional increase in employment (17%) while both the other demand component in real terms and savings in real terms increase by 10% as did the initial stimulus. We have a fixed wage rate as the numeraire. The profit rate moves up since capital becomes the scarce factor, and therefore the general price level, depending on production costs, increases.

An opposite effect comes from a 10% increase in capital supply. In this case, an increase in capital supply reduces the profit rate while wages are fixed since their level is the numeraire of the model. The change in the ratio r/w causes the isocost to become smoother so that the tangency condition holds with a higher isoquant (or in other words, a north-eastern isoquant). In nominal terms production increases, but higher production cost means higher final price of output. In real terms GDP is lower than in the benchmark. By assumption, rentiers' income as well as their real consumption is higher.

The rotation of the isocost has another implication: the new productive technique employs a different combination of inputs with higher capital and lower labour. Therefore m declines, creating more unemployment and reducing workers' income.

A lower workers' income reduces consumption as a consequence of the higher final prices. Our simulation quantifies these changes. An increase in capital supply reduces real output by more than 1.5 percentage points and employment can decline by up to 3 points.

Comparing the consequences of the two shocks, we may assert that a Keynesian model (or in this case "Bastard Keynesian") is a demand- driven system. This result is particularly clear if we analyse the effects on GDP under different shocks. When a demand component (i.e. investments) increases, GDP moves in the same direction, both in real and in nominal terms. A supply side shock (i.e. a capital supply increase) causes an increase of merely nominal GDP while even real GDP declines. This effect is due solely to a price increase.

c) The Neo-Keynesian (Structuralist) closure for a closed economy

In the *Neo- Keynesian* (*Kaldorian*) *closure* factors of production are not remunerated according to their marginal productivity. The adjusting mechanism is based on the forced savings model of Kaldor (1956). Practically, this means that the nominal wage rate is fixed while production is a function of labour and capital supplies as usual.

As the wage is fixed and the price level endogenous, the equality between savings and investments still holds only if there is a change in income distribution. This transfer takes place from households with a weaker saving propensity to households with a higher saving propensity. This reallocation of income means a reallocation of demand. If income moves from weaker saving propensity households (namely wage earners) to higher propensity households (capitalists), this leads to a reduction in consumption. The compositional effect on demand is coherent with the total production determined by initial endowments in factors of production.

In this paper we analyse one of the possible closures, the Structuralist closure, with a formal presentation given in box 3.

In this framework we assume that there is only one factor of production, labour, while capital is considered to be a stable mark-up over variable costs. The production function is a Leontief where labour is employed proportionally to the output (according to the output/labour coefficient *b*), coherently with relation (3). The output price is formed through a mark- up rule where a fixed mark- up rate (*tau*) is considered over variable production costs[12] (relation 1). From this mark- up rate we derive the profit rate (*r* is a function of *tau* and the output/capital ratio *u*). In this way remunerations of capital and labour are not equal to their marginal productivity but instead are fixed in the short run since they depend on "history" (relations (3) and (4)). Simply, they depend on the production techniques available in a specific time and the mark-up decisions carried out by the producers. Income distribution becomes a social phenomenon.

The system is demand driven so a multiplier effect still holds. The material balance works as usual (relation 2), and workers and rentiers have to satisfy their income budget constraints (relationships (5) and (6)).

[12] In this simplest case variable production costs are assumed to be only the labour costs but when we extend the model to an open economy we will also have costs for imported intermediates and related tariffs.

Box 13: The MCP format model for a Structuralist/ Post Keynesian closed economy model without government

$$(1+tau)\cdot w\cdot b = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT)/PX) + INV \quad (2)$$

$$m\cdot LS = G = b\cdot GDP \quad (3)$$

$$KS = G = tau\cdot w\cdot b\cdot GDP \quad (4)$$

$$WORK = E = w\cdot m\cdot LS - alphaz\cdot(PX\cdot INV) \quad (5)$$

$$RENT = E = r\cdot KS - (1-alphaz)\cdot(PX\cdot INV) \quad (6)$$

$$WORK = L = (1-s_w)\cdot w\cdot m\cdot LS \quad (7)$$

$$RENT = L = (1-s_r)\cdot r\cdot KS \quad (8)$$

GDP= total domestic production, *PX*= output price, *w*= wage rate, *b*= output/ labour ratio, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *tau*= mark up rate, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers, *m*= endogenous labour supply multiplier.
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: *Authors' own model*

To clarify the causal chain in this class of models, we will refer to the simulation whose results are summarized in tables 5 and 6. A fundamental assumption to be stated is that capacity constraint does not exist in this economy, therefore employment may go to a full employment level.

Supposing an exogenous investment level exists, we increase it by 10%. Because of the multiplier effect, an increase in a demand component means an increase in total production.

But, since labour is employed in a fixed proportion with total production (the so- called labour- output coefficient), employment also increases with the same proportion. Moreover, profits are derived as a mark-up over variable costs.

In this simplest framework labour is all that enters into the variable costs so that if employment increases, the mark-up income follows in the same direction. It is evident that from this causal chain output, employment and mark-up income all increase by the same percentage (10%).

As usual, we have two social classes, wage earners and rentiers. The wage bill has increased and a fixed share is saved. The same happens for the rentiers. The main difference is in their saving propensities: wage earners save a lower fraction of their income with respect to rentiers. This is coherent with the macroeconomic balance of the model. An increase in investments requires more available savings. Obviously this extra savings comes mainly from rentiers rather than from workers because of the higher saving propensity.

In this case we do not have a direct reference to capital. We call the capital income "mark-up income" referring to its nature. If we want to implement a supply side shock, we must change the parameter *tau* which modifies the total mark-up income. Namely we assume a 10 percent increase (results are in table 6). Simulation results are quite similar to the ones of the

"Bastard Keynesian" model. Also in this case real production declines, as does employment, although in the structuralist case this decline is less evident (1.6 per cent against 2.8 percent). Because of the increase in mark- up, there is income redistribution in favour of rentiers. Despite the 10% increase in *tau*, rentiers' income increases less than proportionally because of the interaction with *w*. Rentiers consume and save higher fractions in nominal terms. For workers the story is the contrary: their nominal consumption decreases and their nominal savings slightly increase. However, this increase is derived only from a price effect: savings in real terms are not affected and remain stable at their benchmark level.

Although both the "Bastard Keynesian" closure and the Structuralist/ Post Keynesian closure work through a multiplier effect, their results are very different. This is due to an element already cited: the pricing rule.

In the "Bastard Keynesian" case, labour income and capital income are distinguished so that when employment increases, only wage earners gain. In the structuralist closure the mark- up pricing rule ensures that the same effects occur for both social classes.

Table 4: Results of a 10% increase in real investments

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***
Volumes				
GDP	100	100	117.2	110
Labour	60	60	70.3	66
Capital	40	40	46.9	44
Investments	35	38.5	41	38.5
Workers' consumption	45	43.5	52.7	49.5
Rentiers' consumption	20	18	23.5	22
Private total consumption	65	61.5	76.2	71.5
Values				
GDP	100	100	110	110
Investments	35	38.5	38.5	38.5
Workers' savings	15	16.5	16.5	16.5
Rentiers' savings	20	22	22	22
Private total savings	35	38.5	38.5	38.5
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	1.1722	1
Output price	1	1	1.0656	1

Source: ***Author's own calculations***

Table 5: Results of a 10% increase in capital supply

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***[13]
Volumes				
GDP	100	104	101.1	102.3
Labour	60	60	58.33	59
Capital	40	44	42.77	43.3
Investments	35	36.4	36	36.4
Workers' consumption	45	44.4	42.9	43.4
Rentiers' consumption	20	23.2	22.2	22.5
Private total consumption	65	67.6	65.1	65.9
Values				
GDP	100	100	98.4	98.4
Investments	35	35	34.6	35
Workers' savings	15	15	14.8	15
Rentiers' savings	20	20	19.8	20
Private total savings	35	35	34.6	35
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	0.9721	0.984
Output price	1	1.04	1.0276	1.04

Source: ***Author's own calculations***

[13] In this case we simulate a 10% increase in *tau*.

III. The closure rule problem in a closed economy with government

Starting from the core version of the CGE discussed by Sen, when we introduce the government as a new agent, we adopt a similar framework to quantify the effects of both supply side and demand side shocks. It is a source of savings as well. In this simple model there is still only one productive sector which produces one good employing capital and labour. There are two classes of households (workers, and capitalists) and the government. Households differ due to their propensity to save: workers have a weaker propensity than capitalists and for their tax rate on income (they pay a higher tax rate). This is an "archetype economy" used to study the effects of the closure choice combined with different shocks on the supply and the demand side. The numerical representation of this economy is a revised closed version of the model presented in Taylor and Lisy (1979) and Rattsø (1982).

The introduction of the government as a new actor complicates the analysis. In this case, a new basic macro- balance is introduced: the government deficit. In the previous model we dealt with only the saving- investments balance which was reduced at its basic form where investments where only balanced by private savings. Now savings include the government's (or deficit) but at the same time we have to set a rule for their determination. Specifically, this means deciding which behavioural target the government pursues. Mainly two rules are commonly adopted in CGE building: fixed government savings (with endogenous real spending) or fixed government expenditures (and endogenous government deficit).

This choice greatly affects the model results not only from a quantitative perspective but also from a theoretical point of view. This decision assumes a modeller's interpretation of the causal chain which directly affects the interpretation of fiscal revenue.

Here we will describe firstly the theory at the basis of this choice and then we will return to our original model to study the impact of the different closures.

Let us suppose we have a more simplified framework with respect to our original model where there is only a consumer and only direct tax revenue for the government's fiscal receipt. The two fundamental macroeconomic balances are:

$$PX \cdot I = PX \cdot (S^P + S^G)$$

$$PX \cdot S^G = Y^G - PX \cdot GZ$$

The first one is the revised version of the saving- investments balance, where investments in equilibrium should be equal to the available savings from the different agents in the economy. In this case there are both households and government. The second relation

describes how government savings are produced, and their links with the other government macro- aggregates.

When government deficit is fixed the relations appear in this way:

$$PX \cdot I = PX \cdot S^P + PX \cdot S^G$$
$$PX \cdot \overline{S^G} = Y^G - PX \cdot GZ$$

where the bar means "*its level is fixed*". To clearly understand this mechanism we suppose there is a change in the real public expenditure level. In this case GZ increases but we have assumed fixed savings so the only way to satisfy the second equation is an increase in fiscal revenue. Since taxes are defined as a fraction of income, endogenous taxes mean income redistribution, lower savings and a likely crowding out of private investments.

The second option is mathematically summed up in this way:

$$PX \cdot I = PX \cdot S^P + PX \cdot S^G$$
$$PX \cdot S^G = Y^G - PX \cdot \overline{GZ}$$

In this case government deficit adjusts when the total tax revenue changes and its expenditures are considered irrepressible, as if there is a minimum level of spending that is optimal for the economy. Therefore, savings follow the revenue receipts trend.

Now we turn to our simulation. The numerical values are as follows: total output, X = 100, is divided among private consumption of the two household groups, C_w = 40 and C_r = 15, investments, I =30, and public expenditures, G = 15. All prices are set equal to one at the base level. Total output is produced employing labour, L =60, and capital, K=40. The savings propensities are assumed to be s_r = 0.571 (or 20/35) for capitalists, and s_w = 0.11 (or 5/45) for workers. Tax rates on personal income are t_r = 0.125 (or 5/40) and t_w = 0.25 or (15/60). For sake of simplicity we assume that our production function is a Cobb- Douglas production function and at this point we suppose that consumption is simply as a residual of tax payments and savings decisions. A concise representation of the economy is given in the MCM in table 7.

Box 14: An illustrative MCM for a closed economy with government

	ACT	WORK	RENT	GOVT	INV
PX	100	-40	-15	-15	-30
w	-60	60			
r	-40		40		
SAV		-5	-20	-5	30
dtax		-15	-5	20	

Source: ***Author's own model***

We are interested in studying the mechanisms at the basis of each closure rule and in the magnitudes of the effects. Moreover we want to analyse whether or not the same closure reacts in the same way if the shock is on the supply or the demand side. Hence, we model two shocks of the same magnitude: a 10% increase in investments and a 10% increase in capital supply.

a) The Neoclassical closure with government

The dataset of this model has already been presented above. Here we start with the description of the model, variables, and equations. In box 4 we list all the equations building the model, and then we describe them in detail.

Box 15: The MCP format for the Neoclassical closed model with government

$$w^{\beta} \cdot r^{(1-\beta)} = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT + GOVT) / PX) + INV \quad (2)$$

$$LS = G = GDP \cdot \beta \cdot \left(\frac{r}{w}\right)^{(1-\beta)} \quad (3)$$

$$KS = G = GDP \cdot (1-\beta) \cdot \left(\frac{w}{r}\right)^{\beta} \quad (4)$$

$$WORK = E = w \cdot LS \cdot (1-t_w) - alphaz \cdot (PX \cdot INV - PX \cdot GSAV) \quad (5)$$

$$RENT = E = r \cdot KS \cdot (1-t_r) - (1-alphaz) \cdot (PX \cdot INV - PX \cdot GSAV) \quad (6)$$

$$GOVT = E = t_w \cdot w \cdot LS + t_r \cdot r \cdot KS - PX \cdot GSAV \quad (7)$$

$$WORK = L = (1-s_w) \cdot (1-t_w) \cdot w \cdot LS \quad (8)$$

$$RENT = L = (1-s_r) \cdot (1-t_r) \cdot r \cdot KS \quad (9)$$

GDP= total domestic production, *PX*= output price, w= wage rate, *r*= rental rate of capital, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers, *GOVT*= government nominal expenditures, *GSAV*= real Government saving.
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: ***Author's own model***

In this economy, output is produced using capital and labour (eq. (1)), both of which are paid a fraction of total production (eq. (2) and (3)) therefore the amount depends on their marginal productivity. This representation is formally known as *Shepard's lemma*: the share of wages (or profits) with respect to total production is equal to the partial derivative of the production function itself with respect to the related factor (labour or capital) The productive

factors are fully employed (eq. (4) and (5)). Labour income is accrued to workers who decide to pay taxes according to a marginal rate t_W, and save a fraction s_W (eq. (7) and (8)). The same happens for rentiers whose tax rate and savings rate are respectively t_R and s_R (eq. (8) and (9)). Therefore, we identify two aggregates defined as total private savings (S^P) and total tax revenue (Y^G) which are simply the sums of households' savings and direct taxes, respectively. Then, government itself has an income constraint to satisfy. It is the relationship between deficit, tax receipts, and government spending (eq.12). Finally, two accounting identities must be fulfilled: the saving- investments balance and the material balance. Investments adjust and are totalled according to the total saving supply in the economy. This amount is decided both by the government and the households. The material balance ensures that the total supplied production is completely devoted to the demand components (consumption of both social classes, government spending, and investments).

In this context we analyse two scenarios: one is a 10 percent increase in investments and the other is a 10 percent increase in capital supply. Both must be studied with fixed government savings or fixed government expenditures.

The effects of a 10 percent increase in investments are exactly the same in both closures. From the production perspective there are no changes: total GDP is stable, and labour and capital fixed in their demands. The investments' increase is absorbed by an increase in total private savings: both wage earners and rentiers increase their savings in the same proportion (a 12% increase).

Since input demand (and consequently income) does not change, both closures have a fixed real amount of public savings and expenditures. Tax revenue is linked to income levels because the government fixes a tax rate. But if income is fixed there is no change in tax revenue and consequently the other macro-aggregates: government consumption and deficit remain unchanged.

When we suppose a supply side shock occurs, the story goes differently. Here, the closure rule matters. A common feature is the productive side: with more capital there is movement of the productive frontier toward the north-eastern corner and this means an increase in total production by 4 percent. This increase, however, is only the effect of the higher output price: real production is unchanged. Finally, there is income redistribution towards the earners of the abundant factor: rentiers.

But how the demand side responds is different. Workers maintain their income since labour has not increased. This means that total nominal taxes are at their benchmark level.

We defined savings as a fraction (by definition fixed) of disposable income (income minus tax payments). Therefore savings in real terms is unchanged in this case. Nonetheless, it increases because of a price effect (a 4 percent increase). Plus, consumption declines both in real and nominal terms. The decline is more evident in real terms because it declines more than 4 percentage points while in nominal terms the reduction is limited to half a percentage point (in this case we also count the effect of higher price).

Rentiers' income increases by assumption. This means higher income taxes as the tax base broads (tax increases proportionally to the tax base). Savings increase in nominal terms (price effect) but they are fixed in real terms at their benchmark level as it is for wage earners as well. But consumption for this social class increases in both nominal and real terms (13 and 18 percentage points respectively).

Government obtains higher nominal tax revenue (a 2.5 percent increase). The constraint is the real level of savings which is fixed at 5, although in nominal terms it increases. To close the identity, nominal public expenditure grows because of a price effect while its real value is lower. Thus, the Government behavioral rule is a kind of "*fiscal responsibility*": higher nominal expenditures are allowed only if there are more nominal tax receipts.

Finally, we have to highlight a peculiarity of the saving- investment balance: changes take place only in nominal terms because of the increase in output price, but in real terms the balance does not differ from the benchmark situation. The main effects of this closure, supposing there is a change in capital supply, are mainly nominal effects. The only real effects are compositional effects on real demand components: private expenditures grow against a reduction in real public consumption.

The demand side works differently when we suppose fixed government expenditures. From a productive point of view, there is no change from the case of fixed public savings. As before, a higher capital supply means a higher rentiers' income, so they increase their nominal tax payments and what remains is divided between savings and consumption. The workers' situation is unchanged: they continue to be paid with the same wage bill and they pay the same income tax. Because of a higher output price, they reduce consumption in favour of savings. Therefore, real savings increase by 2 percent while real consumption declines by 4.5 percentage points. Investments increase only in nominal terms because of the increase in output price.

A different situation presents itself for the government. Here, real expenditures are fixed so that only nominal value increases (a 4 percent increase). Although in nominal terms total fiscal receipts increase, in real terms they decline. This means that to satisfy the saving-investments balance private savings have to increase while the public participation declines. In fact, in this simulation we show that in absolute values the decline in real government

savings is completely fulfilled by the increase in aggregate real private savings. From a demand point of view we have only price effects since each demand component in real terms is at its benchmark level.

b) The "Bastard Keynesian" closure with government

We immediately start to present the main differences between this closure, the neoclassical example we have just shown and the closed version without government of the "Bastard Keynesian" (henceforth BK) closure. With respect to the model presented in box 2, here there is a new actor: the government. This affects both the demand side and the saving-investment balance. In fact, it is a component of the aggregate demand (*GOVT*) but is at the same time, a source of savings (*GSAV*). Furthermore, as we have analysed in the previous section, the introduction of the government requires a new constraint which explicitly defines the relationship between fiscal revenue, expenditures and public deficit (in box 5 it is represented by relation (7)). Except for these differences, the system acts as any Keynesian system would: with the same macroeconomic causality discussed in section 2b.

In respect to the Neoclassical version of the model, here the main introduction is m, the endogenous labour supply multiplier. Its role was already discussed when the BK closure in its simplest version was introduced. In this context it is worthy to point out that now tax revenue is also a function of m. In fact, nominal income depends on the share of supplied work (expressed by m), and tax revenues are counted as a fixed proportion of this income.

Tax rates enter the Keynesian constraint as relations (8) and (9) shown below.

Box 16: The MCP format for the "Bastard Keynesian" closed economy with government

$$w^{\beta} \cdot r^{(1-\beta)} = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT + GOVT)/PX) + INV \quad (2)$$

$$m \cdot LS = G = GDP \cdot \beta \cdot \left(\frac{r}{w}\right)^{(1-\beta)} \quad (3)$$

$$KS = G = GDP \cdot (1-\beta) \cdot \left(\frac{w}{r}\right)^{\beta} \quad (4)$$

$$WORK = E = w \cdot m \cdot LS \cdot (1-tw) - alphaz \cdot (PX \cdot INV - PX \cdot GSAV) \quad (5)$$

$$RENT = E = r \cdot KS \cdot (1-t_r) - (1-alphaz) \cdot (PX \cdot INV - PX \cdot GSAV) \quad (6)$$

$$GOVT = E = t_w \cdot w \cdot m \cdot LS + t_r \cdot r \cdot KS - PX \cdot GSAV \quad (7)$$

$$WORK = L = (1-s_w) \cdot (1-t_w) \cdot w \cdot m \cdot LS \quad (8)$$

$$RENT = L = (1-s_r) \cdot (1-t_r) \cdot r \cdot KS \quad (9)$$

GDP= total domestic production, *PX*= output price, w= wage rate, *r*= rental rate of capital, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers, *GOVT*= government nominal expenditures, *GSAV*= real Government saving, *m*= labour supply multiplier.
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: *Author's own model*

In this case we also simulate the usual two shocks used to evaluate how the model acts.

First, we suppose a 10% increase in investments. Both government closures ensure expansionary results: real, not solely nominal, GDP increases. However, the magnitude of this change is different: when government savings are fixed, GDP increases more than when public expenditures are fixed (20.8 percent against 11.3 percent). The reason is apparent: a change in investments stimulates a higher production level. This extra production may be absorbed by private consumption or public expenditures. When we suppose a fixed deficit, public expenditures are allowed to increase and absorb a share of the extra production. In real terms, private consumption is allowed to increase by 12 percent and at the same time government consumption increases by 16 percent.

Otherwise, when expenditures are fixed, only private consumption may increase (by 11.2 percent) and therefore the increase in production must be lowered so as to be absorbed.

Clearly, labour supply withstands the same effect which increases more in the fixed government savings case than in the fixed government expenditure case (more than 20 percent against 11 percent). Higher production means higher output prices because of the higher quantity of employed labour (due to the increase in m, while w is the numeraire) and the higher rental rate of capital (because the real quantity of employed capital is fixed).

Another interesting aspect to be detected is the savings- investments account. When government savings are fixed, the increase in investments is totally absorbed by private savings that increase by 12 percent, which is the same percentage increase in real GDP. In the other case, both sources of savings work: private savings increase only by 7.2 percent while public deficit increases by 24 percent. In absolute terms, private savings continue to absorb more than half of the investments shock.

Next, we analyse the effects of an increase in capital supply. Here, the results are a bit surprising at first glance. In the case of fixed government expenditures, as predicted, expansionary effects on GDP are evident only in nominal variables as a result of the increase of final prices. In fact the increase in capital supply stimulates the capital costs and therefore final prices increase because of the higher production costs. An interesting aspect to detect and analyse are the different results in the case of fixed government savings. This is a comparison of the effects on prices. In this case shifts in final prices and rental rates of capital are symmetric. Both variables move by 3 percent in different directions: the increase in final prices is counterbalanced by a decrease of the same amount in r. The mechanism is the same as the other shock. Because government saving is free to move, it decreases and the necessary extra savings is supplied by the two households according to the *alphaz* share. The private consumption component increases because of the higher nominal incomes accruing to the two classes. The total tax revenue declines in real terms but nominal tax receipts are unchanged

from the benchmark. The fall is different between the two consumers: workers diminish their real tax payments because of the lower employment level. Rentiers slightly increase their fiscal payments. Because of our assumption of fixed government expenditures in real terms, the consequence is a real decline in public savings. This is further proof of the relationship between the public sector's variables.

In the fixed government savings case, the numerical results are surprising. Because of the closure rule and stable real investments, the saving - investment balance is unchanged from the benchmark.

As usual, a supply side shock reduces real GDP but this time nominal GDP decreases as well. This effect is due to the response of prices. In the previous case we described a symmetrical movement of output price and capital rental rate. Here it no longer appears. Final commodity price increases by 1.1 per cent (a higher increase than the one in the previous case) but the fall in *r* is greater too (6.3 percent). It offsets the expansionary effect of the final price and reduces the nominal magnitude.

With respect to the fixed government expenditure case, employment levels are lower (*m* is now 0.93 against 0.97). This means a lower income for workers and a reduction in their nominal fiscal payments. Because of fixed government saving, this reduction causes a decline in government spending. The effects on rentiers' income have an opposite sign: their income increases and therefore so do their tax payments. However, this is not enough to contain the decline in total fiscal revenue because the fall in workers' income is higher than the increase in rentiers' income (6.3 per cent against 3 per cent).

Finally, the closure rule affects the consumption of the agents. Declines in income are not counterbalanced by reductions in savings in order to avoid worsening the consumption behaviour by much. Taking into account the case of workers, a 6.3 percent reduction in income means a tax payment constant according to a fixed tax rate; there is now a lower disposable income. The savings decision is made prior to the consumption one and the total amount of real savings must be constant because of the closure rule. This means a slight increase in nominal terms. Therefore, as a residual, consumption declines. Numerically, although the income reduction is 6.3 percent (in nominal terms), consumption falls more (7.25 percent in nominal terms). This already considerable value worsens if we consider it in real terms. In fact it becomes 8.25 because there is not the 1.1 percent increase in prices.

c) The Johansen closure with government

The *Johansen closure*, in its original exposition, expands the model of Sen by introducing the government as an important source of savings. In this context government consumption or the tax rate become endogenous. Supposing, as did Johansen (1960, 1974), that personal tax

rates are endogenous breaks the link between production and demand. Without this element the Johansen model is not far from the Neoclassical one in its working system. But, when introducing endogenous tax rates on income, the net remuneration of the factors of production and the net disposable income are no longer equal. Therefore there is a distinction between the production stage and demand.

Savings depend on the tax rate and so when we adjust the variable, we may free enough savings to close the saving- investment gap. The total effect is solely a reallocation of demand because we assume output at the full- employment level.

Nowadays, supposing endogenous tax rates is one of the hallmarks of one of the most worldwide used CGE model: the World Bank LINKAGE model. Quoting the technical notes of the model "*Government collects income taxes,* […] . *Aggregate government expenditures are linked to changes in real GDP. The real government deficit is exogenous. Closure therefore implies that some fiscal instrument is endogenous in order to achieve a given government deficit. The standard fiscal closure rule is that the marginal income tax rate adjusts to maintain a given government fiscal stance*" (van der Mensbrugghe, 2005).

Although widely adopted, we reject the hypothesis of endogenous tax rate, both in the idea of personal taxation and indirect taxes. Macroeconomics and political economics have always defined taxes as a governmental instrument used to pursue a certain goal. However, the imposition of a tax is also a political process with its own timing and procedures. Supposing, as in the LINKAGE model, that tax rates are endogenous contradicts the idea of taxes as a policy instrument. It is not likely to suppose that tax rates move instantaneously in order to reach the equilibrium in the model.

This critique is supported by many scholars. For instance von Arnim and Taylor (2006) suggest "*there is neither an economic theory nor actual country experience that supports this kind of adjustment. Governments cannot spontaneously increase taxes to balance the budget* […]".

d) The Structuralist / Post Keynesian closure with government

Fundamentally, the Structuralist model presented in its MCP format in box 6, is not far from the "Bastard Keynesian" model already discussed. The only difference is the production side which determines the pricing rule and the inputs demand (relations (1), (3), and (4)). Government does not enter these relationships. Therefore, they are exactly the same as we described in detail when we spoke about a closed economy without government.

Results from our simulations are very close to the "Bastard Keynesian" outcomes: the system is still demand driven and employment is endogenous. However, main differences stem from the magnitudes of endogenous variables' variations. In this case, in the investments'

disturbance, prices are fixed at their benchmark level and are unchanged because we have supposed the wage rate to be the numeraire of the model. In fact, when we allow *tau* to move, we demonstrate that the output price level also changes (from relation (1) in the MCP format of the model in box 6).

Fundamentally, the only slight difference between the "Bastard Keynesian" results and the Structuralist model is in absolute terms in the saving- investments balance. As in the former closure, the role of public and private savings is maintained. The main difference is a sort of forced savings mechanisms taking place between consumers, especially in the case of fixed government expenditures.

Results mainly differ in nominal terms, as we have already discussed, because of the different pricing rule. Only in the cases of supply side shocks is the output price different from the unit.

Box 17: The MCP format for the Structuralist/ Post Keynesian closed economy with government

$$(1+tau)\cdot w\cdot b = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT + GOVT)/PX) + INV \quad (2)$$

$$m\cdot LS = G = b\cdot GDP \quad (3)$$

$$KS = G = tau\cdot w\cdot b\cdot GDP \quad (4)$$

$$WORK = E = w\cdot m\cdot LS\cdot(1-tw) - alphaz\cdot(PX\cdot INV - PX\cdot GSAV) \quad (5)$$

$$RENT = E = r\cdot KS\cdot(1-t_r) - (1-alphaz)\cdot(PX\cdot INV - PX\cdot GSAV) \quad (6)$$

$$GOVT = E = t_w\cdot w\cdot m\cdot LS + t_r\cdot r\cdot KS - PX\cdot GSAV \quad (7)$$

$$WORK = L = (1-s_w)\cdot(1-t_w)\cdot w\cdot m\cdot LS \quad (8)$$

$$RENT = L = (1-s_r)\cdot(1-t_r)\cdot r\cdot KS \quad (9)$$

GDP= total domestic production, *PX*= output price, w= wage rate, *r*= rental rate of capital, *WORK*= nominal workers' consumption, *RENT*= nominal rentiers' consumption, *INV*= real investment, *LS*= labour supply, *KS*= capital supply, *tau*= mark up rate, *alphaz*= workers' saving share on total private saving, s_w= saving rate for workers, s_r= saving rate for rentiers, *GOVT*= government nominal expenditures, *GSAV*= real Government saving, *m*= labour supply multiplier, *b*= output/labour ratio,
= *G* = means greater than, = *E* = means strictly equal, and = *L* = means lower than.

Source: ***Author's own model***

Table 6 : Results of a 10% increase in real investments with fixed government expenditures

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***
Volumes				
GDP	100	100	111.4	106.7
Labour	60	60	66.8	64
Capital	40	40	44.6	42.7
Investments	30	33	34.5	33
Workers' consumption	40	39.4	44.5	42.7
Rentiers' consumption	15	12.6	16.7	16
Total private consumption	55	52	61.2	58.7
Government consumption	15	15	15.7	15
Values				
GDP	100	100	106.7	106.7
Investments	30	33	33	33
Workers' savings	5	5.6	5.4	5.3
Rentiers' savings	20	22.4	21.4	21.4
Total private savings	25	28	26.8	26.7
Government savings	5	5	6.2	6.3
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	1.1136	1
Output price	1	1	1.044	1

Source: ***Author's own calculations***

Table 7: Results of a 10% increase in real investments with fixed government savings

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***
Volumes				
GDP	100	100	120.8	112
Labour	60	60	72.5	67.2
Capital	40	40	48.3	44.8
Investments	30	33	35.6	33
Workers' consumption	40	39.4	48.3	44.8
Rentiers' consumption	15	12.6	18.1	16.8
Total private consumption	55	52	66.4	61.6
Government consumption	15	15	18.8	17.4
Values				
GDP	100	100	112	112
Investments	30	33	33	33
Workers' savings	5	5.6	5.6	5.6
Rentiers' savings	20	22.4	22.4	22.4
Total private savings	25	28	28	28
Government savings	5	5	5	5
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	1.2079	1.12
Output price	1	1	1.0785	1

Source: ***Author's own calculations***

Table 8 : Results of a 10% increase in capital supply with fixed government expenditures

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian[14]***
Volumes				
GDP	100	104	101.4	102.5
Labour	60	60	58.5	59.1
Capital	40	44	42.9	43.4
Investments	30	31.2	30.9	31.2
Workers' consumption	40	39.7	38.6	39.4
Rentiers' consumption	15	17.5	16.5	16.3
Total private consumption	55	57.2	55.1	55.7
Government consumption	15	15.6	15.4	15.6
Values				
GDP	100	100	98.5	98.5
Investments	30	30	30	30
Workers' savings	5	5.1	5.1	4.8
Rentiers' savings	20	20.2	20.4	20.8
Total private savings	25	25.3	25.5	25.6
Government savings	5	4.7	4.5	4.4
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	0.97	0.98
Output price	1	1.04	1.03	1.04

Source: ***Author's own calculations***

Table 9: Results of a 10% increase in capital supply with fixed government savings

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist / Post Keynesian[15]***
Volumes				
GDP	100	104	97.4	100.1
Labour	60	60	56.2	57.7
Capital	40	44	41.2	42.4
Investments	30	31.2	30.4	31.2
Workers' consumption	40	39.8	37.1	38.5
Rentiers' consumption	15	17.7	15.8	15.9
Total private consumption	55	57.5	52.9	54.4
Government consumption	15	15.3	14.1	14.5
Values				
GDP	100	100	96.3	96.3
Investments	30	30	30	30
Workers' savings	5	5	5	4.6
Rentiers' savings	20	20	20	20.4
Total private savings	25	25	25	25
Government savings	5	5	5	5
Price				
Wage	1	1	1	1
Rental rate of capital	1	1	0.94	0.97
Output price	1	1.04	1.01	1.04

Source: ***Author's own calculations***

[14] In this case the 10% increase is in *tau*.

[15] In this case the 10% increase is in *tau*.

IV. The closure rule problem in an open economy

As already presented, the model becomes more complex with the introduction of new agents while the closure rule problem becomes more rigid and complex. This has been evident passing from the closed economy without government to the closed version with government. Here, we introduce a new agent we call the "foreigners". It represents the opening up of our economy towards the rest of the World.

Therefore we must now introduce new features to our basic CGE and analyse new relationships among the macro aggregates of these new agents. New possible choices then arise.

First of all we have to describe the basic relationships within this new aspect. In this way we derive the fundamental accounting values and how they are linked together. Namely, we have to consider the concepts of net exports, foreign savings, and then the concepts of export and import functions.

In any standard textbook on international trade we derive these fundamental identities:

$$NEXP = EXP - IMP$$

$$NEXP = FSAV$$

We may say that the first identity is the trade balance. To simply define that, a variable called "net exports" is the difference between exports of final goods and relative imports. Supposing no financial variables, its value equals the foreign savings, that is, the amount of money to be lent to the rest of the World. By definition *NEXP* should be equal to or greater than zero.

The relationship between foreign savings and net exports is clear and understandable. When foreigners sell imports they receive monetary payments which they use partly to buy other goods (exports) and partly to lend to the rest of the World. Supposing they want to demand more exports, they should make a decision and reduce their disposable savings for the rest of the World.

Analysing the issue in this way we implicitly assume that *NEXP* are strictly positive but this is not true. However, in this way we may model shocks on net exports and foreign savings in a proper way and obtain reasonable results.

The model is presented in its MCP format in box 7 where we assume a model similar to the one of the closed economy with only two exceptions. The first exception (which is absolutely apparent) is when we introduce a new agent called *ROW*, which is the foreign sector. The second is that we assume that the productive sector employs primary factors as well as

imported intermediates in fixed proportion. In this example we assume a Neoclassical system for the sake of simplicity.

We should take a moment to describe the fundamental and innovative aspects of the models described up until now. The introduction of imported intermediates complicates the production function that is now a "*nested production function*". This means that there are more steps to aggregate before obtaining the final production. In this example we have assumed that the final product is a Leontief function of value added and intermediates[16], according to a parameter *b*. This is defined as "*value added share on total production*" so that value added and intermediates, with respect to (1-*b*), are both proportional to GDP. In this case, value added is the aggregation of labour and capital through a Cobb- Douglas function. Its formulation, however, is a bit different because there is a new coefficient, the inverse of *b*, an efficiency parameter.

Box 18: The MCP format for an open economy model (option 1)

$$\left[GDP \cdot b \cdot \left(r^{\alpha} \cdot w^{(1-\alpha)}\right)\right] + \left(a0 \cdot PWM \cdot e \cdot (1+tm) \cdot GDP\right) = G = PX \quad (1)$$

$$GDP = G = ((WORK + RENT + GOVT + ROW)/PX) + INV \quad (2)$$

$$LS = G = \left(\frac{GDP}{a}\right) \cdot \left[(1-\alpha) \cdot w^{\alpha} \cdot r^{(1-\alpha)} / r\right] \quad (3)$$

$$KS = G = \frac{GDP}{a}\left[\alpha \cdot w^{\alpha} \cdot \left(r^{(1-\alpha)} / w\right)\right] \quad (4)$$

$$WORK = E = wLS \cdot (1-t_w) - alphaz \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (5)$$

$$RENT = E = rKS \cdot (1-t_r) - (1-alphaz) \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (6)$$

$$GOVT = E = t_w \cdot w \cdot LS + t_r \cdot r \cdot KS + tmi \cdot e \cdot IMP - PX \cdot GSAV \quad (7)$$

$$ROW = E = e \cdot a0 \cdot PWM \cdot GDP - e \cdot FSAV \quad (8)$$

$$WORK = E = \left(1-s_w\right) \cdot (1-t_w) \cdot w \cdot LS \quad (9)$$

$$RENT = E = \left(1-s_r\right) \cdot (1-t_r) \cdot r \cdot KS \quad (10)$$

GDP= real production, *LS*= labour supply, *KS*= capital supply, *a*=capital share in value added, *e*= exchange rate, *tm*= tariff rate, *a0*= input-output coefficient for intermediates, *PWM*= World price for imports, *b*= value added share in total GDP, *a*= efficiency parameter in nested-production function, *r*= rental rate of capital, *w*=wage rate, *WORK*= nominal workers' consumption, *RENT*= nominal capitalists' consumption, *ROW*= foreigners' consumption, s_w= saving propensity for workers, s_r= saving propensity for capitalists, *INV*= real investments, *PX*= output price, *GOVT*= nominal government consumption, t_w= direct tax rate on workers, t_r= direct tax rate on capitalists, *GSAV*= nominal government saving.
=*G*= means greater than or equal to, =*E*= means equal to, =*L*= means lower than or equal to.

Source: ***Author's own model***

Although this model is formally correct, it is limited by the assumption of strictly positive net exports. The assumption appears too strong, such that it may be contradicted. We have to find a solution and we must find modelling exports and imports separately. In this way both

[16] In this example intermediates are completely imported but the reasoning is the same if we assume domestic intermediates, or if we assume an Armington approach.

aggregates may be assumed positive but we do not make such an assumption on net exports. Practically, this aim may be obtained by creating two fictitious productive sectors: one for exports and one for imports. The former works as follows: it employs as input domestic exports at domestic currency and "produces" a new good which is the "foreign demand for exports" whose price is now in foreign currency. The latter acts in the same manner: it employs foreign goods that are imports at foreign price, and it is changed into the "domestic demand for imports" which has a domestic price. A fundamental aspect is how we treat prices, especially foreign prices. They depend on the exchange rate and on World prices according to the assumption of the small open economy. We suppose that the economy is small enough not to have market power and determine the World price.

In this way we have solved the previous model and we have a great advantage. In adding international prices, we may analyse a new class of shocks. A typical exercise of this kind is the modelling of an oil shock where the oil price increases. We are interested in detecting which will be the impact on the productive sectors when employing them as input.

Obviously, this different approach is formalized differently from box 7 because now we have two more productive sectors and two more markets to be cleared.

A likely application of this idea is the Armington assumption. In his renowned 1963 paper, Paul Armington built a coherent framework to formally represent the cross- hauling phenomenon. Looking at statistical data, it is easy to detect both exports and imports of the same commodity because they were imperfect substitutes. His idea was exactly this: goods from different countries may be imperfect substitutes. Practically, this means that domestic productive sectors may decide to produce for the inner market or for the foreign markets in order to have maximized revenues. At the same time, imports are decided in order to minimize costs so domestic agents may decide the origin of their imports.

Mathematically, this leads to the formulation of new aggregates. Firstly, exports and imports are now CET (Constant Elasticity of Transformation) and CES (Constant Elasticity of Substitution) functions.

The former exhibits fixed elasticities of transformation between domestic products sold both domestically and abroad. The idea is that a producer makes these choices according to the comparison of internal and external prices. If external prices are higher, the producer decides to export more; the opposite happens when the domestic price are higher. The latter aggregates a composite supply, composed of imports and domestic products. As in the previous case, the choice is made based on prices. If imports are more competitive than domestic commodities, this aggregate supply will mainly be composed of imports and a lower fraction will be domestic products.

Box 19: The MCP format for an open economy model (option 2)
In this case we have assumed an Armington framework.

$$\left[GDP \cdot b \cdot \left(r^{\alpha} \cdot w^{(1-\alpha)}\right)\right] + \left(a0 \cdot P \cdot GDP\right) = G = Q \quad (1)$$

$$Q \cdot (GDP - ROW) + e \cdot PWM \cdot M \cdot (1 + tm) = G = P \quad (2)$$

$$GDP = G = (SUP \cdot P - (e \cdot PWM \cdot M \cdot (1 + tm)) + ROW) / Q \quad (3)$$

$$SUP = G = (WORK + RENT + GOVT) / P + (a0 \cdot GDP) + INV \quad (4)$$

$$LS = G = \left(\frac{GDP}{a}\right) \cdot \left[(1-\alpha) \cdot w^{\alpha} \cdot r^{(1-\alpha)} / r\right] \quad (5)$$

$$KS = G = \frac{GDP}{a}\left[\alpha \cdot w^{\alpha} \cdot \left(r^{(1-\alpha)} / w\right)\right] \quad (6)$$

$$WORK = E = wLS \cdot (1 - t_w) - alphaz \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (7)$$

$$RENT = E = rKS \cdot (1 - t_r) - (1 - alphaz) \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (8)$$

$$GOVT = E = t_w \cdot w \cdot LS + t_r \cdot r \cdot KS + tmi \cdot e \cdot IMP - PX \cdot GSAV \quad (9)$$

$$ROW = E = e \cdot a0 \cdot PWM \cdot GDP - e \cdot FSAV \quad (10)$$

$$WORK = E = \left(1 - s_w\right) \cdot (1 - t_w) \cdot w \cdot LS \quad (11)$$

$$RENT = E = \left(1 - s_r\right) \cdot (1 - t_r) \cdot r \cdot KS \quad (12)$$

GDP= real domesticproduction, *LS*= labour supply, *KS*= capital supply, *a*=capital share in value added, *e*= exchange rate, *a0*= input-output coefficient for intermediates, *PWM*= World price for imports, *b*= value added share in total GDP, *tm*= tariff rate, *a*= efficiency parameter in nested-production function, *r*= rental rate of capital, *w*=wage rate, *SUP*= real composite supply, *P*= price index for composite supply, *Q* =price index for domestic production, *WORK*= nominal workers' consumption, *RENT*= nominal capitalists' consumption, *ROW*= foreigners' consumption, s_w= saving propensity for workers, s_r= saving propensity for capitalists, *INV*= real investments, *GOVT*= nominal government consumption, t_w= direct tax rate on workers, t_r= direct tax rate on capitalists, *GSAV*= nominal government saving.
=*G*= means greater than or equal to, =*E*= means equal to, =*L*= means lower than or equal to.

Source: ***Author's own model***

A third option allows the modelling of other shocks. This may be obtained through the explicit formulation of export and import functions. These functions are built according to traditional textbook international economics. Real exports are modelled through a function combining two components, exogenous components, *EZ*, and a part which is a function of relative prices and exchange rate according to a certain export elasticity with respect to the exchange rate. Formally, the relation is:

$$RX = EZ \cdot \left(e \cdot \frac{\overline{p}}{p}\right)^{\sigma}$$

Final imports are modelled similarly:

$$IMP = MZ \cdot \left(e \cdot \frac{P_w}{p}\right)^{\gamma}$$

These assumptions are very schematic and many other features may be inserted to make the functions more complete. However, assuming these simple functions we are already able to model other new shocks on international trade moving the autonomous components of these functions or setting different elasticities.

Box 20: The MCP format for an open economy (option 3)
In this case we have made explicit the export function

$$\left[GDP \cdot b \cdot \left(r^{\alpha} \cdot w^{(1-\alpha)}\right)\right] + \left(a0 \cdot P \cdot GDP\right) = G = Q \quad (1)$$

$$\mathrm{Q} \times (\mathrm{GDP} - \mathrm{ROW}) + \mathrm{e} \times \mathrm{PWM} \times \mathrm{M} \times (1 + \mathrm{tm}) = \mathrm{G} = \mathrm{P} \quad (2)$$

$$GDP = G = (SUP \cdot P - (e \cdot PWM \cdot M \cdot (1 + tm)) + ROW) / Q \quad (3)$$

$$SUP = G = (WORK + RENT + GOVT) / P + (a0 \cdot GDP) + INV \quad (4)$$

$$LS = G = \left(\frac{GDP}{a}\right) \cdot \left[\left(1 - \alpha\right) \cdot w^{\alpha} \cdot r^{(1-\alpha)} / r\right] \quad (5)$$

$$KS = G = \frac{GDP}{a}\left[\alpha \cdot w^{\alpha} \cdot \left(r^{(1-\alpha)} / w\right)\right] \quad (6)$$

$$WORK = E = wLS \cdot (1 - t_w) - alphaz \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (7)$$

$$RENT = E = rKS \cdot (1 - t_r) - (1 - alphaz) \cdot \left[\left(PX\left(INV - GSAV\right)\right) - \left(e \cdot FSAV\right)\right] \quad (8)$$

$$GOVT = E = t_w \cdot w \cdot LS + t_r \cdot r \cdot KS + tmi \cdot e \cdot IMP - PX \cdot GSAV \quad (9)$$

$$ROW = E = e \cdot a0 \cdot PWM \cdot GDP - e \cdot FSAV \quad (10)$$

$$WORK = E = \left(1 - s_w\right) \cdot (1 - t_w) \cdot w \cdot LS \quad (11)$$

$$RENT = E = \left(1 - s_r\right) \cdot (1 - t_r) \cdot r \cdot KS \quad (12)$$

$$ROW / P = E = EZ \cdot \left(\frac{PWE}{P} \cdot e\right)^{sigma}$$

GDP= real domesticproduction, *LS*= labour supply, *KS*= capital supply, *a*=capital share in value added, *e*= exchange rate, *a0*= input-output coefficient for intermediates, *PWM*= World price for imports, *b*= value added share in total GDP, *tm*= tariff rate, *a*= efficiency parameter in nested-production function, *r*= rental rate of capital, *w*=wage rate, *SUP*= real composite supply, *P*= price index for composite supply, *Q* =price index for domestic production, *WORK*= nominal workers' consumption, *RENT*= nominal capitalists' consumption, *ROW*= foreigners' consumption, s_w= saving propensity for workers, s_r= saving propensity for capitalists, *INV*= real investments, *GOVT*= nominal government consumption, t_w= direct tax rate on workers, t_r= direct tax rate on capitalists, *GSAV*= nominal government saving, *EZ*= exogenous component of export demand, *PWE*= world price for exports, Sigma0= elasticity of exports respect to exchange rate.
=*G*= means greater than or equal to, =*E*= means equal to, =*L*= means lower than or equal to.

Source: ***Author's own model***

After the modelling of foreign trade, we must consider another relation that is fundamental to establishing equilibrium: the saving- investment balance. As compared to the other models, this relation includes one more saving source. It is the foreign saving, *FSAV*. Therefore the balance condition becomes:

$$PX \cdot I = PX \cdot S^{P} + PX \cdot S^{G} + e \cdot FSAV$$

From a mathematical point of view we may set *FSAV* both exogenous and endogenous without any problem because the aim is making the system square. So, we may treat the

variable as we wish if the system has an equal number of variables and equations. But our assumption is based on mainstream applications. Most of the empirical applications assume that foreigners' decisions are made without being affected by domestic resident behaviour. In other words, foreign residents decide how much they want to save; in this case, they look merely at their own interests. Moreover, for developing countries donors, international institutons and foreign agencies decide how much aid to allocate to a country. Practically, residents have only to accept a decision taken outside the country.

Although this is the mainstream position, a criticism has been made by Taylor (2004). From an accounting point of view, he suggests that modellers usually call "foreign savings" an aggregate that is the "net external position". If it is so, it contains not only savings from foreigners but also a bowl of assets at foreign currency owning by domestic residents. Even if we are thinking of a developing country there is a small fraction of population wealth enough to have finanacial assets. They may decide by themselves to sell these foreign assets. Thay are accounting as foreign savings in the SAM although they are detained by domestic households. In such a situation, foreign savings are no longer exogenous but they become endogenously determined because the decision is taken by domestic actors and not only foreign donors.

In our model we have decided to expose both import and export functions. In this economy there is only one productive sector whose production is X= 100, employing primary factors, L= 50 and K= 35, and intermediate inputs that are fully imported ($INTMZ$= 12). A 25 percent import duty is levied on them (tmi= 0.25). There are two agents: workers, owning labour, and capitalists owning capital. Furthermore, workers pay a tax rate t_w= 0.2 on income and save according to a s_w= 0.125 saving propensity. The residual income is spent (C_w= 35). Rentiers pay a tax rate t_r= (5/35) on their income and save half of their disposable income (s_r= 0.5). The residual part is totally spent (C_r= 15). Government collects taxes on income and imports, saves a fraction ($GSAV$= 3) and spends the rest (GZ= 15). Foreigners ask for exports according to a specific export function (EX= 10) and obtain income from intermediate imports (note that there is no import of final goods). Part of their income is saved as foreign saving ($FSAV$= 2). Finally, all disposable saving is employed to buy investment goods (INV= 25).

These data may be summarized into an MCM like the one below.

Box 21: An illustrative MCM for an open economy

	ACT	WORK	RENT	GOT	ROW	INV
Q	100	-35	-15	-15	-10	-25
w	-50	50				
r	-35		35			
e	-12				-12	
taxM	-3			3		
dtax		-10	-5	15		
SAV		-5	-15	-3	-2	25

Source: ***Author's own model***

a) The neoclassical closure in an open economy[17]

As in previous models, in this case we will analyse both a demand side and a supply side shock for each closure in order to trace differences. In this neoclassical closure we firstly suppose a demand side shock which we model as a 10 percent increase in the autonomous component of the export function. The modeller chooses the exchange rate as numeraire. As any supply driven system, the effect of an increase in exports is only a compositional effect on final demand. In the foreign sector, imports are fixed because they are intermediates according to a fixed fraction with respect to domestic production (in our case production is not affected by the shock), and exports increase by 10 percent. This reduces the foreign savings (now it is 1 and not 2). Income of the two classes is unchanged as are their tax payments. This leads to constant revenue for government. Its spending is fixed and consequently the same happens for its savings. Therefore the reduction in foreign savings should be compensated by an increase in private domestic savings in order to maintain the saving-investment balance. Therefore, workers' and rentiers' consumptions decline (by 0.7 percent and 5 percent, respectively) so that they free available savings to restore the equilibrium. The demand side shock does not affect any price.

As usual the supply side shock is a 10 percent increase in capital supply. After the shock, capital is the more abundant factor so its rental rate declines. The same happens to the wage rate. Both of them lessen by nearly 4 percent (3.95 percent). This means that total costs for primary factors is unchanged and the same happens for imported intermediates. Exchange rate is the numeraire so that imports are as costly as before the shock. Therefore, total production is fixed at the benchmark level.

However, income distribution has now become opposite the initial situation: labour income is lower than capital income. This means that workers have a lower income, their tax payment declines (by 4 percent) like their consumption but their savings remain unchanged. Rentiers'

[17] Formally the model is the one presented in box 9 with only one exception. In this case intermediates are only imported and they are not a composite of domestic and imported goods.

income increases by 5.7 percent. Their income tax increases as well because of the higher tax base and consumption also increases by 10.6 percent. However, in absolute terms, the increase in income (which is 2) is mainly devoted to consumption (1.6). Only a small fraction is devoted to income tax (0.3) and the residual 0.1 is the increase in savings.

As a consequence, to restore the saving- investment balance, government saving should decline to a 2.9 level. This may be explained by looking at the situation of the public sector. Total tax revenue slightly declines because, although import tariffs are unaffected, the fall in workers' tax payment offsets the increase in rentiers' payment. By assumption government expenditures are at their benchmark level so that only public saving may move and in this situation it should decline.

b) The "Bastard Keynesian" closure in an open economy

An increase in exports[18] is a stimulus to the aggregate demand. This means that real production increases. This causes an increase in labour demand to get to a higher production level. Labour increases by 1.6 percentage points. Since now labour is relatively abundant with respect to capital, the rental rate of capital is higher. In the production sphere an increase in total final products means an increase in imported intermediates. In fact, intermediates are in fixed proportion to production (Leontief production function). The increase in imports is lower than the initial shock on exports (note the difference with the Neoclassical case). Therefore, foreign saving declines. Now from a distributive point of view, workers own a higher nominal income. Because of the broader tax base, income taxes increase in the same proportion. But there are also higher saving and consumption levels (+1.14% and +3.4% respectively). In absolute terms, the 0.8 increase in income is allocated in this way: a 0.2 goes to income tax, a 0.4 to consumption, and 0.2 to savings.

Although these calculations are on a nominative basis, the same trend is shown in real variables. In this case quantities are a bit lower because we have eliminated the price effect.

Rentiers have a higher income level because of the higher remuneration of the same amount of capital. This leads to higher taxes and savings (+1.6% and +4%) while consumption lowers (-1.3%). In absolute terms, the 0.5 increase is devoted to tax (+0.1), savings (+0.6), and consumption (-0.2).

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[18] Our shock is on the exogenous component of the export function. However, the increase in exports is higher than the initial stimulus (+10% increase in the foreign component of exports, and 9.7% increase in final real exports).

This is caused by the iteration of the two components. When the exogenous component increases, the general price level increases as well so that the fraction (PEW/Q) declines.

Government has fixed spending (by assumption) but its fiscal revenue is higher. This means that government savings has to increase as well (+9.3%).

Disposable nominal savings are now at a higher level as are nominal investments. However, in real terms the saving- investment balance is unchanged. The loss in foreign savings is counterbalanced by all the domestic saving sources.

Since the system is demand driven, an increase in capital supply stimulates only an increase in nominal terms because of the higher general price level as a consequence of higher price of inputs. Specifically, labour declines (-0.6%). At the same time there is an increase in capital remuneration (+9.1%). Because of the functional form of intermediates, they slightly decline (-0.4%). Workers have a reduced income (-0.6%) so tax payments and consumption are lower (-0.6% and -1.5%). Savings increase by 2.4 % in real terms. Conversely, rentiers increase their income, which is devoted to taxes (+ 9%), savings (+ 5.3%), and consumption (+ 12.6%).

Government expenditures increase only in nominal terms while real expenditures are fixed by assumption. Tax revenue increases in nominal terms while they decline in real terms. Then, public saving declines.

In the foreign sector, imports are lower while exports increase only in nominal terms. Real exports have declined because of the reduction in the term (PEW/Q). As a consequence, foreign savings shrink by 15%.

The saving- investment balance changes only in nominal terms with a slight increase in investment and consequently in nominal total savings. But in real terms investment is unaffected and the reduction in public and foreign sectors' savings is counterbalanced by an increase in real private saving.

c) The Structuralist / Post- Keynesian closure for an open economy

This closure is very similar to the "Bastard Keynesian" model just described both in its achievements and in its logical construction. The main difference is the pricing rule which affects the quantitative results of the simulations. The effects of the export shock are quite simple because they are proportional to the initial shock. Therefore, a 10 percent increase in the autonomous component of exports leads to a 1 percent increase in GDP. Because the nested production function is a two- stage Leontief function, labour and mark- up (what we have called capital income up to this point) increase by the same percentage (+1%). Intermediates are aggregated according to $a0$ and therefore also imported intermediates increase by the same percentage.

There are no nominal effects because final price is a function of wage rate (for the labour costs) and exchange rate (for non-labour costs). In our model both these variables are assumed

to be numeraires of the model. Both workers' and rentiers' incomes increase. For workers, the increase in income means a proportional increase in taxes. Then consumption and savings increase as well (+0.64% and +3.5% respectively). In absolute terms, the income increase is 0.5 and it is allocated in 0.225 for consumption, 0.1 in taxes, and 0.175 for saving. For rentiers the situation is quite similar. However, in this case savings and taxes increase while consumption declines. The extra saving is necessary to maintain the saving- investment balance. Government tax revenue gets higher while, for the chosen closure rule, government spending is fixed. This means an increase in public saving.

In the external sector both exports and imports increase but in absolute terms the export increase is higher so that foreign saving declines as in "Bastard Keynesian". It is worth noting that the decline is lower than in the "Bastard Keynesian" case because now we have no price effect.

The supply side shock is an increase in *tau*. The effect on real GDP is negative: it declines by 0.38%. This makes labour employment diminish while mark- up income increases because of the shock. The imported intermediates decline too. Because we have increased *tau*, final price increases as well. This fact has a direct implication about the foreign sector. In fact, real exports decline because of the increase in final price. This leads foreign savings to decline from the initial benchmark level. The effects on workers' and rentiers' income are very similar to the ones in the "Bastard Keynesian" model. Trends are exactly the same but nominal variables are different because in the Structuralist model price increases more than in the "Bastard Keynesian" model.

Table 10: Results of a 10% increase in exports

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***
Volumes				
GDP	100	100	101.53	101
Labour	50	50	50.82	50.5
Capital	35	35	35.57	35.35[a)]
Imported intermediates	12	12	12.12	12.12
Workers' consumption	35	34.75	35.43	35.225
Rentiers' consumption	15	14.25	14.81	14.775
Total private consumption	50	50	50.24	50
Government consumption	15	15	15.11	15
Exports	10	10	11.04	11
Investments	25	25	25.14	25
Values				
GDP	100	100	100,96	101
Investments	25	25	25	25
Workers' saving	5	5	5.19	5.175
Rentiers' saving	15	15	15.59	15.525
Total private saving	20	20	20.78	20.7
Government saving	3	3	3.14	3.18
Foreign saving	2	1	1.08	1.12
Prices				
Output price	1	1	1.0057	1
Wage rate	1	1	1	1
Rental rate of capital	1	1	1.0164	1
Exchange rate	1	1	1	1

Source: ***Author's own calculations***

Table 11: Results of a 10% increase in capital supply

	Benchmark	***Neoclassical***	***"Bastard Keynesian"***	***Structuralist/ Post Keynesian***
Volumes				
GDP	100	100	102.87	103.11
Labour	50	48	49.68	49.81
Capital	35	37	38.25	38.36[a)]
Imported intermediates	12	12	11.95	11.95
Workers' consumption	35	33.4	34.46	34.54
Rentiers' consumption	15	16.6	16.93	16.96
Total private consumption	50	50	51.39	51.50
Government consumption	15	15	15.48	15.525
Exports	10	10	10.19	10.21
Investments	25	25	25.81	25.875
Values				
GDP	100	100	99.64	99.62
Investments	25	25	25	25
Workers' saving	5	5	5.12	5.13
Rentiers' saving	15	15.1	15.35	15.31
Total private saving	20	20.1	20.47	20.44
Government saving	3	2.9	2.82	2.81
Foreign saving	2	2	1.73	1.75
Prices				
Output price	1	1	1.0325	1.035
Wage rate	1	0.9605	1	1
Rental rate of capital	1	0.9605	0.9936	1
Exchange rate	1	1	1	1

Source:***Author's own calculations***

[a)] In this case income from capital is defined as mark-up income to remember its nature.