UNIVERSITÀ CATTOLICA DEL SACRO CUORE
MILANO

Scuola di Dottorato in Economia (DEFAP)

ciclo XXIX

S.S.D: SECS-P/01

ESSAYS ON MONETARY AND FISCAL POLICY TRANSMISSIONS IN DEVELOPING COUNTRIES WITH SHADOW ECONOMY

Tesi di Dottorato di : Eric Amoo BONDZIE
Matricola : 4212339

Anno Accademico 2015/2016

Scuola di Dottorato in Economia (DEFAP)

ciclo XXIX

S.S.D: SECS-P/01

ESSAYS ON MONETARY AND FISCAL POLICY TRANSMISSIONS IN DEVELOPING COUNTRIES WITH SHADOW ECONOMY

Tesi di Dottorato di : Eric Amoo BONDZIE

Matricola : 4212339

Supervisore: Prof. Patrizio TIRELLI

Coordinatore: Ch.mo Prof. Gianluca FEMMINIS

Anno Accademico 2015/2016

Declaration

I hereby declare that this submission is my own work towards the award of the Doctor of Philosophy degree in Economics and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Professor Patrizio Tirelli, for his guidance and advice. I also acknowledges the useful comments provided by Prof. Giovanni Di Bartolomeo and Prof. Rafealle Rossi.

Foreword

The role played by shadow economy in monetary and fiscal policy transmissions have received considerable attention from researchers and policy-makers because they shed more light on the transmission processes. Many theoretical literatures have suggested that shadow economy or the informal sector is a powerful buffer which absorbs large proportions of the transmission channels of macroeconomic policies. Recent development in Dynamic Stochastic General Equilibrium (DSGE) models have gained momentum due to its ability to evaluate alternative macroeconomic policy measures. However, such developments were tailored towards the advanced economies and therefore lacked the prerequisite ingredients to be used for modelling developing and emerging economies where certain features of the advanced countries are lacking. Empirical literature on developing countries suggest that most of these economies are characterised by weak financial sector, large proportions of liquidity constrained individuals, existence of large informal sector, external shock vulnerabilities and weak economic and political institutions. Given this background, our main aim is to develop a theoretical DSGE model with shadow economy and investigate their impact on the transmissions of monetary and fiscal policies in developing and emerging countries. Our baseline model follows Smets and Wouters (2003, 2007), and we include the necessary altercations to suits our research interest. The dissertation is organised in three chapters as follows.

Chapter one seeks to examine the transmission effects and efficacy of monetary policy and other structural shocks in a standard new Keynesian DSGE model with the interaction of shadow economy. Our model determines whether the presence of shadow economy affects the responses of the official economy and also clarifies the changes in the transmission mechanism within both sectors. The chapter contained five exogenous processes in the official sector namely the risk premium shock, investment specific shock, total factor productivity, price mark-up shock and the conventional monetary policy shock. In effects, our model showed that the presence of shadow economy induces factor flows across sectors and crowding-out of formal sector's activities into the shadow sector when there are negative transmissions of the shock in the formal sector. This strengthens the existing notion that shadow sector serves as a cyclical buffer in a two-sector model.

The second chapter describes a new Keynesian DSGE model with shadow economy and investigate the role of fiscal policies over the aggregate business cycle. In this chapter, we sought to elucidate whether the presence of shadow economy dampens or amplifies the effect of fiscal policy transmissions. We further tried to understand whether fiscal policies can be used to stabilise the economy in response to shocks. We concluded that, tax hikes in an economy with relatively large informal sector lead to a sizeable tax evasion and a boost in the shadow economy making standard aggregate estimates of fiscal policies ineffective while government spending shock slows down the activities in the shadow sector. We also found that the presence of shadow sector lead to factor inputs reallocation across sectors during fiscal policy shocks. It also turns out that the incorporation of shadow sector significantly reduces the government spending multipliers whereas the labour income tax multipliers are increased. Our results from the fiscal feedbacks on government spending (income taxes) stabilized the economy by reducing (raising) output levels and these results even become stronger with the presence of shadow economy.

In chapter three, we study the interplay of rule-of-thumb consumers and the presence of shadow economy focusing on fiscal policy disturbances. Our basic motivation is to know whether the incorporation of shadow economy weakens the amplifying effect of rule-of-thumb consumers on fiscal multipliers. Our results indicated that the amplifying mechanism caused by rule-of-thumb consumers becomes irrelevant given that the disposable income of the rule-of-thumb households as a weighted average of labour incomes earned from the two sectors is virtually unaffected by the fiscal shocks.

In a nut shell, our model contributes to provide a theoretical background to policy-oriented literature that sees consumer and sectoral heterogeneity as an important component of future macroeconomic policy framework. Our results have shown that shadow economies play a significant role in both monetary and fiscal policy analysis and it therefore become paramount to incorporate them in DSGE models especially in economies with relatively larger share of shadow activities. This would help policy makers to understand the underlying transmission processes to make informed decisions.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Implicit in much of the literature on the shadow economy is the view that shadow activity is undesirable...However, it is not certain that all shadow economic activities should be discouraged-

(Matthew H. Fleming).

This page is intentionally left blank

CHAPTER 1

Monetary Policy Transmissions in Developing Countries: A DSGE Model with Shadow Economy.

1.1 Introduction

Dynamic Stochastic General Equilibrium (henceforth DSGE) models are basically the extension of the Real Business Cycle (henceforth RBC) models with the introduction of price and wage rigidities. The most important recent contributions in terms of specification and standardization of modelling procedures involved in DSGE modelling are due to Smets and Wouters (2003, 2007) and Christiano et al. (2005). As a result of this significant improvement in DSGE modelling literature, many central banks of advanced countries have already developed DSGE models for policy analysis and forecasting. These models have succeeded in replicating business cycles features of developed economies and with considerable importance for policy analysis and forecasting at central banks. However, for developing and low-income countries, the adoption of such models require a significant amount of altercations to be coherent with relevant micro evidence. Most developing countries are characterised by weak financial sector, existence of large informal sector, external shock vulnerability, and weak economic and political institutions. The challenge of data inconsistency and unavailability in most developing and low-income countries also become a problem. For most of the existing literature on DSGE models for emerging economies, key parameters are borrowed from the advanced economy literature and data transformation remains inadequate. It therefore becomes erroneous to implement the same DSGE models built for developed economies in the developing countries without the necessary considerations of developing countries microeconomic features. The role played by informal sector on monetary policy transmission and economic activities have received considerable attention in recent times among academic researchers and policy makers. In spite of this, relatively little has been written on the conduct of monetary policy transmissions on economies with large informal sectors.[1] The study of informal sectors in the economy have become paramount because they shed more light on the transmission processes of monetary policy in both developed and developing countries. It must also be emphasized that informal sectors

[1] Informality is described here as the unregistered, hidden, shadow or unofficial economic activities which are not under the purview of policy makers. The terms are used interchangeably with informal sector or shadow economy in this literature.

are mostly observed in developing and low-income countries especially in Sub-Sahara Africa, the Caribbean, the Asia and slightly observed in some advanced countries like the eastern European countries. Given this background, the study seeks to examine the transmission effects and efficacy of macroeconomic policies with informal sectors and further introduces other structural shocks to capture certain economic features of a standard DSGE model. Our model would determine whether the presence of shadow economy affects the responses of the official economy and also to clarify the changes in the transmission mechanism from the official to shadow sector.

DSGE literature is scant on developing countries and over the years efforts are been made by policy makers to capture the salient features of developing countries. Batini et al. (2011) has recently developed a DSGE model for Indian economy with informality in goods market in the presence of credit constraints. They also introduced labour market frictions in the formal sector using Zenou (2008). With the use of Bayesian technique for estimating parameters, they showed that the inclusion of informal sector and financial frictions improved their model fitness. Peiris and Saxegaard (2007) introduced credit frictions in the presence of informality with an assumption that part of the inputs used in the production are financed through borrowing at a premium over deposits from the informal sector. The study was aimed at evaluating monetary policy trade-offs in low-income countries with informal lending sources. Conesa et al. (2002) incorporated informal goods producing sector with differentiated technology in a simple real business cycle model. In this model, sectoral trade-off is allowed in the presence of a wage premium in the formal sector. Furthermore, labour is assumed to be indivisible in the formal sector and households can choose working between the two sectors with a given probability. Aruoba (2010), and Aruoba and Schorfheide (2011) also introduced cash-in-advance constraint to differentiate informal sector from the formal sector by assuming that money is the only medium of exchange used in the informal sector. They found that large informal sector gets smaller in size and overall tax collection becomes higher under rising inflation. Mattesini and Rossi (2009) analysed the monetary policy in a dual economy in the new Keynesian framework with one competitive (informal) and one unionized (formal) sector. They concluded that, the level of output is associated with the relative size of the two sectors. Castillo and Montoro (2008) modelled their economy with frictions in the labour market by introducing formal and informal labour contracts and analysed the interaction between the two sectors and monetary policy. They introduced informality through hiring costs owing to labour market. In their model, firms in the wholesale sector are assumed to balance the high productivity in formal sector with the lower hiring costs faced by the informal sector. The main finding of this theoretical framework is the cyclical behaviour of informal sector. Through this channel a link between informality, the inflation dynamics and monetary policy is established and the study supports the idea of informal labour market being a buffer for an economy. Colombo et al. (2016) investigates the response of the shadow economy to banking crises. Their empirical analysis based on a large sample of countries suggests that the informal sector is a powerful buffer, which expands during banking crises and absorbs a large proportion of the fall in the official output. They assumed limited access to external finance and production technology to be relatively more labour intensive in the informal sector. Following a banking shock in the official sector, the model predicted a large negative transmission to the unofficial economy.

In view of this, we build a qualitative DSGE model with formal and informal sector in the goods market based on Smets and Wouters (2003, 2007). Smets and Wouters model built on Christiano et al. (2005), but features a number of frictions that appear to be necessary to capture the empirical persistence in the main Euro area macroeconomic data. Many of these frictions have become quite standard in the DSGE literature. Smets and Wouters (2007) model exhibits both sticky nominal prices and wages that adjust following a Calvo mechanism but we deviate from that and model goods producer's prices using Rotemberg (1982) framework with full indexation of prices.[2] Our model also incorporates a variable capital utilisation rate which tends to smooth the adjustment of the rental rate of capital in response to changes in output. As in Smets and Wouters (2007), the cost of adjusting the utilisation rate is expressed in terms of consumption goods. The cost of adjusting the capital stock is modelled as a function of the changes in investment, rather than the level of investments. An important feature of our model is a competitive labour market where firms in the two sectors pay the same consumption real wage. This assumption is motivated by the theoretical contributions from Amaral and Quintin (2006); Pratap and Quintin (2006) and supported by Maloney (1999, 2004). The contributions by Pratap and Quintin (2006) on developing countries provided evidence against labour market segmentation and suggested that labour market arguments are not necessary to account for the silent features of labour market in developing countries. Another deviation from Smets and Wouters model is the calibration of technologies and price mark-ups parameters in each sector. The model introduces several structural shocks asymmetrically in the formal sector that include risk premium, investment, technological, price mark-up and the conventional monetary policy shock.

The rest of this chapter is organised as follows; in the next section (section two) we present and extensively discuss the features of the theoretical DSGE model, section three involves the description of parameters used for calibrating the model to fit developing and low-income countries and analyse the results of the model and the last section (section four) concludes the

[2] In fact, Smets and Wouters (2003, 2007) follows a partial indexation of prices.

research.

1.2 The Model

In this section we introduce and discusses the qualitative DSGE model based on Smets and Wouters (2003, 2007) with two-sectors, official sector and the shadow sector. The model features the following types of agents: households, intermediate goods producers and final goods producers operating in each sector of the economy. We then introduce the standard monetary policy rule set by the Central Bank to complete the model. Households are standard and maximises a utility function over a time horizon, supply the same level of labour services to goods producers in each sector. In fact, we do not explicitly model the financial sector, however, as argued in Justiniano et al. (2011), investment specific shocks may be interpreted as a proxy for more fundamental disturbances to the functioning of the financial sector. Households wealth is accumulated by purchasing government bonds and investment in production. Households also decide on how much capital to accumulate based on the capital adjustment cost and capital utilisation. The intermediate goods producers supply their intermediate goods to final goods producers who differentiate and repackage them into final goods for households' consumption. Final goods producers in both sectors are able to reset their prices *ala* Rotemberg model. And finally, we model the central bank to follow strictly inflation targeting policy, this enable us to follow carefully the monetary transmission mechanisms in the economy.

1.2.1 Households

There is a continuum of households of measure unity who supply labour services to firms in both sectors of the economy. Each household is composed by individuals who work in the official and unofficial sectors. Each household member consumes, work and return the wages they earn to the household. Households hold their financial wealth in the form of government bond and also by supplying capital to goods producers in both sectors. The remaining part of their income is spent on consumption goods obtained from final goods producers and investment in physical capital. Households total income therefore consist of labour income, plus cash flow from participating in state-contingent securities offered by the government or the central bank, the returns on physical capital stock and profit derived from investing in goods producers. Within each household, there is a mutual consumption risk sharing so individual consumption decisions are the same and independent from their working conditions. The representative agent's lifetime utility is characterize by:

$$U_t^i = E_t \sum_{n=0}^{\infty} \beta^n \left\{ \ln(c_{t+n}^i) - \chi \frac{l_t^{i(1+\phi)}}{1+\phi} \right\} \tag{1.1}$$

where χ is a parameter that regulates the disutility of work and ϕ defines the Frisch elasticity of substitution for labour. Household members, for each sector, own the goods producers, hold physical capital and choose their investment to both sectors of the economy. As a result of consumption risk sharing of sectoral employments, consumption and investment decisions are identical across individuals. Households can increase the supply of rental services from capital by investing in additional capital and also through their capital utilisation.

Households consumption basket c_t is described as a constant elasticity of substitution (CES) aggregate over the two-sector's consumption bundle:

$$c_t = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^o)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^u)^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{1.2}$$

Furthermore, each c_t is also defined as:

$$c_t = \left(\int_0^1 c_t^{i\left(\frac{\epsilon^i - 1}{\epsilon^i}\right)} dz^i \right)^{\frac{\epsilon^i}{\epsilon^i - 1}}$$

where φ_c indicates official sector consumption goods bias and $\epsilon_c > 1$ is the measure of elasticity of substitution between official and unofficial consumption bundles (c_t^o) and (c_t^u) whereas $\epsilon^i > 1$ measures the elasticity of substitution among the differentiated goods that form c_t. Minimizing total consumption expenditure subject to the consumption bundle given above yields the following demand function for each good:

$$c_t^o = \varphi_c \left(\frac{P_t^o}{P_t} \right)^{-\epsilon_c} c_t \tag{1.3}$$

$$c_t^u = (1-\varphi_c) \left(\frac{P_t^u}{P_t} \right)^{-\epsilon_c} c_t \tag{1.4}$$

The consumption price index is given as:

$$P_t = \left[\varphi_c \left(P_t^o\right)^{1-\epsilon_c} + (1-\varphi_c)\left(P_t^u\right)^{1-\epsilon_c}\right]^{\frac{1}{1-\epsilon_c}} \tag{1.5}$$

In a symmetric way, households provide labour services to both sectors of the economy and we assume nominal wages to be flexible in both sectors, thus labour market equilibrium requires that the marginal rate of substitution between total labour supplied to both sectors equals the real consumption wage in the economy.[3]

Their intertemporal budget constraint is:[4]

$$c_t + \frac{P_t^o}{P_t} i_t^o + \frac{P_t^u}{P_t} i_t^u + \frac{B_t}{P_t R_t \varepsilon_t^{RISK}} = \frac{P_t^o}{P_t} w_t^o l_t^o + \frac{P_t^u}{P_t} w_t^u l_t^u + \frac{P_t^o}{P_t} r_t^{k,o} u_t^o \bar{k}_t^o +$$

$$+ \frac{P_t^u}{P_t} r_t^{k,u} u_t^u \bar{k}_t^u + \frac{B_{t-1}}{P_t} - \frac{P_t^o}{P_t} a(u_t^o)\bar{k}_t^o - \frac{P_t^u}{P_t} a(u_t^u)\bar{k}_t^u + \frac{P_t^o}{P_t}\Pi_t^o + \frac{P_t^u}{P_t}\Pi_t^u \tag{1.6}$$

where B_t is government bond that pays one unit of currency in period $t-1$ and R_t is the gross nominal interest rate. We define a number of sectoral variables: the relative goods prices P_t^i, the capital k_t^i, labour l_t^i, the returns on capital $r_t^{k,i}$, the utilisation rate of capital $a(u_t^i)$ and Π_t^i being the profit received from investment in goods production. ε_t^{RISK} is the risk premium in the returns to bonds, which might reflect a premium that households require to hold on one period bond and it follows an AR(1) stochastic process with an $i.i.d$ error term given as:

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{1.7}$$

Households sectoral capital accumulation is driven by the standard dynamic equation for capital given respectively as:

$$\bar{k}_{t+1}^o = (1-\delta)\bar{k}_t^o + \varepsilon_t^{INV}\left[1 - S\left(\frac{i_t^o}{i_{t-1}^o}\right)\right] i_t^o \tag{1.8}$$

$$\bar{k}_{t+1}^u = (1-\delta)\bar{k}_t^u + \left[1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right)\right] i_t^u \tag{1.9}$$

where $S(.)$ is the capital adjustment cost function and δ is the depreciation rate.[5] ε_t^{INV} is the stochastic shock to the price of investment relative to consumption goods and it also follows an exogenous process with an $i.i.d.$ error term given as:

$$ln\varepsilon_t^{INV} = \rho^{INV} ln\varepsilon_{t-1}^{INV} + \xi_t^{INV} \tag{1.10}$$

Here we note that, exogenous investment shock affects only investment in the official sector and not the shadow sector investment. Households in addition choose the utilisation rate of capital with the amount of effective capital given as:[6]

$$k_t^i = u_t^i \bar{k}_{t-1}^i \tag{1.11}$$

[3]The labour market equilibrium requires that $w_t = mrs_t$, where $mrs_t = -U_{l,t}/U_{c,t}$ is the marginal rate of substitution between consumption and labour supplied in period $t+n$ for the households. This means that the official and shadow sector would pay the same consumption wage to workers (Gali, 2008).

[4]Here we ignore superscript i.

[5]In the steady state, $S(1) = S'(1) = 0$, $S''(1) > 0 \equiv \varpi$ with ϖ being the adjustment cost parameter.

[6]In the steady state, utilisation cost function implies that: $u_s^i = 1$ and $a(1) = 0$.

Households face the usual maximization problem of maximizing their expected discounted sum of instantaneous utility (1.1) subject to equations (1.6), (1.8), (1.9) and (1.11). Letting λ_t denote the Lagrangian multiplier for the household's budget constraint and $\lambda_t Q_t^i$ the Lagrange multiplier for the capital accumulation equations whereby Q_t^i is the Tobin's q which is equal to one, when there are no capital adjustment costs. It can be interpreted as the shadow relative price of one unit of capital with respect to one unit of consumption. The first order conditions with respect to consumption (c_t), government bond (B_t), sectoral labour (l_t^i), sectoral capital ($\bar{k}_{t+1}^i$), sectoral investment (i_t^i) and capital utilisation (u_t^i) are respectively given below.[7] The intertemporal marginal utility of consumption is:

$$U_{c,t} = \lambda_t = \frac{1}{c_t} \tag{1.12}$$

The consumption Euler equation from government bond is:

$$\lambda_t = \varepsilon_t^{RISK} R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{1.13}$$

In competitive labour market, the standard labour supply conditions hold as:

$$U_{l,t}^o = \frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o\phi}}{\lambda_t} \tag{1.14}$$

$$U_{l,t}^u = \frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u\phi}}{\lambda_t} \tag{1.15}$$

The arbitrage condition in the labour market ensures that both sectors pay the same level of real wage as:

$$\frac{P_t^o}{P_t} w_t^o = \frac{P_t^u}{P_t} w_t^u \tag{1.16}$$

The competitive capital supplied to each sector is accordingly given as:

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_{t+1}^o}{P_{t+1}} \left[r_{t+1}^{k,o} u_{t+1}^o - a(u_{t+1}^o) \right] + Q_{t+1}^o (1-\delta) \right] \tag{1.17}$$

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_{t+1}^u}{P_{t+1}} \left[r_{t+1}^{k,u} u_{t+1}^u - a(u_{t+1}^u) \right] + Q_{t+1}^u (1-\delta) \right] \tag{1.18}$$

The first order conditions for investments supplied to each sector is given as:

$$\frac{P_t^o}{P_t} = Q_t^o \varepsilon_t^{INV} \left(1 - S\left(\frac{i_t^o}{i_{t-1}^o} \right) - S'\left(\frac{i_t^o}{i_{t-1}^o} \right) \frac{i_t^o}{i_{t-1}^o} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^o \varepsilon_{t+1}^{INV} S'\left(\frac{i_{t+1}^o}{i_t^o} \right) \left(\frac{i_{t+1}^o}{i_t^o} \right)^2 \tag{1.19}$$

[7] A detailed derivations of all the first order conditions are in the appendix.

$$\frac{P_t^u}{P_t} = Q_t^u\left(1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right) - S'\left(\frac{i_t^u}{i_{t-1}^u}\right)\frac{i_t^u}{i_{t-1}^u}\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^u S'\left(\frac{i_{t+1}^u}{i_t^u}\right)\left(\frac{i_{t+1}^u}{i_t^u}\right)^2 \tag{1.20}$$

And finally, the following equations also gives the first order conditions for effective capital utilised:

$$r_t^{k,o} = a'(u_t^o) \tag{1.21}$$

$$r_t^{k,u} = a'(u_t^u) \tag{1.22}$$

solving equations (1.12) and (1.13) for c_t we obtain the consumption Euler equation.

1.2.2 Official Sector Goods Producers

The official sector firms produce intermediate goods and sell them at the competitive intermediate goods price $P_t^{I,o}$ to final goods producers. The production function for a representative firm is given as:

$$y_t^o = A_t^o k_t^{o(\alpha^o)} l_t^{o(1-\alpha^o)} \tag{1.23}$$

where y_t^o, k_t^o and l_t^o respectively denote sectoral output, capital and labour inputs. A_t^o is the official sector productivity shock which is defined as an AR(1) process with *i.i.d* error term. Official sector firms maximize their market value by choosing labour (l_t^o) and capital (k_t^o) taking into account their production output level. Firms market value (Π_t^o) is expressed as:

$$\Pi_t^o = \frac{P_t^{I,o}}{P_t}\left[y_t^o - w_t^o l_t^o - r_t^{k,o} k_t^o\right] \tag{1.24}$$

where w_t^o and $r_t^{k,o}$ are respectively sectoral real wage rate and real returns from capital. $\frac{P_t^{I,o}}{P_t} y_t^o$ represent the firm's revenue from selling output, and $\frac{P_t^{I,o}}{P_t}(w_t^o l_t^o + r_t^{k,o} k_t^o)$ are the repayments made by firms to households which consist of the wage bill and cost of physical capital. The following equations respectively represent the first order conditions for official sector labour and capital:

$$w_t^o = (1-\alpha^o) A_t^o \left(\frac{k_t^o}{l_t^o}\right)^{\alpha^o} \tag{1.25}$$

$$r_t^{k,o} = \alpha^o A_t^o \left(\frac{k_t^o}{l_t^o}\right)^{-(1-\alpha^o)} \tag{1.26}$$

This implies an official sector capital-labour ratio given as:

$$\frac{r_t^{k,o}}{w_t^o} = \frac{\alpha^o}{1-\alpha^o}\frac{l_t^o}{k_t^o} \tag{1.27}$$

Solving equations (1.25) and (1.26) yield official sector's real marginal cost as:

$$mc_t^{I,o} = \left(\frac{r_t^{k,o}}{\alpha^o}\right)^{\alpha^o}\left(\frac{w_t^o}{1-\alpha^o}\right)^{1-\alpha^o} \tag{1.28}$$

1.2.3 Shadow Sector Goods Producers

Informal sector goods producers obtain working capital from households each period in order to start production of shadow goods. The production function of the representative goods producer in the shadow sector is given as:

$$y_t^u = k_t^{u(\alpha^u)} l_t^{u(1-\alpha^u)} \tag{1.29}$$

where y_t^u, k_t^u and l_t^u respectively define sectoral output, capital and labour. α^u is the usual capital share used in production activities. They choose capital obtained from households and labour optimally in each period to maximize their market value (Π_t^u) given as:

$$\Pi_t^u = \frac{P_t^{I,u}}{P_t} y_t^u - \frac{P_t^{I,u}}{P_t} w_t^u l_t^u - \frac{P_t^{I,u}}{P_t} r_t^{k,u} k_t^u$$

where the first term on the *r.h.s* of the equation above represents the revenue obtained from selling shadow intermediate goods to final goods producers and the remaining part represent the periodic repayments to households. The first order conditions for labour and capital are given respectively as:

$$w_t^u = (1-\alpha^u)\left(\frac{k_t^u}{l_t^u}\right)^{\alpha^u} \tag{1.30}$$

$$r_t^{k,u} = \alpha^u\left(\frac{k_t^u}{l_t^u}\right)^{-(1-\alpha^u)} \tag{1.31}$$

This in turn yield the shadow sector capital-labour input as:

$$\frac{r_t^{k,u}}{w_t^u} = \frac{\alpha^u}{1-\alpha^u}\frac{l_t^u}{k_t^u} \tag{1.32}$$

Solving equations (1.30) and (1.31) yield unofficial sector's real marginal cost as:

$$mc_t^{I,u} = \left(\frac{r_t^{k,u}}{\alpha^u}\right)^{\alpha^u}\left(\frac{w_t^u}{1-\alpha^u}\right)^{1-\alpha^u} \tag{1.33}$$

1.2.4 Final Goods Producers

We assume a sticky price specification based on Rotemberg (1982) quadratic adjustment cost in both sectors of the economy. We index their prices to a combination of both current and past inflation with a weight equal to θ_π. The final goods producers maximize their profit function by choosing their final goods prices taking into account the quadratic adjustment cost given as:

$$\frac{\kappa^p}{2}\left(\frac{P_t^i/P_{t-1}^i}{(\pi_{t-1}^i)^{\theta_\pi}}-1\right)^2 y_t^i$$

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost in adjusting its nominal prices that can be measured in terms of the final goods with κ^p being the price stickiness parameter which accounts for the negative effects of price changes on the customer-firm relation and θ_π representing the price indexation parameter.

The official sector final goods producers are subject to price mark-up shocks, hence in a symmetric equilibrium, the Rotemberg version of non-linear New Keynesian Phillips Curve (NKPC) is derived as:

$$(1-mc_t^o)\epsilon_t^o = 1-\kappa^p\left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}}-1\right)\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}}-1\right)\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}}\frac{y_{t+1}^o}{y_t^o}\right] \tag{1.34}$$

where ϵ_t^o is now a stochastic parameter which determines the time-varying mark-up in the official goods markets. In light of this, following Smets and Wouters (2003), the official sector final goods producers' actual mark-up hovers around its desired level over time. This desired level comprises of an endogenous and exogenous components which is assumed to follow an AR(1) process given as:

$$ln\epsilon_t^o = ln\epsilon^o + ln\varepsilon_t^p$$

$$ln\varepsilon_t^p = ln\varepsilon_{t-1}^p + \xi_t^p \tag{1.35}$$

with ξ_t^p being an *i.i.d.* In a symmetric equilibrium, the price adjustment rule satisfies the following first order condition for the shadow goods producers given as:

$$(1-mc_t^u)\epsilon^u = 1-\kappa^p\left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}-1\right)\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}-1\right)\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}\frac{y_{t+1}^u}{y_t^u}\right] \tag{1.36}$$

where $mc_t^i = \frac{P_t^{I,i}}{P_t^i}$, defines the real marginal cost in terms of the sectoral final goods price. Here we assume that shadow sector goods producers have limited market power. The above equations represent the Rotemberg version of non-linear NKPCs that relate sectoral current inflation to future expected inflation and to the level of relative outputs. The following equations respectively allow to identify the sectoral price levels and the inflation rate for the consumption price index:

$$P_t^o = \pi_t^o P_{t-1}^o \tag{1.37}$$

$$P_t^u = \pi_t^u P_{t-1}^u \quad (1.38)$$

$$P_t = \pi_t P_{t-1} \quad (1.39)$$

where P_t is defined by equation (1.5).

1.2.5 Monetary Policy

We close the model by describing a simple structure for the monetary policy rule. The Central bank is assumed to follow a pure inflation targeting rule and set a standard Taylor-type monetary policy instrument so that the nominal interest rate is adjusted in response to the movement in inflation gap with interest rate smoothing. The policy rule is characterised by the following Taylor rule:

$$R_t = R_{t-1}^{(\rho^R)} (\pi_t^o)^{\mu_\pi (1-\rho^R)} \varepsilon_t^R \quad (1.40)$$

where R_t is the nominal interest rate, ρ^R is interest rate smoothing parameter, μ_π denotes Taylor coefficient in response to inflation gap[8] with ε_t^R denoting monetary policy shock, which is a standard *i.i.d* innovation. In this context, the monetary policy shock is thought of as unexpected deviation of the nominal interest rate via Taylor rule at period t. The exogenous shock to monetary policy enters the nominal interest rate as ε_t^R. The central bank supplies the money demanded by the household to support the desired nominal interest rate.

1.2.6 Market Clearing and Resource Constraint

The labour market is in equilibrium when the demand for labour services by goods producers equal the differentiated labour services supplied by households at the various wage rates. Similarly, the market for physical capital is in equilibrium when the demand for capital services by goods producers equals the capital produced in each sector at the market rental rate which is used for investments. We note here that in the Rotemberg model, the aggregate resource constraint takes the price adjustment cost into account which creates an inefficiency between output and consumption. Therefore, aggregate resource constraint in each sector is defined as:[9]

$$y_t^i = c_t^i + i_t^i + a(u_t^i)\bar{k}_{t-1}^i + \frac{\kappa^p}{2}\left(\frac{\pi_t^i}{{\pi_{t-1}^i}^{\theta_\pi}} - 1\right)^2 y_t^i \quad (1.41)$$

The last two terms of the equations represent household's capital utilisation cost and goods producers price adjustment cost. Letting $\Theta_t^i = \left[1 - \frac{\kappa^p}{2}\left(\frac{\pi_t^i}{{\pi_{t-1}^i}^{\theta_\pi}} - 1\right)^2\right]^{-1}$ and solving for y_t^i,

[8] That is, deviation of inflation rate from the inflation target.

[9] In the Rotemberg model, the cost of nominal rigidities, i.e. the adjustment cost, creates a wedge between aggregate consumption and aggregate output, because part of the output goes in the price adjustment cost. If trend inflation is zero, this wedge vanishes and the model is equivalent up to Calvo mechanism up to first-order. See Ascari and Rossi (2011).

we obtain sectoral aggregate resource constraint as:

$$y_t^i = \Theta_t^i \big[c_t^i + i_t^i + a(u_t^i)\bar{k}_{t-1}^i\big]$$

The aggregate consumption is given by equation (1.2) and aggregation labour constraint is computed as follows:

$$l_t = l_t^o + l_t^u \tag{1.42}$$

Following Colombo et al. (2016), we introduce into the model the relative size of the shadow sector (SH_t) which will be useful when deriving the steady states of the model. From the sectoral resource constraint, SH_t is obtained from a straight forward manipulations using (1.3) and (1.4) as:

$$SH_t = \frac{y_t^u}{y_t^o} \tag{1.43}$$

1.3 Model Dynamics and Results

In this section, we calibrate the theoretical DSGE model derived in the previous section for developing and low-income countries by imposing several structural shocks. The aim is to show how our model is coherent to the new Keynesian DSGE models and to highlight the role played by the informal sector on the economy's dynamics. We do so by examining the various channels and explain their practical relevance to developing and low-income economies.

The equations listed in the previous section represent agents' behaviour and identities that altogether form the non-linear system. These include the first order conditions of households with capital accumulation and investment adjustment cost, intermediate goods producers, final goods producers, agents' budget constraints, the monetary policy rule and equations describing the exogenous processes driving the economy. The current set up involves five exogenous processes in the official sector namely the risk premium shock, investment specific shock, total factor productivity, price mark-up shock and the conventional monetary policy shock. In order to find the solution of the model, we start by focusing on the symmetric equilibrium for prices and quantities, then derive all the log-linearised equations of the model by taking log-linear approximations around the steady state.[10] The linearised DSGE model involves two equations for output inflation in both sectors of the economy. The main difference between these equations is that, the official sector output inflation equation is subject to the price mark-up shock while shadow sector goods producers have limited market power and structural shocks are asymmetrical to the official sector. The coefficients of the log-linear model depend on the

[10] The full set of the first order conditions and log-linearised equations are in the appendix.

primitive parameters of the model as well as steady state values of the variables. We further use the steady state conditions of the model to solve out for number of parameters.

1.3.1 Model Calibrations and Parameterization

The structural parameters of the model are taken in correspondence with developing and low-income countries averages. The conventions in the model calibrations consist of parameters values mostly borrowed from Smets and Wouters (2007) and Colombo et al. (2016) and most current literature with similar modelling structure. This would serve as references when assessing the dynamics of some key macroeconomic variables. They are selected in order to capture specific ratios in the steady state and for most developing and low-income economies as close as possible. Parameters whose information relates to developing countries are calibrated using values and data from the developing countries literature. The complete list of parameters and their values are in table (1.1).

Parameters characterizing the household's preferences are fairly standard. The subjective discount factor β, is set to 0.99 which is consistent with Smets and Wouters (2007), the same value was used in Colombo et al. (2016) to achieve an annual steady state interest rate of 4%. The elasticity of substitution between official and informal consumption bundles is set at $\epsilon_c = 1.5$ as was described in Batini et al. (2011). We set the steady state share of shadow economy at $SH = 0.47$, a value common to several developing and low-income countries to obtain the value for official consumption goods bias φ_c as in Colombo et al. (2016); and Khan and Khan (2011). The coefficient of Frisch elasticity of substitution for labour supply in the utility function is fixed at $\phi = 2$, a value consistent with the posterior mean reported by Smets and Wouters (2007). The steady state elasticity of capital utilisation cost parameter τ is fixed at 0.2696 to indicate a mean of 0.2 for the capital utilisation cost function as suggested by King and Rebelo (2000). The elasticity of the cost of adjusting investment is also fixed at $\varpi = 6.0144$ to be as close as the value estimated by Christiano et al. (2005) and Smets and Wouters (2003). Turning to the goods producer's structural parameters for both sectors, from Schmitt-Grohe and Uribe (2004), we take the price elasticity of demand for differentiated goods parameter $\epsilon^o = 6$, a value consistent with a 20% price mark up in the official sector and the shadow sector value is set at $\epsilon^u = 20$ which also implies a 5% price mark up. The degree of inflation indexation parameter is set to $\theta_\pi = 1$ to indicates a full indexation of inflation. The degree of price stickiness parameter is fixed at $\kappa^p = 4.37$. Available literature suggests no evidence of nominal rigidities in the shadow sector, therefore the benchmark values for inflation indexation and degree of price stickiness are set in accordance with the official sector values. The depreciation rate is set to equal to $\delta = 0.025$ per quarter which implies an annual depreciation

Table 1.1: Model calibration parameters.

Parameter	Value	Description
β	0.99	Subjective discount rate
ϕ	2	Frisch elasticity of substitution for labour
ϵ_c	1.5	Elasticity of substitution between official and unofficial consumption
ϵ_o	6	Official sector price elasticity of demand
ϵ_u	20	Shadow sector price elasticity of demand
κ^p	4.37	Degree of price stickiness
θ_π	1	Inflation indexation
δ	0.025	Capital depreciation
α^o	0.36	Official sector's capital Share
α^u	0.28	Shadow sector's capital Share
ρ^R	0.9	Interest rate smoothing parameter
μ_π	1.5	Taylor coefficient to inflation gap
ρ^A	0.8	Productivity shock autocorrelation
ρ^p	0.8	Innovation to price markup shock
ρ^{RISK}	0.7	Innovation to risk premium shock
ρ^{INV}	0.85	Innovation to investment shock
ρ^ε	0.2	Innovation to interest rate Shock

on capital equal to 10%. We additionally set the official sector capital share to $\alpha^o = 0.36$ to capture a high capital intensity in the official sector than the informal sector. The informal sector firm's capital share parameter is calibrated to capture a relatively low capital intensity in their production function, so we choose a capital share of $\alpha^u = 0.28$ as in Koreshkova (2006). This is in line with data from many developing and low-income countries where most of their production activities are labour intensive.

The conventional parameters characterizing the monetary policy instrument are set accordingly as: the Taylor rule interest smoothing rate parameter $\rho^R = 0.9$ and inflation gap parameter $\mu_\pi = 1.5$. The parameters describing the shock processes are calibrated as follows, innovation to interest rate shock $\rho^\varepsilon = 0.2$ is set to account for the temporal shock to monetary policy as reported by Smets and Wouters (2003, 2007), the persistence of technology shock $\rho^A = 0.8$, and the persistence to price mark-up shock is also set at $\rho^p = 0.8$. Similarly, the persistence to investment and risk premium shock parameters are respectively set at $\rho^{INV} = 0.85$ and $\rho^{RISK} = 0.7$. To achieve a stable steady state, we conventionally set the aggregate labour supply to 0.25. It is paramount to note that, the steady state relative prices are determined by the supply side effects of the model, namely mark ups and technological parameters. Bearing this in mind, the price of shadow sector goods is always relatively higher due to the high cost of capital. The rest of steady state values are calibrated using the assumptions and the structural parameters.

1.3.2 Analysis and Discussion of Results

In this section, we analyse and discusses the various impulse response functions (IRFs) regarding some key macroeconomic variables using the baseline calibration to asymmetric shocks in the official sector. Figures (1.1)-(1.5) represent the various responses following productivity, investment specific shock, price mark-up, risk-premium and monetary policy shocks to the official sector. We further analyse how those shocks transmit into the shadow sector and finally analyse how the presence of shadow sector affects the economy at large. We note here that, variables are already expressed as percentage deviations from the steady state values and the continuous red lines in the figures represent the behaviour of shocks in one sector (official) economy as in Smets and Wouters (2003) while the continuous blue line defines the behaviour of variables in the two sector (both official and shadow) economy. The baseline calibration results in the current set up are in line with the existing New Keynesian DSGE models with two sectors and we describe them accordingly.

Figure (1.1) shows that following a positive technological shock to official sector considering the model without shadow sector (the red line), output, consumption, investment, real wage and capital demand rise and labour employment falls. The fall in labour employment indicates a much stronger effect of the shock on the model without shadow sector. The main qualitative difference when we introduce shadow sector (the blue line) is the rise in both official and shadow sector labour employment and decline in the shadow sector real wage. The reason being that the productivity shock raises the product wage in the official sector and lowers the official sector price level. The lower interest rate raises total consumption demand and consumption of shadow sector goods whose price increases which also explains the fall in the shadow product wage. Moreover, demand for official goods is much stronger in the model with shadow sector because the relative price of official goods falls which also explains why official sector labour employment increases, the transmission of the productivity shock is now positive across sectors.

Turning to investment specific shock, figure (1.2) shows that a positive investment shock increases labour employment in the official sector which induces a rise in output and a fall in consumption in either models (with and without shadow economy). We note here that the presence of the shadow sector amplifies the impact of the shock on the official sector. An important difference is the fall in product wage in the official sector which results from the rise in official sector price levels. The higher interest rate reduces aggregate consumption and shadow consumption goods thereby increasing investment. This mechanism explains the rise in shadow sector product wage. In response to official sector price mark-up shock, our results replicate what is in most DSGE literature as shown by figure (1.3). The responses from the model without shadow sector indicate a fall in official sector consumption, investment, labour employment, real wages, real returns on capital and output. A major discrepancy with the introduction of shadow sector is the rise in official product wage which is attributed to the lower levels of the official sector prices. The increase in shadow sector labour employment is as a result of the impact of the shock which lowers shadow sector product wage and price levels. As shadow sector labour employment increases with a falling aggregate inflation, demand for shadow sector goods increases. Figure (1.4) represents the IRFs for a positive risk premium shock to the official sector, output, consumption, investment, labour employment, capital demand and real returns on capital fall as expected. The same results are replicated in the official sector when we introduce the informal sector except shadow sector labour employment which rises and shadow sector product wage declines. This occurs because the shock raises official sector product wage and lowers official sector price levels. With higher levels of interest rate total consumption declines and the fall in shadow prices raise shadow consumption. This explains the fall in the shadow sector product wage. The reduction in price of capital raises capital utilization and capital demand in the shadow sector thereby increasing real returns from capital.

Table 1.2: Transmission effects of expansionary shocks in official sector to the shadow sector.

Shocks	**Official Output**	**Shadow Output**
Monetary policy	-	+
Risk premium	-	+
Price mark-up	-	+
Investment specific	+	-
Total factor productivity	+	+

Note: (-) Negative transmission effect, (+) Positive transmission effects.

Following a tightening of the monetary policy in the official sector which increases the nominal interest rate, figure (1.5) shows that output, consumption, investment, capital demand, labour employment, real wage and inflation decrease in the model without shadow sector. The difference with the introduction of shadow economy is the rise in official sector product wage which is attributed to the lower official sector price levels. The shock raises interest rate which gives the hump-shaped fall in official sector output. The decline in official consumption is a rational behaviour since households' substitute consumption for investment in government bond whereas private investment in the official sector firms declines due to higher interest rate.[11] However, policy impact on the shadow economy is somewhat asymmetrical which happens because of sectoral price elasticity of demand making shadow prices relatively more flexible. From the various signs shown on table (1.2), we can conclude here that shadow sector indeed

[11]See Batini et al. (2011) for further discussions.

serves as a buffer to the shocks considered in this analysis. As indicated on table (1.2), the negative transmission effects of monetary policy, risk premium and price mark-up shocks in the official sector are assimilated by the informal sector with a diametric response. On the other hand, positive investment specific shock transmissions in the formal sector triggers a negative reaction in the shadow sector while the total factor productivity shock induces positive effects in both sectors of the economy.

1.3.3 Sensitivity Analysis

Deriving a meaningful closed-form solution for the size of the shadow sector SH is complex given the kind of model we have built. We therefore perform sensitivity analysis on the size value of SH to changes in technological parameter (α^i) and in relative price mark-ups (ϵ^i) to understand transmission processes of the model in relation to the size of the shadow sector. We do so by calibrating the value of φ_c at the values consistent with two alternative shares of the shadow economy: one characterised by a high share of the shadow economy ($SH = 0.47$ a value for several developing and low-income countries) as the baseline model and one characterised by a low share of the shadow economy ($SH = 0.10$ an average value for advanced countries). We do this exercise to ascertain the changing effects of the size of shadow economy and its importance to the model. In the appendix we show that in steady state:[12]

$$SH = \frac{y_s^u}{y_s^o} = \frac{1-\varphi_c}{\varphi_c}\left(\frac{P_s^u}{P_s^o}\right)^{-\epsilon_c}\frac{(1-\frac{i_s^o}{y_s^o})}{(1-\frac{i_s^u}{y_s^u})} =$$

$$SH = \frac{1-\varphi_c}{\varphi_c}\left(\frac{\left(\frac{\epsilon^u-1}{\epsilon^u}\right)^{\frac{1}{1-\alpha^u}}\left(\frac{r_s^{k,o}}{\alpha^o}\right)^{\frac{\alpha^o}{1-\alpha^o}}(1-\alpha^u)}{\left(\frac{\epsilon^o-1}{\epsilon^o}\right)^{\frac{1}{1-\alpha^o}}\left(\frac{r_s^{k,u}}{\alpha^u}\right)^{\frac{\alpha^u}{1-\alpha^u}}(1-\alpha^o)}\right)^{\epsilon_c}\frac{\left(1-\delta\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha^o}\right)\right)}{\left(1-\delta\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha^u}\right)\right)}$$

The relative size of shadow sector is obtained by substituting equations (1.3) and (1.4) into equation (1.41) for the solution at (1.43) through a straight forward manipulation. Following this exercise, an increase in the shadow sector retail price mark-ups unambiguously reduce the size of the shadow sector and official sector consumption goods bias parameter which affects the relative goods prices. We must emphasise that relative prices are entirely determined by supply side effects of the model namely mark-ups and technology parameters. In fact with the same price elasticity of demand ($\epsilon^o = \epsilon^u$) in both sectors, we obtain an identical results and responses equivalent to the benchmark model. The relatively high cost of capital makes the price of shadow goods always relatively higher and for that matter when we increase the shadow sector technology parameter to the level of official sector parameter ($\alpha^o = \alpha^u$), the value of SH

[12] The complete steady state derivation of SH is in the appendix.

proportionally doubles with official goods bias and the price of shadow goods becomes relatively lower. This strengthens the argument that consumption wages do not necessarily matter for the determination of relative prices but technological parameters greatly have the effects of adversely changing the relative price of goods and capital in the shadow sector. It also explains how marginal cost with the same technology becomes identical with higher mark-ups in the formal sector. The results from the sensitivity analysis indicate that the size of shadow sector has a greater influence on the shadow sector relative goods prices.

1.4 Conclusions

In this paper, we have introduced and extended to the new Keynesian DSGE model of Smets and Wouters (2003, 2007) with the shadow economy mainly to ascertain the role they play in the transmission of several macroeconomic shocks. This was motivated by the current debate on informal economy's activities which are not under the regulatory purview of policy makers. Households are shown to be standard with all households consuming from the same basket of goods and services and also supplying labour services to both sectors of the economy. There are perfect competitive goods producers operating in both sectors as well as monopolistic competitive final goods producers. We finally close the model with a Central bank which implement monetary policy instruments. The macroeconomic properties of our variables, its directions and the transmission pattern with respect to all the shocks are theoretically sound and match the patterns reported in the existing conventional new Keynesian DSGE models with shadow sectors. Our calibrated model showed that the presence of informal markets with asymmetric shock to the official sector induces factor flow across sectors and crowding-out of formal sector's activities into the shadow sector when there are negative transmissions of the shock to the formal sector. This strengthens the existing notion that shadow sector serves as a cyclical buffer. A sensitivity analysis with changes in technology and price mark-up parameters indicated that the size of shadow sector has an influence on shadow sector relative goods prices. The results suggest that shadow economy play important role in channelling shocks into the real sectors of the economy.

References

Amaral, P. S. & Quintin, E. (2006). A competitive model of the informal sector. *Journal of Monetary Economics*, 53 (7), 1541-1553.

Aruoba, S. (2010). Informal Sector, Government Policies and Institutions. *University of Maryland*. (Mimeographed).

Aruoba, B., & Schorfheide, F. (2011). Sticky prices versus monetary frictions: An estimation of policy trade-offs. *American Economic Journal: Macroeconomics*, 3(1), 60-90.

Ascari, G., & Rossi, L. (2011). Real wage rigidities and disinflation dynamics: Calvo vs. Rotemberg pricing. *Economics Letters*, 110(2), 126-131.

Batini, N., Levine, P., Lotti, E., & Yang B. (2011). Informality, frictions and monetary policy. *School of Economics Discussion Papers 711.*

Castillo, P., & Montoro, C. (2008). Monetary policy in the presence of informal labour markets. *Banco Central de Reserva del Peru, Mimeo.*

Colombo, E., Onnis, L., & Tirelli, P. (2016). Shadow economies at times of banking crises: Empirics and Theory. *Journal of Banking and Finance*, 62, 180-190.

Conesa, J. C., Diaz-Moreno, C., & Galdon-Sanchez, J. E. (2002). Explaining Cross-Country Differences in Participation Rates and Aggregate Fluctuations, *Journal of Economic Dynamics and Control*, 26, 333-345.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1), 1-45.

Gali, J. (2008). Introduction to monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework. *Introductory Chapters, Princeton University Press.*

Galí, J., & Rabanal, P. (2004). Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar US Data?. *NBER Macroeconomics Annual*, 19, 225-288.

Justiniano, A., Primiceri, G., & Tambalotti, A. (2011). Investment Shocks and the relative price of investment. *Review of Economic Dynamics*, 14(1), 101-121.

asif Khan, S., & Khan, S. (2011). *Optimal Taxation, Inflation and the Formal and Informal Sectors* (No. 40). State Bank of Pakistan, Research Department.

King, R. G. & Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of macroeconomics*, 1, 927-1007.

Koreshkova, T. (2006). A Quantitative Analysis of Inflation as a Tax on the Underground Economy. *Journal of Monetary Economy*, 53(4), 773-796.

Maloney, W. (1999). Does informality imply segmentation in urban labour markets? Evidence from sectoral transitions in Mexico. *World Bank Economic Review*, 13 (2), 275-302.

Maloney, W. (2004). Informality revisited. *World Development*, 32 (7), 1159-1178.

Peiris, S. J., & Saxegaard, M. (2007). An estimated DSGE model for monetary policy analysis in low-income countries. International Monetary Fund, 2007.

Pratap, S. & Quintin, E. (2006). The informal sector in developing countries: Output, assets and employment. Working Papers RP2006/130, World Institute for Development Economic Research (UNU-WIDER).

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4), 517-531.

Schmitt-Grohe, S., & Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2), 198-230.

Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association*, 1(5), 1123-1175.

Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3), 586-606.

Zenou, Y. (2008). Job search and mobility in developing countries. Theory and policy implications. *Journal of Development Economics*, 86(2), 336-355.

Figure 1.1: Response to Productivity Shock

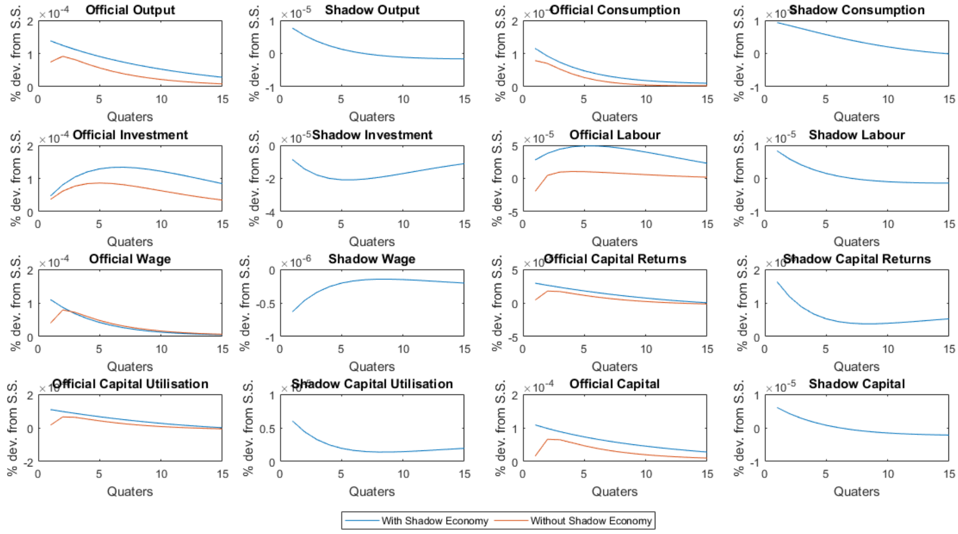


Figure 1.2: Response to Investment Shock

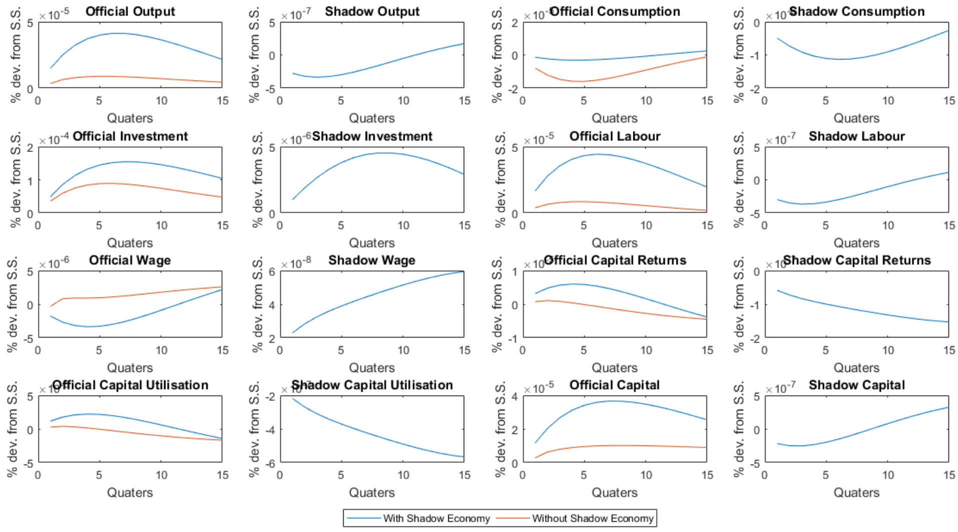


Figure 1.3: Response to Price Mark-up Shock

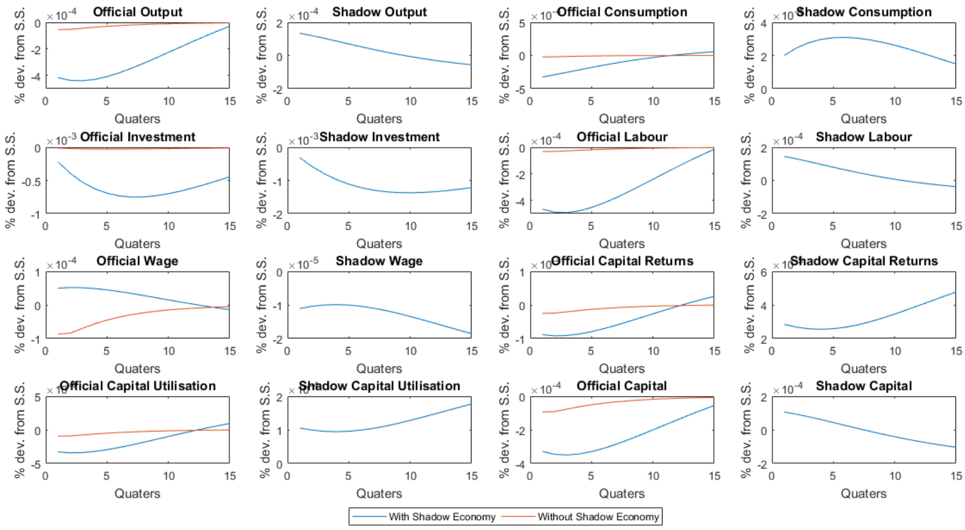


Figure 1.4: Response to Risk Premium Shock

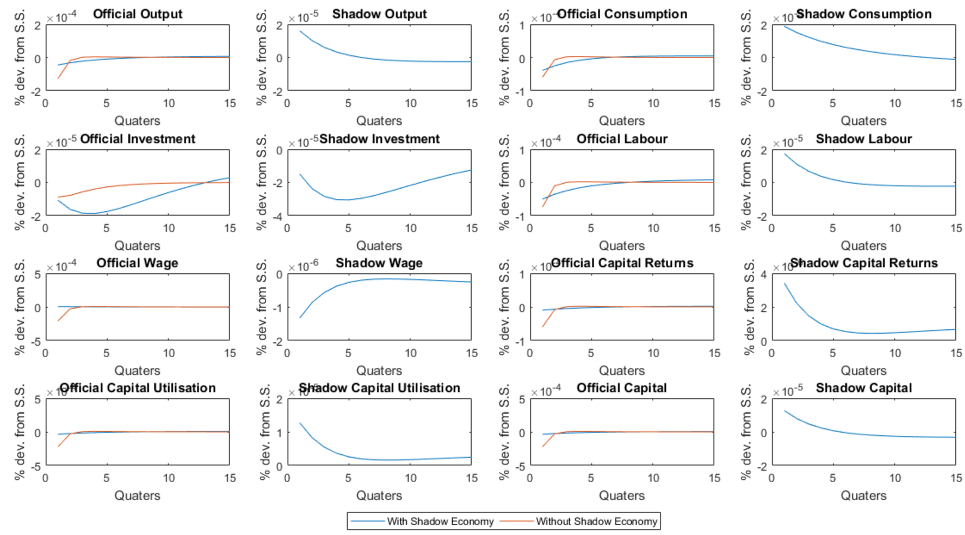
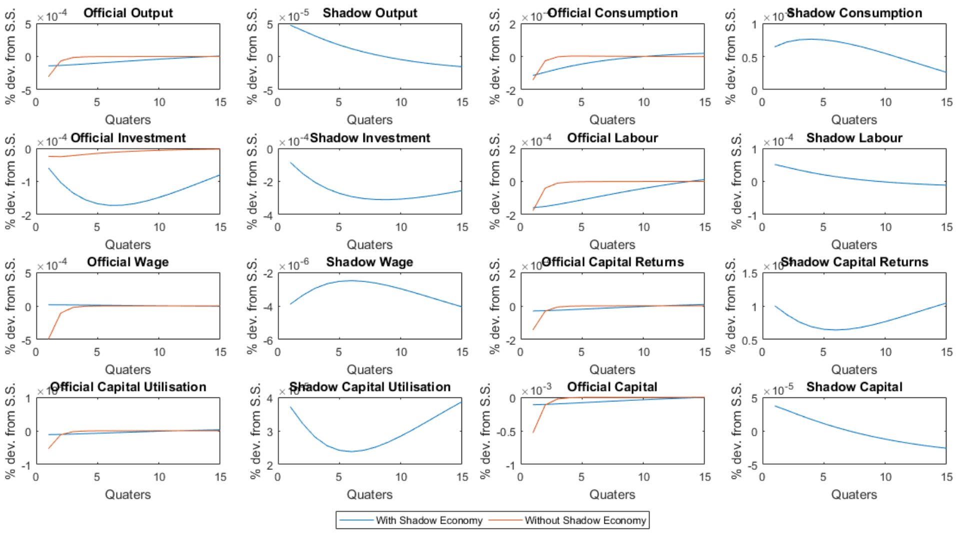


Figure 1.5: Response to Monetary Policy Shock



CHAPTER 2

Effectiveness of Fiscal Policy shocks and feedbacks in a DSGE model with Shadow Economy in Developing Countries.

2.1 Introduction

Over the last decades, we have witnessed the development of a new version of macroeconomic modelling - new Keynesian DSGE models - that explicitly builds on microfounded literature. The advancement in this estimation methodology allows for estimating variants of models that are able to compete with more standard time-series models. Given the microfounded nature of DSGE models, it particularly become suitable for evaluating the effects of alternative macroeconomic policies. However, most literature on DSGE models have largely focused on using variants of the new Keynesian DSGE models in analysing the effects of monetary policy transmissions on other macroeconomic variables. In fact, most benchmark models such as Christiano et al. (2005); Smet and Wouters (2003, 2007), provided evidence showing that an optimisation-based model with nominal and real frictions could account for the effects of a monetary policy shocks.[1] Most of these benchmark DSGE models have sometimes paid little or no attention to the role played by fiscal policy thereby minimising any possible interaction of fiscal policies with monetary policy transmissions. The aftermath of the recent financial crisis saw a large scale of fiscal policy responses in most advanced economies which led to a sizeable increase in fiscal deficits and debt levels (Coenen, 2012). The paradigm shift was due to the inability of monetary policy to avoid the recession making fiscal tools an important and debatable topic in macroeconomic policy modelling. Fiscal expansion particularly became large in the US and in the UK whilst many governments in the Euro area were criticized by the IMF for taking slow actions in the 2007-2009 period and for the "austerity" measures that were imposed onto peripheral countries after the beginning of the Greek crisis in 2010 (Krugman, 2012; Stiglitz et al., 2014). Following this, vast majority of DSGE literature have discussed the role of fiscal policies on macroeconomic variables and its determination on the real business cycle.

[1] Other benchmark DSGE models include Bernanke et al., (1999); Kiyotaki and Moore, (1997); Iacoviello, (2005); Christiano et al., (2003, 2008); Goodfriend and McCallum, (2007).

Real business cycle models seem not to suit the study of government spending effects due to its frictionless nature. Theoretical RBC models predict that increase in government spendings crowd out the private consumption and reduces the real wage. In this regard, Baxter and King (1993) showed that an increase in public expenditure bring about a direct increase in the discounted future value of taxes as government need to finance its intertemporal budget. This lead to the negative wealth effects on households which reduces private consumption, increase in labour supply and output and a fall in real wage in RBC models. Perotti et al. (2007) reviewed this literature and delivered results that are generally consistent with RBC models and argued that the response of private consumption to government spending shock is positive which leaves the debate on effects of fiscal policy shock unsettled. Rotemberg and Woodford (1992), Ramsey and Shapiro (1998) and Cavallo (2005) found that increases in government spending for national defence reduce private consumption, the real wage and increase employment as well as nonresidential investments. New Keynesian paradigm which mainly include real frictions and normal rigidities to RBC framework displays the same wealth-effects mechanism that entails a reduction in private consumption and expansion in labour supply following a government spending shock (Goodfriend and King, 1997; Linnemann and Schabert, 2003). However, in the new Keynesian paradigm real wages may increase due to an outward shift of the labour demand induced by the expansion of demand with sticky prices as in Forni et al. (2009). They typically predicted that an increase in government expenditure will increase labour demand, generating an increase in the real wage and output. The response of private consumption is mainly determined by the negative wealth effect induced by increase in government spending (see, e.g., Linnemann and Schaubert, 2003); and "rule-of-thumb" consumers must also be present to generate an increase in private consumption as in Gali et al. (2007). Indeed, Kumhof and Laxton (2007) have developed a very comprehensive model for the analysis of fiscal policies, which incorporates four non-Ricardian features. In their analysis of the effects of a permanent increase in the US fiscal deficits and debt, they find medium and long-term effects that differ significantly from those of liquidity constrained agents. Furthermore, they find deficits to have a significant effect on the current account. Pappa (2009) studied the transmission of fiscal shocks in the labour market by employing a prototype RBC and new Keynesian model with structural vectoral autoregressive (VAR) model predicted that shocks to government consumption, investment and employment must raise output and deficit. In effects, shocks to government consumption and investment increased real wages and employment simultaneously, however, the dynamics of employment shocks were mix. Other standard empirical version of the new Keynesian DSGE model also typically predict a positive or at least no significant negative response of private consumption to government spending shocks (as in Perotti, 2002; Fatàs and Mihov, 2001; Canzoneri et al., 2002; Mountford and Ulig, 2001). Most of these theoretical and empirical literature that analyse the impact of fiscal policy on economic activities focus mainly on the size and sensitivity of fiscal multipliers as in Cogan et al. (2010); Christiano et al. (2011); and Coenen (2012) without the consideration of the shadow sector. Standard DSGE literature like the ones introduced above lack the prerequisite modelling ingredients for most developing and emerging countries which makes replications of such models in the developing countries questionable. The adoption of such model requires a significant amount of altercations to be coherent with relevant micro evidence. It therefore becomes erroneous to implement the same DSGE models built for the advanced economies for the developing economies without the necessary considerations.[2] The informal sector forms an integral part of many economies in the world and are of larger proportions in most developing and emerging economies. However, most DSGE models neglect the role played by informal sectors in affecting macroeconomic transmission processes (except Arouba, 2010; Batini et al., 2011; Khan and Khan, 2011; Colombo et al., 2016 on financial crisis). On the other hand, DSGE literature which model fiscal policy instruments and stimulus such as Pappa (2009), Christiano et al. (2011), Coenen (2012, 2013) and Albonico et al. (2016) all do not include the shadow sector despite their role in the transmission process.

In this work, we investigate the role of fiscal policies over aggregate business cycle in the presence of shadow economy. In effect, we seek to elucidate whether the presence of shadow economy dampens or amplifies the effects of fiscal policy transmissions. Secondly, we try to identify the role of alternative fiscal shocks and feedbacks on GDP by comparing the cases with and without shadow economy. A major policy elaboration is whether government should target the participation rate with fiscal policies or whether government should implicitly target sectoral relative prices. In order to do this, we follow Smets and Wouters (2003, 2007) and introduce shadow sector to ascertain their role in transmitting fiscal policy shocks. The main difference between the two sectors concern the calibrations of technology and price mark-ups. Fiscal policy packages are computed as an average effective tax rates on labour income, capital income and consumption tax following Melina et al. (2016) which are consistent with data collected by the International Bureau of Fiscal Documentation in 2005-06. On the expenditure side, we take into consideration the variable mostly used in the literature, that is the government consumption from National Income data which includes both purchases of goods and services as well as compensations to government employees as in Rotemberg and Woodford (1992). The model is characterised by a competitive labour market where firms in the two sectors pay the

[2]The reason being that, most developing countries are characterised by weak financial sector, existence of large informal sector, external shock vulnerability, and weak economic and political institutions. The challenge of data inconsistency and unavailability in most developing and low-income countries also become a problem.

same consumption real wage. This assumption is motivated by the theoretical contributions from Amaral and Quintin (2006); Pratap and Quintin (2006) and supported by Maloney (1999, 2004). The contributions by Pratap and Quintin (2006) on developing countries provided evidence against labour market segmentation and suggested that labour market arguments are not necessary to account for the silent features of labour market in developing countries. Our model features a number of real (adjustment cost in investment and utilization rate of capital) and nominal (price adjustment) frictions that appear to be necessary to capture the empirical persistence in the main macroeconomic data which have become quite standard in the DSGE literature. Smets and Wouters (2007) model exhibits both sticky nominal prices and wages that adjust following a Calvo mechanism but we deviate from that and model goods producer's prices using Rotemberg (1982) model with full indexation of prices.[3] Our model also incorporates a variable capital utilisation rate which tends to smooth the adjustment of the rental rate of capital in response to changes in output. As in Smets and Wouters (2007), the cost of adjusting the utilisation rate is expressed in terms of consumption goods and the cost of adjusting the capital stock is modelled as a function of the changes in investment, rather than the level of investments. The model introduces several structural shocks that are asymmetric to the formal sector which include government spending shock, labour income tax shock and capital income tax shock. We further provide estimates of output multipliers for the alternative fiscal instruments from both models (with and without shadow economy) to highlight what happens to the multipliers when we incorporate a shadow economy relative to the benchmark case of one sector economy (without shadow economy). In each case, we consider the longrun and the shortrun multiplier effects of fiscal policy instrument on real GDP. Key finding from our multiplier computations indicate that both shortrun and longrun tax multipliers are typically smaller than one in absolute value than government spending multipliers. Moreover, the introduction of shadow economy further weakens the size of government expenditure multiplier and strengthens the adverse labour and capital tax multipliers. These happen because fiscal multipliers are determined by the private sector's responses to fiscal shocks, generating a spill-over effects onto the shadow sector. The effects of relative consumption prices and factor inputs prices together determines the reallocation of labour and capital into the informal sector. More broadly, our multiplier computations are related to a large and growing set of studies that examine the size of fiscal multipliers within a two-sector macroeconomic models. Most recent and prominent examples include Basile et al. (2016), Pappa et al. (2015), Hayat et al. (2016) and Junior et al. (forthcoming).[4]

[3] In fact, Smets and Wouters (2003, 2007) followed a partial indexation of prices.

[4] Other fiscal multiplier literature within the single sector model include Cogan et al. (2010), Christiano et al. (2011), Eggertsson (2011) and Woodford (2011).

The remainder of this chapter is structured as follows, section 2 provides an overview of the model, while section 3 reports on the parameters and steady state ratios used for calibrating the model, presents the results and some sensitivity analysis. Finally, section 4 concludes.

2.2 The Model

The framework of our model is very close to Smets and Wouters (2003, 2007); and Albonico et al. (2016) with the main difference being the introduction of the shadow economy. The model is characterised by households, intermediate goods producers and final goods producers operating in each sector of the economy. It is then completed by the standard Central Bank and government fiscal instruments. Households are standard and maximise the utility function over a time horizon, supply the same level of labour services to goods producers in each sector. We do not explicitly model the financial sector, however, as argued in Justiniano et al. (2011), investment specific shocks may be interpreted as a proxy for more fundamental disturbances to the functioning of the financial sector. Households wealth is accumulated by purchasing government bonds and investment in firms; they also decide on how much capital to accumulate based on the capital adjustment cost and capital utilisation. The intermediate goods producers supply their intermediate goods to final goods producers who differentiate and repackage them into final goods for households' consumption. Households and goods producers face nominal and real frictions, which have been identified as important in generating empirically plausible dynamics. Real frictions are introduced through generalised adjustment costs in investment, variable capacity utilisation and nominal frictions arise from staggered price-setting *la* Rotemberg (1982), along with full dynamic indexation of price contracts. In addition, there exist financial frictions in the form of domestic risk premium and investment specific shock. Specifically, final goods producers in both sectors are able to reset their prices *ala* Rotemberg (1982) model. We postulate the central banks to follow strictly inflation targeting policy and finally, we capture the fiscal feedback rules by introducing consumption tax, labour and capital income taxes which are used by the government to finance its expenditure.

2.2.1 Households

There is a continuum of households of measure unity who supply labour services to firms. Households are made up of individuals who consume, work in both sectors of the economy and return the wages they earn to the household. Households supply the same amount of undifferentiated labour services to each sector of the economy thereby setting their real wages to the marginal rate of substitution between consumption and labour supplied. Their savings and investment are made through purchasing of government bonds and supplying of capital to sectoral goods producers. Following earlier contributions by (Merz 1995; Andolfatto 1996) we assume that household members perfectly share the risk of sectoral consumption so individuals consumption decisions are the same and independent from their working conditions. The lifetime utility of representative agents is characterised by:

$$U_t^i = E_t \sum_{n=0}^{\infty} \beta^n \left\{ \ln(c_{t+n}) - \chi \frac{l_t^{i(1+\phi)}}{1+\phi} \right\} \tag{2.1}$$

where χ is the parameter that regulates the disutility of work and ϕ defines the Frisch elasticity of labour. For each sector, household members, own goods producers, hold physical capital and choose their investment to both sectors. Households can increase the supply of rental services from capital by investing in additional capital taking into account the adjustment cost of capital. Their intertemporal budget constraint is:[5]

$$c_t + \frac{P_t^o}{P_t} i_t^o + \frac{P_t^u}{P_t} i_t^u + \frac{B_t}{P_t R_t \varepsilon_t^{RISK}} = \frac{P_t^o}{P_t}(1-\tau_t^w) w_t^o l_t^o + \frac{P_t^u}{P_t} w_t^u l_t^u + \frac{P_t^o}{P_t}(1-\tau_t^k) r_t^{k,o} u_t^o \bar{k}_t^o +$$

$$+ \frac{P_t^u}{P_t} r_t^{k,u} u_t^u \bar{k}_t^u + \frac{B_{t-1}}{P_t} - \frac{P_t^o}{P_t}(1-\tau_t^k) a(u_t^o) \bar{k}_t^o + \frac{P_t^o}{P_t} \tau_t^k \delta \bar{k}_t^o - \frac{P_t^u}{P_t} a(u_t^u) \bar{k}_t^u + \frac{P_t^o}{P_t} \Pi_t^o + \frac{P_t^u}{P_t} \Pi_t^u - \frac{P_t^o}{P_t} T_t \tag{2.2}$$

where B_t is government bond that pays one unit of currency in period $t-1$ and R_t is the gross nominal interest rate. We define a number of sectoral variables: the relative goods prices P_t^i, the capital k_t^i (where a bar on top of capital indicates physical units of capital), labour services l_t^i, the returns on capital $r_t^{k,i}$, the utilisation rate of capital u_t^i, Π_t^i represent the profit received from investment in goods production and product wage w_t^i. The term $a(u_t^i)$ defines the real cost of using the capital stock with intensity u_t^i. The fiscal authority makes net lump-sum taxes T_t which allows to deal with debt accumulation, and finances its expenditures by issuing bonds and by levying taxes on labour income τ_t^w and capital income τ_t^k. ε_t^{RISK} is the risk premium shock on the returns to bonds that affects the intertemporal margin, creating a wedge between the interest rate controlled by the central bank and the return on assets held by the households, which follows an AR(1) stochastic process with an $i.i.d$ error term given as:

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{2.3}$$

[5]Here we ignore superscript i.

Households' stock of physical capital in each sector is driven by the standard dynamic equation for capital given respectively as:

$$\bar{k}^o_{t+1} = (1-\delta)\bar{k}^o_t + \varepsilon^{INV}_t \left[1 - S\left(\frac{i^o_t}{i^o_{t-1}}\right)\right] i^o_t \tag{2.4}$$

$$\bar{k}^u_{t+1} = (1-\delta)\bar{k}^u_t + \left[1 - S\left(\frac{i^u_t}{i^u_{t-1}}\right)\right] i^u_t \tag{2.5}$$

where $S(.)$ introduces the investment adjustment cost function.[6] δ is the depreciation rate and only capital used in period $u^i_t \bar{k}^i_t$ is subject to depreciation. ε^{INV}_t is the stochastic shock to the price of investment relative to consumption goods and follows an exogenous process with an $i.i.d.$ error term as:

$$ln\varepsilon^{INV}_t = \rho^{INV} ln\varepsilon^{INV}_{t-1} + \xi^{INV}_t \tag{2.6}$$

Households in addition choose the utilisation rate of capital with the amount of effective capital in each sector given as:[7]

$$k^i_t = u^i_t \bar{k}^i_{t-1} \tag{2.7}$$

Households consumption basket c_t is described as CES aggregate over the two sectors consumption bundle:

$$c_t = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c^o_t)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c^u_t)^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{2.8}$$

Furthermore, each c_t is also defined as:

$$c_t = \left(\int_0^1 c_t^{i\left(\frac{\epsilon^i - 1}{\epsilon^i}\right)} dz^i\right)^{\frac{\epsilon^i}{\epsilon^i - 1}}$$

where φ_c indicates official sector consumption goods bias and $\epsilon_c > 1$ is the measure of elasticity of substitution between official consumption (c^o_t) and unofficial consumption (c^u_t) bundles whereas $\epsilon^i > 1$ measures the elasticity of substitution among the differentiated goods that form c_t. Minimizing total consumption expenditure subject to the consumption bundle given above yields the following demand function for each good:[8]

$$c^o_t = \varphi_c \left(\frac{P^o_t(1+\tau^c)}{P_t}\right)^{-\epsilon_c} c_t \tag{2.9}$$

[6] The investment adjustment cost function is given by:

$$S\left(\frac{i^i_t}{i^i_{t-1}}\right) = \frac{\kappa^I}{2}\left(\frac{i^i_t}{i^i_{t-1}} - 1\right)^2$$

In the steady state, $S(1) = S'(1) = 0$, $S''(1) > 0 \equiv \varpi$ with ϖ being the adjustment cost parameter.

[7] In the steady state, utilisation cost function implies that: $u^i_s = 1$ and $a(1) = 0$.

[8] In the official sector, consumption tax drives a wedge between final goods price set by firms and the corresponding consumption price.

$$c_t^u = (1 - \varphi_c)\left(\frac{P_t^u}{P_t}\right)^{-\epsilon_c} c_t \tag{2.10}$$

where τ^c is a consumption tax levied by the government on official sector consumption goods to finance its expenditure. The aggregate consumption price index is given as:

$$P_t = \left[\varphi_c\left(P_t^o(1+\tau^c)\right)^{1-\epsilon_c} + (1-\varphi_c)\left(P_t^u\right)^{1-\epsilon_c}\right]^{\frac{1}{1-\epsilon_c}} \tag{2.11}$$

In a symmetric way, we assume wages obtained by households from supplying labour services to be flexible in both sectors, thus labour market equilibrium requires that the marginal rate of substitution between total labour supplied to each sector equals the wage.[9]

Households face the usual maximization problem of maximizing their expected discounted sum of instantaneous utility (2.1) subject to equations (2.2), (2.4), (2.5) and (2.7). Letting λ_t denote the Lagrangian multiplier for the household's budget constraint and $\lambda_t Q_t^i$ the Lagrange multiplier for the capital accumulation equations whereby Q_t^i is the Tobin's q which is equal to one when there are no capital adjustment cost. It can be interpreted as the one unit shadow relative price of capital with respect to one unit of consumption. The first order conditions with respect to consumption (c_t), government bond (B_t), sectoral labour (l_t^i), sectoral capital ($\bar{k}_{t+1}^i$), sectoral investment (i_t^i) and capital utilisation (u_t^i) are respectively given given below.[10]

The intertemporal marginal utility of consumption is:

$$U_{c,t} = \lambda_t = \frac{1}{c_t} \tag{2.12}$$

The consumption Euler equation from government bond is:

$$\lambda_t = \varepsilon_t^{RISK} R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{2.13}$$

In competitive labour market, the standard labour supply conditions hold as:

$$(1-\tau_t^w)\frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o\phi}}{\lambda_t} \tag{2.14}$$

$$\frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u\phi}}{\lambda_t} \tag{2.15}$$

The arbitrage condition in the labour market ensures that both sectors pay the same level of

[9] See Gali (2008). The labour market equilibrium requires that $w_t^i = mrs_t^i$, where $mrs_t = -U_{l,t}^i / U_{c,t}^i$ is the marginal rate of substitution between consumption and labour supplied in period $t+n$ for the households. This means that the official and shadow sector would pay the same consumption wage to workers.

[10] A detailed derivations of all the first order conditions are available upon request.

real wage as:

$$\frac{P_t^o}{P_t} w_t^o (1 - \tau_t^w) = \frac{P_t^u}{P_t} w_t^u \tag{2.16}$$

The competitive capital supplied to each sector is accordingly given as:

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_{t+1}^o}{P_{t+1}} \left((1 - \tau_{t+1}^k) \left[r_{t+1}^{k,o} u_{t+1}^o - a(u_{t+1}^o) \right] + \tau_{t+1}^k \delta \right) + Q_{t+1}^o (1 - \delta) \right] \tag{2.17}$$

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_{t+1}^u}{P_{t+1}} \left[r_{t+1}^{k,u} u_{t+1}^u - a(u_{t+1}^u) \right] + Q_{t+1}^u (1 - \delta) \right] \tag{2.18}$$

The first order conditions for investments supplied to each sector is given as:

$$\frac{P_t^o}{P_t} = Q_t^o \varepsilon_t^{INV} \left(1 - S\left(\frac{i_t^o}{i_{t-1}^o} \right) - S'\left(\frac{i_t^o}{i_{t-1}^o} \right) \frac{i_t^o}{i_{t-1}^o} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^o \varepsilon_{t+1}^{INV} S'\left(\frac{i_{t+1}^o}{i_t^o} \right) \left(\frac{i_{t+1}^o}{i_t^o} \right)^2 \tag{2.19}$$

$$\frac{P_t^u}{P_t} = Q_t^u \left(1 - S\left(\frac{i_t^u}{i_{t-1}^u} \right) - S'\left(\frac{i_t^u}{i_{t-1}^u} \right) \frac{i_t^u}{i_{t-1}^u} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^u S'\left(\frac{i_{t+1}^u}{i_t^u} \right) \left(\frac{i_{t+1}^u}{i_t^u} \right)^2 \tag{2.20}$$

And finally, the following equations also gives the first order conditions for effective capital utilised:

$$r_t^{k,o} = a'(u_t^o) \tag{2.21}$$

$$r_t^{k,u} = a'(u_t^u) \tag{2.22}$$

solving equations (2.12) and (2.13) for c_t we obtain the consumption Euler equation.

2.2.2 Intermediate Goods Producers

In each sector $i \in (o, u)$, goods producers produce intermediate goods and sell them at the competitive intermediate price $P_t^{I,i}$ to final goods producers. The production function for a representative firm is given as:

$$y_t^i = A_t^i k_t^{i(\alpha^i)} l_t^{i(1-\alpha^i)} \tag{2.23}$$

where y_t^i, k_t^i and l_t^i respectively denote sectoral output, capital and labour inputs. α^i is the sectoral capital share used in productive activities. A_t^o is the official sector productivity shock which is defined as an AR(1) process with $i.i.d$ error term as:

$$lnA_t^o = \rho^A lnA_{t-1}^o + \xi_t^A \tag{2.24}$$

Goods producers in each sector maximize their market value by choosing labour (l_t^i) and capital (k_t^i) taking into account their production output level. Their market value (Π_t^i) is expressed as:

$$\Pi_t^i = \frac{P_t^{I,i}}{P_t} \left[y_t^i - w_t^i l_t^i - r_t^{k,i} k_t^i \right] \tag{2.25}$$

where w_t^i and $r_t^{k,i}$ are respectively sectoral real wage rate and real returns from capital. $\frac{P_t^{I,i}}{P_t} y_t^i$ represent the firm's revenue from selling output and $\frac{P_t^{I,i}}{P_t}(w_t^i l_t^i + r_t^{k,i} k_t^i)$ are the repayments made by goods producers to households which consist of the wage bill and cost of physical capital. The following equations respectively define the first order conditions for sectoral labour and capital:

$$w_t^i = (1-\alpha^i) A_t^i \left(\frac{k_t^i}{l_t^i} \right)^{\alpha^i} \tag{2.26}$$

$$r_t^{k,i} = \alpha^i A_t^i \left(\frac{k_t^i}{l_t^i} \right)^{-(1-\alpha^i)} \tag{2.27}$$

This implies a capital-labour ratio given as:

$$\frac{r_t^{k,i}}{w_t^i} = \frac{\alpha^i}{1-\alpha^i} \frac{l_t^i}{k_t^i} \tag{2.28}$$

Solving equations (2.26) and (2.27) yield sectoral real marginal cost as:

$$mc_t^{I,i} = \left(\frac{r_t^{k,i}}{\alpha^i} \right)^{\alpha^i} \left(\frac{w_t^i}{1-\alpha^i} \right)^{1-\alpha^i} \tag{2.29}$$

2.2.3 Final Goods Producers

We assume a sticky price specification based on Rotemberg (1982) quadratic adjustment cost in both sectors of the economy. We index their prices to a combination of both current and past inflation with a weight equal to θ_π. The final goods producers maximize their profit function by choosing their final goods prices taking into account the quadratic adjustment cost given as:

$$\frac{\kappa^p}{2} \left(\frac{\pi_t^i}{{\pi_{t-1}^i}^{\theta_\pi}} - 1 \right)^2 y_t^i$$

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost in adjusting its nominal prices that can be measured in terms of the final goods with κ^p being the price stickiness parameter which accounts for the negative effects of price changes on the customer-firm relation and θ_π representing the price indexation parameter.

The official sector final goods producers are subject to price mark-up shocks, hence in a symmetric equilibrium, the Rotemberg version of non-linear New Keynesian Phillips Curve

(NKPC) is derived as:

$$(1 - mc_t^o)\epsilon_t^o = 1 - \kappa^p \left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} - 1 \right) \frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} + \kappa^p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}} - 1 \right) \frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}} \frac{y_{t+1}^o}{y_t^o} \right] \tag{2.30}$$

where ϵ_t^o is now a stochastic parameter which determines the time-varying mark-up in the official goods markets. As in Smets and Wouters (2003, 2007), the official sector final goods producers' actual mark-up hovers around its desired level over time. This desired level comprises of an endogenous and exogenous components which is assumed to follow an AR(1) process given as:

$$ln\epsilon_t^o = ln\epsilon^o + ln\varepsilon_t^p$$

$$ln\varepsilon_t^p = \rho^p ln\varepsilon_{t-1}^p + \xi_t^p \tag{2.31}$$

with ξ_t^p being an *i.i.d.* normal innovation term. In a symmetric equilibrium, the price adjustment rule satisfies the following first order condition for the shadow goods producers given as:

$$(1 - mc_t^u)\epsilon^u = 1 - \kappa^p \left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} - 1 \right) \frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} + \kappa^p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}} - 1 \right) \frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}} \frac{y_{t+1}^u}{y_t^u} \right] \tag{2.32}$$

where $mc_t^i = \frac{P_t^{I,i}}{P_t^i}$, defines the real marginal cost in terms of the sectoral final goods price. Here we assume shadow sector goods producers to have limited market power. The above equations represent the Rotemberg version of non-linear NKPCs that relate sectoral current inflation to future expected inflation and to the level of relative outputs. The following equations respectively allow to identify the sectoral price levels and the inflation rate for the consumption price index:

$$P_t^o = \pi_t^o P_{t-1}^o \tag{2.33}$$

$$P_t^u = \pi_t^u P_{t-1}^u \tag{2.34}$$

$$P_t = \pi_t P_{t-1} \tag{2.35}$$

where P_t is defined by equation (2.11).

2.2.4 Government Policies

In this section we introduce and discuss the various government policies in regulating the real sector. It comprises of the fiscal tools used by the government and a Central Bank who oversees the implementation of monetary instruments.

Fiscal Policy

The government supplies an exogenous amount of public goods (g_t) which is defined in terms of the official sector goods. Government expenditure is financed through the taxes (levied on consumption goods, labour and capital income) and the issuance of one period nominally risk free bonds. The government budget constraint is of the form:

$$g_t + \frac{B_{t-1}}{P_t^o} = \tau_t^w w_t^o l_t^o + \tau_t^k \left[r_t^{k,o} u_t^o - a(u_t^o) - \delta \right] \bar{k}_t^o + \tau^c c_t^o + \frac{B_t}{P_t^o R_t} + T_t \tag{2.36}$$

where T_t are lump-sum taxes which also appear in the household's budget constraint and it explicitly ensures solvency in government deficit. Government spending (g_t) follows a stochastic process with *i.i.d.* error term given as:[11]

$$lng_t = \rho^G lng_{t-1} + \xi_t^G \tag{2.37}$$

As an illustration of the fiscal rules on the revenue side, we set fiscal rules for labour and capital income taxes to follow an AR(1) process given respectively as:[12]

$$ln\tau_t^W = \rho^W ln\tau_{t-1}^W + \xi_t^W \tag{2.38}$$

$$ln\tau_t^K = \rho^K ln\tau_{t-1}^K + \xi_t^K \tag{2.39}$$

where both ξ_t^W and ξ_t^K represent their respective error term defined as an *i.i.d.*

Monetary Policy

We close the model by describing a simple structure for the monetary policy rule. The Central bank is assumed to follow a pure inflation targeting rule and set a standard Taylor-type monetary policy instrument so that the nominal interest rate is adjusted in response to the movement in inflation gap with interest rate smoothing. The policy rule is characterised by the following Taylor rule:

$$R_t = R_{t-1}^{(\rho^R)} (\pi_t^o)^{\mu_\pi (1-\rho^R)} \varepsilon_t^R \tag{2.40}$$

where R_t is the nominal interest rate, ρ^R is interest rate smoothing parameter, μ_π denotes Taylor coefficient in response to inflation gap[13] with ε_t^R denoting monetary policy shock, with a

[11] In the steady state, we impose that $\frac{g_s}{y_s^o} = \bar{g}_s$ in order to obtain the public consumption-output ratio.

[12] See Coenen et al. (2012, 2013) for similar discussions. Here we do not necessarily consider feedback on debt and output but we assume the economy to react to fiscal shocks. Albonico et al. (2016) argued that a more restricted model without the feedbacks is better specified than models with fiscal reaction functions. Therefore, by considering fiscal shocks the model stability is obtained because the implicit lump-sum taxation ensures government solvency. We later consider feedback on output (automatic stabilizer) as a sensitivity test.

[13] That is, deviation of inflation rate from the inflation target.

standard $i.i.d$ innovation. In this context, the monetary policy shock is thought of as unexpected deviation of the nominal interest rate via Taylor rule at period t. The exogenous shock to monetary policy enters the nominal interest rate as ε_t^R. The central bank supplies money demanded by household to support the desired nominal interest rate.

2.2.5 Market Clearing and Resource Constraint

The aggregate resource constraints are given respectively as:[14]

$$y_t^o = c_t^o + i_t^o + g_t + a(u_t^o)\bar{k}_{t-1}^o + \frac{\kappa^p}{2}\left(\frac{\pi_t^o}{\pi_{t-1}^o{}^{\theta_\pi}} - 1\right)^2 y_t^o \tag{2.41}$$

$$y_t^u = c_t^u + i_t^u + a(u_t^u)\bar{k}_{t-1}^u + \frac{\kappa^p}{2}\left(\frac{\pi_t^u}{\pi_{t-1}^u{}^{\theta_\pi}} - 1\right)^2 y_t^u \tag{2.42}$$

The last two terms of each equation represent household's capital utilisation cost and goods producers price adjustment cost. The aggregate consumption is given by equation (2.8) and aggregation labour constraint is computed as follows:

$$l_t = l_t^o + l_t^u \tag{2.43}$$

Following Colombo et al. (2016), we introduce into the model the relative size of the shadow sector (SH_t) which will be useful when deriving the steady states of the model. From the sectoral resource constraints we obtain SH_t as:[15]

$$SH_t = \frac{y_t^u}{y_t^o}$$

2.3 Model Dynamics and Results

In order to ascertain the role played by fiscal feedbacks in a DSGE model, we calibrate the theoretical model in the previous section by focusing on fiscal impulses. The aim is to show how our model is coherent to the new Keynesian DSGE models and to highlight the role played by various fiscal packages in the presence of informal economy. We do so by examining the various channels and factors which influence the dynamic properties of the model. The benchmark simulation involves exogenous processes asymmetric to the official sector consisting of government spending shock, labour and capital income tax shocks. The model is solved by

[14] We note here that, the official sector resource constraint incorporates the government expenditure.
[15] Through a straight forward manipulations using (2.9) and (2.10) we obtain steady state SH as:

$$SH = \frac{y_s^u}{y_s^o} = \frac{1-\varphi_c}{\varphi_c}\left(\frac{P_s^u}{P_s^o(1+\tau^c)}\right)^{-\epsilon_c} \frac{(1 - i_s^o/y_s^o - g_s)}{(1 - i_s^u/y_s^u)}$$

focusing on the constraints and first order conditions for prices and quantities, we then derive all the log-linearised equations of the model by taking log-linear approximations around the steady state.[16] The coefficients of the log-linear model depend on the calibration parameters of the model as well as steady state values and we further use the steady state conditions of the model to solve out for number of other parameters.

2.3.1 Model Calibrations

The structural parameters of the model consist of parameter values mostly borrowed from Smets and Wouters (2007) and Colombo et al. (2016). These parameters are selected in order to capture specific ratios in the steady state of the model. Parameters whose information relates to developing countries are calibrated using values and data from the developing countries literature. Other parameters and steady state ratios are chosen to match values from the developing countries, complete list of parameters and their values are in table (2.1).

Parameters characterizing the household's preferences and the official sector firms are fairly standard. The subjective discount factor β, is set to 0.99 which is consistent with Smets and Wouters (2007), the same value was used in Colombo et al. (2016) to achieve an annual steady state interest rate of 4%. The elasticity of substitution between official and informal consumption bundles is set at $\epsilon_c = 1.5$ as was described in Batini et al. (2011). The coefficient of Frisch elasticity of substitution for labour supply in the utility function is fixed at $\phi = 2$, a value consistent with the posterior mean reported by Smets and Wouters (2007). The steady state elasticity of capital utilisation cost parameter τ is fixed at 0.2696 to indicate a mean of 0.2 for the capital utilisation cost function as suggested by King and Rebelo (2000). The elasticity of the cost of adjusting investment is also fixed at $\varpi = 6.0144$ to be as close as the value estimated by Christiano et al. (2005) and Smets and Wouters (2003). In order to characterise the empirical findings of the shadow sector in the developing countries, we set the steady state share of the shadow economy $SH = 0.47$ (a value common to several developing and low-income countries) to calibrate the value of official consumption goods bias φ_c at 0.54. Turning to the goods producer's structural parameters for both sectors, from Schmitt-Grohe and Uribe (2004), we take the price elasticity of demand for differentiated goods parameter $\epsilon^o = 6$, a value consistent with a 20% price mark up in the official sector and the shadow sector value is set at $\epsilon^u = 20$ which also implies a 5% price mark up. The degree of inflation indexation parameter is set to $\theta_\pi = 1$ to indicates a full indexation of inflation. The degree of price stickiness parameter is fixed at $\kappa^p = 4.37$. Available literature suggest no evidence of nominal rigidities in the shadow sector, therefore the benchmark values for inflation indexation and degree of price stickiness

[16]The full set of the first order conditions, steady state derivations and log-linearised equations are presented in the appendix.

Table 2.1: Model calibration parameters.

Parameter	Value	Description
Preferences & Technology		
β	0.99	Subjective discount rate
ϕ	2	Frisch elasticity of substitution for labour
ϵ_c	1.5	Elasticity of substitution between official and unofficial consumption
ϵ_o	6	Official sector price elasticity of demand
ϵ_u	20	Shadow sector price elasticity of demand
κ^p	4.37	Degree of price stickiness
θ_π	1	Inflation indexation
δ	0.025	Capital depreciation
α^o	0.36	Official sector's capital Share
α^u	0.28	Shadow sector's capital Share
Monetary policy		
ρ^R	0.9	Interest rate smoothing parameter
μ_π	1.5	Taylor coefficient to inflation gap
Shock innovation		
ρ^G	0.7	Innovation to government spending shock
ρ^W	0.8	Innovation to labour income tax shock
ρ^K	0.75	Innovation to capital income tax shock

are used for both sectors as in Colombo et al. (2016). The depreciation rate is set to equal to $\delta = 0.025$ per quarter which implies an annual depreciation on capital equal to 10%. We additionally set the official sector capital share to $\alpha^o = 0.36$ to capture a high capital intensity in the official sector than the informal sector. The informal sector firm's capital share parameter is calibrated to capture a relatively low capital intensity in their production function, so we choose a capital share of $\alpha^u = 0.28$ as in Koreshkova (2006). The conventional parameters

Table 2.2: Fiscal policy steady state ratios.

Fiscal ratio	Value	Description
τ^c	0.10	Consumption tax
τ^k_s	0.20	Capital income tax
τ^w_s	0.15	Labour income tax
$\bar{g}_s$	0.14	Government consumption

characterizing the monetary policy instrument are set accordingly as: the Taylor rule interest smoothing rate parameter $\rho^R = 0.9$ and inflation gap parameter $\mu_\pi = 1.5$. The parameters describing the shock processes are calibrated as follows, innovation to government spending shock parameters is set at $\rho^G = 0.7$. Innovations to labour and capital income tax shocks are set at $\rho^W = 0.8$ and $\rho^K = 0.75$ respectively. The ratios of fiscal variables to GDP and the steady state tax rates as shown on table (2.2) were taken from Melina et al. (2016) which are consistent with data collected by the International Bureau of Fiscal Documentation in 2005-06 for developing countries. The steady state values for τ^c, τ_s^k, and τ_s^w are respectively fixed at 10%, 20% and 15%; and finally, government spending to GDP ratio ($\bar{g}_s$) is set at 14%. To achieve a stable steady state, we conventionally set the aggregate labour supply $l_s = 0.25$ and aggregate price $P_s = 1$ respectively. It is paramount to note that, the steady state relative prices are determined by mark-ups, technological parameters and various tax rates. The steady state relative price of the shadow sector goods is higher than the official sector relative price due to the high cost of factor inputs. The rest of steady state values are calibrated using the other structural parameters.

2.3.2 Analysis and Discussion of Results

This section analyses and discusses the various impulse response functions regarding some key macroeconomic variables with the baseline calibration to asymmetric shocks in the official sector. Figures (2.1)-(2.6) represent the various responses following government spending, labour and capital income tax shocks. We analyse how fiscal shocks are transmitted into the shadow sector and how the presence of shadow sector affects the economy at large. We note here that, variables are already expressed as percentage deviations from the steady state values and the continuous red lines in the figures represent the behaviour of shocks in one-sector (official sector only) economy as in Smets and Wouters (2003, 2007) while the continuous blue line defines the behaviour of variables in the two-sector (both official and shadow sector) economy. The baseline calibration results in the current set up are in line with the existing new Keynesian DSGE models with two sectors and we describe them accordingly.

Figures (2.1) and (2.2) show a positive response to government spending shock and as expected from the one-sector model, labour employment, capital demand and product wage rise which induce an increase in official output. Similar results are obtained in official sector of the two-sector model, with the main qualitative difference being the fall in official sector product wage. The government expenditure shock comes in with two effects; a positive demand effects for official sector firms leading to a rise in labour employment and a negative wealth effects for official consumers which decreases private consumption and investment in both models. With the rise in labour demand and supply, labour employment increases and wage rate falls as a result of a fall in relative goods prices in the two-sector model. The reduction in the relative price of official goods lead to the reallocation of factor demand towards official goods and the reduction in official wage in the two-sector model brings about an inflow of employment toward the formal sector making labour reallocation an integral part of the model. The reallocation of labour employment towards the formal sector causes the shadow sector employment as a share of total employment to fall. This is also attributed to the rise in capital stock in the formal sector which further increases the productivity relative to the shadow sector making agents to reallocate to the formal sector. On the response of labour employment and real wages, see Pappa et al. (2015) for similar analysis on four Euro zone economies. The persistent rise in nominal interest rate, though relative sectoral prices are falling, contributes to the fall in private consumption in both models and the increasing official sector inflation leads to sharp rise in the official sector output. The fall in official sector private consumption in both models is mainly caused by the negative wealth effects which arises from the increase in consumers anticipation of a higher taxes in the future due to government spending hikes. The large positive effects of government spending shock on official output is mainly determined by the substantial rise in labour employment and capital flows in the official sector. The results from this calibration is consistent with Hayat and Qadeer (2016) for Asian countries; and Khan and Khan (2011) for Pakistani economy. On the other hand, the government spending shocks decreases the shadow sector output, consumption, labour employment and capital demand which indicate a dampened effect of the shock on the shadow sector. These results are expected since most of the government spendings are geared toward the official sector. The strong role played by the shadow sector impact on the aggregate variables. The government spending multiplier is substantial in the model in both periods considered and the result replicate that of Forni et al. (2009), which obtained smaller multipliers and a subsequent fall in private consumption. To deepen the consistency of the qualitative results, our government spending multipliers provide us with further reason to believe in the transmissions at work. As table (2.3) shows, there is a positive effect of government spending shock in the one-sector model. However, the two-sector model multipliers indicated a positive spending shock effect in the official sector and a negative spending effect in the shadow sector. This again strengthens the reallocation effects toward the formal sector in the presence of shadow economy.

Figures (2.3) and (2.4) present the impulse responses of labour income tax hikes on the economy. We immediately observe that the shock has a contractionary effect in the one-sector model and the official sector of the two-sector model thereby reducing output, private consumption, investment, labour employment and capital demand. The one-sector model shows that higher taxes raises the product wage, marginal cost and subsequently the inflation. The fall in aggregate demand makes firms to reduce their labour demand leading to a fall in labour employment and output in the official sector. The two-sector model also indicates a negative transmissions of the shock in the official sector. However, the presence of shadow economy has a powerful amplifying effects on the shadow sector output, consumption, investment and labour

employment. The presence of shadow economy has a reallocation effects of factors services toward the shadow sector which is caused by the relative consumption and factor input prices (wages) thus resulting in increasing demand for shadow goods and services. We note here that, aggregate variables such as output, labour employment and consumption all rise in accordance to the shadow sector variables which indicate the powerful role played by the shadow sector.
We now focus on capital income tax shocks which on the impact results in negative transmission of the shock to the official economy as figures in (2.5) and (2.6). Official sector inflation and nominal interest rate falls inducing a decrease in real interest rate leading to an outflow of capital from the official sector to the shadow sector. This in turn lowers the official sector labour employment thereby inducing an increase in the shadow sector capital demand and labour employment as a result of a fall in shadow wages. This means that factor inputs with higher taxes in the official sector would outflow to the unofficial sector making the other factor inputs more scares in the shadow economy and this attract an inflow of the scares inputs as well. In the case of capital tax shock, the adjustment of capital is sufficient to generate the equilibrium dynamics. This mechanism explains the reasons for the lower capital tax multipliers (almost near to zero) even in the presence of the shadow economy. Overall, we could say that, tax hikes lead to factor inputs reallocation into the shadow sector. Higher taxes in the official sector provide incentives for firms and individuals to evade taxes in an economy with large shadow sector. Moreover, the fall in investment and capital stock in the formal sector lowers the productivity differentials between the two sectors hence agents reallocate to the shadow sector leading to an increase in shadow sector labour employment and capital demand. An empirical work by Pappa et al. (2015) provide similar results with VAR and DSGE models for some Euro zone economies. Both tax multipliers as in table (2.3) shows a negative effect of the shock in the one-sector model with two-sector model multipliers indicating a negative tax effect in the official sector and a positive effect in the shadow sector strengthening the factor reallocations toward the informal sector.

2.3.3 Fiscal Multipliers

We now discuss the quantitative assessment of the key factors that determine the GDP effects associated with alternative fiscal instruments with the computed fiscal multipliers. Our aim at this point is to highlight what happens to the multipliers when we incorporate shadow economy relative to the benchmark case of one-sector economy. We further use these estimates to understand whether fiscal policies can be used to stabilise the economy in response to shocks. According to Pappa et al. (2015) and Basile et al. (2016), estimates of fiscal policy multipliers in countries with large unreported production sector cannot be relied upon unless the dynamics of the hidden and regular components of the GDP are taken into consideration. Table (2.3) summarizes the computed multipliers of the three fiscal instruments on output in both the short-run and long-run periods. Following Faia et al. (2013) and Coenen et al. (2013), the short-run multipliers (impact multipliers) are calculated as output effects during the impact period divided by the cost during the impact period.[17] Long-run multipliers (cumulative multipliers) are computed as the discounted output effects divided by the discounted costs over the periods considered.[18] Computing the size and the sign of the fiscal multipliers are essential for designing

Table 2.3: Fiscal multipliers for various fiscal packages based on impulse response.

	Gov. spending	**Labour tax**	**Capital tax**
Without shadow economy (One-sector model)			
Output			
Short run	0.525	-0.123	-0.002
Long run	0.254	-0.221	-0.008
With shadow economy (Two-sector model)			
Official output			
Short run	0.158	-0.193	-0.001
Long run	0.205	-0.300	-0.003
Shadow output			
Short run	-0.364	0.607	0.003
Long run	-1.120	1.560	0.014

and the implementation of fiscal policies. In case a government spending multiplier is smaller than expected, then expansionary fiscal policy would fail to sufficiently boost the economic activity of a country and it would also increase the public indebtedness as a percentage of GDP. The second important component of fiscal policy is that a tax multiplier that have a larger than expected value may depress the economy more than anticipated and it will destroy the tributary base from which all the taxes are collected (Hayat and Qadeer, 2016). Key findings from our computations are that both short-run and long-run labour income tax and government multipliers are quite sizeable, although generally smaller than one while capital tax multipliers are near to zero.[19] The government spending multipliers show a positive effect in

[17]For government spending, we compute the short-run multiplier as:

$$M^g_{SR} = \frac{y_t - y_s}{g_t - g_s}$$

where y_s and g_s denote steady state variables.

[18]For government spending, we compute the long-run multiplier as:

$$M^g_{LR} = \frac{\sum_{t=0}^{\infty} \beta^t (y_t - y_s)}{\sum_{t=0}^{\infty} \beta^t (g_t - g_s)}$$

[19]The reason for the smaller size of the multipliers are: firstly, multipliers computed from impulse responses give smaller multipliers relative to those computed based on the standardised fiscal stimulus. Secondly,

the one-sector model as expected while the two-sector model government spending multipliers indicated a positive government spending shock effect in the official sector and a negative spending effect in the shadow sector. This occurs as a result of the reallocation effects of factor inputs from the shadow sector toward the formal sector. Moreover, tax multipliers as shown on table (2.3) indicate a negative effects of the income tax shock in the one-sector model, however, the presence of the shadow sector offsets the negative multiplier because there is a reallocation of factor inputs in response to the shocks. This further strengthens the role played by shadow economy in the transmission processes. In a nut shell, the presence of shadow economy reduces and weakens the government expenditure multiplier, whiles the income tax multipliers are strengthened as a result of the private sector's response to the shocks. The amplifying income tax effects occur simply because the effects of relative consumption and factor inputs prices create a spill-over effects onto the shadow sector which determines the sectoral factor allocation in the model.

2.3.4 Fiscal Stabilizers

In this section we compare both models (with and without shadow economy) with the situation where fiscal instruments react to some fiscal feedback rule. Specifically, we assume that all the three fiscal instruments react to their own lagged values and to the official sector output (y_t^o) where the latter feedback is thought to reflect the notion of automatic stabilizers. A more realistic treatment of fiscal policy suggest that the inclusion of fiscal stabilizers might be important in assessing the stability of the model (see recent contributions by Coenen et al. (2012, 2013); Albonico et al. 2016). We therefore perform this robustness check by allowing for fiscal stabilizer on government spendings, labour and capital income taxation. We model government spending along the lines of Albonico et al. (2016) given as:

$$lng_t = \rho^G lng_{t-1} + \theta_y^G lny_t^o + \xi_t^G \tag{2.44}$$

Similarly, as an illustration of the fiscal rules on the revenue side, both labour and capital income tax rules are given respectively as:

$$ln\tau_t^W = \rho^W ln\tau_{t-1}^W + \theta_y^W lny_t^o + \xi_t^W \tag{2.45}$$

developing countries usually have lower tax bases. Typically, tax collection is very low in low-income countries around 10%-20% of their GDP. Thirdly, low-income countries mostly have many small-scale firms and large informal sector. It therefore makes it difficult to impose proper taxes on large informal and small-scaled sector of the poor economies, such as village shops and street vendors, because there is no formal record of their incomes and transactions. These countries normally have agrarian economies in which farmer's incomes are seasonal and unstable, so it is difficult to calculate base for an income tax. Fourthly, governments of developing countries have alternative sources of revenues such as foreign aid, which are sometimes larger than domestically generated tax revenues and a significant fraction of GDP. And lastly, incomes are unevenly distributed in developing countries and there are institutional lapses such as a lack of efficient, well trained and well-educated tax administrators.

$$ln\tau_t^K = \rho^K ln\tau_{t-1}^K + \theta_y^K lny_t^o + \xi_t^K \tag{2.46}$$

where parameters θ_y^G, θ_y^W and θ_y^K are the stabilization parameters which measure the responsiveness of the fiscal rules to official sector output. The feedback rule parameters are such that θ_y^G is strictly negative and θ_y^W and θ_y^K are strictly positive according to prior estimates from the literature. An empirical estimation from the literature on feedback coefficients in both expenditure and revenue rules are generally not well specified by the data, we therefore follow Coenen et al. (2012, 2013) and Corsetti et al. (2012), to set $\theta_y^G = -0.06$, $\theta_y^W = \theta_y^K = 0.21$ which were rationalized based on empirical estimations.[20] These rules allow government spending and other fiscal instruments to depend on the level of official sector output and it postulate that whenever official output is below its long-run level, fiscal spending increases and income tax rates fall. The main reason for this check is to examine whether the role played fiscal stabilizers in stabilizing the model is impaired or enhanced following the introduction of shadow economy into the model. We do so by imposing a stabilization policy and compare with the model without fiscal stabilizers in both one-sector and two-sector models. We then compute a fiscal gap variable for the two cases in each model.[21] Figures (2.7)-(2.9) show the various fiscal gap variables for output under the three fiscal instruments considered. The blue bars represent the fiscal gap variable in the model without shadow economy and the orange bars represent the fiscal gap variable in the model with shadow economy.

The results from figure (2.7) shows that fiscal feedback stabilizes the positive government spending shock effects by reducing the output and this result is further strengthened in the model with shadow economy. This is why the fiscal gap variable in this case is negative during the periods considered. It strengthens the argument that fiscal stabilizers moderate overheating economies in periods of booms without affecting the underlying soundness of budgetary positions as long as fluctuations remain balanced. On the other hand, during recessions like our case with income tax hikes, fiscal stabilizers are to support economic activities. As shown on figures (2.8) and (2.9), our fiscal feedbacks on taxes stabilize the economy by raising output in both models. The stabilization policy are strengthened with the introduction of shadow economy. This indicated that fiscal feedbacks on government spending (income taxes) stabilized the economy by reducing (raising) output levels and these results even become stronger with the presence of shadow economy. These robustness checks further strengthens the role and the importance of introducing shadow sector into the model.

[20]To the best of our knowledge, none of the literature on developing and emerging countries have estimates for these feedback rule parameters. We therefore use those values as a proxy for our experiment.

[21]Fiscal gap variable is defined as the difference between the variable value (output) in the case with fiscal stabilizers and the variable value in the case without fiscal stabilizers.

2.4 Conclusions

This paper investigates the effects of fiscal policies in a new Keynesian DSGE model in the presence of shadow economies. We do so by examining the various channels and factors which influence the dynamic properties of the model. So far, most theoretical and empirical DSGE models have mainly focused on monetary policy analysis and the few ones which focus on fiscal policy rules do so without the consideration of shadow economies. We add to the literature by extending the DSGE model by Smets and Wouters (2003, 2007) and explicitly model fiscal policies to interact with monetary policy in the presence of shadow economy to allow for appropriate specifications and analysis. Our simulation involves exogenous processes that are asymmetric to the official sector consisting of government spending shock, labour and capital income tax shocks.

Our results message is that, shadow economies play a significant role in determining the behaviour of the real business cycle specifically the behaviour of the aggregate variables. Our results show that in economies with large share of informal sector, tax hikes lead to a sizeable tax evasion and a boost in the shadow economy making the standard aggregate estimates of fiscal policies ineffective while government spending shocks slow down the activities in the shadow sector. Moreover, there is negative transmission of income tax shock to the official economy as a result of the role played by interest rate and sectoral relative prices. Our results showed quantitatively that, the presence of shadow economy dampens the official sector's response to income tax shocks leading to the reallocation of factor inputs across sectors. We also found that changes in public spending generate a reduction in economic activities in the shadow sector thereby increasing official sector GDP more proportionately, shrinking the shadow sector component of the economy. This crowding-out effect of the output and inputs reallocation respectively contribute to obscure the effectiveness of government spending and income tax on the total GDP. Our results are consistent with VAR evidence obtained for Italian economy, government spendings reduce the size of the informal economy, while formal sector tax hikes increase the size of informal sector. It also turns out that the incorporation of shadow sector significantly reduces the government spending multipliers whereas the labour income tax multipliers are increased. The amplifying income tax effects occur simply because the effects of relative consumption and factor inputs prices create a spill-over effects onto the shadow sector which determines the sectoral factor allocation in the model. Our results from the fiscal feedbacks on government spending (income taxes) stabilized the economy by reducing (raising) output levels and these results even become stronger with the presence of shadow economy.

References

Albonico, A., Paccagnini, A., & Tirelli, P. (2016). In search of the Euro area fiscal stance. *Journal of Empirical Finance*, 39, 254-264.

Amaral, P. S. & Quintin, E. (2006). A competitive model of the informal sector. *Journal of Monetary Economics*, 53 (7), 1541-1553.

Andolfatto, D. (1996). Business cycles and labour market search. *The American Economic Review* 86(1), 112-132.

Aruoba, S. (2010). Informal Sector, Government Policies and Institutions. *University of Maryland*. (Mimeographed).

Basile, R., Chiarini, B., De Luca, G., & Marzano, E. (2016). Fiscal multipliers and unreported production: evidence for Italy. *Empirical Economics*, 51(3), 877-896.

Batini, N., Levine, P., Lotti, E., & Yang B. (2011). Informality, frictions and monetary policy. *School of Economics Discussion Papers 711.*

Baxter, M., & King, R.G. (1993). Fiscal policy in general equilibrium. *The American Economic Review*, 83(3), 315-334

Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1, 1341-1393.

Canzoneri, M. B., Cumby R., and Diba, B. (2002). Should the European Central Bank and the Federal Reserve be concerned about fiscal policy, and in rethinking stabilization policy?, Symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, pp.333-389.

Cavallo, M. (2005). Government employment expenditure and the effects of fiscal policy shocks. *Federal Reserve Bank of San Francisco Working Paper, 16*, 2005.

Christiano, L. J., Motto R,. & Rostagno M,. (2003). The Great Depression and the Friedman-Schwartz Hypothesis. *Journal of Money, Credit, and Banking*, 35(6 - part 2), 1119-1197.

Christiano L. J., Eichenbaum, M., & Evans C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy*, 113(1), 1-45.

Christiano, L., Motto, R., & Rostagno, M. (2008). Shocks, structures or monetary policies? The Euro Area and US after 2001. *Journal of Economic Dynamics and control*, 32(8), 2476-2506.

Christiano, L. J., Eichenbaum, M., & Rebelo, S. (2011). When is the government spending multiplier large?, *Journal of Political Economy*, 119, 78-121.

Cogan, J. F., Cwik, T., Taylor, J. B., & Wieland, V. (2010). New Keynesian versus old Keynesian government spending multipliers, *Journal of Economic Dynamics and Control*, 34(3), 281-295.

Coenen, G., & Straub, R. (2005). Does government spending crowd in private consumption? Theory and empirical evidence for the euro area. *International Finance*, 8(3), 435-470.

Coenen, G., Straub, R., & Trabandt, M. (2012). Fiscal policy and the great recession in the Euro area. *American Economic Review Papers and Proceedings*, 102(3), 71-76.

Coenen, G., Straub, R., & Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the Euro area. *Journal of Economic Dynamics and Control*, 37(2), 367-386.

Colombo, E., Onnis, L., & Tirelli, P. (2016). Shadow economies at times of banking crises: Empirics and Theory. *Journal of Banking and Finance*, 62, 180-190.

Corsetti, G., Meier, A., & Muller, G. J. (2012). Fiscal stimulus with spending reversals. *Review of Economics and Statistics*, 94(4), 878-895.

Eggertsson, G. B. (2011). What fiscal policy is effective at zero interest rates?. *NBER Macroeconomics Annual*, 25(1), 59-112.

Faia, E., Lechthaler, W., Merkl, C., (2013). Fiscal stimulus and labour market policies in Europe. *Journal of Economic Dynamics and Control*, 37(3), 483-499.

Fatàs, A. & Mihov, I. (2001), The Effects of Fiscal Policy on consumption and employment: Theory and Evidence, Manuscript, INSEAD.

Forni , L., Monteforte, L., & Sessa, L. (2009). The general equilibrium effects of fiscal policy: estimates for the Euro area. *Journal of Public Economics*, 93(3-4), 559-585.

Gali, J., Lòpez-Salido, D., & Vallès, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1), 227-270.

Gali, J. (2008). Introduction to monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework. *Introductory Chapters, Princeton University Press.*

Goodfriend, M., & King, R. G. (1997). The new neoclassical synthesis and the role of monetary policy. *NBER Macroeconomics Annual*, 12, 231-283.

Goodfriend, M., & McCallum, B. T. (2007). Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics*, 54(5), 1480-1507.

Hayat, M. A., & Qadeer, H. (2016). SIZE AND IMPACT OF FISCAL MULTIPLIERS: An Analysis of Selected South Asian Countries. *Pakistan Economic and Social Review*, 54(2), 205-231.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *The American Economic Review*, 95(3), 739-764.

Junior, C. J. C., Garcia-Cintado, A. C., & Usabiaga, C. (forthcoming). *Fiscal Adjustments and the Shadow Economy in an Emerging Market.*

Justiniano, A., Primiceri, G., & Tambalotti, A. (2011). Investment Shocks and the relative price of investment. *Review of Economic Dynamics*, 14(1), 101-121.

asif Khan, S., & Khan, S. (2011). *Optimal Taxation, Inflation and the Formal and Informal Sectors* (No. 40). State Bank of Pakistan, Research Department.

King, R. G. & Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of macroeconomics*, 1, 927-1007.

Kiyotaki, N., & Moore, J. (1997). Credit cycles. *Journal of political economy*, 105(2), 211-248.

Koreshkova, T. (2006). A quantitative analysis of inflation as a tax on the underground economy, *Journal of Monetary Economy* 53(4), 773-796.

Krugman, P. (2012). Europe's austerity madness. *New York Times, 28*, A35.

Laxton, M. D., & Kumhof, M. M. (2007). *A party without a hangover? On the effects of US government deficits (No. 7-202).* International Monetary Fund.

Linnemann, L. & Schabert, A. (2003). Fiscal policy in the new Neoclassical synthesis, *Journal of Money, Credit and Banking*, 35(6), 911-929.

Maloney, W. (1999). Does informality imply segmentation in urban labour markets? Evidence from sectoral transitions in Mexico. *World Bank Economic Review*, 13 (2), 275-302.

Maloney, W. (2004). Informality revisited. *World Development*, 32 (7), 1159-1178.

Mankiw, N. G. (2000). The Savers-Spenders theory of fiscal policy, *The American Economic Review*, 90, 120-125.

Melina, G., Yang, S. C. S., & Zanna, L. F. (2016). Debt sustainability, public investment, and natural resources in developing countries: The DIGNAR model. *Economic Modelling*, 52, 630-649.

Merz, M. (1995). Search in the labour market and the real business cycle. *Journal of Monetary Economics*, 36(2), 269-300.

Mountford, A., & Uhlig, H. (2009). What are the effects of fiscal policy shocks?. *Journal of applied econometrics*, 24(6), 960-992.

Oh, H., & Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59, S50-S64.

Pappa, E. (2009). The effects of fiscal shocks on employment and the real wage. *International Economic Review*, 50(1), 217-244.

Pappa, E., Sajedi, R., & Vella, E. (2015). Fiscal consolidation with tax evasion and corruption. *Journal of International Economics*, 96, S56-S75.

Perotti, R., Reis, R., & Ramey, V. (2007). In search of the transmission mechanism of fiscal policy [with Comments and Discussion]. *NBER Macroeconomics Annual*, 22, 169-249.

Pratap, S. & Quintin, E. (2006). The informal sector in developing countries: Output, assets and employment. Working Papers RP2006/130, World Institute for Development Economic Research (UNU-WIDER).

Ramey, V. A., & Shapiro, M. D. (1998, June). Costly capital reallocation and the effects of government spending. In *Carnegie-Rochester Conference Series on Public Policy (Vol. 48, 145-194).* North-Holland.

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4), 517-531.

Rotemberg, J.J., & Woodford, M. (1992). Oligopolistic pricing and the effects of aggregate demand on economic activity. *Journal of Political Economy*, 100(6), 1153-1207

Schmitt-Grohe, S., & Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114(2), 198-230.

Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association*, 1(5), 1123-1175.

Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3), 586-606.

Stiglitz, J. E., Fitoussi, J. P., Bofinger, P., Esping-Andersen, G., Galbraith, J. K., & Grabel, I. (2014). A Call for Policy Change in Europe. *Challenge*, 57(4), 5-17.

Uhlig, H., (2010). Some Fiscal Calculus. *American Economic Review Papers and Proceedings*, 100(2), 30-34.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics, 3(1)*, pp.1-35.

Figure 2.1: Response to Government Spending Shock

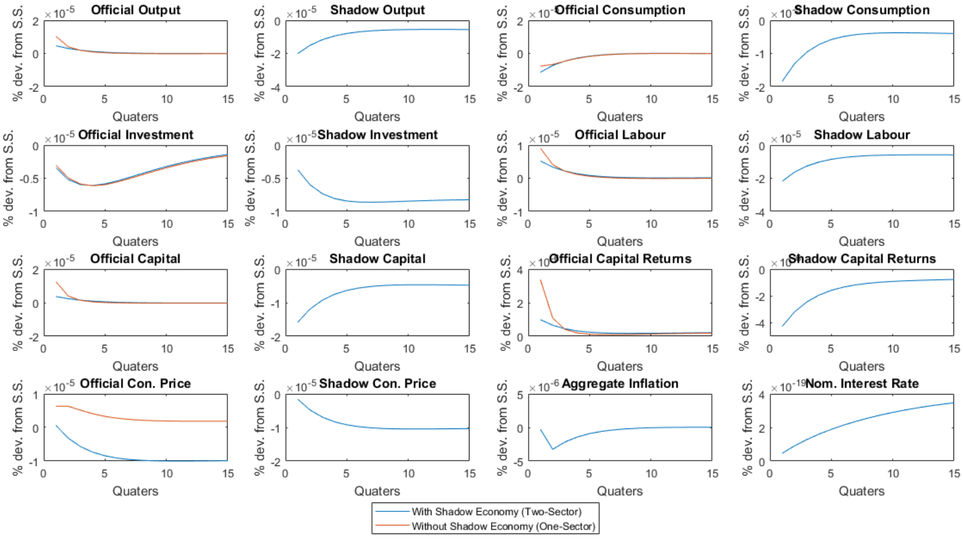


Figure 2.2: Response to Government Spending Shock (cont'd)

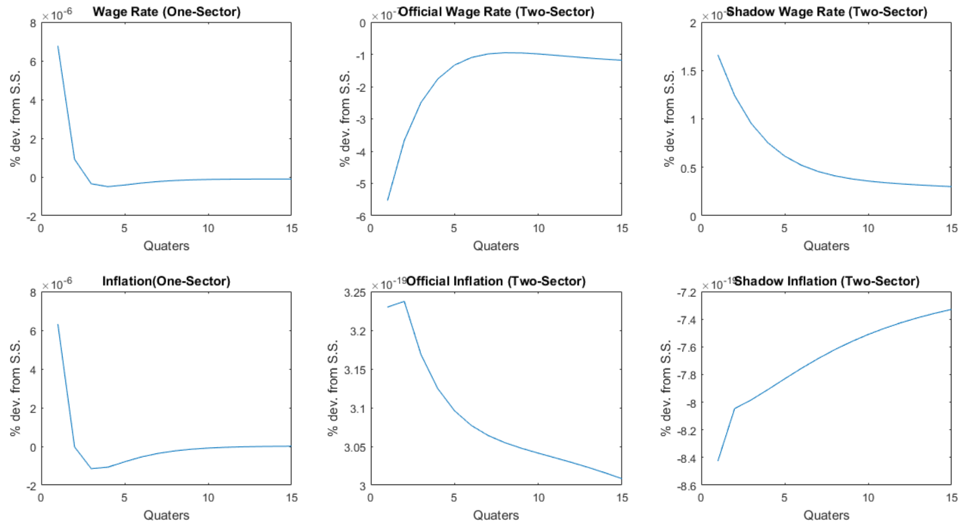


Figure 2.3: Response to Labour Income Tax Shock

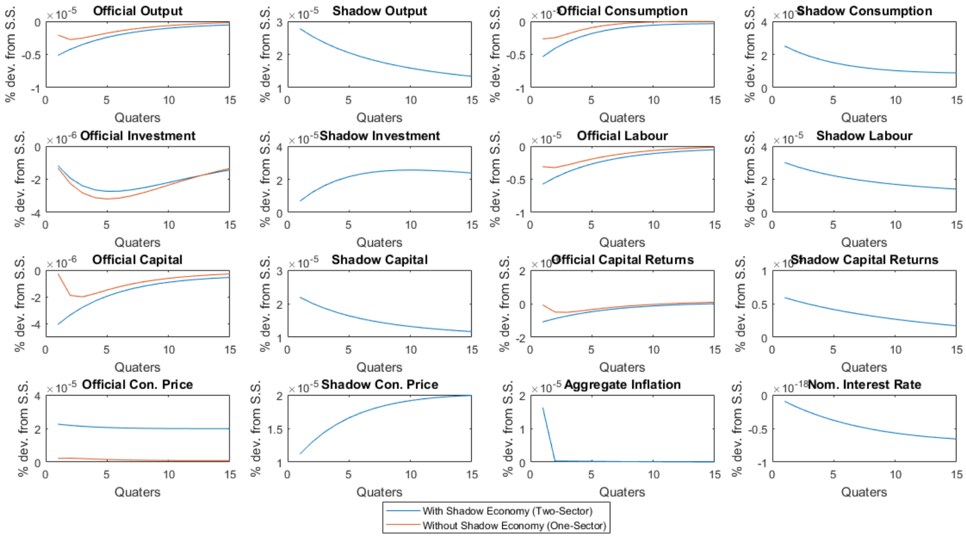


Figure 2.4: Response to Labour Income Tax Shock (cont'd)

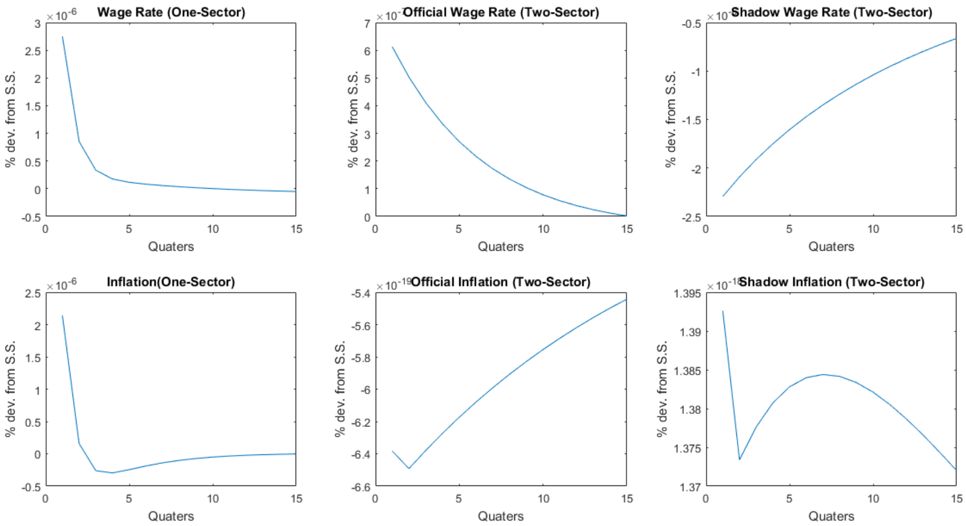


Figure 2.5: Response to Capital Income Tax Shock

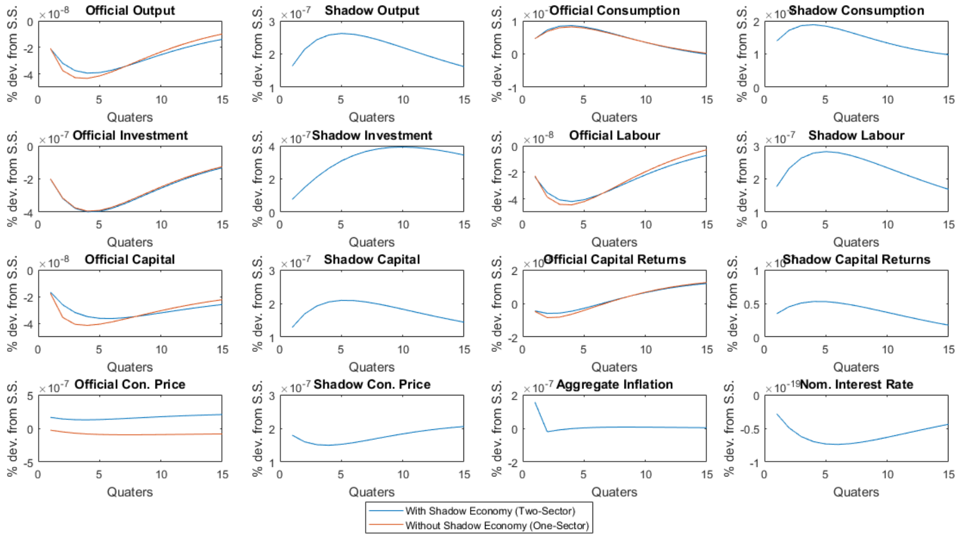


Figure 2.6: Response to Capital Income Tax Shock (cont'd)

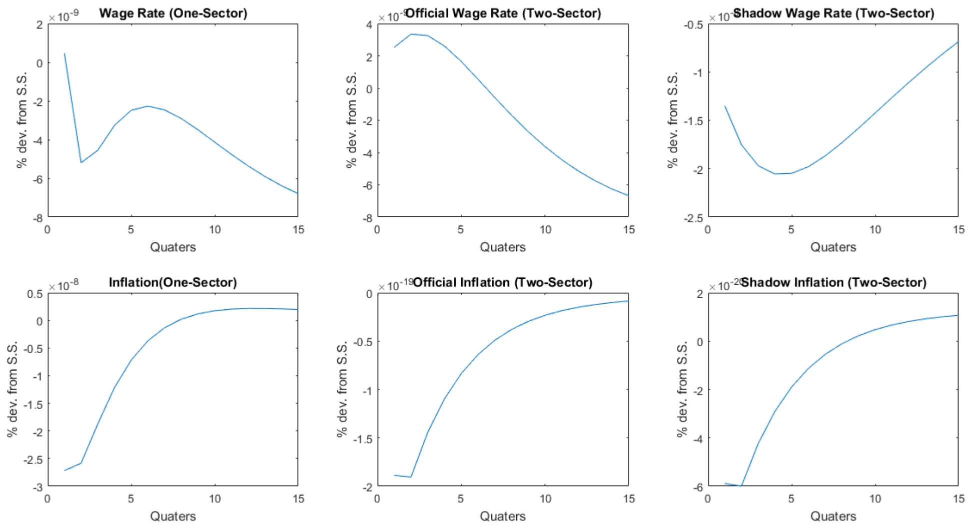


Figure 2.7: Fiscal stabilizer gap - gov. spending shock

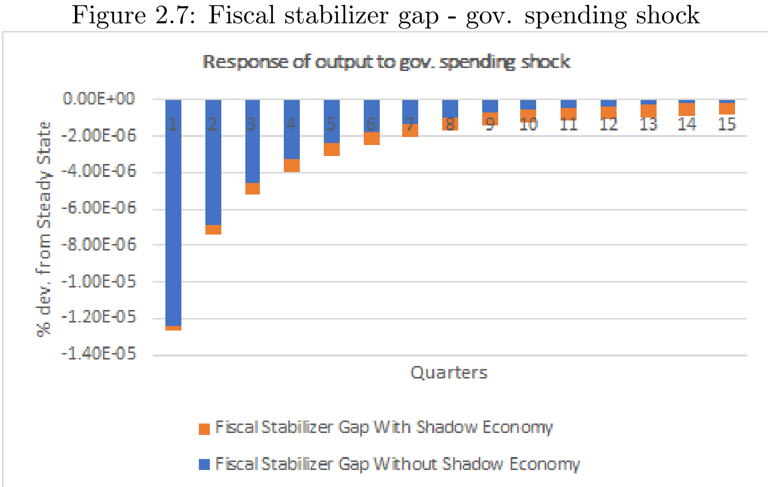


Figure 2.8: Fiscal stabilizer gap - capital income tax shock

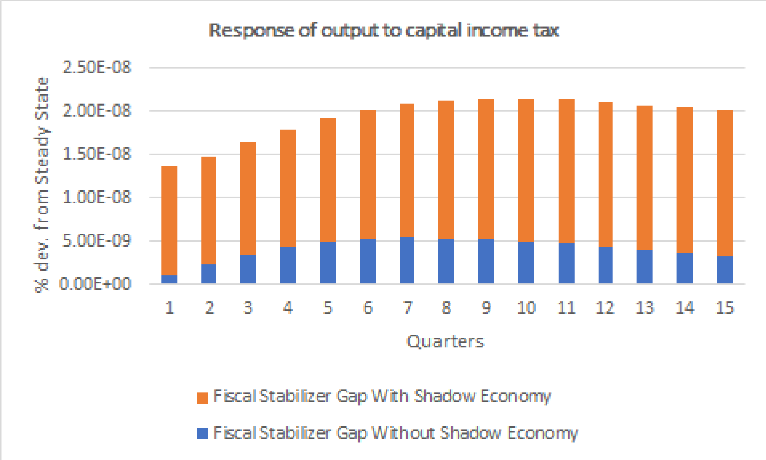
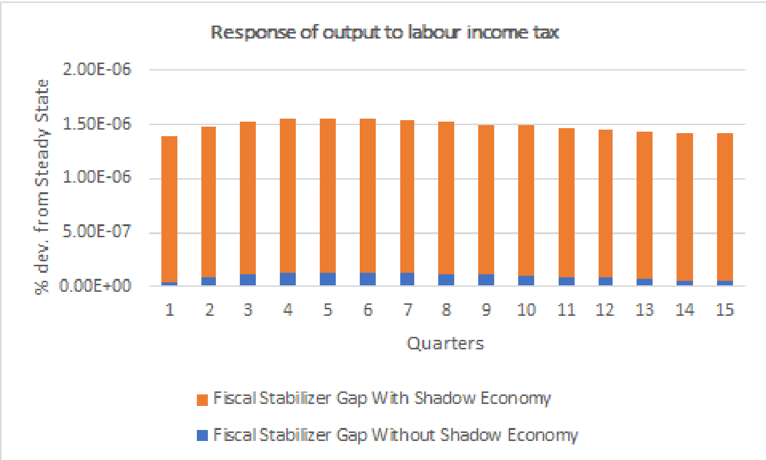


Figure 2.9: Fiscal stabilizer gap - labour income tax shock



CHAPTER 3

Fiscal Policy shocks in Developing Countries: A DSGE model with Rule-of-Thumb Consumers and Shadow Economy.

3.1 Introduction

Fiscal policies over the years have become an important macroeconomic tool for countries of all income levels following the inability of monetary policies to avoid the financial crisis that hit the world economies in 2007. In wake of this, new views on the role played by fiscal policies and its expansion were particularly large in the US and the UK (Krugman, 2012). European countries such as Germany, Greece, Portugal, Spain and others; on the other hand, put forward consolidated plans that include cuts in public employment, public wages and public investments as well as increases in VAT and labour income tax rates. The increase in taxes and cutting of public spending have been the main fiscal tools used by fiscal authorities to bring fiscal imbalances back on track over the years. Most developing and emerging countries over years have followed similar fiscal policy regimes to create an equitable distribution of income and wealth in the society. Though developing and emerging countries' economic structures are different from industrialized countries in many respects; for instance, in developing countries public transfers are typically small, and the biggest share of government spending over the years have been represented by consumption of goods and services, and by government wages; on the revenue side, and consumption taxes often are the biggest component. In the face of this, most developing and emerging countries still use fiscal expansions and recovery plans as a way of achieving economic growth and development.

Recent development in dynamic stochastic general equlibrium models has gained momentum due to its reliability to evaluate alternative macroeconomic policy measures. The applications of DSGE models have included the assessment of temporary versus permanent fiscal instruments, the assessment of structural changes in government taxes and spending policy, the analysis of fiscal packages, the analysis of fiscal multipliers and the role of private demand as well as the interactions of fiscal and monetary policies. Such DSGE literature include, Gali and Monacelli (2008); Coenen et al. (2008); Colciago et al. (2008); and Erceg and Lindè (2010). In the context of fiscal policy, models by Coenen and Straub (2005) and Gali et al. (2007) focus primarily on the implications of government spending, and deficit is adjusted using lump-sum taxes. Coenen et al. (2013) focus on the implications of European Economic Recovery Plan (EERP), whereas Cogan et al. (2010); Drautzburg and Uhlig (2015) focused on the American Recovery and Reinvestment Act (ARRA). Christiano et al. (2011) and Eggertsson (2011) study the effects of fiscal stimulus at the zero-lower bound interest rate. Most of these DSGE models were built upon the real business cycle models that had always produced negative wealth effects on households which in turn reduce private consumption, increase labour supply and decrease real wages (Baxter and King, 1993), new Keynesian DSGE models with real frictions and nominal rigidities also display the same wealth effects mechanism (Goodfriend and King, 1997; Linnemann and Schabert, 2003). Other standard version of the new Keynesian DSGE models also typically predict a positive or at least no significant negative response of private consumption to government spending shocks (Perotti, 2002; Fatàs and Mihov, 2001; Canzoneri et al. 2002; Mountford and Uhlig, 2009). The literature has identified this sharp contrast between the implications of the theory on one hand, and empirical results on the other, as a puzzle. Following this gap and uncertainty in results shown by previous literature, Mankiw (2000) argue that a fiscal policy model where both Ricardian and non-Ricardian agents coexist provide a better fiscal policy analysis relative to neoclassical and overlapping generation models.[1] The notion that fraction of consumers consume all their current disposable incomes each period, while the remaining fraction optimise intertemporally, was first developed by Hall (1978) as an alternative to the permanent income hypothesis. Campbell and Mankiw (1989, 1991) rejected the permanent income hypothesis against this alternative, and later, Mankiw (2000) suggested that rule-of-thumb consumers should be included in models that are built for fiscal policy analysis. In particular, Gali et al. (2007) show that private consumption may rise after a positive shock to government spending if the so-called rule-of-thumb consumers are allowed to co-exist with Ricardian consumers. In the model, Ricardian consumers decrease their consumption following a government spending shock because they correctly anticipate a decline in income through taxation. But rule-of-thumb consumers increase their consumption when current disposable income increases. This happens in the model when the government finances its budget at least partially through the issuance of bonds, under assumptions of sticky prices and an imperfectly competitive labour market. They concluded that, for empirically plausible calibrations of the fraction of rule-of-thumb consumers, the degree of price stickiness, and the

[1] Ricardian or intertemporal optimizing consumers are the distinction between fraction of households who own assets and are able to smooth consumption over time while the remaining fraction of households (non-Ricardian) do not participate in the financial market and thereby entirely consume their current disposable income. The term non-Ricardian consumers are used interchangeably with hand-to-mouth consumers, rule-of-thumb consumers or Limited Asset Market Participation (LAMP). The reasons provided to justify the presence of non-Ricardian consumers are miopia, fear of saving, financial insecurities and transaction costs on financial markets.

extent of deficit financing determine whether an increase in government spending raises or lowers consumption. Coenen and Straub (2005) and Forni et al. (2009), established that the share of non-Ricardian households is an important element to affect a positive reaction of private consumption to public expenditure shocks. Coenen (2012, 2013), focused on the role played by fiscal policies during the 2008-2009 recession periods with a smaller share of non-Ricardian households and found that Coenen (2005) results are mainly determined by complementarity between private and public consumptions in household's preferences. Anderson et al. (2016) use US data to estimate individual-level impulse responses as well as multipliers for government spending and tax policy shocks. Their findings were that the wealthiest individuals behave according to the predictions of standard DSGE models, but the poorest individuals tend to neglect interest rate changes and adopt consumption patterns that closely follow their current disposable income dynamics. They therefore suggested that DSGE models should incorporate the LAMP consumers where a fraction of non-Ricardian households do not hold any wealth and consume their disposable labour income in each period. Rossi (2007), showed that the introduction of non-Ricardian consumers can reverse the traditional predictions of a change in government spending on the economy as a whole: under a reasonable parametrization of the model, an increase in government spending can lead, against the common Keynesian wisdom, to a decrease in total output. The introduction of a distortive fiscal policy leads to a negative response of private consumption to a positive government spending shock. According to Motta and Tirelli (2012), the aftermath of the global financial crisis in 2007 witnessed growing concerns for income inequality and for the distributional effects of macroeconomic policies. They argued that, redistributive actions have been the domain of fiscal policies, but in recent years even monetary policies have come under scrutiny for their effects on inequality. Their work hammered on the importance of including rule-of-thumb consumers in fiscal policy analysis to interact with monetary policy. Albonico et al. (2014, 2016) introduce LAMP to a standard new Keynesian model which allowed to incorporate the possibility that public consumption shocks simulate private consumption and that transfer shocks provide a demand stimulus as in Oh and Reis (2012). They showed that the presence of both sticky prices and LAMP are necessary elements to have a positive response of private consumption to government spending shocks. Sticky prices make it possible for real wages to increase as the price mark-ups may adjust sufficiently downward to absorb the resulting gap. LAMP in part insulate aggregate demand from the negative wealth effects as a result of the higher taxes needed to finance the fiscal expansion. Albonico et al. (2014) results further showed that LAMP is sizeable in the Euro area (39% of households over the 1993-2012 sample) and LAMP were important to understand European Monetary Union business cycle, especially, in light of the recent financial crisis. Bhattarai and

Trzeciakiewicz (2017) show that independently whether public consumption is introduced into the utility of households or not, in a closed or open-economy setup, public consumption and public investment are the most effective fiscal instruments in the short-run, whereas capital income tax and the public investment are such instruments in the longer horizon. The results from their model revealed a negative response of private consumption to government spending shocks and public transfers yield relatively lower multipliers when compared with the remaining fiscal policy instruments they considered.

Most of these standard DSGE models that examine the role of fiscal policy and LAMP have mainly focused on the features of advanced countries and hence lacked the prerequisite ingredients for modelling developing and emerging economies. It therefore becomes irrational to replicate such models with advanced countries' features to the developing and Low-Income Countries (LIC). The reason being that, empirical literature on most of these developing and LIC countries suggest that developing countries are characterised by weak financial sector, large proportions of liquidity constrained consumers, existence of large informal sectors, external shock vulnerability and weak economic and political institutions. In light of this, our major aim and contribution to the existing literature on DSGE models with fiscal policy is the introduction of shadow sector and their interaction with rule-of-thumb consumers. We further seek to know whether the incorporation of shadow economy weakens the amplifying effect of rule-of-thumb consumers on fiscal multipliers. The consideration of rule-of-thumb consumers is very significant in our model because they constitute the larger share of the consumption population in most of the developing and LIC countries. For instance, based on data collected in 2011, Demirguc-Kunt and Klapper (2012) reported that on average only 24% of adults in Sub-Sahara Africa (SSA) countries have an account in the formal financial institutions. Melina et al. (2016) also argue that depending on the degree of financial development of a particular developing country, the measure of intertemporal optimizing households can be lower than 40% in SSA countries. Given this background, the paper seeks to investigate the role of fiscal policy over the real business cycle in the presence of shadow economy with a larger share of the rule-of-thumb consumers. We specifically follow Smets and Wouters (2003, 2007) and introduce these two features to ascertain their role in transmitting fiscal policies into the real sector. Our model is characterised by a competitive labour market where firms in the two sectors pay the same consumption real wage. This assumption is motivated by the theoretical contributions from Amaral and Quintin (2006); Pratap and Quintin (2006) and supported by Maloney (1999, 2004). The contributions by Pratap and Quintin (2006) on developing countries provided evidence against labour market segmentation and suggested that labour market arguments are not necessary to account for the silent features of labour market in developing countries. A major disparity that exist between the sectors (formal and informal sectors) are the calibrations of technology and price mark-up parameters. We choose a smaller capital share to capture a low capital intensity in the shadow sector and we also set the price elasticity of demand parameter to also capture low levels of price mark-ups in the shadow sector. A major policy implication is whether the fiscal authorities should target the participation rate with fiscal policy in the presence of shadow economy with liquidity constrained individuals. We do so by formulating a fiscal macroeconomic DSGE model with large share of shadow economy and LAMP consumers. We compute our fiscal variables (an average effective tax rates on labour, capital income and consumption) following Melina et al. (2016) which are consistent with data collected by the International Bureau of Fiscal Documentation in 2005-2006. Our model features a number of real and nominal frictions that capture the empirical persistence in the literature. It again deviates a little from the Smets and Wouters (2003, 2007) by modelling goods producer's prices *ala* Rotemberg (1982) with full inflation indexation. We further provide computations of output multipliers for the alternative fiscal instruments from both models to highlight the happenings of the model multipliers with the incorporation of shadow economy and LAMP consumers. This would help us answer the question of whether shadow economy weakens the amplifying effect of rule-of-thumb consumers on fiscal policy measures.

The rest of the chapter is structured as follows, section 2 provides an overview of the model with non-Ricardian characteristics, section 3 reports on the parameters and steady state ratios used for calibrating the model. It also presents the results and some sensitivity analysis. Finally, section 4 concludes.

3.2 The Model

There is a continuum of households of measure unity who consume a bundle of two goods and supply labour services at the same wage rate to each sector of the economy ($i \in [o, u]$). Each household group is made up of a share $1 - \theta$ of households (Ricardian households, $j = r$) who can access financial markets, trade in government bonds, accumulate physical capital and rent capital services to firms in each sector. The remaining fraction θ of households (non-Ricardian or LAMP households, $j = rt$) do not have access to financial markets and consume all their disposable labour income and transfers from the government. Each individual in the household supplies the bundle of labour services that firms demand in each sector of the economy at a given unique wage rate in consumption units. Following earlier contributions by Merz (1995) and Andolfatto (1996), we assume that household members perfectly share the risk of sectoral consumption so individual's consumption decisions are independent from their working conditions across sectors. Ricardian households wealth is accumulated by purchasing government bonds and investment in firms; they also decide on how much capital to accumulate based on the capital adjustment cost and capital utilisation. The intermediate goods producers supply their intermediate goods to final goods producers who differentiate and repackage them into final goods for households. Final goods producers in both sectors are able to reset their prices *ala* Rotemberg model. Finally, we capture the financial sector's impact through capital and investment frictions and we close the model by assuming a central bank who follows strictly inflation targeting policy.

Households consumption basket c_t^j is described as a CES aggregate over the two sectors consumption bundle with $j \in [r, rt]$:

$$c_t^j = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^{o,j})^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^{u,j})^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{3.1}$$

Furthermore, each c_t^j is also defined as:

$$c_t^{i,j} = \left(\int_0^1 c_t^{i,j\left(\frac{\epsilon^i - 1}{\epsilon^i}\right)} dz^{i,j}\right)^{\frac{\epsilon^i}{\epsilon^i - 1}}$$

where φ_c indicates official sector consumption goods bias and $\epsilon_c > 1$ is the measure of elasticity of substitution between official consumption $(c_t^{o,j})$ and unofficial consumption $(c_t^{u,j})$ bundles whereas $\epsilon^i > 1$ measures the elasticity of substitution among the differentiated goods that form c_t^j. Minimizing total consumption expenditures subject to the consumption bundle given above yields the following demand function for each good:[2]

$$c_t^{o,j} = \varphi_c \left(\frac{P_t^o(1+\tau^c)}{P_t}\right)^{-\epsilon_c} c_t^j \tag{3.2}$$

$$c_t^{u,j} = (1-\varphi_c)\left(\frac{P_t^u}{P_t}\right)^{-\epsilon_c} c_t^j \tag{3.3}$$

where τ^c is a consumption tax levied by the government on official sector consumption goods to finance its expenditure. The aggregate consumption price index is given as:

$$P_t = \left[\varphi_c\big(P_t^o(1+\tau^c)\big)^{1-\epsilon_c} + (1-\varphi_c)\big(P_t^u\big)^{1-\epsilon_c}\right]^{\frac{1}{1-\epsilon_c}} \tag{3.4}$$

In a symmetric way, we assume wages obtained by households from supplying labour services in both sectors to be flexible, thus labour market equilibrium requires that the marginal rate of substitution between total labour supplied to each sector equals the wage.[3]

[2] In the official sector, consumption tax drives a wedge between final goods price set by firms and the corresponding consumption price.

[3] The labour market equilibrium requires that $w_t^{i,j} = mrs_t^{i,j}$, where $mrs_t^j = -U_{l,t}^{i,j}/U_{c,t}^{i,j}$ is the marginal rate

3.2.1 Ricardian households

Ricardian households are made up of individuals who consume, work in both sectors of the economy and return the wages they earn to the household. Their savings and investments are made through purchasing of government bonds and supplying of capital to sectoral goods producers. For each sector, Ricardian household members, own goods producers, hold physical capital and choose their investment to both sectors. Ricardian households can increase the supply of rental services from capital by investing in additional capital taking into account the adjustment cost of capital. The lifetime utility of Ricardian households is characterised by:

$$U_t^r = E_t \sum_{n=0}^{\infty} \beta^n \left\{ \ln(c_{t+n}^r) - \chi \frac{l_t^{i,r(1+\phi)}}{1+\phi} \right\} \tag{3.5}$$

where χ is a parameter that regulates the disutility of work and ϕ defines the Frisch elasticity of labour. Their intertemporal budget constraint is:[4]

$$c_t^r + \frac{P_t^o}{P_t} i_t^{o,r} + \frac{P_t^u}{P_t} i_t^{u,r} + \frac{B_t^r}{P_t R_t \varepsilon_t^{RISK}} = \frac{P_t^o}{P_t}(1-\tau_t^w) w_t^o l_t^{o,r} + \frac{P_t^u}{P_t} w_t^u l_t^{u,r} + \frac{P_t^o}{P_t}(1-\tau_t^k) r_t^{k,o} u_t^o \bar{k}_t^{o,r} + \frac{P_t^u}{P_t} r_t^{k,u} u_t^u \bar{k}_t^{u,r} +$$

$$+ \frac{B_{t-1}^r}{P_t} - \frac{P_t^o}{P_t}(1-\tau_t^k) a(u_t^o) \bar{k}_t^{o,r} + \frac{P_t^o}{P_t} \tau_t^k \delta \bar{k}_t^{o,r} - \frac{P_t^u}{P_t} a(u_t^u) \bar{k}_t^{u,r} + \frac{P_t^o}{P_t} \Pi_t^o + \frac{P_t^u}{P_t} \Pi_t^u + \frac{P_t^o}{P_t} TR_t^r - \frac{P_t^o}{P_t} T_t^r \tag{3.6}$$

where c_t^r is Ricardian household consumption bundle from both sectors, B_t^r is government bond that pays one unit of currency in period $t-1$ and R_t is the gross nominal interest rate. We define a number of sectoral variables: the relative goods prices P_t^i, the capital $k_t^{i,r}$ (where the bar indicates physical units of capital), labour services $l_t^{i,r}$, the return on capital $r_t^{k,i}$, the utilisation rate of capital u_t^i, Π_t^i being the profit received from investment in goods production and product wage w_t^i. The term $a(u_t^i)$ defines the real cost of using the capital stock with intensity u_t^i. The fiscal authority makes net lump-sum taxes T_t^r which allows to deal with debt accumulation and finances its expenditures by issuing bonds and by levying taxes on labour income τ_t^w and capital income τ_t^k. The public transfers TR_t^r ensure that consumption at the steady state is the same for the two types of households (Gali et al., 2007). ε_t^{RISK} is the risk premium shock on the return to bonds that affects the intertemporal margin, creating a wedge between the interest rate controlled by the central bank and the return on assets held by the households, which follows an AR(1) stochastic process with an $i.i.d$ error term given as:

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{3.7}$$

of substitution between consumption and labour supplied in period $t+n$ for the households. This means that the official and shadow sector would pay the same consumption wage to workers (Gali 2008).

[4]Here we ignore superscript i.

Households' stock of physical capital in each sector is driven by the standard dynamic equation for capital given respectively as:

$$\bar{k}^{o,r}_{t+1} = (1-\delta)\bar{k}^{o,r}_{t} + \varepsilon^{INV}_{t}\left[1 - S\left(\frac{i^{o,r}_{t}}{i^{o,r}_{t-1}}\right)\right] i^{o,r}_{t} \tag{3.8}$$

$$\bar{k}^{u,r}_{t+1} = (1-\delta)\bar{k}^{u,r}_{t} + \left[1 - S\left(\frac{i^{u,r}_{t}}{i^{u,r}_{t-1}}\right)\right] i^{u,r}_{t} \tag{3.9}$$

where $S(.)$ introduces the investment adjustment cost function.[5] δ is the depreciation rate and only capital used in period $u^{i}_{t}\bar{k}^{i,r}_{t}$ is subject to depreciation. ε^{INV}_{t} is the stochastic shock to the price of investment relative to consumption goods and follows an exogenous process with an *i.i.d.* error term as:

$$ln\varepsilon^{INV}_{t} = \rho^{INV} ln\varepsilon^{INV}_{t-1} + \xi^{INV}_{t} \tag{3.10}$$

Households in addition choose the utilisation rate of capital with the amount of effective capital in each sector given as:[6]

$$k^{i,r}_{t} = u^{i}_{t}\bar{k}^{i,r}_{t-1} \tag{3.11}$$

Households face the usual maximization problem of maximizing their expected discounted sum of instantaneous utility (3.5) subject to equations (3.6), (3.8), (3.9) and (3.11). Letting λ^{r}_{t} denote the Lagrangian multiplier for the Ricardian household budget constraint and $\lambda^{r}_{t}Q^{i}_{t}$ the Lagrange multiplier for the capital accumulation equations whereby Q^{i}_{t} is the Tobin's q which is equal to one when there are no capital adjustment costs. It can be interpreted as the one unit shadow relative price of capital with respect to one-unit of consumption. The first order conditions with respect to consumption (c^{r}_{t}), government bond (B^{r}_{t}), sectoral labour ($l^{i,r}_{t}$), sectoral capital ($\bar{k}^{i,r}_{t+1}$), sectoral investment ($i^{i,r}_{t}$) and capital utilisation (u^{i}_{t}) are respectively given below.[7] The intertemporal marginal utility of consumption is:

$$U^{r}_{c,t} = \lambda^{r}_{t} = \frac{1}{c^{r}_{t}} \tag{3.12}$$

The consumption Euler equation from government bond is:

$$\lambda^{r}_{t} = \varepsilon^{RISK}_{t} R_{t}\beta E_{t}\frac{\lambda^{r}_{t+1}}{\pi_{t+1}} \tag{3.13}$$

[5]The investment adjustment cost function is given by:

$$S\left(\frac{i^{i,r}_{t}}{i^{i,r}_{t-1}}\right) = \frac{\kappa^{I}}{2}\left(\frac{i^{i,r}_{t}}{i^{i,r}_{t-1}} - 1\right)^{2}$$

In the steady state, $S(1) = S'(1) = 0$, $S''(1) > 0 \equiv \varpi$ with ϖ being the adjustment cost parameter.

[6]In the steady state, utilisation cost function implies that: $u^{i}_{s} = 1$ and $a(1) = 0$.

[7]A detailed derivations of all the first order conditions are available upon request.

In competitive labour market, the standard labour supply conditions hold as:

$$(1-\tau_t^w)\frac{P_t^o}{P_t}w_t^o = \frac{\chi l_t^{o,r(\phi)}}{\lambda_t^r} \tag{3.14}$$

$$\frac{P_t^u}{P_t}w_t^u = \frac{\chi l_t^{u,r(\phi)}}{\lambda_t^r} \tag{3.15}$$

The arbitrage condition in the labour market ensures that both sectors pay the same level of real wage as:

$$\frac{P_t^o}{P_t}w_t^o(1-\tau_t^w) = \frac{P_t^u}{P_t}w_t^u \tag{3.16}$$

The competitive capital supplied to each sector is accordingly given as:

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r}\left[\frac{P_{t+1}^o}{P_{t+1}}\left((1-\tau_{t+1}^k)\left[r_{t+1}^{k,o}u_{t+1}^o - a(u_{t+1}^o)\right] + \tau_{t+1}^k\delta\right) + Q_{t+1}^o(1-\delta)\right] \tag{3.17}$$

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r}\left[\frac{P_{t+1}^u}{P_{t+1}}\left[r_{t+1}^{k,u}u_{t+1}^u - a(u_{t+1}^u)\right] + Q_{t+1}^u(1-\delta)\right] \tag{3.18}$$

The first order conditions for investments supplied to each sector is given as:

$$\frac{P_t^o}{P_t} = Q_t^o\varepsilon_t^{INV}\left(1-S\left(\frac{i_t^{o,r}}{i_{t-1}^{o,r}}\right)-S'\left(\frac{i_t^{o,r}}{i_{t-1}^{o,r}}\right)\frac{i_t^{o,r}}{i_{t-1}^{o,r}}\right)+\beta E_t\frac{\lambda_{t+1}^r}{\lambda_t^r}Q_{t+1}^o\varepsilon_{t+1}^{INV}S'\left(\frac{i_{t+1}^{o,r}}{i_t^{o,r}}\right)\left(\frac{i_{t+1}^{o,r}}{i_t^{o,r}}\right)^2 \tag{3.19}$$

$$\frac{P_t^u}{P_t} = Q_t^u\left(1-S\left(\frac{i_t^{u,r}}{i_{t-1}^{u,r}}\right)-S'\left(\frac{i_t^{u,r}}{i_{t-1}^{u,r}}\right)\frac{i_t^{u,r}}{i_{t-1}^{u,r}}\right)+\beta E_t\frac{\lambda_{t+1}^r}{\lambda_t^r}Q_{t+1}^uS'\left(\frac{i_{t+1}^{u,r}}{i_t^{u,r}}\right)\left(\frac{i_{t+1}^{u,r}}{i_t^{u,r}}\right)^2 \tag{3.20}$$

And finally, the following equations also gives the first order conditions for effective capital utilised:

$$r_t^{k,o} = a'(u_t^o) \tag{3.21}$$

$$r_t^{k,u} = a'(u_t^u) \tag{3.22}$$

solving the first order conditions for c_t^r and B_t^r defines the consumption Euler equation.

3.2.2 Non-Ricardian households

Rule-of-thumb households have the same lifetime utility function as that of intertemporal optimizing households given as:

$$U_t^{rt} = E_t\sum_{n=0}^{\infty}\beta^n\left\{\ln(c_{t+n}^{rt}) - \chi\frac{l_t^{i,rt(1+\phi)}}{1+\phi}\right\} \tag{3.23}$$

LAMP households consume their disposable labour income and transfers in each period, therefore their consumption is determined by the budget constraint:[8]

$$c_t^{rt} = \frac{P_t^o}{P_t}(1-\tau_t^w)w_t^o l_t^{o,rt} + \frac{P_t^u}{P_t}w_t^u l_t^{u,rt} + \frac{P_t^o}{P_t}TR_t^{rt} \tag{3.24}$$

where c_t^{rt} is non-Ricardian households consumption bundle from both sectors and labour services $l_t^{i,rt}$ defines sectoral labour supplied. Letting λ_t^{rt} denote the Lagrangian multiplier for the non-Ricardian household's budget constraint, the first order conditions with respect to consumption (c_t^{rt}) and sectoral labour ($l_t^{i,rt}$) are respectively given below. The intertemporal marginal utility of consumption is:

$$U_{c,t}^{rt} = \lambda_t^{rt} = \frac{1}{c_t^{rt}} \tag{3.25}$$

In competitive labour market, the standard labour supply conditions hold as:

$$(1-\tau_t^w)\frac{P_t^o}{P_t}w_t^o = \frac{\chi l_t^{o,rt(\phi)}}{\lambda_t^{rt}} \tag{3.26}$$

$$\frac{P_t^u}{P_t}w_t^u = \frac{\chi l_t^{u,rt(\phi)}}{\lambda_t^{rt}} \tag{3.27}$$

3.2.3 Intermediate Goods Producers

In each sector $i \in (o,u)$, goods producers produce intermediate goods and sell them at the competitive intermediate price $P_t^{I,i}$ to final goods producers. The production function for a representative firm is given as:

$$y_t^i = A_t^i k_t^{i(\alpha^i)} l_t^{i(1-\alpha^i)} \tag{3.28}$$

where y_t^i, k_t^i and l_t^i respectively denote sectoral output, capital and labour inputs. α^i is the sectoral capital share used in productive activities. A_t^o is the official sector productivity shock which is defined as an AR(1) process with *i.i.d* error term as:

$$lnA_t^o = \rho^A lnA_{t-1}^o + \xi_t^A \tag{3.29}$$

Goods producers in each sector maximize their market value by choosing labour (l_t^i) and capital (k_t^i) taking into account their production output level. Their market value (Π_t^i) is expressed as:

$$\Pi_t^i = \frac{P_t^{I,i}}{P_t}\left[y_t^i - w_t^i l_t^i - r_t^{k,i} k_t^i\right] \tag{3.30}$$

[8] Here we ignore superscript i.

where $P_t^{I,i}$, w_t^i and $r_t^{k,i}$ are respectively sectoral goods price, real wage rate and the real returns from capital. $\frac{P_t^{I,i}}{P_t} y_t^i$ represent the firm's revenue from selling output, and $\frac{P_t^{I,i}}{P_t}(w_t^i l_t^i + r_t^{k,i} k_t^i)$ are the repayments made by goods producers to households which consist of the wage bill and cost of physical capital. The following equations respectively define the first order conditions for sectoral labour and capital:

$$w_t^i = (1-\alpha^i) A_t^i \left(\frac{k_t^i}{l_t^i}\right)^{\alpha^i} \tag{3.31}$$

$$r_t^{k,i} = \alpha^i A_t^i \left(\frac{k_t^i}{l_t^i}\right)^{-(1-\alpha^i)} \tag{3.32}$$

This implies a capital-labour ratio given as:

$$\frac{r_t^{k,i}}{w_t^i} = \frac{\alpha^i}{1-\alpha^i} \frac{l_t^i}{k_t^i} \tag{3.33}$$

Solving equations (3.31) and (3.32) yield sectoral real marginal cost as:

$$mc_t^{I,i} = \left(\frac{r_t^{k,i}}{\alpha^i}\right)^{\alpha^i} \left(\frac{w_t^i}{1-\alpha^i}\right)^{1-\alpha^i} \tag{3.34}$$

3.2.4 Final Goods Producers

We assume a sticky price specification based on Rotemberg (1982) quadratic adjustment cost in both sectors of the economy. We index their prices to a combination of both current and past inflation with a weight equal to θ_π. The final goods producers maximize their profit function by choosing their final goods prices taking into account the quadratic adjustment cost given as:

$$\frac{\kappa^p}{2} \left(\frac{\pi_t^i}{{\pi_{t-1}^i}^{\theta_\pi}} - 1\right)^2 y_t^i$$

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost in adjusting its nominal prices that can be measured in terms of the final goods with κ^p being the price stickiness parameter which accounts for the negative effects of price changes on the customer-firm relation and θ_π representing the price indexation parameter.

The official sector final goods producers are subject to price mark-up shocks, hence in a symmetric equilibrium, the Rotemberg version of non-linear New Keynesian Phillips Curve (NKPC) is derived as:

$$(1-mc_t^o)\epsilon_t^o = 1 - \kappa^p \left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} - 1\right) \frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} + \kappa^p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}} - 1\right) \frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}} \frac{y_{t+1}^o}{y_t^o}\right] \tag{3.35}$$

where ϵ_t^o is now a stochastic parameter which determines the time-varying mark-up in the official goods markets. As in Smets and Wouters (2003, 2007), the official sector final goods producers' actual mark-up hovers around its desired level over time. This desired level comprises of an endogenous and exogenous components which is assumed to follow an AR(1) process given as:

$$ln\epsilon_t^o = ln\epsilon^o + ln\varepsilon_t^p$$

$$ln\varepsilon_t^p = \rho^p ln\varepsilon_{t-1}^p + \xi_t^p \tag{3.36}$$

with ξ_t^p being an $i.i.d.$ normal innovation term. In a symmetric equilibrium, the price adjustment rule satisfies the following first order condition for the shadow goods producers given as:

$$(1 - mc_t^u)\epsilon^u = 1 - \kappa^p \left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} - 1 \right) \frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} + \kappa^p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}} - 1 \right) \frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}} \frac{y_{t+1}^u}{y_t^u} \right] \tag{3.37}$$

where $mc_t^i = \frac{P_t^{I,i}}{P_t^i}$, defines the real marginal cost in terms of the sectoral final goods price. Here we assume shadow sector goods producers to have limited market power. The above equations represent the Rotemberg version of non-linear NKPCs that relate sectoral current inflation to future expected inflation and to the level of relative outputs. The following equations respectively allow to identify the sectoral price levels and the inflation rate for the consumption price index:

$$P_t^o = \pi_t^o P_{t-1}^o \tag{3.38}$$

$$P_t^u = \pi_t^u P_{t-1}^u \tag{3.39}$$

$$P_t = \pi_t P_{t-1} \tag{3.40}$$

where P_t is defined by equation (3.4).

3.2.5 Government Policies

In this section we introduce and discuss the various government policies in regulating the real sector. It comprises of the fiscal tools used by the government and a Central Bank who oversees the implementation of monetary instruments.

Fiscal Policy

The government supplies an exogenous amount of public goods (g_t) which is defined in terms of the official sector goods. Government expenditure is financed through the taxes (levied on consumption goods, labour and capital income) and the issuance of one period nominally risk-free bonds. The government budget constraint is of the form:

$$g_t + \frac{B_{t-1}}{P_t^o} + TR_t = \tau_t^w w_t^o l_t^o + \tau_t^k \left[r_t^{k,o} u_t^o - a(u_t^o) - \delta \right] \bar{k}_t^o + \tau^c c_t^o + \frac{B_t}{P_t^o R_t} + T_t \tag{3.41}$$

where TR_t are public transfers and T_t are lump-sum taxes which also appear in the household's budget constraint and explicitly ensure solvency in government deficit. Government spending follows a stochastic process with *i.i.d.* error term given as:[9]

$$ln g_t = \rho^G ln g_{t-1} + \xi_t^G \tag{3.42}$$

As an illustration of the fiscal rules on the revenue side, we set fiscal rules for the labour and capital income taxes to follow an AR(1) process given respectively as:

$$ln\tau_t^W = \rho^w ln\tau_{t-1}^W + \xi_t^W \tag{3.43}$$

$$ln\tau_t^k = \rho^K ln\tau_{t-1}^K + \xi_t^W \tag{3.44}$$

where both ξ_t^W and ξ_t^K represent the respective error term defined as an *i.i.d.*

Monetary Policy

We close the model by describing a simple structure for the monetary policy rule. The Central bank is assumed to follow a pure inflation targeting rule and set a standard Taylor-type monetary policy instrument so that the nominal interest rate is adjusted in response to the movement in inflation gap with interest rate smoothing. The policy rule is characterised by the following Taylor rule:

$$R_t = R_{t-1}^{(\rho^R)} (\pi_t^o)^{\mu_\pi (1-\rho^R)} \varepsilon_t^R \tag{3.45}$$

where R_t is the nominal interest rate, ρ^R is interest rate smoothing parameter, μ_π denotes Taylor coefficient in response to inflation gap[10] with ε_t^R denoting monetary policy shock, with a standard *i.i.d* innovation. In this context, the monetary policy shock is thought of as unexpected deviation of the nominal interest rate via Taylor rule at period t. The exogenous shock to monetary policy enters the nominal interest rate as ε_t^R. The central bank supplies the money demanded by the household to support the desired nominal interest rate.

[9] In the steady state, we impose that $\frac{g_s}{y_s^o} = \bar{g}_s$ in order to obtain the public consumption-output ratio.
[10] That is, deviation of inflation rate from the inflation target.

3.2.6 Aggregation

With two types of households, aggregate consumption, labour, investment, capital, privately-owned government bonds and transfers from the government are computed as follows for the respective sectors in the economy:

$$c_t^o = (1-\theta)c_t^{o,r} + \theta c_t^{o,rt} \tag{3.46}$$

$$c_t^u = (1-\theta)c_t^{u,r} + \theta c_t^{u,rt} \tag{3.47}$$

$$c_t = (1-\theta)c_t^r + \theta c_t^{rt} \tag{3.48}$$

$$l_t^o = (1-\theta)l_t^{o,r} + \theta l_t^{o,rt} \tag{3.49}$$

$$l_t^u = (1-\theta)l_t^{u,r} + \theta l_t^{u,rt} \tag{3.50}$$

$$l_t^r = l_t^{o,r} + l_t^{u,r} \tag{3.51}$$

$$l_t^{rt} = l_t^{o,rt} + l_t^{u,rt} \tag{3.52}$$

$$l_t = (1-\theta)l_t^r + \theta l_t^{rt} \tag{3.53}$$

$$i_t^o = (1-\theta)i_t^{o,r} \tag{3.54}$$

$$i_t^u = (1-\theta)i_t^{u,r} \tag{3.55}$$

$$k_t^o = (1-\theta)k_t^{o,r} \tag{3.56}$$

$$k_t^u = (1-\theta)k_t^{u,r} \tag{3.57}$$

$$B_t = (1-\theta)B_t^r \tag{3.58}$$

$$T_t = (1-\theta)T_t^r \tag{3.59}$$

$$TR_t = (1-\theta)TR_t^r + \theta TR_t^{rt} \tag{3.60}$$

3.2.7 Market Clearing and Resource Constraint

The aggregate resource constraints are given respectively as:[11]

$$y_t^o = c_t^o + i_t^o + g_t + a(u_t^o)\bar{k}_{t-1}^o + \frac{\kappa^p}{2}\left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} - 1\right)^2 y_t^o \tag{3.61}$$

[11]We note here that, the official sector resource constraint incorporates the government expenditure.

$$y_t^u = c_t^u + i_t^u + a(u_t^u)\bar{k}_{t-1}^u + \frac{\kappa^p}{2}\left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} - 1\right)^2 y_t^u \tag{3.62}$$

3.3 Model Dynamics and Results

In this section, we calibrate the theoretical model by imposing several fiscal impulses. The aim, as explained in the introductory section, is to investigate the role played by rule-of-thumb consumers in a new Keynesian DSGE model with shadow economy during fiscal transmissions. We do so by examining the various channels of fiscal shocks on the dynamic properties of the model. Model calibrations involve the fiscal exogenous processes asymmetric to the official sector consisting of government spending shock, labour and capital income tax shocks. We accordingly solve the model by focusing on the constraints and first order conditions for prices and quantities, we then derive all the log-linearised equations of the model by taking log-linear approximations around the steady state.[12] The next subsection presents the results and analysis of the impulse responses to the three exogenous shocks.

3.3.1 Model Calibrations

Most of the conventional structural parameters are borrowed from Smets and Wouters (2007) and Colombo et al. (2016). These parameters are selected in order to capture specific ratios in the steady state of the model. Other structural parameters whose information are particularly related to the developing countries are calibrated using values and data from literature on developing and low-income countries which is the focus of this research. The complete list of parameters and their values are in table (3.1).

Conventional household's preference and technological parameters are fairly standard. We set the subjective discount factor β to 0.99 which is consistent to achieve an annual steady state interest rate of 4%. The elasticity of substitution between official and informal consumption bundles is set at $\epsilon_c = 1.5$ as described in Batini et al. (2011). The coefficient of Frisch elasticity of substitution for labour supply in the utility function is fixed at $\phi = 2$ to ensure determinacy of the equilibrium. Turning to the goods producer's structural parameters, from Schmitt-Grohe and Uribe (2004), we take the price elasticity of demand for differentiated goods parameter $\epsilon^o = 6$, a value consistent with a 20% price mark up in the official sector. Its shadow sector counterpart value is set at $\epsilon^u = 20$ to imply a 5% price mark up. The degree of inflation indexation parameter is set to $\theta_\pi = 1$ to indicates a full indexation of inflation. The degree of price stickiness parameter is fixed at $\kappa^p = 4.37$. Available literature suggests no evidence of nominal rigidities in the shadow sector, therefore the benchmark values for

[12] The appendix shows the full set of the first order conditions and the log-linearised equations.

inflation indexation and degree of price stickiness are used for both sectors as in Colombo et al. (2016). The depreciation rate is set to equal $\delta = 0.025$ quarterly which implies an annual depreciation on capital of 10%. We additionally set the official sector capital share to $\alpha^o = 0.36$ to capture a high capital intensity in the official sector than the informal sector share which is valued at $\alpha^u = 0.28$ as in Koreshkova (2006). In order to characterise the

Table 3.1: Model calibration parameters.

Parameter	Value	Description
Preferences & Technology		
β	0.99	Subjective discount rate
ϕ	2	Frisch elasticity of substitution for labour
ϵ_c	1.5	Elasticity of substitution between official and unofficial consumption
θ	0.60	Share of non-Ricardian households
ϵ_o	6	Official sector price elasticity of demand
ϵ_u	20	Shadow sector price elasticity of demand
κ^p	4.37	Degree of price stickiness
θ_π	1	Inflation indexation
δ	0.025	Capital depreciation
α^o	0.36	Official sector's capital Share
α^u	0.28	Shadow sector's capital Share
Monetary policy		
ρ^R	0.9	Interest rate smoothing parameter
μ_π	1.5	Taylor coefficient to inflation gap
Fiscal ratio		
τ^c	0.10	Consumption tax
τ_s^k	0.20	Capital income tax
τ_s^w	0.15	Labour income tax
$\bar{g}_s$	0.14	Government consumption
Shock innovation		
ρ^G	0.7	Innovation to government spending shock
ρ^W	0.8	Innovation to labour income tax shock
ρ^K	0.75	Innovation to capital income tax shock

empirical findings of the shadow sector in the developing countries, we set the steady state share of the shadow economy $SH = 0.47$ (a value common to several developing and low-income countries) to calibrate the value of official consumption goods bias φ_c. Another crucial parameter which influence the response of the shocks is the share of non-Ricardian consumers in the model. Most empirical estimates on developing and low-income countries report that large proportions of households in these areas are liquidity constrained. For instance, Ardic et al. (2013) reported that only 25% of the poor population in developing countries has a bank account with the financial institutions and 23% of people living under \$2 a day have account in formal financial institutions. Therefore, depending on the degree of financial development of these countries, the measure of intertemporal optimizing households can be lower than 40% in most Sub Sahara African (SSA) countries. Based on Global Findex Database, Demirguc-Kunt and Klapper (2012), reported that the percentage of adults with formal bank account in SSA is 45% in the rich quantile countries and only 12% in the poorest quantile. Upon this background we set the size of non-Ricardian households $\theta = 0.60$ as used by Melina et al. (2016). The ratios of fiscal variables to GDP and the steady state tax rates were taken from Melina et al. (2016) which are consistent with data collected by the International Bureau of Fiscal Documentation in 2005-06 for developing and low-income countries. The steady state values for τ^c, τ^k_s, and τ^w_s are respectively fixed at 10%, 20% and 15%; and finally, government spending to GDP ratio ($\bar{g}_s$) is set at 14%. The steady state distribution of transfers is set to obtain a steady state consumption ratio between the two household groups as $c_t^{i,RT} = 0.8c_t^{i,R}$. To achieve a stable steady state, we conventionally set the aggregate labour supply $l^j_s = 0.25$ and aggregate price $P_s = 1$ respectively. It is paramount to note that, the steady state relative prices are determined by mark-ups, technological parameters and various tax rates. The steady state relative price of the shadow sector goods is higher than the official sector relative price due to the high cost of factor inputs. The rest of steady state values are calibrated using the other structural parameters. The conventional parameters characterizing the monetary policy instrument are set accordingly as: the Taylor rule interest smoothing rate parameter $\rho^R = 0.9$ and inflation gap parameter $\mu_\pi = 1.5$. The parameters describing the shock processes are calibrated as follows, innovation to government spending shock parameters is set at $\rho^G = 0.7$. Innovations to labour and capital income tax shocks are set at $\rho^W = 0.8$ and $\rho^K = 0.75$ respectively.

3.3.2 Analysis and Discussion of Results

This section discusses the impulse response functions to various fiscal policy instruments (government spending, labour and capital income taxes) with rule-of-thumb consumers and the shadow economy. The shocks are asymmetric to the official sector and the continuous blue line in figures (3.1)-(3.6) represent the behaviour of the macroeconomic variables in two-sector model while the broken red line defines the behaviour of the shock in the one-sector model. The responses of the fiscal shocks to macroeconomic variables are shown in a percentage deviation from the steady states.

Figures (3.1)-(3.2) present the IRFs from an increase in government spending shock. The expenditure increase has an expansionary effect in the official sector of both models. The shock predicted an increase in government demand for goods and services leading to a higher capital utilisation and increase in demand for labour employment in the official sector which put pressure on official rental rate of capital. This in effect, leads to an increase in the official sector marginal cost which translate into higher inflation which means that monetary authorities increase the nominal interest rate. Moreover, the surge in government spending means that government would increase taxes in the future to finance its budget and the Ricardian consumers anticipate this increase in taxes by reducing their current consumption levels. This leads to the so called negative wealth effects associated to government spending shocks as documented in several fiscal policy literatures. Our model is also able to account for the crowding-out of private investment as in the literature.[13] The reason for the fall in Ricardian consumption may be associated to the fact that our model does not necessarily consider government debt and labour market imperfections as demonstrated by Gali et al. (2007). The negative wealth effects do not affect non-Ricardian consumers and therefore their consumption levels rise. The boom and the subsequent rise in relative consumption and factor input prices in the official sector lead to the flow of factor inputs from the shadow sector into the official sector, triggering a fall in the shadow sector capital utilisation and labour employment demanded as well as shadow sector's private consumption and aggregate demand. One striking qualitative difference is the fall in the official sector product wage which could be attributed to the influx of labour employment from the shadow sector to the formal sector. The steady state arbitrage condition for wage rate ensures factor services move freely in the economy. In fact, the mechanism indicates that the presence of shadow economies weakens the effectiveness of government expenditure shock as most of the shadow activities are slowed down due to reallocating effects of the shock. Our computed government spending multiplier reiterate this negative effect of the shadow sector as shown in table (3.2).

Figures (3.3)-(3.4) show the responses for labour income tax hikes to the official sector which on the impact shows a contractionary effects in the one-sector and the official sector of the two-sector model. The shock decreases official sector inflation, thus reducing nominal interest rate and the real interest rate. This in turn, leads to a positive response of official sector investment in both models. Moreover, the fall in relative goods and factor prices in the shadow sector lead to a factor input reallocation between the two sectors, raising shadow labour employment, capital utilisation and capital demand. Additionally, as shadow sector labour employment and capital demand increase, the fall in shadow consumption goods price makes shadow consumption and aggregate demand to increase. Therefore, in the presence of shadow economy, the negative transmission effects from asymmetric labour income tax shocks in the official sector are absorbed

[13] Gabriel et al. (2010), showed a similar mechanism in a model with two-sector and rule-of-thumb consumers for the Indian economy. Coenen and Straub (2005); Gali et al. (2007); Rossi (2007) and Colciago (2011) also reported similar results though none of these literatures considered the shadow economy.

by the shadow economy. This mechanism strengthens the relative importance of the existence of shadow sector in wake of income tax shocks. Given the size of our non-Ricardian consumers in the model, it become paramount to note their relative importance to the transmission process of the model. The presence of non-Ricardian consumers cannot be underestimated, they make the labour tax multiplier on consumption to be negative in both models which in effect means that non-Ricardian consumers weaken the labour tax multiplier. Turning to the capital income tax shock, figures (3.5)-(3.6) show that, an increase in capital income tax shock like the labour income tax, produces a contractionary effects in the official sector of both models. Thus, official sector output, investment, labour employment and capital demand fall. The income tax on the impact, reduces official sector capital demand inducing a fall in official labour employment. Changes in factor prices in the shadow sector make both shadow labour employment and capital demand rises due to factor reallocation effects. On the demand side, the changes in relative goods and increase in labour employment induces higher consumption for Ricardian consumers. This in turn, triggers a positive response of shadow sector consumption, investment, aggregate demand and output.

3.3.3 Fiscal Multipliers

The computed fiscal multipliers provide the quantitative assessment of key factors that determine the GDP effects associated with the use of alternative fiscal instruments. We do so to further understand whether fiscal policies can be used to stabilise the economy in response to shocks and also to ascertain whether the presence of shadow economy weaken the amplifying effect of rule-of-thumb consumers on fiscal multipliers. Table (3.2) summarizes the computed multipliers of the three fiscal instruments on output in both the short-run and long-run periods. The short-run multipliers (impact multipliers) are calculated as output effects during the impact period divided by the cost during the impact period. Long-run multipliers (cumulative multipliers) are computed as the discounted output effects divided by the discounted costs over the periods considered.[14]

The computed government spending multipliers from both models show a positive effect in the official sector as expected. However, the presence of the shadow economy resulted in a negative government spending effect in the shadow sector. In effects, we observe that shadow economy significantly reduces the government spending multiplier. Moreover, income tax multipliers

[14]For government spending, we compute the short-run multiplier as:

$$M_{SR}^{g} = \frac{y_t - y_s}{g_t - g_s}$$

and the long-run multiplier as:

$$M_{LR}^{g} = \frac{\sum_{t=0}^{\infty} \beta^t (y_t - y_s)}{\sum_{t=0}^{\infty} \beta^t (g_t - g_s)}$$

where y_s and g_s denote steady state variables.

as shown by table (3.2) indicate a negative effects in the one-sector model, however, in the two sector model the presence of the shadow sector offsets the negative effects because there is a reallocation of factor inputs in response to the shocks. This postulates that shadow economy increases the tax multipliers specifically labour income tax multiplier with a significant amount. Another interesting result from our computed multipliers is that the amplifying effects

Table 3.2: Fiscal multipliers for various fiscal packages based on impulse response.

	Gov. spending	**Labour tax**	**Capital tax**
Without shadow economy (One-sector model)			
Output			
Short run	0.431	-0.109	-0.0011
Long run	0.142	-0.085	-0.0035
Ricardian cons.			
Short run	-0.292	0.025	0.0023
Long run	-0.196	0.016	0.0071
Non-Ricardian cons.			
Short run	0.450	-0.206	-0.0007
Long run	0.127	-0.180	-0.0025
With shadow economy (Two-sector model)			
Official output			
Short run	0.065	-0.056	-0.0001
Long run	0.024	-0.025	-0.0004
Shadow output			
Short run	-0.210	0.211	0.0004
Long run	-0.223	0.264	0.002
Official Ricardian cons.			
Short run	-0.396	0.188	0.0006
Long run	-0.255	0.078	0.002
Shadow Ricardian cons.			
Short run	-0.651	0.476	0.001
Long run	-0.378	0.260	0.009
Official non-Ricardian cons.			
Short run	0.009	-0.205	0.000004
Long run	-0.067	-0.150	-0.0001
Shadow non-Ricardian cons.			
Short run	0.203	0.093	-0.0003
Long run	0.054	0.022	-0.001

associated with rule-of-thumb consumers on fiscal multipliers are weakened in our model when we incorporate shadow sector. From the computed multipliers, we observe that the two-sector model multipliers with non-Ricardian consumers are reduced for all fiscal instruments considered compared to the single sector model making the amplifying effect induced by rule-of-thumb consumers to be irrelevant. This may be attributed to the fact that disposable income of the rule-of-thumb households as a weighted average of labour incomes earned in the two sectors is virtually unaffected by the fiscal shocks. In short, the presence of shadow economy weakens the government expenditure multiplier and strengthens the income tax multipliers because the effects of relative consumption and factor inputs prices produce factor inputs reallocation in the model. Our results also suggested that an economy with relatively large share of informal sector, fiscal policy specifically income taxes can be used to stabilise the economy in response to shocks.

3.4 Conclusions

This paper has introduced the rule-of-thumb consumers in a standard new Keynesian DSGE model with shadow economy. It is motivated by the role played by rule-of-thumb consumers in ensuring effective fiscal policy analysis and their importance in understanding the real business cycle in the aftermath of the recent financial crisis. In order to achieve the aim of the paper, we adopt the Smets and Wouters (2003, 2007) model and incorporate rule-of-thumb consumers and the informal sector. We explicitly model fiscal policies to interact with monetary policy to allow for appropriate analysis and the various channels of the fiscal policy transmissions. The calibration involves exogenous processes that are asymmetric to the official sector consisting of government spending shock, labour and capital income tax shocks. We specifically set the share of the informal sector and the rule-of-thumb consumers to match the steady state share of developing and emerging countries.

Our results have shown that, shadow economies play an important role in determining the transmission channels of the fiscal policy on the real business cycle. We showed that in economies with large informal sector, government spending shocks expand the formal sector and slow down the activities in the shadow sector while tax hikes lead to a boost in the shadow economy making the standard aggregate estimates of fiscal policies ineffective. Our model account for the reallocation of factor services between the two economies as a result of the role played by relative sectoral prices. Moreover, the various multipliers provided an interesting result in the presence of rule-of-thumb consumers and shadow economy. We found that shadow economy weakens the government expenditure multiplier whereas income tax multipliers amplifies the effect of fiscal shocks specifically the labour income tax multiplier which increase significantly. Finally, our results indicated that the amplifying mechanism caused by rule-of-thumb consumers becomes significantly irrelevant because the disposable income of the rule-of-thumb households is virtually unaffected by the fiscal shocks. In effect, our model presented a reduction in the various rule-of-thumb consumer multipliers when we introduce shadow sector. Our model contributes to provide a theoretical background to policy-oriented literature that sees households and sectoral heterogeneity as an important component of future macroeconomic policy framework.

Future research should add and investigate the role of financial intermediaries (explicit financial frictions and banking sector) in both sectors of the economy to regulate the financial market in a medium-scaled model. This would enable us to understand the interactions of the two type of households on their liquidity decisions especially the shadow savers and the financial market's interactions with fiscal and monetary rules.

References

Albonico, A., Paccagnini, A., & Tirelli, P. (2014). Estimating a DSGE model with Limited Asset Market Participation for the Euro Area. University of Milan Bicocca Department of Economics, Management and Statistics Working Paper No. 286. Available at SSRN: https://ssrn.com/abstract=2526800 or http://dx.doi.org/10.2139/ssrn.2526800

Albonico, A., Paccagnini, A., & Tirelli, P. (2016). In search of the Euro area fiscal stance. *Journal of Empirical Finance*, 39, 254-264.

Amaral, P. S. & Quintin, E. (2006). A competitive model of the informal sector. *Journal of Monetary Economics*, 53 (7), 1541-1553.

Anderson, E., Inoue, A., & Rossi, B. (2016). Heterogeneous consumers and fiscal policy shocks. *Journal of Money, Credit and Banking*, 48(8), 1877-1888.

Andolfatto, D. (1996). Business cycles and labour market search. *The American Economic Review*, 86(1), 112-132.

Ardic, O., Imboden, K., & Latortue, A. (2013, June). Financial access 2012: Getting to a more comprehensive picture. In *Access to Finance Forum Reports by CGAP and Its Patners* (No. 6).

Basile, R., Chiarini, B., De Luca, G., & Marzano, E. (2016). Fiscal multipliers and unreported production: evidence for Italy. *Empirical Economics*, 51(3), 877-896.

Batini, N., Levine, P., Lotti, E., & Yang B. (2011). Informality, frictions and monetary policy. *School of Economics Discussion Papers 711.*

Baxter, M., & King, R.G. (1993). Fiscal policy in general equilibrium. *The American Economic Review*, 83(3), 315-334.

Bhattarai, K., and Trzeciakiewicz, D. (2017). Macroeconomic impacts of fiscal policy shocks in the UK: A DSGE analysis. *Economic Modelling*, 61, 321-338.

Campbell, J. Y., & Mankiw, N. G. (1989). Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER Macroeconomics Annual*, 4, 185-216.

Campbell, J. Y., & Mankiw, N. G. (1991). The response of consumption to income: a cross-country investigation. *European Economic Review*, 35(4), 723-756.

Canzoneri, M. B., Cumby R., and Diba, B. (2002). Should the European Central Bank and the Federal Reserve be concerned about fiscal policy, and in rethinking stabilization policy?, Symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, pp.333-389.

Christiano, L. J., Eichenbaum, M., & Rebelo, S. (2011). When is the government spending multiplier large?, *Journal of Political Economy*, 119, 78-121.

Cogan, J. F., Cwik, T., Taylor, J. B., & Wieland, V. (2010). New Keynesian versus old Keynesian government spending multipliers, *Journal of Economic Dynamics and Control*, 34(3), 281-295.

Coenen, G., & Straub, R. (2005). Does government spending crowd in private consumption? Theory and empirical evidence for the Euro area. *International Finance*, 8(3), 435-470.

Coenen, G., McAdam, P., & Straub, R. (2008). Tax reform and labour-market performance in the Euro area: A simulation-based analysis using the New Area-Wide Model. *Journal of Economic Dynamics and Control*, 32(8), 2543-2583.

Coenen, G., Straub, R., & Trabandt, M. (2012). Fiscal policy and the great recession in the Euro area. *American Economic Review Papers and Proceedings*, 102(3), 71-76.

Coenen, G., Straub, R., & Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the Euro area. *Journal of Economic Dynamics and Control*, 37(2), 367-386.

Colciago, A., Ropele, T., Muscatelli, V. A., & Tirelli, P. (2008). The role of fiscal policy in a monetary union: are national automatic stabilizers effective?. *Review of International Economics*, 16(3), 591-610.

Colciago, A. (2011). Rule of Thumb Consumers Meet Sticky Wages. *Journal of Money, Credit and Banking*, 43, 325–53.

Colombo, E., Onnis, L., & Tirelli, P. (2016). Shadow economies at times of banking crises: Empirics and Theory. *Journal of Banking and Finance*, 62, 180-190.

Demirguc-Kunt, A., Klapper, L., (2012). Measuring financial inclusion: the Global Findex Database. World Bank Policy Research Working Paper No. 6025.

Drautzburg, T., & Uhlig, H. (2015). Fiscal stimulus and distortionary taxation. *Review of Economic Dynamics*, 18(4), 894-920.

Eggertsson, G. B. (2011). What fiscal policy is effective at zero interest rates?. *NBER Macroeconomics Annual*, 25(1), 59-112.

Erceg, C. J., & Lindè, J. (2010). Asymmetric shocks in a currency union with monetary and fiscal handcuffs. In *NBER International Seminar on Macroeconomics 2010 (pp. 95-135)*. University of Chicago Press.

Faia, E., Lechthaler, W., & Merkl, C., (2013). Fiscal stimulus and labour market policies in Europe. *Journal of Economic Dynamics and Control*, 37(3), 483-499.

Fatàs, A. & Mihov, I. (2001), The Effects of Fiscal Policy on consumption and employment: Theory and Evidence, Manuscript, INSEAD.

Forni , L., Monteforte, L., & Sessa, L. (2009). The general equilibrium effects of fiscal policy: estimates for the Euro area. *Journal of Public Economics*, 93(3-4), 559-585.

Gabriel, V., Levine, P., Pearlman, J., & Yang, B., (2010), An Estimated DSGE Model for Indian Economy, Working Paper, University of Surrey, DP No. 12/10.

Gali, J., Lòpez-Salido, D., & Vallès, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1), 227-270.

Gali, J. (2008). Introduction to monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework. *Introductory Chapters, Princeton University Press.*

Gali, J., & Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1), 116-132.

Goodfriend, M., & King, R. G. (1997). The new neoclassical synthesis and the role of monetary policy. *NBER Macroeconomics Annual*, 12, 231-283.

Hall, R. E. (1978). Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence. *Journal of Political Economy*, 86(6), 971-987.

Koreshkova, T. (2006). A quantitative analysis of inflation as a tax on the underground economy, *Journal of Monetary Economy*, 53(4), 773-796.

Krugman, P. (2012). Europe's austerity madness. *New York Times, 28*, A35.

Linnemann, L. & Schabert, A. (2003). Fiscal policy in the new Neoclassical synthesis, *Journal of Money, Credit and Banking*, 35(6), 911-929.

Maloney, W. (1999). Does informality imply segmentation in urban labour markets? Evidence from sectoral transitions in Mexico. *World Bank Economic Review*, 13 (2), 275-302.

Maloney, W. (2004). Informality revisited. *World Development*, 32 (7), 1159-1178.

Mankiw, N. G. (2000). The Savers-Spenders theory of fiscal policy, *The American Economic Review*, 90, 120-125.

Melina, G., Yang, S. C. S., & Zanna, L. F. (2016). Debt sustainability, public investment, and natural resources in developing countries: The DIGNAR model. *Economic Modelling*, 52, 630-649.

Merz, M. (1995). Search in the labour market and the real business cycle. *Journal of Monetary Economics*, 36(2), 269-300.

Motta, G., & Tirelli, P. (2012). Optimal Simple Monetary and Fiscal Rules under Limited Asset Market Participation, *Journal of Money, Credit and Banking*, 44(7), 1351-1374.

Mountford, A., & Uhlig, H. (2009). What are the effects of fiscal policy shocks?. *Journal of Applied Econometrics*, 24(6), 960-992.

Oh, H., & Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59, S50-S64.

Pappa, E., Sajedi, R., & Vella, E. (2015). Fiscal consolidation with tax evasion and corruption. *Journal of International Economics*, 96, S56-S75.

Perotti, R. (2002). Estimating the effects of fiscal policy in OECD countries, Manuscript, European University Institute.

Pratap, S. & Quintin, E. (2006). The informal sector in developing countries: Output, assets and employment. Working Papers RP2006/130, World Institute for Development Economic Research (UNU-WIDER).

Rossi, R. (2007). *Rule of thumb consumers, public debt and income tax* (No. 2007-44).

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4), 517-531.

Schmitt-Grohe, S., & Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2), 198-230.

Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association*, 1(5), 1123-1175.

Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3), 586-606.

Figure 3.1: Response to Government Spending Shock (Official Economy)

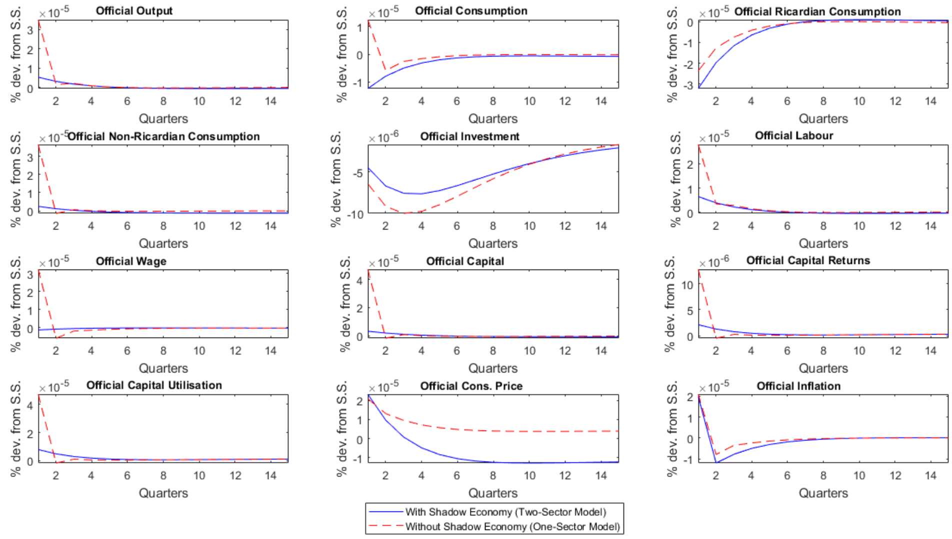


Figure 3.2: Response to Government Spending Shock (Shadow Economy)

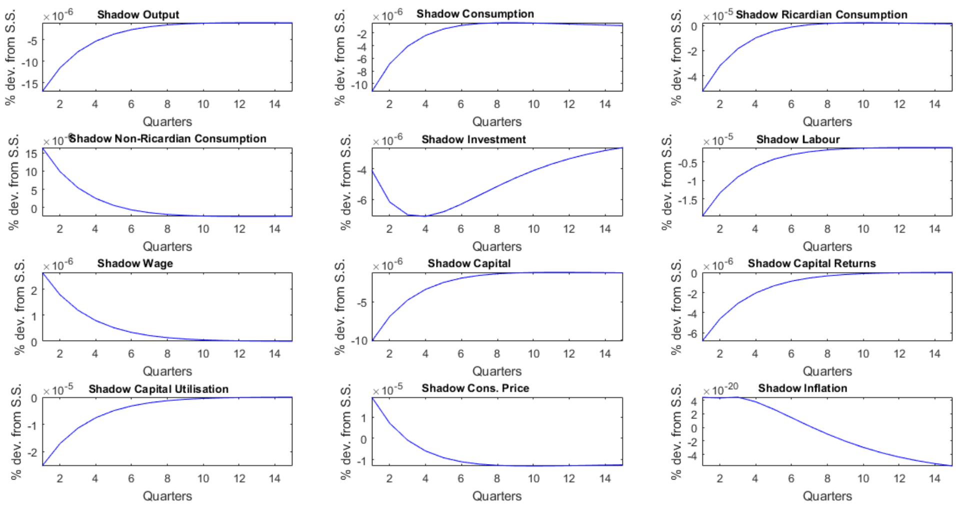


Figure 3.3: Response to Labour Income Tax Shock (Official Economy)

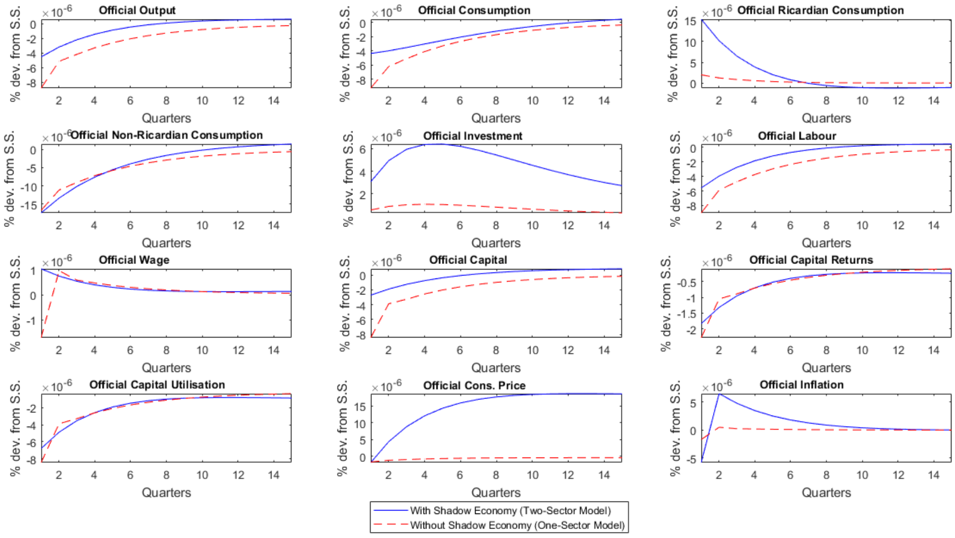


Figure 3.4: Response to Labour Income Tax Shock (Shadow Economy)

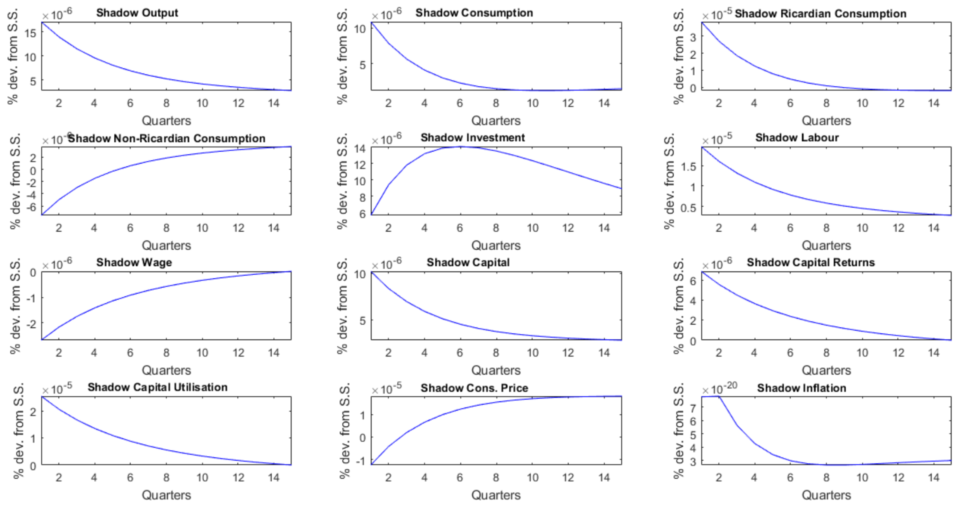


Figure 3.5: Response to Capital Income Tax Shock (Official Economy)

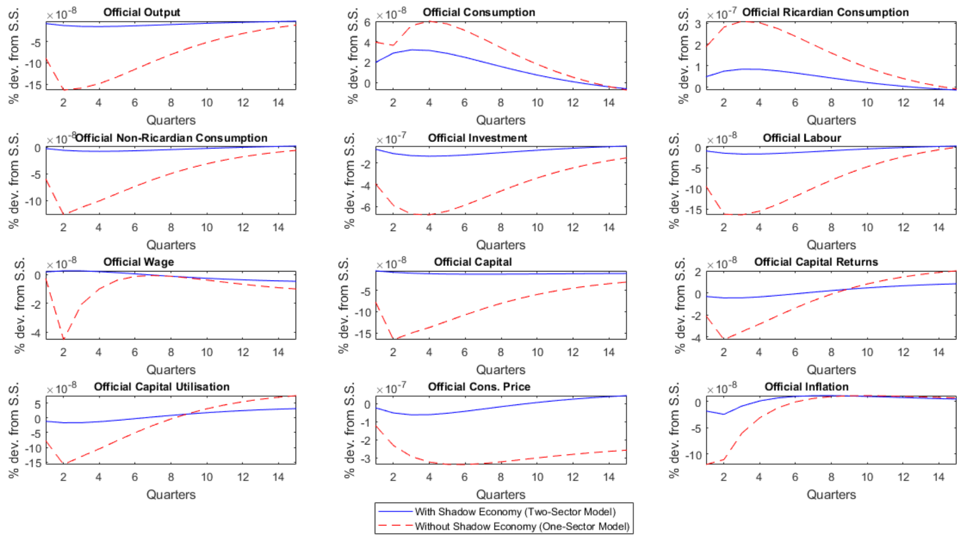
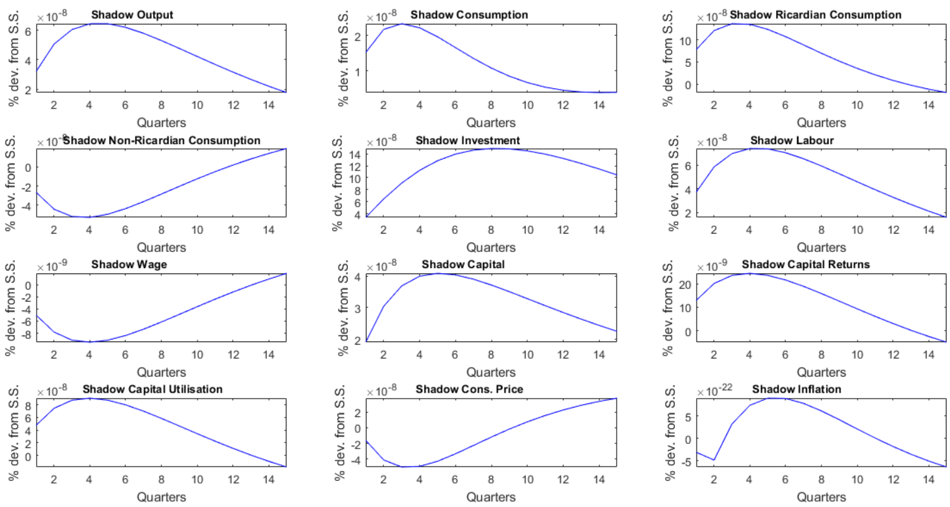


Figure 3.6: Response to Capital Income Tax Shock (Shadow Economy)



Appendix A

Technical Appendices

A.1 Appendix to Chapter One

A.1.1 Symmetric Equilibrium of the Model

Households

Consumption in official sector

$$c_t^o = \varphi_c \left(\frac{P_t^o}{P_t}\right)^{-\epsilon_c} c_t \tag{A.1}$$

Consumption in unofficial sector

$$c_t^u = (1-\varphi_c) \left(\frac{P_t^u}{P_t}\right)^{-\epsilon_c} c_t \tag{A.2}$$

Consumption price index

$$P_t = \left[\varphi_c \left(P_t^o\right)^{1-\epsilon_c} + (1-\varphi_c)\left(P_t^u\right)^{1-\epsilon_c}\right]^{\frac{1}{1-\epsilon_c}} \tag{A.3}$$

Marginal utility of the consumption bundle

$$\lambda_t = \frac{1}{c_t} \tag{A.4}$$

Consumption Euler equation

$$\lambda_t = \varepsilon_t^{RISK} R_t \beta \frac{E_t \lambda_{t+1}}{\pi_{t+1}} \tag{A.5}$$

Labour supplied to official sector

$$\frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o\phi}}{\lambda_t} \tag{A.6}$$

Labour supplied to unofficial sector

$$\frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u\phi}}{\lambda_t} \tag{A.7}$$

Labour market arbitrage condition

$$\frac{P_t^o}{P_t} w_t^o = \frac{P_t^u}{P_t} w_t^u \tag{A.8}$$

Official sector capital

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{P_{t+1}^o}{P_{t+1}}\left[r_{t+1}^{k,o} u_{t+1}^o - a(u_{t+1}^o)\right] + Q_{t+1}^o(1-\delta)\right] \tag{A.9}$$

Unofficial sector capital

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{P_{t+1}^u}{P_{t+1}}\left[r_{t+1}^{k,u} u_{t+1}^u - a(u_{t+1}^u)\right] + Q_{t+1}^u(1-\delta)\right] \tag{A.10}$$

Official sector investment

$$\frac{P_t^o}{P_t} = Q_t^o \varepsilon_t^{INV}\left(1 - S\left(\frac{i_t^o}{i_{t-1}^o}\right) - S'\left(\frac{i_t^o}{i_{t-1}^o}\right)\frac{i_t^o}{i_{t-1}^o}\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^o \varepsilon_{t+1}^{INV} S'\left(\frac{i_{t+1}^o}{i_t^o}\right)\left(\frac{i_{t+1}^o}{i_t^o}\right)^2 \tag{A.11}$$

Unofficial sector investment

$$\frac{P_t^u}{P_t} = Q_t^u\left(1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right) - S'\left(\frac{i_t^u}{i_{t-1}^u}\right)\frac{i_t^u}{i_{t-1}^u}\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1}^u S'\left(\frac{i_{t+1}^u}{i_t^u}\right)\left(\frac{i_{t+1}^u}{i_t^u}\right)^2 \tag{A.12}$$

Official sector capital utilisation

$$r_t^{k,o} = a'(u_t^o) \tag{A.13}$$

Unofficial sector capital utilisation

$$r_t^{k,u} = a'(u_t^u) \tag{A.14}$$

Official sector capital

$$\bar{k}_{t+1}^o = (1-\delta)\bar{k}_t^o + \varepsilon_t^{INV}\left[1 - S\left(\frac{i_t^o}{i_{t-1}^o}\right)\right] i_t^o \tag{A.15}$$

Unofficial sector capital

$$\bar{k}_{t+1}^u = (1-\delta)\bar{k}_t^u + \left[1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right)\right] i_t^u \tag{A.16}$$

Official sector capital utilisation

$$k_t^o = u_t^o \bar{k}_{t-1}^o \tag{A.17}$$

Unofficial sector capital utilisation

$$k_t^u = u_t^u \bar{k}_{t-1}^u \tag{A.18}$$

Official Sector Goods Producers

Official sector output

$$y_t^o = A_t^o k_t^{o(\alpha^o)} l_t^{o(1-\alpha^o)} \tag{A.19}$$

Official sector labour demand

$$w_t^o = (1-\alpha^o)A_t^o\left(\frac{k_t^o}{l_t^o}\right)^{\alpha^o} \tag{A.20}$$

official sector capital demand

$$r_t^{k,o} = \alpha^o A_t^o\left(\frac{k_t^o}{l_t^o}\right)^{-(1-\alpha^o)} \tag{A.21}$$

Official sector marginal cost

$$mc_t^{I,o} = \left(\frac{r_t^{k,o}}{\alpha^o}\right)^{\alpha^o}\left(\frac{w_t^o}{1-\alpha^o}\right)^{1-\alpha^o} \tag{A.22}$$

Shadow Sector Goods Producers

Shadow sector output

$$y_t^u = k_t^{u(\alpha^u)} l_t^{u(1-\alpha^u)} \tag{A.23}$$

Unofficial sector labour demand

$$w_t^u = (1-\alpha^u)\left(\frac{k_t^u}{l_t^u}\right)^{\alpha^u} \tag{A.24}$$

Unofficial sector capital demand

$$r_t^{k,u} = \alpha^u\left(\frac{k_t^u}{l_t^u}\right)^{-(1-\alpha^u)} \tag{A.25}$$

Unofficial sector marginal cost

$$mc_t^{I,u} = \left(\frac{r_t^{k,u}}{\alpha^u}\right)^{\alpha^u}\left(\frac{w_t^u}{1-\alpha^u}\right)^{1-\alpha^u} \tag{A.26}$$

Final Goods Producers

Official sector NKPC

$$(1-mc_t^o)\epsilon_t^o = 1-\kappa^p\left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}}-1\right)\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}}-1\right)\frac{\pi_{t+1}^o}{{\pi_t^o}^{\theta_\pi}}\frac{y_{t+1}^o}{y_t^o}\right] \tag{A.27}$$

Unofficial sector NKPC

$$(1-mc_t^u)\epsilon^u = 1-\kappa^p\left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}-1\right)\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}-1\right)\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}\frac{y_{t+1}^u}{y_t^u}\right] \tag{A.28}$$

Aggregate inflation

$$P_t = \pi_t P_{t-1} \tag{A.29}$$

Official sector inflation

$$P_t^o = \pi_t^o P_{t-1}^o \tag{A.30}$$

Unofficial sector inflation

$$P_t^u = \pi_t^u P_{t-1}^u \tag{A.31}$$

Monetary Policy

Taylor's rule

$$R_t = R_{t-1}^{(\rho^R)} (\pi_t^o)^{\mu_\pi (1-\rho^R)} \varepsilon_t^R \tag{A.32}$$

Market Clearing and Resource Constraint

Official sector resource

$$y_t^o = c_t^o + i_t^o + a(u_t^o)\bar{k}_{t-1}^o + \frac{\kappa^p}{2}\left(\frac{\pi_t^o}{\pi_{t-1}^{o\;\theta_\pi}} - 1\right)^2 y_t^o \tag{A.33}$$

Shadow sector resource

$$y_t^u = c_t^u + i_t^u + a(u_t^u)\bar{k}_{t-1}^u + \frac{\kappa^p}{2}\left(\frac{\pi_t^u}{\pi_{t-1}^{u\;\theta_\pi}} - 1\right)^2 y_t^u \tag{A.34}$$

Aggregate consumption

$$c_t = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^o)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^u)^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{A.35}$$

Aggregate labour

$$l_t = l_t^o + l_t^u \tag{A.36}$$

Shock Processes

Risk premium shock

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{A.37}$$

Investment shock

$$ln\varepsilon_t^{INV} = \rho^{INV} ln\varepsilon_{t-1}^{INV} + \xi_t^{INV} \tag{A.38}$$

Official sector productivity shock

$$lnA_t^o = \rho^A lnA_{t-1}^o + \xi_t^A \tag{A.39}$$

Price markup

$$ln\varepsilon_t^p = \rho^p ln\varepsilon_{t-1}^p + \xi_t^p \tag{A.40}$$

Monetary policy shock

$$ln\varepsilon_t^R = \rho^\varepsilon ln\varepsilon_{t-1}^R + \xi_t^\varepsilon \tag{A.41}$$

A.1.2 Steady States of the model

In this section, we derive the steady states of the symmetric model whereby a variable with subscript "s"represents the steady state of that variable. We therefore recursively derive the steady states of the model whereby in the steady state, all variables are assumed to be constant. Given the following properties about the capital adjustment cost function and capital utilisation: $S(.) = 0$, $S'(.) = 0$ and $a(.) = 0$. From the capital utilisation equations $(A.17)$ and $(A.18)$, we have in the steady state,

$$k_s^i = \bar{k}_s^i$$

which implies that,

$$u_s^i = 1$$

From equation $(A.5)$, and assuming zero inflation steady state, it holds that the steady state return on government bond:

$$R_s = \frac{1}{\beta}$$

and from equations $(A.9)$ and $(A.10)$, we obtain the steady state sectoral real return on capital as:

$$r_s^{k,i} = \frac{1}{\beta} - (1 - \delta)$$

It also implies from equations $(A.13)$ and $(A.14)$,

$$a'(u_s^i) = r_s^{k,i}$$

This implies that households expect the same rate of returns from investing in the formal and shadow sector capital. Assuming steady state exogenous shocks to be equal to one and given $r_s^{k,i}$, from equation $(A.21)$ and $(A.25)$, the steady state capital-labour ratio in the official sector is obtained accordingly as:

$$\frac{k_s^i}{l_s^i} = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{-\frac{1}{1-\alpha^i}}$$

The steady state output-capital ratio is also obtained accordingly from equation $(A.19)$ and $(A.23)$ as:

$$\frac{y_s^i}{k_s^i} = \left(\frac{k_s^i}{l_s^i}\right)^{\alpha^i - 1}$$

From equations $(A.15)$ and $(A.16)$, we obtain steady state investment-capital ratio in both sectors as:

$$\frac{i_s^i}{k_s^i} = \delta$$

From equation $(A.33)$, $(A.34)$ and given that $a(.) = 0$ as well as steady state price adjustment cost collapsing to zero, we obtain that:

$$\frac{c_s^i}{y_s^i} = 1 - \frac{i_s^i}{y_s^i}$$

which implies that,

$$\frac{c_s^i}{y_s^i} = 1 - \frac{i_s^i}{k_s^i}\frac{k_s^i}{y_s^i}$$

and steady state investment-output ratio as:

$$\frac{i_s^i}{y_s^i} = \delta\left(\frac{k_s^i}{l_s^i}\right)^{1-\alpha^i}$$

From equations $(A.29)$-$(A.31)$, it implies a steady state aggregate and sectoral inflation is $\pi_s = \pi_s^o = \pi_s^u = 1$. It also emerges from the final goods producers NKPC $(A.27)$ and $(A.28)$, that the steady state average mark-up is given by:

$$\frac{1}{mc_s^i} = \frac{\epsilon^i}{\epsilon^i - 1}$$

This implies that equations $(A.22)$ and $(A.26)$ can be defined in terms of their sectoral price mark-ups as:

$$mc_s^i = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{\alpha^i}\left(\frac{w_s^i}{1-\alpha^i}\right)^{1-\alpha^i}$$

$$\frac{\epsilon^i - 1}{\epsilon^i} = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{\alpha^i}\left(\frac{w_s^i}{1-\alpha^i}\right)^{1-\alpha^i}$$

From equations $(A.20)$ and $(A.24)$, the nominal wage (w_s^i) can be obtained as:

$$w_s^i = \left(\frac{\epsilon^i - 1}{\epsilon^i}\right)^{\frac{1}{1-\alpha^i}}\left(\frac{r_s^{k,i}}{\alpha^i}\right)^{-\frac{\alpha^i}{1-\alpha^i}}(1-\alpha^i)$$

Given the arbitrage condition in the labour market,

$$\frac{P_s^o}{P_s}w_s^o = \frac{P_s^u}{P_s}w_s^u$$

Thus, the relative prices are determined as:

$$\frac{P_s^o}{P_s^u} = \frac{w_s^u}{w_s^o}$$

By substitution,

$$\frac{P_s^o}{P_s^u} = \frac{\left(\frac{\epsilon^u - 1}{\epsilon^u}\right)^{\frac{1}{1-\alpha^u}} \left(\frac{r_s^{k,o}}{\alpha^o}\right)^{\frac{\alpha^o}{1-\alpha^o}} (1-\alpha^u)}{\left(\frac{\epsilon^o - 1}{\epsilon^o}\right)^{\frac{1}{1-\alpha^o}} \left(\frac{r_s^{k,u}}{\alpha^u}\right)^{\frac{\alpha^u}{1-\alpha^u}} (1-\alpha^o)}$$

From the relative size of the shadow sector equation $SH = \frac{y_s^u}{y_s^o}$, we have that:

$$SH = \frac{y_s^u}{y_s^o} = \frac{1-\varphi_c}{\varphi_c}\left(\frac{P_s^u}{P_s^o}\right)^{-\epsilon_c} \frac{(1 - \frac{i_s^o}{y_s^o})}{(1 - \frac{i_s^u}{y_s^u})}$$

$$SH = \frac{1-\varphi_c}{\varphi_c}\left(\frac{\left(\frac{\epsilon^u - 1}{\epsilon^u}\right)^{\frac{1}{1-\alpha^u}} \left(\frac{r_s^{k,o}}{\alpha^o}\right)^{\frac{\alpha^o}{1-\alpha^o}} (1-\alpha^u)}{\left(\frac{\epsilon^o - 1}{\epsilon^o}\right)^{\frac{1}{1-\alpha^o}} \left(\frac{r_s^{k,u}}{\alpha^u}\right)^{\frac{\alpha^u}{1-\alpha^u}} (1-\alpha^o)}\right)^{\epsilon_c} \frac{\left(1 - \delta\left(\frac{\frac{1}{\beta} - (1-\delta)}{\alpha^o}\right)\right)}{\left(1 - \delta\left(\frac{\frac{1}{\beta} - (1-\delta)}{\alpha^u}\right)\right)}$$

Given the aggregate labour constraint $l_s = l_s^o + l_s^u$ and calibrating $l_s = 0.25$, from equation $(A.23)$, we have that:

$$l_s^u = \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} y_s^u$$

However, $SH = \frac{y_s^u}{y_s^o}$; therefore,

$$l_s^u = \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SH y_s^o$$

and from equation $(A.19)$, steady state official sector labour is:

$$l_s^o = \left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} y_s^o$$

Finally, from equation $(A.36)$, we obtain that:

$$0.25 = \left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} y_s^o + \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SH y_s^o$$

solving for y_s^o, we obtain:

$$0.25 = \left[\left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} + \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SH\right] y_s^o$$

This enable us to obtain the steady state for other variables as:

$$y_s^u = SH y_s^o$$

$$i_s^i = \delta\left(\frac{k_s^i}{l_s^i}\right)^{1-\alpha^i} y_s^i$$

From equations $(A.33)$ and $(A.34)$,

$$c_s^i = y_s^i - i_s^i$$

$$l_s^i = \left(\frac{k_s^i}{l_s^i}\right)^{-\alpha^i} y_s^i$$

From the consumption price index equation $(A.3)$ and setting $P_s = 1$,

$$\frac{1}{P_s^u} = \left[\varphi_c \left(\frac{P_s^o}{P_s^u}\right)^{1-\epsilon_c} + (1-\varphi_c)\right]^{\frac{1}{1-\epsilon_c}}$$

From labour market arbitrage condition,

$$P_s^o = P_s^u \frac{w_s^u}{w_s^o}$$

which also implies that,

$$Q_s^i = \frac{P_s^i}{P_s}$$

From the aggregate consumption index $(A.35)$ is given as:

$$c_s = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_s^o)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_s^u)^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}}$$

From equations $(A.6)$ and $(A.7)$ we calibrate for χ as:

$$\chi = \frac{P_s^i}{P_s} \frac{w_s^i}{l_s^{i\phi} c_s}$$

A.1.3 Log-Linearised Model

The log-linearised relations are derived in accordance with the non-linear equilibrium relationships where a variable with "*hat*"represent the log-deviations of that variable around its steady state.

Households

The households consumption demand in both sectors, equations $(A.1)$, $(A.2)$ and price index $(A.3)$ give the following log-linearised equations:

$$\widehat{c}_t^o = \widehat{c}_t - \epsilon_c(\widehat{P}_t^o - \widehat{P}_t) \tag{A.42}$$

$$\widehat{c}_t^u = \widehat{c}_t - \epsilon_c(\widehat{P}_t^u - \widehat{P}_t) \tag{A.43}$$

$$\widehat{P}_t = \varphi_c \left(\frac{P_s^o}{P_s}\right)^{1-\epsilon_c} \widehat{P}_t^o + (1-\varphi_c)\left(\frac{P_s^u}{P_s}\right)^{1-\epsilon_c} \widehat{P}_t^u \tag{A.44}$$

From the first order conditions of the households maximization problem for consumption $(A.4)$, government bond $(A.5)$ and labour $(A.6)$ we solve by substitution to obtain the following equations:

The consumption Euler equation is obtained by solving equations $(A.4)$ and $(A.5)$ for c_t as:

$$\widehat{c}_t = \widehat{c}_{t+1} + E_t\widehat{\pi}_{t+1} - \widehat{R}_t - \widehat{\varepsilon}_t^{RISK} \tag{A.45}$$

The equilibrium labour supplied is also obtained by substituting equation $(A.4)$ into equation $(A.6)$ and $(A.7)$ for real wage rate (w_t) as:

$$\widehat{P}_t^o - \widehat{P}_t + \widehat{w}_t^o = \phi\widehat{l}_t^o + \widehat{c}_t \tag{A.46}$$

$$\widehat{P}_t^u - \widehat{P}_t + \widehat{w}_t^u = \phi\widehat{l}_t^u + \widehat{c}_t \tag{A.47}$$

where real wage is equal to the marginal rate of substitution between total labour supplied and consumption. The arbitrage condition in the labour market ensures that both sectors pay the same level of real wage given as:

$$\widehat{P}_t^o + \widehat{w}_t^o = \widehat{P}_t^u + \widehat{w}_t^u \tag{A.48}$$

From the official sector capital supplied $(A.9)$ and unofficial sector capital supplied $(A.10)$, the log-linearised version is derived symmetrically as follows:

$$Q_s^i\widehat{Q}_t^i = \left(\beta\frac{P_s^i}{P_s}\big(r_s^{k,i}u_s^i - a(u_s^i)\big) + Q_s^i(1-\delta)\right)(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\frac{P_s^i}{P_s}\left(r_s^{k,i}u_s^i - a(u_s^i)\right)\big(\widehat{P}_{t+1}^i - \widehat{P}_{t+1}\big) +$$

$$+\beta\frac{P_s^i}{P_s}r_s^{k,i}u_s^i\widehat{r}_{t+1}^{k,i} + \beta\frac{P_s^i}{P_s}\big[r_s^{k,i} - a'(u_s^i)\big]u_s^i\widehat{u}_{t+1}^i + \beta(1-\delta)Q_s^i\widehat{Q}_{t+1}^i$$

Dividing through by Q_s^i and noting from the sectoral investment equations $(A.9)$ and $(A.10)$, it holds in the steady state that $Q_s^i = \frac{P_s^i}{P_s}$,

$$\widehat{Q}_t^i = \{\beta\big(r_s^{k,i}u_s^i - a(u_s^i)\big) + (1-\delta)\}(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\big(r_s^{k,i}u_s^i - a(u_s^i)\big)\big(\widehat{P}_{t+1}^i - \widehat{P}_{t+1}\big) +$$

$$+\beta r_s^{k,i}u_s^i\widehat{r}_{t+1}^{k,i} + \beta\big[r_s^{k,i} - a'(u_s^i)\big]u_s^i\widehat{u}_{t+1}^i + \beta(1-\delta)\widehat{Q}_{t+1}^i$$

In the steady state, the following conditions must hold which simplifies the derivation of log-linearised sectoral capital and investments. The sectoral capital utilisation equation in the steady state is:

$$u_s^i = 1$$

And we also identify the following properties from the capital adjustment cost function: $S(.) = 0$, $S'(.) = 0$, $S''(.) = \varpi$, $a(.) = 0$, $a'(.) = r_s^{k,i}$ and $\frac{a''(.)}{a'(.)} = \frac{a''(1)}{a'(1)} = \tau$. Given the above conditions, we can continue as:

$$\widehat{Q}_t^i = \beta\big(r_s^{k,i} + (1-\delta)\big)(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta r_s^{k,i}\big(\widehat{P}_{t+1}^i - \widehat{P}_{t+1}\big) + \beta r_s^{k,i}\widehat{r}_{t+1}^{k,i} + \beta(1-\delta)\widehat{Q}_{t+1}^i$$

From the steady state capital equation, it holds that,

$$1 = \beta[r_s^{k,i} + (1-\delta)]$$

$$\frac{1}{\beta} = r_s^{k,i} + (1-\delta)$$

$$r_s^{k,i} = \frac{1}{\beta} - (1-\delta)$$

Therefore,

$$\widehat{Q}_t^i = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^i - \widehat{P}_{t+1}\big) + [1-\beta(1-\delta)]r_{t+1}^{k,i} + \beta(1-\delta)\widehat{Q}_{t+1}^i$$

Substituting the log-linearised marginal utility of consumption equation $\widehat{\lambda}_t = -\widehat{c}_t$,

$$\widehat{Q}_t^i = \widehat{c}_t - \widehat{c}_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^i - \widehat{P}_{t+1} + \widehat{r}_{t+1}^{k,i}\big) + \beta(1-\delta)\widehat{Q}_{t+1}^i$$

Log-linearised Euler equation for capital supplied to both official and shadow sector are given respectively as:

$$\widehat{Q}^o_t = \widehat{c}_t - \widehat{c}_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}^o_{t+1} - \widehat{P}_{t+1} + \widehat{r}^{k,o}_{t+1}\big) + \beta(1-\delta)\widehat{Q}^o_{t+1} \tag{A.49}$$

$$\widehat{Q}^u_t = \widehat{c}_t - \widehat{c}_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1} + \widehat{r}^{k,u}_{t+1}\big) + \beta(1-\delta)\widehat{Q}^u_{t+1} \tag{A.50}$$

Following the first order conditions for sectoral investment equations $(A.11)$ and $(A.12)$, we obtain the following log-linear equations:

$$\frac{P^i_s}{P_s}\big(\widehat{P}^i_t - \widehat{P}_t\big) = \Bigg[Q_s\bigg(-S'(1)\frac{1}{i^i_s} - S''(1)\frac{1}{i^i_s} - S'(1)\frac{1}{i^i_s}\bigg) + \beta Q_s\bigg(S''(1)\bigg(-\frac{i^i_s}{i^{i2}_s}\bigg) + 2S'(1)\bigg(-\frac{i^i_s}{i^{i2}_s}\bigg)\bigg)\Bigg] i^i_s \widehat{i}^i_t +$$

$$+Q_s\Bigg[-S'(1)\bigg(-\frac{i^i_s}{i^{i2}_s}\bigg) - S''(1)\bigg(-\frac{i^i_s}{i^{i2}_s}\bigg) - S'(1)\bigg(-\frac{i^i_s}{i^{i2}_s}\bigg)\Bigg] i^i_s i^i_{t-1} + \beta Q_s\bigg(S''(1)\frac{1}{i^i_s} + 2S'(1)\frac{1}{i^i_s}\bigg) i^i_s \widehat{i}^i_{t+1} +$$

$$+\bigg(1 - S(1) - S'(1)\bigg) Q_s \widehat{Q}^i_t + \beta S'(1) Q_s \widehat{Q}^i_{t+1} + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

$$\frac{P^i_s}{P_s}\big(\widehat{P}^i_t - \widehat{P}_t\big) = \bigg(1 - S(1) - S'(1)\bigg) Q_s \widehat{Q}^i_t + \Bigg[Q_s\bigg(-S'(1) - S''(1) - S'(1)\bigg) + \beta Q_s\bigg(-S''(1) - 2S'(1)\bigg)\Bigg]\widehat{i}^i_t +$$

$$+Q_s\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i^i_{t-1} + \beta Q_s\bigg(S''(1) + 2S'(1)\bigg)\widehat{i}^i_{t+1} + \beta S'(1) Q_s \widehat{Q}^i_{t+1} + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

Dividing through by Q^i_s and noting that $Q^i_s = \frac{P^i_s}{P_s}$,

$$\widehat{P}^i_t - \widehat{P}_t = \bigg(1 - S(1) - S'(1)\bigg)\widehat{Q}^i_t + \Bigg[\bigg(-S'(1) - S''(1) - S'(1)\bigg) + \beta\bigg(-S''(1) - 2S'(1)\bigg)\Bigg]\widehat{i}^i_t +$$

$$+\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i^i_{t-1} + \beta\bigg(S''(1) + 2S'(1)\bigg)\widehat{i}^i_{t+1} + \beta S'(1)\widehat{Q}^i_{t+1} + \beta S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

Following the above steady state conditions,

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - \bigg(S''(1) + \beta S''(1)\bigg)\widehat{i}^i_t + S''(1) i^i_{t-1} + \beta S''(1)\widehat{i}^i_{t+1}$$

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - (1+\beta) S''(1)\widehat{i}^i_t + S''(1)\widehat{i}^i_{t-1} + \beta S''(1)\widehat{i}^i_{t+1}$$

Given that $S''(1) = \varpi$ and solving for i^i_t,

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - (1+\beta)\varpi\widehat{i}^i_t + \varpi i^i_{t-1} + \beta\varpi\widehat{i}^i_{t+1}$$

solving for i_t^i,

$$\widehat{i}_t^i = \frac{\widehat{Q}_t^i}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^i}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^i - \frac{1}{(1+\beta)\varpi}\left(\widehat{P}_t^i - \widehat{P}_t\right)$$

Therefore, sectoral investments are given as:

$$\widehat{i}_t^o = \frac{1}{(1+\beta)\varpi}\widehat{Q}_t^o + \frac{\widehat{i}_{t-1}^o}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^o - \frac{1}{(1+\beta)\varpi}\left(\widehat{P}_t^o - \widehat{P}_t\right) + \frac{1}{(1+\beta)\varpi}\widehat{\varepsilon}_t^{INV} \quad \text{(A.51)}$$

$$\widehat{i}_t^u = \frac{\widehat{Q}_t^u}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^u}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^u - \frac{1}{(1+\beta)\varpi}\left(\widehat{P}_t^u - \widehat{P}_t\right) \quad \text{(A.52)}$$

The log-linearised equation for the sectoral capital utilisation cost equations $(A.13)$ and $(A.14)$ give the following:

$$r_s^{k,i}\widehat{r}_t^{k,i} = a''(u_t^i)u_s^i$$

$$\widehat{r}_t^{k,i} = \frac{a''(1)}{a'(1)}\widehat{u}_t^i$$

Given that $\frac{a''(1)}{a'(1)} = \tau$, the sectoral first order conditions for capital utilisation cost is given respectively as:

$$\widehat{r}_t^{k,o} = \tau\widehat{u}_t^o \quad \text{(A.53)}$$

$$\widehat{r}_t^{k,u} = \tau\widehat{u}_t^u \quad \text{(A.54)}$$

The log-linearised equation for both official and shadow sector capital accumulation equations $(A.15)$ and $(A.16)$ are given respectively as:

$$\widehat{\bar{k}}_{t+1}^o = (1-\delta)\widehat{\bar{k}}_t^o + \delta\widehat{i}_t^o + \delta\widehat{\varepsilon}_t^{INV} \quad \text{(A.55)}$$

$$\widehat{\bar{k}}_{t+1}^u = (1-\delta)\widehat{\bar{k}}_t^u + \delta\widehat{i}_t^u \quad \text{(A.56)}$$

whereby in the steady state, $\frac{i_s^i}{k_s^i} = \delta$. The log-linearisation of capital utilisation equations $(A.17)$ and $(A.18)$ give:

$$\widehat{k}_t^o = \widehat{u}_t^o + \widehat{\bar{k}}_{t-1}^o \quad \text{(A.57)}$$

$$\widehat{k}_t^u = \widehat{u}_t^u + \widehat{\bar{k}}_{t-1}^u \quad \text{(A.58)}$$

Official Sector Goods Producers

The first order conditions for the official sector goods producers give the following log-linearised equations in accordance to equations $(A.19)$, $(A.20)$, $(A.21)$ and $(A.22)$ as:

$$\widehat{y}_t^o = \widehat{A}_t^o + \alpha^o \widehat{k}_t^o + (1-\alpha^o)\widehat{l}_t^o \tag{A.59}$$

$$\widehat{w}_t^o = \widehat{A}_t^o + \alpha^o(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.60}$$

$$\widehat{r}_t^{k,o} = \widehat{A}_t^o - (1-\alpha^o)(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.61}$$

$$\widehat{mc}_t^{I,o} = \alpha^o \widehat{r}_t^{k,o} + (1-\alpha^o)\widehat{w}_t^o \tag{A.62}$$

Only the first three equations are needed for calibrations.

Shadow Sector Goods Producers

From the informal goods producers output and the first order conditions derived, the log-linearised version of the various equations $(A.23)$, $(A.24)$, $(A.25)$ and $(A.26)$ are derived respectively as:

$$\widehat{y}_t^u = \alpha^u \widehat{k}_t^u + (1-\alpha^u)\widehat{l}_t^u \tag{A.63}$$

$$\widehat{w}_t^u = \alpha^u(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.64}$$

$$\widehat{r}_t^{k,u} = -(1-\alpha^u)(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.65}$$

$$\widehat{mc}_t^{I,u} = \alpha^u \widehat{r}_t^{k,u} + (1-\alpha^u)\widehat{w}_t^u \tag{A.66}$$

Only the first three equations are needed for calibrations.

Final Goods Producers

The standard NKPC is derived accordingly from equations $(A.27)$ and $(A.28)$ for both sectors. Log-linearising the non.linear equations under a zero steady state inflation, the Rotemberg-pricing yield the following generalised NKPC respectively for official and shadow sectors as:

$$\widehat{\pi}_t^o = \beta E_t \widehat{\pi}_{t+1}^o + \frac{\epsilon^o - 1}{\kappa^p}(\widehat{mc}_t^o + \widehat{\varepsilon}_t^p) \tag{A.67}$$

and

$$\widehat{\pi}_t^u = \beta E_t \widehat{\pi}_{t+1}^u + \frac{\epsilon^u - 1}{\kappa^p}\widehat{mc}_t^u \tag{A.68}$$

It emerges from the steady state that the average mark-up is given by the inverse of the real marginal cost as:

$$mc_s^i = \frac{\epsilon^i - 1}{\epsilon^i}$$

The log-linearised version of aggregate and sectoral inflation, equations $(A.29)$, $(A.30)$ and $(A.31)$ are respectively given as:

$$\widehat{\pi}_t + \widehat{P}_{t-1} = \widehat{P}_t \tag{A.69}$$

$$\widehat{\pi}_t^o + \widehat{P}_{t-1}^o = \widehat{P}_t^o \tag{A.70}$$

$$\widehat{\pi}_t^u + \widehat{P}_{t-1}^u = \widehat{P}_t^u \tag{A.71}$$

Monetary Policy

The log-linearisation of the monetary policy instrument $(A.32)$ that is set by the central bank is:

$$\widehat{R}_t = \rho^R \widehat{R}_{t-1} + (1 - \rho^R)(\mu_\pi)\widehat{\pi}_t^o + \widehat{\epsilon}_t^R \tag{A.72}$$

Resource constraints

The log-linearised version of the aggregate resource constraints in both sectors $(A.33)$, $(A.34)$, aggregate consumption $(A.35)$ and the labour resource constraint $(A.36)$, yield the following log-linearised equations:

$$\widehat{y}_t^o = \frac{c_s^o}{y_s^o}\widehat{c}_t^o + \frac{i_s^o}{y_s^o}\widehat{i}_t^o + \frac{r_s^{k,o}k_s^o}{y_s^o}\widehat{u}_t^o \tag{A.73}$$

$$\widehat{y}_t^u = \frac{c_s^u}{y_s^u}\widehat{c}_t^u + \frac{i_s^u}{y_s^u}\widehat{i}_t^u + \frac{r_s^{k,u}k_s^u}{y_s^u}\widehat{u}_t^u \tag{A.74}$$

$$\widehat{c}_t = \varphi_C^{\frac{1}{\epsilon_c}}\left(\frac{c_s^o}{c_s}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}}\widehat{c}_t^o + (1 - \varphi)^{\frac{1}{\epsilon_c}}\left(\frac{c_s^u}{c_s}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}}\widehat{c}_t^u \tag{A.75}$$

$$\widehat{l}_t = \frac{l_s^o}{l_s}\widehat{l}_t^o + \frac{l_s^u}{l_s}\widehat{l}_t^u \tag{A.76}$$

Shock Processes

The log-linearised equations for shocks in the official sector $(A.37)$, $(A.38)$, $(A.39)$, $(A.40)$ and $(A.41)$ that are considered in the model are given as:

Risk premium shock

$$\widehat{\varepsilon}_t^{RISK} = \rho^{RISK}\widehat{\varepsilon}_{t-1}^{RISK} + \widehat{\xi}_t^{RISK} \tag{A.77}$$

Investment shock

$$\widehat{\varepsilon}_t^{INV} = \rho^{INV}\widehat{\varepsilon}_{t-1}^{INV} + \widehat{\xi}_t^{INV} \tag{A.78}$$

Official sector productivity shock

$$\widehat{A}_t^o = \rho^A \widehat{A}_{t-1}^o + \widehat{\xi}_t^A \tag{A.79}$$

Price mark-up shock

$$\widehat{\varepsilon}_t^p = \rho^p \widehat{\varepsilon}_{t-1}^p + \widehat{\xi}_t^p \tag{A.80}$$

Monetary policy shock

$$\widehat{\epsilon}_t^R = \rho^{\varepsilon}\widehat{\epsilon}_{t-1}^R + \widehat{\xi}_t^{\varepsilon} \tag{A.81}$$

A.2 Appendix to Chapter Two

A.2.1 Symmetric Equilibrium of the Model

Households

Consumption in official sector

$$c_t^o = \varphi_c \left(\frac{P_t^o(1+\tau^c)}{P_t} \right)^{-\epsilon_c} c_t \tag{A.82}$$

Consumption in unofficial sector

$$c_t^u = (1-\varphi_c) \left(\frac{P_t^u}{P_t} \right)^{-\epsilon_c} c_t \tag{A.83}$$

Consumption price index

$$P_t = \left[\varphi_c \left(P_t^o(1+\tau^c) \right)^{1-\epsilon_c} + (1-\varphi_c) \left(P_t^u \right)^{1-\epsilon_c} \right]^{\frac{1}{1-\epsilon_c}} \tag{A.84}$$

Marginal utility of the consumption bundle

$$\lambda_t = \frac{1}{c_t} \tag{A.85}$$

Consumption Euler equation

$$\lambda_t = \varepsilon_t^{RISK} R_t \beta \frac{E_t \lambda_{t+1}}{\pi_{t+1}} \tag{A.86}$$

Labour supplied to official sector

$$(1-\tau_t^w) \frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o\phi}}{\lambda_t} \tag{A.87}$$

Labour supplied to unofficial sector

$$\frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u\phi}}{\lambda_t} \tag{A.88}$$

Labour market arbitrage condition

$$\frac{P_t^o}{P_t} w_t^o (1-\tau_t^w) = \frac{P_t^u}{P_t} w_t^u \tag{A.89}$$

Official sector capital

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{P_{t+1}^o}{P_{t+1}}\left((1-\tau_{t+1}^k)\left[r_{t+1}^{k,o}u_{t+1}^o - a(u_{t+1}^o)\right] + \tau_{t+1}^k\delta\right) + Q_{t+1}^o(1-\delta)\right] \tag{A.90}$$

Unofficial sector capital

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{P_{t+1}^u}{P_{t+1}}\left[r_{t+1}^{k,u}u_{t+1}^u - a(u_{t+1}^u)\right] + Q_{t+1}^u(1-\delta)\right] \tag{A.91}$$

Official sector investment

$$\frac{P_t^o}{P_t} = Q_t^o\varepsilon_t^{INV}\left(1 - S\left(\frac{i_t^o}{i_{t-1}^o}\right) - S'\left(\frac{i_t^o}{i_{t-1}^o}\right)\frac{i_t^o}{i_{t-1}^o}\right) + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}^o\varepsilon_{t+1}^{INV}S'\left(\frac{i_{t+1}^o}{i_t^o}\right)\left(\frac{i_{t+1}^o}{i_t^o}\right)^2 \tag{A.92}$$

Unofficial sector investment

$$\frac{P_t^u}{P_t} = Q_t^u\left(1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right) - S'\left(\frac{i_t^u}{i_{t-1}^u}\right)\frac{i_t^u}{i_{t-1}^u}\right) + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}^u S'\left(\frac{i_{t+1}^u}{i_t^u}\right)\left(\frac{i_{t+1}^u}{i_t^u}\right)^2 \tag{A.93}$$

Official sector capital utilisation

$$r_t^{k,o} = a'(u_t^o) \tag{A.94}$$

Unofficial sector capital utilisation

$$r_t^{k,u} = a'(u_t^u) \tag{A.95}$$

Official sector capital

$$\bar{k}_{t+1}^o = (1-\delta)\bar{k}_t^o + \varepsilon_t^{INV}\left[1 - S\left(\frac{i_t^o}{i_{t-1}^o}\right)\right]i_t^o \tag{A.96}$$

Unofficial sector capital

$$\bar{k}_{t+1}^u = (1-\delta)\bar{k}_t^u + \left[1 - S\left(\frac{i_t^u}{i_{t-1}^u}\right)\right]i_t^u \tag{A.97}$$

Official sector capital utilisation

$$k_t^o = u_t^o\bar{k}_{t-1}^o \tag{A.98}$$

Unofficial sector capital utilisation

$$k_t^u = u_t^u\bar{k}_{t-1}^u \tag{A.99}$$

Official Sector Goods Producers

Official sector output

$$y_t^o = A_t^o k_t^{o(\alpha^o)} l_t^{o(1-\alpha^o)} \tag{A.100}$$

Official sector labour demand

$$w_t^o = (1-\alpha^o)A_t^o\left(\frac{k_t^o}{l_t^o}\right)^{\alpha^o} \tag{A.101}$$

official sector capital demand

$$r_t^{k,o} = \alpha^o A_t^o\left(\frac{k_t^o}{l_t^o}\right)^{-(1-\alpha^o)} \tag{A.102}$$

Official sector marginal cost

$$mc_t^{I,o} = \left(\frac{r_t^{k,o}}{\alpha^o}\right)^{\alpha^o}\left(\frac{w_t^o}{1-\alpha^o}\right)^{1-\alpha^o} \tag{A.103}$$

Shadow Sector Goods Producers

Shadow sector output

$$y_t^u = k_t^{u(\alpha^u)} l_t^{u(1-\alpha^u)} \tag{A.104}$$

Unofficial sector labour demand

$$w_t^u = (1-\alpha^u)\left(\frac{k_t^u}{l_t^u}\right)^{\alpha^u} \tag{A.105}$$

Unofficial sector capital demand

$$r_t^{k,u} = \alpha^u\left(\frac{k_t^u}{l_t^u}\right)^{-(1-\alpha^u)} \tag{A.106}$$

Unofficial sector marginal cost

$$mc_t^{I,u} = \left(\frac{r_t^{k,u}}{\alpha^u}\right)^{\alpha^u}\left(\frac{w_t^u}{1-\alpha^u}\right)^{1-\alpha^u} \tag{A.107}$$

Final Goods Producers

Official sector NKPC

$$(1-mc_t^o)\epsilon_t^o = 1-\kappa^p\left(\frac{\pi_t^o}{\pi_{t-1}^{o\ \theta_\pi}}-1\right)\frac{\pi_t^o}{\pi_{t-1}^{o\ \theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^o}{\pi_t^{o\theta_\pi}}-1\right)\frac{\pi_{t+1}^o}{\pi_t^{o\theta_\pi}}\frac{y_{t+1}^o}{y_t^o}\right] \tag{A.108}$$

Unofficial sector NKPC

$$(1-mc_t^u)\epsilon^u = 1-\kappa^p\left(\frac{\pi_t^u}{\pi_{t-1}^{u\ \theta_\pi}}-1\right)\frac{\pi_t^u}{\pi_{t-1}^{u\ \theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^u}{\pi_t^{u\theta_\pi}}-1\right)\frac{\pi_{t+1}^u}{\pi_t^{u\theta_\pi}}\frac{y_{t+1}^u}{y_t^u}\right] \tag{A.109}$$

Aggregate inflation

$$P_t = \pi_t P_{t-1} \tag{A.110}$$

Official sector inflation

$$P_t^o = \pi_t^o P_{t-1}^o \tag{A.111}$$

Unofficial sector inflation

$$P_t^u = \pi_t^u P_{t-1}^u \tag{A.112}$$

Monetary Policy

Taylor's rule

$$R_t = R_{t-1}^{(\rho^R)} (\pi_t^o)^{\mu_\pi (1-\rho^R)} \varepsilon_t^R \tag{A.113}$$

Market Clearing and Resource Constraint

Official sector resource

$$y_t^o = c_t^o + i_t^o + g_t + a(u_t^o)\bar{k}_{t-1}^o + \frac{\kappa^p}{2}\left(\frac{\pi_t^o}{\pi_{t-1}^{o}{}^{\theta_\pi}} - 1\right)^2 y_t^o \tag{A.114}$$

Shadow sector resource

$$y_t^u = c_t^u + i_t^u + a(u_t^u)\bar{k}_{t-1}^u + \frac{\kappa^p}{2}\left(\frac{\pi_t^u}{\pi_{t-1}^{u}{}^{\theta_\pi}} - 1\right)^2 y_t^u \tag{A.115}$$

Aggregate consumption

$$c_t = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^o)^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^u)^{\frac{\epsilon_c - 1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{A.116}$$

Aggregate labour

$$l_t = l_t^o + l_t^u \tag{A.117}$$

Shock Processes

Risk premium shock

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{A.118}$$

Investment shock

$$ln\varepsilon_t^{INV} = \rho^{INV} ln\varepsilon_{t-1}^{INV} + \xi_t^{INV} \tag{A.119}$$

Official sector productivity shock

$$lnA_t^o = \rho^A lnA_{t-1}^o + \xi_t^A \tag{A.120}$$

Price mark-up

$$ln\varepsilon_t^p = \rho^p ln\varepsilon_{t-1}^p + \xi_t^p \tag{A.121}$$

Monetary policy shock

$$ln\varepsilon_t^R = \rho^\varepsilon ln\varepsilon_{t-1}^R + \xi_t^\varepsilon \tag{A.122}$$

Government spending shock

$$lng_t = \rho^G lng_{t-1} + \xi_t^G \tag{A.123}$$

Labour income tax shock

$$ln\tau_t^W = \rho^W ln\tau_{t-1}^W + \xi_t^W \tag{A.124}$$

Capital income tax shock

$$ln\tau_t^K = \rho^K ln\tau_{t-1}^K + \xi_t^K \tag{A.125}$$

A.2.2 Steady States of the model

The steady states of the symmetric model is derived recursively whereby a variable with subscript "s"represents the steady state of that variable. The following properties hold about capital adjustment cost function and capital utilisation: $S(.) = 0$, $S'(.) = 0$ and $a(.) = 0$. We set the steady state values for the fiscal variable τ_s^k, τ_s^w, τ^c and $\bar{g}_s$ as a percentage of GDP. From the capital utilisation equations $(A.98)$ and $(A.99)$, we have in the steady state,

$$k_s^i = \bar{k}_s^i$$

which implies that,

$$u_s^i = 1$$

From equation $(A.86)$, and assuming zero inflation steady state, it holds that the steady state return on government bond:

$$R_s = \frac{1}{\beta}$$

and from equations $(A.90)$ and $(A.91)$, we obtain the steady state sectoral real return on capital as:

$$r_s^{k,o} = \frac{1}{(1-\tau_s^k)}\left[\frac{1}{\beta} - \tau_s^k\delta - (1-\delta)\right]$$

$$r_s^{k,u} = \frac{1}{\beta} - (1-\delta)$$

It also implies from equations $(A.94)$ and $(A.95)$ that,

$$a'(u_s^i) = r_s^{k,i}$$

This implies that households expect the same rate of returns from investing in the formal and shadow sector capital. Assuming steady state exogenous shocks to be equal to one and given $r_s^{k,i}$, from equation $(A.102)$ and $(A.106)$, the steady state capital-labour ratio in the official sector is obtained accordingly as:

$$\frac{k_s^i}{l_s^i} = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{-\frac{1}{1-\alpha^i}}$$

The steady state output-capital ratio is also obtained accordingly from equation $(A.100)$ and $(A.104)$ as:

$$\frac{y_s^i}{k_s^i} = \left(\frac{k_s^i}{l_s^i}\right)^{\alpha^i - 1}$$

From equations $(A.96)$ and $(A.97)$, we obtain steady state investment-capital ratio in both sectors as:

$$\frac{i_s^i}{k_s^i} = \delta$$

From equation $(A.114)$, $(A.115)$, and given that $a(.) = 0$. We impose $\frac{g_s}{y_s^o} = \bar{g}_s$ to obtain steady state consumption-output ratio as:

$$\frac{c_s^o}{y_s^o} = 1 - \frac{i_s^o}{y_s^o} - \bar{g}_s$$

$$\frac{c_s^u}{y_s^u} = 1 - \frac{i_s^u}{y_s^u}$$

where,

$$\frac{i_s^i}{y_s^i} = \frac{i_s^i}{k_s^i}\frac{k_s^i}{y_s^i}$$

which implies a steady state investment-output ratio as:

$$\frac{i_s^i}{y_s^i} = \delta\left(\frac{k_s^i}{l_s^i}\right)^{1-\alpha^i}$$

From equations $(A.110)$-$(A.112)$, it implies a steady state aggregate and sectoral inflation is $\pi_s = \pi_s^o = \pi_s^u = 1$. It also emerges from the final goods producers NKPC $(A.108)$ and $(A.109)$ that the steady state average mark-up is given by:

$$\frac{1}{mc_s^i} = \frac{\epsilon^i}{\epsilon^i - 1}$$

This implies that equations $(A.103)$ and $(A.107)$ can be defined in terms of their sectoral mark-up prices as:

$$mc_s^i = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{\alpha^i}\left(\frac{w_s^i}{1-\alpha^i}\right)^{1-\alpha^i}$$

$$\frac{\epsilon^i - 1}{\epsilon^i} = \left(\frac{r_s^{k,i}}{\alpha^i}\right)^{\alpha^i}\left(\frac{w_s^i}{1-\alpha^i}\right)^{1-\alpha^i}$$

From equations $(A.101)$ and $(A.105)$, the nominal wage (w_s^i) can be obtained as:

$$w_s^i = \left(\frac{\epsilon^i - 1}{\epsilon^i}\right)^{\frac{1}{1-\alpha^i}}\left(\frac{r_s^{k,i}}{\alpha^i}\right)^{-\frac{\alpha^i}{1-\alpha^i}}(1-\alpha^i)$$

Given the arbitrage condition in the labour market,

$$\frac{P_s^o}{P_s} w_s^o (1 - \tau_s^w) = \frac{P_s^u}{P_s} w_s^u$$

The relative prices are determined as:

$$\frac{P_s^o}{P_s^u} = \frac{w_s^u}{w_s^o(1-\tau_s^w)}$$

By substitution,

$$\frac{P_s^o}{P_s^u} = \frac{\left(\frac{\epsilon^u-1}{\epsilon^u}\right)^{\frac{1}{1-\alpha^u}}\left(\frac{r_s^{k,o}}{\alpha^o}\right)^{\frac{\alpha^o}{1-\alpha^o}}(1-\alpha^u)}{\left(\frac{\epsilon^o-1}{\epsilon^o}\right)^{\frac{1}{1-\alpha^o}}\left(\frac{r_s^{k,u}}{\alpha^u}\right)^{\frac{\alpha^u}{1-\alpha^u}}(1-\alpha^o)(1-\tau_s^w)}$$

From the relative size of the shadow sector equation $SH = \frac{y_s^u}{y_s^o}$, we have that:

$$SH = \frac{y_s^u}{y_s^o} = \frac{1-\varphi_c}{\varphi_c}\left(\frac{P_s^u}{P_s^o(1+\tau^c)}\right)^{-\epsilon_c}\frac{(1-\frac{i_s^o}{y_s^o}-\bar{g}_s)}{(1-\frac{i_s^u}{y_s^u})}$$

$$SH = \frac{y_s^u}{y_s^o} = \frac{1-\varphi_c}{\varphi_c}\left(\frac{P_s^o}{P_s^u}\right)^{\epsilon_c}(1+\tau^c)^{\epsilon_c}\frac{(1-\frac{i_s^o}{y_s^o}-\bar{g}_s)}{(1-\frac{i_s^u}{y_s^u})}$$

$$SH = \frac{1-\varphi_c}{\varphi_c}\left(\frac{\left(\frac{\epsilon^u-1}{\epsilon^u}\right)^{\frac{1}{1-\alpha^u}}\left(\frac{\frac{1}{(1-\tau_s^k)}\left[\frac{1}{\beta}-\tau_s^k\delta-(1-\delta)\right]}{\alpha^o}\right)^{\frac{\alpha^o}{1-\alpha^o}}(1-\alpha^u)}{\left(\frac{\epsilon^o-1}{\epsilon^o}\right)^{\frac{1}{1-\alpha^o}}\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha^u}\right)^{\frac{\alpha^u}{1-\alpha^u}}(1-\alpha^o)}\right)^{\epsilon_c} *$$

$$*\left(\frac{1+\tau^c}{1-\tau_s^w}\right)^{\epsilon_c}\frac{\left(1-\delta\left(\frac{\frac{1}{(1-\tau_s^k)}\left[\frac{1}{\beta}-\tau_s^k\delta-(1-\delta)\right]}{\alpha^o}\right)-\bar{g}_s\right)}{\left(1-\delta\left(\frac{\frac{1}{\beta}-(1-\delta)}{\alpha^u}\right)\right)}$$

The above conditions allow to calibrate the steady state value for φ_c. Given the aggregate labour constraint $l_s = l_s^o + l_s^u$ and calibrating $l_s = 0.25$, from equation $(A.104)$, we have that:

$$l_s^u = \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} y_s^u$$

However, $SH = \frac{y_s^u}{y_s^o}$; therefore,

$$l_s^u = \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SHy_s^o$$

and from equation $(A.100)$, steady state official sector labour is:

$$l_s^o = \left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} y_s^o$$

Finally, from equation $(A.117)$, we obtain that:

$$0.25 = \left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} y_s^o + \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SHy_s^o$$

solving for y_s^o, we obtain:

$$0.25 = \left[\left(\frac{k_s^o}{l_s^o}\right)^{-\alpha^o} + \left(\frac{k_s^u}{l_s^u}\right)^{-\alpha^u} SH\right] y_s^o$$

This enable us to obtain the steady state for other variables as:

$$y_s^u = SHy_s^o$$

$$i_s^i = \delta\left(\frac{k_s^i}{l_s^i}\right)^{1-\alpha^i} y_s^i$$

From equations $(A.114)$ and $(A.115)$,

$$c_s^o = y_s^o - i_s^o - \bar{g}_s y_s^o$$

$$c_s^u = y_s^u - i_s^u$$

$$l_s^i = \left(\frac{k_s^i}{l_s^i}\right)^{-\alpha^i} y_s^i$$

From the consumption price index equation $(A.84)$ and setting $P_s = 1$,

$$\frac{1}{P_s^u} = \left[\varphi_c\left(\frac{P_s^o}{P_s^u}\right)^{1-\epsilon_c}(1+\tau^c)^{1-\epsilon_c} + (1-\varphi_c)\right]^{\frac{1}{1-\epsilon_c}}$$

From labour market arbitrage condition,

$$P_s^o = P_s^u \frac{w_s^u}{w_s^o(1-\tau_s^w)}$$

which also implies that from the investment equations $(A.92)$ and $(A.93)$,

$$Q_s^i = \frac{P_s^i}{P_s}$$

From the aggregate consumption index $(A.116)$ is given as:

$$c_s = \left[\varphi_c^{\frac{1}{\epsilon_c}}(c_s^o)^{\frac{\epsilon_c-1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}}(c_s^u)^{\frac{\epsilon_c-1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c-1}}$$

From equations $(A.88)$ we calibrate for χ as:

$$\chi = \frac{P_s^u}{P_s}\frac{w_s^u}{l_s^{u\phi} c_s}$$

A.2.3 Log-Linearised Model

The log-linearised relations are derived in accordance with the non-linear equilibrium relationships where a variable with "*hat*"represent the log-deviations of that variable around its steady state.

Households

The households consumption demand in both sectors, equations ($A.82$), ($A.83$) and price index ($A.84$) give the following log-linearised equations:

$$\widehat{c}_t^o = \widehat{c}_t - \epsilon_c(\widehat{P}_t^o(1+\tau^c) - \widehat{P}_t) \tag{A.126}$$

$$\widehat{c}_t^u = \widehat{c}_t - \epsilon_c(\widehat{P}_t^u - \widehat{P}_t) \tag{A.127}$$

$$\widehat{P}_t = \varphi_c\left(\frac{P_s^o}{P_s}\right)^{1-\epsilon_c}(1+\tau^c)^{1-\epsilon_c}\widehat{P}_t^o + (1-\varphi_c)\left(\frac{P_s^u}{P_s}\right)^{1-\epsilon_c}\widehat{P}_t^u \tag{A.128}$$

From the first order conditions of the households maximization problem for consumption ($A.85$), government bond ($A.86$) and labour ($A.87$) we solve by substitution to obtain the following equations:

The consumption Euler equation is obtained by solving equations ($A.85$) and ($A.86$) for c_t as:

$$\widehat{c}_t = \widehat{c}_{t+1} + E_t\widehat{\pi}_{t+1} - \widehat{R}_t - \widehat{\varepsilon}_t^{RISK} \tag{A.129}$$

The equilibrium labour supplied is also obtained by substituting equation ($A.85$) into equation ($A.87$) and ($A.88$) for real wage rate (w_t) as:

$$\widehat{P}_t^o - \widehat{P}_t + \widehat{w}_t^o - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\widehat{\tau}_t^w = \phi\widehat{l}_t^o + \widehat{c}_t \tag{A.130}$$

$$\widehat{P}_t^u - \widehat{P}_t + \widehat{w}_t^u = \phi\widehat{l}_t^u + \widehat{c}_t \tag{A.131}$$

where real wage is equal to the marginal rate of substitution between total labour supplied and consumption. The arbitrage condition in the labour market ensures that both sectors pay the same level of real wage given as:

$$\widehat{P}_t^o + \widehat{w}_t^o - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\widehat{\tau}_t^w = \widehat{P}_t^u + \widehat{w}_t^u \tag{A.132}$$

From the official sector capital supplied $(A.90)$ the log-linearised version is derived as follows:

$$Q_s^o\widehat{Q}_t^o = \beta\left[\frac{P_s^o}{P_s}\left((1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\right) + Q_s^o(1-\delta)\right](\widehat{\lambda}_{t+1} - \widehat{\lambda}_t)+$$

$$+\beta\frac{P_s^o}{P_s}\big[(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) - \beta\frac{P_s^o}{P_s}\tau_s^k r_s^{k,o}u_s^o(\widehat{\tau}_{t+1}^k)+$$

$$+\beta\frac{P_s^o}{P_s}(1-\tau_s^k)r_s^{k,o}u_s^o\widehat{r}_{t+1}^{k,o} + \beta\frac{P_s^o}{P_s}(1-\tau_s^k)\big[r_s^{k,o} - a'(u_s^o)\big]u_s^o\widehat{u}_{t+1}^o + \beta\frac{P_s^o}{P_s}\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)Q_s^o\widehat{Q}_{t+1}^o$$

Dividing through by Q_s^i and noting from the sectoral investment equations $(A.90)$ and $(A.91)$, it holds in the steady state that $Q_s^i = \frac{P_s^i}{P_s}$,

$$\widehat{Q}_t^o = \beta\{(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta + (1-\delta)\}(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\big[(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big)+$$

$$-\beta\tau_s^k r_s^{k,o}u_s^o(\widehat{\tau}_{t+1}^k) + \beta(1-\tau_s^k)r_s^{k,o}u_s^o\widehat{r}_{t+1}^{k,o} + \beta(1-\tau_s^k)\big[r_s^{k,o} - a'(u_s^o)\big]u_s^o\widehat{u}_{t+1}^o + \beta\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o$$

In the steady state, the following conditions must hold which simplifies the derivation of log-linearised sectoral capital and investments. The sectoral capital utilisation equation in the steady state is:

$$u_s^i = 1$$

And we also identify the following properties from the capital adjustment cost function: $S(.) = 0$, $S'(.) = 0$, $S''(.) = \varpi$, $a(.) = 0$, $a'(u_s^i) = r_s^{k,i}$ and $\frac{a''(.)}{a'(.)} = \frac{a''(1)}{a'(1)} = \tau$. Given the above conditions, we can continue as:

$$\widehat{Q}_t^o = \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)\big](\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big)+$$

$$-\beta\tau_s^k r_s^{k,o}\widehat{\tau}_{t+1}^k + \beta(1-\tau_s^k)r_s^{k,o}\widehat{r}_{t+1}^{k,o} + \beta\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o$$

From the steady state capital equation, it holds that,

$$1 = \beta[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)]$$

$$\frac{1}{\beta} = (1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)$$

$$1 - \beta(1-\delta) = \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta\big]$$

$$1 - \beta\tau_s^k\delta - \beta(1-\delta) = \beta(1-\tau_s^k)r_s^{k,o}$$

$$r_s^{k,o} = \frac{1}{(1-\tau_s^k)}\left[\frac{1}{\beta} - \tau_s^k\delta - (1-\delta)\right]$$

Therefore,

$$\widehat{Q}_t^o = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) + \beta(1-\tau_s^k)r_s^{k,o}r_{t+1}^{k,o} + \beta(\delta - r_s^{k,o})\tau_s^k\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o$$

Substituting the log-linearised marginal utility of consumption equation $\widehat{\lambda}_t = -\widehat{c}_t$, we obtain Log-linearised Euler equation for capital supplied to the official sector as:

$$\widehat{Q}_t^o = \widehat{c}_t - \widehat{c}_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) + \beta(1-\tau_s^k)r_s^{k,o}\widehat{r}_{t+1}^{k,o} + \beta(\delta - r_s^{k,o})\tau_s^k\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o \tag{A.133}$$

From unofficial sector capital supplied $(A.91)$, the log-linearised version is derived as follows:

$$Q_s^u\widehat{Q}_t^u = \left(\beta\frac{P_s^u}{P_s}\big(r_s^{k,u}u_s^u - a(u_s^u)\big) + Q_s^u(1-\delta)\right)(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\frac{P_s^u}{P_s}\left(r_s^{k,u}u_s^u - a(u_s^u)\right)\big(\widehat{P}_{t+1}^u - \widehat{P}_{t+1}\big) +$$

$$+\beta\frac{P_s^u}{P_s}r_s^{k,u}u_s^u\widehat{r}_{t+1}^{k,u} + \beta\frac{P_s^u}{P_s}\left[r_s^{k,u} - a'(u_s^u)\right]u_s^u\widehat{u}_{t+1}^u + \beta(1-\delta)Q_s^u\widehat{Q}_{t+1}^u$$

Dividing through by Q_s^u and noting that $Q_s^u = \frac{P_s^u}{P_s}$,

$$\widehat{Q}_t^u = \{\beta\big(r_s^{k,u}u_s^u - a(u_s^u)\big) + (1-\delta)\}(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta\big(r_s^{k,u}u_s^u - a(u_s^u)\big)\big(\widehat{P}_{t+1}^u - \widehat{P}_{t+1}\big) +$$

$$+\beta r_s^{k,i}u_s^u\widehat{r}_{t+1}^{k,u} + \beta\big[r_s^{k,u} - a'(u_s^u)\big]u_s^u\widehat{u}_{t+1}^u + \beta(1-\delta)\widehat{Q}_{t+1}^u$$

In the steady state, the conditions and properties about capital utilisation equation and the capital adjustment cost function also hold in the shadow sector. This simplifies the derivation of log-linearised shadow capital as:

$$\widehat{Q}_t^u = \beta\big(r_s^{k,u} + (1-\delta)\big)(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \beta r_s^{k,u}\big(\widehat{P}_{t+1}^u - \widehat{P}_{t+1}\big) + \beta r_s^{k,u}\widehat{r}_{t+1}^{k,u} + \beta(1-\delta)\widehat{Q}_{t+1}^u$$

From the steady state capital equation, it also holds that,

$$1 = \beta[r_s^{k,u} + (1-\delta)]$$

$$\frac{1}{\beta} = r_s^{k,u} + (1-\delta)$$

$$r_s^{k,u} = \frac{1}{\beta} - (1-\delta)$$

Therefore,

$$\widehat{Q}_t^u = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^u - \widehat{P}_{t+1}\big) + [1-\beta(1-\delta)]r_{t+1}^{k,u} + \beta(1-\delta)\widehat{Q}_{t+1}^u$$

Substituting the log-linearised marginal utility of consumption equation $\widehat{\lambda}_t = -\widehat{c}_t$, we obtain Log-linearised Euler equation for capital supplied to the unofficial sector as:

$$\widehat{Q}^u_t = \widehat{c}_t - \widehat{c}_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1} + \widehat{r}^{k,u}_{t+1}\big) + \beta(1-\delta)\widehat{Q}^u_{t+1} \tag{A.134}$$

Following the first order conditions for sectoral investment equations $(A.92)$ and $(A.93)$, we obtain the following log-linear equations:

$$\frac{P^i_s}{P_s}\big(\widehat{P}^i_t - \widehat{P}_t\big) = \Bigg[Q_s\Bigg(-S'(1)\frac{1}{i^i_s} - S''(1)\frac{1}{i^i_s} - S'(1)\frac{1}{i^i_s}\Bigg) + \beta Q_s\Bigg(S''(1)\Bigg(-\frac{i^i_s}{i^{i2}_s}\Bigg) + 2S'(1)\Bigg(-\frac{i^i_s}{i^{i2}_s}\Bigg)\Bigg)\Bigg] i^i_s\widehat{i}^i_t +$$

$$+Q_s\Bigg[-S'(1)\Bigg(-\frac{i^i_s}{i^{i2}_s}\Bigg) - S''(1)\Bigg(-\frac{i^i_s}{i^{i2}_s}\Bigg) - S'(1)\Bigg(-\frac{i^i_s}{i^{i2}_s}\Bigg)\Bigg] i^i_s i^i_{t-1} + \beta Q_s\Bigg(S''(1)\frac{1}{i^i_s} + 2S'(1)\frac{1}{i^i_s}\Bigg) i^i_s\widehat{i}^i_{t+1} +$$

$$+\Bigg(1 - S(1) - S'(1)\Bigg)Q_s\widehat{Q}^i_t + \beta S'(1)Q_s\widehat{Q}^i_{t+1} + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

$$\frac{P^i_s}{P_s}\big(\widehat{P}^i_t - \widehat{P}_t\big) = \Bigg(1 - S(1) - S'(1)\Bigg)Q_s\widehat{Q}^i_t + \Bigg[Q_s\Bigg(-S'(1) - S''(1) - S'(1)\Bigg) + \beta Q_s\Bigg(-S''(1) - 2S'(1)\Bigg)\Bigg]\widehat{i}^i_t +$$

$$+Q_s\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i^i_{t-1} + \beta Q_s\Bigg(S''(1) + 2S'(1)\Bigg)\widehat{i}^i_{t+1} + \beta S'(1)Q_s\widehat{Q}^i_{t+1} + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

Dividing through by Q^i_s and noting that $Q^i_s = \frac{P^i_s}{P_s}$,

$$\widehat{P}^i_t - \widehat{P}_t = \Bigg(1 - S(1) - S'(1)\Bigg)\widehat{Q}^i_t + \Bigg[\Bigg(-S'(1) - S''(1) - S'(1)\Bigg) + \beta\Bigg(-S''(1) - 2S'(1)\Bigg)\Bigg]\widehat{i}^i_t +$$

$$+\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i^i_{t-1} + \beta\Bigg(S''(1) + 2S'(1)\Bigg)\widehat{i}^i_{t+1} + \beta S'(1)\widehat{Q}^i_{t+1} + \beta S'(1)\big[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\big]$$

Following the above steady state conditions,

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - \Bigg(S''(1) + \beta S''(1)\Bigg)\widehat{i}^i_t + S''(1)i^i_{t-1} + \beta S''(1)\widehat{i}^i_{t+1}$$

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - (1+\beta)S''(1)\widehat{i}^i_t + S''(1)\widehat{i}^i_{t-1} + \beta S''(1)\widehat{i}^i_{t+1}$$

Given that $S''(1) = \varpi$ and solving for i^i_t,

$$\widehat{P}^i_t - \widehat{P}_t = \widehat{Q}^i_t - (1+\beta)\varpi\widehat{i}^i_t + \varpi i^i_{t-1} + \beta\varpi\widehat{i}^i_{t+1}$$

solving for i_t^i,

$$\widehat{i}_t^i = \frac{\widehat{Q}_t^i}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^i}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^i - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^i - \widehat{P}_t\big)$$

Therefore, sectoral investments are given as:

$$\widehat{i}_t^o = \frac{1}{(1+\beta)\varpi}\widehat{Q}_t^o + \frac{\widehat{i}_{t-1}^o}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^o - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^o - \widehat{P}_t\big) + \frac{1}{(1+\beta)\varpi}\widehat{\varepsilon}_t^{INV} \quad \text{(A.135)}$$

$$\widehat{i}_t^u = \frac{\widehat{Q}_t^u}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^u}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^u - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^u - \widehat{P}_t\big) \quad \text{(A.136)}$$

The log-linearised equation for the sectoral capital utilisation cost equations $(A.94)$ and $(A.95)$ give the following:

$$r_s^{k,i}\widehat{r}_t^{k,i} = a''(u_t^i)u_s^i$$

$$\widehat{r}_t^{k,i} = \frac{a''(1)}{a'(1)}\widehat{u}_t^i$$

Given that $\frac{a''(1)}{a'(1)} = \tau$, the sectoral first order conditions for capital utilisation cost is given respectively as:

$$\widehat{r}_t^{k,o} = \tau\widehat{u}_t^o \quad \text{(A.137)}$$

$$\widehat{r}_t^{k,u} = \tau\widehat{u}_t^u \quad \text{(A.138)}$$

The log-linearised equation for both official and shadow sector capital accumulation equations $(A.96)$ and $(A.97)$ are given respectively as:

$$\widehat{\bar{k}}_{t+1}^o = (1-\delta)\widehat{\bar{k}}_t^o + \delta\widehat{i}_t^o + \delta\widehat{\varepsilon}_t^{INV} \quad \text{(A.139)}$$

$$\widehat{\bar{k}}_{t+1}^u = (1-\delta)\widehat{\bar{k}}_t^u + \delta\widehat{i}_t^u \quad \text{(A.140)}$$

whereby in the steady state, $\frac{i_s^i}{k_s^i} = \delta$. The log-linearisation of capital utilisation equations $(A.98)$ and $(A.99)$ give:

$$\widehat{k}_t^o = \widehat{u}_t^o + \widehat{\bar{k}}_{t-1}^o \quad \text{(A.141)}$$

$$\widehat{k}_t^u = \widehat{u}_t^u + \widehat{\bar{k}}_{t-1}^u \quad \text{(A.142)}$$

Official Sector Goods Producers

The first order conditions for the official sector goods producers give the following log-linearised equations in accordance to equations ($A.100$), ($A.101$), ($A.102$) and ($A.103$) as:

$$\widehat{y}_t^o = \widehat{A}_t^o + \alpha^o \widehat{k}_t^o + (1 - \alpha^o)\widehat{l}_t^o \tag{A.143}$$

$$\widehat{w}_t^o = \widehat{A}_t^o + \alpha^o(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.144}$$

$$\widehat{r}_t^{k,o} = \widehat{A}_t^o - (1 - \alpha^o)(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.145}$$

$$\widehat{mc}_t^{I,o} = \alpha^o \widehat{r}_t^{k,o} + (1 - \alpha^o)\widehat{w}_t^o \tag{A.146}$$

Only the first three equations are needed for calibrations.

Shadow Sector Goods Producers

From the informal goods producers output and the first order conditions derived, the log-linearised version of the various equations ($A.104$), ($A.105$), ($A.106$) and ($A.107$) are derived respectively as:

$$\widehat{y}_t^u = \alpha^u \widehat{k}_t^u + (1 - \alpha^u)\widehat{l}_t^u \tag{A.147}$$

$$\widehat{w}_t^u = \alpha^u(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.148}$$

$$\widehat{r}_t^{k,u} = -(1 - \alpha^u)(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.149}$$

$$\widehat{mc}_t^{I,u} = \alpha^u \widehat{r}_t^{k,u} + (1 - \alpha^u)\widehat{w}_t^u \tag{A.150}$$

Only the first three equations are needed for calibrations.

Final Goods Producers

The standard NKPC is derived accordingly from equations ($A.108$) and ($A.109$) for both sectors. Log-linearising the non.linear equations under a zero steady state inflation, the Rotemberg-pricing yield the following generalised NKPC respectively for official and shadow sectors as:

$$\widehat{\pi}_t^o = \beta E_t \widehat{\pi}_{t+1}^o + \frac{\epsilon^o - 1}{\kappa^p}(\widehat{mc}_t^o + \widehat{\varepsilon}_t^p) \tag{A.151}$$

and

$$\widehat{\pi}_t^u = \beta E_t \widehat{\pi}_{t+1}^u + \frac{\epsilon^u - 1}{\kappa^p}\widehat{mc}_t^u \tag{A.152}$$

It emerges from the steady state that the average mark-up is given by the inverse of the real marginal cost as:

$$mc_s^i = \frac{\epsilon^i - 1}{\epsilon^i}$$

The log-linearised version of aggregate and sectoral inflation, equations $(A.110)$, $(A.111)$ and $(A.112)$ are respectively given as:

$$\widehat{\pi}_t + \widehat{P}_{t-1} = \widehat{P}_t \tag{A.153}$$

$$\widehat{\pi}_t^o + \widehat{P}_{t-1}^o = \widehat{P}_t^o \tag{A.154}$$

$$\widehat{\pi}_t^u + \widehat{P}_{t-1}^u = \widehat{P}_t^u \tag{A.155}$$

Monetary Policy

The log-linearisation of the monetary policy instrument $(A.113)$ that is set by the central bank is:

$$\widehat{R}_t = \rho^R \widehat{R}_{t-1} + (1 - \rho^R)(\mu_\pi)\widehat{\pi}_t^o + \widehat{\epsilon}_t^R \tag{A.156}$$

Resource constraints

The log-linearised version of the aggregate resource constraints in both sectors $(A.114)$, $(A.115)$, aggregate consumption $(A.116)$ and the labour resource constraint $(A.117)$, yield the following log-linearised equations:

$$\widehat{y}_t^o = \bar{g}_s \widehat{g}_t + \frac{c_s^o}{y_s^o}\widehat{c}_t^o + \frac{i_s^o}{y_s^o}\widehat{i}_t^o + \frac{r_s^{k,o} k_s^o}{y_s^o}\widehat{u}_t^o \tag{A.157}$$

$$\widehat{y}_t^u = \frac{c_s^u}{y_s^u}\widehat{c}_t^u + \frac{i_s^u}{y_s^u}\widehat{i}_t^u + \frac{r_s^{k,u} k_s^u}{y_s^u}\widehat{u}_t^u \tag{A.158}$$

$$\widehat{c}_t = \varphi_c^{\frac{1}{\epsilon_c}} \left(\frac{c_s^o}{c_s}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}} \widehat{c}_t^o + (1 - \varphi)^{\frac{1}{\epsilon_c}} \left(\frac{c_s^u}{c_s}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}} \widehat{c}_t^u \tag{A.159}$$

$$\widehat{l}_t = \frac{l_s^o}{l_s}\widehat{l}_t^o + \frac{l_s^u}{l_s}\widehat{l}_t^u \tag{A.160}$$

Shock Processes

The log-linearised equations for shocks in the official sector $(A.118)$, $(A.119)$, $(A.120)$, $(A.121)$, $(A.122)$, $(A.123)$, $(A.124)$ and $(A.125)$ that are considered in the model are given as:

Risk premium shock

$$\widehat{\varepsilon}_t^{RISK} = \rho^{RISK}\widehat{\varepsilon}_{t-1}^{RISK} + \widehat{\xi}_t^{RISK} \tag{A.161}$$

Investment shock

$$\widehat{\varepsilon}_t^{INV} = \rho^{INV}\widehat{\varepsilon}_{t-1}^{INV} + \widehat{\xi}_t^{INV} \tag{A.162}$$

Official sector productivity shock

$$\widehat{A}_t^o = \rho^A\widehat{A}_{t-1}^o + \widehat{\xi}_t^A \tag{A.163}$$

Price mark-up shock

$$\widehat{\varepsilon}_t^p = \rho^p\widehat{\varepsilon}_{t-1}^p + \widehat{\xi}_t^p \tag{A.164}$$

Monetary policy shock

$$\widehat{\epsilon}_t^R = \rho^\varepsilon\widehat{\epsilon}_{t-1}^R + \widehat{\xi}_t^\varepsilon \tag{A.165}$$

Government spending shock

$$\widehat{g}_t = \rho^G\widehat{g}_{t-1} + \widehat{\xi}_t^G \tag{A.166}$$

Labour income tax shock

$$\widehat{\tau}_t^W = \rho^W\widehat{\tau}_{t-1}^W + \widehat{\xi}_t^W \tag{A.167}$$

Capital income tax shock

$$\widehat{\tau}_t^K = \rho^K\widehat{\tau}_{t-1}^K + \widehat{\xi}_t^K \tag{A.168}$$

A.3 Appendix to Chapter Three

A.3.1 Symmetric Equilibrium of the Model

Ricardian households

Ricardian official consumption

$$c_t^{o,r} = \varphi_c \left(\frac{P_t^o (1+\tau^c)}{P_t} \right)^{-\epsilon_c} c_t^r \tag{A.169}$$

Ricardian unofficial consumption

$$c_t^{u,r} = (1-\varphi_c) \left(\frac{P_t^u}{P_t} \right)^{-\epsilon_c} c_t^r \tag{A.170}$$

Ricardian consumption index

$$c_t^r = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^{o,r})^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^{u,r})^{\frac{\epsilon_c - 1}{\epsilon_c}} \right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{A.171}$$

Ricardian marginal utility of consumption

$$\lambda_t^r = \frac{1}{c_t^r} \tag{A.172}$$

Ricardian consumption Euler equation

$$\lambda_t^r = \varepsilon_t^{RISK} R_t \beta E_t \frac{\lambda_{t+1}^r}{\pi_{t+1}} \tag{A.173}$$

Ricardian labour supplied to official sector

$$(1-\tau_t^w) \frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o,r(\phi)}}{\lambda_t^r} \tag{A.174}$$

Ricardian labour supplied to unofficial sector

$$\frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u,r(\phi)}}{\lambda_t^r} \tag{A.175}$$

Labour market arbitrage condition

$$\frac{P_t^o}{P_t} w_t^o (1-\tau_t^w) = \frac{P_t^u}{P_t} w_t^u \tag{A.176}$$

Ricardian official sector capital

$$Q_t^o = \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r} \left[\frac{P_{t+1}^o}{P_{t+1}} \left((1 - \tau_{t+1}^k) \left[r_{t+1}^{k,o} u_{t+1}^o - a(u_{t+1}^o) \right] + \tau_{t+1}^k \delta \right) + Q_{t+1}^o (1 - \delta) \right] \tag{A.177}$$

Ricardian unofficial sector capital

$$Q_t^u = \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r} \left[\frac{P_{t+1}^u}{P_{t+1}} \left[r_{t+1}^{k,u} u_{t+1}^u - a(u_{t+1}^u) \right] + Q_{t+1}^u (1 - \delta) \right] \tag{A.178}$$

Ricardian official sector investment

$$\frac{P_t^o}{P_t} = Q_t^o \varepsilon_t^{INV} \left(1 - S\left(\frac{i_t^{o,r}}{i_{t-1}^{o,r}} \right) - S'\left(\frac{i_t^{o,r}}{i_{t-1}^{o,r}} \right) \frac{i_t^{o,r}}{i_{t-1}^{o,r}} \right) + \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r} Q_{t+1}^o \varepsilon_{t+1}^{INV} S'\left(\frac{i_{t+1}^{o,r}}{i_t^{o,r}} \right) \left(\frac{i_{t+1}^{o,r}}{i_t^{o,r}} \right)^2 \tag{A.179}$$

Ricardian unofficial sector investment

$$\frac{P_t^u}{P_t} = Q_t^u \left(1 - S\left(\frac{i_t^{u,r}}{i_{t-1}^{u,r}} \right) - S'\left(\frac{i_t^{u,r}}{i_{t-1}^{u,r}} \right) \frac{i_t^{u,r}}{i_{t-1}^{u,r}} \right) + \beta E_t \frac{\lambda_{t+1}^r}{\lambda_t^r} Q_{t+1}^u S'\left(\frac{i_{t+1}^{u,r}}{i_t^{u,r}} \right) \left(\frac{i_{t+1}^{u,r}}{i_t^{u,r}} \right)^2 \tag{A.180}$$

Ricardian official sector capital returns

$$r_t^{k,o} = a'(u_t^o) \tag{A.181}$$

Ricardian unofficial sector capital returns

$$r_t^{k,u} = a'(u_t^u) \tag{A.182}$$

Ricardian official sector capital accumulation

$$\bar{k}_{t+1}^{o,r} = (1 - \delta)\bar{k}_t^{o,r} + \varepsilon_t^{INV} \left[1 - S\left(\frac{i_t^{o,r}}{i_{t-1}^{o,r}} \right) \right] i_t^{o,r} \tag{A.183}$$

Ricardian unofficial sector capital accumulation

$$\bar{k}_{t+1}^{u,r} = (1 - \delta)\bar{k}_t^{u,r} + \left[1 - S\left(\frac{i_t^{u,r}}{i_{t-1}^{u,r}} \right) \right] i_t^{u,r} \tag{A.184}$$

Ricardian official sector capital utilisation

$$k_t^{o,r} = u_t^i \bar{k}_{t-1}^{o,r} \tag{A.185}$$

Ricardian unofficial sector capital utilisation

$$k_t^{u,r} = u_t^i \bar{k}_{t-1}^{u,r} \tag{A.186}$$

Non-Ricardian households

Non-Ricardian official consumption

$$c_t^{o,rt} = \varphi_c \left(\frac{P_t^o (1+\tau^c)}{P_t} \right)^{-\epsilon_c} c_t^{rt} \tag{A.187}$$

Non-Ricardian unofficial consumption

$$c_t^{u,rt} = (1-\varphi_c) \left(\frac{P_t^u}{P_t} \right)^{-\epsilon_c} c_t^{rt} \tag{A.188}$$

Non-Ricardian consumption index

$$c_t^{rt} = \left[\varphi_c^{\frac{1}{\epsilon_c}} (c_t^{o,rt})^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} (c_t^{u,rt})^{\frac{\epsilon_c - 1}{\epsilon_c}} \right]^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{A.189}$$

Non-Ricardian consumption marginal utility

$$\lambda_t^{rt} = \frac{1}{c_t^{rt}} \tag{A.190}$$

Non-Ricardian official labour supplied

$$(1-\tau_t^w) \frac{P_t^o}{P_t} w_t^o = \frac{\chi l_t^{o,rt(\phi)}}{\lambda_t^{rt}} \tag{A.191}$$

Non-Ricardian unofficial labour supplied

$$\frac{P_t^u}{P_t} w_t^u = \frac{\chi l_t^{u,rt(\phi)}}{\lambda_t^{rt}} \tag{A.192}$$

Non-Ricardian budget constraint

$$c_t^{rt} = \frac{P_t^o}{P_t} (1-\tau_t^w) w_t^o l_t^{o,rt} + \frac{P_t^u}{P_t} w_t^u l_t^{u,rt} + \frac{P_t^o}{P_t} TR_t^{rt} \tag{A.193}$$

Consumption price index

$$P_t = \left[\varphi_c \big(P_t^o (1+\tau^c) \big)^{1-\epsilon_c} + (1-\varphi_c) \big(P_t^u \big)^{1-\epsilon_c} \right]^{\frac{1}{1-\epsilon_c}} \tag{A.194}$$

Official Sector Goods Producers

Official sector output

$$y_t^o = A_t^o k_t^{o(\alpha^o)} l_t^{o(1-\alpha^o)} \tag{A.195}$$

Official sector labour demand

$$w_t^o = (1-\alpha^o) A_t^o \left(\frac{k_t^o}{l_t^o}\right)^{\alpha^o} \tag{A.196}$$

official sector capital demand

$$r_t^{k,o} = \alpha^o A_t^o \left(\frac{k_t^o}{l_t^o}\right)^{-(1-\alpha^o)} \tag{A.197}$$

Official sector marginal cost

$$mc_t^{I,o} = \left(\frac{r_t^{k,o}}{\alpha^o}\right)^{\alpha^o} \left(\frac{w_t^o}{1-\alpha^o}\right)^{1-\alpha^o} \tag{A.198}$$

Shadow Sector Goods Producers

Shadow sector output

$$y_t^u = k_t^{u(\alpha^u)} l_t^{u(1-\alpha^u)} \tag{A.199}$$

Unofficial sector labour demand

$$w_t^u = (1-\alpha^u) \left(\frac{k_t^u}{l_t^u}\right)^{\alpha^u} \tag{A.200}$$

Unofficial sector capital demand

$$r_t^{k,u} = \alpha^u \left(\frac{k_t^u}{l_t^u}\right)^{-(1-\alpha^u)} \tag{A.201}$$

Unofficial sector marginal cost

$$mc_t^{I,u} = \left(\frac{r_t^{k,u}}{\alpha^u}\right)^{\alpha^u} \left(\frac{w_t^u}{1-\alpha^u}\right)^{1-\alpha^u} \tag{A.202}$$

Final Goods Producers

Official sector NKPC

$$(1-mc_t^o)\epsilon_t^o = 1 - \kappa^p \left(\frac{\pi_t^o}{\pi_{t-1}^{o}{}^{\theta_\pi}} - 1\right) \frac{\pi_t^o}{\pi_{t-1}^{o}{}^{\theta_\pi}} + \kappa^p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\left(\frac{\pi_{t+1}^o}{\pi_t^{o\theta_\pi}} - 1\right) \frac{\pi_{t+1}^o}{\pi_t^{o\theta_\pi}} \frac{y_{t+1}^o}{y_t^o}\right] \tag{A.203}$$

Unofficial sector NKPC

$$(1-mc_t^u)\epsilon^u = 1-\kappa^p\left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}-1\right)\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}}+\kappa^p\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[\left(\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}-1\right)\frac{\pi_{t+1}^u}{{\pi_t^u}^{\theta_\pi}}\frac{y_{t+1}^u}{y_t^u}\right] \quad (A.204)$$

Aggregate inflation

$$P_t = \pi_t P_{t-1} \quad (A.205)$$

Official sector inflation

$$P_t^o = \pi_t^o P_{t-1}^o \quad (A.206)$$

Unofficial sector inflation

$$P_t^u = \pi_t^u P_{t-1}^u \quad (A.207)$$

Government Policy

Government budget constraint

$$g_t + \frac{B_{t-1}}{P_t^o} + TR_t = \tau_t^w w_t^o l_t^o + \tau_t^k\left[r_t^{k,o}u_t^o - a(u_t^o)-\delta\right]\bar{k}_t^o + \tau^c c_t^o + \frac{B_t}{P_t^o R_t} + T_t \quad (A.208)$$

Taylor's rule

$$R_t = R_{t-1}^{(\rho^R)}(\pi_t^o)^{\mu_\pi(1-\rho^R)}\varepsilon_t^R \quad (A.209)$$

Aggregation

Aggregate official consumption

$$c_t^o = (1-\theta)c_t^{o,r} + \theta c_t^{o,rt} \quad (A.210)$$

Aggregate unofficial consumption

$$c_t^u = (1-\theta)c_t^{u,r} + \theta c_t^{u,rt} \quad (A.211)$$

Aggregate consumption

$$c_t = (1-\theta)c_t^r + \theta c_t^{rt} \quad (A.212)$$

Aggregate official labour supplied

$$l_t^o = (1-\theta)l_t^{o,r} + \theta l_t^{o,rt} \quad (A.213)$$

Aggregate unofficial labour supplied

$$l_t^u = (1-\theta)l_t^{u,r} + \theta l_t^{u,rt} \quad (A.214)$$

Ricardian labour supplied

$$l_t^r = l_t^{o,r} + l_t^{u,r} \tag{A.215}$$

Non-Ricardian labour supplied

$$l_t^{rt} = l_t^{o,rt} + l_t^{u,rt} \tag{A.216}$$

Aggregate labour

$$l_t = (1-\theta)l_t^r + \theta l_t^{rt} \tag{A.217}$$

Aggregate official investment

$$i_t^o = (1-\theta)i_t^{o,r} \tag{A.218}$$

Aggregate unofficial investment

$$i_t^u = (1-\theta)i_t^{u,r} \tag{A.219}$$

Aggregate official capital

$$k_t^o = (1-\theta)k_t^{o,r} \tag{A.220}$$

Aggregate unofficial capital

$$k_t^u = (1-\theta)k_t^{u,r} \tag{A.221}$$

Aggregate transfers

$$TR_t = (1-\theta)TR_t^r + \theta TR_t^{rt} \tag{A.222}$$

Market Clearing and Resource Constraint

Official sector resource

$$y_t^o = c_t^o + i_t^o + g_t + a(u_t^o)\bar{k}_{t-1}^o + \frac{\kappa^p}{2}\left(\frac{\pi_t^o}{{\pi_{t-1}^o}^{\theta_\pi}} - 1\right)^2 y_t^o \tag{A.223}$$

Shadow sector resource

$$y_t^u = c_t^u + i_t^u + a(u_t^u)\bar{k}_{t-1}^u + \frac{\kappa^p}{2}\left(\frac{\pi_t^u}{{\pi_{t-1}^u}^{\theta_\pi}} - 1\right)^2 y_t^u \tag{A.224}$$

Shock Processes

Risk premium shock

$$ln\varepsilon_t^{RISK} = \rho^{RISK} ln\varepsilon_{t-1}^{RISK} + \xi_t^{RISK} \tag{A.225}$$

Investment shock

$$ln\varepsilon_t^{INV} = \rho^{INV} ln\varepsilon_{t-1}^{INV} + \xi_t^{INV} \tag{A.226}$$

Official sector productivity shock

$$lnA_t^o = \rho^A lnA_{t-1}^o + \xi_t^A \tag{A.227}$$

Price mark-up

$$ln\varepsilon_t^p = \rho^p ln\varepsilon_{t-1}^p + \xi_t^p \tag{A.228}$$

Monetary policy shock

$$ln\varepsilon_t^R = \rho^\varepsilon ln\varepsilon_{t-1}^R + \xi_t^\varepsilon \tag{A.229}$$

Government spending shock

$$lng_t = \rho^G lng_{t-1} + \xi_t^G \tag{A.230}$$

Labour income tax shock

$$ln\tau_t^W = \rho^W ln\tau_{t-1}^W + \xi_t^W \tag{A.231}$$

Capital income tax shock

$$ln\tau_t^K = \rho^K ln\tau_{t-1}^K + \xi_t^K \tag{A.232}$$

A.3.2 Log-Linearised Model

The log-linearised relations are derived in accordance with the non-linear equilibrium relationships where a variable with "*hat*"represent the log-deviations of that variable around its steady state.

Ricardian households

The households consumption demand in both sectors, equations $(A.169)$, $(A.170)$ and aggregate consumption index $(A.171)$ give the following log-linearised equations:

$$\widehat{c}_t^{o,r} = \widehat{c}_t^r - \epsilon_c(\widehat{P}_t^o(1+\tau^c) - \widehat{P}_t) \tag{A.233}$$

$$\widehat{c}_t^{u,r} = \widehat{c}_t^r - \epsilon_c(\widehat{P}_t^u - \widehat{P}_t) \tag{A.234}$$

$$\widehat{c}_t^r = \varphi_c^{\frac{1}{\epsilon_c}} \left(\frac{c_s^{o,r}}{c_s^r}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}} \widehat{c}_t^{o,r} + (1-\varphi_c)^{\frac{1}{\epsilon_c}} \left(\frac{c_s^{u,r}}{c_s^r}\right)^{\frac{\epsilon_c - 1}{\epsilon_c}} \widehat{c}_t^{u,r} \tag{A.235}$$

From the first order conditions of the households maximization problem for consumption $(A.172)$, government bond $(A.173)$ and labour $(A.174)$ we solve by substitution to obtain the following equations:

The consumption Euler equation is obtained by solving equations $(A.172)$ and $(A.173)$ for c_t as:

$$\widehat{c}_t^r = \widehat{c}_{t+1}^r + E_t\widehat{\pi}_{t+1} - \widehat{R}_t - \widehat{\varepsilon}_t^{RISK} \tag{A.236}$$

The equilibrium labour supplied is also obtained by substituting equation $(A.172)$ into equation $(A.174)$ and $(A.175)$ for real wage rate (w_t) as:

$$\widehat{P}_t^o - \widehat{P}_t + \widehat{w}_t^o - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\widehat{\tau}_t^w = \phi\widehat{l}_t^{o,r} + \widehat{c}_t^r \tag{A.237}$$

$$\widehat{P}_t^u - \widehat{P}_t + \widehat{w}_t^u = \phi\widehat{l}_t^{u,r} + \widehat{c}_t^r \tag{A.238}$$

where real wage is equal to the marginal rate of substitution between total labour supplied and consumption. The arbitrage condition in the labour market equation $(A.176)$ ensures that both sectors pay the same level of real wage given as:

$$\widehat{P}_t^o + \widehat{w}_t^o - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\widehat{\tau}_t^w = \widehat{P}_t^u + \widehat{w}_t^u \tag{A.239}$$

From the official sector capital supplied $(A.177)$ the log-linearised version is derived as follows:

$$Q_s^o\widehat{Q}_t^o = \beta\left[\frac{P_s^o}{P_s}\left((1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\right) + Q_s^o(1-\delta)\right](\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r) +$$

$$+\beta\frac{P_s^o}{P_s}\big[(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) - \beta\frac{P_s^o}{P_s}\tau_s^k r_s^{k,o}u_s^o(\widehat{\tau}_{t+1}^k) +$$

$$+\beta\frac{P_s^o}{P_s}(1-\tau_s^k)r_s^{k,o}u_s^o\widehat{r}_{t+1}^{k,o} + \beta\frac{P_s^o}{P_s}(1-\tau_s^k)\big[r_s^{k,o} - a'(u_s^o)\big]u_s^o\widehat{u}_{t+1}^o + \beta\frac{P_s^o}{P_s}\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)Q_s^o\widehat{Q}_{t+1}^o$$

Dividing through by Q_s^i and noting from the sectoral investment equations $(A.90)$ and $(A.91)$, it holds in the steady state that $Q_s^i = \frac{P_s^i}{P_s}$,

$$\widehat{Q}_t^o = \beta\{(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta + (1-\delta)\}(\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r) + \beta\big[(1-\tau_s^k)\big(r_s^{k,o}u_s^o - a(u_s^o)\big) + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) +$$

$$-\beta\tau_s^k r_s^{k,o}u_s^o(\widehat{\tau}_{t+1}^k) + \beta(1-\tau_s^k)r_s^{k,o}u_s^o\widehat{r}_{t+1}^{k,o} + \beta(1-\tau_s^k)\big[r_s^{k,o} - a'(u_s^o)\big]u_s^o\widehat{u}_{t+1}^o + \beta\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o$$

In the steady state, the following conditions must hold which simplifies the derivation of log-linearised sectoral capital and investments. The sectoral capital utilisation equation in the steady state is:

$$u_s^i = 1$$

And we also identify the following properties from the capital adjustment cost function: $S(.) = 0$, $S'(.) = 0$, $S''(.) = \varpi$, $a(.) = 0$, $a'(u_s^i) = r_s^{k,i}$ and $\frac{a''(.)}{a'(.)} = \frac{a''(1)}{a'(1)} = \tau$. Given the above conditions, we can continue as:

$$\widehat{Q}_t^o = \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)\big](\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r) + \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta\big]\big(\widehat{P}_{t+1}^o - \widehat{P}_{t+1}\big) +$$

$$-\beta\tau_s^k r_s^{k,o}\widehat{\tau}_{t+1}^k + \beta(1-\tau_s^k)r_s^{k,o}\widehat{r}_{t+1}^{k,o} + \beta\tau_s^k\delta\widehat{\tau}_{t+1}^k + \beta(1-\delta)\widehat{Q}_{t+1}^o$$

From the steady state capital equation, it holds that,

$$1 = \beta[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)]$$

$$\frac{1}{\beta} = (1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta + (1-\delta)$$

$$1 - \beta(1-\delta) = \beta\big[(1-\tau_s^k)r_s^{k,o} + \tau_s^k\delta\big]$$

$$1 - \beta\tau_s^k\delta - \beta(1-\delta) = \beta(1-\tau_s^k)r_s^{k,o}$$

$$r_s^{k,o} = \frac{1}{(1-\tau_s^k)}\left[\frac{1}{\beta} - \tau_s^k\delta - (1-\delta)\right]$$

Therefore,

$$\widehat{Q}^o_t = \widehat{\lambda}^r_{t+1} - \widehat{\lambda}^r_t + [1-\beta(1-\delta)]\big(\widehat{P}^o_{t+1} - \widehat{P}_{t+1}\big) + \beta(1-\tau^k_s) r^{k,o}_s r^{k,o}_{t+1} + \beta(\delta - r^{k,o}_s)\tau^k_s \widehat{\tau}^k_{t+1} + \beta(1-\delta)\widehat{Q}^o_{t+1}$$

Substituting the log-linearised marginal utility of consumption equation $\widehat{\lambda}^r_t = -\widehat{c}^r_t$, we obtain Log-linearised Euler equation for capital supplied to the official sector as:

$$\widehat{Q}^o_t = \widehat{c}^r_t - \widehat{c}^r_{t+1} + [1-\beta(1-\delta)]\big(\widehat{P}^o_{t+1} - \widehat{P}_{t+1}\big) + \beta(1-\tau^k_s) r^{k,o}_s \widehat{r}^{k,o}_{t+1} + \beta(\delta - r^{k,o}_s)\tau^k_s \widehat{\tau}^k_{t+1} + \beta(1-\delta)\widehat{Q}^o_{t+1} \tag{A.240}$$

From unofficial sector capital supplied ($A.178$), the log-linearised version is derived as follows:

$$Q^u_s \widehat{Q}^u_t = \left(\beta \frac{P^u_s}{P_s}\big(r^{k,u}_s u^u_s - a(u^u_s)\big) + Q^u_s(1-\delta)\right)\big(\widehat{\lambda}^r_{t+1} - \widehat{\lambda}^r_t\big) + \beta \frac{P^u_s}{P_s}\left(r^{k,u}_s u^u_s - a(u^u_s)\right)\big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1}\big) +$$

$$+\beta \frac{P^u_s}{P_s} r^{k,u}_s u^u_s \widehat{r}^{k,u}_{t+1} + \beta \frac{P^u_s}{P_s}\big[r^{k,u}_s - a'(u^u_s)\big] u^u_s \widehat{u}^u_{t+1} + \beta(1-\delta) Q^u_s \widehat{Q}^u_{t+1}$$

Dividing through by Q^u_s and noting that $Q^u_s = \frac{P^u_s}{P_s}$,

$$\widehat{Q}^u_t = \{\beta\big(r^{k,u}_s u^u_s - a(u^u_s)\big) + (1-\delta)\}\big(\widehat{\lambda}^r_{t+1} - \widehat{\lambda}^r_t\big) + \beta\big(r^{k,u}_s u^u_s - a(u^u_s)\big)\big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1}\big) +$$

$$+\beta r^{k,i}_s u^u_s \widehat{r}^{k,u}_{t+1} + \beta\big[r^{k,u}_s - a'(u^u_s)\big] u^u_s \widehat{u}^u_{t+1} + \beta(1-\delta)\widehat{Q}^u_{t+1}$$

In the steady state, the conditions and properties about capital utilisation equation and the capital adjustment cost function also hold in the shadow sector. This simplifies the derivation of log-linearised shadow capital as:

$$\widehat{Q}^u_t = \beta\big(r^{k,u}_s + (1-\delta)\big)\big(\widehat{\lambda}^r_{t+1} - \widehat{\lambda}^r_t\big) + \beta r^{k,u}_s \big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1}\big) + \beta r^{k,u}_s \widehat{r}^{k,u}_{t+1} + \beta(1-\delta)\widehat{Q}^u_{t+1}$$

From the steady state capital equation, it also holds that,

$$1 = \beta[r^{k,u}_s + (1-\delta)]$$

$$\frac{1}{\beta} = r^{k,u}_s + (1-\delta)$$

$$r^{k,u}_s = \frac{1}{\beta} - (1-\delta)$$

Therefore,

$$\widehat{Q}^u_t = \widehat{\lambda}^r_{t+1} - \widehat{\lambda}^r_t + [1-\beta(1-\delta)]\big(\widehat{P}^u_{t+1} - \widehat{P}_{t+1}\big) + [1-\beta(1-\delta)] r^{k,u}_{t+1} + \beta(1-\delta)\widehat{Q}^u_{t+1}$$

Substituting the log-linearised marginal utility of consumption equation $\widehat{\lambda}_t^r = -\widehat{c}_t^r$, we obtain Log-linearised Euler equation for capital supplied to the unofficial sector as:

$$\widehat{Q}_t^u = \widehat{c}_t^r - \widehat{c}_{t+1}^r + [1-\beta(1-\delta)]\big(\widehat{P}_{t+1}^u - \widehat{P}_{t+1} + \widehat{r}_{t+1}^{k,u}\big) + \beta(1-\delta)\widehat{Q}_{t+1}^u \tag{A.241}$$

Following the first order conditions for sectoral investment equations $(A.179)$ and $(A.180)$, we obtain the following log-linear equations:

$$\frac{P_s^i}{P_s}\big(\widehat{P}_t^i - \widehat{P}_t\big) = \Bigg[Q_s\Bigg(-S'(1)\frac{1}{i_s^{i,r}} - S''(1)\frac{1}{i_s^{i,r}} - S'(1)\frac{1}{i_s^{i,r}}\Bigg) + \beta Q_s\Bigg(S''(1)\Bigg(-\frac{i_s^{i,r}}{i_s^{i,r2}}\Bigg) + 2S'(1)\Bigg(-\frac{i_s^{i,r}}{i_s^{i,r2}}\Bigg)\Bigg)\Bigg] i_s^{i,r}\widehat{i}_t^{i,r} +$$

$$+Q_s\Bigg[-S'(1)\Bigg(-\frac{i_s^{i,r}}{i_s^{i,r2}}\Bigg) - S''(1)\Bigg(-\frac{i_s^{i,r}}{i_s^{i,r2}}\Bigg) - S'(1)\Bigg(-\frac{i_s^{i,r}}{i_s^{i,r2}}\Bigg)\Bigg] i_s^{i,r} i_{t-1}^{i,r} + \beta Q_s\Bigg(S''(1)\frac{1}{i_s^{i,r}} + 2S'(1)\frac{1}{i_s^{i,r}}\Bigg) i_s^{i,r}\widehat{i}_{t+1}^{i,r} +$$

$$+\Bigg(1 - S(1) - S'(1)\Bigg)Q_s\widehat{Q}_t^i + \beta S'(1)Q_s\widehat{Q}_{t+1}^i + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r\big]$$

$$\frac{P_s^i}{P_s}\big(\widehat{P}_t^i - \widehat{P}_t\big) = \Bigg(1 - S(1) - S'(1)\Bigg)Q_s\widehat{Q}_t^i + \Bigg[Q_s\Bigg(-S'(1) - S''(1) - S'(1)\Bigg) + \beta Q_s\Bigg(-S''(1) - 2S'(1)\Bigg)\Bigg]\widehat{i}_t^{i,r} +$$

$$+Q_s\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i_{t-1}^{i,r} + \beta Q_s\Bigg(S''(1) + 2S'(1)\Bigg)\widehat{i}_{t+1}^{i,r} + \beta S'(1)Q_s\widehat{Q}_{t+1}^i + \beta Q_s S'(1)\big[\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r\big]$$

Dividing through by Q_s^i and noting that $Q_s^i = \frac{P_s^i}{P_s}$,

$$\widehat{P}_t^i - \widehat{P}_t = \Bigg(1 - S(1) - S'(1)\Bigg)\widehat{Q}_t^i + \Bigg[\Bigg(-S'(1) - S''(1) - S'(1)\Bigg) + \beta\Bigg(-S''(1) - 2S'(1)\Bigg)\Bigg]\widehat{i}_t^{i,r} +$$

$$+\Bigg[S'(1) + S''(1) + S'(1)\Bigg] i_{t-1}^{i,r} + \beta\Bigg(S''(1) + 2S'(1)\Bigg)\widehat{i}_{t+1}^{i,r} + \beta S'(1)\widehat{Q}_{t+1}^i + \beta S'(1)\big[\widehat{\lambda}_{t+1}^r - \widehat{\lambda}_t^r\big]$$

Following the above steady state conditions,

$$\widehat{P}_t^i - \widehat{P}_t = \widehat{Q}_t^i - \Bigg(S''(1) + \beta S''(1)\Bigg)\widehat{i}_t^{i,r} + S''(1) i_{t-1}^{i,r} + \beta S''(1)\widehat{i}_{t+1}^{i,r}$$

$$\widehat{P}_t^i - \widehat{P}_t = \widehat{Q}_t^i - (1+\beta)S''(1)\widehat{i}_t^{i,r} + S''(1)\widehat{i}_{t-1}^{i,r} + \beta S''(1)\widehat{i}_{t+1}^{i,r}$$

Given that $S''(1) = \varpi$ and solving for $i_t^{i,r}$,

$$\widehat{P}_t^i - \widehat{P}_t = \widehat{Q}_t^i - (1+\beta)\varpi\widehat{i}_t^{i,r} + \varpi i_{t-1}^{i,r} + \beta\varpi\widehat{i}_{t+1}^{i,r}$$

solving for $i_t^{i,r}$,

$$\widehat{i}_t^{i,r} = \frac{\widehat{Q}_t^i}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^{i,r}}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^{i,r} - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^i - \widehat{P}_t\big)$$

Therefore, sectoral investments are given as:

$$\widehat{i}_t^{o,r} = \frac{1}{(1+\beta)\varpi}\widehat{Q}_t^o + \frac{\widehat{i}_{t-1}^{o,r}}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^{o,r} - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^o - \widehat{P}_t\big) + \frac{1}{(1+\beta)\varpi}\widehat{\varepsilon}_t^{INV} \quad \text{(A.242)}$$

$$\widehat{i}_t^{u,r} = \frac{\widehat{Q}_t^u}{(1+\beta)\varpi} + \frac{\widehat{i}_{t-1}^{u,r}}{(1+\beta)} + \frac{\beta}{(1+\beta)}\widehat{i}_{t+1}^{u,r} - \frac{1}{(1+\beta)\varpi}\big(\widehat{P}_t^u - \widehat{P}_t\big) \quad \text{(A.243)}$$

The log-linearised equation for the sectoral capital utilisation cost equations $(A.181)$ and $(A.182)$ give the following:

$$r_s^{k,i}\widehat{r}_t^{k,i} = a''(u_t^i)u_s^i$$

$$\widehat{r}_t^{k,i} = \frac{a''(1)}{a'(1)}\widehat{u}_t^i$$

Given that $\frac{a''(1)}{a'(1)} = \tau$, the sectoral first order conditions for capital utilisation cost is given respectively as:

$$\widehat{r}_t^{k,o} = \tau\widehat{u}_t^o \quad \text{(A.244)}$$

$$\widehat{r}_t^{k,u} = \tau\widehat{u}_t^u \quad \text{(A.245)}$$

The log-linearised equation for both official and shadow sector capital accumulation equations $(A.183)$ and $(A.184)$ are given respectively as:

$$\widehat{\bar{k}}_{t+1}^{o,r} = (1-\delta)\widehat{\bar{k}}_t^{o,r} + \delta\widehat{i}_t^{o,r} + \delta\widehat{\varepsilon}_t^{INV} \quad \text{(A.246)}$$

$$\widehat{\bar{k}}_{t+1}^{u,r} = (1-\delta)\widehat{\bar{k}}_t^{u,r} + \delta\widehat{i}_t^{u,r} \quad \text{(A.247)}$$

whereby in the steady state, $\frac{i_s^{i,r}}{k_s^{i,r}} = \delta$. The log-linearisation of capital utilisation equations $(A.185)$ and $(A.186)$ give:

$$\widehat{k}_t^{o,r} = \widehat{u}_t^o + \widehat{\bar{k}}_{t-1}^{o,r} \quad \text{(A.248)}$$

$$\widehat{k}_t^{u,r} = \widehat{u}_t^u + \widehat{\bar{k}}_{t-1}^{u,r} \quad \text{(A.249)}$$

Non-Ricardian households

The non-Ricardian households consumption demand in both sectors, equations ($A.187$), ($A.188$) and aggregate consumption index ($A.189$) give the following log-linearised equations:

$$\widehat{c}_t^{o,rt} = \widehat{c}_t^{rt} - \epsilon_c(\widehat{P}_t^o(1+\tau^c) - \widehat{P}_t) \tag{A.250}$$

$$\widehat{c}_t^{u,rt} = \widehat{c}_t^{rt} - \epsilon_c(\widehat{P}_t^u - \widehat{P}_t) \tag{A.251}$$

$$\widehat{c}_t^{rt} = \varphi_c^{\frac{1}{\epsilon_c}}\left(\frac{c_s^{o,rt}}{c_s^{rt}}\right)^{\frac{\epsilon_c-1}{\epsilon_c}}\widehat{c}_t^{o,rt} + (1-\varphi_c)^{\frac{1}{\epsilon_c}}\left(\frac{c_s^{u,rt}}{c_s^{rt}}\right)^{\frac{\epsilon_c-1}{\epsilon_c}}\widehat{c}_t^{u,rt} \tag{A.252}$$

The equilibrium labour supplied is also obtained by substituting equation ($A.190$) into equation ($A.191$) and ($A.192$) for real wage rate (w_t) as:

$$\widehat{P}_t^o - \widehat{P}_t + \widehat{w}_t^o - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\widehat{\tau}_t^w = \phi\widehat{l}_t^{o,rt} + \widehat{c}_t^{rt} \tag{A.253}$$

$$\widehat{P}_t^u - \widehat{P}_t + \widehat{w}_t^u = \phi\widehat{l}_t^{u,rt} + \widehat{c}_t^{rt} \tag{A.254}$$

where real wage is equal to the marginal rate of substitution between total labour supplied and consumption. The log-linearised non-Ricardian budget constraint ($A.193$) gives:

$$\widehat{c}_t^{rt} = (1-\tau_s^w)\left(\frac{w_s^o l_s^{o,rt}}{c_s^{rt}}\right)(\widehat{w}_t^o + \widehat{l}_t^{o,rt}) - \left(\frac{\tau_s^w}{1-\tau_s^w}\right)\left(\frac{w_s^o l_s^{o,rt}}{c_s^{rt}}\right)\widehat{\tau}_t^w + \left(\frac{w_s^u l_s^{u,rt}}{c_s^{rt}}\right)(\widehat{w}_t^u + \widehat{l}_t^{u,rt}) \tag{A.255}$$

The log-linearised aggregate consumption price index, equation ($A.194$) gives:

$$\widehat{P}_t = \varphi_c\left(\frac{P_s^o}{P_s}\right)^{1-\epsilon_c}(1+\tau^c)^{1-\epsilon_c}\widehat{P}_t^o + (1-\varphi_c)\left(\frac{P_s^u}{P_s}\right)^{1-\epsilon_c}\widehat{P}_t^u \tag{A.256}$$

Official Sector Goods Producers

The first order conditions for the official sector goods producers give the following log-linearised equations in accordance to equations ($A.195$), ($A.196$), ($A.197$) and ($A.198$) as:

$$\widehat{y}_t^o = \widehat{A}_t^o + \alpha^o\widehat{k}_t^o + (1-\alpha^o)\widehat{l}_t^o \tag{A.257}$$

$$\widehat{w}_t^o = \widehat{A}_t^o + \alpha^o(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.258}$$

$$\widehat{r}_t^{k,o} = \widehat{A}_t^o - (1-\alpha^o)(\widehat{k}_t^o - \widehat{l}_t^o) \tag{A.259}$$

$$\widehat{mc}_t^{I,o} = \alpha^o \widehat{r}_t^{k,o} + (1-\alpha^o)\widehat{w}_t^o \tag{A.260}$$

Only the first three equations are needed for calibrations.

Shadow Sector Goods Producers

From the informal goods producers output and the first order conditions derived, the log-linearised version of the various equations ($A.199$), ($A.200$), ($A.201$) and ($A.202$) are derived respectively as:

$$\widehat{y}_t^u = \alpha^u \widehat{k}_t^u + (1-\alpha^u)\widehat{l}_t^u \tag{A.261}$$

$$\widehat{w}_t^u = \alpha^u(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.262}$$

$$\widehat{r}_t^{k,u} = -(1-\alpha^u)(\widehat{k}_t^u - \widehat{l}_t^u) \tag{A.263}$$

$$\widehat{mc}_t^{I,u} = \alpha^u \widehat{r}_t^{k,u} + (1-\alpha^u)\widehat{w}_t^u \tag{A.264}$$

Only the first three equations are needed for calibrations.

Final Goods Producers

The standard NKPC is derived accordingly from equations ($A.203$) and ($A.204$) for both sectors. Log-linearising the non.linear equations under a zero steady state inflation, the Rotemberg-pricing yield the following generalised NKPC respectively for official and shadow sectors as:

$$\widehat{\pi}_t^o = \beta E_t \widehat{\pi}_{t+1}^o + \frac{\epsilon^o - 1}{\kappa^p}(\widehat{mc}_t^o + \widehat{\varepsilon}_t^p) \tag{A.265}$$

and

$$\widehat{\pi}_t^u = \beta E_t \widehat{\pi}_{t+1}^u + \frac{\epsilon^u - 1}{\kappa^p}\widehat{mc}_t^u \tag{A.266}$$

It emerges from the steady state that the average mark-up is given by the inverse of the real marginal cost as:

$$mc_s^i = \frac{\epsilon^i - 1}{\epsilon^i}$$

The log-linearised version of aggregate and sectoral inflation, equations ($A.205$), ($A.206$) and ($A.207$) are respectively given as:

$$\widehat{\pi}_t + \widehat{P}_{t-1} = \widehat{P}_t \tag{A.267}$$

$$\widehat{\pi}_t^o + \widehat{P}_{t-1}^o = \widehat{P}_t^o \tag{A.268}$$

$$\widehat{\pi}_t^u + \widehat{P}_{t-1}^u = \widehat{P}_t^u \tag{A.269}$$

Monetary Policy

The log-linearisation of the monetary policy instrument ($A.209$) that is set by the central bank is:

$$\widehat{R}_t = \rho^R \widehat{R}_{t-1} + (1-\rho^R)(\mu_\pi)\widehat{\pi}_t^o + \widehat{\epsilon}_t^R \tag{A.270}$$

Aggregation

The log-linearised equations for the aggregate equations ($A.210$) to ($A.222$) are given respectively as follows:

$$\widehat{c}_t^o = (1-\theta)\widehat{c}_t^{o,r} + \theta\widehat{c}_t^{o,rt} \tag{A.271}$$

$$\widehat{c}_t^u = (1-\theta)\widehat{c}_t^{u,r} + \theta\widehat{c}_t^{u,rt} \tag{A.272}$$

$$\widehat{c}_t = (1-\theta)\widehat{c}_t^{r} + \theta\widehat{c}_t^{rt} \tag{A.273}$$

$$\widehat{l}_t^o = (1-\theta)\widehat{l}_t^{o,r} + \theta\widehat{l}_t^{o,rt} \tag{A.274}$$

$$\widehat{l}_t^u = (1-\theta)\widehat{l}_t^{u,r} + \theta\widehat{l}_t^{u,rt} \tag{A.275}$$

$$\widehat{l}_t^r = \frac{l_s^{o,r}}{l_s^r}\widehat{l}_t^{o,r} + \frac{l_s^{u,r}}{l_s^r}\widehat{l}_t^{u,r} \tag{A.276}$$

$$\widehat{l}_t^{rt} = \frac{l_s^{o,rt}}{l_s^{rt}}\widehat{l}_t^{o,rt} + \frac{l_s^{u,rt}}{l_s^{rt}}\widehat{l}_t^{u,rt} \tag{A.277}$$

$$\widehat{l}_t = (1-\theta)\widehat{l}_t^{r} + \theta\widehat{l}_t^{rt} \tag{A.278}$$

$$\widehat{i}_t^o = (1-\theta)\widehat{i}_t^{o,r} \tag{A.279}$$

$$\widehat{i}_t^u = (1-\theta)\widehat{i}_t^{u,r} \tag{A.280}$$

$$\widehat{k}_t^o = (1-\theta)\widehat{k}_t^{o,r} \tag{A.281}$$

$$\widehat{k}_t^u = (1-\theta)\widehat{k}_t^{u,r} \tag{A.282}$$

$$\widehat{TR}_t = (1-\theta)\widehat{TR}_t^r + \theta\widehat{TR}_t^{rt} \tag{A.283}$$

Resource constraints

The log-linearised version of the aggregate resource constraints in both sectors $(A.223)$ and $(A.224)$ yield the following log-linearised equations:

$$\widehat{y}_t^o = \bar{g}_s\widehat{g}_t + \frac{c_s^o}{y_s^o}\widehat{c}_t^o + \frac{i_s^o}{y_s^o}\widehat{i}_t^o + \frac{r_s^{k,o}k_s^o}{y_s^o}\widehat{u}_t^o \tag{A.284}$$

$$\widehat{y}_t^u = \frac{c_s^u}{y_s^u}\widehat{c}_t^u + \frac{i_s^u}{y_s^u}\widehat{i}_t^u + \frac{r_s^{k,u}k_s^u}{y_s^u}\widehat{u}_t^u \tag{A.285}$$

Shock Processes

The log-linearised equations for asymmetric shocks in the official sector equations $(A.225)$ to $(A.232)$ that are considered in the model are given as:

Risk premium shock

$$\widehat{\varepsilon}_t^{RISK} = \rho^{RISK}\widehat{\varepsilon}_{t-1}^{RISK} + \widehat{\xi}_t^{RISK} \tag{A.286}$$

Investment shock

$$\widehat{\varepsilon}_t^{INV} = \rho^{INV}\widehat{\varepsilon}_{t-1}^{INV} + \widehat{\xi}_t^{INV} \tag{A.287}$$

Official sector productivity shock

$$\widehat{A}_t^o = \rho^A\widehat{A}_{t-1}^o + \widehat{\xi}_t^A \tag{A.288}$$

Price mark-up shock

$$\widehat{\varepsilon}_t^p = \rho^p\widehat{\varepsilon}_{t-1}^p + \widehat{\xi}_t^p \tag{A.289}$$

Monetary policy shock

$$\widehat{\epsilon}_t^R = \rho^{\varepsilon}\widehat{\epsilon}_{t-1}^R + \widehat{\xi}_t^{\varepsilon} \tag{A.290}$$

Government spending shock

$$\widehat{g}_t = \rho^G\widehat{g}_{t-1} + \widehat{\xi}_t^G \tag{A.291}$$

Labour income tax shock

$$\widehat{\tau}_t^W = \rho^W\widehat{\tau}_{t-1}^W + \widehat{\xi}_t^W \tag{A.292}$$

Capital income tax shock

$$\widehat{\tau}_t^K = \rho^K\widehat{\tau}_{t-1}^K + \widehat{\xi}_t^K \tag{A.293}$$