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Correlation and portfolio analysis of financial contagion and capital flight

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Doctoral Thesis of Siwat NAKMAI

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of Università Cattolica del Sacro Cuore, Milan, Italy is solely my own work other than where I have clearly indicated that it is the work of others.

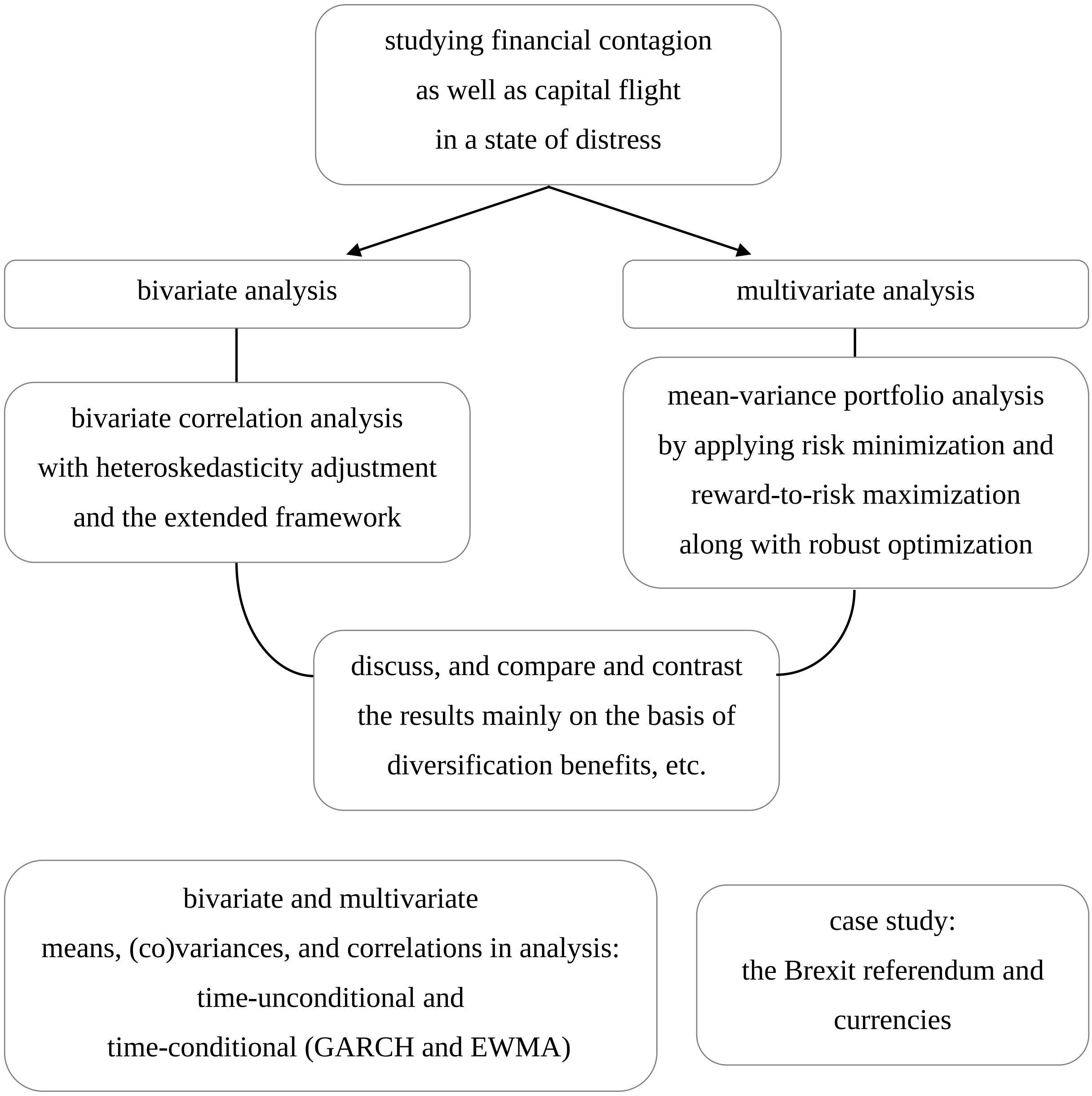
I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Foreword

This dissertation mainly studies correlation and then portfolio analysis of financial contagion and capital flight, focusing on currency co-movements around the political uncertainty due to the Brexit referendum on 26 June 2016. The correlation, mean, and covariance computations in the analysis are both time-unconditional and time-conditional, and the generalized autoregressive conditional heteroskedasticity (GARCH) and exponentially weighted moving average (EWMA) methods are applied.

The correlation analysis in this dissertation (Chapter 1) extends the previous literature on contagion testing based on a single global factor model, bivariate correlation analysis, and heteroskedasticity bias correction. Chapter 1 proposes an alternatively extended framework, assuming that intensification of financial correlations in a state of distress could coincide with rising global-factor-loading variability, provides simple tests to verify the assumptions of the literature and of the extended framework, and considers capital flight other than merely financial contagion. The outcomes show that, compared to the literature, the extended framework can be deemed more verified to the Brexit case. Empirically, with the UK being the shock-originating economy and the sterling value plummeting on the US dollar, there exist contagions to some other major currencies as well as a flight to quality, particularly to the yen, probably suggesting diversification benefits. When the correlation coefficients are time-conditional, or depend more on more recent data, the evidence shows fewer contagions and flights since the political uncertainty in question disappeared gradually over time. After relevant interest rates were partialled out, some previous statistical contagion and flight occurrences became less significant or even insignificant, possibly due to the significant impacts of the interest rates on the corresponding currency correlations.

The portfolio analysis in this dissertation (Chapter 2) examines financial contagion and capital flight implied by portfolio reallocations through mean-variance portfolio analysis, and builds on the correlation analysis in Chapter 1. In the correlation analysis, correlations are bivariate, whereas in the portfolio analysis they are multivariate and the risk-return tradeoff is also vitally involved. Portfolio risk minimization and reward-to-risk maximization are the two analytical cases of portfolio optimality taken into consideration. Robust portfolio optimizations, using shrinkage estimations and newly proposed risk-based weight constraints, are also applied. The evidence demonstrates that the portfolio analysis outcomes regarding currency contagions and flights, implying diversification benefits, vary and are noticeably dissimilar from the correlation analysis outcomes of Chapter 1. Subsequently, it could be inferred that the diversification benefits deduced from the portfolio and correlation analyses differ owing to the dominance, during market uncertainty, of the behaviors of the means and (co)variances of all the shock-originating and shock-receiving returns, over the behaviors of just bivariate correlations between the shock-originating and shock-receiving returns. Moreover, corrections of the heteroskedasticity bias inherent in the shock-originating returns, overall, do not have an effect on currency portfolio rebalancing. Additionally, hedging demands could be implied from detected structural portfolio reallocations, probably as a result of variance-covariance shocks rising from Brexit.


studying financial contagion as well as capital flight in a state of distress
bivariate analysis
multivariate analysis
bivariate correlation analysis with heteroskedasticity adjustment and the extended framework
mean-variance portfolio analysis by applying risk minimization and reward-to-risk maximization along with robust optimization
discuss, and compare and contrast the results mainly on the basis of diversification benefits, etc.
bivariate and multivariate means, (co)variances, and correlations in analysis: time-unconditional and time-conditional (GARCH and EWMA)
case study: the Brexit referendum and currencies

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Contents

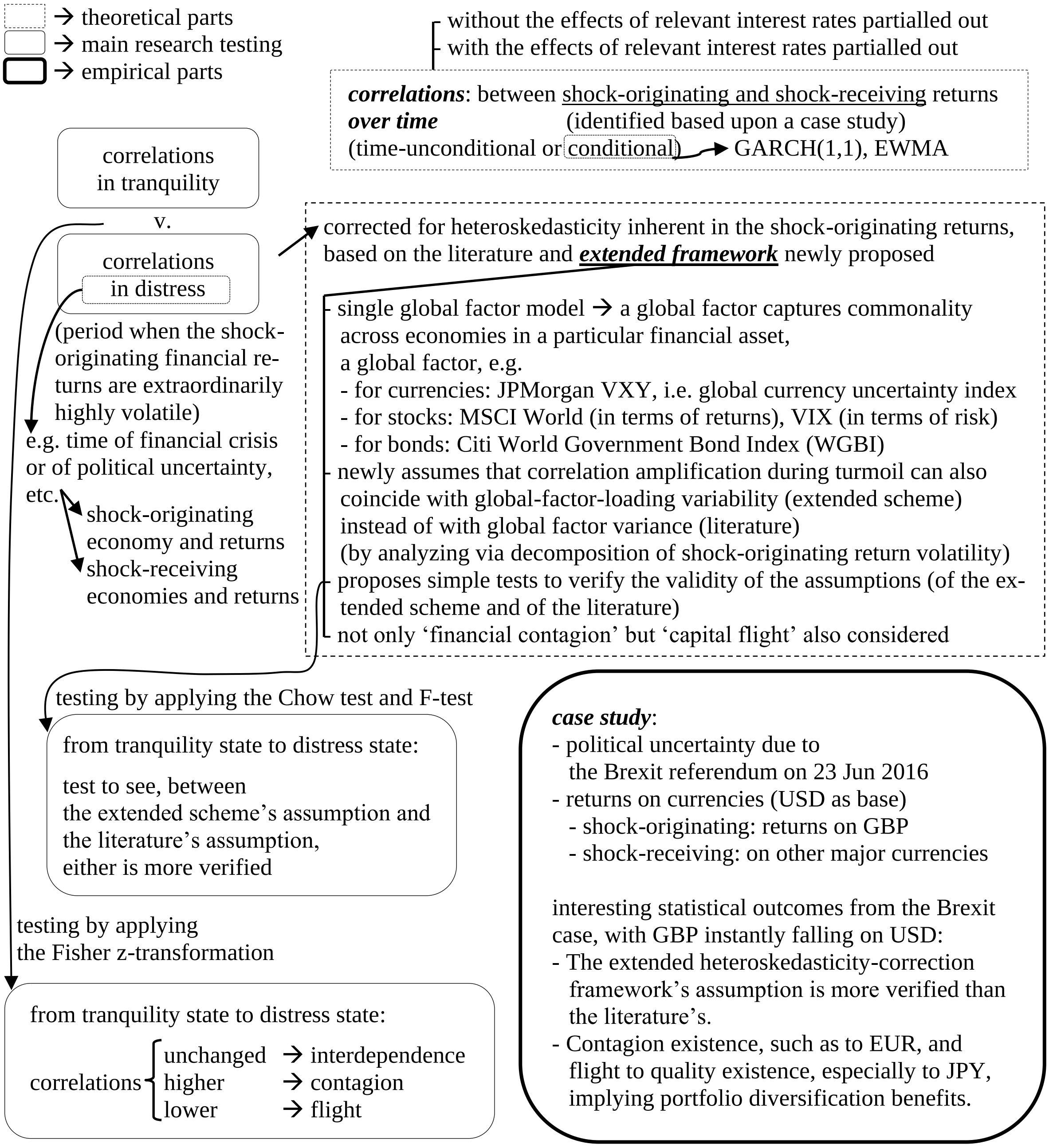
Chapter 1 Extended correlation analysis of financial contagion and capital flight: evidence from Brexit and currency co-movements

Abstract

This paper extends the former literature on financial contagion based on a single global factor model, bivariate correlation analysis, and heteroskedasticity bias correction, by reconsidering and alternatively reassuming the relation between a correlation coefficient and a global factor such that amplifying financial correlations in a state of distress could coincide with soaring global-factor-loading variability. On top of that, this chapter provides simple tests to verify the assumptions of the literature and of the extended framework, and also hypothetically considers capital flight other than financial contagion. Empirically, this chapter concentrates on currency co-movements around the political uncertainty arising from the Brexit referendum on 23 June 2016. The assumption-verification tests suggest that, for the Brexit case, the extended framework can be considered more suitable than the literature. Under the assumption that a contagion or a flight can be said to be detected if a heteroskedasticity-corrected correlation intensifies significantly, the evidence reveals that the fall of the sterling against the US dollar was relatively contagious to some other major currencies, such as the euro, and there exists a fairly strong currency flight to quality, particularly to the yen. Hence, diversification benefits seem obtainable in the currency market in times when they are needed most. After the effects of relevant interest rates are removed, some significant contagion and flight occurrences reduce, possibly due to the significant impacts of the interest rates on the corresponding currency correlations. Besides this, because the financial panics and herding owing to the UK political uncertainty have faded away over time, contagions and flights become less detectable when correlation computations are time-conditional or weight newer information more heavily (also see **Figure 1.1** on the next page for an infographic brief outline of Chapter 1).

Keywords: bivariate correlation analysis, dynamic conditional correlation, heteroskedasticity, financial contagion, capital flight, Brexit, political uncertainty

Figure 1.1: diagram outline or mind map of Chapter 1



1.1 Introduction

Financial crises or political uncertainty, leading to periods in which the shock-originating financial returns, that is financial returns in the economy or economies in which the shock originated, are extraordinarily highly volatile, emerge across economies from time to time. It is therefore useful to develop relevant methodologies and tests, so as to be able to make advantageous statistical inferences, given that financial contagion is one of the more popular topics of investigation, and also the primary focus of this dissertation. This chapter will mainly extend financial contagion literature, using bivariate correlation analysis, by re-assuming the association between a correlation coefficient and a global factor, offering simple assumption testing, and investigating a recent political event, Brexit, with currencies, on the basis of both time unconditionality and conditionality. These are from the following three main areas of motivation and contribution that inspire this chapter.

The first motivation is to extend Corsetti et al.'s (2005) work on financial contagion, which applies a single global factor model, bivariate correlation analysis, and heteroskedasticity bias. This chapter attempts to make Corsetti et al.'s (2005) contagion testing more flexible and comprehensive, by newly delivering an alternative set of assumptions along with simple assumption-verification tests, as well as studying capital flight in addition to financial contagion. The global factor captures commonality across economies in terms of a particular financial asset. This chapter alternatively assumes that correlation amplification in a state of distress or during highly uncertain times could coincide with rising global-factor-loading variability instead of rising global factor variance, as is supposed by Corsetti et al. (2005). Whether the extended framework or that of Corsetti et al. (2005) can be considered more appropriate for a specific case can be verified by the simple assumption tests newly proposed in this chapter.

The second aim is to fill some gaps in existing contagion studies. Unlike stock or bond co-movements during financial crises, which have been widely examined in the literature, currency co-movements under political uncertainty have not yet been studied extensively. Accordingly, this chapter examines currency correlations and the political uncertainty caused by a recent major political event, namely the Brexit referendum on 23 June 2016 which produced an unexpected result.[1]

The third motivation is to attempt to obtain statistical outcomes and make inferences under time unconditionality as opposed to time conditionality. This would enable us to see how conclusions could be drawn from evidence generated by different time-weighting schemes, i.e. those more or less dependent on newer or older information.

The remaining sections of this chapter deal with background, definitions and data description; unconditional and dynamic conditional correlations; correlation analysis of financial contagion and capital flight; evidence from the Brexit referendum, and concluding

[1] The result of the Brexit referendum on 23 June 2016, 51.9% to 48.1% in favor of leaving the EU, was unexpected because opinion polls had anticipated that the UK would vote to remain in the EU.

remarks (apart from the chapter's structure described, also see **Figure 1.1** for the chapter's infographic brief outline).

1.2 Background, definitions and data description

The background and definitions of financial contagion and capital flight based on correlation analysis and the data descriptions are as follows.

1.2.1 Background

In a state of distress or during periods of crisis or high uncertainty, or when financial returns in the economy of origin of the shock are extraordinarily highly volatile, the magnitudes of financial return co-movements across countries and across markets are likely to intensify. The return co-movement intensification brings into question whether, in the move from a state of tranquility to a state of distress, the international transmission mechanism of financial shocks indeed shifts. If such a mechanism shifts, '***contagion***' occurs.[2] On the other hand, if it does not shift, there exists only '***interdependence***', that is two specific economies have strong linkages in all states (Corsetti et al., 2005; Forbes & Rigobon, 2002).

Several scholars have analyzed and performed tests on financial contagion, with the aim of dealing with the issue of contagion versus interdependence. The literature includes Forbes and Rigobon (2002) and Corsetti et al. (2005). In Forbes and Rigobon (2002), contagion is considered to be detected if the magnitude of a crisis-time correlation with heteroskedasticity adjustment goes beyond that of a tranquility-state correlation. With similar intuition, Corsetti et al. (2005) test whether the magnitude of a crisis-time correlation significantly exceeds a theoretical measure of interdependence. Interdependence is a tranquil-time correlation with a statistical correction subject to a model and a set of assumptions. The model and assumptions proposed by Forbes and Rigobon (2002) correct for the entire heteroskedasticity of the returns in the shock-originating economy.[3] Arguing that Forbes and Rigobon (2002) implement an overcorrection with their restriction on the shock-originating idiosyncratic noise, Corsetti et al. (2005) propose another model and set of assumptions without that

[2] Other relevant contagion definitions may include (1) a significant rise in the likelihood of a crisis in an economy, conditional on a crisis happening in another economy, (2) asset price volatility spillovers from a crisis economy to others, and (3) spillovers unexplainable by fundamentals, etc. (Pericoli & Sbracia, 2003).

[3] Boyer et al. (1999) also propose a correlation correction, similar to that of Forbes and Rigobon (2002).

restriction. The two research works conduct comparable tests on the international impacts of the Hong Kong stock market crash in 1997.[4] Forbes and Rigobon (2002) find no contagion, only interdependence, whereas Corsetti et al. (2005) discover some contagion and some interdependence.

There are several other studies on financial contagions or flights, focusing upon certain financial crises. Most of those studies involve stocks and bonds rather than other types of assets. Other works on stock contagion across economies include King and Wadhwani (1990), Baig and Goldfajn (1999), Bae et al. (2003), Baur and Schulze (2005), and others. The literature on international bond contagion includes Baig and Goldfajn (1999), Dungey et al. (2006), and Claeys and Vašíček (2014), among other works. Those dealing with stock-bond contagions and flights, include, among others, Gonzalo and Olmo (2005), Baur and Lucey (2009), and Papavassiliou (2014). Besides these, the literature that involves dynamic conditional correlation analysis of financial contagion includes Baur and Lucey (2009), who look at several crisis episodes, Chiang et al. (2007) who focus on the 1997 Asian crisis, and Papavassiliou (2014) who studies the Greek debt crisis.

1.2.2 Contagion and flight definitions

As aforementioned, if a correlation magnitude in distress rises in excess of an interdependence measure, contagion occurs, whereas the opposite implies interdependence or a strong connection between two particular economies in all states.

Nevertheless, a rise in the magnitude of a correlation does not always imply that the correlation is increasing. The correlation could be negative and decreasing. A correlation coefficient that increases significantly, if it ends up positive in a state of distress, implies the presence of a ***contagion***. A correlation coefficient that falls significantly, ending up negative during market uncertainty, indicates the presence of a ***flight*** (Baur & Lucey, 2009). Furthermore, a contagion can be negative or positive, with two asset prices falling or rising together, respectively. A flight can be to or from quality, from riskier to safer or vice versa (Baur & Lucey, 2009). In a state of distress or during a crisis or political uncertainty, there are a

[4] Forbes and Rigobon (2002) also study other financial crises: the 1994 Mexican peso crisis and the 1987 US stock market crash.

shock-originating economy, and shock-receiving economies. The definitions of interdependence, financial contagion, and capital flight can thereby be summarized and applied to currencies as in **Table 1.1**.

change in a correlation coefficient ($\Delta\rho$) (correlation between the shock-originating economy's financial returns and another shock-receiving economy's financial returns)	**instant change in the shock-originating economy's financial asset value, from a state of tranquility (T) to a state of distress (D)**	
	shock-originating economy's financial asset value falling, e.g. currency depreciation	shock-originating economy's financial asset value rising, e.g. currency appreciation
insignificant $\Delta\rho$	interdependence	interdependence
significant $\Delta\rho > 0$, given $\rho_D > 0$ [5]	negative contagion (e.g. both currencies depreciate)	positive contagion (e.g. both currencies appreciate)
significant $\Delta\rho < 0$, given $\rho_D < 0$	flight to quality (e.g. shock-originating one depreciates, shock-receiving one appreciates)	flight from quality (e.g. shock-originating one appreciates, shock-receiving one depreciates)

Table 1.1: summary of interdependences, contagions, and flights between shock-originating and shock-receiving currencies, for bivariate correlation analysis, currencies considered against a base currency (e.g. USD, etc.) or as currency indices

Now consider the Brexit referendum of 23 June 2016.[6] Since the majority unexpectedly voted in favor of the UK leaving the EU, political and related uncertainties, specifically regarding the potential long-term damage to the British economy, have risen substantially. Thus, the UK represents the shock-originating economy in this scenario, while other affected countries are shock-receiving. That the referendum results were unexpected and viewed negatively rather than positively, or seen as a negative shock, is possibly due to financial herding taking place during the period of political uncertainty, which can be inferred from several occurrences following the Brexit referendum. The sterling value against the US dollar slumped (as in **Figure 1.2**), even down by 13% to the lowest level in 30 years during the early hours of 24 June 2016, according to Financial Times.[7] The values of other major foreign exchanges shifted unusually (as in **Figure 1.3**). Sterling volatility increased markedly (as in **Figure 1.A.1**).[8] And, post-referendum correlations between the sterling and other major currencies behave considerably differently than before (as in **Figure 1.4**, **Figure 1.5**, and

[5] ρ_D: a correlation coefficient in distress (D) or after a breakpoint.

[6] Brexit: Britain to leave the EU, Bremain: Britain to remain in the EU.

[7] 'Pound tumbles to 30-year low as Britain votes Brexit', Financial Times (https://www.ft.com/content/8d8a100e-38c2-11e6-a780-b48ed7b6126f)

[8] **Figure 1.A.1** showing rising currencies' volatilities is supplementary and placed in Appendices, unlike the figures (**Figure 1.4**, **Figure 1.5**, and **Figure 1.6**) showing shifts in currencies correlations that are placed in the main context, because correlations are vitally more focused in Chapter 1.

Figure 1.6, more details in **1.3.3 Time-varying currency correlations around the Brexit referendum**).

The case of the instant depreciation of the shock-originating economy's currency, which is the sterling plunging instantly on the US dollar, is then taken into analysis. To be more specific, the existence of negative contagions and flights to quality as a result of the UK political uncertainty that led to a harsh drop in the sterling is to be investigated.

The case of the sudden appreciation of a shock-originating economy's currency at a particular breakpoint can also occur. Taking the opposite situation to Brexit, for example, suppose the prior opinion polls had indicated that the UK was likely to leave the EU, but the results were unexpectedly in favor of the UK remaining. The results could thereby be seen as a positive shock. Then, political uncertainty should have reduced considerably and the sterling would likely have risen instantly against the US dollar. Thus, for this, possible occurrences of positive contagions and flights from quality would be the case.

Other unanticipated financial or political events with other asset classes or types could also be applied in line with the definitions provided.

1.2.3 Data description

The examined currencies are the shock-originating economy's currency, GBP, and various shock-receiving economies' currencies, the EUR, CHF, CAD, JPY, AUD, and NZD. All are considered with respect to the USD as the base currency and their daily last prices are used.[9] The shock-receiving currencies were chosen because they are floating against the USD and are among the most traded, internationally.[10,11]

As well as the currencies of shock-originating and shock-receiving economies, a currency global factor is also included in the analysis; JPMorgan's global currency implied

[9] USD: US dollar (currency base), GBP: Great British pound (shock-originating economy's currency), EUR: euro, CHF: Swiss franc, CAD: Canadian dollar, JPY: Japanese yen, AUD: Australian dollar, and NZD: New Zealand dollar (the other major currencies, sorted by geography in terms of Europe, the Americas, and Asia Pacific). The currencies are analyzed against USD as the base currency: namely, how much can one unit of the counter currency be exchanged for in USD.

[10] According to the International Monetary Fund (IMF) 2017 Annual Report on Exchange Arrangements and Exchange (https://www.imf.org/~/media/Files/Publications/AREAER/areaer-2017-overview.ashx)

[11] Currency turnover is reported by the Bank for International Settlements (BIS) (https://www.bis.org/publ/rpfx16fx.pdf). CHF is floating against the USD and also has not been pegged with the EUR since 15 January 2015 (https://www.snb.ch/en/mmr/reference/pre_20150115/source/pre_20150115.en.pdf). The Chinese currency, RMB or CNY (or CHY offshore), despite now being highly traded and included in the SDR (Special Drawing Rights: an international reserve asset, created by the IMF, and defined as a weighted average of several major currencies) since October 2016, is still relatively managed, and not yet floating, against the USD. Thus, the RMB is not a good choice for the currency correlation analyses.

volatility index is employed, on a risk perspective, as a proxy for the global factor, denoted by VXY (not the VXY based on G7 currencies or the VXY based on only emerging economies' currencies, but the VXY globally based on both major and emerging currencies).[12] The VXY index is computed based on three-month at-the-money volatilities implied from foreign exchange options with currency basket weighting, across both major and emerging currencies (subject to currency-pair turnover, all against USD, reported in the latest two BIS Triennial Central Bank Surveys), all against the USD. VXY thereby reflects foreign exchange market participants' forward-looking views on currency uncertainties and values.[13] The higher is VXY, the more uncertainty is expected in the currency market. Just as VIX (expected stock market volatility index implied by S&P 500 index options, published by the Chicago Board Options Exchange (CBOE)) fundamentally reflects common financial uncertainties across stock markets[14], VXY does so across currencies (Corte et al., 2016; Gonzalez-Perez, 2015). Accordingly, VXY can be perceived as a global factor in the foreign exchange market. Furthermore, Corte et al. (2016), in their research on currency premiums and global imbalances, also employ VXY to capture global risk aversion in the foreign exchange market (also see in sub-section **Additional discussions on the currency global factor**, right after the following descriptive statistics).

The daily data of the studied currencies and VXY were obtained from the Bloomberg Terminal. The tranquility-state time span is one year up to the Brexit referendum (24 June 2015 – 23 June 2016, or 262 trading days), and the distress-state time span is one month (24 June – 22 July 2016, or 21 trading days following the Brexit referendum) (these are similar to Forbes and Rigobon (2002) and Corsetti et al. (2005)).[15] The descriptive statistics of the

[12] JPMorgan's global currency implied volatility index is actually denoted by JPMVXYGL in Bloomberg Terminal. It has been introduced after JPMVXYG7 and JPMVXYEM: G7- and emerging-currency implied volatilities, respectively (additional reference: 'JPMorgan Chase Adds Global Currency Option Volatility Index', Bloomberg (https://www.bloomberg.com/news/articles/2011-03-25/jpmorgan-chase-adds-a-global-currency-option-volatility-index-to-offerings)).

[13] Weights used in calculating JPMorgan's VXY are based on the global currency turnover (also see the referenced Bloomberg document in Footnote 12), averaged across the latest two BIS triennial surveys (also see the referenced BIS document in Footnote 10). Thus, the weights used in calculating VXY should well reflect global foreign exchange trades. Consequently, the weighted foreign-exchange-option implied-volatility index, namely VXY, should also well reflect global foreign exchange uncertainty on the basis of forward looking.

[14] VIX index could be a global factor on a risk perspective if stock, rather than currency, co-movements are studied. MSCI World index could also be a global factor, but on a return perspective, for stock-market-return co-movements, similarly to what is examined by Forbes and Rigobon (2002) and Corsetti et al. (2005). While, Citi World Government Bond Index (WGBI), could be a global factor, but on a return perspective, for bond-market-return co-movements. Additionally, VIX index could possibly also be a global factor for stock-bond co-movements.

[15] In the later hypothesis tests on currency contagions and flights, other nearby time spans are also taken into account to check the robustness, namely tranquility-state time spans of 15, 12, and 9 months, with the same distress-state time span of 1 month only (see section **1.3.3 Time-varying currency correlations around the Brexit referendum** and **Appendix 1.G Robustness checks on the extended framework**), but also with distress-state time spans of 0.5 and 0.75 month, to check contagion and flight persistence during 1 month following the Brexit referendum date (i.e. distress-state time span of 1

daily log returns (i.e. $\Delta \ln(\cdot)$) of the investigated currencies and the global factor in the tranquility and distress states are provided in **Table 1.2**, with the movements in the examined exchange rates depicted in **Figure 1.2** and **Figure 1.3**.[16]

tranquility (1 year)	**GBP**	**EUR**	**CHF**	**CAD**	**JPY**	**AUD**	**NZD**	**VXY**
min	-0.017	-0.018	-0.017	-0.016	-0.021	-0.021	-0.025	-0.125
max	0.025	0.029	0.025	0.025	0.027	0.028	0.038	0.101
mean	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
annualized mean	-0.058	0.015	-0.025	-0.035	0.152	-0.017	0.055	0.042
annualized s.d.	0.088	0.097	0.093	0.097	0.104	0.124	0.139	0.377
normality[17]	R,le***	R,le***	R,le***	R,le***	R,le***	normal	R,le**	L,le***
distress (1 month)	**GBP**	**EUR**	**CHF**	**CAD**	**JPY**	**AUD**	**NZD**	**VXY**
min	-0.080	-0.022	-0.013	-0.017	-0.022	-0.016	-0.015	-0.085
max	0.023	0.006	0.004	0.011	0.034	0.016	0.014	0.213
mean	-0.006	-0.002	-0.001	-0.001	0.000	-0.001	-0.002	0.003
annualized mean	-1.464	-0.403	-0.347	-0.318	-0.044	-0.197	-0.404	0.705
annualized s.d.	0.323	0.089	0.062	0.097	0.182	0.131	0.150	0.854
normality[17]	L,le***	L,le***	L,le**	normal	R,le**	normal	normal	R,le***

Table 1.2: tranquility-state and distress-state (pre- and post-Brexit referendum) (covering 262 and 21 trading days) descriptive statistics of currency and VXY daily log returns (USD as base)[18]
(*, **, and ***: non-normal at 0.1, 0.05, and 0.01 significance levels, respectively;
L: left-skewed, R: right-skewed; le: leptokurtic, pl: platykurtic)

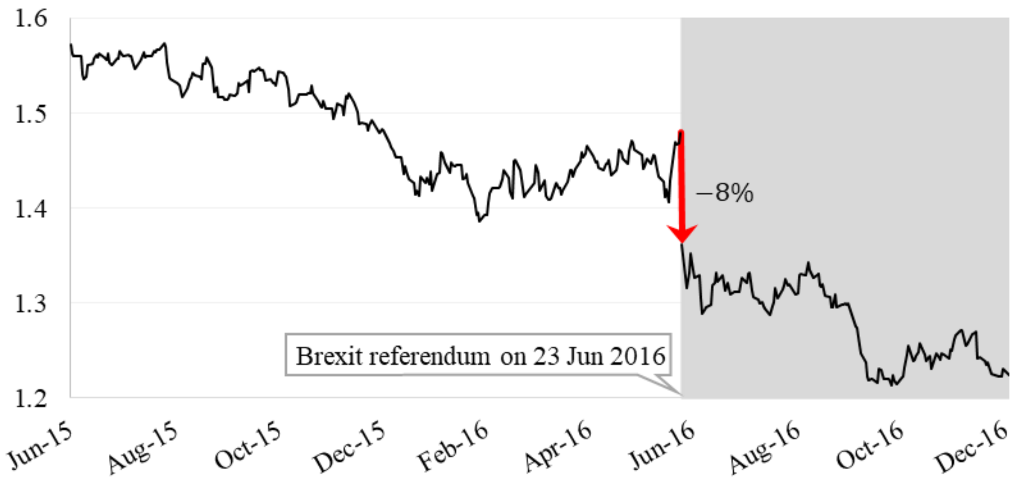


Figure 1.2: daily GBPUSD (USD as base) around the Brexit referendum, from Jun 2015 to Dec 2016 (the shaded areas: after the Brexit referendum)

month) (see section **Persistence of currency negative contagions and flights to quality**, under section **1.5.2 Evidence of currency contagions and flights**).

[16] Levels rather than returns of the exchange rates are illustrated in **Figure 1.2** and **Figure 1.3** so as to see tendencies in the values over the period of political uncertainty rather than just returns' white noise processes with substantial jumps immediately after the Brexit referendum date.

[17] Normality is tested via the Jarque-Bera (JB) test: $JB = (\tau/6)[S^2 + (K-3)^2/4]$, τ: sample size, S: skewness, K: kurtosis (Gujarati & Porter, 2009). Chi-squared p-values are calculated from JB statistics, using Monte Carlo simulation via MATLAB's jbtest for more precision since JB test outcomes are considered to be only approximate when $\tau < 2000$, and then compared against 0.1, 0.05, and 0.01 significance levels (https://www.mathworks.com/help/stats/jbtest.html).

[18] Currency log returns and standard deviations are annualized by multiplying by 252 and $\sqrt{252}$, respectively.

Due to the amplification of British political risk immediately after the Brexit surprise or the unexpected Brexit result, GBP fell with respect to USD by 8%, from 1.481 to 1.367. Meanwhile, VXY, which indicates the forward view of global uncertainty regarding currencies, jumped up by 21.3%. With regard to the means, the values in tranquility approach zero, while the values in distress are obviously negative, particularly that of GBP returns. Concerning the return volatilities, those of the major currencies ranged over $6\% - 15\%$ on a yearly basis across both the tranquility and distress states, with the exception of GBP and JPY after the referendum. Owing to the political uncertainty, the post-breakpoint returns on GBP and JPY saw increased dispersion of 32.3% and 18.2% per annum on average, respectively.

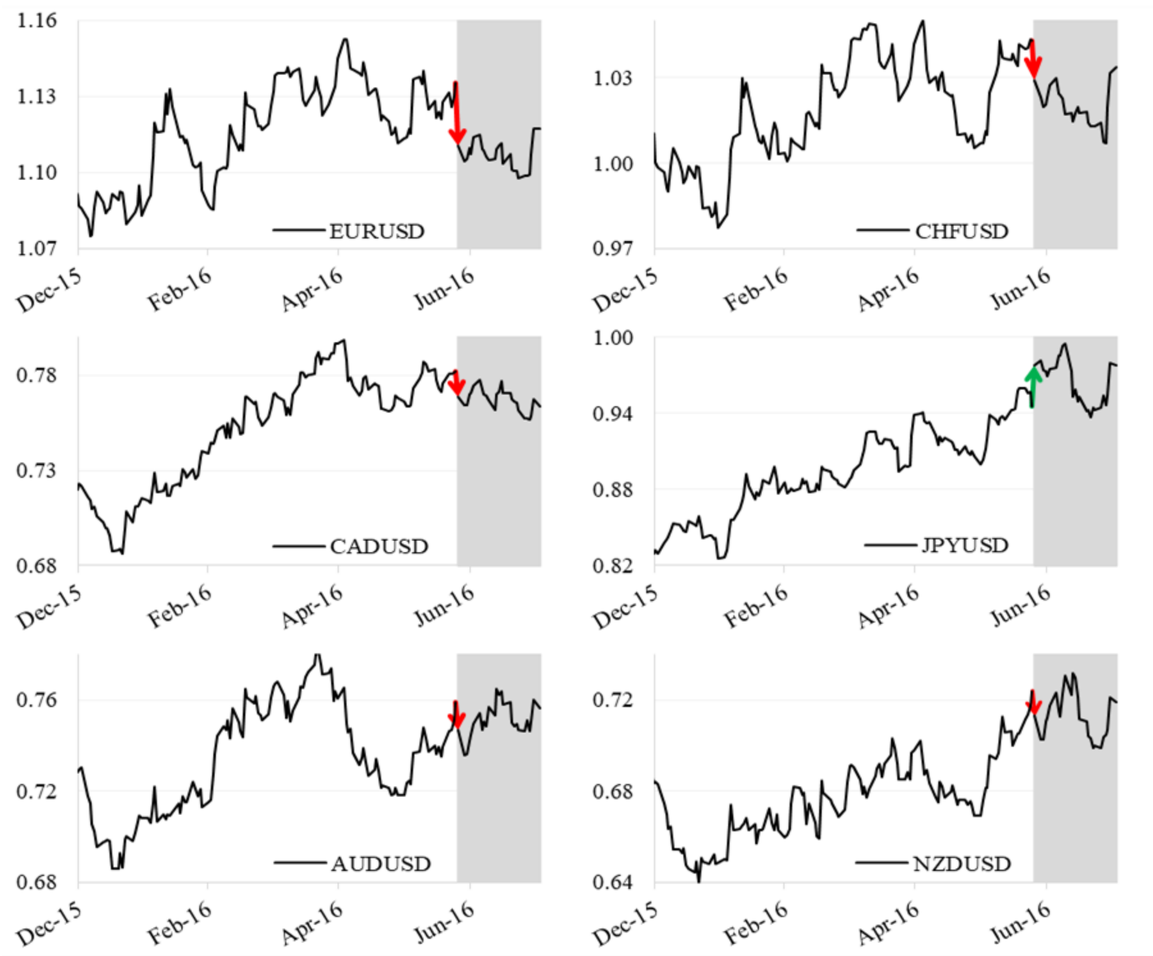


Figure 1.3: daily movements of the studied major currencies (USD as base) around the Brexit referendum, from Dec 2015 to Jul 2016 (the shaded areas: after the Brexit referendum)[21]

After the first and second moments of the data were briefly discussed, the third and fourth moments will be done so. As to learn the normality, skewness, and kurtosis of the data, normality is tested using the Jarque-Bera (JB) test.[19] Some returns are shown to be statistically normally distributed, whereas others are not. Most of the detected non-normal returns, like typical financial time series, are heavy-tailed, which could be inferred from detected leptokurtosis.[20] During a short time period after the referendum, GBP, EUR, and CHF returns became left skewed, while JPY and VXY turned out to be right skewed since JPY rose and uncertainty increased in the foreign exchange market.

Pertaining to the shock-originating economy's currency, GBP (see **Figure 1.2**), its value dropped harshly on the USD instantly following the Brexit referendum. Subsequently, GBP's depreciation and lower levels persisted for a number of months. The persistence may have been due to continuing concerns about potential detrimental effects on the UK economy.

Regarding the studied shock-receiving economies' currencies (see **Figure 1.3**)[21], it is obvious that, while JPY jumped up, all other currencies jumped down along with the sharp and sudden decline in the value of GBP. Nonetheless, although the EUR and CHF depreciations against USD seem to have continued for some time after the political incident, the post-breakpoint movements of the other major currencies do not show clear falling or rising tendencies.

Thus, it could be interesting to further discuss and analyze the correlations of GBP with those other major currencies around this time of political uncertainty.

In calculating the currency correlation coefficients, the effects of the overnight interbank interest rates related to the currencies will be uncontrolled (i.e. without interest rates partialled out), but afterwards controlled (i.e. with interest rates partialled out) in this dissertation (see section **Controlling for the effects of relevant interest rates**, under section **1.4.4 Hypothesis tests** for more detail on interest rates being partialled out). The overnight tenor is used in order to correspond to the employed daily foreign exchange rates. These overnight interbank interest rate data were collected from the corresponding central banks' data resources.

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[19] The Fisher z-transformation, later used in this chapter's the hypothesis tests, is considered quite robust to non-normality (Kocherlakota & Singh, 1982).

[20] Leptokurtosis is a distribution with positive excess kurtosis that is slender and heavier-tailed than normal distribution.

[21] Note that JPYUSD = (USD per JPY) × 100, while no scaling occurs for the other currencies.

Additional discussions on the currency global factor

The follows are additional discussions on the currency global factor: endogeneity issue and other possible global factors.

As discussed previously, VXY is chosen to be the currency global factor, which is also applied by Corte et al. (2016). In this chapter, an endogeneity between the global factor and the shock-originating economy's currency, however, might be raised as a concerning issue. Nonetheless, calculating the expected attribution of GBP to VXY and testing Granger causality between GBP and VXY around the Brexit referendum possibly implies that there should not exist such an endogeneity issue (see **Appendix 1.A Impact of GBP on VXY around the Brexit referendum**). The results from calculating attribution of currencies to VXY (see **Appendix 1.A.1 Attribution of GBP to VXY** and **Table 1.A.1**) show that, even augmented by the UK political uncertainty, the impact of GBP implied volatility on VXY is still less than JPY's, EUR's, and even the total impact of emerging currencies, which usually follow alike trends and fundamentally have interlinkages. This could be because of the option tenor used to compute VXY, which is three-month. Prior to the Brexit referendum date, at any given time point, even if it was closer to the date, where the three-month horizon covered the referendum date, VXY was not quite influenced by Brexit since opinion polls relatively rather suggested Bremain. And, immediately subsequent to the referendum, VXY was still not remarkably affected by Brexit because currency option market participants probably expected that the uncertainty would not significantly persist up to as long as three months. Additionally, even using realized historical volatilities, both time-unconditional and time-conditional, as implied-volatility proxies could to a certain extent still reaffirm the earlier implication. With these proxies, although the impact of GBP on VXY became higher due to the UK political uncertainty, GBP's effect on VXY was still less than JPY's, close to EUR's, and not quite different from the total effect of others, whose about half attributes to emerging-market currencies. Moreover, the results of the pairwise Granger causality tests of GBP and VXY also statistically show no endogeneity between them (see **Appendix 1.A.2 Granger causality between GBP and VXY**).

Other possible and interesting currency global or common factors could also be discussed as follows.

With concentration on investments and excess returns in the foreign exchange forward market, currency common factors could be carry trade and dollar risk factors, as discussed by Lustig et al. (2011). The excess returns are differences between forward and underlying spot foreign exchange rates (alternatively, viewed as forward forecasting errors), given going short in the USD in the forward market and long in other currencies. Carry trade risk factor is the difference between the average excess return on highest interest rate currencies and that on lowest interest rate currencies. Dollar risk factor is the average excess return on all currencies available in the forward market. Respectively, these two risk factors represent currency returns in carry trades and in the USD, determined by the fluctuations of the USD against other currencies. Nonetheless, unlike VXY weighted based on global currency turnover, the computations of these two common risk factors weighted equally across currencies ignore which currency should be more concentrated on across economies.

Another factor that could capture commonality in currency excess returns, especially in advanced economies, is global crash or disaster risk, according to Farhi et al. (2009). In addition to Gaussian or normal time risk factor in explaining the excess returns, the disaster risk premium is defined as the disaster risk exposure, less the anticipated loss during a disaster. They estimate the disaster risk factor from currency option values, whose volatility smiles are highly asymmetric during disasters, by obtaining the parameters from minimizing the sum of squared option price differences (between actual and the model) across countries and exercise prices.

Apart from the currency global factors discussed earlier, a financial systemic risk measure so-called conditional value-at-risk (CoVaR), proposed by Adrian and Brunnermeier (2016) and ably somewhat applied to the foreign exchange market, may also be mentioned here. CoVaR of a unit in a system is the unit's value-at-risk (VaR) conditional on other units being in distress. Adrian and Brunnermeier (2016) use CoVaR to measure the aggregate tail risk of individual financial institutions. Borri (2018) adopts CoVaR to measure the vulnerability of individual countries to overall systemic risk in the aggregate market for government bonds denominated in local currencies.

1.3 Unconditional and dynamic conditional correlations

Correlation coefficients, in correlation analysis of financial contagion and capital flight, may be computed unconditionally or conditionally, as explained below.

1.3.1 Unconditional correlations

Let $\mathbf{R}_t$ be a column vector of returns at time t of N financial assets, $\mathbf{R}_t = (r_{1,t}, \dots, r_{N,t})'$. The matrix of time-unconditional or static correlation coefficients of $\mathbf{R}_t$, denoted by $\widehat{\mathbf{P}}_{uncond}$, can be expressed as (1.1) (Pearson, 1920; Rodgers & Nicewander, 1988) (Although the correlation analysis of financial contagion and capital flight is bivariate in this chapter, correlations are discussed in multivariate form because the generalized autoregressive conditional heteroskedasticity (GARCH) correlations, expressed later in (1.2), require joint maximum likelihood estimation (MLE) (as in (1.4)) on the corresponding GARCH variances and covariances (see (1.3))) (Multivariate correlations will also be applied and analyzed in **Chapter 2 Mean-variance portfolio analysis of financial contagion and capital flight: evidence from Brexit and currencies**.).

$$\widehat{\mathbf{P}}_{uncond} = \left(\operatorname{diag}\left(\widehat{\mathbf{V}}_{uncond}\right)\right)^{-\frac{1}{2}} \widehat{\mathbf{V}}_{uncond} \left(\operatorname{diag}\left(\widehat{\mathbf{V}}_{uncond}\right)\right)^{-\frac{1}{2}} \tag{1.1}$$

where $\widehat{\mathbf{V}}_{uncond} = E[(\mathbf{R}_t - E[\mathbf{R}_t])(\mathbf{R}_t - E[\mathbf{R}_t])'], t \in [1, \tau]$, τ being the sample time span[22], is the $N \times N$ matrix of unconditional or static variances and covariances of $\mathbf{R}_t$ and $\operatorname{diag}(\widehat{\mathbf{V}}_{uncond})$ is the $N \times N$ diagonal matrix of $\widehat{\mathbf{V}}_{uncond}$.[23] Computationally, the variances and covariances in $\widehat{\mathbf{V}}_{uncond}$ are historically equally weighted. Consider time series data. The weakness is that older observations, which are less relevant, are viewed as just as important as more recent ones, which are more relevant. To cope with this weakness, dynamic conditional models, which are more reliant on newer data, are taken into account. These models are also particularly suitable for financial price changes, including returns on exchange rates, etc. This is because financial time series typically produce time-varying second moments and exhibit a characteristic so-called volatility clustering (Cont, 2007; Gujarati & Porter, 2009; Poon, 2005).[24]

1.3.2 Dynamic conditional correlations

[22] Unlike T, which denotes the time span of a state of tranquility.

[23] The diagonal elements of $\widehat{\mathbf{V}}_{uncond}$ or those of $\operatorname{diag}(\widehat{\mathbf{V}}_{uncond})$ are the variances of $r_{1,t}$, ..., $r_{N,t}$.

[24] Volatility clustering: persistence of periods of financial time series with wide swings, followed by periods with relative calms, i.e. large financial returns tend to cluster together and so do small ones.

Dynamic conditional models include those such as GARCH and exponentially weighted moving average (EWMA).

GARCH

A GARCH covariance depends on two main components: products of disturbances or innovations from previous time periods and conditional covariances from previous time periods.[25] The information is lagged a certain number of steps for each component. When the information is lagged by just one period for both components, GARCH takes its simplest form, written as GARCH(1,1) (Bollerslev, 1986). The matrix of multivariate GARCH(1,1) correlation coefficients of R_t, denoted by $\widehat{\mathbf{P}}_{t,GARCH}$, can be expressed as

$$\widehat{\mathbf{P}}_{t,GARCH} = \left(\mathrm{diag}\left(\widehat{\mathbf{V}}_{t,GARCH}\right)\right)^{-\frac{1}{2}} \widehat{\mathbf{V}}_{t,GARCH} \left(\mathrm{diag}\left(\widehat{\mathbf{V}}_{t,GARCH}\right)\right)^{-\frac{1}{2}} \tag{1.2}$$

with the following $N \times N$ matrix of GARCH(1,1) variance-covariance, $\widehat{\mathbf{V}}_{t,GARCH}$, which is a scalar BEKK in this context, for simplicity. The scalar BEKK is considered to be a simplified version of Engle's (2002) direct dynamic conditional correlation (DCC) since it requires more convenient MLE because returns are simply viewed as innovations (Caporin & McAleer, 2008; Engle, 2002).[26]

$$\widehat{\mathbf{V}}_{t,GARCH} = (1 - \alpha_1 - \alpha_2)\overline{\mathbf{V}} + \alpha_1(\mathbf{R}_{t-1}\mathbf{R}'_{t-1}) + \alpha_2\widehat{\mathbf{V}}_{t-1,GARCH} \tag{1.3}$$

where $\overline{\mathbf{V}}$ denotes the $N \times N$ unconditional variance-covariance matrix such that $\overline{\mathbf{V}} = \widehat{\mathbf{V}}_{uncond}$ in (1.1) with $(1 - \alpha_1 - \alpha_2)\overline{\mathbf{V}}$ viewed as the intercept of the model, $\mathbf{R}_{t-1}\mathbf{R}'_{t-1}$ the $N \times N$ matrix of previous-period products of innovations[27], $\widehat{\mathbf{V}}_{t-1,GARCH}$ the $N \times N$ previous-

[25] A GARCH variance is composed of squared innovations from previous time periods and conditional variances from previous time periods.

[26] Indirect versus direct DCC:
- Scalar BEKK or indirect DCC: returns viewed as innovations in $\widehat{\mathbf{V}}_{t,GARCH}$ (1.3) with $\widehat{\mathbf{P}}_t$ requiring only one-step MLE (Bauwens et al., 2006; Silvennoinen & Teräsvirta, 2009).
- Engle (2002) or direct DCC: returns transformed into standardized residuals $\boldsymbol{\varepsilon}_t = \left(\mathrm{diag}\left(\widehat{\mathbf{V}}_{t,GARCH}\right)\right)^{-1/2}\mathbf{R}_t$. Then, $\widehat{\mathbf{V}}_t$ becomes $\widehat{\mathbf{V}}_t = (1 - \alpha_1 - \alpha_2)\overline{\mathbf{V}} + \alpha_1(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) + \alpha_2\widehat{\mathbf{V}}_{t-1}$ with the corresponding $\widehat{\mathbf{P}}_t$ requiring two-step MLE.

[27] Explanation of how a financial return (r_t) equates to an innovation (e_t), i.e. $r_t = e_t$:
- Let the natural log of the exchange rate ($\{\ln FX_t\}$) follow a random walk: $\ln FX_t - \ln FX_{t-1} = e_t$, i.e. a change in $\ln FX_t$ or 'e_t' is a residual or innovation. Accordingly, the natural log return (r_t) is equal to the innovation ($r_t = \ln FX_t -$

period conditional variance-covariance matrix with $\widehat{\mathbf{V}}_{0,GARCH}$ equal to $\widehat{\mathbf{V}}_{uncond}$ of returns over a time interval prior to the sample of $[1,\tau]$, and α_1 and α_2 the non-negative scalar persistence parameters.[28] In addition, $\alpha_1 + \alpha_2 < 1$ for stationarity and mean reversibility of the model (Engle, 2002; Jorion, 2011). The higher is α_1, the more persistent are the earlier squared innovations. The higher is α_2, the more persistent are the earlier conditional variances and covariances. $\alpha_1 + \alpha_2$ (< 1) also determines the speed of mean reversion ($\widehat{\mathbf{V}}_{t,GARCH}$ reverting to $\overline{\mathbf{V}}$ with $1 - \alpha_1 - \alpha_2$, indicating how persistent $\overline{\mathbf{V}}$ is over time) (Bollerslev, 1986; Jorion, 2011). Additionally, the characteristic decay time of $\widehat{\mathbf{V}}_{t,GARCH}$ (i.e. time that $\widehat{\mathbf{V}}_{t,GARCH}$ takes to decay) is $-1/\ln\alpha_2$ (Zumbach, 2007).

The optimal values of the parameters α_1 and α_2 can be figured out via the following MLE (1.4) (where $L(\cdot)$ denotes a log likelihood function), given $\mathbf{R}_t \sim N\left(\mathbf{0}, \widehat{\mathbf{V}}_{t,GARCH}\right)$ (i.e. conditional multivariate normality) (Bauwens et al., 2006; Silvennoinen & Teräsvirta, 2009):

$$\begin{aligned}&\underset{\alpha_1,\alpha_2}{\arg\max}\, L(\alpha_1,\alpha_2),\\ &L(\alpha_1,\alpha_2) = -\tfrac{1}{2}\textstyle\sum_{t=1}^{T}\left[N\log(2\pi) + \log\left|\widehat{\mathbf{V}}_{t,GARCH}\right| + \mathbf{R}'_{t-1}\widehat{\mathbf{V}}_{t,GARCH}^{-1}\mathbf{R}_{t-1}\right]\end{aligned} \tag{1.4}$$ [29]

Beyond GARCH in forms of the scalar BEKK, used in this dissertation for simplicity, Engle's (2002) DCC mentioned formerly, and others, the econometrics of dynamic correlation analysis has been continually growing over time for multivariate GARCH improvements, which could be briefly mentioned here. For instance, Gouriéroux et al. (2009) offer the Wishart autoregressive (WAR) process of multivariate stochastic volatility. Developed as an alternative to multivariate GARCH as well as stochastic volatility models, WAR becomes a multivariate dynamic stochastic volatility-covolatility model. Gouriéroux et al.

$\ln FX_{t-1} = e_t$).
- Since $\mathbf{R}_t \sim N\left(\mathbf{0}, \widehat{\mathbf{V}}_{t,GARCH}\right)$, $E[r_t] = 0$ so $r_t - E[r_t] = r_t - 0 = e_t$.

[28] For $\mathbf{V}_0$, in this dissertation on the Brexit case study, a time interval of one calendar year (2011) is applied for both GARCH(1,1) and later EWMA. Then, $\widehat{\mathbf{V}}_{t,GARCH}$ (1.3) and $\widehat{\mathbf{V}}_{t,EWMA}$ (1.6) are dynamically computed up to the sample time span starting in June 2015. This is done to ensure that $\widehat{\mathbf{V}}_{t,GARCH}$ and $\widehat{\mathbf{V}}_{t,EWMA}$ not far prior to the sample period are indeed dynamically conditional on updated information.

[29] The normal-distribution MLE (1.4) on $\mathbf{V}_{t,GARCH}$ is a combination of univariate MLE on the variances and bivariate MLE on the covariances. Given $x, y \in [1, \ldots, N]$, financial returns $r_{x,t}$, $r_{y,t}$, variance σ^2, and correlation ρ,
the univariate log likelihood is $\ln\left[\frac{1}{\sqrt{2\pi}\sigma_{x,t,GARCH}}\exp\left(-\frac{r_{x,t}^2}{2\sigma_{x,t,GARCH}^2}\right)\right]$
and the bivariate log likelihood is $\ln\left[\frac{1}{2\pi\sigma_{x,t,GARCH}\sigma_{y,t,GARCH}\sqrt{1-\rho_{x,y,t,GARCH}}}\exp\left(-\frac{\zeta}{2\sqrt{1-\rho_{x,y,t,GARCH}}}\right)\right]$,
$\zeta := \left(\frac{r_{x,t}^2}{\sigma_{x,t,GARCH}^2} - 2\rho_{x,y,t,GARCH}\frac{r_{x,t}r_{y,t}}{\sigma_{x,t,GARCH}\sigma_{y,t,GARCH}} + \frac{r_{y,t}^2}{\sigma_{y,t,GARCH}^2}\right)$.

(2009) also ensure the positivity and symmetry of WAR volatility matrices. In addition, WAR estimation depends on the method of moments and MLE.

EWMA

Despite the advantages of GARCH(1,1) over the unconditional approach in terms of the second-moment properties of time variability, volatility clustering, and mean reversion, the special case of GARCH(1,1), EWMA, should also be mentioned as it provides more simplicity and convenience in practice than GARCH(1,1) (Zumbach, 2007; Jorion, 2011).

GARCH(1,1) becomes EWMA when we allow $\alpha_1 + \alpha_2 = 1$, implying there is no longer mean reversion (Engle, 2002; Jorion, 2011).[30] Let the so-called persistence parameter α_2 be denoted by φ instead.[31] Then, the matrix of multivariate EWMA correlation coefficients of $\mathbf{R}_t$, denoted by $\widehat{\mathbf{P}}_{t,EWMA}$, can be expressed as

$$\widehat{\mathbf{P}}_{t,EWMA} = \left(\mathrm{diag}\left(\widehat{\mathbf{V}}_{t,EWMA}\right)\right)^{-\frac{1}{2}} \widehat{\mathbf{V}}_{t,EWMA} \left(\mathrm{diag}\left(\widehat{\mathbf{V}}_{t,EWMA}\right)\right)^{-\frac{1}{2}} \tag{1.5}$$

with the following EWMA variance-covariance matrix, denoted by $\widehat{\mathbf{V}}_{t,EWMA}$:

$$\widehat{\mathbf{V}}_{t,EWMA} = (1 - \varphi)(\mathbf{R}_{t-1}\mathbf{R}'_{t-1}) + \varphi \widehat{\mathbf{V}}_{t-1,EWMA} \tag{1.6}$$

where $\mathbf{R}_{t-1}\mathbf{R}'_{t-1}$ is the $N \times N$ matrix of the latest products of innovations, $\widehat{\mathbf{V}}_{t-1,EWMA}$ is the $N \times N$ latest conditional variance-covariance matrix, with $\widehat{\mathbf{V}}_{0,EWMA}$ equal to the $\widehat{\mathbf{V}}_{uncond}$ of returns over a time interval prior to $[1, \tau]$, and $0 < \varphi < 1$ is the decay parameter.[28,32] Similarly to α_2 in GARCH(1,1), φ in EWMA indicates the proportional dependence of previous conditional (co)variances. With a larger φ, older conditional variances and covariances decay more slowly. Moreover, the characteristic decay time of $\widehat{\mathbf{V}}_{t,EWMA}$ is $-1/\ln \varphi$ (Zumbach, 2007).

[30] $\alpha_1 + \alpha_2 = 1$ indicates permanent persistence, i.e. shocks to second moments do not decay. However, the permanent persistence only matters significantly at considerably longer extrapolations of EWMA. In other words, permanent persistence in EWMA only differs from mean reversibility in GARCH over considerably longer extrapolations (Jorion, 2011).

[31] The typical EWMA notation of λ will be used in the later correlation analysis of financial contagion and capital flight; hence, φ is used here instead.

[32] A rule of thumb suggested by Alexander (2008) is that it should range between about 0.75 and 0.98.

For simple and convenient application, RiskMetrics determines the decay parameter φ as 0.94 for daily data and 0.97 for monthly data, which provide relatively good forecasts across all financial assets in practice (Zumbach, 2007; Jorion, 2011). The investigated currency data series are daily, hence, following RiskMetrics, $\varphi = 0.94$, giving $\widehat{\mathrm{V}}_{t,EWMA}$'s characteristic decay time of $-1/\ln 0.94 \cong 16.2$ trading days, will be applied.

Nevertheless, with GARCH(1,1) or EWMA, as older information dies out more quickly with a smaller α_2 or φ, less information is reflected in the forecast estimator. As a result, the variance of the forecast estimator increases (in this dissertation, GARCH(1,1) is applied on a theoretical basis, whereas EWMA on a more simply and conveniently practical basis, as aforementioned). Therefore, advantageously, while the conditional methods convey more up-to-date information, the unconditional approach could take into account more observations (backwards on a timeline) (as better statistics in general require the use of as much information as possible) with lower estimator variance (Zumbach, 2007).

1.3.3 Time-varying currency correlations around the Brexit referendum

Time-varying unconditional, GARCH(1,1), and EWMA correlation coefficients around the Brexit referendum on 23 June 2016 are visually presented in **Figure 1.4**, **Figure 1.5**, and **Figure 1.6**, respectively.[33] As lower GBP values persist for several months (see **Figure 1.2**), the post-breakpoint time span may cover up to several months as well so as to get a rough idea of how contagions and flights might persist through a period of political uncertainty.

Regarding all the examined time-varying unconditional and dynamic conditional correlations over the year before the Brexit referendum, GBP looks to have co-moved with other major currencies quite positively, overall, except in the case of JPY. The East Asian currency in contrast becomes negatively or divergently associated with sterling from time to time, especially according to the dynamic conditional calculations. In addition, there is no clear, instant, massive jump in any of those correlations during the pre-breakpoint year. Thus, the prior year may be considered tranquil or normal, i.e. there was probably no major disruption.

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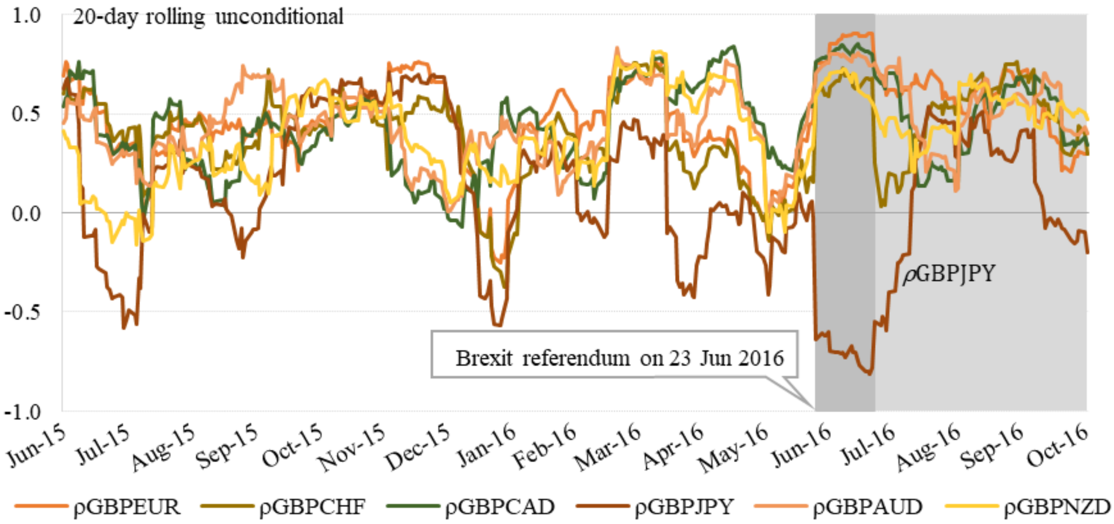
[33] In **Figure 1.5**, the joint MLE (for $\widehat{\mathbf{P}}_{t,GARCH}$ (1.2)) over the tranquility-state time span of one year (excluding the distress-state period so as not to incorporate uncommon behaviors of currency returns and correlations) optimally yields $\alpha_1 \cong 0.054$, and $\alpha_2 \cong 0.854$ (using the generalized reduced gradient (GRG) nonlinear method in Excel Solver). For the optimal values of α_1 and α_2 over the other time spans, see **Appendix 1.E Analytical and hypothesis-testing parameters**.



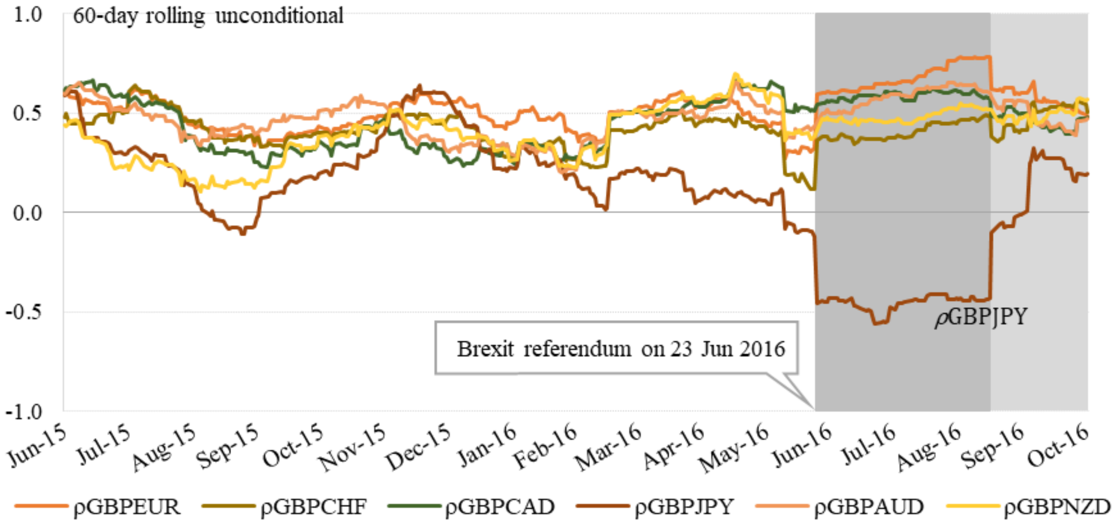


Figure 1.4: time-varying 20-day and 60-day rolling unconditional correlation coefficients from Jun 2015 to Oct 2016, shaded areas: after the Brexit referendum, areas with darker shading (covering about 20 and 60 trading days, in the figures above and below, respectively): possible negative-contagion and flight-to-quality existence, areas with lighter shading: possible contagion and flight disappearance

Given that some financial market participants presumed the Brexit ex ante, despite the fact that the Bremain (i.e. the British remaining in the EU) was rather predicted (according to opinion polls), on account of the expectation of risk intensification, they could gain some advance knowledge of this by observing overall pre-breakpoint currency co-movements. Prior positive correlations (of GBP with the major currencies other than JPY) might have been expected to shoot up abruptly, whereas the occasionally divergent relationship (between GBP and JPY) would perhaps have been expected to drop instantaneously.

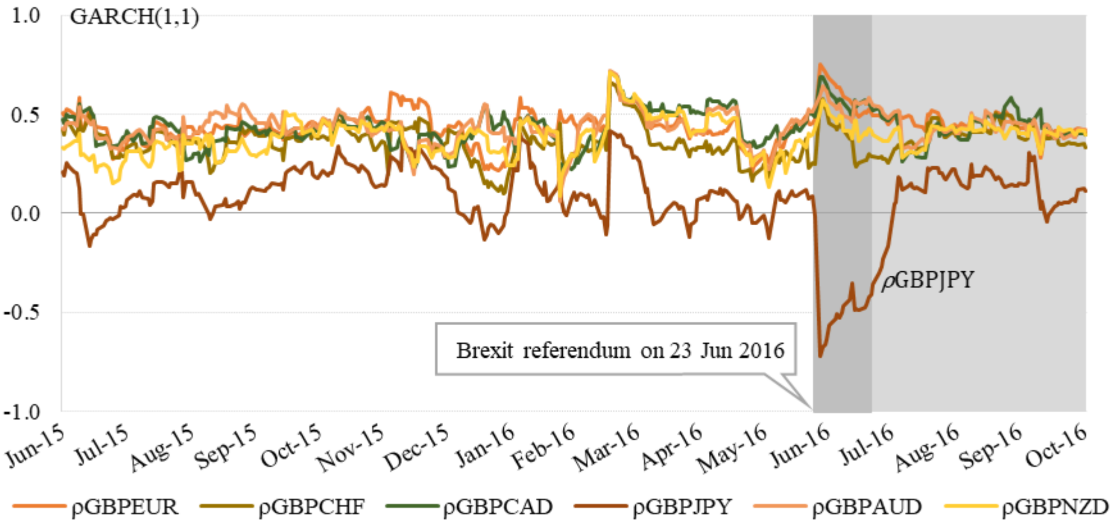


Figure 1.5: time-varying GARCH(1,1) dynamic conditional correlation coefficients[33] from Jul 2015 to Oct 2016, shaded areas: after the Brexit referendum, area with darker shading (covering slightly less than a month): possible negative-contagion and flight-to-quality existence, area with lighter shading: possible contagion and flight disappearance

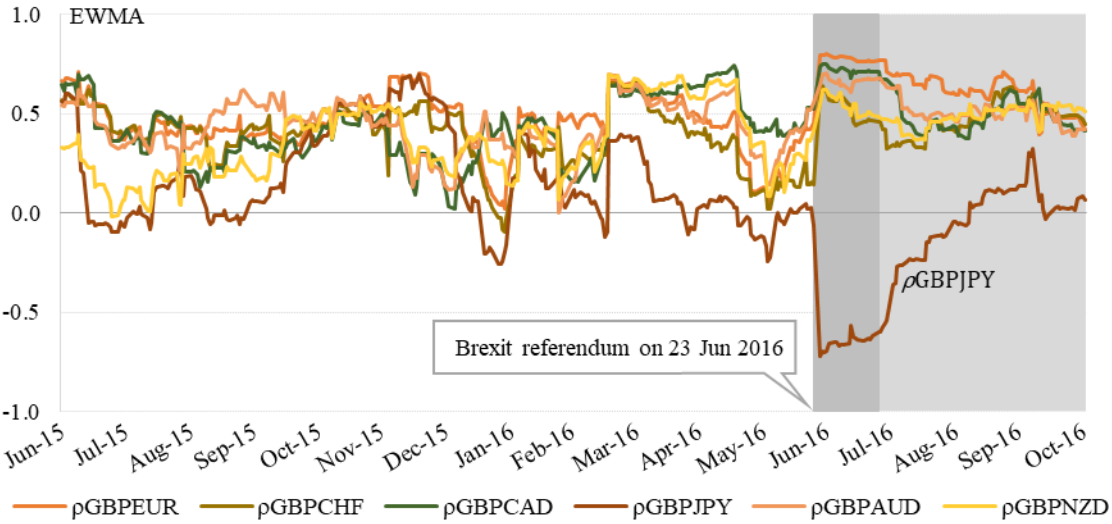


Figure 1.6: time-varying EWMA ($\varphi = 0.94$) dynamic conditional correlation coefficients from Jul 2015 to Oct 2016, shaded areas: after the Brexit referendum, area with darker shading (covering about a month or about 20 trading days): possible negative-contagion and flight-to-quality existence, area with lighter shading: possible contagion and flight disappearance

Accordingly, consider the ex post visualizations of currency co-movements. While the correlations of GBP with the major currencies other than JPY rise instantly, the co-movement with JPY plummets. Moreover, both the up and down shifts in the GARCH(1,1) and EWMA correlations over the period of political uncertainty seem more sizeable than in the unconditional computations. In particular, the GARCH(1,1) and EWMA GBP-JPY correlations fall rapidly from near to zero down to about -0.7 around the Brexit referendum event.

This is because up-to-date information on rising political and related risks immediately after the political event is better reflected in the conditional than the unconditional approaches.

Consider how the correlation shifts persist over the period of political uncertainty but seemingly converge to previous levels thereafter. The unconditional correlation shifts look to be possibly persistent over up to as many as 60 trading days, or approximately three months (by 60-day rolling computations). Such persistence over several months could be due to overweighting of the older information that captures the financial panics that existed following the referendum. With regard to more forward-looking correlation computations, with newer information taken into account, the persistence based on GARCH(1,1) and EWMA seems to last only around a month. More specifically, the GARCH(1,1) correlations appear to converge more quickly than the EWMA correlations due to a lower persistence parameter and shorter theoretical characteristic decay time ($\alpha_2 \cong 0.854 < \varphi = 0.94$ yield decay time of $6.4 < 16.2$ trading days, also see **Appendix 1.E Analytical and hypothesis-testing parameters**). Accordingly, across all three correlation methodologies, the plots (particularly the areas with the darkest shading) may also imply that the relevant time span of the state of distress or political uncertainty, in testing contagions and flights, is approximately one month in duration (the one-month crisis time span is also observed in Forbes and Rigobon (2002) and Corsetti et al. (2005)).

These observations motivate exploration of whether those correlation shifts over the period of political uncertainty are indeed statistically significant or not, under each correlation approach.

1.4 Correlation analysis of financial contagion and capital flight

Analyzing financial contagions and flights in this context entails a single global factor model, bivariate correlation analysis, and heteroskedasticity bias correction (Forbes & Rigobon 2002; Corsetti et al., 2005). In this dissertation, four interdependence measures will be discussed and derived from different models and sets of assumptions for financial returns: the unadjusted measure, the measures used by Corsetti et al. (2005) and Forbes and Rigobon (2002), and a measure derived from the extended framework. They will be introduced and explained one by one interconnectedly.

1.4.1 Interdependence measurement

Start with a standard single-factor model of financial returns at time t in economies x and y, denoted by $r_{x,t}$ and $r_{y,t}$, respectively (Corsetti et al., 2005):

$$\begin{aligned} r_{x,t} &= \alpha_x + \beta_x g_t + \varepsilon_{x,t} \\ r_{y,t} &= \alpha_y + \beta_y g_t + \varepsilon_{y,t} \end{aligned} \qquad (1.7)^{34}$$

where α_x and α_y denote country-specific or idiosyncratic constants, β_x and β_y global-factor loadings, $g_t \sim \left(0, \sigma_g^2\right)$ the global or common factor at time t, and $\varepsilon_{x,t} \sim \left(0, \sigma_{\varepsilon_x}^2\right)$ and $\varepsilon_{y,t} \sim \left(0, \sigma_{\varepsilon_y}^2\right)$ country-specific or idiosyncratic shocks at time t.

Assume that the single-factor model is well specified. Then, there exists non-endogeneity or orthogonality between g_t and the disturbance terms (i.e. $g_t \perp \varepsilon_{x,t}$ and $g_t \perp \varepsilon_{y,t}$) and contemporaneous residual independence or orthogonality between $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ (i.e. $\varepsilon_x \perp \varepsilon_y$). Accordingly, the corresponding covariances are zero, i.e. $\sigma_{g,\varepsilon_x} = \sigma_{g,\varepsilon_y} = \sigma_{\varepsilon_x,\varepsilon_y} = 0$. As a result, the variances, covariance, correlation coefficient, and global-factor loadings, respectively, are

$$\begin{aligned} &\sigma_x^2 = \beta_x^2 \sigma_g^2 + \sigma_{\varepsilon_x}^2, \sigma_y^2 = \beta_y^2 \sigma_g^2 + \sigma_{\varepsilon_y}^2, \\ &\sigma_{x,y} = \beta_x \beta_y \sigma_g^2, \\ &\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}, \text{and} \\ &\beta_x = \frac{\sigma_{x,g}}{\sigma_g^2}, \beta_y = \frac{\sigma_{y,g}}{\sigma_g^2}. \end{aligned} \qquad (1.8)$$

As bivariate co-movements are of interest in analyzing financial contagion, the correlation coefficient between economies x and y, $\rho_{x,y}$ (or ρ for short) $\in [-1,1]$, is considered, especially in a state of distress. Unlike Forbes and Rigobon (2002) and Corsetti et al. (2005), who only focus on the positive sign of ρ, i.e. contagions, this dissertation considers both positive and negative signs, i.e. both contagions and flights, respectively. Hence, the 'magnitude', which is the absolute value and captures either a positive or negative sign of ρ, is used.

[34] Consistent with a classical equilibrium model named the capital asset pricing model (CAPM), which is also a single-factor model incorporating one common factor.

Relevant research works assert that heteroskedasticity is inherent, particularly in the shock-originating economy's financial returns, as their volatility naturally rises over a period of market distress (see **Appendix 1.B Heteroskedasticity in the shock-originating returns**). Thus, the magnitudes of the related correlation coefficients in a state of distress are biased (Boyer et al., 1999; Forbes & Rigobon, 2002). Let economy x be the shock-originating economy and y the shock-receiving economy (in this study, therefore, x's currency is GBP and y's is EUR, CHF, CAD, JPY, AUD, or NZD). Thus, heteroskedasticity exists in $r_{x,t}$ such that the natural increase in σ_x^2 in a state of distress biases the magnitude of ρ.

Next, consider what could cause σ_x^2, which influences the size of ρ, to vary mechanically over time, through the ***decomposition of shock-originating return volatility***. Since the single factor model yields $\sigma_x^2 = \beta_x^2\sigma_g^2 + \sigma_{\varepsilon_x}^2$ (see (1.8)), it follows that σ_x^2 can be decomposed into global and country-specific components: $\beta_x^2\sigma_g^2$ and $\sigma_{\varepsilon_x}^2$, respectively. Hence, a rising σ_x^2 could be owing to an increase in β_x, σ_g^2, or $\sigma_{\varepsilon_x}^2$. Accordingly, ρ is expressed in terms of β_x, σ_g^2, and $\sigma_{\varepsilon_x}^2$ as below (derived from assumptions (1.8)):

$$\rho = \frac{\beta_x\beta_y\sigma_g^2}{\sqrt{\beta_x^2\sigma_g^2+\sigma_{\varepsilon x}^2}\sqrt{\beta_y^2\sigma_g^2+\sigma_{\varepsilon y}^2}} = \left(1+\frac{\sigma_{\varepsilon x}^2}{\beta_x^2\sigma_g^2}\right)^{-\frac{1}{2}}\left(1+\frac{\sigma_{\varepsilon y}^2}{\beta_y^2\sigma_g^2}\right)^{-\frac{1}{2}} \quad (1.9)$$ [35]

Consider the impact of the heteroskedasticity in $r_{x,t}$ (or simply rising $\sigma_x^2 = \beta_x^2\sigma_g^2 + \sigma_{\varepsilon_x}^2$) on ρ in terms of global in relation to country-specific variations, i.e. $\beta_x^2\sigma_g^2$ relative to $\sigma_{\varepsilon_x}^2$ (or simply the reciprocal of $\sigma_{\varepsilon_x}^2/\left(\beta_x^2\sigma_g^2\right)$ inside the first $(\cdot)^{-1/2}$ term in (1.9)). Expression (1.9) shows that either (1) higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$ or (2) higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ could coincide with an increase in the magnitude of ρ, especially during a period of turmoil. This coincidence relates to strong connections through states, i.e. from tranquility to distress, between $r_{x,t}$ and $r_{y,t}$, which is known as ***interdependence***, whose measure is denoted by ϕ.[36,37] To deal with the heteroskedasticity bias, the matter of the coincidence is to be incorporated when measuring the interdependence between $r_{x,t}$ and $r_{y,t}$ in a state of distress.

---

[35] As $\rho_{x,y} \in [-1,1]$ and $\forall\ \beta_x, \beta_y \gtreqless 0$, both the positive and negative roots from the terms $(\cdot)^{-1/2}$ in expression (1.9) are considered. $\beta_x, \beta_y > 0$ implies $\rho > 0$, $\beta_x, \beta_y < 0$ implies $\rho > 0$, while $\beta_x < 0$ or $\beta_y < 0$ implies $\rho < 0$.

[36] Also see 'independence' descriptions in **1.2.1 Background**'s first paragraph and in **Table 1.1**.

[37] In Corsetti et al. (2005), ϕ is an interdependence measure while, in Forbes and Rigobon (2002), it is regarded as a correlation coefficient conditional on the heteroskedasticity, while its inverse transformation is a correlation coefficient unconditional on heteroskedasticity.

Without correcting for the heteroskedasticity at all, ϕ is unadjusted (*1st interdependence measure*). With a heteroskedasticity correction, ϕ is defined in three separate ways in this dissertation. Case (2) above (higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$) is the framework of Corsetti et al. (2005) (*2nd interdependence measure*). Case (2) but implicitly leaving out $\sigma_{\varepsilon_x}^2$ (as calirified by Corsetti et al. (2005)) is from Forbes and Rigobon (2002) (*3rd interdependence measure*)[38] (Explicitly, Forbes and Rigobon (2002) assume that the entire σ_x^2 coincides with the interdependence but in fact they analyze it using a different framework from Corsetti et al. (2005)).[39] Lastly, case (1) above (higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$) represents the extended framework (*4th interdependence measure*).

With the ***first interdependence measure***, when moving from a state of tranquility (T) to a state of distress (D), the unadjusted or non-corrected interdependence measure (denoted by ϕ_{unadj}) is simply equal to the correlation coefficient during the tranquil period (denoted by ρ_T):

$$\phi_{unadj} = \rho_T = \frac{\sigma_{x,y|T}}{\sigma_{x|T}\sigma_{y|T}} \tag{1.10}$$

Expression (1.10) says that the interdependence between $r_{x,t}$ and $r_{y,t}$ remains unchanged over time, when moving from a state of tranquility to a state of distress. If the magnitude of the correlation coefficient in distress, $\rho_D = \sigma_{x,y|D}/(\sigma_{x|D}\sigma_{y|D})$, is statistically larger than that of ϕ_{unadj}, it implies that a contagion or a flight occurs.

The ***second interdependence measure*** applies a way of correcting for the heteroskedasticity, proposed by Corsetti et al. (2005). They assume that a higher magnitude of ρ in a state of distress, i.e. interdependence, naturally coincides with a rising σ_g^2 relative to $\sigma_{\varepsilon_x}^2$. The corresponding set of assumptions is

[38] This implicit consideration is in line with Corsetti et al. (2005).

[39] Forbes and Rigobon (2002) employ univariate linear regression without endogeneity or variable omission: $r_{y,t} = a + br_{x,t} + v_t$ where x explains y unidirectionally.

$$
\begin{aligned}
&\frac{\sigma^2_{g|D}}{\sigma^2_{g|T}} \gtreqless 1,\\
&\frac{\sigma^2_{\varepsilon_x|D}}{\sigma^2_{\varepsilon_x|T}} \gtreqless 1,\\
&\sigma^2_{\varepsilon_y} = \sigma^2_{\varepsilon_y|T} = \sigma^2_{\varepsilon_y|D}, \text{ and}\\
&\sigma_{\varepsilon_x,\varepsilon_y} = \sigma_{\varepsilon_x,\varepsilon_y|T} = \sigma_{\varepsilon_x,\varepsilon_y|D} = 0.
\end{aligned}
\tag{1.11}
$$

Consider (1.9) together with (1.11). If an increase in the magnitude of ρ over the period of market turmoil is in line with the model and set of assumptions (1.7) and (1.11), it implies that $r_{x,t|D}$ and $r_{y,t|D}$ are fundamentally interdependent. On the contrary, if the rise in the magnitude of ρ in a state of distress becomes too strong compared to what is inferred by (1.7) and (1.11), it will imply that the cross-market interconnection between $r_{x,t}$ and $r_{y,t}$ has changed structurally, i.e. a contagion or a flight is occurring. That is to say, it is too strong to be described by the behavior of σ^2_g relative to $\sigma^2_{\varepsilon_x}$. The excessive behavior of σ^2_g in relation to $\sigma^2_{\varepsilon_x}$ is by construction, because of a significant increase in the magnitude of parameter β_x.[40]

On the basis of the model and set of assumptions (1.7) and (1.11), in correcting for the heteroskedasticity bias, the interdependence measure proposed by Corsetti et al. (2005) (denoted by ϕ_{CPS}) can be written as follows (see full derivation in Corsetti et al. (2005)):

$$
\phi_{CPS} = \rho_T \frac{1+\lambda_T}{1+\lambda_D}\sqrt{\frac{1+\delta}{1+\rho_T^2(1+\lambda_T)\left[(1+\delta)\frac{1+\lambda_T}{1+\lambda_D}-1\right]}} \qquad (1.12)[41]
$$

where ρ_T denotes the correlation coefficient in a state of tranquility, λ_T the idiosyncratic-global variance ratio in a state of tranquility, λ_D the idiosyncratic-global variance ratio in a

[40] Although β_y by construction may be a cause of contagion or flight, only the decomposed components of σ^2_x (i.e. $\beta^2_x\sigma^2_g$ and $\sigma^2_{\varepsilon_x}$) are focused upon. This is because correcting for the heteroskedasticity in $r_{x,t}$ is theoretically and primarily of interest in measuring interdependence. Besides, β_y could be viewed as unchanged in a state of distress. Moreover, the empirical tests on β_y show insignificant structural change as expected. In the tests on β_y over the six investigated currencies of shock-receiving economies (y) and over 15-, 12-, and 9-month tranquility-state time spans and a one-month distress-state time span, as many as 12 out of 18 tests show β_y to be unchanged statistically.

[41] The heteroskedasticity correction part of ϕ_{CPS} is $\frac{1+\lambda_T}{1+\lambda_D}\sqrt{\frac{1+\delta}{1+\rho_T^2(1+\lambda_T)\left[(1+\delta)\frac{1+\lambda_T}{1+\lambda_D}-1\right]}}$ (also see **Appendix 1.E Analytical and hypothesis-testing parameters**).

state of distress, and δ the relative increase in the shock-originating economy's financial returns.[42] That is,

$$\lambda_T = \frac{\sigma^2_{\varepsilon_{x|T}}}{\beta_x^2 \sigma^2_{g|T}}, \lambda_D = \frac{\sigma^2_{\varepsilon_{x|D}}}{\beta_x^2 \sigma^2_{g|D}}, 1 + \lambda_T = \frac{\sigma^2_{x|T}}{\beta_x^2 \sigma^2_{g|T}}, 1 + \lambda_D = \frac{\sigma^2_{x|D}}{\beta_x^2 \sigma^2_{g|D}}, \text{ and } \delta = \frac{\sigma^2_{x|D}}{\sigma^2_{x|T}} - 1.$$

Holding the other parameters constant, in the analytical closed-form solution (1.12) it can be observed that the magnitude of ϕ_{CPS} decreases in λ_D thus increasing in $\sigma^2_{g|D}$.[43] If σ_g^2 increases substantially in a state of distress (i.e. $\sigma^2_{g|T} < \sigma^2_{g|D}$), the interdependence measure will coincide with the magnitude of the correlation coefficient in distress (i.e. $\phi_{CPS} \cong \rho_D$).[44] As a result, a contagion or a flight may not exist. In contrast, if σ_g^2 does not intensify enough (i.e. $\sigma^2_{g|T} \cong \sigma^2_{g|D}$), it likely follows that $|\phi_{CPS}| < |\rho_D|$. This specifies the possible presence of a contagion or a flight by means of a structural break in the global-factor loading, that is β_x possibly becomes significantly more sizeable (i.e. $|\beta_{x|T}| < |\beta_{x|D}|$). Thus, ceteris paribus, higher $\sigma^2_{g|D}$ leading to lower λ_D and higher magnitude of ϕ_{CPS} indicates a higher likelihood of interdependence but a lower likelihood of a contagion or a flight. With ϕ_{CPS}, Corsetti et al. (2005) evidently find some contagion and some interdependence in the stock markets in the case of the 1997 Asian financial crisis.

The ***third interdependence measure*** is in line with Forbes and Rigobon (2002). Corsetti et al. (2005) show that, through their single common factor model and correlation analyses, Forbes and Rigobon (2002) implicitly neglect the shock-originating economy's idiosyncratic term, i.e. $\varepsilon_{x,t}$ in (1.7) is absent ($\varepsilon_{x,t} = 0$). It follows that $r_{x,t} = \alpha_x + \beta_x g_t$ leading to $\sigma_x^2 = \beta_x^2 \sigma_g^2$. The omission of the shock-originating economy's idiosyncratic noise implies shock-originating residual variability of zero ($\sigma^2_{\varepsilon_x} = \sigma^2_{\varepsilon_{x|T}} = \sigma^2_{\varepsilon_{x|D}} = 0$) and a residual-global variance ratio of zero ($\lambda_T = \lambda_D = 0$) (i.e. $\varepsilon_{x,t} = 0 \therefore \sigma^2_{\varepsilon_x} = \sigma^2_{\varepsilon_{x|T}} = \sigma^2_{\varepsilon_{x|D}} = 0 \therefore \lambda_T = \lambda_D =$

[42] $1 + \lambda = 1 + \frac{\sigma^2_{\varepsilon_x}}{\beta_x^2 \sigma_g^2} = \frac{\beta_x^2 \sigma_g^2 + \sigma^2_{\varepsilon_x}}{\beta_x^2 \sigma_g^2} = \frac{\sigma_x^2}{\beta_x^2 \sigma_g^2}$, i.e. return variance relative to global-component variability, may be called a return-global variance ratio. Then, $1 + \lambda_T$ and $1 + \lambda_D$ denote return-global variance ratios in a state of tranquility and a state of distress, respectively.

[43] Writing ϕ_{CPS} as $\rho_T \sqrt{\left(\frac{1+\lambda_T}{1+\lambda_D}\right)^2 \frac{1+\delta}{1+\rho_T^2(1+\lambda_T)\left[(1+\delta)\frac{1+\lambda_T}{1+\lambda_D} - 1\right]}}$, it can be observed that $\frac{\partial}{\partial \lambda_C} |\phi_{CPS}| < 0$.

[44] $\cong$ implies coincidence or no difference in statistical terms.

0). As a consequence, this framework could bring about an overcorrection in the heteroskedasticity, claim Corsetti et al. (2005).

Based on the model and set of assumptions (1.7) and (1.11), together with the absence of the shock-originating disturbance, the interdependence measure proposed by Forbes and Rigobon (2002) (denoted by ϕ_{FR}) becomes $\phi_{FR} = \phi_{CPS} | \lambda_T = \lambda_D = 0$, drawn from (1.12)).[45]

$$\phi_{FR} = \rho_T \sqrt{\frac{1+\delta}{1+\rho_T^2 \delta}}. \quad (1.13)^{46}$$

ϕ_{FR} is also viewed as the correlation coefficient conditional on the heteroskedasticity, differently derived by Forbes and Rigobon (2002).[47] In the analytical solution (1.13) it can be noticed that the magnitude of ϕ_{FR} increases in δ.[48] If δ rises (i.e. $\sigma_{x|D}^2 > \sigma_{x|T}^2$) significantly, ϕ_{FR} will converge ρ_D possibly indicating neither contagion nor flight, and vice versa. This implies that the interdependence coincides with a rising σ_x^2 while a contagion or a flight exists when ρ increases in excess of σ_x^2 during the market distress. Hence, the higher is δ, the higher will be the magnitude of ϕ_{FR}, and the higher the probability of interdependence or the lower the probability of a contagion or a flight. With ϕ_{FR}, Forbes and Rigobon (2002) empirically discover no contagion and only interdependence in the equity markets in the cases of the 1997 Asian financial crisis, the 1994 Mexican peso crisis, and the 1987 US stock market crash. Such findings of no contagion and only interdependence can be explained by

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[45] This set-up, i.e. (1.7) and (1.11) with the omission of the shock-originating residual, is considered implicitly equivalent to the framework of Forbes and Rigobon (2002).

[46] $\phi_{FR} \in [-1,1]$. The heteroskedasticity correction part of ϕ_{FR} is $\sqrt{\frac{1+\delta}{1+\rho_T^2 \delta}}$ (also see **Appendix 1.E Analytical and hypothesis-testing parameters**).

[47] The correlation coefficient conditional on the heteroskedasticity, ϕ_{FR}, is originally and explicitly derived from $r_{y,t} = a + br_{x,t} + v_t$ with no endogeneity or variable omission (this framework can be rewritten as (1.7) with $\varepsilon_{x,t}$ omitted (Corsetti et al., 2005) (also see Footnote 39). Accordingly, $\sigma_y^2 = b^2\sigma_x^2 + \sigma_v^2$ and, when moving from a state of tranquility to a state of distress, $\sigma_{x|D}^2 / \sigma_{x|T}^2 = \sigma_{x,y|D} / \sigma_{x,y|T} = 1 + \delta$. Then, the heteroskedasticity in $r_{x,t}$ is taken into account, where a jump in the entire σ_x^2 is assumed to coincide with amplifying correlations in a distress state. Finally, ϕ_{FR} is as in (1.13) (see full derivation in Forbes and Rigobon (2002)).

[48] $\frac{\partial}{\partial \delta} |\phi_{FR}| \begin{cases} > 0, |\rho_T| < 1 \\ = 0, |\rho_T| = 1 \end{cases}.$

the heteroskedasticity being overcorrected due to the implicit restriction imposed on the shock-originating idiosyncratic noise (Corsetti et al., 2005).[49]

1.4.2 Extended framework

The last interdependence measure is derived from the framework that builds on Forbes and Rigobon (2002) and Corsetti et al. (2005). A single global factor model, bivariate correlation analysis, and heteroskedasticity bias correction are taken into account. The main assumption under the extended framework was also mentioned by Corsetti et al. (2005, 2011) as a further research contribution.

The ***fourth interdependence measure*** stems from the presumption that interdependence in a state of distress corresponds to a rise in the shock-originating economy's global-factor-loading variability relative to idiosyncratic noise variance, i.e. β_x^2 relative to $\sigma_{\varepsilon_x}^2$ (also see (1.9) and its subsequent explanations). Correspondingly, the set of assumptions is

$$\begin{aligned}
&\frac{\beta_{x|D}^2}{\beta_{x|T}^2} \gtreqless 1,\\
&\frac{\sigma_{\varepsilon_{x|D}}^2}{\sigma_{\varepsilon_{x|T}}^2} \gtreqless 1,\\
&\beta_y^2 = \beta_{y|T}^2 = \beta_{y|D}^2,\\
&\sigma_{\varepsilon_y}^2 = \sigma_{\varepsilon_{y|T}}^2 = \sigma_{\varepsilon_{y|D}}^2, \text{and}\\
&\sigma_{\varepsilon_x,\varepsilon_y} = \sigma_{\varepsilon_x,\varepsilon_{y|T}} = \sigma_{\varepsilon_x,\varepsilon_{y|D}} = 0.
\end{aligned} \qquad (1.14)^{40}$$

Consider (1.9) alongside (1.14) by focusing on σ_x^2 and its decomposed global and country-specific components.[40] Then, the interdependence between $r_{x,t}$ and $r_{y,t}$ is defined as the intensification of ρ in a state of distress being consistent with the model and set of assumptions (1.7) and (1.14). In contrast, if the increase in the magnitude of ρ over the period of market turmoil is too strong in relation to what is implied by (1.7) and (1.14), this will imply a regime shift in the transmission mechanism between $r_{x,t}$ and $r_{y,t}$, i.e. that a contagion or a flight is happening. In other words, the amplification of ρ is too strong to be

[49] Such overcorrection may be explained as the entire σ_x^2 (but with a different framework in Forbes and Rigobon (2002)) rather than the global-factor relative to country-specific variations (Corsetti et al. (2005)) being taken into account in the correction of heteroskedasticity.

explained by the behavior of β_x^2 relative to $\sigma_{\varepsilon_x}^2$. By construction, such excessive behavior by β_x^2 relative to $\sigma_{\varepsilon_x}^2$ is driven by a significant shift in σ_g^2.[40]

Subject to the model and set of assumptions (1.7) and (1.14) for coping with the heteroskedasticity bias, the interdependence measure (denoted by $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$) can be written as follows (see full algebraic derivation in **Appendix 1.C Derivations of the extended interdependence measure**):

$$\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)} = \rho_T \sqrt{\frac{\beta_{x|D}^2}{\beta_{x|T}^2}\frac{1}{1+\delta}}. \tag{1.15}$$ [50]

Holding the other parameters constant, from the analytical closed-form solution (1.15) it can be seen that the magnitude of $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$ increases in $\beta_{x|D}^2$.[51] When moving from a state of tranquility to a state of distress, if β_x intensifies markedly (i.e. $\beta_{x|T}^2 < \beta_{x|D}^2$), the interdependence measure will likely coincide with the magnitude of the correlation coefficient in distress (i.e. $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)} \cong \rho_D$). Consequently, a contagion or a flight might not exist. On the contrary, if the magnitude of β_x is unchanged structurally (i.e. $\beta_{x|T}^2 \cong \beta_{x|D}^2$), then $\left|\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}\right| < |\rho_D|$ is obtained. This implies that a contagion or a flight may well exist in the form of a regime shift in σ_g^2, that is σ_g^2 shifting up significantly (i.e. $\sigma_{g|T}^2 < \sigma_{g|D}^2$) probably causes a contagion or flight. Therefore, ceteris paribus, larger $\beta_{x|D}^2$ relative to $\beta_{x|T}^2$, amplifying the size of ϕ_{CPS}, indicates a higher probability of interdependence but a lower probability of contagion or flight. The contagion and flight test results thereby primarily rely on $\beta_{x|D}^2$ relative to $\beta_{x|T}^2$.

Furthermore, the dependence of the test outcomes on the $\beta_{x|D}^2$-to-$\beta_{x|T}^2$ ratio can be intuitively and analytically illustrated by the plotting of the inverse transformation of $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$ (denoted by $\phi'_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$) over various values of the $\beta_{x|D}^2$-to-$\beta_{x|T}^2$ ratio (see **Figure 1.7**

---

[50] The heteroskedasticity correction part of $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$ is $\sqrt{\frac{\beta_{x|D}^2}{\beta_{x|T}^2}\frac{1}{1+\delta}}$ (also see **Appendix 1.E Analytical and hypothesis-testing parameters**). $\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}$ is written in terms of β_x^2 rather than β_x so as to be aligned with the extended framework's assumption of interdependence due to higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$.

[51] $\frac{\partial}{\partial \beta_{x|D}^2}\left|\phi_{(\widetilde{\beta}_x,\widetilde{\sigma}_{\varepsilon_x}^2)}\right| > 0$.

and **Figure 1.8** as examples of positive and negative co-movements in distress, respectively). Such graphs of $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})}$ are also directly comparable with similar graphs found in Forbes and Rigobon (2002) and Corsetti et al. (2005). The inverse transformation of $\phi_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})}$ (1.15) can be written as

$$\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})} = \rho_D\sqrt{\frac{1+\delta}{\theta_\beta}} \tag{1.16}$$

where θ_β denotes the squared beta ratio, $\beta^2_{x|D}/\beta^2_{x|T}$. Thence, $\sqrt{\theta_\beta}$, which can be termed the beta-magnitude ratio, captures the relative change in the magnitude of the shock-originating-economy global-factor loading in a state of distress, or the period of political uncertainty in this dissertation.

In general, the inverse interdependence measure, ϕ', is known as the adjusted or unconditional correlation coefficient in Forbes and Rigobon (2002) (i.e. the correlation coefficient that is adjusted for or unconditional on the heteroskedasticity in $r_{x,t}$ in a state of distress). By construction, $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})}$ is basically equal to ρ_D plus an inverse adjustment for the heteroskedasticity bias. Thus, it follows that $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})} \cong \rho_T$ implies interdependence, while $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})} > \rho_T > 0$ and $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})} < \rho_T < 0$ could indicate a contagion and a flight, respectively.

With higher $\sqrt{\theta_\beta}$, i.e. a larger magnitude of $\beta_{x|D}$ relative to $\beta_{x|T}$, the adjustment for the heteroskedasticity in $r_{x,t}$ is more sizeable. **Figure 1.7** and **Figure 1.8** demonstrating examples of positive and negative shifts in correlations, respectively, present the time-varying GBP-EUR and GBP-JPY correlations, respectively.

With considerably high adjustments (as high as $\sqrt{\theta_\beta} = 6$ for the positive shift example of GBP-EUR and $\sqrt{\theta_\beta} = 20$ for the negative shift example of GBP-JPY), the adjusted correlation coefficients, $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\varepsilon_x})}$, appear to coincide more with the tranquility-state correlation coefficients, ρ_T. With sufficiently low adjustments (as low as $\sqrt{\theta_\beta} = 4$ for both GBP-EUR and GBP-JPY examples), on the other hand, it generally coincides with ρ_D, possibly signifying a contagion for positive shift cases and a flight for negative shift cases.

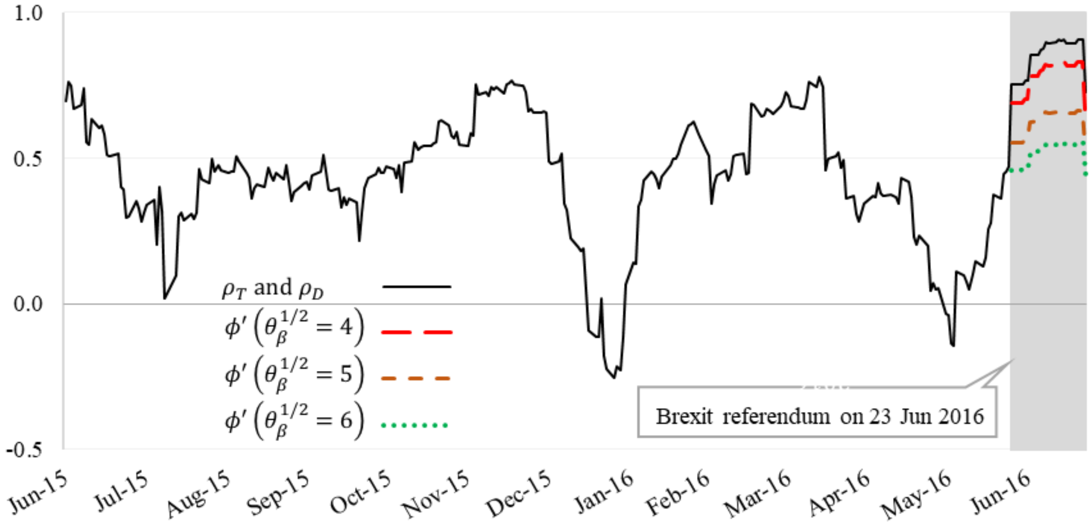


Figure 1.7: an example of a ***positive shift*** in correlation correlations in a state of distress: GBP-EUR's time-varying tranquility-state and distress-state 20-day rolling unconditional (on time) correlation coefficients (i.e. time-varying ρ_T and ρ_D) and time-varying inverse interdependence (i.e. time-varying ϕ') with different beta-magnitude ratios ($\sqrt{\theta_\beta}$) over the UK political uncertainty

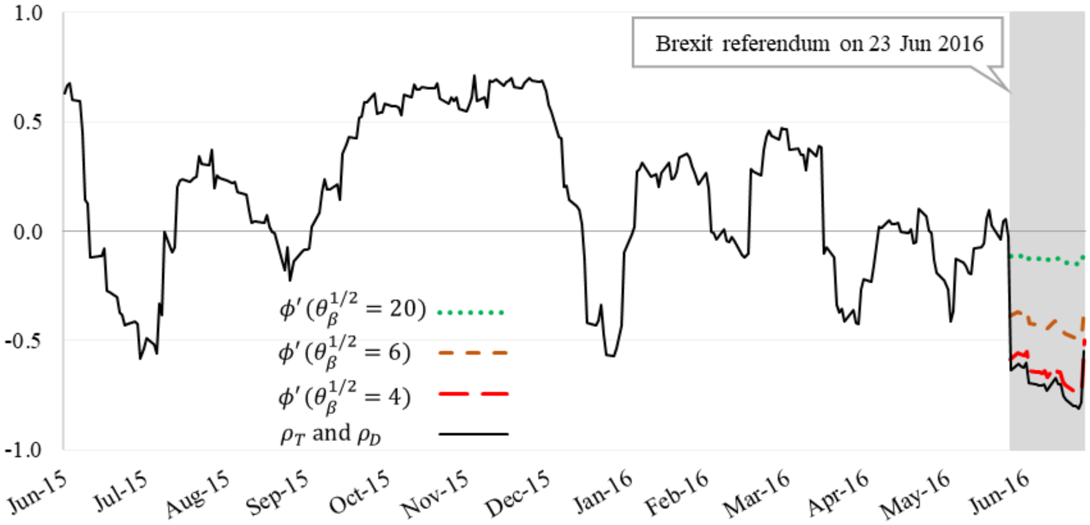


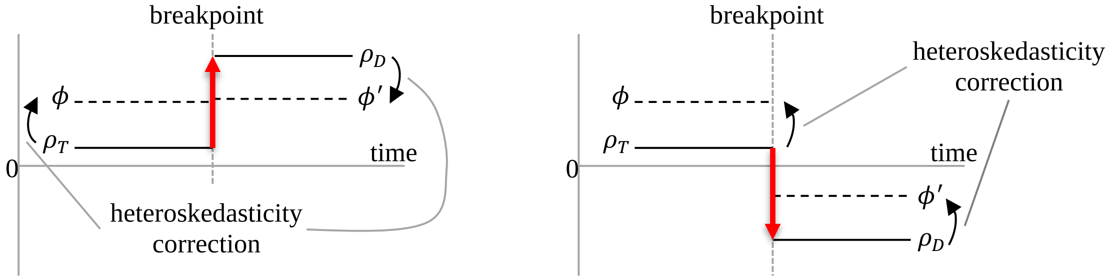
Figure 1.8: an example of a ***negative shift*** in correlation correlations in a state of distress: GBP-JPY's time-varying tranquility-state and distress-state 20-day rolling unconditional (on time) correlation coefficients (i.e. time-varying ρ_T and ρ_D) and time-varying interdependence (i.e. time-varying ϕ') with different beta-magnitude ratios ($\sqrt{\theta_\beta}$) over the UK political uncertainty

For the Brexit case, the value of $\sqrt{\theta_\beta} = 4.511$ is obtained, implying that $\phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ may be closer to ρ_D than to ρ_T. Hence, a contagion and a flight might exist in respect to the EUR and JPY, respectively.

1.4.3 Inverse interdependence measures as the problem solver

Now, the interdependence measure, ϕ, will be reconsidered. Although they beneficially convey information on the extent to which two variables are theoretically interdependent, especially in a state of distress, they could potentially become problematic when ρ_T, ρ_D, and ϕ are further analyzed by means of signs, particularly when ρ_T and ρ_D have opposite signs. ϕ evaluates a degree of interdependence by adjusting a correlation coefficient in tranquility, ρ_T, with a heteroskedasticity correction term. ϕ will then have the same sign as ρ_T. However, intuitively, ϕ should also have the same sign as the correlation coefficient in distress, ρ_D, which sometimes has the opposite sign to ρ_T, especially when ρ_T is approaching zero. This issue obviously arises when ϕ is used in testing for the presence of contagion or flight in financial correlations, with the possibility of both positive and negative signs, not only either sign (in Corsetti et al. (2005), the problem of using ϕ does not emerge because stock indices are in general only positively correlated, particularly in a state of distress or during a crisis, so both ρ_T and ρ_D have the same positive sign).

Supposedly and theoretically, a greater degree of heteroskedasticity correction increases the interdependence likelihood but decreases the contagion or flight likelihood (i.e. ϕ closer to ρ_D). However, it instead mistakenly decreases the interdependence likelihood but increases the contagion or flight likelihood when ρ_T has the opposite sign to ρ_D. For instance, when $\rho_T > 0$ but $\rho_D < 0$, an oversized heteroskedasticity correction, amplifying ϕ (where $\phi > 0$ since $\rho_T > 0$), conversely decreases the probability of interdependence since a higher ϕ (> 0) is even further away from ρ_D (< 0) (i.e. ρ_T-v.-ρ_D differential is wrongly smaller than ϕ-v.-ρ_D differential since ρ_T-v.-ρ_D differential supposedly outweighs ϕ-v.-ρ_D differential) (see **Figure 1.9**). Thus, consequential outcomes can be misleading. For example, flight to quality could be too significantly or too often detected for the GBP-JPY case (see **Figure 1.9**).



ρ_T and ρ_D with the same sign ($\rho_T > 0$, $\rho_D > 0$, alike the case of GBP-EUR correlation around the Brexit referendum): not an issue in tests using ϕ

ρ_T and ρ_D with opposite signs ($\rho_T > 0$, $\rho_D < 0$, alike the case of GBP-JPY correlation around the Brexit referendum): problematic in tests using ϕ

Figure 1.9: visualization showing that use of ϕ brings about misleading statistics when ρ_T and ρ_D have opposite signs, as wrongly making $|\rho_D - \phi|$ even larger than $|\rho_D - \rho_T|$ ($|\rho_D - \phi| < |\rho_D - \rho_T|$, supposedly), while there will be no issue for any case in tests using ϕ' (interdependence null hypothesis: $\phi \cong \rho_D$, or $\phi' \cong \rho_T$)

To solve this issue, that is to make the impact of a heteroskedasticity correction on the probability of interdependence more sensible in all cases (i.e. signs) of ρ_T and ρ_D, an inverse rather than direct interdependence measure is used (thus in line with the tests in Forbes and Rigobon (2002)). With an inverse interdependence measure, ϕ', which was mentioned earlier when the extended framework and its crucial parameter were analytically visualized (see **Figure 1.7**, **Figure 1.8**, and **Figure 1.9**), interdependence satisfies $\phi' \cong \rho_T$ instead of $\phi \cong \rho_D$. Computationally, ϕ' is basically ρ_D plus an inverse heteroskedasticity adjustment. In other words, it is ρ_D with the shock-originating-return heteroskedasticity removed. Accordingly, the four interdependence measures, ϕ_{unadj} (1.10), ϕ_{CPS} (1.12), ϕ_{FR} (1.13), and $\phi_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}x})}$ (1.15) can be inversely transformed into the following expressions, respectively:[52]

$$\phi'_{unadj} = \rho_D = \frac{\sigma_{x,y|D}}{\sigma_{x|D}\sigma_{y|D}} \qquad (1.17)$$

$$\phi'_{CPS} = \rho_D \sqrt{\left(\frac{1+\lambda_D}{1+\lambda_T}\right)\frac{1}{1+\delta-\rho_D^2[(1+\delta)(1+\lambda_T)+1]}} \qquad (1.18)$$

$$\phi'_{FR} = \rho_D \sqrt{\frac{1}{1+\delta\left(1-\rho_D^2\right)}} \qquad (1.19)$$

$$\phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}x})} = \rho_D \sqrt{\frac{\beta^2_{x|T}}{\beta^2_{x|D}}(1+\delta)} \qquad \text{(recall (1.16))}$$

From these inverse analytical solutions it can be observed that a smaller inverse heteroskedasticity adjustment (i.e. a larger direct heteroskedasticity correction) attached to ρ_D lowers the magnitude of ϕ' (which makes interdependence more likely) even when ρ_T and ρ_D have opposite signs.

Suppose an inverse heteroskedasticity adjustment is less sizeable. If $\rho_T > 0$ while $\rho_D < 0$, then the value of ϕ' (where $\phi' < 0$ since $\rho_D < 0$) is, although higher in value,

[52] The specific inverse heteroskedasticity correction part for the latter three ϕ''s are $\sqrt{\left(\frac{1+\lambda_D}{1+\lambda_T}\right)\frac{1}{1+\delta-\rho_D^2[(1+\delta)(1+\lambda_T)+1]}}$, $\sqrt{\frac{1}{1+\delta\left(1-\rho_D^2\right)}}$, and $\sqrt{\frac{\beta^2_{x|T}}{\beta^2_{x|D}}(1+\delta)}$, respectively.

smaller in absolute terms. Hence, ϕ' coincides with ρ_T, indicating that interdependence is more probable. On the other hand, if $\rho_T < 0$ while $\rho_D > 0$, then ϕ' (where $\phi' > 0$ since $\rho_D > 0$) falls in both value and absolute terms. Thus, ϕ' coincides with ρ_T, leading to a greater possibility of interdependence.

1.4.4 Hypothesis tests

The hypothesis tests on the assumptions and on financial contagion and capital flight and controlling for the effects of interest rate are as follows.

Assumption hypothesis testing

Before discussing the hypothesis testing on financial contagions and flights, simple statistical testing on the suitability of a specific interdependence presumption is proposed. First, restating that the heteroskedasticity is inherent in $r_{x,t}$ being in distress, with the composition $\sigma_x^2 = \beta_x^2\sigma_g^2 + \sigma_{\varepsilon_x}^2$ (as in (1.8)), recall the term 'interdependence' or that either higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$ or higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ could naturally coincide with an intensification of a financial correlation, particularly in a state of distress (also see (1.9) and the explanations around it, especially for the decomposition of shock-originating return volatility, and for interdependence). The case of higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$ is the interdependence assumption of the extended framework yielding $\phi_{(\tilde{\beta}_x,\tilde{\sigma}_{\varepsilon_x}^2)}$ (1.15), whereas the case of higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ is that of Corsetti et al. (2005) yielding ϕ_{CPS} (1.12) (and implicitly that of Forbes and Rigobon (2002)). For preliminary and simple consideration of the appropriateness of these assumptions, the Chow test and the F-test could be conducted to examine a significant change in the magnitude of β_x and that of σ_g^2 relative to $\sigma_{\varepsilon_x}^2$, respectively, over a distress-state period of interest. The former test does not need to be scaled by $\sigma_{\varepsilon_x}^2$ since terms associated with $\sigma_{\varepsilon_x|T}^2$ and $\sigma_{\varepsilon_x|D}^2$ are all implicitly cancelled out in $\phi_{(\tilde{\beta}_x,\tilde{\sigma}_{\varepsilon_x}^2)}$ (1.15). However, σ_g^2 in the latter test does need to be scaled by $\sigma_{\varepsilon_x}^2$ since $\sigma_{\varepsilon_x|T}^2$ and $\sigma_{\varepsilon_x|D}^2$ implicitly exist in λ_T and λ_D, respectively, in ϕ_{CPS} (1.12).

Structural-break Chow test:[53] $H_0\colon |\beta_{x|T}| \geq |\beta_{x|D}|$

$H_1\colon |\beta_{x|T}| < |\beta_{x|D}|$

Variance-ratio-inequality F-test:[54] $H_0\colon \sigma^2_{g|T}/\sigma^2_{\varepsilon_x|T} \geq \sigma^2_{g|D}/\sigma^2_{\varepsilon_x|D}$

$H_1\colon \sigma^2_{g|T}/\sigma^2_{\varepsilon_x|T} < \sigma^2_{g|D}/\sigma^2_{\varepsilon_x|D}$

If the structural-break Chow test's null hypothesis is rejected, such that the magnitude of β_x is significantly larger over time (from a state of tranquility to a state of distress), it will imply that the extended framework and its consequent explanations seem practical. If the variance-ratio-inequality F-test's null hypothesis is rejected, such that σ^2_g relative to $\sigma^2_{\varepsilon_x}$ is significantly higher over time (from a state of tranquility to a state of distress), it will imply that Corsetti et al.'s (2005) framework and its implications should be valid. If both the Chow and the F-test's null hypotheses are concurrently rejected, it will imply that the test results for interdependences, contagions, or flights should be explainable in the different forms of the models and assumptions: the extended framework and Corsetti et al. (2005).

Contagion and flight hypothesis testing

Now the hypothesis testing for financial contagions and flights will be presented. As discussed in earlier sections, especially **1.2.2**, **1.4.1**, **1.4.2**, and **1.4.3** , the null and alternative hypotheses for detecting financial contagions and flights can be written as follows (also see interdependence, contagion, and flight definitions in **Table 1.1**, in which the Brexit case, in particular, results in the cases of negative contagion and flight to quality):

[53] Test $H_0\colon |\beta_{x|T}| \geq |\beta_{x|D}|$ and $H_1\colon |\beta_{x|T}| < |\beta_{x|D}|$ using the Chow test with F-distribution (Gujarati & Porter, 2009).
$F_{Chow} = \frac{(RSS_R - RSS_{UR})/k}{RSS_{UR}/(\tau_T+\tau_D-2k)} \sim F_{[k,\tau_T+\tau_D-2k]}$,
τ_T: tranquility-state time span, τ_D: distress-state time span, $k = 2$ for single-factor regressions, RSS: residual sum of squares, RSS_R: restricted or pooled RSS, RSS_{UR} (unrestricted RSS) $= RSS_T + RSS_D$
($\beta_{x|T}$ and $\beta_{x|D}$ having the same sign, i.e. $\forall \beta_{x|T}, \beta_{x|D} > 0$ or $\forall \beta_{x|T}, \beta_{x|D} < 0$, supposedly occurs over samples as β_x shows how a shock-originating economy x is naturally linked (positively or negatively) to the global factor over a certain time period, also see Footnote 57).

[54] Test $H_0\colon \sigma^2_{g|T}/\sigma^2_{\varepsilon_x|T} \geq \sigma^2_{g|D}/\sigma^2_{\varepsilon_x|D}$ and $H_1\colon \sigma^2_{g|T}/\sigma^2_{\varepsilon_x|T} < \sigma^2_{g|D}/\sigma^2_{\varepsilon_x|D}$ applying the F-test with F-distribution (Gujarati & Porter, 2009). $F_{variance\ equality} = \frac{\sigma^2_{g|D}/\sigma^2_{\varepsilon_x|D}}{\sigma^2_{g|T}/\sigma^2_{\varepsilon_x|T}} \sim F_{[\tau_D-1,\tau_T-1]}$, τ_T: tranquility-state time span, τ_D: distress-state time span.

$$
\begin{aligned}
&H_0\colon \rho_D = \phi, \text{i.e. interdependence} \\
&H_1\colon \rho_D \neq \phi\,, \begin{cases} \rho_D > \phi | \rho_D > 0, \text{i.e. contagion,} \\ \rho_D < \phi | \rho_D < 0, \text{i.e. flight} \end{cases}
\end{aligned}
$$

which can be transformed into (1.20) below in order to use ϕ' in place of ϕ (see **1.4.3 Inverse interdependence measures as the problem solver**):

$$
\begin{aligned}
&H_0\colon \phi' = \rho_T, \text{i.e. interdependence} \\
&H_1\colon \phi' \neq \rho_T\,, \begin{cases} \phi' > \rho_T | \rho_D > 0, \text{i.e. contagion} \\ \phi' < \rho_T | \rho_D < 0, \text{i.e. flight} \end{cases}
\end{aligned}
\tag{1.20}
$$

ϕ and ϕ' under the different assumptions are summarized in **Table 1.3**.

frame-work	**interdependence key assumption**	**interdependence measure** (ϕ)	**inverse interdependence measure** (ϕ')
unadjusted	no heteroskedasticity correction	$\rho_T = \frac{\sigma_{x,y\|T}}{\sigma_{x\|T}\sigma_{y\|T}}$	$\rho_D = \frac{\sigma_{x,y\|D}}{\sigma_{x\|D}\sigma_{y\|D}}$
CPS (2005)	σ_g^2 relative to $\sigma_{\varepsilon_x}^2$	$\rho_T \frac{1+\lambda_T}{1+\lambda_D} \sqrt{\frac{1+\delta}{1+\rho_T^2(1+\lambda_T)\left[(1+\delta)\frac{1+\lambda_T}{1+\lambda_D}-1\right]}}$	$\rho_D \sqrt{\frac{1}{1+\delta(1-\rho_D^2)}}$
FR (2002)	σ_g^2 with implicit $\sigma_{\varepsilon_x}^2 = 0$ (via CPS (2005)); or, entire σ_x^2 (via FR (2002))	$\rho_T \sqrt{\frac{1+\delta}{1+\rho_T^2\delta}}$	$\rho_D \sqrt{\left(\frac{1+\lambda_D}{1+\lambda_T}\right)\frac{1}{1+\delta-\rho_D^2[(1+\delta)(1+\lambda_T)+1]}}$
extended $(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)$	β_x^2 relative to $\sigma_{\varepsilon_x}^2$	$\rho_T \sqrt{\frac{\beta_{x\|D}^2}{\beta_{x\|T}^2}\frac{1}{1+\delta}}$	$\rho_D \sqrt{\frac{\beta_{x\|T}^2}{\beta_{x\|D}^2}(1+\delta)}$

Table 1.3: direct and inverse interdependence measures under the different assumptions

Testing (1.20) relies not only on certain crucial parameters for each ϕ' type but also considerably on the sizes of ρ_T and ρ_D (ρ_D with heteroskedasticity adjustment becomes ϕ'). To statistically test (1.20), the Fisher z-transformation, which is considered quite robust to non-normality, is applied (Kocherlakota & Singh, 1982). Fisher developed this methodology to cope with asymmetry in the confidence interval's lower and upper limits for the correlation coefficient sampling distribution (Cohen et al., 2003). Nonetheless, tests using different inverse interdependence measures differ slightly, as follows.

To test (1.20) with ϕ'_{unadj} (1.17) and ϕ'_{FR} (1.19) , which are in $[-1,1]$, the Fisher z-transformation is performed directly (see **Appendix 1.D Applications of the Fisher z-transformation**). However, for the other two interdependence measures, tests are implemented similarly to in Corsetti et al. (2005). Testing (1.20) with ϕ'_{CPS}(1.18) and $\phi'_{(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)}$ (1.16), the Fisher z-transformation is applied indirectly. Based on each of the two inverse interdependence measures, a certain crucial parameter, estimated from the sample, is compared to the statistical thresholds (either floors or ceilings) at standard significance levels. The procedures for each are set out below.

For ϕ'_{CPS} (1.18) (inverse of ϕ_{CPS} (1.12)), the crucial parameter is λ_D (see (1.12) and its explanations), a higher value of which will lower the size of the heteroskedasticity correction (i.e. increasing the inverse heteroskedasticity adjustment) and increase the probability of detecting a contagion or a flight. The floor values of λ_D that reject the null hypothesis (H_0) in (1.20) at different standard significance levels (α), denoted by $\underline{\lambda_D}_{\alpha}$, can then be discovered. $\underline{\lambda_D}_{\alpha}$ can be backed out from equation (1.18) implicitly via the Fisher z-transformation (see **Appendix 1.D Applications of the Fisher z-transformation**). Thus, the hypotheses for ϕ'_{CPS}, which are identical to (1.20), become the following (1.21):

$$
\begin{aligned}
&H_0\colon \lambda_D \le \underline{\lambda_D}_{\alpha}, \text{i.e. interdependence} \\
&H_1\colon \lambda_D > \underline{\lambda_D}_{\alpha} \,|\, \begin{cases} \rho_D > 0, \text{i.e. contagion} \\ \rho_D < 0, \text{i.e. flight} \end{cases}
\end{aligned}
\tag{1.21}
$$

For $\phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ (1.16) (the inverse of $\phi_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ (1.15)), the crucial parameter is $\beta^2_{x|D}$ (see (1.15) and its explanations), a lower value of which reduces the size of the heteroskedasticity correction (i.e. increasing the inverse heteroskedasticity adjustment) and raises the likelihood of contagion or flight detection (also see **Figure 1.7** and **Figure 1.8** and their explanations). The ceiling values of $\beta^2_{x|D}$ that reject the null hypothesis (H_0) in (1.20) at different standard significance levels (α), denoted by $\overline{\beta^2_{x|D}}_{\alpha}$, can then be discovered. $\overline{\beta^2_{x|D}}_{\alpha}$ can be backed out from equation (1.16) implicitly via the Fisher z-transformation (see **Appendix 1.D Applications of the Fisher z-transformation**). Hence, the hypotheses for $\phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$, which are identical to (1.20), become the following (1.22):

$$
\begin{aligned}
&H_0\colon \beta^2_{x|D} \ge \overline{\beta^2_{x|D}}_{\alpha}, \text{i.e. interdependence} \\
&H_1\colon \beta^2_{x|D} < \overline{\beta^2_{x|D}}_{\alpha} \,|\, \begin{cases} \rho_D > 0, \text{i.e. contagion} \\ \rho_D < 0, \text{i.e. flight} \end{cases}
\end{aligned}
\tag{1.22}
$$

The entire set of the inverse interdependence measures specified in **Table 1.3** will be tested and analyzed via the hypotheses (1.22) using the unconditional, GARCH(1,1), and EWMA correlation computations.

Controlling for the effects of relevant interest rates

Interest rates can theoretically and practically affect associated currencies. For instance, changes in interest rates could influence changes in international capital flows and foreign investments, which further relate to changes in exchange rates. Therefore, the tests on financial contagion and capital flight are to be conducted without and then with relevant ***interest rates being partialled out***.[55]

Accordingly, the fundamental no-arbitrage concept for foreign exchange, namely '***uncovered interest rate parity***', is applied. For instance, regarding GBP-EUR co-movements, the interest rates on the GBP and EUR are controlled in order to remove their effects. The USD interest rate, which relates to the base currency and is also assuredly one of the most dominant factors in the monetary and foreign exchange markets, is also controlled. To be more specific, the uncovered interest rate (i) parity of GBP on USD and that of EUR on USD are, respectively, $r_{GBP,t} = \ln GBPUSD_t - \ln GBPUSD_{t-1} \cong i_{t-1}^{GBP} - i_{t-1}^{USD}$ and $r_{EUR,t} = \ln EURUSD_t - \ln EURUSD_{t-1} \cong i_{t-1}^{EUR} - i_{t-1}^{USD}$. Hence, the effects of lagged i^{GBP}, i^{EUR}, and i^{USD} are partialled out when $\rho_{partial,GBP,EUR}$ (i.e. GBP-EUR partial correlation coefficient) is computed. This way of partialling out the impacts of relevant interest rates is similarly applied to all the other examined currencies.

1.5 Evidence from Brexit

1.5.1 Test results on interdependence assumptions

As mentioned previously, over-breakpoint magnitude changes in the shock-originating global-factor loading (β_x) and in the ratio of global-factor variance to shock-originating idiosyncratic noise variance ($\sigma_g^2/\sigma_{\varepsilon_x}^2$) may be tested in order to gain an idea of which set of assumptions might be most suitable (see section **1.4.4 Hypothesis tests**). The empirical outcomes from detecting shifts in the magnitude of β_x and in σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ around the time

[55] The effects of relevant variables are partialled out by using partial correlation coefficient computations. Let $\gamma_{m,n}$ be the $(m,n)^{\text{th}}$ element of an $M \times M$ inverted full correlation coefficient matrix, denoted by P_{full}^{-1}, with the $M \times M$ full correlation coefficient matrix given by $P_{full} = [\rho_{m,n}]$ with $m,n \in \{1,\cdots,M\}, m \gtreqless n$, $\rho_{m,n|m=n} = 1$. Then, the partial correlation coefficient is given by $\rho_{partial,m,n} \in \left\{-\frac{\gamma_{m,n}}{\sqrt{\gamma_{m,m}\gamma_{n,n}}} \in [-1,1] \text{ if } m \neq n, \frac{\gamma_{m,n}}{\sqrt{\gamma_{m,m}\gamma_{n,n}}} = 1 \text{ if } m = n\right\}$. Partialling a correlation coefficient is equivalent to standardizing a regression coefficient (β) (i.e. adjusting such that $\in [-1,1]$).

of the Brexit referendum on 23 June 2016 are shown in **Table 1.4**. Different nearby time intervals are also experimented with for robustness testing.

tranquility (months)	**distress** (month)	$\boldsymbol{\beta_x}$ **magnitude**	$\boldsymbol{\sigma_g^2/\sigma_{\varepsilon_x}^2}$
15		higher***	not higher
12	1	higher***	not higher
9		higher***	higher*

Table 1.4: shifts in the magnitude of β_x and in $\sigma_g^2/\sigma_{\varepsilon_x}^2$ from the state of tranquility (pre-) to the state of distress (post-breakpoint) (*, **, and ***: significant at 0.1, 0.05, and 0.01 levels, respectively) [56]

The evidence in **Table 1.4** clearly demonstrates that an increase in the magnitude of β_x [57] around the breakpoint is statistically significant, while that in $\sigma_g^2/\sigma_{\varepsilon_x}^2$ is considerably not (only significant once, even at 0.1 level, among the three samples). This may imply that the magnitude of β_x rather than σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ naturally shifts up over the period of British political uncertainty, i.e. coinciding with the interdependence between shock-originating and shock-receiving economies. Therefore, the analysis and consequential statistical inferences based on the extended framework would seem to be more valid than those based on the framework of Corsetti et al. (2005) (Forbes and Rigobon (2002) included) for the Brexit case study specifically.

These statistical preliminary verifications of the interdependence coincidence assumptions can be reassessed by simply looking at how $\beta_x^2/\sigma_{\varepsilon_x}^2$ and $\sigma_g^2/\sigma_{\varepsilon_x}^2$ change over the course of the political event.

	$\boldsymbol{\beta_x^2/\sigma_{\varepsilon_x}^2}$	$\boldsymbol{\sigma_g^2/\sigma_{\varepsilon_x}^2}$
tranquility	0.731	20.244
distress	3.760	26.207
relative change	4.141	0.295

Table 1.5: values and relative changes in $\beta_x^2/\sigma_{\varepsilon_x}^2$ and $\sigma_g^2/\sigma_{\varepsilon_x}^2$ around the period of the Brexit referendum (tranquility-state time span: one year, distress-state time span: one month)

The facts, in the move from the tranquility state to the distress state, that the relative change in $\beta_x^2/\sigma_{\varepsilon_x}^2$ substantially exceeds that in $\sigma_g^2/\sigma_{\varepsilon_x}^2$, shown in **Table 1.5**, and that time-varying β_x^2 displays a relatively clearer instant jump and a steeper trendline than time-varying σ_g^2 (see **Figure 1.A.3** in **Appendix 1.F Global factor around the Brexit referendum**), could help reaffirm that higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$ over the period of political uncertainty (the

56 Only a one-month distress-state time span is analyzed due to the observations made in section **1.3.3 Time-varying currency correlations around the Brexit referendum**.

57 In the Brexit case study, $\beta_x < 0$ ($\beta_{x|T} < 0$ and $\beta_{x|D} < 0$) always results from tests performed over several nearby sample time spans, as GBP returns are negatively tied to the global currency uncertainty index, VXY (also see **Appendix 1.E Analytical and hypothesis-testing parameters**).

extended framework) could be a more suitable assumption than higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ (Corsetti et al. (2005)).

Furthermore, the substantially larger relative change in $\beta_x^2/\sigma_{\varepsilon_x}^2$ than in $\sigma_g^2/\sigma_{\varepsilon_x}^2$ could imply that the alternative hypothesis of contagion or flight existence in the form of a regime shift in σ_g^2 relative to $\sigma_{\varepsilon_x}^2$ (i.e. H_1 in (1.22), the extended framework) might be less likely than that in the form of a structural change in β_x^2 relative to $\sigma_{\varepsilon_x}^2$, which is the directly comparable framework (i.e. H_1 in (1.21), Corsetti et al. (2005)).

1.5.2 Evidence of currency contagions and flights

Earlier, currency co-movements were displayed and analyzed in section **1.3.3 Time-varying currency correlations around the Brexit referendum**. Now, the statistical results on currency contagions and flights due to the UK political uncertainty, caused by the Brexit referendum, demonstrating currencies' interactions within the foreign exchange market are presented and discussed: presence and persistence of currency negative contagions and flights to quality.

Presence of currency negative contagions and flights to quality

When related interest rates are not controlled, i.e. the impacts of those interest rates are not partialled out from the correlation coefficients, the empirical outcomes are as shown in **Table 1.6**. Once the effects of those interest rates (i.e. overnight interbank rates on the paired currencies and on USD for each paired correlation) are removed, the empirical results are as presented in **Table 1.7**.

Looking at the empirical evidence overall, to a certain extent, whether without or with relevant interest rates partialled out, whether without or with shock-originating-return heteroskedasticity adjusted (particularly Corsetti et al. (2005) and the extended framework), especially under the unconditional and EWMA methods, the GBP falling on the USD is contagious to the EUR and CAD (EUR looks slightly more noticeable as it has geopolitically strong linkages with EUR), while there is a flight to quality from GBP to JPY. As a consequence, diversification benefits are somewhat present in the foreign exchange market at the time they are needed most.

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
unconditional	unadjusted	C***	C**	C***	F***	C**	I
	FR (2002)	I	I	I	F**	I	I
	CPS (2005)	C***	C**	C***	F***	C***	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C***	C*	C**	F***	C*	I
GARCH(1,1)	unadjusted	I	I	I	F**	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	F*	I	I
EWMA	unadjusted	C**	C*	I	F***	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	C***	I	C**	F***	C**	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C**	I	I	F***	I	I

Table 1.6: currency interdependences (I), contagions (C), and flights (F) of GBP v. the others following the Brexit referendum (tranquility state: one year, distress state: one month), ***without relevant interest rates partialled out***
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
('CPS' is right above 'extended', so their results are more conveniently compared.)

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
unconditional	unadjusted	C***	I	C**	F***	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	C***	I	C**	F***	C*	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C***	I	C*	F***	I	I
GARCH(1,1)	unadjusted	I	I	I	F*	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	I	I	I
EWMA	unadjusted	C**	C*	C*	F***	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	C***	I	C**	F***	C**	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C**	I	I	F**	I	I

Table 1.7: currency interdependences (I), contagions (C), and flights (F) of GBP v. the others following the Brexit referendum (tranquility state: one year, distress state: one month), ***with relevant interest rates partialled out***
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
('CPS' is right above 'extended', so their results are more conveniently compared.)

Next, focusing on the impacts of the heteroskedasticity corrections, the extended framework, with assumption verified (see section **1.5.1 Test results on interdependence assumptions**), particularly in the Brexit case study, overall as expected comes up with slightly fewer or less significant contagions or flights than does the framework of Corsetti et al. (2005), whose main assumption may not be verifiable (in the Brexit case study) (see section **1.5.1 Test results on interdependence assumptions**). Thus, due to the UK political uncertainty, some currency contagions, especially to EUR, and a flight to quality from GBP to JPY were present because of a regime shift in the global-factor volatility, according to the extended framework. With the framework of Forbes and Rigobon (2002), which is considered to overcorrect the heteroskedasticity, almost all statistical outcomes are unsurprisingly interdependences.

Now compare the conditional and unconditional correlation results. As more outdated information fades away over time, the conditional GARCH(1,1)- and EWMA-based outcomes provide fewer or less significant detections of contagions and flights than do the results of the unconditional correlation method.

Consider the conditional correlation computations only by comparing theoretical GARCH(1,1) with practical RiskMetrics EWMA. The lower persistence parameter on previous conditional variances and covariances indicates shorter theoretical characteristic decay time of information (without interest rates partialled out, $\alpha_2 \cong 0.854 < \varphi = 0.94$ yield decay time of $6.4 < 16.2$ trading days, and with interest rates partialled out, $\alpha_2 \cong 0.914 < \varphi = 0.94$ yield decay time of $11.1 < 16.2$ trading days, also see **Appendix 1.E Analytical and hypothesis-testing parameters**). So, the GARCH(1,1)-based tests detect many more interdependences than do the EWMA-based. With a heteroskedasticity correction, the significant effects of the political uncertainty on foreign exchange almost totally disappear based on the optimal characteristics produced by the GARCH(1,1) approach.

Concerning related interest rates being partialled out, removing the effects of the relevant interest rates somewhat reduces the occurrences of contagions or flights, remarkably for the cases of CHF. That is, contagion and flight statistical results are fewer or less significant than when the relevant interest rates are controlled. This possibly infers that the Swiss interbank overnight interest rates, and also some others economies' rates, have significant impacts in the correlations between the sterling and their corresponding currencies (i.e. CHF and some others) over time.

Lastly, the robustness of the contagion/flight tests using the extended framework is tested using different time spans. The results imply that the tests are fairly robust (see **Appendix 1.G Robustness checks on the extended framework**).

Persistence of currency negative contagions and flights to quality

As aforementioned in section **1.3.3 Time-varying currency correlations around the Brexit referendum**, contagion and flight existence using the GARCH(1,1) and EWMA approaches, following the Brexit referendum date, could persist up to about one month. Nearer after the referendum date, although the observed currency correlation shifts are larger than in other time (see **Figure 1.5** and **Figure 1.6**), the shock-originating-return heteroskedasticity is much larger during the same period of time (i.e. nearer after the referendum date)

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
GARCH(1,1)	unadjusted	I	I	I	F**	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	F**	I	I
EWMA	unadjusted	C**	C*	I	F**	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	C**	I	C*	F***	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C**	C*	I	F***	I	I

currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: 0.5 month),
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
GARCH(1,1)	unadjusted	I	I	I	I	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	I	I	I
EWMA	unadjusted	C*	I	I	F**	I	I
	FR (2002)	I	I	I	I	I	I
	CPS (2005)	C*	I	I	F***	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C*	I	I	F**	I	I

currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: 0.5 month),
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)

Table 1.8: time-conditional currency contagions and flights with 0.5-month distress-state time span

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
GARCH(1,1)	unadjusted	I	I	I	F**	I	I
	CPS (2005)	I	I	I	F*	I	I
	FR (2002)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	F**	I	I
EWMA	unadjusted	C**	I	I	F***	I	I
	CPS (2005)	C**	I	C**	F***	C*	I
	FR (2002)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C**	I	I	F***	I	I

currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: 0.75 month),
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)

correlation method	framework	EUR	CHF	CAD	JPY	AUD	NZD
GARCH(1,1)	unadjusted	I	I	I	F*	I	I
	CPS (2005)	I	I	I	I	I	I
	FR (2002)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	I	I	I	I	I	I
EWMA	unadjusted	C**	I	I	F**	I	I
	CPS (2005)	C**	I	C*	F***	C*	I
	FR (2002)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	C*	I	I	F**	I	I

currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: 0.75 month),
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)

Table 1.9: time-conditional currency contagions and flights with 0.75-month distress-state time span

(‘CPS’ is right next to ‘extended’, so their results are more conveniently compared.)

than in other time too (see **Appendix 1.B Heteroskedasticity in the shock-originating returns**). Hence, it could be interesting to see if there existed statistically significant currency contagions or flights during less than one month subsequent to the UK political event, with the heteroskedasticity corrections taken into account.

Accordingly, the detection of currency contagion and flight occurrences during less than one month, inclusive of 0.5- and 0.75- month time spans, is presented and discussed. The results based on unconditional computations are not presented here because daily data points over a less than one-month time horizon (or approximately fewer than 20 trading days) could be statistically considered to be too few. Unlike unconditional, conditional computations includes information dynamically over time. Thus, the outcomes based on GARCH(1,1) and EWMA calculations at any time point could be inferred.

The empirical outcomes of time-conditional currency contagions and flights with 0.5- and 0.75-month distress-state time spans are as shown in **Table 1.8** and in **Table 1.9**, respectively. Each table includes the results both without and with relevant interest rates partialled out.

Comparing each correlation method and framework, one by one, across various distress-state time spans, from shorter to longer (i.e. from 0.5-, 0.75-, to one-month, as in **Table 1.8**, **Table 1.9**, and **Table 1.6** and **Table 1.7**, respectively), the statistical results of interdependences, contagions, and flights of the sterling against others are, overall, quite similar across the different short distress-state time spans. No obvious dissimilar statistical outcomes are shown. Pertaining to each framework with heteroskedasticity adjustment, the similar results across the different short distress-state time periods imply that large correlation shifts are seemingly well offset by the massive heteroskedasticity of shock-originating returns, no matter what form of heteroskedasticity is assumed: Forbes and Rigobon (2002), Corsetti et al. (2005), or the extended framework. Nevertheless, relevant interest rates seem to have more impact on 0.5-month distress time span than 0.75-month one.

The empirical correlation analysis of financial contagion and capital flight, for the case study of Brexit and currencies or other cases, could also be done with financial data at higher frequencies than daily. However, unlike the intraday data of typical financial asset returns that could be available, a global factor's time series data may not be available at intraday frequencies. Hence, mixed-frequency methodologies, such as mixed-data sampling (MIDAS), may be required.

1.6 Concluding remarks

This research extensively builds on the previous literature on contagion, with a single common factor model, bivariate correlation analysis, and heteroskedasticity bias correction. Higher shock-originating-economy factor loading variability relative to country-specific noise variance is assumed to coincide with the interdependence between economies in a state of distress, instead of higher global-factor variance relative to country-specific noise variance. The assumption tests show that the extended framework seems to be a better fit, specifically for the Brexit case study, than the base framework. The evidence on currency negative contagions and flights to quality over the period of British political uncertainty demonstrates that, to some degree, EUR returns are fairly contagious, whereas JPY returns are diverging from GBP depreciations against the USD. Accordingly, currency diversification benefits are considered to exist at the time they are needed most. Interdependences as expected occur more often with conditional than unconditional correlation methods, since concerns over Brexit decay over time, and interdependences are found more often in the testing based on GARCH(1,1) than that in that based on EWMA due to a lower persistence parameter and characteristic decay time. Once related interest rates are partialled out, some significant happenings of contagions and flights decrease, probably owing to the significant impacts of the interest rates on the corresponding currency correlations.

Appendices

Appendix 1.A Impact of GBP on VXY around the Brexit referendum

Appendix 1.A.1 Attribution of GBP to VXY

Since VXY, the global currency uncertainty index, represents weighted currency implied volatility, it could be expressed as

$$VXY_t = \sum_{k=1}^{K} \omega_{k,t}\sigma_{IV,k,t}$$

where $\omega_{k,t}$ denotes weight of currency k's implied volatility at time t, and $\sigma_{IV,k,t}$ currency k's three-month at-the-money (ATM) implied volatility at time t (all against the USD).

Then, the expected impact of each currency's implied volatility on VXY over a time horizon, τ, could be as written as

$$\frac{1}{\tau}\sum_{t=1}^{\tau} \frac{\omega_{k,t}\sigma_{IV,k,t}}{VXY_t}$$

where τ, in assessing the impact of currencies on VXY around the Brexit referendum and obtaining some information on GBP-VXY endogeneity, can be tranquility-state (T) or distress-state (D) time span (i.e. pre- or post-referendum period of 262 and 21 trading days, respectively), $\tau \in \{T, D\}$.

<table>
<tr><th colspan="3"></th><th colspan="9">expected impact of each currency's implied volatility on VXY</th></tr>
<tr><th colspan="3"></th><th>GBP</th><th>EUR</th><th>CHF</th><th>CAD</th><th>JPY</th><th>AUD</th><th>NZD</th><th>others</th><th>EM</th></tr>
<tr><td colspan="3">expected weight in VXY</td><td>0.09</td><td>0.24</td><td>0.03</td><td>0.04</td><td>0.18</td><td>0.06</td><td>0.02</td><td>0.35</td><td>0.16</td></tr>
<tr><td colspan="2" rowspan="2">3-month ATM implied volatility</td><td>tranquility</td><td>0.08</td><td>0.24</td><td>0.03</td><td>0.04</td><td>0.17</td><td>0.07</td><td>0.02</td><td>0.35</td><td>0.16</td></tr>
<tr><td>distress</td><td>0.11</td><td>0.20</td><td>0.03</td><td>0.03</td><td>0.22</td><td>0.06</td><td>0.02</td><td>0.33</td><td>0.15</td></tr>
<tr><td rowspan="6">implied volatility proxy</td><td rowspan="2">unconditional</td><td>tranquility</td><td>0.07</td><td>0.22</td><td>0.03</td><td>0.04</td><td>0.17</td><td>0.07</td><td>0.02</td><td>0.38</td><td>0.17</td></tr>
<tr><td>distress</td><td>0.23</td><td>0.21</td><td>0.02</td><td>0.03</td><td>0.25</td><td>0.07</td><td>0.02</td><td>0.15</td><td>0.07</td></tr>
<tr><td rowspan="2">GARCH(1,1)</td><td>tranquility</td><td>0.08</td><td>0.22</td><td>0.03</td><td>0.04</td><td>0.18</td><td>0.07</td><td>0.02</td><td>0.37</td><td>0.16</td></tr>
<tr><td>distress</td><td>0.17</td><td>0.21</td><td>0.03</td><td>0.04</td><td>0.22</td><td>0.07</td><td>0.02</td><td>0.24</td><td>0.11</td></tr>
<tr><td rowspan="2">EWMA</td><td>tranquility</td><td>0.07</td><td>0.23</td><td>0.03</td><td>0.04</td><td>0.17</td><td>0.07</td><td>0.02</td><td>0.37</td><td>0.17</td></tr>
<tr><td>distress</td><td>0.24</td><td>0.21</td><td>0.02</td><td>0.04</td><td>0.26</td><td>0.07</td><td>0.02</td><td>0.15</td><td>0.07</td></tr>
</table>

Table 1.A.1: tranquility-state and distress-state (pre- and post-Brexit referendum) (covering 262 and 21 trading days) impacts of each currency's implied volatility (USD as base, data from Blomberg) on VXY, others' impact: unity minus the total impact of the examined major currencies, EM: emerging-market currencies, their weight and impact are calculated out of 'others'

Table 1.A.1 shows the expected impacts of currencies on VXY, using three-month ATM implied volatility (as calculated in VXY) and other three implied-volatility proxies: time-unconditional, GARCH(1,1), and EWMA (see them in variance forms: $\widehat{\mathbf{V}}_{uncond}$, $\widehat{\mathbf{V}}_{t,GARCH}$, and $\widehat{\mathbf{V}}_{t,EWMA}$, in section **1.3 Unconditional and dynamic conditional correlations**). Weights are averaged based on currency-pair turnover (all against USD) from the latest two BIS triennial surveys: 2013 and 2016 (as used in VXY calculation, see Footnote 13, arithmetic averaging method is simply assumed).

The information from **Table 1.A.1** could to a certain extent help ease concern over the possible endogeneity between the shock-originating economy's financial returns, GBP, and the global factor, VXY (see sub-section **Additional discussions on the currency global factor**).

In addition, from **Table 1.A.1**, almost half, about 0.16, of the weight of others than the studied major currencies, which is about 0.35, is attributable to emerging currencies.

Appendix 1.A.2 Granger causality between GBP and VXY

The Granger causality tests were implemented to see if a time series of the daily log returns of the shock-originating economy's currency, GBP, ($r_{x,t}$ as in (1.7)) is a Granger cause of a time series of the daily log returns of the global factor, VXY, (g_t as in (1.7)) which could provide information on possible endogeneity in the main framework (as in (1.7)). The test results are provided in **Table 1.A.2**.

period covered null hypothesis	tranquility state (24 Jun 2015 – 23 Jun 2016)	tranquility and distress states (24 Jun – 22 Jul 2016)
VXY does not Granger cause GBP	0.007***	0.002***
GBP does not Granger cause VXY	0.239	0.140

Table 1.A.2: p-values from the pairwise Granger causality tests of the daily log returns of GBP and VXY with tranquility-state and entire (tranquility and distress) time spans (covering 262 and 283 trading days), lags: three and five, respectively, selected based on model selection criteria[58]
(*, **, and ***: significant at 0.1, 0.05, and 0.01 levels, respectively)

VXY significantly Ganger causes GBP, while GBP does not Granger cause VXY although the degree of GBP Granger causing VXY is higher once the distress state is taken into account (implied by lower p-value, 0.239 v. 0.140). Thus, there should not exist the GBP-VXY endogeneity.

[58] The criteria include sequential modified likelihood ratio test statistic (LR), final prediction error (FPE), Akaike information criterion (AIC), Schwarz information criterion (SC), and Hannan-Quinn information criterion (HQ). The lags indicated are chosen by at least three out of the five criteria.

Appendix 1.B Heteroskedasticity in the shock-originating returns

Heteroskedasticity in the shock-originating (GBP) returns is shown in **Figure 1.A.1**.

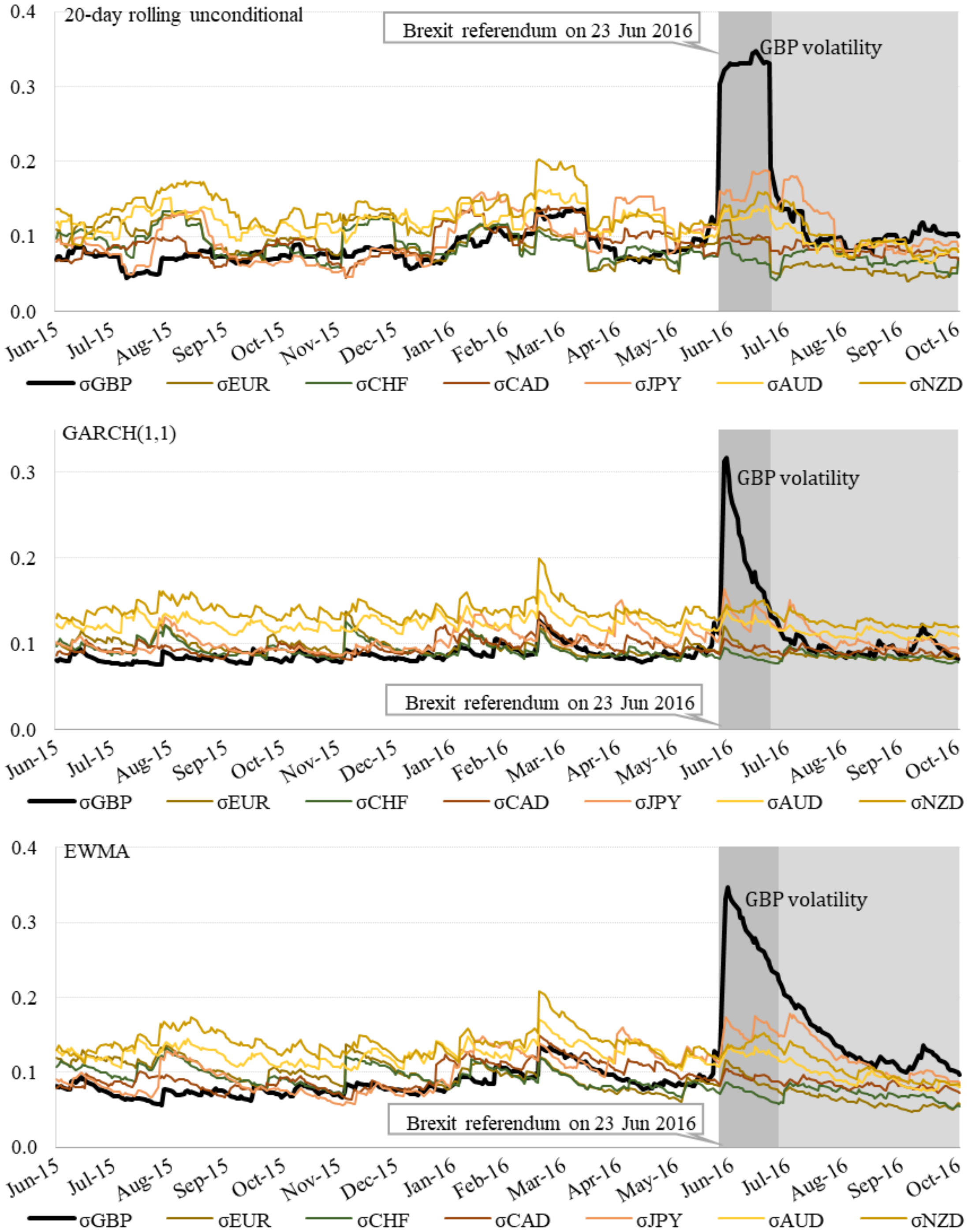


Figure 1.A.1: time-varying unconditional, GARCH(1,1) ($\alpha_1 \cong 0.054$, $\alpha_2 \cong 0.854$), and EWMA ($\varphi = 0.94$) annualized volatilities (σ), from Jun 2015 to Oct 2016, shaded areas: after the Brexit referendum, areas with darker shading (same time spans as in **Figure 1.4**, **Figure 1.5**, and **Figure 1.6**): existing heteroskedasticity or extraordinarily high volatility in $r_{x,t}$ (GBP returns)

Appendix 1.C Derivations of the extended interdependence measure

Full derivation of the extended interdependence measure, $\phi_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})}$ (1.15), which is drawn from the model and set of assumptions (1.7) and (1.14), is achieved as follows (the steps are similar to those by which ϕ_{CPS} is derived in Corsetti et al. (2005)). Initially, from the single global factor model (1.7) and its assumptions (1.8), express the correlation coefficient in the following form for later convenience:

$$\rho = \frac{1}{1+\lambda}\sqrt{\frac{\beta_y^2\sigma_x^2}{\beta_x^2\sigma_y^2}} \tag{1.A.1}$$

[59]

where $\lambda = \sigma^2_{\varepsilon_x}/\left(\beta_x^2\sigma_g^2\right)$ and $1+\lambda = \sigma_x^2/\left(\beta_x^2\sigma_g^2\right)$. For the assumptions in (1.14) specifically, the variance ratios λ and $1+\lambda$ in a state of tranquility (T) and in a state of distress (D) become (with subscript β)

$$\lambda_{\beta|T} = \frac{\sigma^2_{\varepsilon x|T}}{\beta^2_{x|T}\sigma_g^2}, \lambda_{\beta|D} = \frac{\sigma^2_{\varepsilon x|D}}{\beta^2_{x|D}\sigma_g^2}, 1+\lambda_{\beta|T} = \frac{\sigma^2_{x|T}}{\beta^2_{x|T}\sigma_g^2}, \text{ and } 1+\lambda_{\beta|D} = \frac{\sigma^2_{x|D}}{\beta^2_{x|D}\sigma_g^2}.$$

Then, the interdependence measure, $\phi_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})}$, which is ρ_D conditional on the model and set of assumptions (1.7) and (1.14), is derived by starting from the expression (1.A.1):

$$\phi_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})} = \frac{1}{1+\lambda_{\beta|D}}\sqrt{\frac{\beta_y^2\sigma^2_{x|D}}{\beta^2_{x|D}\sigma_y^2}}$$

(since $\sigma_y^2 = \sigma^2_{y|T} = \sigma^2_{y|D}$, due to the assumptions in (1.14) that β_y, σ_g^2, and $\sigma^2_{\epsilon_y}$ supposedly do not coincide with the intensification of correlation in a state of distress, so-called interdependence, also see Footnote 40)

$$= \frac{1}{1+\lambda_{\beta|D}}\sqrt{\frac{\beta_y^2\sigma^2_{x|T}}{\beta^2_{x|D}\sigma_y^2}(1+\delta)}$$

[59] Refer to (1.8) and (1.9), $\rho = \left(1+\frac{\sigma^2_{\varepsilon_x}}{\beta_x^2\sigma_g^2}\right)^{-\frac{1}{2}}\left(1+\frac{\sigma^2_{\varepsilon_y}}{\beta_y^2\sigma_g^2}\right)^{-\frac{1}{2}} = (1+\lambda)^{-\frac{1}{2}}\left((1+\lambda)\frac{\beta_x^2\sigma_y^2}{\beta_y^2\sigma_x^2}\right)^{-\frac{1}{2}} = \frac{1}{1+\lambda}\sqrt{\frac{\beta_y^2\sigma_x^2}{\beta_x^2\sigma_y^2}}$ since $1+\frac{\sigma^2_{\varepsilon_x}}{\beta_x^2\sigma_g^2} = \frac{\sigma_x^2}{\beta_x^2\sigma_g^2} = 1+\lambda$ and $1+\frac{\sigma^2_{\varepsilon_y}}{\beta_y^2\sigma_g^2} = \frac{\sigma_y^2}{\beta_y^2\sigma_g^2} = \frac{\sigma_y^2}{\beta_y^2\sigma_g^2}\frac{\beta_x^2\sigma_x^2}{\beta_x^2\sigma_x^2} = (1+\lambda)\frac{\beta_x^2\sigma_y^2}{\beta_y^2\sigma_x^2}$.

$$= \frac{1}{1+\lambda_{\beta|D}} \rho_T \left(1 + \lambda_{\beta|T}\right) \sqrt{\frac{\beta_{x|T}^2}{\beta_y^2}} \sqrt{\frac{\beta_y^2}{\beta_{x|D}^2}(1+\delta)}$$

(since $\rho_T\left(1 + \lambda_{\beta|T}\right) \sqrt{\frac{\beta_{x|T}^2}{\beta_y^2}} = \sqrt{\frac{\sigma_{x|T}^2}{\sigma_y^2}} \because \rho_T = \frac{1}{1+\lambda_{\beta|T}} \sqrt{\frac{\beta_y^2 \sigma_{x|T}^2}{\beta_{x|T}^2 \sigma_y^2}}$, refer to (1.A.1) and given (1.14))

$$= \rho_T \frac{1+\lambda_{\beta|T}}{1+\lambda_{\beta|D}} \sqrt{\frac{\beta_{x|T}^2}{\beta_{x|D}^2}(1+\delta)}$$

$$= \rho_T \sqrt{\frac{\beta_{x|D}^2}{\beta_{x|T}^2} \frac{1}{1+\delta}}$$

(after plugging $1 + \lambda_{\beta|T}$ and $1 + \lambda_{\beta|D}$ back in and expressing the result in terms of β_x^2 rather than β_x so as to align it with the extended assumption of interdependence due to higher β_x^2 relative to $\sigma_{\varepsilon_x}^2$)

$$\phi_{(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)} = \rho_T \sqrt{\frac{\beta_{x|D}^2}{\beta_{x|T}^2} \frac{1}{1+\delta}} \quad \blacksquare$$

Appendix 1.D Applications of the Fisher z-transformation

The Fisher z-transformation can be applied to investigate whether two correlation measures are statistically different as follows (under the assumption that the two samples are independently normally distributed, but the Fisher z-transformation is considered quite robust to non-normality (Kocherlakota & Singh, 1982)) (Cohen et al., 2003).

First, transform ϕ' and ρ_T into $z(\cdot)$ as follows:

$$z(\phi') = \frac{1}{2}[\ln(1+\phi') - \ln(1-\phi')] \text{ and}$$
$$z(\rho_T) = \frac{1}{2}[\ln(1+\rho_T) - \ln(1-\rho_T)]. \tag{1.A.2}$$

To test whether ϕ' is different from ρ_T (testing (1.20)) and since $z(\phi') - z(\rho_T) \sim N(0, \sigma_z^2)$ is assumed, where $\sigma_z^2 = \left(\tau_{\phi'} - 3\right)^{-1} + \left(\tau_{\rho_T} - 3\right)^{-1}$ ($\tau_{\phi'}$: distress-state time span as ϕ is adjusted on ρ_D (also equivalent to τ_D), τ_{ρ_T}: tranquility-state time span (also equivalent to τ_T)), calculate z and compare to the one-sided critical values (asymmetrical tests typically practiced in the literature on financial contagion) as follows:

$$\frac{z(\phi') - z(\rho_T)}{\sigma_z} = z \text{ v. } z_\alpha \ (z_\alpha\text{: critical } z \text{ at the significance level of } \alpha) \qquad (1.A.3)$$[60]

Statistically, $|z| \leq |z_\alpha|$ implies $\phi' = \rho_T$. Given $\rho_D > 0$, $z > z_\alpha > 0$ implies that $\phi' > \rho_T$ which indicates the presence of contagion. Given $\rho_D < 0$, $z < z_\alpha < 0$, implying $\phi' > \rho_T$, which indicates the presence of flight (see (1.20)).

To detect contagions and flights via the inverse interdependence measures, ϕ'_{unadj} (1.17) and ϕ'_{FR} (1.19), which are in $[-1,1]$, use the standard Fisher z-transformation procedure described above.

To detect contagions and flights via the other two inverse interdependence measures, ϕ'_{CPS} (1.18) and $\phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})}$ (1.16), apply the Fisher z-transformation procedure to test hypotheses (1.21) and (1.22), respectively, as follows (similarly to applications in Corsetti et al. (2005)).

First, figure out ϕ'_α (that is a critical value of ϕ' where $\phi'_\alpha \in \left\{\phi'_{CPS,\alpha}, \phi'_{(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}),\alpha}\right\}$) such that $[z(\phi'_\alpha) - z(\rho_T)]/\sigma_z = z_\alpha$ where $z_\alpha > 0$ if $\rho_D > 0$ for contagion cases and $z_\alpha < 0$ if $\rho_D < 0$ for flight cases. From (1.A.2) and (1.A.3), ϕ'_α can be written as

$$\phi'_\alpha = \frac{\exp\{2[z_\alpha \sigma_z + z(\rho_T)]\} - 1}{\exp\{2[z_\alpha \sigma_z + z(\rho_T)]\} + 1}.$$

Then, for ϕ'_{CPS} (1.18), find a critical floor value of the crucial parameter $\underline{\lambda_D}_\alpha$ (see (1.21) and its explanation) from its function of $\phi'_{CPS,\alpha}$ below (i.e. backing out $\underline{\lambda_D}_\alpha$ from (1.18)):

$$\underline{\lambda_D}_\alpha(\phi'_{CPS,\alpha}) = \frac{\phi'^2_{CPS,\alpha}}{\rho_D^2}(1 + \lambda_T)\{1 + \delta - \rho_D^2[(1 + \delta)(1 + \lambda_T) + 1]\} - 1.$$

Thus, $\underline{\lambda_D}_\alpha$ is the value of λ_D such that $\left[z\left(\phi'_{CPS,\alpha}\right) - z(\rho_T)\right]/\sigma_z = z_\alpha$. Finally, perform the hypothesis testing in (1.21).

[60] One-sided z_α (critical z statistics) are ± 1.28, ± 1.65 and ± 2.33 at 0.1, 0.05, or 0.01 significance levels (α), respectively.

For $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ (1.16), find a critical ceiling value of the crucial parameter $\overline{\beta^2_{x|D}}_{\alpha}$ (see (1.22) and its explanation) from its function of $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x}),\alpha}$ below (i.e. backing out $\overline{\beta^2_{x|D}}_{\alpha}$ from (1.16)):

$$\overline{\beta^2_{x|D}}_{\alpha}\left(\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x}),\alpha}\right) = \frac{\rho_D^2}{\phi'^2_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x}),\alpha}}\beta^2_{x|T}(1+\delta).$$

Hence, $\overline{\beta^2_{x|D}}_{\alpha}$ is the value of $\beta^2_{x|D}$ such that $\left[z\left(\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x}),\alpha}\right) - z(\rho_T)\right]/\sigma_z = z_\alpha$. Finally, implement the hypothesis testing shown in (1.22).

Indeed, tests using $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ can be more simply done by comparing the estimated $\phi'_{(\tilde{\beta}_x,\tilde{\sigma}^2_{\tilde{\varepsilon}_x})}$ (from (1.16)) against ϕ'_α (i.e. critical ϕ'). However, $\overline{\beta^2_{x|D}}_{\alpha}$ is used and (1.22) is tested in this dissertation in alignment with the base framework (Corsetti et al. (2005) with the hypothesis testing (1.21)) and with the illustrative analytics shown in **Figure 1.7** and **Figure 1.8**.

Appendix 1.E Analytical and hypothesis-testing parameters

This section presents the hypothesis testing values and analytical parameters used in this chapter.

For $P_{t,GARCH}$ (1.2), the joint MLEs across all the sampled major currencies and their correlations over various tranquility-state time spans (each excluding the distress-state period so as not to incorporate uncommon behaviors in the currency returns and correlations) optimally yield the α_1 and α_2 values shown in **Table 1.A.3** (obtained using the generalized reduced gradient (GRG) nonlinear method in Excel Solver). Persistence of $\bar{\mathbf{V}}$ $(1-\alpha_1-\alpha_2)$ and the characteristic decay time of conditional (co)variances (computed as $-1/\ln\alpha_2$) are also shown.

	tranquility (months)	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\bar{\mathbf{V}}$ **persistence** $(1-\alpha_1-\alpha_2)$	$\hat{\mathbf{V}}_t$ **decay time** (trading days)
without interest rates partialled out	15	0.041	0.913	0.046	11.0
(joint MLE on the variance-covariance	12	0.054	0.854	0.091	6.4
of the currencies)	9	0.051	0.859	0.090	6.6
with interest rates partialled out	15	0.081	0.916	0.003	11.3
(joint MLE on the variance-covariance of	12	0.081	0.914	0.005	11.1
the currencies and interest rates)	9	0.082	0.910	0.009	10.6

Table 1.A.3: optimal α_1 and α_2 with conditional (co)variance lifetime for $P_{t,GARCH}$ over different time spans

Characteristic decay time increases, i.e. the information dies out more slowly over time, when the effects of relevant interest rates are removed, and does so when more information is included (i.e. longer tranquility-state time is covered in an MLE).

Next are the values of the hypothesis test and the other analytical parameters with one-year tranquility-state and one-month distress-state time spans around the date of the Brexit referendum, 23 June 2016.

The test parameters, namely the tranquility-state and distress-state bivariate correlation coefficients, ρ_T and ρ_D, respectively, for GBP against the other investigated currencies, based on the unconditional, GARCH(1,1), and EWMA approaches, are shown in **Table 1.A.4** and **Figure 1.A.2**.

	correlation method	$\boldsymbol{\rho}$	**correlations with GBP**					
			EUR	**CHF**	**CAD**	**JPY**	**AUD**	**NZD**
without interest rates partialled out	unconditional	ρ_T	0.433	0.348	0.428	0.106	0.432	0.380
		ρ_D	0.908	0.673	0.800	-0.778	0.744	0.551
	GARCH(1,1)	ρ_T	0.452	0.253	0.498	0.083	0.510	0.398
		ρ_D	0.493	0.286	0.520	-0.359	0.545	0.396
	EWMA	ρ_T	0.429	0.158	0.532	0.027	0.527	0.373
		ρ_D	0.764	0.466	0.709	-0.618	0.673	0.492
with interest rates partialled out	unconditional	ρ_T	0.467	0.379	0.441	0.116	0.456	0.406
		ρ_D	0.881	0.239	0.743	-0.751	0.646	0.477
	GARCH(1,1)	ρ_T	0.451	0.248	0.433	0.030	0.493	0.372
		ρ_D	0.481	0.278	0.522	-0.300	0.550	0.383
	EWMA	ρ_T	0.430	0.160	0.459	-0.010	0.518	0.363
		ρ_D	0.768	0.469	0.719	-0.587	0.694	0.521

Table 1.A.4: tranquility-state and distress-state bivariate correlation coefficients of GBP v. the other currencies around the time of the Brexit referendum (tranquility state (T): one year, distress state (D): one month)

It is interesting to note that, for the GBP-JPY correlations around the time of British political uncertainty, $\rho_T > 0$, while $\rho_D < 0$. With opposite signs of ρ_T and ρ_D, the results of tests using ϕ could lead to erroneous inferences. See section **1.4.3 Inverse interdependence measures as the problem solver**. Another noticeable point is that, after the relevant interest rates are partialled out, GBP-CHF correlation shift changes from an increase to a slight decrease, resulting in no contagion detected (see section **Presence of currency negative contagions and** flights to quality, and **Table 1.6** and **Table 1.7**).

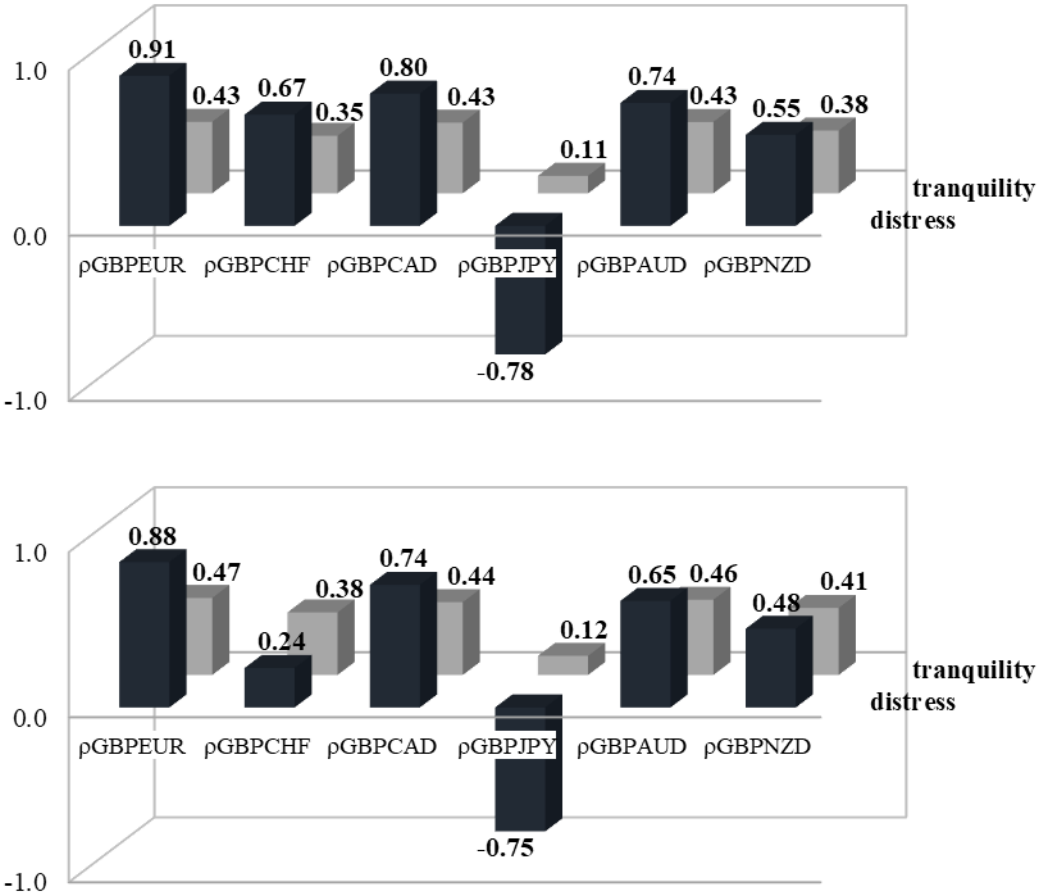


Figure 1.A.2: shifts of unconditional bivariate correlation coefficients (ρ)
(without and with relevant interest rates partialled out: above and below, respectively)
between GBP and the other currencies around the time of the Brexit referendum
(tranquility state (T): one year, distress state (D): one month)

The values of the other hypothesis test parameters: δ, λ_T, λ_D, $\beta^2_{x|T}$, and $\beta^2_{x|D}$, including $\beta_{x|T}$, $\beta_{x|D}$, and $\sqrt{\theta_\beta}$ that were used in the extended inverse interdependence illustration (see **Figure 1.7** and **Figure 1.8**) and $\lambda_{\beta|T}$ and $\lambda_{\beta|D}$ that were used in **Appendix 1.C** are displayed in **Table 1.A.5**.

parameter	value	parameter	value
δ	12.416	$\beta_{x\|T}$	-0.072
λ_T	1.549	$\beta_{x\|D}$	-0.323
λ_D	5.673	$\theta_\beta^{1/2}$	4.511
$\beta^2_{x\|T}$	0.005	$\lambda_{\beta\|T}$	7.117
$\beta^2_{x\|D}$	0.105	$\lambda_{\beta\|D}$	4.351

Table 1.A.5: hypothesis test and analytical parameters
(tranquility state: one year, distress state: one month)

Appendix 1.F Global factor around the Brexit referendum

(This appendix is to mainly support section **1.5.1 Test results on interdependence assumptions**.)

Figure 1.A.3 shows time-varying global-factor-loading variability against global-factor variance, that is β_x^2 versus σ_g^2, while **Figure 1.A.4** shows time-varying global-factor loading, β_x , and R-squared, from regression of g_t on $r_{x,t}$ (as in (1.7)), around the Brexit referendum on 23 June 2016.

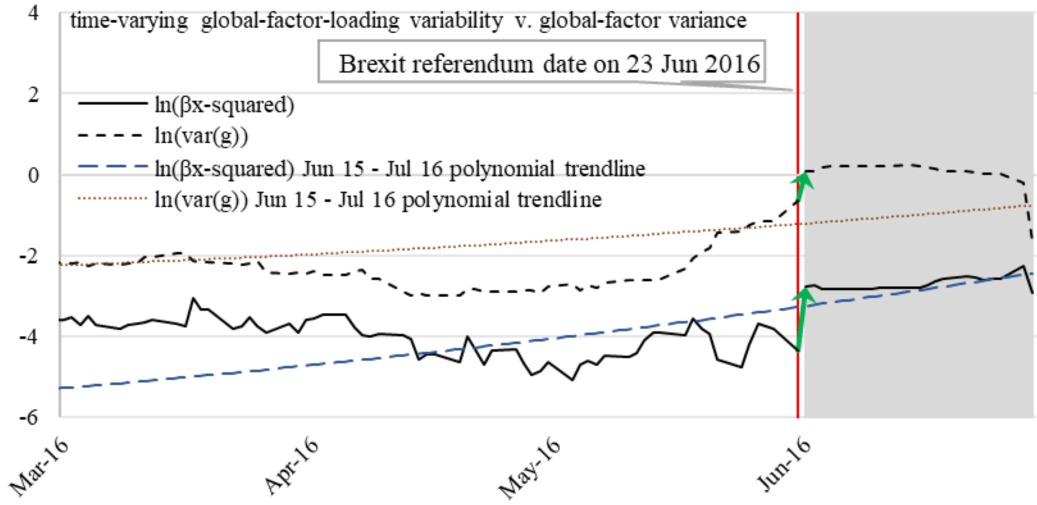


Figure 1.A.3: 20-day-rolling time-varying β_x^2 v. σ_g^2 around the Brexit referendum on 23 Jun 2016 (rescaled logarithmically and zoomed in to be during Mar 16 – Jul 16 to see their jumps clearer), their instant jumps following the date, and their polynomial (order 2) trendlines over Jun 15 – Jul 16 (one-year tranquility followed by one-month distress as in the tests in both Chapter 1 and Chapter 2)

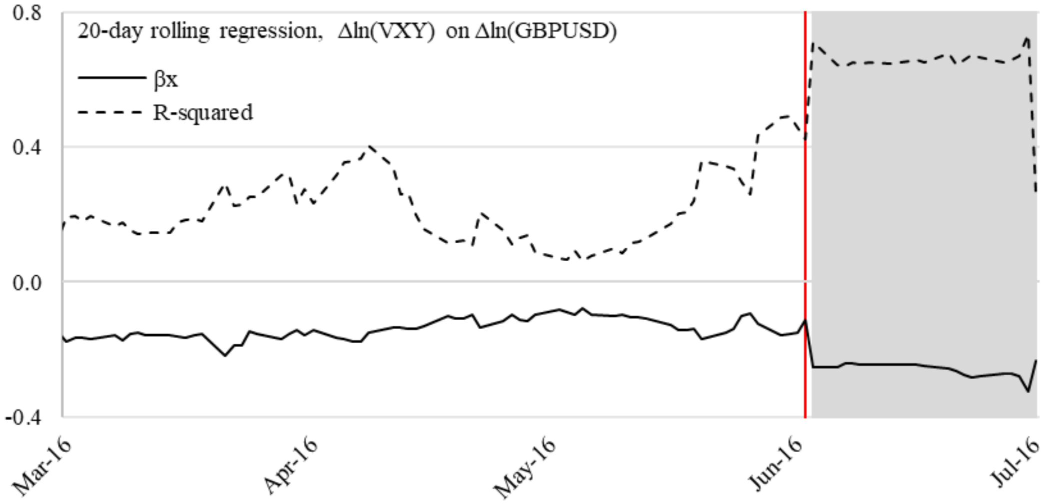


Figure 1.A.4: 20-day-rolling time-varying β_x^2 and R-squared, from regression of g_t on $r_{x,t}$, around the Brexit referendum on 23 Jun 2016

In addition to the expected GBP-VXY non-endogeneity (see sub-section **Additional discussions on the currency global factor** together with **Appendix 1.A Impact of GBP on VXY around the Brexit referendum**), **Figure 1.A.3**, visually showing a bigger instant jump and a steeper trendline of β_x^2 versus σ_g^2, to a certain extent reaffirms the assumption test results in **1.5.1 Test results on interdependence assumptions** such that, following the UK political event, $\beta_x^2/\sigma_{\varepsilon_x}^2$ rather than $\sigma_g^2/\sigma_{\varepsilon_x}^2$ instantly intensifies. VXY also explained GBP more than before (i.e. relatively higher R-squared as in **Figure 1.A.4**) since, globally, foreign exchange market participants were possibly getting more worried about the UK political uncertainty and the sterling than other issues.

Appendix 1.G Robustness checks on the extended framework

Robustness test results from testing other alternative nearby time spans are shown in **Table 1.A.6**.

	correlation method	**tranquility** (months)	**distress** (month)	**I**	**C**	**F**
without interest rates partialled out	unconditional	15	1	1	4	1
		12		1	4	1
		9		1	4	1
	GARCH(1,1)	15	1	5	0	1
		12		5	0	1
		9		5	0	1
	EWMA	15	1	3	2	1
		12		4	1	1
		9		3	2	1
with interest rates partialled out	unconditional	15	1	3	2	1
		12		3	2	1
		9		3	2	1
	GARCH(1,1)	15	1	5	0	1
		12		6	0	0
		9		6	0	0
	EWMA	15	1	2	3	1
		12		4	1	1
		9		2	3	1

Table 1.A.6: numbers of interdependences (I), contagions (C), and flights (F) discovered using the extended framework and different methods and time spans (only the one-month distress-state time span is analyzed due to the observations made in section **1.3.3 Time-varying currency correlations around the Brexit referendum**)

Looking at the cases without or with interest rates partialled out separately, for each correlation approach, the outcomes from the extended framework look quite robust as the numbers of discovered interdependences, contagions, and flights are to a certain extent consistent over different time spans, very obviously for the unconditional and GARCH(1,1) methods.

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Chapter 2 Mean-variance portfolio analysis of financial contagion and capital flight: evidence from Brexit and currencies

Abstract

This chapter analyzes financial contagion and capital flight, which could be implied by portfolio rebalances, by applying mean-variance portfolio analysis and also builds on the correlation analysis and statistical inferences regarding currency contagions and flights to quality due to political uncertainty, presented in Chapter 1. In the correlation analysis, correlations are bivariate while, in the portfolio analysis, correlations become multivariate, under which the risk-return tradeoff also crucially comes into play. Portfolio risk minimization and reward-to-risk maximization are the two analytical cases of portfolio optimality included here. Robust portfolio optimizations, via shrinkage estimations and newly proposed risk-based weight restrictions, are also applied. The statistical results on foreign exchange contagions and flights, inferring diversification advantages, drawn from the portfolio analysis seem considerably different from those obtained from the correlation analysis in Chapter 1. The difference could be due to that, during the political uncertainty, the behaviors of the means and (co)variances among all the shock-originating and shock-receiving returns in the sample, dominate over the behaviors of the bivariate correlations between the shock-originating and shock-receiving returns. In addition, shock-originating-return heteroskedasticity adjustments seem not to have an impact on currency portfolio reallocation. On structural portfolio rebalancing, its statistical detections could infer hedging demands, owing to variance-covariance shocks from the political uncertainty (also see **Figure 2.1** on the next page for an infographic brief outline of Chapter 2).

Keywords: mean-variance portfolio analysis, multivariate correlation, risk minimization, reward-to-risk maximization, heteroskedasticity, financial contagion, capital flight, Brexit, political uncertainty

Figure 2.1: diagram outline or mind map of Chapter 2 (since Chapter 2 mainly builds on Chapter 1, also see **Figure 1.1**: diagram outline or mind map of Chapter 1)

→ theoretical parts
→ main research testing
→ empirical parts

- without the effects of relevant interest rates partialled out
- with the effects of relevant interest rates partialled out

portfolio allocation: mean-variance tradeoff
(e.g. risk minimization, reward-to-risk maximization, etc.)
→ use sample means, covariances, and correlations over a time span
(time-unconditional or conditional) → GARCH(1,1), EWMA
of shock-originating and shock-receiving assets' returns
(identified upon a case study)

for more robust portfolio optimizations,
- apply shrinkage estimations with both sample means and covariances
- make portfolio weights not too extreme
(especially for ***reward-to-risk maximization*** cases)
by constraining portfolio expected return and volatility based on risk management insights by applying mean VaR and volatility VaR

portfolio allocation in tranquility
v.
portfolio allocation in distress

corrected for heteroskedasticity in the shock-originating financial returns, with proposed standardization, based on correlation analysis in Chapter 1

e.g. financial crisis, political uncertainty, etc.
shock-originating economy
shock-receiving economies

testing
portfolio reallocation
- structural (entire portfolio)
- individual asset weight

from tranquility state to distress state, given structural rebalancing is required and shock-originating asset's weight decreases,

then shock-receiving assets' weights:
- unchanged → interdependence
- lower → negative contagion (or flight from quality)
- higher → flight to quality (or positive contagion)

case study:
- political uncertainty due to the Brexit referendum
- an investment portfolio
  - risky assets (including only currencies):
    GBP (shock-originating) and
    other major currencies (shock-receiving)
    (their returns calculated with USD as base)
  - risk-free asset: yielding at the USD overnight interbank interest rate

risk-free asset not taken into account:
- risky portfolio allocation based on ***risk minimization***
→ two reallocation cases:
(1) risk aversion maintenance
(2) portfolio variance maintenance

risk-free asset taken into account:
- riskless-risky portfolio allocation based on ***reward-to-risk maximization***
→ two reallocation cases:
(1) risk aversion maintenance
(2) Sharpe ratio maintenance

propose efficient-frontier ***synthetic*** analytical computations

main findings from Brexit and currencies:
- The portfolio analysis results of contagions and flights, implying diversification benefits, are noticeably dissimilar from the correlation analysis results of Chapter 1.
- Possibly, because the behaviors of the means and (co)variances among all the shock-originating and shock-receiving returns have dominance over the behaviors of the bivariate correlations between the shock-originating and shock-receiving returns.
- Adjustments of the heteroskedasticity bias inherent in the shock-originating returns, overall, do not influence currency portfolio rebalancing, in the move from the tranquility state to the distress state.
- Detected structural portfolio rebalancing, possibly due to variance-covariance shocks, could imply hedging demands.

2.1 Introduction

The tests on shifts in currency correlations due to political uncertainty presented in **Chapter 1 Extended correlation analysis of financial contagion and capital flight: evidence from Brexit and currency co-movements**, which indicated the presence of interdependences, contagions, and flights, could convey information about diversification benefits and portfolio rebalances. Portfolio analysis examining portfolio reallocations could thereby also be challenging but applicable in analyzing financial contagions and flights. Here, some constructed portfolios will be considered in terms of how they would be reallocated in a time of political uncertainty.

Accordingly, the main motivation here is to compare and contrast correlation analysis of financial contagion and capital flight, on the basis of diversification benefits, particularly the discussions and statistical results in Chapter 1, against portfolio analysis (from bivariate to multivariate analysis). The main contribution is to apply mean-variance portfolio analysis to investigate financial contagion and capital flight implied by portfolio reallocations. Additional contributions are first, robust portfolio optimizations, inclusive of shrinkage estimations and newly proposed risk-management weight restrictions, second, different schemes in studying portfolio risk minimization and reward-to-risk maximization, and third, standardizations of correlations corrected for heteroskedasticity.

Widely practiced by researchers and practitioners studying portfolio investments, Markowitz's (1952) modern portfolio theory (MPT) or mean-variance analysis is mainly applied in this chapter. Additionally, rather than other strategies, such as carry trade, momentum investing, and others, mean-variance strategy is adopted here because it essentially involves asset correlations and covariances in addition to risk-return profiles, which could directly build on to yield results to be compared to Chapter 1's bivariate correlation analysis. Without considering asset correlations, carry trade strategy chiefly deals with interest rate differentials, which is also taken into account in this chapter's portfolio analysis by having cases of partialling out interest rate effects (see section **Controlling for the effects of relevant interest rates**, under Chapter 1's section **1.4.4 Hypothesis tests**). Regardless of asset correlation consideration, momentum investing strategy mostly associates with timing and persistence of asset prices, being long in assets, whose prices are momentarily upward, and short in assets, whose prices are momentarily upward. To some extent, this approach could be explained by risk-return compensation, which is already a vital foundation in the MPT (Li et al., 2008).

Regarding the MPT, portfolio risk minimization and portfolio reward-to-risk maximization are employed. Bodnar and Schmid (2007), Bodnar (2009), and Bodnar et al. (2017)'s theoretical discussions of the test statistics and properties of the weights of global-minimum-variance (GMV) portfolios and tangency portfolios, and structural changes in a test portfolio, will also be applied. Empirically, Bodnar and Schmid (2007) test GMV portfolio structural changes on an internationally diversified stock portfolio. Bodnar (2009) investigates GMV portfolio structural changes on a US-diversified stock portfolio. Bodnar et al. (2017) analyze tangency portfolio behaviors via US stocks and treasury bills.

In an attempt to reduce sample estimation errors and improve portfolio optimization robustness, shrinkage estimations, which apply weights to sample and benchmark estimates, are taken into account. Broadly recognized shrinkage methods include the Bayes-Stein mean shrinkage presented by Jorion (1986) and the constant-correlation covariance shrinkage of Ledoit and Wolf (2004). Besides this, from a risk management perspective, additional constraints regarding mean value-at-risk (VaR) and volatility VaR are proposed as indirect weight restrictions in portfolio reward-to-risk maximization.

Further, to adjust for the heteroskedasticity bias in the shock-originating financial returns, the multivariate-correlation matrices will also be rebuilt and standardized (to range from -1 to $+1$) based on the methodologies discussed in the first chapter. Additionally, by means of less against more forward looking, the estimated multivariate-correlation matrices used in the portfolio optimization problems will be both unconditional and dynamically conditional, based on generalized autoregressive conditional heteroskedasticity (GARCH) and exponentially weighted moving average (EWMA) methodologies.

Apart from sections **1.2.3 Data description** and **1.3 Unconditional and dynamic conditional correlations** in the first chapter, which are also applied and referred in the portfolio analysis in this chapter, the remaining sections cover contagion and flight definitions, mean-variance background and estimations, risk minimization, reward-to-risk maximization, standardized correlations with heteroskedasticity correction, evidence from Brexit, and concluding remarks (aside from the chapter's structure described, also see **Figure 2.1** for the chapter's infographic brief outline).

2.2 Contagion and flight definitions and structural portfolio rebalancing

2.2.1 Contagion and flight definitions

In correlation analysis, as discussed in the literature and the previous chapter, financial contagion or capital flight takes place when the international transmission of financial shocks shifts in state of distress or during a crisis. Financial contagion and capital flight, respectively, are measured as significant positive and negative changes in correlation coefficients, accounting for the heteroskedasticity in the shock-originating returns (see **1.2.2 Contagion and flight definitions**). Detections of contagions and flights from the shock-originating to shock-receiving financial returns (denoted by $r_{x,t}$ and $r_{y,t}$, respectively)[1] in correlation analysis could reflect diversification benefits. In portfolio analysis, once there is financial distress, reallocation of the whole portfolio and of each asset may be needed, on the basis of diversification improvement as well as in correlation analysis. Specifically, in either correlation or portfolio analysis, the holdings of the initially shocked assets and of the other assets for which contagion has been detected should be diversified away towards those assets for which flight has been detected.

In spite of the similar intuition, the diversification implications drawn from correlation analysis are not exactly identical to those in portfolio analysis due to different analytical schemes applying. In correlation analysis, the correlations (ρ) taken into account are bivariate, i.e. between the shock-originating and shock-receiving financial returns, $r_{x,t}$ and $r_{y,t}$s, only (only $\rho_{x,y}$s). In portfolio analysis, correlations become multivariate, also involving correlations among shock-receiving financial returns, $r_{y,t}$ (not only $\rho_{x,y}$s but also ρs among $r_{y,t}$s). Additionally, in portfolio analysis, the expected return and the risk are also crucial elements, especially in the mean-variance analysis that is applied in this chapter.

In the move from a state of tranquility (T) to a state of distress (D), any structural changes that occur in the portfolio allocation ($\Delta\mathbf{w} \neq 0$, $\mathbf{w}$: entire portfolio allocation, $\mathbf{w} = (w_1, \dots, w_N)'$, w_i: individual weight, $i \in \{x, y_1, \dots, y_{N-1}\}$), due to the existence of variance-covariance shocks and correlation risk, according to Brandt (2009) and Buraschi et al. (2010), as well as any currency interdependences, contagions, and flights, by means of diversification benefits and comparability to Chapter 1's bivariate correlation analysis, could possibly be defined and summarized in **Table 2.1**.

[1] As before, here, x denotes the shock-originating economy and y a shock-receiving economy.

<table>
<tr><th colspan="2" rowspan="2">portfolio allocation
from a tranquility state (T) to a distress state (D)</th><th colspan="2">instant $\Delta r_{x,t}$</th></tr>
<tr><th>$\Delta r_{x,t} < 0$
$\rightarrow w_{x|T} > w_{x|D}$</th><th>$\Delta r_{x,t} > 0$
$\rightarrow w_{x|T} < w_{x|D}$</th></tr>
<tr><td colspan="2">no structural change in portfolio allocation ($\mathbf{w}_T = \mathbf{w}_D$)</td><td>interdependence</td><td>interdependence</td></tr>
<tr><td rowspan="2">structural change in portfolio allocation
($\mathbf{w}_T \neq \mathbf{w}_D$)</td><td>$w_{y|T} > w_{y|D}$</td><td>negative contagion</td><td>flight from quality</td></tr>
<tr><td>$w_{y|T} < w_{y|D}$</td><td>flight to quality</td><td>positive contagion</td></tr>
</table>

Table 2.1: summary of financial interdependences, contagions, and flights for portfolio analysis

Now consider currency portfolios, with all currencies against the USD (that is, the US can be viewed as the home country), under political uncertainty, specifically the Brexit referendum of 23 June 2016. The result in favor of the UK leaving the EU was relatively unexpected and negatively viewed owing to extensive concerns about long-run adverse effects on the British economy. That the voting result was unanticipated and adversely viewed, or seen as a negative shock, can also be inferred from the dropping GBP returns (see **Figure 1.2**), rocketing GBP return volatilities (see **Figure 1.A.1**), and shifting correlations (see **Figure 1.4**, **Figure 1.5**, and **Figure 1.6**) following the Brexit referendum.

Hence, the case of interest in this dissertation (i.e. the Brexit surprise, seen as a negative shock) is GBP instantly falling on USD (i.e. instant $\Delta r_{x,t} < 0$), possibly causing diversifications away from GBP towards some other currencies in a currency portfolio (i.e. $w_{x|T} > w_{x|D}$), with the statistical results of interdependences, negative contagions, and flights to quality.

Nonetheless, the opposite case (instant $\Delta r_{x,t} > 0$) is also possible, reflecting an unexpectedly positive political event, or seen as a positive shock, with the statistical results of interdependences, positive contagions, and flights from quality.

Other surprising financial or political incidents with other asset classes or types may also be applied similarly.

2.2.2 Structural portfolio rebalancing

As aforementioned that there could exist structural portfolio rebalancing in a state of distress, it may be discussed more on the basis of myopic and dynamic portfolio choice and hedging demands and as follows.

Hypothetically, requiring structural portfolio reallocation could be due to the presence of variance-covariance shocks and correlation risk, referring to Brandt (2009) and Buraschi et al. (2010), which is possibly caused by a surprised positive/negative financial or political incident.

Consider portfolio allocation moving from a state of tranquility, across a surprised financial or political event, to a state of distress. An optimal portfolio allocation in a state of tranquility could represent the myopic portfolio choice, while an optimal portfolio allocation in a state of distress or during a crisis could characterize a dynamic portfolio choice. Owing to emerging (co)variance shocks and correlation risk arising from an unexpected event, the dynamic portfolio choice could be significantly different from the myopic portfolio choice, bringing about investors' hedging demands (Brandt, 2009; Buraschi et al., 2010). In other words, the myopic portfolio choice cannot capture the possible ex ante willingness of financial market participants to hedge against the potential upcoming unusual fluctuations. Thus, structural portfolio rebalancing may be required around the state transition, from tranquility to distress.

2.3 Mean-variance background and estimations

In this chapter, the portfolio constructions used to test financial contagion and capital flight, specifically currency negative contagion and flight to quality due to political uncertainty, are based on the MPT or mean-variance analysis.[2]

2.3.1 Risk-return tradeoff and portfolio mean and covariance

According to the MPT, investors rationally make investment decisions based on a tradeoff between risk and the expected return. In other words, they are characteristically risk averse. The tradeoff is that seeking higher yields requires higher risk to be borne, or that rising risk must be compensated by a greater potential return (i.e. upward-sloping indifference curves (IC) on the return-volatility plane, also see Endnote a). As a consequence, for any given level of expected return, the portfolio with the minimum risk is preferred or alternatively, for any given risk level, the portfolio with the maximum expected return will be

[2] Other portfolio strategies than mean-variance analysis are possibly carry-trade and momentum strategies.

selected. Over a certain time span, τ, the expected return of a financial asset is the expected value change, including any additional yield, while the risk is the return variance, which is standardized as standard deviation or volatility.

Next, the expected return and risk of financial assets and then those of a portfolio will be discussed. Let $\boldsymbol{\mu}$ and $\mathbf{V}$, respectively, denote the true expected return and true variance-covariance matrix of certain financial assets. $\widehat{\boldsymbol{\mu}}$ and $\widehat{\mathbf{V}}$ denote their estimators, respectively. Finally, let μ_p and σ_p^2 denote, respectively, the expected return and variance of a risky portfolio (p).

In this chapter, $\boldsymbol{\mu}$ and $\mathbf{V}$ are estimated in line with section **1.3 Unconditional and dynamic conditional correlations**. Recall the column vector of returns at time t of N financial assets, $\mathbf{R}_t = \left(r_{1,t}, \dots, r_{N,t}\right)'$, having unconditional expected returns $E[\mathbf{R}_t] = \left(E\left[r_{1,t}\right], \dots, E\left[r_{N,t}\right]\right)'$ over a sample time span τ, $t \in [1, \tau]$ (from (1.1)) (in this dissertation, τ can be tranquility-state (T) or distress-state (D) time span, $\tau \in \{T, D\}$). Let $\widehat{\boldsymbol{\mu}}_{uncond}$ denote the $N \times 1$ matrix $E[\mathbf{R}_t]$. Also recall the three forms of unconditional and conditional variance-covariance matrix of the N financial assets: $\widehat{\mathbf{V}}_{uncond}$ (implicit in (1.1)), $\widehat{\mathbf{V}}_{t,GARCH}$ (1.3), and $\widehat{\mathbf{V}}_{t,EWMA}$ (1.6), respectively. The degrees of freedom (df) of $\widehat{\mathbf{V}}_{uncond}$ is adjusted from $\tau - 1$ to $\tau - N - 2$ in this chapter (τ: sample time length, N: number of variables) due to the multivariate generalization of $\widehat{\mathbf{V}}_{uncond}$ from a chi-squared distribution to a Wishart distribution (Brandt, 2009). Then, $\widehat{\boldsymbol{\mu}}$ is proposed on the basis of dynamic conditional computations as well, to bring it in line with $\widehat{\mathbf{V}}_{t,GARCH}$ and $\widehat{\mathbf{V}}_{t,EWMA}$. Corresponding to $\widehat{\mathbf{V}}_{t,GARCH}$ (a scalar BEKK GARCH(1,1) to be specific, see section **1.3.2 Dynamic conditional correlations**), $\widehat{\boldsymbol{\mu}}$ is constructed using the exponential moving average with mean reversion, denoted by $\widehat{\boldsymbol{\mu}}_{t,GARCH}$, as written in (2.1) below (Mukherji, 2011).[3] Consistent with $\widehat{\mathbf{V}}_{t,EWMA}$, $\widehat{\boldsymbol{\mu}}$ based on EWMA, denoted by $\widehat{\boldsymbol{\mu}}_{t,EWMA}$, is written as in (2.2).

$$\widehat{\boldsymbol{\mu}}_{t,GARCH} = (1 - \alpha_1 - \alpha_2)\widehat{\boldsymbol{\mu}}_{uncond} + \alpha_1 \mathbf{R}_{t-1} + \alpha_2 \widehat{\boldsymbol{\mu}}_{t-1,GARCH} \tag{2.1}$$

$$\widehat{\boldsymbol{\mu}}_{t,EWMA} = (1 - \varphi)\mathbf{R}_{t-1} + \varphi \widehat{\boldsymbol{\mu}}_{t-1,EWMA} \tag{2.2}$$

[3] Mukherji (2011) demonstrates that financial, especially stock, returns can be characteristically mean reverting. This may apply to currency returns.

In (2.1), $(1-\alpha_1-\alpha_2)\hat{\boldsymbol{\mu}}_{uncond}$ represents the intercept term, $\mathbf{R}_{t-1}$ the previous-period returns, $\hat{\boldsymbol{\mu}}_{t-1,GARCH}$ the $N \times 1$ previous-period conditional expected-return matrix with $\hat{\boldsymbol{\mu}}_{0,GARCH}$ equal to $\hat{\boldsymbol{\mu}}_{uncond}$ calculated over a time horizon prior to $[1,\tau]$, and α_1 and α_2 the parameters identical to those in $\mathbf{V}_{t,GARCH}$ (1.3) (drawn from MLE (1.4)). $1-\alpha_1-\alpha_2$, α_1, and α_2 indicate the persistence of $\hat{\boldsymbol{\mu}}_{uncond}$, $\mathbf{R}_{t-1}$, and $\hat{\boldsymbol{\mu}}_{t-1,GARCH}$, respectively. $\alpha_1+\alpha_2$ (<1 assumed for model stability) determines the speed of mean reversion ($\hat{\boldsymbol{\mu}}_{t,GARCH}$ reverting to $\hat{\boldsymbol{\mu}}_{uncond}$). In (2.2), there is no mean reversion, for simplicity, and the persistence parameter $\varphi = 0.94$ (following RiskMetrics, for practicality) is the same as in $\hat{\mathbf{V}}_{t,EWMA}$ (1.6). Moreover, $\hat{\boldsymbol{\mu}}_{t,GARCH}$'s and $\hat{\boldsymbol{\mu}}_{t,EWMA}$'s characteristic decay time are $-1/\ln\alpha_2$ and $-1/\ln 0.94 \cong 16.2$ trading days, respectively (see **Appendix 2.A Risk-return profile of the examined currencies**).

Now, consider μ_p and σ_p^2. Regard an investment portfolio of N risky financial assets. Let $\mathbf{w} = (w_1, \ldots, w_N)'$ be a column vector of those N risky assets' weights allocated in the portfolio where $\mathbf{w}'\mathbf{1}_N = 1$ ($\mathbf{1}_N$: $N \times 1$ $(1, \ldots, 1)'$) ($\mathbf{w}'\mathbf{1}_N \neq 1$ possible when risk-free assets also taken into account later). μ_p and σ_p^2 are as (2.3) and (2.4), respectively.

$$\mu_p = \mathbf{w}'\boldsymbol{\mu} \tag{2.3}$$

where the estimator of $\boldsymbol{\mu}$ is $\hat{\boldsymbol{\mu}} \in \{\hat{\boldsymbol{\mu}}_{uncond}, \hat{\boldsymbol{\mu}}_{t,GARCH}, \hat{\boldsymbol{\mu}}_{t,EWMA}\}$, which is improved so as to be a shrinkage estimator, denoted by $\hat{\boldsymbol{\mu}}_{shrunk}$ (see (2.5) and (2.7) in section **2.3.2 Mean and covariance shrinkage estimation**) so as to reduce the estimation risk.

$$\sigma_p^2 = \mathbf{w}'\mathbf{V}\mathbf{w} = \mathbf{w}'[\mathrm{diag}(\mathbf{V})]^{\frac{1}{2}}\mathbf{P}[\mathrm{diag}(\mathbf{V})]^{\frac{1}{2}}\mathbf{w} \tag{2.4}$$

where the estimator of $\mathbf{V}$ is $\hat{\mathbf{V}} \in \{\hat{\mathbf{V}}_{uncond}, \hat{\mathbf{V}}_{t,GARCH}, \hat{\mathbf{V}}_{t,EWMA}\}$ ($\mathrm{diag}(\mathbf{V})$: $N \times N$ diagonal matrix of $\mathbf{V}$), which is improved so as to be a shrinkage estimator denoted by $\hat{\mathbf{V}}_{shrunk}$ (see (2.6), (2.8), and (2.9) in section **2.3.2 Mean and covariance shrinkage estimation**) to reduce the estimation error. The estimator of $\mathbf{P}$ is $\hat{\mathbf{P}} \in \{\hat{\mathbf{P}}_{uncond}, \hat{\mathbf{P}}_{t,GARCH}, \hat{\mathbf{P}}_{t,EWMA}\}$. $\hat{\mathbf{P}}$ is the estimator of the true multivariate-correlation matrix $\mathbf{P}$ and can be calculated unconditionally or conditionally (on time) (see section **1.3 Unconditional and dynamic conditional correlations** and **Appendix 2.B Correlation matrices and heat maps without heteroskedastic correction**). $\hat{\mathbf{P}}$ can also be calculated either without or with relevant interest rates partialled out, as explained in section **Controlling for the effects of relevant interest rates**, under section **1.4.4 Hypothesis tests**.

From σ_p^2 (2.4) it can also be observed that investing in a portfolio with more than a few assets could provide diversification benefits over only holding one or a few assets. The benefits are such that σ_p^2 reduces due to negative covariances (in the matrix $\mathbf{V}$) or correlations below one (in the matrix $\mathbf{P}$).[4]

Now, apply this intuition with currency contagions and flights due to political uncertainty. Diversification could be achieved if a currency portfolio was allocated such that comparatively less was invested in currencies for which financial contagion had been detected and more in currencies for which a flight to quality had been detected, leading to a lower σ_p^2 (see **2.2.1 Contagion and flight definitions**). Nonetheless, portfolio analysis of financial contagion and capital flight incorporates multivariate instead of the bivariate correlations used in the previous chapter's correlation and interdependence analyses. That is, not only correlations between the shock-originating and shock-receiving economies but also those among the shock-receiving economies are taken into account.[5]

In portfolio selection, risky portfolios chosen with minimum σ_p^2 at any level of μ_p or maximum μ_p at any level of σ_p^2 are considered efficient or optimal mean-variance portfolios. The set of all these optimal mean-variance portfolios is known as the Markowitz efficient set, and lies entirely on the ***Markowitz efficient frontier***.[a]

On the Markowitz efficient frontier, the portfolio with the smallest risk is the ***global minimum variance portfolio***, obtained by minimizing the portfolio variance, while the portfolio with the highest reward for risk, given a certain risk-free, rate is the ***tangency portfolio***, obtained by maximizing the portfolio's reward-to-variability ratio.[a]

In this paper, currency contagions and flights based on investment portfolios will accordingly be analyzed on the basis of ***risk minimization*** and ***reward-to-risk maximization***. However, before discussing those two analytical portfolio optimization schemes, the sample shrinkage used to deal with $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{V}}$ estimation errors is presented.

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[4] According to the central limit theorem, σ_p^2 approaches zero when the number of assets in the portfolio approaches infinity (Fabozzi et al., 2007).

[5] In the notation of this dissertation, not only the correlations between the $r_{x,t}$ and ys but also those among the $r_{y,t}$s (using the same notation for economies: x as shock-originating and y as shock-receiving, as in **Chapter 1**).

2.3.2 Mean and covariance shrinkage estimations

Due to sample estimation errors, the use of the sample mean vector and covariance matrix can lead to issues such as high sensitivity to the plugged-in parameters, poor out-of-sample performance, etc., in the later portfolio optimizations. This would imply that the chosen portfolios were probably sub-optimal (Adcock, 2015; Fabozzi et al., 2007). One of the widely implemented methodologies for reducing estimation risk and further enhancing portfolio optimization robustness is sample shrinkage. This simply means shrinking the amount of information drawn from the sample and instead incorporating some drawn from a particular benchmark (i.e. weighting between the sample and the benchmark). The approach can be conducted for both the expected return and variance-covariance estimators. Properly shrunk estimators could diminish the sample estimation variability. This could further lead to more stability and better forecasting of mean-variance optimizations (Brandt, 2009; Fabozzi et al., 2007; James & Stein, 1961; Jorion, 1986; Ledoit & Wolf, 2004).

In general, the sample mean and covariance shrinkage estimations can be written as in (2.5) and (2.6), respectively:

$$\widehat{\boldsymbol{\mu}}_{shrunk} = \left(1 - \theta_{\mu}\right)\widehat{\boldsymbol{\mu}} + \theta_{\mu}\overline{\boldsymbol{\mu}}\mathbf{1}_N \tag{2.5}$$

$$\widehat{\mathbf{V}}_{shrunk} = (1 - \theta_{\mathrm{V}})\widehat{\mathbf{V}} + \theta_{\mathrm{V}}\overline{\mathbf{V}}\mathbf{1}_N \tag{2.6}$$

The estimators, $\widehat{\boldsymbol{\mu}}$ and $\widehat{\mathbf{V}}$, are the sample mean and covariance matrices, respectively (also see (2.3) and (2.4)). $\overline{\boldsymbol{\mu}}$ and $\overline{\mathbf{V}}$ are the benchmarked or targeted mean and covariance matrices, respectively, which are supposedly close to their population matrices ($\boldsymbol{\mu}$ and $\mathbf{V}$, respectively).[6] θ_{μ} and θ_{V} are the expected return and covariance shrinkage intensities (i.e. the degree to which the benchmark is weighted), respectively, and are specific to each analysis, being chosen so as to reduce sample estimation errors.

In this chapter, the Bayes-Stein $\widehat{\boldsymbol{\mu}}_{shrunk}$ proposed by Jorion (1986) (denoted by $\widehat{\boldsymbol{\mu}}_{shrunk,BS}$) and the constant-correlation $\widehat{\mathbf{V}}_{shrunk}$ proposed by Ledoit and Wolf (2004) (denoted by $\widehat{\mathbf{V}}_{shrunk,LW}$) are adopted. The corresponding shrinkage intensities, $\theta_{\mu,BS}$ and $\theta_{\mathrm{V},LW}$, are discussed next.

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[6] In **Chapter 1**, the $\overline{\mathbf{V}}$ in $\widehat{\mathbf{V}}_{t,GARCH}$ (1.3) is specifically $\widehat{\mathbf{V}}_{uncond}$.

In 1986, Jorion proposed a sample mean shrinkage method by applying the empirical Bayes approach to extend the James-Stein shrinkage estimation (which involved lowering the loss function of the latter to estimate $\boldsymbol{\mu}$)[7] and Stein's (1956) results on sample mean estimation inadmissibility. The Bayesian framework solves the issue of the sample mean inappropriately corresponding to an uninformative prior. The mean benchmark ($\bar{\boldsymbol{\mu}}$ in (2.5)) is aimed at the GMV portfolio expected return[8] ($\bar{\boldsymbol{\mu}} = \bar{\boldsymbol{\mu}}_{GMV} = \widehat{\mathbf{w}}'_{GMV}\widehat{\boldsymbol{\mu}}$, see (2.10) and **2.4 Risk minimization**)). Jorion's (1986) Bayes-Stein (BS) mean shrinkage intensity $\theta_{\mu,BS}$ is correspondingly acquired as in (2.7) with shrinkage estimation $\widehat{\boldsymbol{\mu}}_{shrunk,BS} = \left(1 - \theta_{\mu,BS}\right)\widehat{\boldsymbol{\mu}} + \theta_{\mu,BS}\bar{\boldsymbol{\mu}}_{GMV}\mathbf{1}_N$ (also see (2.5)):

$$\theta_{\mu,BS} = \frac{N+2}{N+2+\tau(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}_{GMV}\mathbf{1}_N)'\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}_{GMV}\mathbf{1}_N)} \qquad (2.7)^{9,10}$$

Empirically, the Bayes-Stein mean shrinkage estimation with $\bar{\boldsymbol{\mu}} = \bar{\boldsymbol{\mu}}_{GMV}$, perhaps the most recognized in the financial literature, demonstrates substantial superiority over the sample estimation. It significantly mitigates parameter uncertainty and over-variability in portfolio rebalancing and also improves forecasts (Jorion, 1986; Fabozzi et al., 2007).

In this paper, $\widehat{\boldsymbol{\mu}}_{shrunk,BS}$ ((2.5) and (2.7)) will simply replace $\widehat{\boldsymbol{\mu}}$ in all the portfolio analyses and hypothesis tests.

In 2004, Ledoit and Wolf shrank the sample covariance matrix using the constant correlation matrix (denoted by $\bar{\mathbf{P}}$) (all the correlation coefficients, except for the diagonal coefficients that are ones, are the same constant value, $\bar{\rho}$, averaged across the entire sample,

[7] James and Stein's (1961) (as well as Stein's (1956)) quadratic loss function (L) is applied to estimate $\boldsymbol{\mu}$ (the true expected return column vector, asymptotically computed in L) with $L(\boldsymbol{\mu}, \widehat{\boldsymbol{\mu}}) = (\boldsymbol{\mu} - \widehat{\boldsymbol{\mu}})'\widehat{\mathbf{V}}^{-1}(\boldsymbol{\mu} - \widehat{\boldsymbol{\mu}})$ (while $L(\cdot)$ in **Chapter 1** denotes a log likelihood function). The ***James-Stein*** (JS) mean shrinkage intensity $\theta_{\mu,JS} = \min\left(1, \frac{N-2}{\tau(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}\mathbf{1}_N)'\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}\mathbf{1}_N)}\right)$ is then acquired such that the shrinkage estimation $\widehat{\boldsymbol{\mu}}_{shrunk,JS} = \left(1 - \theta_{\mu,JS}\right)\widehat{\boldsymbol{\mu}} + \theta_{\mu,JS}\bar{\boldsymbol{\mu}}\mathbf{1}_N$ produces lower loss than does the sample (i.e. $L(\boldsymbol{\mu}, \widehat{\boldsymbol{\mu}}_{shrunk}) < L(\boldsymbol{\mu}, \widehat{\boldsymbol{\mu}})$) (Jorion, 1986). Also, $\bar{\boldsymbol{\mu}} = \bar{\boldsymbol{\mu}}_{avg} = (1/N)\sum_{i=1}^{N} E\left[r_{i,t}\right]$ (i.e. averaging across the entire sample) is widely practiced although it can even be arbitrary, explain Fabozzi et al. (2007).

[8] Targeting $\boldsymbol{\mu}$ at $\bar{\boldsymbol{\mu}}_{GMV} = \widehat{\mathbf{w}}'_{GMV}\widehat{\boldsymbol{\mu}}$ could also be explained by the fact that the error in sample mean estimation is typically much larger than that in sample covariance estimation. Thus, the expected return on the GMV portfolio, which is dominantly drawn from $\widehat{\mathbf{V}}$, could to a certain extent be considered a relatively stable benchmark of the portfolio expected return.

[9] $\theta_{\mu,BS}$ is generalized as $\frac{b}{d+\tau(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}\mathbf{1}_N)'\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}}-\bar{\boldsymbol{\mu}}\mathbf{1}_N)}$, $0 \leq b \leq 2(N-2)$ with some weak conditions on d (Jorion, 1986).

[10] Jorion (1986) also adjusts the df of $\widehat{\mathbf{V}}_{uncond}$ from $\tau - 1$ to $\tau - N - 2$, which is also discussed in Brandt (2009) and was mentioned earlier in **2.3.1 Risk-return tradeoff and portfolio mean and covariance**.

$\overline{\rho} = 2/\left(N(N-1)\right)\sum_{i,j=1,i\neq j}^{N} p_{i,j}$).[11] The targeted covariance matrix ($\overline{\mathbf{V}}$ in (2.6)) is then drawn from $\overline{\mathbf{P}}$ as in (2.8). By attempting to minimize a quadratic loss when using shrinkage estimation ($\min\left\|\widehat{\mathbf{V}}_{shrunk} - \mathbf{V}\right\|^2$ with asymptotic $\mathbf{V}$), the Ledoit-Wolf (LW) covariance shrinkage intensity ($\theta_{\mathrm{V},LW}$) is then determined as in (2.9) with shrinkage estimation $\widehat{\mathbf{V}}_{shrunk,LW} = \left(1 - \theta_{\mathrm{V},LW}\right)\widehat{\mathbf{V}} + \theta_{\mathrm{V},LW}\overline{\mathbf{V}}\mathbf{1}_N$ (also see (2.6)).

$$\overline{\mathbf{V}} = \left[\mathrm{diag}\left(\widehat{\mathbf{V}}\right)\right]^{\frac{1}{2}}\overline{\mathbf{P}}\left[\mathrm{diag}\left(\widehat{\mathbf{V}}\right)\right]^{\frac{1}{2}} \tag{2.8}$$

$$\theta_{\mathrm{V},LW} = \max\left\{0, \min\left(1, \frac{\hat{\kappa}}{\tau}\right)\right\} \tag{2.9}$$

$\hat{\kappa}$ represents the difference between the asymptotic summation of variances and that of covariances scaled by the shrinkage target misspecification (Ledoit & Wolf, 2004).[12]

Experimentally, the Ledoit-Wolf covariance shrinkage estimation improves portfolio optimization performance (compared to the sample, Fama-French factor model, etc.) similarly to Sharpe's single-factor method but much more convenient to implement (Fabozzi et al., 2007).

In this paper, $\widehat{\mathbf{V}}_{shrunk,LW}$ ((2.6), (2.8), and (2.9)) will simply substitute for $\widehat{\mathbf{V}}$ in all the portfolio analyses and hypothesis tests.

2.4 Risk minimization

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[11] $\overline{\mathbf{P}} = \begin{bmatrix} 1 & \overline{\rho} & \dots & \overline{\rho} \\ \overline{\rho} & 1 & \ddots & \vdots \\ \vdots & \ddots & 1 & \overline{\rho} \\ \overline{\rho} & \dots & \overline{\rho} & 1 \end{bmatrix}, \overline{\rho} = \frac{\sum_{i,j=1}^{N} p_{i,j}}{N(N-1)/2}, i \neq j$

[12] $\hat{\kappa} = \frac{\hat{\pi} - \hat{c}}{\hat{\gamma}}$ where

$\hat{\pi} = \sum_{i,j=1}^{N}\left\{E\left[\left(r_{i,t} - \bar{r}_i\right)\left(r_{j,t} - \bar{r}_j\right) - \hat{\sigma}_{i,j}\right]^2\right\}$ ($E(\cdot)$: the expected value over a time span),

$\hat{c} = \sum_{i}^{N} E\left[\left(r_{i,t} - \bar{r}_i\right)^2 - \hat{\sigma}_i^2\right]^2 + \sum_{i,j=1,,i\neq j}^{N}\frac{\overline{\rho}}{2}\left(\frac{\hat{\sigma}_j}{\hat{\sigma}_i}\hat{\vartheta}_{ii,ij} + \frac{\hat{\sigma}_i}{\hat{\sigma}_j}\hat{\vartheta}_{jj,ij}\right)$ (see $\overline{\rho}$ in Footnote 11),

$\hat{\gamma} = \sum_{i,j=1}^{N}\left(\overline{\rho}\hat{\sigma}_i\hat{\sigma}_j - \hat{\sigma}_{i,j}\right)^2$,

$\bar{r}_i = E\left[r_{i,t}\right]$, $\hat{\sigma}_{i,j} = E\left[\left(r_{i,t} - \bar{r}_i\right)\left(r_{j,t} - \bar{r}_j\right)\right]$,

$\hat{\vartheta}_{ii,ij} = E\left\{\left[\left(r_{i,t} - \bar{r}_i\right)^2 - \hat{\sigma}_i^2\right]\left[\left(r_{i,t} - \bar{r}_i\right)\left(r_{j,t} - \bar{r}_j\right) - \hat{\sigma}_{i,j}\right]\right\}$, and

$\hat{\vartheta}_{jj,ij} = E\left\{\left[\left(r_{j,t} - \bar{r}_j\right)^2 - \hat{\sigma}_j^2\right]\left[\left(r_{i,t} - \bar{r}_i\right)\left(r_{j,t} - \bar{r}_j\right) - \hat{\sigma}_{i,j}\right]\right\}$.

2.4.1 Discussion

On the Markowitz or risky-asset efficient frontier (the risk-free rate not yet involved), optimality depends on individual risk acceptability. The less risk accepting an investor is (i.e. the steeper is their IC on the return-volatility plane), the closer will be the optimal choice to the GMV portfolio (i.e. the point GMV in Endnote a's figure). Thus, the GMV portfolio is located in the place where risk aversion (denoted by γ, $\gamma \geq 0$ for risk-averse persons)[13] is as high as possible on the frontier (Bodie et al., 2014). More specifically, the optimum tends to be the GMV portfolio if risk aversion approaches infinity ($\gamma \to \infty$).[14]

By minimizing the portfolio risk σ_p^2 (2.4), the analytical solution of the GMV portfolio's allocation ($\mathbf{w}_{GMV}$) can be written as follows (Fabozzi et al., 2007) ($\mathbf{w}_{\underline{\sigma_p^2}}$: the minimum risk portfolio allocation subject to (s.t.) certain constraint(s), $\mathbf{w}^*_{\underline{\sigma_p^2}}$: with the only constraint being $\mathbf{w}'\mathbf{1}_N = 1$, $\underline{\sigma_p^2}$: its variance):

$$\begin{gathered}\min_{\mathbf{w}} \mathbf{w}'\mathbf{V}\mathbf{w} \text{ s.t. } \mathbf{w}'\mathbf{1}_N = 1,\\ \mathbf{w}^*_{\underline{\sigma_p^2}} = \mathbf{w}_{GMV} = \frac{\mathbf{V}^{-1}\mathbf{1}_N}{\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N},\ \mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N \neq 0\end{gathered} \tag{2.10}$$ [15]

where the estimator of $\mathbf{w}_{GMV}$ is $\widehat{\mathbf{w}}_{GMV} = \widehat{\mathbf{V}}^{-1}\mathbf{1}_N/\left(\mathbf{1}'_N\widehat{\mathbf{V}}^{-1}\mathbf{1}_N\right)$, $\mathbf{1}'_N\widehat{\mathbf{V}}^{-1}\mathbf{1}_N \neq 0$.

In the classical scheme, it can be seen that weights are unbounded and short sales are allowed, i.e. the asset weights are not restricted to be non-negative ($\mathbf{w} \geq \mathbf{0}$ not imposed). In this chapter, a currency portfolio is considered. Short selling in the foreign exchange market is distinctive from doing so in other asset markets. It is not necessary to borrow outside the market, such as in money markets, since selling one currency implicitly means buying the paired one. Regarding foreign exchange rates with USD as the base currency, being short in

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[13] γ, denoting risk aversion degree here in Chapter 2, is not the same as that is used in Chapter 1's Footnote 55 to help explain partialling out interest rate effects (Chapter 1's section **Controlling for the effects of relevant interest rates**).

[14] On the Markowitz efficient frontier, by maximizing a utility function $\mu_p - 0.5\gamma\sigma_p^2 = \mathbf{w}'\boldsymbol{\mu} - 0.5\gamma\mathbf{w}'\mathbf{V}\mathbf{w}$, the analytical solution is obtained as $\mathbf{V}^{-1}\mathbf{1}_N/(\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N) + \gamma^{-1}\mathbf{Q}\boldsymbol{\mu}$ where $\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N \neq 0$ and $\mathbf{Q} = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{1}_N\mathbf{1}'_N\mathbf{V}^{-1}/(\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N)$ (Okhrin & Schmid, 2006). Thus, when γ approaches ∞, the optimal portfolio allocation becomes $\mathbf{V}^{-1}\mathbf{1}_N/(\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N)$, which is $\mathbf{w}_{GMV}$ as in (2.10).

[15] First-order condition (FOC): $\frac{\partial}{\partial \mathbf{w}}\mathbf{w}'\mathbf{V}\mathbf{w} = 2\mathbf{w}'\mathbf{V} = \mathbf{0} \Leftrightarrow \mathbf{w}^* = \mathbf{V}^{-1}\mathbf{1}_N$ (where $\mathbf{w}'\mathbf{1}_N = 1$ is not yet considered in this step). Since $\mathbf{w}'\mathbf{1}_N = 1$, then $\mathbf{w}^*_{\underline{\sigma_p^2}} = \mathbf{V}^{-1}\mathbf{1}_N/(\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N)$, $\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N \neq 0$, where $\mathbf{1}'_N\mathbf{V}^{-1}\mathbf{1}_N$ is the sum of all the individual elements of $\mathbf{V}^{-1}\mathbf{1}_N$.
Second-order condition (SOC): $\frac{\partial^2}{\partial \mathbf{w}^2}\mathbf{w}'\mathbf{V}\mathbf{w} = 2V > \mathbf{0}$ (since $\mathbf{V} > \mathbf{0}$), i.e. positive definiteness, satisfying minimization.
In the context of matrices, $\mathbf{0}$ represents a matrix of all zeros with the corresponding size.

a currency (e.g. GBP, EUR, CHF, etc.) always implicitly means being long in USD, and vice versa. Thus, short positions in the currency market are more common across economies than in other financial markets, such as stock and bond.

Okhrin and Schmid (2006) also demonstrate that the estimator, $\widehat{\mathbf{w}}_{GMV} = \widehat{\mathbf{V}}^{-1}\mathbf{1}_N / \left(\mathbf{1}_N'\widehat{\mathbf{V}}^{-1}\mathbf{1}_N\right)$, could be considered an unbiased estimator for the true analytical solution, $\mathbf{w}_{GMV}$ (2.10). In supporting the claim on $\widehat{\mathbf{w}}_{GMV}$'s unbiasedness, it is generally recognized that the estimation error in a sample covariance matrix is considerably smaller, about 10 times so, than that in a sample mean vector. In addition, the estimation error in sample covariances is approximately half the error in sample variances (Fabozzi et al., 2007). Besides this, $\widehat{\mathbf{V}}_{shrunk,LW}$ ((2.6), (2.8), and (2.9)) from **2.3.2 Mean and covariance shrinkage estimations** is also applied to help increase the robustness of the portfolio optimization. Hence, the estimator $\widehat{\mathbf{w}}_{GMV}$ is implementable.

2.4.2 Analytical schemes

In examining minimum risk portfolio reallocations when moving from a state of tranquility (T) to a state of distress (D), specifically due to the Brexit referendum in this chapter, there are two schemes: (1) ***risk aversion maintenance*** and (2) ***risk level maintenance***.

Risk aversion maintenance

Consider (1) ***risk aversion maintenance***. Assume that the minimum risk portfolio holder is fully risk-averse (i.e. $\gamma \to \infty$) such that he or she always seeks the GMV portfolio. Hence, a relatively unbiased $\widehat{\mathbf{w}}_{GMV}$ will accordingly represent $\widehat{\mathbf{w}}_{\underline{\sigma_p^2}}$ over both states of tranquility (T) and distress (D) as follows:

$\gamma_T \cong \gamma_D \to \infty$

$\widehat{\mathbf{w}}_{GMV|T}$ v. $\widehat{\mathbf{w}}_{GMV|D}$

tranquility | distress

Figure 2.2: $\widehat{\boldsymbol{w}}_{\underline{\sigma_p^2}}$ reallocation over time with risk aversion maintenance

$$\begin{aligned} \widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T} &= \widehat{\mathbf{w}}_{GMV|T} = \frac{\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}{\mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}, \text{ then} \\ \widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D} &= \widehat{\mathbf{w}}_{GMV|D} = \frac{\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N}{\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N}. \end{aligned} \tag{2.11}$$

If $\widehat{\mathbf{V}}$ is not invertible, following Bodnar et al. (2017), the Moore-Penrose pseudoinverse is applied (for both risk minimization's and later reward-to-risk maximization's case studies).[16]

Risk level maintenance

Consider (2) ***risk level maintenance***. Suppose that the minimum risk portfolio's risk level is to be maintained over time (i.e. $\underline{\hat{\sigma}_p^2}|T = \underline{\hat{\sigma}_p^2}|D$). To ascertain the portfolio efficiency (i.e. remain on the Markowitz efficient frontier), $\hat{\mu}_{p|D}$ must also be maximized. **Figure 2.3** shows possible ways of maintaining the portfolio risk level when short sales are allowed so that the Markowitz efficient frontier in distress (D) is likely made wider (widened by short positions in assets with negative returns) but achieves the GMV at a lower μ_p that is also negative. This shows how the diversification among assets adjusts to give the same degree of risk. Thus, the portfolio risk minimizations in tranquility (T) and then in distress (D) become

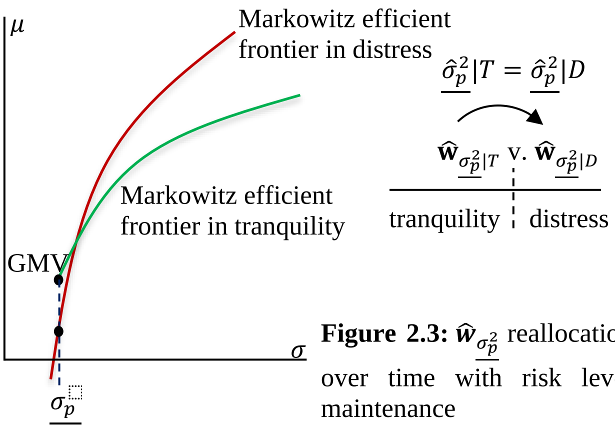

Figure 2.3: $\widehat{\boldsymbol{w}}_{\underline{\sigma_p^2}}$ reallocation over time with risk level maintenance

$$\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T} = \widehat{\mathbf{w}}_{GMV|T} = \frac{\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}{\mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}, \text{ then} \qquad (2.12)^{17,18}$$

[16] $\widehat{\mathbf{V}}$ not invertible appears occasionally in the empirical work for this chapter's portfolio-optimization case studies (**2.4.2 Analytical schemes** and **2.5.2 Analytical schemes**). With linearly independent columns, the left pseudoinverse of $\widehat{\mathbf{V}}$, $(\widehat{\mathbf{V}}'\widehat{\mathbf{V}})^{-1}\widehat{\mathbf{V}}'$, is applied. With linearly independent rows, the right pseudoinverse of $\widehat{\mathbf{V}}$, $\widehat{\mathbf{V}}'(\widehat{\mathbf{V}}'\widehat{\mathbf{V}})^{-1}$, is applied.

[17] If the second expression in (2.12) is not feasible, which can be happening sometimes when correlations and volatilities are time-conditional (GARCH(1,1) or EWMA), i.e. $\underline{\hat{\sigma}_p^2}|D = \widehat{\mathbf{w}}'_{\underline{\sigma_p^2}|D}\widehat{\mathbf{V}}_D\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D}$ cannot be as low as $\underline{\hat{\sigma}_p^2}|T = \widehat{\mathbf{w}}'_{\underline{\sigma_p^2}|T}\widehat{\mathbf{V}}_T\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T}$, the optimization will be re-implemented in reverse instead, as $\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D} = \widehat{\mathbf{w}}_{GMV|D} = \widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N/(\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N)$, then $\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T}$: $\arg\max_{\widehat{\mathbf{w}}} \widehat{\mathbf{w}}'\widehat{\boldsymbol{\mu}}_T$ s.t. $\widehat{\mathbf{w}}'\widehat{\mathbf{V}}_T\widehat{\mathbf{w}} = \underline{\hat{\sigma}_p^2}|D$, $\widehat{\mathbf{w}}'\mathbf{1}_N = 1$.

[18] The numerical tool implemented to perform constrained portfolio optimizations in this paper is Excel Solver with the GRG Nonlinear method, which is used for smooth nonlinear problems. This is also in line with portfolio optimizations, which are nonlinear and quadratic. The algorithm of GRG perceives the slope or gradient of the objective function as changes in the input or decision variables, and determines that the objective has optimized the outcome when the partial derivatives are zero. Nonetheless, the resolution attained from this approach greatly relies on the initial values and might not be globally but instead locally optimal, since the procedure will most likely stop at the local optimum closest to the initial values. In the experiments in this chapter, all the examined currency portfolio optimizations start with the initial values of $1/N$ equal weights or the analytical-solution weights (of the GMV or tangency portfolio).

$\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D}$: $\arg\max_{\widehat{\mathbf{w}}} \widehat{\mathbf{w}}'\widehat{\boldsymbol{\mu}}_D$ s.t. $\widehat{\mathbf{w}}'\widehat{\mathbf{V}}_D\widehat{\mathbf{w}} = \underline{\hat{\sigma}_p^2}|T$, $\widehat{\mathbf{w}}'\mathbf{1}_N = 1$.

For both risk aversion maintenance and risk level maintenance, both the structural ($\widehat{\mathbf{w}}_{\underline{\sigma_p^2}}$) and single-asset ($\widehat{w}_{\underline{\sigma_p^2},i}$) reallocations will be tested by applying the derived tests on GMV portfolios.

2.4.3 Hypothesis tests

With the assumption of normally distributed returns, $\mathbf{R}_t \sim N(\boldsymbol{\mu}, \mathbf{V})$, the GMV portfolio estimator, $\widehat{\mathbf{w}}_{GMV} = \widehat{\mathbf{V}}^{-1}\mathbf{1}_N/(\mathbf{1}_N'\widehat{\mathbf{V}}^{-1}\mathbf{1}_N)$ (2.10), follows a t-distribution with $\tau - N + 1$ degrees of freedom (df) (τ: sample time span, N: number of assets) and has the following mean vector and covariance matrix, respectively (Bodnar, 2009; Bodnar & Schmid, 2007; Okhrin & Schmid, 2006):

$$E(\widehat{\mathbf{w}}_{GMV}) = \mathbf{w}_{GMV} \text{ and } cov(\widehat{\mathbf{w}}_{GMV}) = \frac{1}{\tau-N-1}\frac{\mathbf{Q}}{\mathbf{1}_N'\mathbf{V}^{-1}\mathbf{1}_N}, \mathbf{Q} = \mathbf{V}^{-1} - \frac{\mathbf{V}^{-1}\mathbf{1}_N\mathbf{1}_N'\mathbf{V}^{-1}}{\mathbf{1}_N'\mathbf{V}^{-1}\mathbf{1}_N}.$$

Accordingly, the corresponding hypotheses (H_0 and H_1) and two-sided test statistic (denoted by $T_{\widehat{\mathbf{w}}_{\underline{\sigma_p^2}}}$), respectively, testing whether a minimum risk portfolio needs structural reallocation in the move from a state of tranquility (T) to a state of distress (D) (tranquility time span: τ_T, distress time span: τ_D), can be applied and expressed as follows (applying the GMV portfolio tests in Bodnar and Schmid (2007) and in Bodnar (2009) as if $\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D}$ deviates from the ***benchmark*** $\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T}$):

$$H_0: \mathbf{w}_{\underline{\sigma_p^2}|T} = \mathbf{w}_{\underline{\sigma_p^2}|D}$$
$$H_1: \mathbf{w}_{\underline{\sigma_p^2}|T} \neq \mathbf{w}_{\underline{\sigma_p^2}|D}$$

$$T_{\widehat{\mathbf{w}}_{\underline{\sigma_p^2}}} = \frac{\tau_D - N}{N-1}\left(\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N\right)\left(\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D} - \widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T}\right)'\widehat{\mathbf{Q}}_D^{-1}\left(\widehat{\mathbf{w}}_{\underline{\sigma_p^2}|D} - \widehat{\mathbf{w}}_{\underline{\sigma_p^2}|T}\right)$$

where $T_{\widehat{\mathbf{w}}_{\underline{\sigma_p^2}}} \sim F_{N-1,\tau_D-N}$ and $\widehat{\mathbf{Q}}_D = \widehat{\mathbf{V}}_D^{-1} - \widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}/(\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N)$.

It follows that, if $T_{\hat{\mathbf{w}}_{\underline{\sigma_p^2}}} \leq F_{N-1,\tau_D-N}$, H_0 is not rejected, i.e. a structural change in the GMV portfolio is not required. If $T_{\hat{\mathbf{w}}_{\underline{\sigma_p^2}}} > F_{N-1,\tau_D-N}$, H_0 is rejected, i.e. a change is required.

Then, given that the GMV portfolio structurally alters when moving from T to D (i.e. $\hat{\mathbf{w}}_{\underline{\sigma_p^2}|T} \neq \hat{\mathbf{w}}_{\underline{\sigma_p^2}|D}$), the individual weight ($\hat{w}_{\underline{\sigma_p^2},i} \in \hat{\mathbf{w}}_{\underline{\sigma_p^2}}$) reallocation that occurs can be further analyzed. The hypotheses (H_0 and H_1) and one-sided test statistic (denoted by $T_{\hat{w}_{\underline{\sigma_p^2},i}}$), respectively, can be written as follows (applying Bodnar and Schmid (2007) and Bodnar (2009) as if $\hat{w}_{\underline{\sigma_p^2},i|D}$ deviates from the ***benchmark*** $\hat{w}_{\underline{\sigma_p^2},i|T}$) (also see interdependence, contagion, and flight definitions in **Table 2.1**, in which the Brexit case, in specific, results in the cases of negative contagion and flight to quality):

$$H_0\colon w_{\underline{\sigma_p^2},i|T} = w_{\underline{\sigma_p^2},i|D}, \text{ i.e. interdependence}$$
$$H_1\colon w_{\underline{\sigma_p^2},i|T} \neq w_{\underline{\sigma_p^2},i|D},$$
$$\begin{cases} w_{\underline{\sigma_p^2},i|T} > w_{\underline{\sigma_p^2},i|D}, \text{i. e. negative contagion (or flight from quality)} \\ w_{\underline{\sigma_p^2},i|T} < w_{\underline{\sigma_p^2},i|D}, \text{i. e. flight to quality (or positive contagion)} \end{cases}$$

$$T_{\hat{w}_{\underline{\sigma_p^2},i}} = \sqrt{\tau_D - N}\,\frac{(\mathbf{1}_N'\hat{\mathbf{V}}_D^{-1}\mathbf{1}_N)\left(\hat{w}_{\underline{\sigma_p^2},i|D} - \hat{w}_{\underline{\sigma_p^2},i|T}\right)}{\sqrt{(\mathbf{1}_N'\hat{\mathbf{V}}_D^{-1}\mathbf{1}_N)\hat{\sigma}_{11|D}^{(-)} - \left(\sum_{j=1}^{N}\hat{\sigma}_{1j|D}^{(-)}\right)^2}}$$

where $T_{\hat{w}_{\underline{\sigma_p^2},i}} \sim t_{\tau_D-N}$ and $\hat{\sigma}_{ij|D}^{(-)} \in \hat{\mathbf{V}}_D^{-1}$.

It follows that, if $\left|T_{\hat{w}_{\underline{\sigma_p^2},i}}\right| \leq \left|t_{\tau_D-N}\right|$, H_0 is not rejected. If $\left|T_{\hat{w}_{\underline{\sigma_p^2},i}}\right| > \left|t_{\tau_D-N}\right|$, H_0 is rejected. In addition, $T_{\hat{w}_{\underline{\sigma_p^2},i}} > t_{\tau_D-N} > 0$ (since $\hat{w}_{\underline{\sigma_p^2},i|D} > \hat{w}_{GMV,i|T}$) signifies a negative contagion (for a negative incident, such as Brexit) (but a flight from quality for a positive incident) while $T_{\hat{w}_{\underline{\sigma_p^2},i}} < t_{\tau_D-N} < 0$ (since $\hat{w}_{\underline{\sigma_p^2},i|D} < \hat{w}_{GMV,i|T}$) indicates a flight to quality (but a positive contagion for a positive incident).

2.5 Reward-to-risk maximization

2.5.1 Discussion

When riskless assets, such as cash, a sovereign bond issued by a country with high credit rating, or a highly rated money-market instrument, come into play combined with a Markowitz efficient risky portfolio, efficient combined portfolios are then any combination on the best possible capital allocation line (CAL).[19] On the return-volatility plane, the best possible CAL is the line from the risk-free rate (r_f) tangent to the mean-variance efficient frontier (see Endnote a). The best possible CAL's slope is the Markowitz efficient frontier's highest possible Sharpe ratio (SR), which is the reward to variability or excess return to risk ratio, that is, the maximum $SR_p = \left(\mu_p - r_f\right)/\sigma_p$, which is equal to the SR of an efficient riskless-risky combination (co), $SR_{co} = \left(\mu_{co} - r_f\right)/\sigma_{co}$ (Bodie et al., 2014; Lintner, 1965; Sharpe, 1964, 1966; Tobin, 1958).

Similar to a risky portfolio on the Markowitz efficient frontier, a combined riskless-risky portfolio on the best possible CAL is optimized conditional on an individual risk preference, measured by the degree of risk aversion (γ).[20] All efficient riskless-risky portfolios are at least as preferable as all efficient risky portfolios. This is due to their better risk-averse IC, except when they have the same IC at the tangency portfolio located where the best possible CAL is tangent to the Markowitz efficient frontier (i.e. the point tan_f, letting $r_f = r_b$, in Endnote a's figure).[21] Hence, incorporating the opportunity cost on the risk-free asset is at least as good as not doing so (Bodie et al., 2014; Fabozzi et al., 2007; Lintner, 1965; Sharpe, 1964).

By maximizing $SR_p = \left(\mu_p - r_f\right)/\sigma_p$, the analytical solution of the tangency portfolio's allocation ($\mathbf{w}_{tan}$) can be written as follows (Fabozzi et al., 2007) ($\mathbf{w}_{SR}$: the maximum-SR portfolio allocation s.t. certain constraint(s), $\mathbf{w}_{SR}^*$: with the only constraint $\mathbf{w}'\mathbf{1}_N = 1$):

$$\max_{\mathbf{w}} \frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}'\mathbf{V}\mathbf{w}}} \text{ s.t. } \mathbf{w}'\mathbf{1}_N = 1,$$

[19] The best possible CAL is optimal and becomes the ***capital market line*** (CML) when the Markowitz efficient frontier represents the entire market. Correspondingly, the tangency portfolio becomes known as the ***market portfolio***. Roughly speaking, the terms 'tangency portfolio' and 'market portfolio' could be used interchangeably.

[20] The less risk-averse an investor is (i.e. the flatter is their IC on the return-volatility plane), the less the portfolio is optimally allocated to the risk-free and the more to the risky side. When only risky securities are invested in, the combination becomes the tangency portfolio (i.e., in Endnote a's figure, anywhere between the points tan_f and tan_b on the best possible CAL if $r_f \neq r_b$ or at the point tan_f if $r_f = r_b$).

[21] That is, the best possible CAL is at least as preferable as ($\succcurlyeq$) the Markowitz efficient frontier.

$$\mathbf{w}_{SR}^{*} = \mathbf{w}_{tan} = \frac{\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)}{\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)},\ \mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N) \neq 0.$$ [22]

where the estimator of $\mathbf{w}_{tan}$ is $\widehat{\mathbf{w}}_{tan} = \widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)/\left(\mathbf{1}_N'\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)\right)$.

Next, generalize $\mathbf{w}_{tan}$ by incorporating γ. From the unconstrained maximization of the quadratic utility function $U = \mu_{co} - 0.5\gamma\sigma_{co}^2 = (1 - \mathbf{w}'\mathbf{1}_N)r_f + \mu_p - 0.5\gamma\sigma_p^2$ (since $\sigma_{r_f} = 0$) (i.e. maximizing satisfaction from riskless-risky investment, graphically along the best possible CAL, $\mathbf{w}'\mathbf{1}_N = 1$ (only risky, no riskless assets) is not necessarily the case), a maximum-SR portfolio allocation over different values of γ becomes (Fabozzi et al., 2007; Okhrin & Schmid, 2006)

$$\mathbf{w}_{SR} = \gamma^{-1}\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N),\ \gamma\mathbf{V} \neq 0. \tag{2.13}$$ [23]

$\mathbf{w}_{SR}$ (2.13) with the estimator $\widehat{\mathbf{w}}_{SR} = \gamma^{-1}\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)$, $\gamma\widehat{\mathbf{V}} \neq 0$, is any efficient riskless-risky combination on the best possible CAL with γ identifying the optimum location (also referring to the two-fund separation theorem).[b,24] The optimal riskless-asset weight is then $1 - \mathbf{w}_{SR}'\mathbf{1}_N$. In a special case, the tangency portfolio, $\mathbf{w}_{tan}$, is obtained when $\mathbf{w}_{SR}'\mathbf{1}_N = 1$ (only risky, no riskless assets) thus $\gamma_{tan} = \mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$. If $\gamma > \gamma_{tan}$, $\mathbf{w}_{SR}'\mathbf{1}_N < 1$. If $\gamma < \gamma_{tan}$, $\mathbf{w}_{SR}'\mathbf{1}_N > 1$. Accordingly, an efficient riskless-risky combination has expected

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[22] FOC: $\frac{\partial}{\partial \mathbf{w}}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}'\mathbf{V}\mathbf{w}}} = \left(\frac{\boldsymbol{\mu}}{\sqrt{\mathbf{w}'\mathbf{V}'\mathbf{w}}}\right)' - \left(\frac{1}{2}\frac{\boldsymbol{\mu}'\mathbf{w} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}}\mathbf{w}'\mathbf{V}\right)' - \frac{1}{2}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}}\mathbf{w}'\mathbf{V} = \mathbf{0} \Leftrightarrow \mathbf{w}^* = \mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$ (where $\mathbf{w}'\mathbf{1}_N = 1$ is not yet considered in this step). Since $\mathbf{w}'\mathbf{1}_N = 1$, then $\mathbf{w}_{SR}^* = \mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)/\left(\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)\right)$, $\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N) \neq 0$, where $\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$ is the sum of all the individual elements of $\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$.
SOC: $\frac{\partial^2}{\partial \mathbf{w}^2}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}'\mathbf{V}\mathbf{w}}} = -\left(\frac{\boldsymbol{\mu}\mathbf{w}'\mathbf{V}}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}} + \frac{\mathbf{V}\mathbf{w}\boldsymbol{\mu}'}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}} - \frac{3}{4}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{5/2}}\mathbf{V}\mathbf{w}(\mathbf{w}'\mathbf{V}) - \frac{3}{4}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{5/2}}\mathbf{V}\mathbf{w}(\mathbf{V}\mathbf{w})' + \frac{1}{2}\frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}}\mathbf{V} - \frac{3}{4}\frac{\boldsymbol{\mu}'\mathbf{w} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{5/2}}\mathbf{V}\mathbf{w}(\mathbf{w}'\mathbf{V}) - \frac{3}{4}\frac{\boldsymbol{\mu}'\mathbf{w} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{5/2}}\mathbf{V}\mathbf{w}(\mathbf{V}\mathbf{w})' + \frac{1}{2}\frac{\boldsymbol{\mu}'\mathbf{w} - r_f}{(\mathbf{w}'\mathbf{V}\mathbf{w})^{3/2}}\mathbf{V}\right) < \mathbf{0}$ (since $\mathbf{V} > \mathbf{0}$ and certain sets of terms with possible negativity have the same sign: the terms with $\mathbf{w}'\boldsymbol{\mu}$ or $\boldsymbol{\mu}'\mathbf{w}$ are all positive, and the terms with $\mathbf{w}'\boldsymbol{\mu} - r_f$ or $\boldsymbol{\mu}'\mathbf{w} - r_f$ are all negative), i.e. negative definiteness, satisfying maximization (also see Footnote **Error! Bookmark not defined.** for an alternative way to obtain $\mathbf{w}_{SR}^*$ or $\mathbf{w}_{tan}$ using $\mathbf{w}_{SR}$, which is much more convenient).

[23] FOC: $\frac{\partial}{\partial \mathbf{w}}\left[U = (1 - \mathbf{w}'\mathbf{1}_N)r_f + \mu_p - 0.5\gamma\sigma_p^2\right] = -r_f\mathbf{1}_N + \boldsymbol{\mu} - \gamma\mathbf{V}\mathbf{w} = 0 \Leftrightarrow \mathbf{w}_{SR} = \gamma^{-1}\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$, $\gamma\mathbf{V} \neq 0$ (also recall μ_p and σ_p^2 as in (2.3) and (2.4)).
SOC: $\frac{\partial^2}{\partial \mathbf{w}^2}U = -\gamma\mathbf{V} \leq \mathbf{0}$ (since $\gamma \geq 0$, $\mathbf{V} > \mathbf{0}$), i.e. negative semi-definiteness, satisfying maximization.
In addition, working out via $\mathbf{w}_{SR}$ (just derived above or as in (2.13)) can also alternatively deliver $\mathbf{w}_{SR}^*$ or $\mathbf{w}_{tan}$, which is formerly from the SR maximization problem with $\mathbf{w}'\mathbf{1}_N = 1$ (supplementarily explained in Footnote 22).
Since $\mathbf{w}'\mathbf{1}_N = 1$ for $\mathbf{w}_{SR}^*$ or $\mathbf{w}_{tan}$, then $\mathbf{1}_N'\mathbf{w}_{SR} = \gamma^{-1}\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N) = 1 \Leftrightarrow \gamma = \gamma_{tan} = \mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$. Hence, by plugging γ_{tan} into $\mathbf{w}_{SR}$ (as in (2.13)), $\mathbf{w}_{SR}^* = \mathbf{w}_{tan} = \mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)/\left(\mathbf{1}_N'\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)\right)$.

[24] The two-fund separation theorem is that an investor with quadratic utility can separate his/her asset allocation decision into two steps: first, finding the tangency portfolio, $\mathbf{w}_{SR}^*$ ($= \mathbf{w}_{tan}$), and second, deciding on the optimal riskless-risky combination, $\mathbf{w}_{SR}$, based on his/her risk aversion degree, γ.

return $\mu_{co} = (1 - \mathbf{w}'_{SR}\mathbf{1}_N)r_f + \mathbf{w}'_{SR}\boldsymbol{\mu}$ and variance $\sigma^2_{co} = \mathbf{w}'_{SR}\mathbf{V}\mathbf{w}_{SR}$. Thus, along the best possible CAL, the slope is equal to $SR = (\mu_{co} - r_f)/\sigma_{co} = (\mu_{tan} - r_f)/\sigma_{tan}$ ($\mu_{tan} = \mathbf{w}'_{tan}\boldsymbol{\mu}$, $\sigma^2_{tan} = \mathbf{w}'_{tan}\mathbf{V}\mathbf{w}_{tan}$).

As discussed earlier, the estimation risk in $\widehat{\boldsymbol{\mu}}$ is much greater than that in $\widehat{\mathbf{V}}$, making the estimator $\widehat{\mathbf{w}}_{SR} = \gamma^{-1}\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)$ not quite a good estimator for its true analytical solution, $\mathbf{w}_{SR}$ (2.13). In $\widehat{\mathbf{w}}_{SR}$, the biased $\widehat{\boldsymbol{\mu}}$ and $\widehat{\mathbf{V}}$ (especially $\widehat{\boldsymbol{\mu}}$) lead to over- or under-concentrations on particular allocated assets, especially when γ is considerably low, i.e. when risky assets are strongly preferable to riskless assets. That is, the resultant $\widehat{w}_{SR,i}$s $\in \widehat{\mathbf{w}}_{SR}$ can be mistakenly too extreme. Michaud (1989) also claims that mean-variance optimizers are likely to allocate larger positive (negative) weights to assets with larger positive (negative) mean estimation errors or with larger negative (positive) volatility estimation errors. In fact, with transaction costs incorporated, extreme weights are practically unfeasible as well.

It has been shown in the literature that weight restrictions (in addition to the previously discussed remedies $\widehat{\boldsymbol{\mu}}_{shrunk,BS}$ ((2.5) and (2.7)) and $\widehat{\mathbf{V}}_{shrunk,LW}$ ((2.6), (2.8), and (2.9))) can cope with this issue by diminishing sampling errors and yielding better out-of-sample performance (it has typically been tested on stock portfolios). Relevant restrictions include non-negativity[25], upper bounds[26], the $1/N$ strategy[27], the variance-based constraint[28], and others[29] (Brandt, 2009; Fabozzi et al., 2007; Frost & Savarino, 1988; Jagannathan & Ma, 2003; DeMiguel et al., 2007; Levy & Levy, 2014; Pflug et al., 2012) ($\widehat{\boldsymbol{\mu}}_{shrunk,BS}$ ((2.5) and

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[25] Jagannathan and Ma (2003) demonstrate that non-negative weights, i.e. $\mathbf{w} \geq \mathbf{0}$, to a certain extent improve portfolio optimization. This improvement is implicitly equivalent to optimization using covariance shrinkage estimation.

[26] Frost and Savarino (1988) discuss that estimation biases from over- or under-investments can be reduced by assigning upper limits to the weights, i.e. $w_i \leq \overline{w}_i$.

[27] DeMiguel et al. (2007) prove that optimization models, e.g. GMV, Bayesian, CAPM-based, short-sales restriction, etc., are not consistently better than the $1/N$ strategy (i.e. equal allocation), by means of the Sharpe ratio. Pflug et al. (2012), using the classical Markowitz and conditional-value-at-risk models, show that optimal investment decisions for minimizing portfolio risk tend towards the $1/N$ strategy.

[28] Levy and Levy (2014) extend the $1/N$ strategy by also incorporating $\hat{\sigma}_i/\bar{\sigma}$ as they introduce the variance-based constraint (VBC) (i.e. $|w_i - 1/N|(\hat{\sigma}_i/\bar{\sigma}) \leq \eta$) and global variance-based constraint (GVBC) (i.e. $\sum_{i=1}^{N}(w_i - 1/N)^2(\hat{\sigma}_i/\bar{\sigma}) \leq \eta$) methods. The VBC imposes weights individually, while the GVBC does so globally. Since the VBC imposes locally, a local constraint on an asset with significantly large $\hat{\sigma}_i$ can be too close to η, although such a high $\hat{\sigma}_i$ is often accompanied by a with high $E[r_{i,t}]$. This indicates that the VBC might over-concentrate on risk while somewhat disregarding the return. Unlike the VBC, the GVBC is implicitly more able to convey information on both the risk and the return. The GVBC imposes globally rather than locally, thus providing more flexibility for assets with a considerably high expected return. Therefore, the VBC may be considered relatively more suitable for use with risk minimization whereas the GVBC may be better for reward-to-risk maximization. Both the VBC and GVBC (especially the GVBC) quite considerably outperform other weighting schemes in terms of the Sharpe ratio.

[29] Other practical weight restrictions include turnover limits (i.e. $|w_i| \leq \overline{w}_i$), holding limits (both lower and upper bounds imposed) (i.e. $\underline{w}_i \leq w_i \leq \overline{w}_i$), $1/N$ deviation limits (i.e. $|w_i - 1/N| \leq \eta$), and others (Fabozzi et al., 2007; Levy & Levy, 2014).

(2.7)) and $\widehat{\mathbf{V}}_{shrunk,LW}$ ((2.6), (2.8), and (2.9)) seen in section **2.3.2 Mean and covariance shrinkage estimations** are also supplementary remedies that are applied in this chapter).

Now, for this chapter's purposes, consider the risky portion of a maximum-SR portfolio comprising only currencies. The feature of no short-selling, although regarded as an interesting choice for dealing with over- or underweighting, is not as common (for currencies?) as it is in stock or bond portfolios. With the other mentioned weight restriction methods (upper limits, the $1/N$ scheme, and variance-based approaches), in spite of their optimization improvement capabilities, the bounds to be specified (i.e. $\overline{w}_i$ in Footnote 26 and η in Footnote 28) are fairly arbitrary.

In this chapter, as portfolio diversification is of key interest and plays a crucial role in stabilizing the portfolio risk level, weights are instead constrained indirectly in terms of risk management, using the downside risk measure VaR. Certain sensible VaR constraints are imposed on both the portfolio mean and variance (or standardized as volatility).[30] A portfolio's mean VaR can be defined as its worst potential loss at a particular confidence level, over a certain horizon. Its volatility VaR can be interpreted as its worst volatility at a specified confidence level over the horizon. The confidence level ($1-\alpha$, α: significance level) used will be 99%, while the horizon is one day, corresponding to the daily data used.

Consistent with the risk-return tradeoff insight (also see **2.3.1 Risk-return tradeoff and portfolio mean and covariance**), a portfolio mean VaR (denoted by $\mu_{p,VaR}$) constraint prohibits a too-low portfolio mean, whereas a portfolio volatility VaR (denoted by $\sigma_{p,VaR}$) constraint disallows a too-high portfolio mean. Not letting the portfolio mean be too low or too high could reduce weight extremes (i.e. over- or under-investment), especially with short-sales permitted. This is because a maximum-SR portfolio's weights often end up extreme when both long and short positions are unbounded.

From risk management perspectives, and for simplicity, VaR computations may be heavy-tailed parametric. Although $\mathbf{R}_t \sim N(\boldsymbol{\mu}, \mathbf{V})$ is assumed in the hypothesis tests in both Chapter 1 and Chapter 2, sample financial returns are often heavy-tailed or platykurtic (also see **1.2.3 Data description**). A popular heavy-tailed distribution, the Student's t-distribution with $\tau-1$ degrees of freedom (df), and the sample biasedness correction factor

[30] The literature analyzing the mean-VaR and mean-conditional-VaR constraints imposed on efficient mean-variance portfolios includes Alexander and Baptista (2004), for example.

$\sqrt{(\tau-3)/(\tau-1)}$, is therefore applied (Glasserman et al., 2002; Rozga & Arnerić, 2009).[31] So, let $\mathbf{R}_t \sim t_{\tau-1}$ instead of $\mathbf{R}_t \sim N(\boldsymbol{\mu}, \mathbf{V})$ in computing the VaRs. This is also applied to the $N \times 1$ vector of percentage changes in time-varying volatilities (denoted by $d\boldsymbol{\sigma}_t$, $\widehat{\boldsymbol{\sigma}}_t := \mathrm{col}\left(\mathrm{diag}\left(\widehat{\mathbf{V}}_t\right)\right)$)[32] with true mean vector $\boldsymbol{\mu}_\sigma$ and covariance matrix $\mathbf{V}_\sigma$, that is let $d\boldsymbol{\sigma}_t \sim t_{\tau-1}$. Like $\widehat{\boldsymbol{\mu}}$ and $\widehat{\mathbf{V}}$, $\widehat{\boldsymbol{\mu}}_\sigma$ and $\widehat{\mathbf{V}}_\sigma$ can be either unconditional, GARCH(1,1)-based, or EWMA-based in this chapter as well. Accordingly, the estimators of the heavy-tailed parameters, $\mu_{p,VaR}$ and $\sigma_{p,VaR}$, can be written as

$$\begin{aligned}
&\hat{\mu}_{p,VaR} = \widehat{\mathbf{w}}'\widehat{\boldsymbol{\mu}}_{VaR},\ -\widehat{\boldsymbol{\mu}}_{VaR} = \widehat{\boldsymbol{\mu}} - t_{\alpha,\tau-1}\left[\mathrm{diag}\left(\widehat{\mathbf{V}}\right)\right]^{\frac{1}{2}}\sqrt{\tfrac{\tau-3}{\tau-1}}, \text{ and} \\
&\hat{\sigma}_{p,VaR} = \widehat{\mathbf{w}}'\widehat{\mathbf{V}}_{VaR}\widehat{\mathbf{w}} \\
&,\ \widehat{\mathbf{V}}_{VaR} = \left[\mathrm{diag}\left(\widehat{\mathbf{V}}\right)_{VaR}\right]^{\frac{1}{2}}\mathbf{P}\left[\mathrm{diag}\left(\widehat{\mathbf{V}}\right)_{VaR}\right]^{\frac{1}{2}} \\
&,\ \widehat{\boldsymbol{\sigma}}_{t,VaR} = \widehat{\boldsymbol{\sigma}}_t\left(\mathbf{1}_N + \widehat{\boldsymbol{\mu}}_\sigma + t_{\alpha,\tau-1}\left[\mathrm{diag}\left(\widehat{\mathbf{V}}_\sigma\right)\right]^{\frac{1}{2}}\sqrt{\tfrac{\tau-3}{\tau-1}}\right).
\end{aligned} \tag{2.14}$$

[33,34]

Now, the portfolio mean-VaR and volatility-VaR bounds can be proposed. Consider an undiversified portfolio consisting of only one risky asset i ($i \in \{1, \dots, N\}$ as previously introduced). The portfolio with the highest $\hat{\mu}_{i,VaR}$ or with the highest $\hat{\sigma}_{i,VaR}$ could be one of the worst portfolios on the basis of return or risk, respectively. Hence, the $\hat{\mu}_{p,VaR}$ and $\hat{\sigma}_{p,VaR}$ boundaries are potentially set equal to $\max(\hat{\mu}_{i,VaR})$ and $\max(\hat{\sigma}_{i,VaR})$, respectively. That is, the maximum-SR portfolios are constructed so as to at least not be worse than one of the worst undiversified portfolios. Accordingly, the portfolio mean-VaR and volatility-VaR constraints, also seen as indirect weight restrictions by means of tail risk consideration, can be expressed as

$$\begin{aligned}
&\hat{\mu}_{p,VaR} \geq -\max(\hat{\mu}_{i,VaR}), \text{ and} \\
&\hat{\sigma}_{p,VaR} \leq \max(\hat{\sigma}_{i,VaR}).
\end{aligned} \tag{2.15}$$

[31] The sample biasedness correction factor is equal to $\sqrt{(df-2)/df} = \sqrt{(\tau-3)/(\tau-1)}$.

[32] $\mathrm{col}(\mathrm{diag}(\mathbf{A}))$: a column vector containing the diagonal elements of the matrix $\mathbf{A}$.

[33] For estimation robustness, $\widehat{\mathbf{V}}_{shrunk,LW}$ ((2.6), (2.8), and (2.9)) is also applied to $\widehat{\mathbf{V}}_{VaR}$.

[34] $\hat{\mu}_{i,VaR} \in \widehat{\boldsymbol{\mu}}_{VaR}$, $-\hat{\mu}_{i,VaR} = \hat{\mu}_i - t_{\alpha,\tau-1}\hat{\sigma}_i\sqrt{(\tau-3)/(\tau-1)}$ and
$\hat{\sigma}_{i,VaR} \in \widehat{\boldsymbol{\sigma}}_{t,VaR}$, $\hat{\sigma}_{i,VaR} = \hat{\sigma}_i\left(1 + (\hat{\mu}_\sigma)_i + t_{\alpha,\tau-1}(\hat{\sigma}_\sigma)_i\sqrt{(\tau-3)/(\tau-1)}\right)$.

When a weight restriction (direct or indirect) is imposed in some way in coping with biased $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{V}}$ (especially $\hat{\boldsymbol{\mu}}$), the Markowitz efficient frontier adjusts and can become properly reshaped (also see **Figure 2.4**). As a result, the maximum-SR portfolio can be allocated more appropriately.

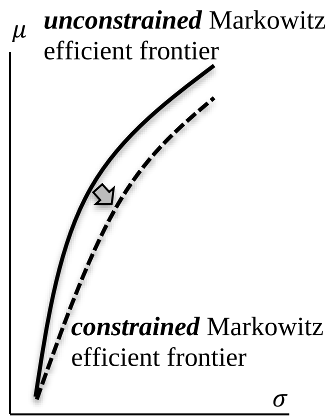


Figure 2.4: constraining the Markowitz efficient frontier

Since, as discussed earlier, the long-only and $1/N$ strategies could improve portfolio optimization, the additional SR constraints applied to ensure that an estimated maximum-SR portfolio also at least outperforms the simple $\mathbf{w} \geq \mathbf{0}$ and $1/N$ approaches, could be

$$\begin{aligned} &\widehat{SR} \geq \widehat{SR}_{\mathbf{w}\geq\mathbf{0}}, \text{ and} \\ &\widehat{SR} \geq \widehat{SR}_{1/N}. \end{aligned} \tag{2.16}$$

These introduced mean-VaR, volatility-VaR, and SR constraints can be applied when the following case studies of maximum-SR portfolio reallocation, in the move from a state of tranquility (T) to a state of distress (D), are examined.

2.5.2 Analytical schemes

In this chapter, in examining the maximum-SR portfolio reallocations when moving from the state of tranquility (T) to the state of distress (D) at the time of the Brexit referendum, two schemes are studied: (1) ***risk aversion maintenance*** and (2) ***SR maintenance***. In both cases, the mean-VaR, volatility-VaR, and SR constraints ((2.14), (2.15), and (2.16))[18] are applied to deal with potentially biased $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{V}}$ (especially $\hat{\boldsymbol{\mu}}$) over both states of tranquility (T) and distress (D).

Risk aversion maintenance

Consider (1) ***risk aversion maintenance***. Assume that the maximum-SR portfolio holder initially invests everything in

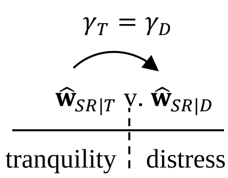


Figure 2.5: $\hat{\boldsymbol{w}}_{SR}$ reallocation over time with risk aversion maintenance

risky assets (i.e. holding the tangency portfolio, $\widehat{\mathbf{w}}'\mathbf{1}_N = 1$, $\gamma_T = \gamma_{tan|T}$), with his or her γ unchanged over time (i.e. $\gamma_T = \gamma_{tan|T} = \gamma_D$, and then $\widehat{\mathbf{w}}'\mathbf{1}_N = 1$ not necessarily the case in the other time span). Accordingly, the portfolio SR maximizations in tranquility (T) and then in distress (D) become

$$\begin{aligned}&\widehat{\mathbf{w}}_{SR|T}\text{: arg}\max_{\widehat{\mathbf{w}}}\frac{\widehat{\mathbf{w}}'(\widehat{\boldsymbol{\mu}}_T - r_f)}{\sqrt{\widehat{\mathbf{w}}'\widehat{\mathbf{V}}_T\widehat{\mathbf{w}}}} \text{ s.t. } \widehat{\mathbf{w}}'\mathbf{1}_N = 1, \text{(2.14), (2.15), and (2.16), then}\\&\widehat{\mathbf{w}}_{SR|D}\text{: arg}\max_{\widehat{\mathbf{w}}}\frac{\widehat{\mathbf{w}}'(\widehat{\boldsymbol{\mu}}_D - r_f)}{\sqrt{\widehat{\mathbf{w}}'\widehat{\mathbf{V}}_D\widehat{\mathbf{w}}}} \text{ s.t. (2.14), (2.15), and (2.16), } \gamma_{tan|T} = \gamma_D.\end{aligned} \tag{2.17}$$

At first, $\widehat{\mathbf{w}}_{SR|T}$ is numerically drawn from the first constrained maximization in (2.17), with $\gamma_T = \gamma_{tan|T}$ to be maintained over time, being implicit. A resolution for identifying the implicit $\gamma_{tan|T}$ and a process for getting from $\widehat{\mathbf{w}}_{SR|T}$ to $\widehat{\mathbf{w}}_{SR|D}$ in (2.17) are proposed as follows.

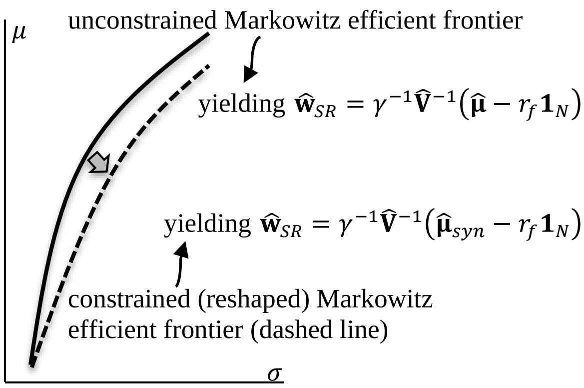


Figure 2.6: the reshaped Markowitz efficient frontier and synthetic $\widehat{\mathbf{w}}_{SR} = \gamma^{-1}\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}}_{syn} - r_f\mathbf{1}_N)$

Recall that additional constraints imposed on a mean-variance portfolio optimization reshape the Markowitz efficient frontier (see **Figure 2.6**). Reconsider the reshaped Markowitz efficient frontier being in tranquility (T) without the additional constraints. It can be inferred that the reshaped mean-variance efficient frontier yields the corresponding (could be called ***synthetic***) unconstrained analytical solution of the maximum-SR portfolio that is equal to the numerical $\widehat{\mathbf{w}}_{SR|T}$ drawn from (2.17). With both $\widehat{\boldsymbol{\mu}}_T$ and $\widehat{\mathbf{V}}_T$ already shrunk, and the estimation risk of the former substantially higher than that of the latter, $\widehat{\mathbf{V}}_T$ is probably allowed to remain at its estimate, while only $\widehat{\boldsymbol{\mu}}_T$ is allowed to be synthetic, and denoted by $\widehat{\boldsymbol{\mu}}_{T,syn}$. Thus, the synthetic analytical closed-form solution, $\widehat{\mathbf{w}}_{SR,syn|T} = \gamma_{tan|T}^{-1}\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)$, is identical to the numerical $\widehat{\mathbf{w}}_{SR|T}$ (2.17) (also see **Figure 2.7**). Since $\widehat{\mathbf{w}}'_{SR,syn|T}\mathbf{1}_N = \widehat{\mathbf{w}}'_{SR|T}\mathbf{1}_N = 1$, it follows that $\gamma_{tan|T} =$

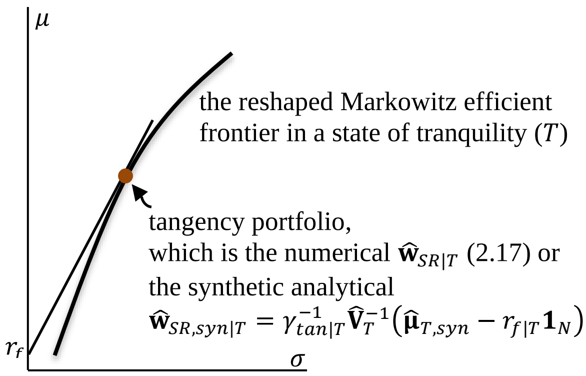


Figure 2.7: the tangency portfolio in a state of tranquility (T) drawn from the reshaped Markowitz efficient frontier

$\mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)$ (also recall the tangency portfolio's γ from section **2.5.1 Discussion**). However, there are now two unknowns, $\gamma_{tan|T}$ and $\widehat{\boldsymbol{\mu}}_{T,syn}$, with only one relationship, $\gamma_{tan|T} = \mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)$. To resolve this, $\widehat{\boldsymbol{\mu}}_{T,syn}$ is figured out before $\gamma_{tan|T}$ using the SR equivalence (between the synthetic analytical and numerical schemes, $\widehat{SR}_{syn|T} = \widehat{SR}_T$).

The SR of the numerical $\widehat{\mathbf{w}}_{SR|T}$ (2.17) is $\widehat{SR}_T = \left((1 - \widehat{\mathbf{w}}_{SR|T}'\mathbf{1}_N)r_f + \widehat{\mathbf{w}}_{SR|T}'\widehat{\boldsymbol{\mu}}_T - r_{f|T}\right) / \sqrt{\widehat{\mathbf{w}}_{SR|T}'\widehat{\mathbf{V}}_T\widehat{\mathbf{w}}_{SR|T}} = \widehat{\mathbf{w}}_{SR|T}'(\widehat{\boldsymbol{\mu}}_T - r_{f|T})/\sqrt{\widehat{\mathbf{w}}_{SR|T}'\widehat{\mathbf{V}}_T\widehat{\mathbf{w}}_{SR|T}}$ while the maximum synthetic SR can be analytically calculated as $\widehat{SR}_{syn|T} = \sqrt{(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)'\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)}$ since the estimated maximum SR of an unconstrained Markowitz efficient frontier is $\sqrt{(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)'\widehat{\mathbf{V}}^{-1}(\widehat{\boldsymbol{\mu}} - r_f\mathbf{1}_N)}$. Consequently, $\widehat{\boldsymbol{\mu}}_{T,syn}$ and then $\gamma_{tan|T}$ can be obtained as follows:

$$\widehat{\boldsymbol{\mu}}_{T,syn}\!: \sqrt{(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)'\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)} = \frac{\widehat{\mathbf{w}}_{SR|T}'(\widehat{\boldsymbol{\mu}}_T - r_{f|T})}{\sqrt{\widehat{\mathbf{w}}_{SR|T}'\widehat{\mathbf{V}}_T\widehat{\mathbf{w}}_{SR|T}}},$$

then substitute $\widehat{\boldsymbol{\mu}}_{T,syn}$ to obtain
$\gamma_{tan|T} = \mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)$.[35]

Next, for the state of distress (D), proceed to obtain the numerical $\widehat{\mathbf{w}}_{SR|D}$ from the second constrained maximization in (2.17) such that $\gamma_{tan|T}$ is maintained over time (i.e. $\gamma_T = \gamma_{tan|T} = \gamma_D$) by applying the SR equivalence (between the synthetic analytical and numerical schemes, $\widehat{SR}_{D,syn} = \widehat{SR}_D$). Besides this, due to several or all $\hat{\mu}_{D,i} \in \widehat{\boldsymbol{\mu}}_D$ being negative and the relatively uncommon behaviors of $\hat{\sigma}_{D,ij} \in \widehat{\mathbf{V}}_D$, simple turnover limits, $|w_i| \leq \overline{w}$, in addition to the VaR constraints, are imposed to help avoid extreme weights, especially when $\widehat{\boldsymbol{\mu}}_D$ and $\widehat{\mathbf{V}}_D$ are time-unconditional.

$\widehat{\mathbf{w}}_{SR|D}$ in (2.17): $|w_i| \leq 1$ (single asset turnover not more than total investment),

$$\sqrt{(\widehat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N)'\widehat{\mathbf{V}}_D^{-1}(\widehat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N)} = \frac{\widehat{\mathbf{w}}_{SR|D}'(\widehat{\boldsymbol{\mu}}_D - r_{f|D})}{\sqrt{\widehat{\mathbf{w}}_{SR|D}'\widehat{\mathbf{V}}_D\widehat{\mathbf{w}}_{SR|D}}}$$

with $\widehat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N = \gamma_{tan|T}\widehat{\mathbf{V}}_D\widehat{\mathbf{w}}_{SR|D}$ ($\gamma_{tan|T}$ in place of γ_D)

[35] Alternatively, $\gamma_{tan|T}\mathbf{1}_N = \widehat{\mathbf{V}}_T^{-1}(\widehat{\boldsymbol{\mu}}_{T,syn} - r_{f|T}\mathbf{1}_N)/\widehat{\mathbf{w}}_{SR|T}$ ((2.13) rearranged in the form of degree of risk aversion).

((2.13) rearranged in terms of the vector of excess returns)

Given the behaviors of $\hat{\mu}_{D,i} \in \hat{\boldsymbol{\mu}}_D$ and $\hat{\sigma}_{D,ij} \in \hat{\mathbf{V}}_D$ as aforementioned, the GMV portfolio typically yields a lower return than r_f. As a result, $\hat{\mathbf{w}}_{SR|D}$ with $\gamma_T = \gamma_{tan|T} = \gamma_D$ drawn from the optimization commonly satisfies $\hat{\mathbf{w}}'_{SR|D}\mathbf{1}_\mathrm{N} < 0$ (see **Figure 2.9**). That is, risky assets are highly diversified away to riskless securities due to market distress, with the relationship changing from $\hat{\mathbf{w}}'_{SR|T}\mathbf{1}_\mathrm{N} = 1$ to $\hat{\mathbf{w}}'_{SR|D}\mathbf{1}_\mathrm{N} < 0$.

Sharpe ratio maintenance

Consider (2) ***SR maintenance***. Assume that a maximum-SR portfolio holder initially has a totally risky investment (i.e. holding the tangency portfolio, $\hat{\mathbf{w}}'\mathbf{1}_N = 1$, $\gamma_T = \gamma_{tan|T}$, also see **Figure 2.7**). The SR is also to be maintained over time (i.e. $\widehat{SR}_T = \widehat{SR}_D$) at as high as $\hat{\mu}_{p|D}$ but satisfying the imposed volatility-VaR constraint. Maintaining the SR is equivalent to maintaining the slope or the shape of the best possible CAL. Therefore, the portfolio risk minimizations in tranquility (T) then in distress (D) become

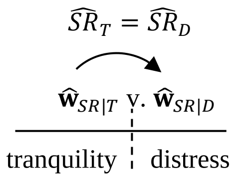


Figure 2.8: $\hat{w}_{SR}$ reallocation over time with SR maintenance

$$\begin{aligned}&\hat{\mathbf{w}}_{\overline{SR}|T}\text{: arg}\max_{\hat{\mathbf{w}}}\frac{\hat{\mathbf{w}}'(\hat{\boldsymbol{\mu}}_T - r_f)}{\sqrt{\hat{\mathbf{w}}'\hat{\mathbf{V}}_T\hat{\mathbf{w}}}} \text{ s.t. } \hat{\mathbf{w}}'\mathbf{1}_N = 1, \text{(2.14), (2.15), and (2.16), then}\\ &\hat{\mathbf{w}}_{\overline{SR}|D}\text{: arg}\max_{\hat{\mathbf{w}}}\hat{\mathbf{w}}'\hat{\boldsymbol{\mu}}_D, \text{(2.14), (2.15), and (2.16), } \frac{\hat{\mathbf{w}}'(\hat{\boldsymbol{\mu}}_D - r_f)}{\sqrt{\hat{\mathbf{w}}'\hat{\mathbf{V}}_D\hat{\mathbf{w}}}} = \widehat{SR}_T.\end{aligned} \tag{2.18}$$

γ_D also needs to be identified as it will be used in the hypothesis tests (in the next section **2.5.3 Hypothesis tests**). Similarly to in the first case study, γ_D can be figured out by again applying the SR equivalence (between the synthetic analytical and numerical schemes, $\widehat{SR}_{D,syn} = \widehat{SR}_D$) where the SR is maintained over time ($\widehat{SR}_T = \widehat{SR}_D$) as follows:

$$\begin{aligned}&\gamma_D\text{: }\sqrt{\left(\hat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N\right)'\hat{\mathbf{V}}_D^{-1}\left(\hat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N\right)} = \frac{\hat{\mathbf{w}}'_{SR|D}(\hat{\boldsymbol{\mu}}_D - r_{f|D})}{\sqrt{\hat{\mathbf{w}}'_{SR|D}\hat{\mathbf{V}}_D\hat{\mathbf{w}}_{SR|D}}}\\ &\qquad\text{with } \hat{\boldsymbol{\mu}}_{D,syn} - r_{f|D}\mathbf{1}_N = \gamma_D\hat{\mathbf{V}}_D\hat{\mathbf{w}}_{SR|D}\end{aligned}$$

Similarly to in the previous case study, the turnover limits $|w_i| \leq 1$ are also added to help prevent extreme allocations in optimizing $\hat{\mathbf{w}}_{SR|D}$, particularly when $\hat{\boldsymbol{\mu}}_D$ and $\hat{\mathbf{V}}_D$ are

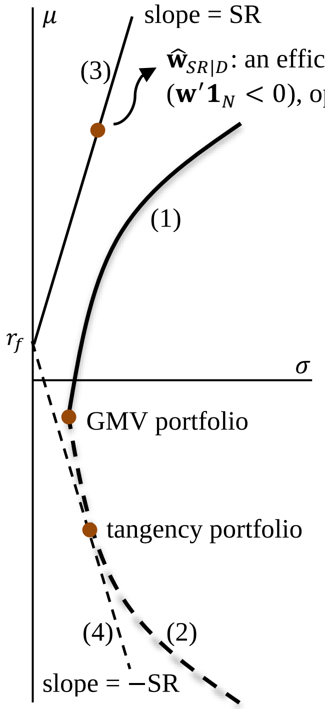


Figure 2.9: a likely situation in a state of ditress (D) (with the additional constraints imposed), which is also the case in this paper (for both unconditional and conditional correlations), where $r_f > \hat{\mu}_{GMV}$ leading to the tangency portfolio located on the lower limb of the hyperbola (Ingersoll, 1987).

investment (in)efficiency upon the risk-return trade-off insight:

(1) upper mean-variance frontier: efficient risky portfolios,
(2) lower mean-variance frontier (dashed): inefficient risky portfolios,

(3) upward sloping CAL with slope = SR: efficient riskless-risky combinations with $\mathbf{w}'\mathbf{1}_N < 0$ (risky) and $1 - \mathbf{w}'\mathbf{1}_N > 1$ (riskless),
(4) downward sloping CAL (dashed) with slope = $-$SR: inefficient riskless-risky combinations

time-unconditional. Besides this, the optimal $\hat{\mathbf{w}}_{SR|D}$ with $\widehat{SR}_T = \widehat{SR}_D$ is assumed to be a short position, i.e. $\hat{\mathbf{w}}'_{SR|D}\mathbf{1}_{\mathrm{N}} < 0$ (see **Figure 2.9**).

Next, for both risk aversion maintenance and SR maintenance, both the structural ($\hat{\mathbf{w}}_{SR}$) and single-asset ($\hat{w}_{SR,i}$) reallocations will be tested by applying the derived tests to the tangency portfolios.

2.5.3 Hypothesis tests

Under the assumption of normally distributed returns $\mathbf{R}_t \sim N(\boldsymbol{\mu}, \mathbf{V})$, the maximum-SR portfolio estimator, $\hat{\mathbf{w}}_{SR} = \gamma^{-1}\hat{\mathbf{V}}^{-1}\left(\hat{\boldsymbol{\mu}} - r_f\mathbf{1}_N\right)$, has the following mean vector and covariance matrix (Adcock, 2015; Bodnar et al., 2017; Okhrin & Schmid, 2006):

$$E(\hat{\mathbf{w}}_{SR}) = \frac{\tau-1}{\tau-N-2}\mathbf{w}_{SR} \text{ and } cov(\hat{\mathbf{w}}_{SR}) = c_1\mathbf{w}_{SR}\mathbf{w}'_{SR} + c_2\mathbf{V}^{-1},$$

$$\text{where } c_1 = \frac{(\tau-N)(\tau-1)^2}{(\tau-N-1)(\tau-N-2)^2(\tau-N-4)} \text{ and } c_2 = \frac{(\tau-1)^2\left[\tau-2+\tau\left(\boldsymbol{\mu}-r_f\mathbf{1}_N\right)'\mathbf{V}^{-1}\left(\boldsymbol{\mu}-r_f\mathbf{1}_N\right)\right]}{\tau(\tau-N-1)(\tau-N-2)(\tau-N-4)\gamma^2}$$

Accordingly, the corresponding hypotheses (H_0 and H_1) and two-sided test statistic (denoted by $T_{\overline{\hat{\mathbf{w}}_{SR}}}$), for testing whether a maximum-SR portfolio ($\hat{\mathbf{w}}_{SR}$) requires structural reallocation when moving from a state of tranquility (T) to a state of distress (D) (tranquility time span: τ_T, distress time span: τ_D), are as follows (applying Adcock (2015) and Bodnar et al. (2017) as if $\widehat{\mathbf{w}}_{SR|D}$ deviates from the ***benchmark*** $\widehat{\mathbf{w}}_{SR|T}$):

$$H_0\colon \mathbf{w}_{SR|T} = \mathbf{w}_{SR|D}$$
$$H_1\colon \mathbf{w}_{SR|T} \neq \mathbf{w}_{SR|D}$$

$$T_{\widehat{\mathbf{w}}_{SR}} = \frac{\tau_D - N}{\tau_D - 1} \frac{\gamma_C^2 \left(\widehat{\mathbf{w}}_{SR|D} - \widehat{\mathbf{w}}_{SR|T}\right)' \left(\widehat{\mathbf{w}}_{SR|D} - \widehat{\mathbf{w}}_{SR|T}\right)}{\left(\mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N\right)\left[\frac{1}{\tau_D} + \frac{1}{\tau_D - 1}\left(\widehat{\boldsymbol{\mu}}_D - r_f \mathbf{1}_N\right)' \widehat{\mathbf{Q}}_D \left(\widehat{\boldsymbol{\mu}}_D - r_f \mathbf{1}_N\right)\right]}$$

where $T_{\widehat{\mathbf{w}}_{SR}} \sim F_{N-1,\tau_D - N}$, γ_D is the degree of risk aversion in a state of distress (D), and $\widehat{\mathbf{Q}}_D = \widehat{\mathbf{V}}_D^{-1} - \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} / \left(\mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N\right)$.

It follows that, if $T_{\widehat{\mathbf{w}}_{SR}} \leq F_{N-1,\tau_D - N}$, H_0 is not rejected, i.e. a structural change to the tangency portfolio is not required. If $T_{\widehat{\mathbf{w}}_{SR}} > F_{N-1,\tau_D - N}$, H_0 is rejected, i.e. structural change is required.

Then, given that the tangency portfolio is structurally adjusted when moving from T to D (i.e. $\widehat{\mathbf{w}}_{SR|T} \neq \widehat{\mathbf{w}}_{SR|D}$), individual weight ($\widehat{w}_{SR,i} \in \widehat{\mathbf{w}}_{SR}$) reallocation over the same time can be further analyzed. The hypotheses (H_0 and H_1) and one-sided test statistic (denoted by $T_{\widehat{w}_{SR,i}}$), can be applied as follows (applying Bodnar et al. (2017) as if $\widehat{w}_{SR,i|D}$ deviates from the ***benchmark*** $\widehat{w}_{SR,i|T}$, for simplicity, the following test is allowed to remain dependent on $\mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N$ in this chapter[36]) (also see interdependence, contagion, and flight definitions in **Table 2.1**, in which the Brexit case, in specific, results in the cases of negative contagion and flight to quality):

$H_0\colon w_{SR,i|T} = w_{SR,i|D}$, i.e. interdependence
$H_1\colon w_{SR,i|T} \neq w_{SR,i|D}$,

$$\begin{cases} w_{SR,i|T} > w_{SR,i|D}, \text{i. e. negative contagion (or flight from quality)} \\ w_{SR,i|T} < w_{SR,i|D}, \text{i. e. flight to quality (or positive contagion)} \end{cases}$$

[36] Without dependence on $\mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N$, which could be considered a nuisance parameter since its true value is unknown, an additional hypothesis, according to Bodnar et al. (2017), if applied to this dissertation, becomes
H_0: $\mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N = \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N$ against H_1: $\mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N \neq \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N$,
with the test statistic, $(\tau_D - 1) \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N / \left(\mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N\right) \sim \chi^2$, and $df = \tau_D - N$.
Then, the globally joint hypothesis, if applied to this dissertation, becomes
H_0: $\widehat{w}_{SR,i|T} = \widehat{w}_{SR,i|D}, \mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N = \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N$ against H_1: $\widehat{w}_{SR,i|T} \neq \widehat{w}_{SR,i|D}, \mathbf{1}_N' \widehat{\mathbf{V}}_T^{-1} \mathbf{1}_N \neq \mathbf{1}_N' \widehat{\mathbf{V}}_D^{-1} \mathbf{1}_N$.

$$T_{\widehat{w}_{SR,i}} = \sqrt{\mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}\,\frac{\frac{\gamma_D\widehat{w}_{tan,i|D}}{\mathbf{1}_N'\widehat{\mathbf{V}}_D^{-1}\mathbf{1}_N}-\frac{\gamma_D\widehat{w}_{tan,i|T}}{\mathbf{1}_N'\widehat{\mathbf{V}}_T^{-1}\mathbf{1}_N}}{\sqrt{\frac{1}{\tau_D}+\frac{1}{\tau_D-1}\left(\widehat{\boldsymbol{\mu}}_D-r_f\mathbf{1}_N\right)'\widehat{\mathbf{Q}}_D\left(\widehat{\boldsymbol{\mu}}_D-r_f\mathbf{1}_N\right)}}$$

where $T_{\widehat{w}_{SR,i}} \sim N(0,1)$.

It follows that, if $\left|T_{\widehat{w}_{SR,i}}\right| \leq |z_\alpha|$, H_0 is not rejected. If $\left|T_{\widehat{w}_{SR,i}}\right| > |z_\alpha|$, H_0 is rejected. In addition, $T_{\widehat{w}_{SR,i}} > z_\alpha > 0$ (since $\widehat{w}_{SR,i|D} > \widehat{w}_{SR,i|T}$) implies a negative contagion (for a negative incident, such as Brexit) (but a flight from quality for a positive incident) while $T_{\widehat{w}_{SR,i}} < z_\alpha < 0$ (since $\widehat{w}_{SR,i|D} < \widehat{w}_{SR,i|T}$) signposts a flight to quality (but a positive contagion for a positive incident).

2.6 Standardized correlations with heteroskedasticity correction

Referring to Chapter 1, the heteroskedasticity inherent in the shock-originating returns is included in the correlation analyses based on Forbes and Rigobon (2002), Corsetti et al. (2005), and the extended framework. This will also be done in the portfolio analyses, so that the resultant statistical discussion is comparable to that in Chapter 1. In this chapter, standardizations of heteroskedasticity-adjusted correlations are proposed. The heteroskedasticity will be incorporated into the multivariate-correlation matrices such that they are standardized, i.e. all the coefficients range from -1 to $+1$.

First, recall that $r_{x,t}$ denotes the shock-originating economy's financial returns while $r_{y,t}$ represents a shock-receiving economy's financial returns. The correlations of interest in Chapter 1 are only between $r_{x,t}$ and the $r_{y,t}$s (i.e. bivariate analysis) whereas those in this chapter also include correlations among the $r_{y,t}$s (i.e. multivariate analysis) and are to be standardized in the mean-variance analysis. Then, standardized correlations with heteroskedasticity correction based on Forbes and Rigobon (2002), Corsetti et al. (2005), and the extended framework can be demonstrated below.

Recall (1.19), or Forbes and Rigobon's (2002) (FR (2002)) distress-state correlation coefficient adjusted for or unconditional on the heteroskedasticity, denoted by $\rho_{x,y|D,adj}$ or $\rho_{D,adj}$. It comes initially from the univariate linear regression without endogeneity or variable omission, of $r_{y,t} = a + br_{x,t} + v_t$, where x explains y unidirectionally (see Footnote 47). Equivalently, $\rho_{D,adj}$ is the inverse interdependence measure ϕ_{FR}' (1.13) in Chapter 1:

FR (2002): $\rho_{D,adj} = \rho_D \sqrt{\frac{1}{1+\delta(1-\rho_D^2)}} \in [-1,1]$ (recall (1.19))[37]

where $\delta = \sigma_{x|D}^2 / \sigma_{x|T}^2 - 1$, which is the relative change in the shock-originating return variance when moving from a state of tranquility (T) to a state of distress (D), reflecting such heteroskedasticity.

Since (1.19) is standardized to range from -1 to $+1$, only modifying δ, which is multiplied by the term $(1 - \rho_D^2) \in [-1,1]$, under a certain assumption about the heteroskedasticity will keep $\rho_{D,adj}$ standardized. Then, the modifications based on Corsetti et al. (2005) and the extended framework could be proposed as follows.

Recall Corsetti et al.'s (2005) single global factor model $r_{x,t} = \alpha_x + \beta_x g_t + \varepsilon_{x,t}$ (1.7). Instead of $r_{y,t} = \alpha_y + \beta_y g_t + \varepsilon_{y,t}$, for simplicity let $r_{y,t} = a + b r_{x,t} + v_t$, in line with Forbes and Rigobon (2002), as aforementioned, and let $\sigma_x^2 = \beta_x^2 \sigma_g^2 + \sigma_{\varepsilon_x}^2$ from (1.8) (also see (1.9) and its explanations).

In accordance with the assumption of Corsetti et al. (2005) (CPS (2005)), that the intensification of $\rho_{x,y}$ in a state of distress coincides with higher σ_g^2 relative to $\sigma_{\varepsilon_x}^2$, δ used to compute $\rho_{D,adj}$ (denoted by δ_g) and the corresponding $\rho_{D,adj}$ (denoted by $\rho_{D,adj,g}$) become

$$\begin{aligned} &\delta_g = \frac{\beta_{x|T}^2 \sigma_{g|D}^2 + \sigma_{\varepsilon_x|D}^2}{\beta_{x|T}^2 \sigma_{g|T}^2 + \sigma_{\varepsilon_x|T}^2} - 1, \text{and} \\ &\text{CPS (2005): } \rho_{D,adj,g} = \rho_D \sqrt{\frac{1}{1+\delta_g(1-\rho_D^2)}} \in [-1,1]. \end{aligned} \qquad (2.19)$$

Consistent with the extended framework's assumption that $\rho_{x,y}$ magnifies in a state of distress in tandem with a larger β_x^2 relative to $\sigma_{\varepsilon_x}^2$, δ used to compute $\rho_{D,adj}$ (denoted by δ_β) and the corresponding $\rho_{D,adj}$ (denoted by $\rho_{D,adj,\beta}$) become

[37] $\rho_{D,adj} \in [-1,1] \because 1 + \delta(1 - \rho_D^2) \geq 0 \because \delta > 0, \rho_D^2 \geq 0$ (similar to $\rho_{D,adj,g}$ (2.19) and $\rho_{D,adj,\beta}$ (2.20))

$$\delta_\beta = \frac{\beta_{x|D}^2 \sigma_{g|T}^2 + \sigma_{\varepsilon_x|D}^2}{\beta_{x|T}^2 \sigma_{g|T}^2 + \sigma_{\varepsilon_x|T}^2} - 1, \text{and}$$
$$\text{extended } \left(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2\right): \rho_{D,adj,\beta} = \rho_D \sqrt{\frac{1}{1+\delta_\beta\left(1-\rho_D^2\right)}} \in [-1,1]. \tag{2.20}$$

Consider Forbes and Rigobon (2002), Corsetti et al. (2005), or the extended framework, respectively. In a distress-state (D) correlation-coefficient matrix estimator $\widehat{\mathbf{P}}_D$ ($\widehat{\mathbf{P}}_D \in \left\{\widehat{\mathbf{P}}_{D,uncond}, \widehat{\mathbf{P}}_{t,D,GARCH}, \widehat{\mathbf{P}}_{t,D,EWMA}\right\}$, (1.1), (1.2), or (1.5)), the adjusted correlations $\rho_{D,adj}$, $\rho_{D,adj,g}$, or $\rho_{D,adj,\beta}$ will replace all the $\rho_{D,x,y}$ in the first row and first column (where x is represented by the first row and the ys come after that in order), while all the rest of the entries (that are $\rho_{y,y}$) will remain the same (see **Appendix 2.C Distress-state correlation matrices and heat maps with heteroskedastic correction**).

2.7 Evidence from Brexit

For comparability, in terms of statistical inferences, to Chapter 1's bivariate correlation analysis, risky assets in Chapter 2's examined portfolios comprise exactly the same major currencies as in Chapter 1's empirical work on Brexit. Thus, interdependences, negative contagions, and flights to quality among the studies major currencies, showing their interactions within the foreign exchange market, can be compared and contrasted. Accordingly, the statistical evidence from currency portfolio analysis of negative contagion, flight to quality, and structural rebalancing (see **1.5.1 Test results on interdependence assumptions**) are as follows. Each portfolio analytical scheme is without and then with relevant interest rates partialled out (see Chapter 1's section **Controlling for the effects of relevant interest rates**, under section **1.4.4 Hypothesis tests**).

2.7.1 Shifts in minimum risk portfolio weights

With risk-free assets not yet taken into account (i.e. only risky assets, which are the studied major currencies, are allocated), minimum risk portfolios are examined with the outcomes pertaining to the two schemes: risk aversion maintenance and risk level maintenance. These are presented and discussed in the following (also see **Figure 2.10**).

Risk aversion maintenance

Consider minimum risk portfolio reallocations due to the UK political uncertainty, with risk aversion maintenance (i.e. the portfolio holder remains fully risk-averse over time). The statistical results are presented on the basis of currency interdependences, negative contagions, and flights to quality, without interest rates partialled out in **Table 2.2** and with interest rates partialled out in **Table 2.3**.

Across different correlation methods or different heteroskedasticity corrections, the results of interdependences, contagions, and flights to quality are overall shown to vary considerably and to be dissimilar from those obtained from the correlation analysis in Chapter 1. As the portfolio risk is being minimized, this is probably due to the focus on not only correlations and covariances but also individual volatilities.

Unlike in the bivariate correlation analysis, flights now happen not only with JPY but also with some other currencies with comparably low volatilities, namely EUR, CHF, and CAD. Unlike JPY, which is the only currency that becomes negatively correlated with the initially shocked currency, GBP, these other currencies (EUR, CHF, and CAD) are indeed even increasingly positively correlated with GBP following the Brexit referendum. Under certain correlation and volatility computations, contagions even exist with JPY because of its noticeably rising volatility, contradicting the results in Chapter 1.

correlation, volatility	**heteroskedasticity correction**	**portfolio reallocation** (structural and individual)							
		structural	x **GBP**	y_1 **EUR**	y_2 **CHF**	y_3 **CAD**	y_4 **JPY**	y_5 **AUD**	y_6 **NZD**
unconditional	unadjusted	**	less***	I	F***	I	C*	I	I
	FR (2002)	***	less***	F**	F**	C**	C**	I	F*
	CPS (2005)	-	(less***)	(F**)	(F**)	(C**)	(C**)	I	(F*)
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	***	less***	F**	F**	C**	C**	I	F*
GARCH(1,1)	unadjusted	-	-	I	I	I	I	I	I
	FR (2002)	-	-	I	I	I	I	I	I
	CPS (2005)	-	-	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	-	-	I	I	I	I	I	I
EWMA	unadjusted	*	less**	I	I	I	I	I	I
	FR (2002)	-	(less***)	I	I	I	I	I	I
	CPS (2005)	-	(less***)	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	-	(less***)	I	I	I	I	I	I

Table 2.2: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***minimum risk portfolio weights***, given ***risk aversion maintenance***,
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

correlation, volatility	**heteroskedasticity correction**	**portfolio reallocation** (structural and individual)							
		structural	$\boldsymbol{x}$	$\boldsymbol{y_1}$	$\boldsymbol{y_2}$	$\boldsymbol{y_3}$	$\boldsymbol{y_4}$	$\boldsymbol{y_5}$	$\boldsymbol{y_6}$
			GBP	**EUR**	**CHF**	**CAD**	**JPY**	**AUD**	**NZD**
unconditional	unadjusted	***	-	I	I	I	I	C*	I
	FR (2002)	***	less***	F*	I	I	C*	I	I
	CPS (2005)	***	less***	I	I	I	C*	I	I
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	I	I	I	C*	I	I
GARCH(1,1)	unadjusted	***	less***	F***	C***	F***	F***	I	C***
	FR (2002)	***	less***	F***	C***	F***	F***	I	C***
	CPS (2005)	***	less***	F***	C***	F***	F***	I	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	F***	C***	F***	F***	I	C***
EWMA	unadjusted	-	(less***)	I	(F***)	I	(C***)	I	(F***)
	FR (2002)	***	less***	C**	F***	I	C***	I	F***
	CPS (2005)	***	less***	C**	F***	I	C***	I	F***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	C**	F***	I	C***	I	F***

Table 2.3: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***minimum risk portfolio weights***, given ***risk aversion maintenance***,
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

Comparing the extended heteroskedasticity-correction framework to the other correction approaches within the same correlation-volatility methodology, the outcomes are not so distinctive.

The portfolio and asset reallocation suggestions produced from the hypothesis tests are also affected by less or more forward-looking correlations and volatilities.

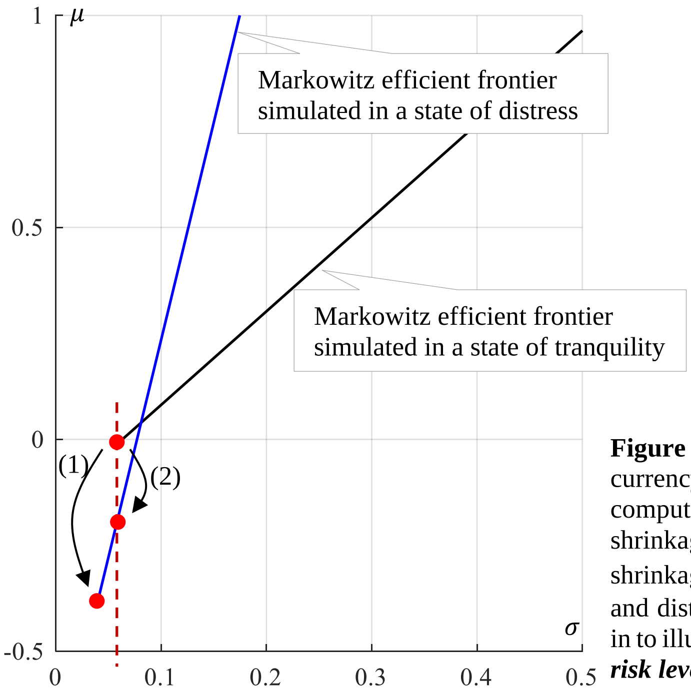


Figure 2.10: Markowitz efficient frontiers of currency portfolios simulated using unconditional computations and estimations of Bayes-Stein mean shrinkage ($\hat{\boldsymbol{\mu}}_{uncond,BS}$) and Ledoit-Wolf covariance shrinkage ($\hat{\boldsymbol{V}}_{uncond,LW}$) over the tranquility-state and distress-state time spans. The figure is zoomed in to illustrate (1) ***risk aversion maintenance*** and (2) ***risk level maintenance***.

Before removing the effects of relevant interest rates, several single-asset reallocations are suggested pertaining to unconditional correlations. However, after the interest rate effects are eliminated, only a few contagions and flights remain detectable. This is the opposite to the cases of more forward-looking correlations under which currency contagions and flights become statistically significant after the interest rates are partialled out.

Risk level maintenance

Now consider minimum risk portfolio reallocations due to the UK political uncertainty but with risk level maintenance. The statistical results are presented, by means of currency interdependences, negative contagions, and flights to quality, without interest rates partialled out in **Table 2.4** and with interest rates partialled out in **Table 2.5**.

Although the investigation of currency interdependences, contagions, and flights has switched from risk aversion maintenance to risk level maintenance, the statistical outcomes seem relatively unchanged, whether in terms of structural and individual asset reallocations or when comparing the heteroskedasticity corrections.

correlation, volatility	**heteroskedasticity correction**	**portfolio reallocation** (structural and individual)							
		structural	x **GBP**	y_1 **EUR**	y_2 **CHF**	y_3 **CAD**	y_4 **JPY**	y_5 **AUD**	y_6 **NZD**
unconditional	unadjusted	-	(less***)	I	(F**)	I	(C*)	I	I
	FR (2002)	***	less*	F*	F***	C**	C***	I	I
	CPS (2005)	***	less**	F*	F***	C*	C***	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	***	less**	F*	F***	C*	C***	I	I
GARCH(1,1)	unadjusted	-	-	I	I	I	I	I	I
	FR (2002)	-	-	I	I	I	I	I	I
	CPS (2005)	-	-	I	I	I	I	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	-	-	I	I	I	I	I	I
EWMA	unadjusted	***	less**	I	I	I	F*	I	I
	FR (2002)	*	less***	I	I	I	F**	I	I
	CPS (2005)	**	less***	I	I	I	F**	I	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	-	(less***)	I	I	I	(F**)	I	I

Table 2.4: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***minimum risk portfolio weights***, given ***risk level maintenance***,
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

Nonetheless, in a state of distress, US-based risk-minimizing investors, in practice, still have the option to reallocate all their wealth into riskless assets, such as cash and USD-denominated money-market instruments. And, this reallocation is an extreme form of flight to quality, whose empirical results can be delivered when US overnight interest rates come into play in the case studies regarding reward-to-risk maximization (see, especially, **Table 2.8** and **Table 2.9** and their discussions, for the case of reward-to-risk maximization with risk aversion maintenance, and, especially, **Table 2.12** and **Table 2.13** and their discussions, for the case of reward-to-risk maximization with Sharpe ratio maintenance).

correlation, volatility	**heteroskedasticity correction**	**portfolio reallocation** (structural and individual)							
		structural	x **GBP**	y_1 **EUR**	y_2 **CHF**	y_3 **CAD**	y_4 **JPY**	y_5 **AUD**	y_6 **NZD**
unconditional	unadjusted	***	-	I	I	I	I	I	I
	FR (2002)	***	less***	F*	I	I	I	I	I
	CPS (2005)	***	less***	F*	I	I	I	I	I
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	F*	I	I	I	I	I
GARCH(1,1)	unadjusted	***	less***	F***	C***	F**	F***	I	C***
	FR (2002)	***	less***	F***	C***	F**	F***	I	C***
	CPS (2005)	***	less***	F***	C***	F**	F***	I	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	F***	C***	F***	F***	I	C***
EWMA	unadjusted	***	less***	I	F***	I	C***	I	F***
	FR (2002)	***	less***	I	F***	I	C***	C**	F***
	CPS (2005)	***	less***	I	F***	I	C***	C**	F***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	I	F***	I	C***	I	F***

Table 2.5: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***minimum risk portfolio weights***, given ***risk level maintenance***,
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

2.7.2 Shifts in maximum reward-to-risk portfolio weights

Now, taking riskless assets into consideration (i.e. both riskless and risky assets, which are USD-denominated money-market instruments and the studied major currencies, are now allocated), maximum-SR portfolios are investigated for the two schemes: risk aversion maintenance and SR maintenance. The results are presented and discussed below.

Risk aversion maintenance

Consider maximum-SR portfolio reallocations due to UK political uncertainty with risk aversion maintenance. The statistical results are presented in terms of currency interdependences, negative contagions, and flights to quality, without interest rates partialled out in **Table 2.6** and with interest rates partialled out in **Table 2.7**.

correlation, volatility	heteroskedasticity correction	portfolio reallocation (structural and individual)							
		structural	x GBP	y_1 EUR	y_2 CHF	y_3 CAD	y_4 JPY	y_5 AUD	y_6 NZD
unconditional	unadjusted	***	-	I	I	C**	C***	I	C**
	FR (2002)	***	-	C***	C***	C**	C**	F***	C***
	CPS (2005)	***	-	C***	C**	C**	C**	F***	C***
	extended ($\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2$)	***	-	C***	C***	C**	C**	F***	C***
GARCH(1,1)	unadjusted	***	less***	F***	C***	F**	I	I	C***
	FR (2002)	***	less***	F***	C***	F**	I	I	C***
	CPS (2005)	***	less***	F***	C***	F**	I	I	C***
	extended ($\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2$)	***	less***	F***	C***	F**	I	I	C***
EWMA	unadjusted	***	less***	F***	C***	F**	C***	F**	C***
	FR (2002)	***	less***	F***	C***	F**	C***	F**	C***
	CPS (2005)	***	less***	F***	C***	F**	C***	F**	C***
	extended ($\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2$)	***	less***	F***	C***	F**	C***	F**	C***

Table 2.6: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***maximum-SR portfolio weights***, given ***risk aversion maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases,
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

Similarly to what was seen in the case studies of portfolio risk minimization, the outcomes for the interdependences, contagions, and flights, among different correlation computations or different heteroskedasticity adjustments, appear to vary quite considerably and to differ from those in Chapter 1's correlation analysis. As the portfolio reward-to-risk is maximized, this may be owing to the consideration of not only correlations but also risk-return tradeoffs.

Compared with the risk minimization case, currency flights to quality are not only occasionally detected for EUR, CAD, and JPY but also sometimes for AUD, which has a relatively worse risk-return tradeoff in the state of distress. As regards JPY, flights are more clearly seen here than in the bivariate correlation analysis (Chapter 1), seemingly because the effect of the risk-return tradeoff outweighs that of the decreasing and negative correlation with GBP, bringing about contagion detection.

As with the portfolio risk minimization, the results from the extended heteroskedasticity-correction framework are quite similar to those produced by the other correction methods, within the same correlation-volatility computation technique.

Also, information persistence (less v. more forward-looking correlation and volatility calculations) influences the statistical outcomes for the portfolio and asset reallocations.

After the effects of interest rates have been removed, there are fewer contagions and flights detected, particularly in the case of CAD.

correlation, volatility	heteroskedasticity correction	portfolio reallocation (structural and individual)							
		structural	x	y_1	y_2	y_3	y_4	y_5	y_6
			GBP	EUR	CHF	CAD	JPY	AUD	NZD
unconditional	unadjusted	***	-	C*	I	C*	C***	I	C*
	FR (2002)	**	-	C*	F**	C*	C**	I	I
	CPS (2005)	***	-	C**	I	C*	C***	I	C*
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	***	-	C**	I	C*	C***	I	C*
GARCH(1,1)	unadjusted	***	less***	F***	C***	I	F*	I	C***
	FR (2002)	***	less***	F***	C***	I	F*	I	C***
	CPS (2005)	***	less***	F***	C***	I	F*	I	C***
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	***	less***	F***	C***	I	F*	I	C***
EWMA	unadjusted	**	-	I	C**	I	C**	F*	I
	FR (2002)	***	less**	C**	C***	I	C***	F***	C**
	CPS (2005)	***	less*	C*	C**	I	C***	F**	I
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	***	less*	C*	C**	I	C***	F**	I

Table 2.7: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***maximum-SR portfolio weights***, given ***risk aversion maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases,
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

Besides this, **Table 2.8** and **Table 2.9** demonstrate how portfolio allocations shift when moving from the state of tranquility (T) to the state of distress (D) due to the UK political uncertainty, such that the risky portfolio becomes completely short, with the holdings diversified away to risk-free assets (also see **Figure 2.9**). With the political uncertainty information fading out more slowly (unconditional, then EWMA, then GARCH(1,1) with the lowest information persistence, also see **Appendix 1.E Analytical and hypothesis-testing parameters**), the proceeds invested in the riskless securities are larger.

correlation, volatility	heteroskedasticity correction	tranquility-state allocation		distress-state allocation	
		riskless	risky	riskless	risky
unconditional	unadjusted	0	1	6.218	-5.218
	FR (2002)	0	1	4.494	-3.494
	CPS (2005)	0	1	4.429	-3.429
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	4.444	-3.444
GARCH(1,1)	unadjusted	0	1	1.542	-0.542
	FR (2002)	0	1	1.595	-0.595
	CPS (2005)	0	1	1.591	-0.591
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	1.592	-0.592
EWMA	unadjusted	0	1	1.596	-0.596
	FR (2002)	0	1	1.696	-0.696
	CPS (2005)	0	1	1.689	-0.689
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	1.690	-0.690

Table 2.8: riskless and risky portfolio allocations in the states of tranquility and distress,
based on ***SR maximization*** and ***risk aversion maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases,
without relevant interest rates partialled out

correlation, volatility	heteroskedasticity correction	tranquility-state allocation		distress-state allocation	
		riskless	risky	riskless	risky
unconditional	unadjusted	0	1	6.308	-5.308
	FR (2002)	0	1	4.115	-3.115
	CPS (2005)	0	1	6.219	-5.219
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	0	1	6.217	-5.217
GARCH(1,1)	unadjusted	0	1	1.328	-0.328
	FR (2002)	0	1	1.375	-0.375
	CPS (2005)	0	1	1.372	-0.372
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	0	1	1.373	-0.373
EWMA	unadjusted	0	1	1.989	-0.989
	FR (2002)	0	1	2.138	-1.138
	CPS (2005)	0	1	2.388	-1.388
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	0	1	2.389	-1.389

Table 2.9: riskless and risky portfolio allocations in the states of tranquility and distress,
based on ***SR maximization*** and ***risk aversion maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases,
with relevant interest rates partialled out

Sharpe ratio maintenance

Now consider the maximum-SR portfolio reallocations due to the UK political uncertainty, with SR maintenance. The statistical results are presented in terms of currency interdependences, negative contagions, and flights to quality, without interest rates partialled out in **Table 2.10** and with interest rates partialled out in **Table 2.11**.

correlation, volatility	heteroskedasticity correction	**portfolio reallocation** (structural and individual)							
		structural	x	y_1	y_2	y_3	y_4	y_5	y_6
			GBP	EUR	CHF	CAD	JPY	AUD	NZD
unconditional	unadjusted	*	-	I	I	I	I	I	I
	FR (2002)	***	-	F**	I	C***	C***	C***	C***
	CPS (2005)	***	-	F**	I	C***	C***	C**	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	-	F**	I	C***	C***	C**	C***
GARCH(1,1)	unadjusted	***	less***	F***	C***	F*	I	I	C***
	FR (2002)	***	less***	F**	C***	F**	I	I	C***
	CPS (2005)	***	less***	F**	C***	F**	I	I	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	F**	C***	F**	I	I	C***
EWMA	unadjusted	***	less***	C***	I	F***	C***	F**	C***
	FR (2002)	***	less***	C***	I	F***	C***	F**	C***
	CPS (2005)	***	less***	C***	I	F***	C***	F**	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	***	less***	C***	I	F***	C***	F**	C***

Table 2.10: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***maximum-SR portfolio weights***, given ***SR maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$),
without relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

correlation, volatility	heteroskedasticity correction	portfolio reallocation (structural and individual)							
		structural	x GBP	y_1 EUR	y_2 CHF	y_3 CAD	y_4 JPY	y_5 AUD	y_6 NZD
unconditional	unadjusted	-	-	I	I	I	I	I	I
	FR (2002)	-	-	I	(F*)	I	I	I	I
	CPS (2005)	-	-	I	I	I	I	I	I
	extended $(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)$	-	-	I	I	I	I	I	I
GARCH(1,1)	unadjusted	***	less***	F***	C**	I	F*	I	C***
	FR (2002)	***	less***	F***	C*	I	F*	I	C***
	CPS (2005)	***	less***	F***	C*	I	F*	I	C***
	extended $(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)$	***	less***	F***	C*	I	F*	I	C***
EWMA	unadjusted	**	-	C**	C*	I	C**	F*	I
	FR (2002)	***	less*	C***	C**	I	C***	F***	C*
	CPS (2005)	***	less*	C***	C*	I	C***	F**	I
	extended $(\tilde{\beta}_x, \tilde{\sigma}_{\varepsilon_x}^2)$	***	less*	C***	C*	I	C***	F**	I

Table 2.11: structural reallocations, shifts in GBP currency weight,
currency interdependences (I), contagions (C), and flights (F) of GBP v. the others
following the Brexit referendum (tranquility state: one year, distress state: one month),
based on ***maximum-SR portfolio weights***, given ***SR maintenance***,
100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases,
with relevant interest rates partialled out
(*, **, and ***: significant at 0.1, 0.05, and 0.01, respectively)
(results of significant individual asset reallocations are in $(\cdot)$ when there is no structural change)

Variations in the contagion and flight statistical results, probably caused by risk-return tradeoffs, do not show significant differences from the risk aversion maintenance case.

Nevertheless, in terms of the overall number of detections of contagion and flight, maintaining the SR leads to fewer cases of such detections than maintaining the risk aversion. This is perhaps because it is difficult to remain at the same degree of risk aversion in the state of distress (D), with the previous portfolio allocation. On the other hand, the SR maintenance (i.e. preserving the best possible CAL shape) seems to require fewer reallocations, possibly because, with short sales allowed, a higher SR can be achieved by shorting assets that have negative expected returns balanced with comparatively not too high volatilities.

As regards heteroskedasticity corrections, the outcomes are similar among the extended and other frameworks, within the same correlation-volatility type.

Concerning interest rates, removal of their effect reduces the amount of single-asset reallocation that is suggested, especially with unconditional correlations and with CAD.

In addition, **Table 2.12** and **Table 2.13** illustrate that there are shifts in the portfolio allocations when moving from the state of tranquility (T) to the state of distress (D), such that the proceeds from short selling the risky portfolio are remarkably diversified away to riskless assets (also see **Figure 2.9**). As in the risk aversion maintenance case, GARCH(1,1), where the political uncertainty information dies out more quickly than with EWMA, and the unconditional schemes yield smaller short positions in the risk portfolio (also see **Appendix 1.E Analytical and hypothesis-testing parameters**).

correlation, volatility	heteroskedasticity correction	tranquility-state allocation		distress-state allocation	
		riskless	risky	riskless	risky
unconditional	unadjusted	0	1	5.069	-4.069
	FR (2002)	0	1	4.521	-3.521
	CPS (2005)	0	1	4.557	-3.557
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	4.548	-3.548
GARCH(1,1)	unadjusted	0	1	1.542	-0.542
	FR (2002)	0	1	1.595	-0.595
	CPS (2005)	0	1	1.591	-0.591
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	1.592	-0.592
EWMA	unadjusted	0	1	1.596	-0.596
	FR (2002)	0	1	1.696	-0.696
	CPS (2005)	0	1	1.689	-0.689
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	1.690	-0.690

Table 2.12: riskless and risky portfolio allocations in the states of tranquility and distress, based on ***SR maximization*** and ***SR maintenance***, 100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases, ***without relevant interest rates partialled out***

correlation, volatility	heteroskedasticity correction	tranquility-state allocation		distress-state allocation	
		riskless	risky	riskless	risky
unconditional	unadjusted	0	1	4.562	-3.562
	FR (2002)	0	1	3.755	-2.755
	CPS (2005)	0	1	3.736	-2.736
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	3.898	-2.898
GARCH(1,1)	unadjusted	0	1	1.328	-0.328
	FR (2002)	0	1	1.375	-0.375
	CPS (2005)	0	1	1.372	-0.372
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	1.373	-0.373
EWMA	unadjusted	0	1	1.989	-0.989
	FR (2002)	0	1	2.138	-1.138
	CPS (2005)	0	1	2.388	-1.388
	extended ($\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x}$)	0	1	2.389	-1.389

Table 2.13: riskless and risky portfolio allocations in the states of tranquility and distress, based on ***SR maximization*** and ***SR maintenance***, 100% distress-state turnover limit ($|w_i| \leq \overline{w} = 1$), particularly for the unconditional cases, ***with relevant interest rates partialled out***

Moreover, complete diversification away from risky to riskless assets (i.e. from being totally long in risky assets to being even over-long in riskless assets in a state of distress), as in **Table 2.8**, **Table 2.9**, **Table 2.12**, and **Table 2.13**), for US-based reward-to-risk-maximizing investors, also represents an extreme form of flight to quality due to emerging UK political uncertainty.

2.7.3 Structural portfolio rebalancing

After the results of shifts in individual asset weights are learned on the basis of negative contagion, flight to quality, and possible diversifications among the major currencies, structural portfolio rebalancing may also be discussed.

From all the risk minimization and reward-to-risk maximization case studies, structural portfolio rebalances are statistically detected in several scenarios (see **Table 2.2**, **Table 2.3**, **Table 2.4**, **Table 2.5**, **Table 2.6**, **Table 2.7**, **Table 2.10**, and **Table 2.11**), especially when portfolio risk is minimized with interest rates partialled out or when portfolio reward-to-risk is maximized without interest rates partialled out.

The requirement of structural portfolio rebalancing could be as a consequence of the existence of variance-covariance shocks and correlation risk, which is possibly caused by the UK political uncertainty, rising from the Brexit referendum, in the case study. The presence of variance-covariance shocks and correlation risk may also be somewhat noticed from the examined currencies' characteristics, which are risk-return profile and correlations, around the Brexit referendum (see **Appendix 2.A** and **Appendix 2.B**).

Pre-Brexit-referendum optimal portfolio allocation (i.e. in the state of tranquility) could represent the myopic portfolio choice, whereas post-Brexit-referendum optimal portfolio allocation (i.e. in the state of distress or during the uncertain time span) could represent the dynamic portfolio choice. Thus, the structural rebalancing needed around the UK political incident also maybe imply the significant difference between the currency myopic portfolio choice and dynamic portfolio choice, leading to investors' hedging demands. Hence, the optimal currency myopic portfolio choice might be unable to capture the possible ex ante willingness of market participants to hedge against potential upcoming high uncertainty in the foreign exchange market.

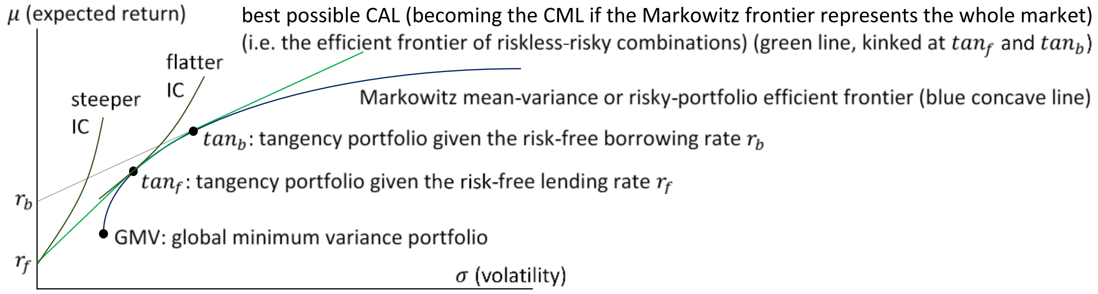
The empirical portfolio analysis of financial contagion and capital flight, for the case study of Brexit and currencies or other cases, could also be conducted with financial data at higher frequencies than daily. Nonetheless, unlike the possible availability of the intraday data of typical financial asset returns, a global factor's intraday data may not be available. Thus, mixed-frequency methodologies, such as mixed-data sampling (MIDAS), may be needed.

2.8 Concluding remarks

This chapter extends the investigation of financial contagion and capital flight in the previous chapter, on the basis of portfolio diversification and rebalancing, by applying mean-variance analysis, taking into account shrinkage estimations and newly offered risk-based weight constraints for robust portfolio optimization, and multivariate instead of bivariate correlations. The Brexit empirical outcomes from computations, primarily based on risk-return tradeoffs and covariances, seem remarkably different from those drawn from focusing purely on the financial-return bivariate correlations between the shock-originating and shock-receiving economies. In the mean-variance portfolio analysis, not only JPY, the currency negatively correlated with the initially shocked currency, GBP, is found to exhibit a possible flight to quality, as happened in Chapter 1. In portfolio risk minimization, a flight to quality also happens with currencies possessing substantially low volatilities. In portfolio reward-to-risk maximization, a flight to quality is also suggested for a currency with a worthwhile risk-return balance. Overall, the portfolio analysis provides inferences on diversification benefits quite differently from the correlation analysis in Chapter 1. Hence, due to the behaviors of means and (co)variances during highly uncertain times, the possible portfolio diversification benefits inferred from a bivariate correlation analysis may not always apply in mean-variance portfolio optimizations. Besides this, the heteroskedasticity-correction methodologies (as in Chapter 1), among each other, do not yield vastly different statistical results when it comes to portfolio reallocation. Furthermore, around the Brexit referendum, the structural reallocation of currency portfolios quite needed can be viewed as investors' hedging demands.

Endnotes:

[a] The following figure represents the Markowitz efficient frontier, the best possible capital allocation line (CAL) and the global minimum variance (GMV) and tangency portfolios (also see '***risk aversion and portfolio choice***' in Endnote b).



The figure shows the GMV and tangency portfolios on the Markowitz efficient frontier and on the best possible CAL when the risk-free lending rate is lower than the risk-free borrowing rate ($r_f < r_b$) although $r_f = r_b$ (infering $tan_f = tan_b$) is possible and also applied in this paper. Besides, riskless-risky combinations beyond tan_f leftward on the best possible CAL implies lending position in the money market. Neither lending nor borrowing is between tan_f and tan_b on the best possible CAL. And, the combinations beyond tan_b rightward on the best possible CAL implies borrowing position in the money market.

[b] ***Risk aversion and portfolio choice*** on the Markowitz efficient frontier and on the best possible CAL (also see Endnote a). On the Markowitz efficient frontier, higher risk aversion (with a steeper indifference curve (IC)) is closer to the GMV point. On the best possible CAL, risk aversion may be either defensive, moderate, or aggressive, described using a riskless-risky portfolio combination, allocated as $w_f + \mathbf{w}'_{SR}\mathbf{1}_N = 1$ (w_f: riskless-asset weight, $\mathbf{w}'_{SR}\mathbf{1}_N$: risky-asset weight, $\mathbf{w}_{SR} = \gamma^{-1}\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$, γ: risk aversion degree, w_f increases in γ while $\mathbf{w}'_{SR}\mathbf{1}_N$ decreases in γ).

- ***defensive*** (with considerably steep IC): partially long risk-free and partially long risky assets ($w_f < 1, \mathbf{w}'_{SR}1_N < 1$), i.e. lending in the money market. In the case of several or all negative risky-asset returns, possibly short selling the risky portfolio to invest in the risk-free asset more than the initial endowment ($w_f > 1, \mathbf{w}'_{SR}1_N < 0$).
- ***moderate***: no risk-free but only risky assets ($w_f = 0, \mathbf{w}'_{SR}\mathbf{1}_N = 1$), i.e. neither lending nor borrowing but investing in the tangency portfolio ($\gamma = \mathbf{1}'_N\mathbf{V}^{-1}(\boldsymbol{\mu} - r_f\mathbf{1}_N)$).
- ***aggressive*** (with considerably flat IC): short risk-free and over-long risky assets ($w_f < 0, \mathbf{w}'_{SR}\mathbf{1}_N > 1$), i.e. borrowing in the money market in order to increasingly invest in risky assets where the borrowing rate (r_b) may be higher than r_f flattening the best possible CAL when it is beyond the tangency to r_b.

Appendices

Appendix 2.A Risk-return profile of the examined currencies

Table 2.A.1 shows the tranquility-state and distress-state risk-return profile, that is the expected returns and volatilities, of the studied currencies' daily returns (all against the USD), based on different methods: time-unconditional, GARCH(1,1) ($\alpha_2 \cong 0.854$), GARCH(1,1) ($\alpha_2 \cong 0.914$), and RiskMetrics EWMA (see **2.3.1 Risk-return tradeoff and portfolio mean and covariance**) (for GARCH(1,1), different α_2 values are pertaining to the MLE involving currency correlations without and then with interest rates partialled out, see **Appendix 1.E Analytical and hypothesis-testing parameters**).

The expected returns and volatilities (i.e. annualized means and standard deviations) could possibly demonstrate how the currencies individually behave over time, when moving from tranquility to distress states.

method	statistic	state	GBP	EUR	CHF	CAD	JPY	AUD	NZD
unconditional	return	tranquility	-0.058	0.015	-0.025	-0.035	0.152	-0.017	0.055
		distress	-1.464	-0.403	-0.347	-0.318	-0.044	-0.197	-0.404
	volatility	tranquility	0.088	0.097	0.093	0.097	0.104	0.124	0.139
		distress	0.323	0.089	0.062	0.097	0.182	0.131	0.150
GARCH(1,1) ($\alpha_2 \cong 0.854$)	return	tranquility	0.661	0.129	0.305	0.188	0.587	0.580	0.807
		distress	-0.112	-0.198	-0.221	-0.228	-0.335	-0.223	-0.743
	volatility	tranquility	0.113	0.091	0.083	0.089	0.102	0.115	0.128
		distress	0.133	0.085	0.078	0.090	0.124	0.121	0.138
GARCH(1,1) ($\alpha_2 \cong 0.914$)	return	tranquility	0.397	0.068	0.264	0.154	0.483	0.394	0.629
		distress	-0.322	-0.189	-0.180	-0.152	-0.274	-0.049	-0.415
	volatility	tranquility	0.129	0.083	0.072	0.081	0.107	0.105	0.115
		distress	0.221	0.074	0.057	0.085	0.156	0.120	0.142
EWMA	return	tranquility	0.278	0.043	0.209	0.126	0.422	0.268	0.512
		distress	-0.405	-0.171	-0.147	-0.117	-0.146	0.011	-0.227
	volatility	tranquility	0.122	0.082	0.073	0.085	0.111	0.108	0.117
		distress	0.236	0.078	0.061	0.088	0.158	0.122	0.141

Table 2.A.1: tranquility-state and distress-state (pre- and post-Brexit referendum) (covering 262 and 21 trading days) means and annualized volatilities of the studied currencies (USD as base), based on different methods: time-unconditional, GARCH(1,1) ($\alpha_2 \cong 0.854$), GARCH(1,1) ($\alpha_2 \cong 0.914$), and EWMA

Appendix 2.B Correlation matrices and heat maps without heteroskedastic correction

The matrices of unconditional, GARCH(1,1), and EWMA correlation coefficients in tranquility and distress, ρ_T and ρ_D (ρ in this context not only between GBP each of the other currencies (as in Chapter 1) but also pairwise among the other currencies), respectively, ***without heteroskedastic correction*** and ***without interest rates partialled out***, and the heat maps, are illustrated in **Table 2.A.2**. The heat maps may convey information on possible diversification among those currencies.

	unconditional							**GARCH(1,1)**							**EWMA**						
ρ_T	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
GBP	1							1							1						
EUR	0.433	1						0.452	1						0.429	1					
CHF	0.348	0.845	1					0.253	0.789	1					0.158	0.739	1				
CAD	0.428	0.253	0.216	1				0.498	0.365	0.296	1				0.532	0.595	0.543	1			
JPY	0.106	0.535	0.541	-0.012	1			0.083	0.483	0.455	0.021	1			0.027	0.474	0.395	0.181	1		
AUD	0.432	0.285	0.194	0.646	0.042	1		0.510	0.422	0.282	0.665	0.063	1		0.527	0.702	0.538	0.704	0.236	1	
NZD	0.380	0.347	0.277	0.604	0.176	0.746	1	0.398	0.379	0.258	0.581	0.250	0.717	1	0.373	0.528	0.354	0.564	0.512	0.683	1
ρ_D	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
GBP	1							1							1						
EUR	0.908	1						0.493	1						0.764	1					
CHF	0.673	0.812	1					0.286	0.817	1					0.466	0.751	1				
CAD	0.800	0.840	0.697	1				0.520	0.367	0.270	1				0.709	0.741	0.598	1			
JPY	-0.778	0.535	-0.510	-0.559	1			-0.359	0.180	0.308	-0.154	1			-0.618	-0.357	-0.133	-0.341	1		
AUD	0.744	0.735	0.505	0.785	-0.500	1		0.545	0.395	0.211	0.671	-0.126	1		0.673	0.709	0.416	0.732	-0.332	1	
NZD	0.551	0.559	0.339	0.638	-0.236	0.785	1	0.396	0.393	0.234	0.583	0.051	0.733	1	0.492	0.545	0.261	0.591	-0.056	0.743	1

Table 2.A.2: Unconditional, GARCH(1,1) ($\alpha_1 \cong 0.054$, $\alpha_2 \cong 0.854$), and EWMA ($\varphi = 0.94$) lower-triangular correlation matrix (upper-triangular elements left out), ***without heteroskedastic correction*** and ***without interest rates partialled out***,
around the time of the Brexit referendum of 23 June 2016, moving from a one-year tranquility-state (T) to a one-month distress-state (D) time span,
heat maps from green to red: ρ closer to -1, green, ρ closer to 0, yellow, and ρ closer to $+1$, red
(greener: higher potential diversification benefits, redder: higher potential diversification disadvantages)

The matrices of unconditional, GARCH(1,1), and EWMA correlation coefficients in tranquility and distress, ρ_T and ρ_D, respectively, ***without heteroskedastic correction*** and ***with interest rates partialled out***, and the heat maps, are illustrated in **Table 2.A.3**. The heat maps may convey information on possible diversification among those currencies.

	unconditional							**GARCH(1,1)**							**EWMA**						
ρ_T	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
GBP	1							1							1						
EUR	0.467	1						0.451	1						0.430	1					
CHF	0.379	0.844	1					0.248	0.795	1					0.160	0.741	1				
CAD	0.441	0.288	0.249	1				0.433	0.340	0.270	1				0.459	0.545	0.492	1			
JPY	0.116	0.534	0.542	-0.010	1			0.030	-0.164	0.407	-0.042	1			-0.010	0.689	0.354	0.089	1		
AUD	0.456	0.313	0.244	0.652	0.024	1		0.493	0.425	0.253	0.631	0.010	1		0.518	0.721	0.535	0.664	0.221	1	
NZD	0.406	0.381	0.324	0.618	0.161	0.758	1	0.372	0.382	0.192	0.572	0.165	0.685	1	0.363	0.531	0.310	0.523	0.491	0.658	1
ρ_D	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
GBP	1							1							1						
EUR	0.881	1						0.481	1						0.768	1					
CHF	0.239	0.572	1					0.278	0.813	1					0.469	0.747	1				
CAD	0.743	0.822	0.640	1				0.522	0.357	0.259	1				0.719	0.733	0.589	1			
JPY	-0.751	-0.694	-0.185	-0.442	1			-0.300	-0.062	0.315	-0.158	1			-0.587	-0.890	-0.169	-0.391	1		
AUD	0.646	0.762	0.432	0.809	-0.412	1		0.550	0.398	0.211	0.675	-0.143	1		0.694	0.719	0.423	0.736	-0.352	1	
NZD	0.477	0.655	0.246	0.664	-0.159	0.834	1	0.383	0.386	0.208	0.610	-0.025	0.743	1	0.521	0.554	0.241	0.615	-0.131	0.755	1

Table 2.A.3: Unconditional, GARCH(1,1) ($\alpha_1 \cong 0.081$, $\alpha_2 \cong 0.914$), and EWMA ($\varphi = 0.94$) lower-triangular correlation matrix (upper-triangular elements left out), ***without heteroskedastic correction*** and ***with interest rates partialled out***,
around the time of the Brexit referendum of 23 June 2016, moving from a one-year tranquility-state (T) to a one-month distress-state (D) time span,
heat maps from green to red: ρ closer to -1, green, ρ closer to 0, yellow, and ρ closer to $+1$, red
(greener: higher potential diversification benefits, redder: higher potential diversification disadvantages)

Appendix 2.C Distress-state correlation matrices and heat maps with heteroskedastic correction

The matrices of unconditional, GARCH(1,1), and EWMA correlation coefficients in distress, ρ_D, ***with heteroskedastic correction*** and ***without interest rates partialled out***, and the heat maps, are illustrated in **Table 2.A.4**. The heat maps may convey information on possible diversification among those currencies (only $\rho_{x,y}$s are corrected, see section **2.6 Standardized correlations with heteroskedasticity correction**).

		unconditional							**GARCH(1,1)**							**EWMA**						
	ρ_D	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
FR (2002)	GBP	1							1							1						
	EUR	0.414	1						0.118	1						0.242	1					
	CHF	0.188	0.812	1					0.063	0.817	1					0.110	0.751	1				
	CAD	0.269	0.840	0.697	1				0.127	0.367	0.270	1				0.207	0.741	0.598	1			
	JPY	-0.252	0.535	-0.510	-0.559	1			-0.081	0.180	0.308	-0.154	1			-0.163	-0.357	-0.133	-0.341	1		
	AUD	0.228	0.735	0.505	0.785	-0.500	1		0.135	0.395	0.211	0.671	-0.126	1		0.188	0.709	0.416	0.732	-0.332	1	
	NZD	0.138	0.559	0.339	0.638	-0.236	0.785	1	0.090	0.393	0.234	0.583	0.051	0.733	1	0.118	0.545	0.261	0.591	-0.056	0.743	1
CPS (2005)	GBP	1							1							1						
	EUR	0.493	1						0.147	1						0.296	1					
	CHF	0.231	0.812	1					0.078	0.817	1					0.136	0.751	1				
	CAD	0.329	0.840	0.697	1				0.157	0.367	0.270	1				0.254	0.741	0.598	1			
	JPY	-0.308	0.535	-0.510	-0.559	1			-0.100	0.180	0.308	-0.154	1			-0.201	-0.357	-0.133	-0.341	1		
	AUD	0.279	0.735	0.505	0.785	-0.500	1		0.168	0.395	0.211	0.671	-0.126	1		0.231	0.709	0.416	0.732	-0.332	1	
	NZD	0.170	0.559	0.339	0.638	-0.236	0.785	1	0.112	0.393	0.234	0.583	0.051	0.733	1	0.146	0.545	0.261	0.591	-0.056	0.743	1
extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	GBP	1							1							1						
	EUR	0.476	1						0.140	1						0.284	1					
	CHF	0.222	0.812	1					0.074	0.817	1					0.130	0.751	1				
	CAD	0.316	0.840	0.697	1				0.150	0.367	0.270	1				0.243	0.741	0.598	1			
	JPY	-0.296	0.535	-0.510	-0.559	1			-0.096	0.180	0.308	-0.154	1			-0.193	-0.357	-0.133	-0.341	1		
	AUD	0.268	0.735	0.505	0.785	-0.500	1		0.160	0.395	0.211	0.671	-0.126	1		0.222	0.709	0.416	0.732	-0.332	1	
	NZD	0.163	0.559	0.339	0.638	-0.236	0.785	1	0.107	0.393	0.234	0.583	0.051	0.733	1	0.140	0.545	0.261	0.591	-0.056	0.743	1

Table 2.A.4: Unconditional, GARCH(1,1) ($\alpha_1 \cong 0.054$, $\alpha_2 \cong 0.854$), and EWMA ($\varphi = 0.94$) lower-triangular correlation matrix (upper-triangular elements left out), ***with heteroskedastic correction*** and ***without interest rates partialled out***, of a one-month distress-state (D) time span,
heat maps from green to red: ρ closer to -1, green, ρ closer to 0, yellow, and ρ closer to $+1$, red
(greener: higher potential diversification benefits, redder: higher potential diversification disadvantages)

The matrices of unconditional, GARCH(1,1), and EWMA correlation coefficients in distress, ρ_D, ***with heteroskedastic correction*** and ***with interest rates partialled out***, and the heat maps, are illustrated in **Table 2.A.5**. The heat maps may convey information on possible diversification among those currencies (only $\rho_{x,y}$s are corrected, see section **2.6 Standardized correlations with heteroskedasticity correction**).

		unconditional							**GARCH(1,1)**							**EWMA**						
	ρ_D	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD	GBP	EUR	CHF	CAD	JPY	AUD	NZD
FR (2002)	GBP	1							1							1						
	EUR	0.364	1						0.115	1						0.244	1					
	CHF	0.052	0.572	1					0.061	0.813	1					0.111	0.747	1				
	CAD	0.227	0.822	0.640	1				0.128	0.357	0.259	1				0.212	0.733	0.589	1			
	JPY	-0.232	-0.694	-0.185	-0.442	1			-0.066	-0.062	0.315	-0.158	1			-0.151	-0.890	-0.169	-0.391	1		
	AUD	0.175	0.762	0.432	0.809	-0.412	1		0.137	0.398	0.211	0.675	-0.143	1		0.199	0.719	0.423	0.736	-0.352	1	
	NZD	0.113	0.655	0.246	0.664	-0.159	0.834	1	0.087	0.386	0.208	0.610	-0.025	0.743	1	0.127	0.554	0.241	0.615	-0.131	0.755	1
CPS (2005)	GBP	1							1							1						
	EUR	0.438	1						0.142	1						0.299	1					
	CHF	0.064	0.572	1					0.075	0.813	1					0.137	0.747	1				
	CAD	0.278	0.822	0.640	1				0.158	0.357	0.259	1				0.261	0.733	0.589	1			
	JPY	-0.285	-0.694	-0.185	-0.442	1			-0.082	-0.062	0.315	-0.158	1			-0.186	-0.890	-0.169	-0.391	1		
	AUD	0.216	0.762	0.432	0.809	-0.412	1		0.170	0.398	0.211	0.675	-0.143	1		0.244	0.719	0.423	0.736	-0.352	1	
	NZD	0.141	0.655	0.246	0.664	-0.159	0.834	1	0.108	0.386	0.208	0.610	-0.025	0.743	1	0.158	0.554	0.241	0.615	-0.131	0.755	1
extended $(\tilde{\beta}_x, \tilde{\sigma}^2_{\varepsilon_x})$	GBP	1							1							1						
	EUR	0.422	1						0.136	1						0.287	1					
	CHF	0.061	0.572	1					0.072	0.813	1					0.131	0.747	1				
	CAD	0.267	0.822	0.640	1				0.151	0.357	0.259	1				0.250	0.733	0.589	1			
	JPY	-0.273	-0.694	-0.185	-0.442	1			-0.078	-0.062	0.315	-0.158	1			-0.178	-0.890	-0.169	-0.391	1		
	AUD	0.207	0.762	0.432	0.809	-0.412	1		0.162	0.398	0.211	0.675	-0.143	1		0.234	0.719	0.423	0.736	-0.352	1	
	NZD	0.134	0.655	0.246	0.664	-0.159	0.834	1	0.103	0.386	0.208	0.610	-0.025	0.743	1	0.151	0.554	0.241	0.615	-0.131	0.755	1

Table 2.A.5: Unconditional, GARCH(1,1) ($\alpha_1 \cong 0.081$, $\alpha_2 \cong 0.914$), and EWMA ($\varphi = 0.94$) lower-triangular correlation matrix (upper-triangular elements left out), ***with heteroskedastic correction*** and ***with interest rates partialled out***, of a one-month distress-state (D) time span,
heat maps from green to red: ρ closer to -1, green, ρ closer to 0, yellow, and ρ closer to $+1$, red
(greener: higher potential diversification benefits, redder: higher potential diversification disadvantages)

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