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Strong and Weak Hypotheses

Abstract. In this paper, we investigate the nature of empirical hypotheses used in scientific reasoning and the act of formulating hypotheses. This is achieved through a novel logical framework in which we provide specific semantics for two types of hypotheses: a *strong* and a *weak* sense of hypothesis, each characterized by different logical structures. This framework enables us to better characterize certain aspects of hypothetical reasoning in scientific practice, especially when we attempt to rationally deny the content of an empirical hypothesis.

Keywords: hypothesis, evidence; denial; scientific reasoning; semantics; illocutionary acts

1. Introduction

In this paper we investigate the concept of empirical hypothesis and the act of hypothesizing within a framework provided by the pragmatic logic (hereafter, $\mathcal{PL}$) [10, 11]. The act of formulating hypotheses is an essential component of rational activity and plays a fundamental role in the scientific enterprise. It is almost trivial to note that a significant part of the core of the scientific method consists of formulating hypotheses that attempt to describe patterns of regularity in natural or social phenomena. For this reason, working with hypotheses — formulating, comparing, and rejecting them — constitutes a set of basic operations at the heart of scientific rationality. As observed by Carl Gustav Hempel:

> As is well known, empirical science decides upon the acceptability of a proposed hypothesis by means of suitable tests. Sometimes such a

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> test may involve nothing more than what might be called direct observation of pertinent facts. [...] But most of the important hypotheses in empirical science cannot be tested in this simple manner. Direct observation does not suffice to decide, for example, whether to accept or to reject the hypotheses that the earth is a sphere, that hereditary characteristics are transmitted by genes, that all Indo-European languages developed from one common ancestral language, that light is an electromagnetic wave process, and so forth. With hypotheses such as these, science resorts to indirect methods of test and validation. While these methods vary greatly in procedural detail, they all have the same basic structure and rationale. [13, p. 83]

The idea of characterizing the basic inferential processes through which we reason with hypotheses allows for the explicit articulation of the dynamics of hypothesis-making. This, beyond possessing intrinsic theoretical interest, enables the provision of a normative account of scientific practice and, in particular, offers a logical background for framing scientific disagreement.

This logic can be used to develop various frameworks in which the logical features of illocutionary acts such as asserting, denying, conjecturing, hypothesizing and so on can be characterized. Depending on both syntactical and semantic characteristics of the frameworks we can formulate different $\mathcal{PL}$ systems (there are some recent works developing the growing family of pragmatic logics [see, e.g., 2, 5, 6, 9]).

Since they are acts, illocutionary acts are not truth-bearers; it would be in fact rather strange to claim that the act of asserting that it is raining today is true or false. Acts are not things that possess the property of being true or false; rather, the *content* of this illocutionary act, i.e. the proposition that it is raining today, is clearly a truth bearer; it may be true or false that it's raining today. Of course, the truth value of the content of one's assertion affects some properties of the assertion. The first two characteristics that a $\mathcal{PL}$ must therefore satisfy are the following:

(i) there is a fundamental distinction between (illocutionary) act and content,
(ii) only the content has semantic value.

These two characteristics are both reflected in the syntax and semantics of a $\mathcal{PL}$; that is, the language of $\mathcal{PL}$ presents two types of signs: the first type expresses the acts and their logical structure (for example, pragmatic conjunctions, disjunctions, and negations among acts). The second type of signs describes the contents of acts and their logical structure (for example, conjunctions, disjunctions, negations of propositions that are asserted, hypothesized, and so on). In what follows, we will use classical propositional logic as a language that describes the content of illocutionary acts (nothing prevents, of course, making the logic of illocutionary contents more complex and fine-grained, using, for example, first-order logic, or some modal logics).

Pragmatic formulas, i.e. those that describe illocutionary acts, are subject to pragmatic interpretations which usually ascribe two values of justification to the formulas: justified (J) and unjustified (U). Obviously, the conditions under which a given illocutionary act is J or U depend on the system chosen and the act in question.[1]

The third characteristic of $\mathcal{PL}$ is:

(iii) the non-iterability of pragmatic operators.

The intuitive justification of (iii) is straightforward; an assertion, say, cannot be the object of another assertion. When one says, for example, "I assert to assert that p", the second occurrence of the assertion is actually a *nominalization* of the act of asserting that p; it is therefore a proposition (the subject x asserts that p) and as such it has a truth value and belongs to the semantic part of $\mathcal{PL}$. The systematic analysis of the possibility of naming illocutionary acts and, thus, obtaining some forms of iterability is an interesting and not very much explored possibility.[2]

The aim of this paper is to provide a logical characterization of two types of *hypothesis* operators and a *denial* operator within a novel modal-probabilistic semantics. Our framework can be utilized to offer a logical reconstruction of empirical hypotheses, particularly during stages of scientific inquiry when hypotheses have not yet undergone testing in any statistical trial. The paper is structured in the following way. Section 2 presents, from a semantic perspective, a new logical system for two types of acts of hypothesis. Section 3 explores the logical relations between these two types of hypothesis and the act of denial. Finally, Section 4 concludes the paper indicating some future lines of research.

[1] In some cases, depending on the general purposes of the work at play, the conditions of justification can be left intuitive (e.g.: the assertion of A is justified if and only if there is a proof (or conclusive evidence) that A is true).

[2] For an epistemic application of a system of pragmatic logic with a more fine-grained analysis of contents, see [7].

2. Pragmatic logic of hypotheses

In the previous section we have introduced the very general elements of $\mathcal{PL}$. In light of them, it is therefore quite natural to try to formulate a system of pragmatic logic in order to handle the formal behaviour of the illocutionary act of hypothesizing. Indeed, what are, *prima facie*, the conditions under which a hypothesis is justified? There are (at least) two insights at play here and they are quite different.

On the one hand, the conditions that guarantee that a subject is justified in making an empirical hypothesis are rather light, since, precisely, the subject is simply hypothesizing (that p), i.e. one has only a weak commitment to the truth (or even the plausibility) of p[3]. Let us consider the following example:

> Although there is no direct evidence, given the astronomical conditions of the extrasolar planet X, it is possible that X has all the conditions to support life.

In that case, we are not committing to the actual existence of extra-solar forms of life nor are we saying that it is a plausible option; instead, we are just assuming that there is no conclusive contrary evidence.

On the other hand, the use of hypotheses commonly made in other provinces of scientific practice presupposes some more robust justification or admissibility conditions. For example:

> The hypothesis that liquid water is found among the internal states of Titan is supported by the reports of the Cassini spacecraft.

Here, we have reasons that effectively support the existence of liquid water. In hypothesizing that, in other terms, we have a commitment towards a specific inner constitution of Titan.

The two above examples clarify the intuitions we have about the justification conditions of our hypotheses. Sometimes, hypothesizing simply means assuming something for the sake of discussion, with (almost) no commitment to the truth (or plausibility) of the hypotheses' content.

[3] It is worth noting that this sense of the act of hypothesis-making is different from the act of making an assumption (as it happens, for instance, in natural deduction). The content of an assumption can be also false and can be used (when a contradiction follows by the assumption) in an indirect proof to prove the negation of the content of the assumption.

In other contexts, hypotheses are conjectures for which we have some degree of confirmation but which, however, do not reach the expected threshold to abandon the status of a hypothesis and become part of the set of scientific knowledge on that field. Thus, we call hypotheses of the first kind *weak hypotheses* and hypotheses of the second kind *strong hypotheses*.

In what follows we will therefore provide a twofold framework capable of characterizing the two insights we have just presented. The language of the logic of hypothesis consists of a set of signs about acts and their logical relationships:

$\mathcal{H}^+$ – is the sign for the hypothesis in the strong sense
$\mathcal{H}^-$ – is the sign for the hypothesis in a weak sense
$\sqcap$ – is the sign for the conjunction of two hypotheses
$\sqcup$ – is the sign for the disjunction of two hypotheses
$\sqsupset$ – is the sign for the implication of two hypotheses

So, there are signs that concern the logical structure of the content of the hypotheses:

- Set of propositional letters: p, q, r, ...
- Classical Boolean connectives: $\wedge, \vee, \neg, \rightarrow$
- Brackets: (,)

We will therefore have that $\mathcal{H}^+p$ describes the hypothesis (in the strong sense) that p is true while $\mathcal{H}^-q$ describes the hypothesis (in the weak sense) that q is true.

Let us now establish a pragmatic interpretation of our language. The ingredients of the interpretation are the following:

(i) A set of informational scenarios: s_1, s_2, s_3, ..
(ii) A bulleted scenario (which we can assume as the scenario in which the subject is ideally placed): @
(iii) An accessibility relation defined on the sets of scenarios: D. Therefore, with $D(@)$ we indicate the set of scenarios that are available starting from the "current" scenario
(iv) A functional term $e(...)$ such that, given a scenario s, $e(s)$ indicates the *evidence available* in that scenario
(v) A pragmatic evaluation function indexed to the @ scenario: $\pi_@$

We have now all the ingredients in order to outline the pragmatic interpretation of the language. As we mentioned above, no particular property of the relation D is specified. With the concept of evidence available in a given scenario we include all forms of evidence although the underlying idea remains that of empirical evidence.

2.1. Justification conditions of the hypothesis in a strong sense

In this section we introduce and critically discuss the justification conditions for the hypotheses in a strong sense.

(h_1) $\pi_{@}(\mathcal{H}^{+}p) = J$ iff $\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s))$,
(h_2) $\pi_{@}(\mathcal{H}^{+}p) = U$ iff $\exists s \in D(@), Pr(p|e(s)) \leqslant Pr(\neg p|e(s))$.

According to (h_1), the hypothesis that p is true is justified if and only if the available evidence makes p more probable than $\neg p$. In the example of Titan, the available evidence (thanks to data from the Cassini spacecraft and a lot of underlying theory about the composition of planets) makes the existence of water more probable than the non-existence of water. Even if the tone of the examples suggests a subjectivist interpretation of probability, our framework is neutral with respect to the interpretations of probability. It is important to underline the contrastive component of the semantics of the $\mathcal{H}^{+}$ operator.[4] In other words: a strong hypothesis is never justified or unjustified *tout court*, but always in comparison with the rival hypothesis consisting of its negation. Condition (h_2) makes clear the unjustification condition of hypothesis in this strong sense, i.e. a hypothesis in a strong sense is unjustified when in at least a scenario the probability of p given the evidence available in that scenario is lower or equal to the probability of $\neg p$ in the same scenario.

One might indeed question[5] the actual cogency of the strong hypothesis justification clause within the context of current scientific practice. In particular, it should be noted that for any data/evidence set, there will be an infinite number of hypotheses consistent with that set. Therefore, the objection continues, there will never be a situation in which a hypothesis (considered as a conjunction of propositions) is more likely than its negation.

We acknowledge that this is a genuine issue. However, our system stipulates that the justification of a hypothesis is tied to its higher probability compared to its rival hypothesis. Now, what is the rival hypothesis? It depends. It depends on many factors, including the context, the

[4] A review of different probabilistic operators is [23]. See also the seminal work of Rescher on the relations among logic, evidence and probability [18].

[5] We thank an anonymous referee for emphasizing this point.

epistemic standards adopted, and the aims driving the scientific inquiry. Identifying a rival hypothesis is, therefore, not straightforward. For the purpose of logical characterisation, we have chosen to define the rival hypothesis as the mere negation of the hypothesis itself, fully aware that this is a simplification that might prove not fully adequate in certain contexts.

Based on (h_1) and (h_2), we have that:

(i) $\pi_{@}(\mathcal{H}^{+}p) = J \Rightarrow \pi_{@}(\mathcal{H}^{+}\neg p) = U$,
(ii) $\pi_{@}(\mathcal{H}^{+}p) = U \not\Rightarrow \pi_{@}(\mathcal{H}^{+}\neg p) = J$.

Principle (i) is a consistency requirement; by the fact that the strong hypothesis of the presence of water on Titan is justified follows that the opposite strong hypothesis is not justified by the available evidence. Principle (ii) shows that from the antecedent $\exists s \in D(@), Pr(p|e(s)) \leqslant Pr(\neg p|e(s))$ does not follow that $\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s))$. In fact, the condition of unjustification for the hypothesis p is compatible with the case of indeterminacy such as the following

(iii) $\pi_{@}(\mathcal{H}^{+}p) = U$ and $\pi_{@}(\mathcal{H}^{+}\neg p) = U$.

This is the case when $\exists s \in D(@), Pr(p|e(s)) \leqslant Pr(\neg p|e(s))$ and $\exists s \in D(@)$, $Pr(\neg p|e(s)) \leqslant Pr(p|e(s))$. These gappy situations arise when $Pr(p|e(s)) = Pr(\neg p|e(s))$, i.e., when the available evidence is unable to decide between two rival hypotheses. This is an aspect of paracompleteness of our framework that mirrors well-known and explored cases in scientific practice.[6]

2.1.1. Conjunction

Once the conditions for justifying the act of hypothesizing in a strong sense have been established, we can analyze the possible logical composition of these acts. For example, when are we justified in making the hypothesis that liquid water can be found in Titan's deep layers *and* that Saturn's core consists of metallic hydrogen? Note that, from the point of view of natural language, the difference between the following sentences can be difficult to disambiguate:

[6] For other logics of evidence showing paraconsistency and paracompleteness, see [4]. For another logical system designed to handle the methodology of scientific discovery from a more fallibilist perspective, see [20]. In another paper we will investigate the relations of our logical perspectives on evidence and science dynamics with the aforementioned systems.

(1) Ada hypothesizes that p and hypothesizes that q
(2) Ada hypothesizes that $(p \wedge q)$

Let us consider (1), that is a case of conjunction of hypotheses. The justification clause is the following:

(h_3) $\pi_{@}(\mathcal{H}^{+}p \sqcap \mathcal{H}^{+}q) = J$ iff $(\mathcal{H}^{+}p) = J$ and $(\mathcal{H}^{+}q) = J$, that is, $\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s)) \wedge Pr(q|e(s)) > Pr(\neg q|e(s))$.

According to (h_3) the pragmatic, or external, conjunction between hypotheses is nothing more than the conjunction of the conditions of justification of the two hypotheses.

Things are different, however, if we provide an internal reading of the conjunction; in this case, the illocutionary act at play is just one while the content of the hypothesis has the logical structure of the conjunction.

Internal conjunction is particularly relevant in the case of scientific hypotheses, given that, one could argue, a hypothesis is almost always a conjunction of propositions. However, it is important to note that the rival hypothesis, which in our logical framework is the negation of the conjunction, is compatible — as we shall see shortly — with various alternative combinations.

Let us therefore consider the case (2): Ada hypothesizes that $(p \wedge q)$. Now, according to our semantics we have:

(h_4) $\pi_{@}(\mathcal{H}^{+}(p \wedge q)) = J$ iff $\forall s \in D(@), Pr(p \wedge q|e(s)) > Pr(\neg(p \wedge q)|e(s))$.

We can now consider again our previous astronomic example, i.e., the hypothesis that Titan has profoundly water (p) and that Saturn has a metallic hydrogen core (q) is justified if and only if the available evidence makes $p \wedge q$ more likely than their negation. However, a problem arises here: the negation of $p \wedge q$ is logically compatible with three alternatives:

- $p \wedge \neg q$ – there is water on Titan but Saturn's core is not made of metallic hydrogen.
- $\neg p \wedge q$ – there is no water on Titan and Saturn's core is made of metallic hydrogen.
- $\neg p \wedge \neg q$ – there is no water on Titan and Saturn's core is not made of metallic hydrogen.

So, the information we obtain from the internal justification of a conjunction is unable to tell us which combination is made less likely by the available evidence. In other words, given the astronomical observations we are justified in hypothesizing both the presence of water on Titan and the nature of Saturn's core. But if we understand this hypothesis as a single conjunctive hypothesis, our logic is unable to capture which least likely combination is ruled out by the available evidence.

Note that in extreme cases — those involving pragmatic contradictions and tautologies — we get results that are perfectly in line with our intuitions. In fact, the external conjunction of two contradictory hypotheses is never justified:

(h_5) $\pi_@(\mathcal{H}^+p \sqcap \mathcal{H}^+\neg p) = J$ iff $\pi_@(\mathcal{H}^+p) = J$ and $\pi_@(\mathcal{H}^+\neg p) = J$, i.e.
$\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s)) \wedge Pr(\neg p|e(s)) > Pr(p|e(s))$.

Obviously, there is no probability assignment such that $Pr(p) > Pr(\neg p)$ and that $Pr(\neg p) > Pr(p)$. But even if we consider the conjunction from the internal point of view we have a similar result:

(h_6) $\pi_@(\mathcal{H}^+(p \wedge \neg p)) = J$ iff
$\forall s \in D(@), Pr(p \wedge \neg p|e(s)) > Pr(\neg(p \wedge \neg p)|e(s))$.

Since the probability of $p \wedge \neg p$ is equal to 0 it follows that the hypothesis of the contradiction is never justified.

2.1.2. Disjunction

The case of disjunction of hypotheses is quite intuitive:

(h_7) $\pi_@(\mathcal{H}^+p \sqcup \mathcal{H}^+q) = J$ iff $\pi_@(\mathcal{H}^+p) = J$ or $\pi_@(\mathcal{H}^+q) = J$, i.e.
$\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s))$ or $Pr(q|e(s)) > Pr(\neg q|e(s))$.

It is not difficult to show that the excluded middle is not a pragmatically valid formula, that is

(h_8) $\pi_@(\mathcal{H}^+p \sqcup \mathcal{H}^+\neg p) = J$ iff $\pi_@(\mathcal{H}^+p) = J$ or $\pi_@(\mathcal{H}^+\neg p) = J$, i.e.
$\forall s \in D(@), Pr(p|e(s)) > Pr(\neg p|e(s))$ or $Pr(\neg p|e(s)) > Pr(p|e(s))$.

The countermodel is the case in which $Pr(p) = Pr(\neg p)$, i.e., the case in which the available evidence is not able to decide about the proposition p. Since we are dealing with a strong conception of hypothesis, the excluded middle $(\mathcal{H}^+p \sqcup \mathcal{H}^+\neg p)$ does not hold. What happens if we intend the disjunction internally? Analogously to what was seen for conjunction, we will have the following clause:

(h_9) $\pi_@(\mathcal{H}^+(p \vee q)) = J$ iff
$\forall s \in D(@), Pr((p \vee q)|e(s)) > Pr(\neg(p \vee q)|e(s))$.

In this case, however, unlike the conjunction, $\neg(p \vee q)$ is compatible with only one case, namely with $\neg p \wedge \neg q$. We will therefore say that the hypothesis of a disjunction is justified when the available evidence makes the probability of $p \vee q$ greater than the probability of $\neg p \wedge \neg q$. This has a glimmer of plausibility; moreover, in this interpretation, excluded middle (that is, $\mathcal{H}^+(p \vee \neg p)$) becomes valid since it means to make the hypothesis of a tautology. The logical behavior of the hypothesis operator seems to be different if we consider its logical structure from an internal or external point of view. This seems even more evident in the case of implication.

2.1.3. Implication

Also for the conditional, we distinguish between the external and internal case. Regarding the former, we will have the following clause:

$$(h_{10})\ \ \pi_{@}(\mathcal{H}^+ p \sqsupset \mathcal{H}^+ q) = J \text{ iff } \forall s \in D(@), Pr(p|e(s)) \leqslant Pr(q|e(s)).$$

The meaning of (h_{10}) is that if we are justified in hypothesizing that p then we are justified in hypothesizing that q; this means that the evidence that confirms p is, by itself, able to justify q as well. A little different is the case where the content of our hypothesis has a conditional form[7]. Here, based on our semantics, we have that:

$$(h_{10})\ \ \pi_{@}(\mathcal{H}^+(p \to q)) = J \text{ iff}$$
$$\forall s \in D(@), Pr((\neg(p \wedge \neg q))|e(s)) > Pr((p \wedge \neg q)|e(s)).$$

Let us imagine the following reasoning:

> We have good reasons to hypothesize that if John went to the cinema (p), then he bought popcorn (q).

Consider the hypothetical nature of the reasoning: we are not asserting (in the sense that we do not have very strong or even complete evidence) that John went to the cinema — perhaps the available evidence tends to exclude this possibility. What is meant is that it is reasonable to formulate the hypothesis that the scenario in which John went to the cinema without buying popcorn is less probable than the opposite scenario. In other words, in conditional hypothetical reasoning, if p then q, two opposing scenarios are compared: in the first, it is not the case that p and $\neg q$, while, in the second, p is the case, but q is not. If the

[7] For a recent review on the probability of conditionals, see [14].

first scenario is favored by the available evidence, then this allows us to justify our hypothetical inference.

It is worth exploring the behaviour of the two conditionals with respect to the logical rule of *modus ponens*. Let us consider *modus ponens* rule for the implication of two strong hypotheses. We have: $\mathcal{H}^+p$; $\mathcal{H}^+p \sqsupset \mathcal{H}^+q$; therefore $\mathcal{H}^+q$. Indeed, in virtue of the first premise, we have that given the available evidence $e(s)$, $Pr(p|e(s)) > Pr(\neg p|e(s))$. Then, in virtue of the conditional formula we have that $Pr(p|e(s)) \leqslant Pr(q|e(s))$ and it is easy to show that the conclusion $Pr(q|e(s)) > Pr(\neg q|e(s))$ follows by the premises. Therefore, *modus pones* holds for the implication of strong hypothesis.[8]

Let us consider now the *modus pones* rule for strong hypothesis with conditional content, that is: $\mathcal{H}^+(p)$, $\mathcal{H}^+(p \to q)$; therefore, $\mathcal{H}^+(q)$. From the first premise we have $Pr(p|e(s)) > Pr(\neg p|e(s))$, then by the second premise we have that $Pr(\neg(p \wedge \neg q)|e(s))$ must be greater than $Pr((p \wedge \neg q)|e(s))$. But from this we have no information about the relationship between the probabilities of q and $\neg q$. However, the following specific version of *modus pones* holds: $\mathcal{H}^+(p \to q)$, p is certain; therefore, $\mathcal{H}^+q$. By the first premise we have that $Pr(\neg(p \wedge \neg q)|e(s)) > Pr((p \wedge \neg q)|e(s))$; but given that p is certain, the probability of $p \wedge \neg q$ is equal to the probability of $\neg q$ and, by consequence, the probability of $\neg(p \wedge \neg q)$ is the probability of $\neg\neg q$, that is, the probability of q. Likewise the probability of $p \wedge \neg q$ is equal to the probability of $\neg q$. So, it follows that $Pr(q) > Pr(\neg q)$.

2.2. Justification conditions for hypotheses in the weak sense

A weaker notion of hypothesis is sometimes adopted in science and common reasoning and is connected with a notion of evidential (or epistemic) possibility, in the sense that we can make a hypothesis in any case unless the negation of the hypothesis is conclusively proven. It is a much weaker sense of hypothesis that has to do with the mere possibility of formulating conjectures. After all, it seems admissible in a dialectical context to hypothesize p under the assumption that p is at least possible given the evidence available. This is, for instance, also the sense of hypothesis involved in Peirce's idea of abduction, intended as an invitation to further

[8] On some probability-preserving properties of inferences, see [1].

investigate a hypothesis.[9] The dynamics of scientific knowledge almost always begins with (weak) hypothesis searching for confirmation.

Even in this case, it could be argued, the mere possibility (i.e., a probability greater than zero) of a given content is not sufficient to guarantee the legitimacy of the hypothesis.[10] Certainly, many other factors may contribute to determining which hypotheses, and in what order, should be tested. It is therefore possible to introduce a threshold that represents a lower bound of probability, below which it would not be rational to consider the hypothesis in question. The precise value of this threshold is clearly a matter of convention, once again tied to contextual and pragmatic elements. In our proposal, which is logical in character, we have chosen to adopt the minimal possible threshold, thereby assuming the most liberal perspective imaginable. A hypothesis, in the weak sense, is justified if and only if the probability of its content is considered greater than 0.

The conditions under which we can hypothesize something in a weak sense are the following:

(h_{12}) $\pi_{@}(\mathcal{H}^{-}p) = J$ iff $\forall s \in D(@), Pr(p|e(s)) \neq 0$,
(h_{13}) $\pi_{@}(\mathcal{H}^{-}p) = U$ iff $\exists s \in D(@), Pr(p|e(s)) = 0$.

The pragmatic justification requirements for the act of hypothesizing in a weak sense are connected to the idea of the *mere* compatibility with the evidence available starting from the reference scenario. Obviously, in the case when $p \equiv \bot$ we have that $\mathcal{H}^{-}p$ is always unjustified.

Moreover, it is trivial to show that from the justification of the strong hypothesis $\mathcal{H}^{+}p$ it is possible to infer the justification of the weak hypothesis $\mathcal{H}^{-}p$.

Let's now see the logical behavior of the $\mathcal{H}^{-}$ operator in a way similar to what was done for the strong hypothesis operator. As in the previous case, the formal language allows us to disambiguate cases of

[9] The evaluation of abducted hypotheses is related to the so-called principle of economy of research. Regarding this principle, Peirce pointed out that "now economy, in general, depends upon three kinds of factors; cost; the value of the thing proposed, in itself; and its effect upon other projects. Under the head of cost, if a hypothesis can be put to the test of experiment with very little expense of any kind, that should be regarded as a recommendation for giving it precedence in the inductive procedure" (CP 7.220, 1901) [17]. On Peirce's economy of research and some of its developments, see, e.g., [16, 19, 22].

[10] We again thank an anonymous referee for having drawn attention to this point.

internal interpretation from those of external interpretation. In general, the following relationships hold:

(a) $\pi_{@}(\mathcal{H}^{-}p \sqcap \mathcal{H}^{-}q) = J$ iff
$\forall s \in D(@), Pr(p|e(s)) \neq 0$ and $\forall s \in D(@), Pr(q|e(s)) \neq 0$,
(b) $\pi_{@}(\mathcal{H}^{-}p \sqcup \mathcal{H}^{-}q) = J$ iff
$\forall s \in D(@), Pr(p|e(s)) \neq 0$ or $\forall s \in D(@), Pr(q|e(s)) \neq 0$,
(c) $\pi_{@}(\mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q) = J$ iff $\forall s \in D(@), Pr(p|e(s)) \leqslant Pr(q|e(s))$.

By following the justification clauses, we can also analyse cases of contents with logical structure:

(a_1) $\pi_{@}(\mathcal{H}^{-}(p \wedge q)) = J$ iff $\forall s \in D(@), Pr(p \wedge q|e(s)) \neq 0$.

Stated differently: a conjunction can be weakly hypothesized when its conjuncts are compossible. It follows, indeed, that if $\mathcal{H}^{-}(p \wedge q)$ is justified, then $Pr(p|e(s))$ and $Pr(q|e(s))$ are different from 0. This allows us to establish the following bridge principle:

(a_2) $\mathcal{H}^{-}(p \wedge q) \Rightarrow \mathcal{H}^{-}p \sqcap \mathcal{H}^{-}q$.

The converse of (a_2) is not valid; just because two states are possible, it doesn't follow that their conjunction is possible. It is weakly reasonable to hypothesize when tossing a coin that head will come up and it is weakly reasonable to hypothesize that tail will come up. But it is not weakly reasonable to hypothesize that both head and tail will come up together.

In regard to the disjunction, things are even more straightforward:

(b_1) $\pi @(\mathcal{H}^{-}(p \vee q)) = J$ iff $\forall s \in D(@), Pr(p \vee q|e(s)) \neq 0$.

Based on probability calculus, we have the following complete bridge principle:

(b_2) $\mathcal{H}^{-}(p \vee q) \Leftrightarrow \mathcal{H}^{-}p \sqcup \mathcal{H}^{-}q$.

Finally, let us consider the conditional. The clause of the weak hypothesis for a conditional content is the following:

(c_1) $\pi_{@}(\mathcal{H}^{-}(p \to q)) = J$ iff $\forall s \in D(@), Pr(p \to q|e(s)) \neq 0$.

This means that in every scenario the available evidence excludes the case of $p \wedge \neg q$; namely that $Pr(p \wedge \neg q) = 0$. Also in this case we have just one sense of the bridge principle, that is,

(c_2) $\mathcal{H}^{-}(p \to q) \Rightarrow \mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q$.

Let us assume the antecedent. Hence, as we just said, in every scenario, $Pr((p \wedge \neg q)|s(e)) = 0$; let us assume $\mathcal{H}^{-}p$; thus, the probability of p given the available evidence is different from 0. But then it follows that $Pr(\neg q|s(e)) = 0$ and, consequently, that $Pr(q|s(e)) \neq 0$; therefore, $\mathcal{H}^{-}q$. The converse, however, does not hold. Let us consider $\mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q$. We have that $Pr(p|e(s)) \leqslant Pr(q|e(s))$. Let us consider $\mathcal{H}^{-}(p \to q)$. This formula is justified when $Pr((\neg(p \wedge \neg q))|e(s)) \neq 0)$, that is, $Pr((p \wedge \neg q)|e(s)) = 0)$. But this information cannot be derived by the simple fact that $Pr(p|e(s)) \leqslant Pr(q|e(s))$. Therefore, this sense of the bridge principle does not hold.

We can summarize and list below the aforementioned results:

- $\mathcal{H}^{-}(p \wedge q) \Rightarrow \mathcal{H}^{-}p \sqcap \mathcal{H}^{-}q$,
- $\mathcal{H}^{-}p \sqcap \mathcal{H}^{-}q \nRightarrow \mathcal{H}^{-}(p \wedge q)$,
- $\mathcal{H}^{-}(p \vee q) \Leftrightarrow \mathcal{H}^{-}p \sqcup \mathcal{H}^{-}q$,
- $\mathcal{H}^{-}(p \to q) \Rightarrow \mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q$,
- $\mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q \nRightarrow \mathcal{H}^{-}(p \to q)$.

Finally, we can investigate the validity of *modus pones* rule for weak hypotheses when the conditional is placed in the content of the hypothesis or when there is a pragmatic implication between hypotheses.

In the first case we have: $\mathcal{H}^{-}p$, $\mathcal{H}^{-}(p \to q)$; therefore $\mathcal{H}^{-}q$. In virtue of the first premise, we have that $Pr(p|e(s)) \neq 0$. The second premise is justified when $Pr(\neg(p \wedge \neg q)|e(s))) \neq 0$. This means that $Pr(p \wedge \neg q|e(s)) = 0$. Since we know by the first premise that $Pr(p|e(s)) \neq 0$, we have that $Pr(\neg q|e(s)) = 0$, i.e. $Pr(q|e(s)) = 1$ and, therefore, different from zero. This means that this form of *modus pones* holds.

The second type of *modus ponens* for weak hypotheses is the following: $\mathcal{H}^{-}p$, $\mathcal{H}^{-}p \sqsupset \mathcal{H}^{-}q$; therefore, $\mathcal{H}^{-}q$. According to the first premise $Pr(p|e(s)) \neq 0$, while according to the second premise we have that $Pr(p|e(s)) \leqslant Pr(q|e(s))$. This means that also $Pr(q|e(s)) \neq 0$, coherently with the conclusion of this form of *modus ponens*. So, also this second form of *modus ponens* holds.

3. Acts of hypothesis and denial

It is quite natural to extend the probabilistic framework we have developed to define an operator of (rational) denial that expresses the act of denying (or rejecting) the content of a hypothesis. We have already explored this route in [8]; here, however, the conditions for justifying the denial are different and follow the semantics of the hypotheses.

Intuitively we are justified in rejecting p when p is inconsistent with all the available evidence. Indicating with "$\dashv$" the sign of the denial, we have the following clauses[11]

- $\pi_{@}(\dashv p) = J$ iff $\forall s \in D(@), Pr(p|e(s)) = 0$,
- $\pi_{@}(\dashv p) = U$ iff $\exists s \in D(@), Pr(p|e(s)) \neq 0$.

Now, it is worthwhile to compare the clause of unjustification of the weak hypothesis with the clause of justification of denial for their logical resemblance:

- $\pi_{@}(\mathcal{H}^{-}p) = U$ iff $\exists s \in D(@), Pr(p|e(s)) = 0$,
- $\pi_{@}(\dashv p) = J$ iff $\forall s \in D(@), Pr(p|e(s)) = 0$.

In other words, in the first clause we are not justified in hypothesizing that there is extra-solar life because there is at least one case in which this option is incompatible with the available evidence. In the second clause, however, we are justified in rejecting the hypothesis that there is extra-solar life because this fact conflicts with all the scenarios we consider.

So, we can indicate a sort of gradient. Let, as usual, $e(s)$ be the available evidence and p the proposition expressing the content we are interested in:

(1) In all scenarios $Pr(p|e(s)) \neq 0$, i.e., $\pi_{@}(\mathcal{H}^{-}p) = J$,
(2) In at least one scenario $Pr(p|e(s)) \neq 0$, i.e., $\pi_{@}(\dashv p) = U$,
(3) In at least one scenario $Pr(p|e(s)) = 0$, i.e., $\pi_{@}(\mathcal{H}^{-}p) = U$,
(4) In all scenarios $Pr(p|e(s)) = 0$, i.e., $\pi_{@}(\dashv p) = J$.

It is not difficult to verify that the following relations hold: (4) $\Rightarrow$ (3) and (1) $\Rightarrow$ (2).

An important point to discuss concerns the so-called Duhem-Quine thesis. In a nutshell, the idea is that it is not possible to refute a hypothesis in isolation, as it is always accompanied by a set of auxiliary hypotheses, assumptions, laws, and so on. Our framework thus stipulates that $\dashv (H \wedge A_1 \wedge \cdots \wedge A_n) = J$ iff in all available scenarios $Pr(H \wedge A_1 \wedge \cdots \wedge A_n) = 0$.

However, as is easy to observe, since we do not know whether the hypothesis under test and the auxiliary information are independent of

[11] This sign was firstly introduced as a sign of rejection in [15].

each other or not, we cannot determine which part of the content of our hypothesis we are assigning a probability of 0. This is consistent with the intuitions surrounding Duhem-Quine's thesis, and our logic reflects a genuine limitation in the rational reconstruction of scientific inquiry.

Another objection concerns the assignment of extreme probabilistic values (1 and 0) to empirical statements in light of well-known fallibilist intuitions. Even in this case, our logic allows for the adoption of thresholds (both upper and lower) beyond which it is not "permissible" to go. For the sake of simplicity, and without loss of generality, we have assumed the extreme probabilistic values. The choice of these thresholds may depend on epistemic as well as pragmatic reasons, similarly to what happens with the concept of *inductive risk* [12]. Once the choice is made, however, the logic of the hypothesis should rationally reconstruct (even from a normative perspective) our inferential practice concerning hypotheses.[12]

3.1. Relationships between denial and weak hypothesis

Denying a hypothesis is a crucial element of scientific methodology and common reasoning. This is why the interplay between the acts of denial and hypothesis is a key illocutionary aspect of ordinary and scientific languages. The following two principles connecting hypothesis and denial are notable:

(1) $\pi_{@}(\dashv p) = J \Rightarrow \pi_{@}(\mathcal{H}^{-}\neg p) = J$.

i.e., if the denial of p is justified then it is also justified to weakly hypothesize that $\neg p$. However, we have that:

(2) $\pi_{@}(\mathcal{H}^{-} p) = J \not\Rightarrow \pi_{@}(\dashv \neg p) = J$.

[12] A referee points out that the importance of some scientists taking on the role of actively investigating/endorsing positions which are not those of the majority in the community. In other words, the issue raised is whether this system forces us to consider minority positions in science as irrational. Although this topic exceeds both our expertise and the scope of this paper, we believe that our framework provides a straightforward way to characterise disagreement among research communities. Such disagreement may lie in the differing probability assignments to hypotheses based, potentially, on different pieces of evidence. For instance, one community might justify hypothesis H_1 as being more probable than the rival hypothesis H_2 given a certain set of evidence; conversely, another scientific community might consider H_2 to be the more rational hypothesis. However, we emphasise that the disagreement concerns the assignment of probabilities and the way evidence is selected and interpreted. Our logical framework aims to account for this dynamic.

From the fact that in all the scenarios the probability of p is different from 0, it does not follow that the probability of $\neg p$ is equal to 0. With reference to (1), its converse obviously also holds, i.e.,

(1*) $\pi_{@}(\mathcal{H}^{-}\neg p) = U \Rightarrow \pi_{@}(\dashv p) = U.$

The antecedent, indeed, states there is at least one scenario such that $Pr(\neg p|e(s)) = 0$ but then, in that very same scenario, we obtain that $Pr(p|e(s)) \neq 0$ and, therefore, $\pi_{@}(\dashv p) = U$.[13]

3.2. Relationships between denial and strong hypotheses

Given the probabilistic semantics that we have formulated, we also obtain that:

(3) $\pi_{@}(\dashv p) = J \Rightarrow \pi_{@}(\mathcal{H}^{+}\neg p) = J.$

If the denial of p is justified then strongly hypothesizing $\neg p$ is also justified. This can be, perhaps, problematic in some contexts because it amounts to say that the fact that the available evidence excludes p represents a sufficient reason to hypothesize $\neg p$ in a strong way. Similarly,

(4) $\pi_{@}(\mathcal{H}^{+}p) = J \not\Rightarrow \pi_{@}(\dashv \neg p) = J.$

[13] From a Bayesian perspective the clauses for the denial of a (weak) hypothesis can be modified in order to comply with a Bayesian form of hypothesis testing. This is the case since the justification of the denial of a hypothesis is usually done in statistics in comparison with an alternative hypothesis. Let assume that there are two hypotheses H_0 and H_1 and α represents a specific level of Bayes factor required to justify the denial of a hypothesis.

Bayes factor is the ratio of the probability of the evidence of the data conditional on two competing hypotheses and is expressed as follows:

- $\dfrac{Pr(e(s)|H_1)}{Pr(e(s)|H_o)}$ and is equal to $\dfrac{Pr(H_1|e(s))/Pr(H_1)}{Pr(H_0|e(s))/Pr(H_0)}$.

Therefore, the clauses for the denial can be modified in the following way:

- $\pi_{@}(\dashv H_1) = J$ iff $\forall s \in D(@), \dfrac{Pr(H_1|e(s))/Pr(H_1)}{Pr(H_0|e(s))/Pr(H_0)} < \alpha,$
- $\pi_{@}(\dashv H_1) = U$ iff $\exists s \in D(@), \dfrac{Pr(H_1|e(s))/Pr(H_1)}{Pr(H_0|e(s))/Pr(H_0)} \geqslant \alpha.$

It is worth noting that different general-purpose interpretations of Bayes factor values are available in the literature and different decisions rules for statistically accepting or denying hypotheses are possible [see 21].

From the fact that in all scenarios the evidence makes p more likely than the opposite hypothesis $\neg p$, it does not follow that in all scenarios the probability of $\neg p$ is equal to 0. Since on the basis of (3) the justification of the strong hypothesis of $\neg p$ follows from the denial of p, and since a strong hypothesis entails a weak hypothesis with the same content, therefore — as we have seen before — we have that from the denial of p follows the weak hypothesis that $\neg p$.

Finally, let us consider the following last relation:

(5) $\pi_{@}(H^{+}p) = U \Rightarrow \pi_{@}(\dashv \neg p) = U$.

The antecedent means that there is at least a scenario in which the available evidence makes $Pr(p) \leqslant Pr(\neg p)$; the consequent, on the other hand, means that there is a scenario in which the available evidence makes the $Pr(\neg p|e(s))$ different from 0. But in order to be false the conditional, it must be false the consequent, that is, $Pr(\neg p|e(s))$ must be 0. But then it is impossible that the antecedent is true.

4. Conclusion

Formulating, accepting, denying empirical hypotheses are essential acts of scientific dynamics. In this paper, we have investigated, from a semantic perspective, the possibility to develop a logical framework for the acts of weak and strong hypothesis. A justification of a weak hypothesis requires its probability to be just different from zero. From an intuitive perspective, an operator of weak hypothesis can be interpreted as a kind of an evidential (or epistemic) possibility. We then investigated the logical behaviour of the justification conditions for a strong hypothesis with content p, which is justified when the probability of p is greater than the probability of $\neg p$. Both types of hypotheses are usually identifiable in scientific practice, particularly from a fallibilist perspective on science dynamics. Subsequently, we critically discussed the semantic conditions for the deniability of both types of hypotheses. Finally, we have applied our novel semantics for hypotheses and denial to handle different aspects of hypothetical reasoning in science. Future research endeavors should focus on refining our logical framework to accommodate the nuances of different forms of statistical hypothesis testing and investigate the complexities of abductive inference in science.

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