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Hybrid platforms with free entry: demand-enhancing activities

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Abstract
We study the decision of a platform as to the quantity and quality of the products to sell directly on its own marketplace, where also third-party sellers decide how much to invest in the quality of their products. Using a representative agent framework that is based on a quasi-linear quality-augmented indirect utility function, we show that, under free entry, the quality investments of sellers do not change with platform entry, while the number of joining sellers does. Moreover, contrarily to what is found in the received literature, the platform may go hybrid even in the case it does not enjoy a competitive advantage vis-á-vis third-party sellers. We then study the welfare implications of a platform's entry decision and show that promoting sellers' investments and/or contrasting platform entry may lead to a larger as well as a lower consumer welfare. This depends on the platform's response in equilibrium, both in terms of changes of its quality-enhancing investments and the fee charged on the revenues of third-party sellers.

Keywords Hybrid marketplace · Free entry · Monopolistic competition · Demand enhancement · Consumer welfare

JEL Classification D42 · L12 · L13 · L40

1 Introduction

In 2022, selling revenues of the top online marketplaces amounted to \$3.25 trillion.[1] In particular, third-party sales through platforms like those operated by Alibaba, Amazon.com Inc., eBay Inc. and others accounted for 77.5% of total gross

[1] Data provided by Digital Commerce 360, 2023.

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merchandise value among the Top 100 online marketplaces. Yet, in the same year, growth came from hybrid marketplaces, namely, platforms enabling interactions between buyers and sellers while simultaneously being retailers (resellers) of their own products (either private label products or branded products). Overall, gross merchandise volume (GMV) of marketplaces grew by 2.9%, but hybrids like Amazon, Walmart Inc., and JD.com Inc. grew 7.2%, outperforming pure marketplaces like eBay, Wish.com, and Alibaba's Chinese marketplaces Tmall and TaoBao. Furthermore, pure players' GMV declined by 0.9%. These trends reveal that the hybrid outperformed the pure marketplace model, which is very telling about its significant and growing role in the Internet economy.

Essentially, in the hybrid platform business model, in addition to the third-party sellers (platform users), the platform owners themselves launch their own varieties. Since they interact with the same consumers visiting the platform, platform owners are thus in competition with the platform users.

The economics of the hybrid platform integrates a vertical with a horizontal element. Specifically, in line with the classic agency model, the hybrid model involves a vertical supply chain in which suppliers (the platform users) set final retail prices by interacting with consumers, and sales revenue is split between suppliers and intermediaries (the platform owners) according to endogenously determined ad-valorem commissions (see Johnson (2017)). In addition, the platform owners and the third-party sellers are in a horizontal competitive relationship for the customers on the platform itself.

This specific setting has raised significant antitrust concerns lately. The combination of vertical and horizontal factors in the hybrid model provides both the means and the motivations for the hybrid platform to undertake harmful conduct for competition in the marketplace. First, the dominant position originating from the role performed in the vertical relation might allow the marketplace to leverage its market power to promote its own labels by affecting the costs of the competitors or even by preventing them from entering the platform. In this regard, the US House Majority Report[2] remarks that "As Amazon, Apple, Facebook, and Google have captured control over key channels of distribution, they have come to function as gatekeepers. A large swath of businesses across the US economy now depends on these gatekeepers to access users and markets." Ultimately, the report holds responsible each of these platforms for "using its gatekeeper position to maintain its market power". Secondly, a recurrent concern addresses how the platform might steer competition in favor of its own products by promoting them in the marketplace and penalizing third-party products. To this end, in 2020, the European Commission opened a formal antitrust investigation (European Commission, 2020) into the possible preferential treatment of Amazon's own retail offers. Finally, platform entry could be motivated by the anti-competitive goal of capturing the value generated by the most successful sellers (e.g., cloning their products and selling them at lower prices), free riding on their innovation, and R &D efforts. If perceived as systemic, this conduct might lead the sellers not to invest any longer as they anticipate they would be deprived of an appropriate rate of return.

[2] Report of the Subcommittee on antitrust (2020).

Despite the concerns previously mentioned, an alternative explanation with radically different welfare implications may justify platform entry decisions. The argument gives prominence to indirect network effects and leverages on the fact that consumers and third-party sellers benefit from mutual participation in the platform. Since platforms profit by selling to customers and collecting commissions on the interactions between sellers and consumers, they do not have any incentive to harm either constituency. Under this perspective, platform entry aims to stimulate marketplace competition, promoting consumers' attendance and sellers' participation, thereby improving its own appeal against alternative distributive channels. This argument also suggests that sellers' incentives to innovate need not be diminished. On the one hand, an ample customer base increases sellers' profitability and thus returns from innovating; on the other, higher competition from platform varieties might lead sellers to invest more to gain market shares.

By proposing a comprehensive setting in which all the aspects previously mentioned are incorporated (namely, fluid entry, platform promotion of its own varieties, third-party sellers' investments), our analysis attempts to contribute to the growing body of literature concerning the welfare and policy implications of the hybrid business model. Specifically, in this paper, we analyze the incentives of a hybrid marketplace to undertake demand-enhancing activities in favor of its own labels, while considering sellers' response in terms of entry decisions and investments in quality. Indeed, firms operating in the marketplace (both platform owners and third-party sellers) typically perform a wide range of such activities to boost their sales (e.g., choice of product portfolio, informative advertising, persuasive advertising, quality-related upgrades). The problem is complicated in a dual setting due to the underlying agency model. In particular, we consider a platform choice about whether or not to sell private labels (i.e., go hybrid) and the related choice of the number of its varieties and their quality. Indeed, when the platform undertakes such demand-enhancing activities, it has to consider not only the returns from its own reseller arm but also the effect on demand allocation across all the products of the platform (including third-party products) affecting its revenues from commissions. These aspects are particularly significant in the hybrid platform debate, as the literature (see Dryden et al. (2020)) typically contrasts the pro-competitive view of the platform becoming an effective participant in the distribution market and increasing the intensity of competition in the marketplace, against the value capture theory concerned with a reduction in sellers' investments.

We adopt a quality-augmented variant of the quasi-linear indirect utility function employed by, for instance, Etro (2023b), in which quality weights are responsive to changes in the elasticity of substitution. Furthermore, we allow the platform to undertake demand-enhancing activities as a first mover. More specifically, we let the platform decide the number of its varieties and their quality at a prior stage compared to when seller's entry, demand-enhancing, and pricing decisions take place. This extends the set of instruments available to the platform (along with the ad-valorem commission and prices of its own varieties) to divert competition in the marketplace by affecting sellers' entry, investments, and pricing.

Our setting allows us to build on what is found in Etro (2023b) by showing that a platform may well be willing to enter the market even in case third-party sellers enjoy a competitive advantage relative to the platform's varieties. This result depends on the interplay between the relative importance of the fees vs. own products revenue channels and the endogeneity of demand-enhancing investments.

We notice that Shopova (2023) also finds that a platform may go hybrid offering a low-quality product. However, in her work, a one-product marketplace faces a single seller, there is no entry, and consumers are heterogeneous in their willingness to pay for quality. In such a model, the platform's decision to enter with low quality is driven by its need to differentiate from the seller targeting those consumers that the latter does not serve, ending in efficient market segmentation. While the outcome is similar, the driving forces are very different.

Having observed platform decisions, sellers decide whether or not to enter and how much to invest in the quality of their product. We show that, under free entry with monopolistic competition, the sellers' demand-enhancing investments do not depend on any of the platform decisions. Rather, investment incentives only depend (beyond costs) on the parameters affecting sellers' competition, namely the elasticity of substitution among their products. As a consequence, the platform hybrid decision or the scope of its demand-enhancing activities do not affect the quality of the products offered by sellers. They have consequences though, since any investment in the platform reseller arm makes the number of entering sellers decline for given fee.

From a welfare perspective, our setting provides insights into the conditions upon which the hybrid mode is detrimental to consumers and into the effectiveness of the instruments available to policymakers to counter potential anti-competitive effects. We extend to a setting in which agents undertake demand-enhancing investments the finding from Etro (2023b) according to which the platform owner might increase or decrease the fee upon entry depending on the relative degree of differentiation between its own varieties and those of the sellers. Our setting allows us to discuss the opportunity of influencing agents' incentive to invest as an instrument to mold the welfare effects of the hybrid mode. When sellers' products are highly differentiated, leading to a higher fee and a negative impact on consumers due to platform entry, policymakers could increase the platform costs associated with investments. This measure would prevent the hybrid model from occurring and clearly improve consumer welfare. In such cases, encouraging sellers' investments lowers the fee and enhances consumer welfare, provided the external surplus is sufficiently larger than the marketplace's surplus. Conversely, if platform products are seen as more differentiated than those of the sellers so that the hybrid model increases welfare, promoting sellers' investments might reduce consumer welfare due to a raise in the fee. Finally, when the differentiation between the platform and sellers' products is roughly the same, supporting sellers' investments benefits consumers despite an increase in the commission.

The rest of the paper is organized as follows. Section 2 summarizes the relevant literature. Section 3 sets out the formal model. Section 4 characterizes the equilibrium. Section 5 conducts a welfare analysis and studies policy implications. Section 6 concludes. Proofs are in the Appendix.

2 Literature

Our paper contributes to the recently growing literature on the welfare implications of the hybrid platform business model (see Etro (2023a) for a comprehensive and up-to-date survey). It is possible to distinguish between a stream of the literature supporting welfare-enhancing effects (Anderson & Bedre-Defolie, 2022; Dryden et al., 2020; Hagiu et al., 2022; Shelegia & Hervas-Drane, 2022; Shopova, 2023; Zennyo, 2022), another suggesting that the hybrid mode is detrimental to consumer surplus (Anderson & Bedre-Defolie, 2023; Jiang et al., 2011; Padilla et al., 2022), and one delivering mixed results (Etro, 2021, 2023). Empirical evidence is surveyed by Zhu (2019) and has been recently advanced by Lee and Musolf (2023), and it is also mixed. There is therefore no consensus on the welfare implications of the hybrid model, which needs to be further investigated. We borrow from the literature two features: differentiated products and indirect network effects. Firstly, allowing for differentiated products guarantees incentives for both the platform's products and third-party products to survive and be remunerative for the marketplace simultaneously; that is, it allows us to treat a fully-fledged hybrid platform. Secondly, indirect network effects represent the focal point of an entire stream of literature supporting the agreement between private and social motives in the entry decision of hybrid marketplaces (see among others Dryden et al. (2020)). Furthermore, putting network externalities out of the picture might lead to biased conclusions, as competing on platforms is different from competing on traditional markets (Evans & Schmalensee, 2013). Both aspects are incorporated in the setup formalized by Anderson and Bedre-Defolie (2023) and Etro (2023b).

In our paper, we analyze the incentives of a hybrid marketplace to undertake demand-enhancing activities in favor of its own labels while taking into account the possible effect on third-party sellers' investments. This aspect is crucial to properly assess the welfare implications of the platform's entry as suggested by Dryden et al. (2020). In their paper, the authors sustain that the hybrid marketplace has to invest in the appeal of its own reseller arm so that it can steer price and non-price competition in the store, thereby increasing the store's traffic, which is a key dimension in terms of its sustainability and attractiveness vis-à-vis competing marketplaces and other distribution channels in light of significant network effects. Nevertheless, platform entry could determine a decline in sellers' investment negatively affecting welfare. On this point, the empirical literature delivers ambiguous results. Zhu and Liu (2018) and Wen and Zhu (2019) sustain that platform entry damages sellers' incentives to innovate, while Foerderer et al. (2018) and Li and Agarwal (2017) find the opposite result.

In such a setting, we investigate how the structural features of the market and the products characteristics shape the entry-related decisions of the platform and its welfare implications. In this regard, Zhu (2019)'s survey of empirical studies delivers multifaceted results concerning the impact of entry on complementors, suggesting that industry characteristics may intervene as moderating factors and influence the platform's strategies. To this end, differently from Etro (2023b)'s demand specification, we augment the baseline indirect utility function with preference weights assigned to each traded good. In doing so, we obtain a demand function featuring scale parameters that include the term measuring the elasticity of substitution among products; thus, the parameters are responsive to changes in the elasticity of substitution. Overall, on the one hand, we preserve the neutral property of aggregative games characterizing all the demand systems that satisfy the independence of irrelevant alternatives (IIA) property under free entry as far as consumer surplus measures are concerned (Anderson et al., 2020). On the other, we allow both the platform's products and the third-party products to be endowed with different quality levels, and the demand to be sensitive to quality changes to different degrees according to the elasticity of substitution among products. As a result, we effectively assess the implications on agents' decisions and welfare of the level of competition in the market and products characteristics.

3 The model

We consider a sizable platform hosting multiple (n) differentiated sellers, each of which supplies a single variety under monopolistic competition. Each seller incurs a fixed cost K to produce and participate in the marketplace (e.g., costs of entering into a contract with the platform or setting up necessary logistics to be able to sell on the platform) and has to pay an ad-valorem percentage fee, t. Seller i, if she joins the platform, sets the quality of the good supplied ω_i which costs $C(\omega_i) = b\frac{\omega_i^2}{2}$, and incurs marginal cost of production c_i.

If the marketplace operates in hybrid mode, the platform provides M varieties with a marginal cost of production c_k for each variety k. The quality of variety k is ω_k. Demand-enhancing investments M and ω_k, $k = 1, \dots, M$ are chosen by the platform at cost $C(\omega_k, M) = b_A\frac{\omega_k^2}{2} + f\frac{M^2}{2}$.[3]

Following the literature, we express preferences through an indirect utility that is a convex function of the prices of all products sold on the marketplace. In particular, we deploy the following quality-augmented demand for each seller i charging price p_i

$$q_i = \frac{\left(\sigma_1 - 1\right)\omega_i^{\sigma_1 - 1} p_i^{-\sigma_1}}{\Sigma_{j=1}^{n}\omega_j^{\sigma_1 - 1} p_j^{1-\sigma_1} + \Sigma_{k=1}^{M}\omega_k^{\sigma_2 - 1} p_k^{1-\sigma_2} + \theta} l \tag{1}$$

where l is the market size (e.g. the number of active consumers with disposable income normalized to 1, see for instance (Bertoletti & Etro, 2017)), p_j and p_k are the prices charged by seller j and by the platform for its variety k, θ is the outside surplus and the whole expression at the denominator (which includes the price aggregators of third party sellers and private labels) is called the "aggregator" and it is a sufficient statistic for the consumer surplus. Note that we allow the platform and sellers' items to respond to different elasticities, $\sigma_1 > 1$ and $\sigma_2 > 1$, in line with Etro (2023b)'s demand specification. However, differently from Etro, qualities ω_j and ω_k

[3] The separability assumption, $C_{M\omega_A} = 0$, is not necessary for the results, but simplifies the exposition.

are sensitive to changes in the elasticity of substitution. This feature is shared with, for instance, Baldwin and Harrigan (2011); Feenstra and Romalis (2014); Hottman et al. (2016); Redding and Weinstein (2020), and Alfaro and Lander (2021).

The timing of the game is as follows:

1. The platform sets an ad-valorem percentage fee t;
2. If the marketplace is hybrid, the platform undertakes the demand enhancing activity, simultaneously deciding the number of its own varieties, M, and their quality levels ω_k; then, it sets the prices of its own products p_k;
3. Entry of sellers takes place;
4. If sellers do enter, they incur the fixed cost, K, and set their quality ω_j and prices, p_j, under monopolistic competition.

We derive the subgame perfect free entry Nash equilibrium of the described sequential game by backward induction in the two platform modes, pure and hybrid. We characterize sellers' quality investments and, in the hybrid mode, the platform demand-enhancing investments. Finally, we contrast the equilibrium strategies and welfare implications under the two modes, studying the conditions under which a hybrid platform is beneficial to consumers.

4 Equilibrium analysis

4.1 Pure marketplace

We start by studying the pure marketplace mode. In this setting the platform does not sell its own products and only hosts third-party products. By the choice of its fee, t, the platform can control the number of sellers and affect their price. Given a fee set by the platform, we now consider first the monopolistic competitors (sellers) decisions.

Throughout the analysis we assume that $1 < \sigma_1 < 2$ and $1 < \sigma_2 < 2$. While the lower bound is a standard requirement, in the Appendix we show that the upper bounds are sufficient conditions for the optimization problem in quality-related investments of sellers and (hybrid) platform to be well-behaved under our cost functions and for the optimization problem of a hybrid platform when setting the ad-valorem fee to be well-behaved in case $\sigma_1 > \sigma_2$.

4.1.1 Seller's problem

In line with Anderson and Bedre-Defolie (2023), we assume symmetric sellers: all sellers face the same marginal cost, c, and fixed entry cost, K. If seller i enters, its demand is (1) with $M = \omega_k = 0$ and profits are

$$\Pi_i = (p_i(1-t) - c)q_i - b\frac{\omega_i^2}{2} - K \tag{2}$$

Maximizing profits with respect to price (not accounting for the marginal impact on the aggregator) yields the following price

$$p_i = p(t) = \frac{\sigma_1}{\sigma_1 - 1}\frac{c}{1-t} \tag{3}$$

Under monopolistic competition, the equilibrium price charged by the sellers is equal to the effective marginal cost of selling on the platform, $\frac{c}{1-t}$, plus a mark-up that increases as the elasticity of substitution, σ_1, decreases (or, equivalently, the product differentiation increases). As expected, the higher the ad-valorem fee charged by the platform, t, the higher the price imposed by sellers. In line with the monopolistic competition assumption, the price is independent of n and quality investment ω_i.

Incorporating the equilibrium price $p(t)$, each seller's demand and profits are

$$q_i = \frac{\left(\sigma_1 - 1\right)\omega_i^{\sigma_1 - 1}p(t)^{-\sigma_1}}{n\omega^{\sigma_1 - 1}p(t)^{1-\sigma_1} + \theta}l \equiv \left(\sigma_1 - 1\right)\frac{V(t, \omega_i)}{A(t)}l \tag{4}$$

$$\Pi = c\frac{V(t, \omega_i)}{A(t)}l - b\frac{\omega_i^2}{2} - K \tag{5}$$

where $V(t, \omega_i)$ and $A(t)$ denote, respectively, the numerator and denominator of q_i/l.

The optimal quality level for each seller ω_i^* satisfies the first order condition

$$\frac{\partial \Pi}{\partial \omega_i} = c\frac{\left(\sigma_1 - 1\right)\omega_i^{*\sigma_1 - 2}p(t)^{-\sigma_1}}{A(t)}l - b\omega_i^* = 0$$

where it is immediate to show that it must be $\sigma_1 < 3$ for ω_i^* to be a maximum. The first order condition can be conveniently rewritten as

$$c\frac{V(t, \omega_i^*)}{A(t)}l = \frac{b\omega_i^{*2}}{\sigma_1 - 1} \tag{6}$$

Note that in a free entry equilibrium each seller makes zero profits, that is

$$c\frac{V(t, \omega_i^*)}{A(t)}l - b\frac{\omega_i^{*2}}{2} - K = 0 \tag{7}$$

Plugging the LHS of the first order condition (6) into the zero profit condition (7) and solving for a seller's investment one obtains the equilibrium value of ω_i^*. Plugging this into $V\left(t, \omega_i^*\right)$ from the definition of V in (4) one finds its equilibrium value. The equilibrium value of the aggregator is found plugging $V\left(t, \omega_i^*\right)$ and ω_i^* in the first order condition (6) and solving for $A(t)$. Finally, the number of sellers joining the platforms is obtained from the definition of A in (4).

$$\omega^* = \left(\frac{2(\sigma_1 - 1)}{3 - \sigma_1} \frac{K}{b} \right)^{\frac{1}{2}} \tag{8}$$

$$V(t) = \left(\frac{2(\sigma_1 - 1)}{3 - \sigma_1} \frac{K}{b} \right)^{\frac{\sigma_1 - 1}{2}} p(t)^{-\sigma_1} \tag{9}$$

$$A(t) = \left(\frac{2(\sigma_1 - 1)}{3 - \sigma_1} K \right)^{-\frac{3-\sigma_1}{2}} \left(\sigma_1 - 1\right) b^{-\frac{\sigma_1 - 1}{2}} p(t)^{-\sigma_1} cl \tag{10}$$

$$n(t) = \frac{1 - t}{K + b\frac{\omega^{*2}}{2}} \frac{\sigma_1 - 1}{\sigma_1} l - \frac{\theta}{\omega^{*\sigma_1 - 1} p(t)^{1-\sigma_1}} \tag{11}$$

We can now state the first result of our analysis, which extends to a wider class of cost functions.

Proposition 1 *Under demand system (1) with no platform entry, the optimal seller investment ω^* does not depend on the ad-valorem fee charged by the platform for any power cost function $C(\omega) = b\frac{\omega^x}{x}$ with $x > \sigma_1 - 1$. Moreover, ω^* does not depend on the market size l, decreases in b, and increases in the entry cost K and in the elasticity of substitution σ_1.*

To get tractable results and focus our discussion on the main intuitions, we consider quadratic cost functions ($x = 2$), so that the equilibrium values of the aggregate, a seller surplus, its optimal investment and the number of entering sellers are given by (10) and (11).

The fact that ω^* does not depend on the ad-valorem fee follows from the properties of aggregative games with free entry. In particular, it relies on the fact that, as the fee changes, so does the price $p(t)$—in such a way that the mark-up stays constant—and the number of sellers joining the platform, which decreases so as to keep the demand of each participating seller constant. Hence, the marginal benefit of investing in quality is unaffected by the change in t and so does the optimal investment. Similarly, the market size does not affect the equilibrium quality investment decision. While a seller's demand increases in the market size, so does the number of sellers joining the platform, effectively making the marginal benefit of quality investment independent of the market size.

However, sellers invest more in quality the higher the degree of competition (measured by σ_1) and the higher the entry cost K. This happens because, as σ_1 increases, the price $p(t)$ decreases reducing the mark-up and, similarly, an increase in K reduces profits for given investment in quality. This profit erosion on the intensive margin is countered by a reduction in the number of entering sellers, which in turn leads to a higher demand for each seller and, as a consequence, a higher incentive to invest in quality. Finally, as obvious, ω^* decreases in the cost parameter b.

To make our analysis interesting, we consider a viable platform, that is one which is able to attract a positive number of sellers at the lowest fee, $t = 0$.

Assumption 1 $n(0) > 0$, or

$$K + b\frac{\omega^{*2}}{2} < \frac{cV(0)}{\theta}l \tag{Ass.1}$$

Hence, under (Ass.1) and given an ad-valorem fee t, a number of sellers defined by (11) join the platform, set quality (8) and produce quantity (4) at price (3).

4.1.2 Platform's problem

We now study the platform optimal choice of the ad-valorem fee. The platform profit is the total amount of revenue generated from fees collected from third-party sales

$$\Pi = tp(t)n(t)q(t) = t\frac{A(t) - \theta}{A(t)}\left(\sigma_1 - 1\right)l$$

Rearranging the first order condition and noting that $A'(t) = -\frac{\sigma_1}{1-t}A(t) < 0$, the optimal fee is implicitly derived from

$$\frac{\theta}{A(t)}\left(1 + \frac{t}{1-t}\sigma_1\right) = 1 \tag{12}$$

We remark that the optimal fee decreases with the outside surplus θ, the investment cost b, the entry cost K and the marginal production cost c. The platform reduces the ad-valorem fee in order to prop-up entry whenever the profitability of sellers decreases because of exogenous factors (θ, b, K and c). These results are standard in the received literature and proofs are omitted.

4.2 Hybrid platform

We now characterize the hybrid mode. In the hybrid mode, the platform sells its own products competing alongside third-party products. We assume symmetric platform varieties: all platform products share the same quality, ω_A, and marginal cost, c_A. As already mentioned, introducing M varieties with quality ω_A entails an increasing and convex separable cost. Alongside the number of own varieties M, and their quality ω_A, the platform sets both the price of its varieties and the fee, trading-off the entry deterrence effects of increasing the fee and competing with third-party sellers with the additional revenue coming from directly selling to customers.

4.2.1 Seller's problem

Seller i's demand is (1). The aggregator (its denominator) differs from that of Sect. 4.1 as it incorporates the platform's products. Since in monopolistic competition fringe sellers do not internalize the impact of their price decisions on the aggregator, they set the same price and quality investment as in the pure marketplace mode, defined in (3) and (8). Given the equilibrium price and investment, seller's demand and profits are again (4) and (5) with the caveat that the aggregator is now different. In fact, since by symmetry it will be $p_k = p_A$ for all k, the aggregator can be written as $A_h(t) = n\omega^{*\sigma_1 - 1}p(t)^{1-\sigma_1} + M\omega_A^{\sigma_2-1}p_A^{1-\sigma_2} + \theta$.

The zero-profit entry condition ties down the equilibrium value of the aggregator. Since, for given fee t, sellers' quality investments and price decisions are unchanged, this condition is (7) and implies that the equilibrium aggregator of a hybrid platform takes the same value as the aggregator of the pure marketplace, in line with the theory of aggregative games. Hence, $A_h(t) = A(t)$.

While it takes the same value, the composition of the aggregator is now different, since the entry of the platform products reduces the number of active sellers. To derive the number of active sellers in the hybrid mode, n_h, we use $V(t) = \omega^{*\sigma_1-1}p(t)^{-\sigma_1}$ to rewrite the aggregator as

$$A_h(t) = n_h V(t)p(t) + M\omega_A^{\sigma_2-1}p_A^{1-\sigma_2} + \theta \tag{13}$$

We then combine (7) with (13) and solve for n

$$n_h(t) = \frac{1-t}{K + b\frac{\omega^{*2}}{2}}\frac{\sigma_1 - 1}{\sigma_1}l - \frac{\theta + M\omega_A^{\sigma_2-1}p_A^{1-\sigma_2}}{\omega^{*\sigma_1-1}p(t)^{1-\sigma_1}}$$

We note that, for given fee, $n_h(t) < n(t)$ since the introduction of platform varieties crowds out some fringe sellers. Intuitively, the number of active sellers increases in the price set by the platform, p_A, and decreases in the quantity M and quality ω_A chosen by the hybrid platform.

4.2.2 Platform's problem

We now study the hybrid platform optimization problem. We first derive the optimal price p_A of the platform's varieties, which, owing to the properties of aggregative games, is independent of their number and quality. Given price p_A, the overall demand for the platform varieties and the platform profits are

$$q_A = \frac{\left(\sigma_2 - 1\right)M\omega_A^{\sigma_2-1}p_A^{-\sigma_2}}{n_h V(t)p(t) + M\omega_A^{\sigma_2-1}p_A^{1-\sigma_2} + \theta}l = \left(\sigma_2 - 1\right)\frac{V_A(t, \omega_A, M)}{A_h(t)}l \tag{14}$$

$$\Pi_h = tp(t)n_h(t)\frac{V(t)}{A_h(t)}\left(\sigma_1 - 1\right)l + (p_A - c_A)\frac{V_A(t, \omega_A, M)}{A_h(t)}\left(\sigma_2 - 1\right)l - C(\omega_A, M) \tag{15}$$

where $n_h(t)\frac{V(t)}{A_h(t)}\left(\sigma_1 - 1\right)l$ is the total quantity sold by the $n_h(t)$ platform users at price $p(t)$, and $V_A(t, \omega_A, M)$ is the surplus generated by the sale of platform varieties. Note that the profit is the sum of two terms: the revenues from collecting the fees from the sellers entering the platform, and the revenues originating from the platform own reseller arm.

It is useful to rewrite the profit using (13) as

$$\begin{aligned}\Pi_h =& t\frac{A_h(t) - V_A(t, \omega_A, M)p_A - \theta}{A_h(t)}\left(\sigma_1 - 1\right)l \\ &+ (p_A - c_A)\frac{V_A(t, \omega_A, M)}{A_h(t)}\left(\sigma_2 - 1\right)l - C(\omega_A, M)\end{aligned} \tag{16}$$

Since the aggregator $A_h(t)$, which takes the same value as in the pure mode, does not depend on p_A, the price maximizing profits is

$$p_A(t) = \frac{\sigma_2}{\sigma_2 - 1}\frac{c_A}{1 - t}. \tag{17}$$

The result that the price of the platform only depends on its effective marginal cost and the elasticity of substitution is, again, a property of aggregative games. More interestingly, the platform sets the same mark-up as the sellers as long as they face the same elasticity of substitution ($\sigma_1 = \sigma_2$). Conversely, when the platform faces a more (less) elastic demand than the sellers, it enjoys a lower (higher) mark-up.

As in the pure mode analysis, we are interested in a viable platform capable of attracting sellers. With endogenous platform entry, it is not obvious that $n_h(t)$ decreases in t. Let's thus define $\hat{t} \equiv \arg\max_{t\in(0,1)} n_h(t)$. Then, we make the following assumption.

Assumption 2 $n_h(\hat{t}) > 0$, or

$$K + b\frac{\omega^{*2}}{2} < \frac{cV(\hat{t})}{\theta + V_A(\hat{t}, \omega_A, M)p_A(\hat{t})}l \tag{Ass.2}$$

Comparing (Ass.1) and (Ass.2), we note that the entry condition becomes tighter when the platform goes hybrid (which follows from $V\left(\hat{t}\right) \leq V(0)$ and $V_A p_A \geq 0$).

We now turn to the optimal choice of quality ω_A and number M of platform varieties. Given the optimal price (17), the hybrid platform profit function becomes

$$\begin{aligned}\Pi_h =& t\frac{A_h(t) - \theta}{A_h(t)}\left(\sigma_1 - 1\right)l \\ &+ \left(t\left(\sigma_2 - \sigma_1\right) + (1 - t)\frac{\sigma_2 - 1}{\sigma_2}\right)\frac{\sigma_2}{\sigma_2 - 1}\frac{c_A}{1 - t}\frac{V_A(t, \omega_A, M)}{A_h(t)}l - C(\omega_A, M)\end{aligned} \tag{18}$$

where the first addendum represents the revenues from the commission on third-party sellers, while the second one is the revenue on own varieties net of the commission lost on the crowded-out third-party sellers.

Differentiating (18) with respect to ω_A and M and combining the first-order conditions, we obtain that the optimal quality ω_A^* and number M^* of the platform varieties solve the system

$$C_M = K_\omega \Delta \frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}} \frac{\omega_A^{*\,\sigma_2-1}}{\omega^{*\sigma_1-1}} \tag{19}$$

$$\frac{C_M}{C_{\omega_A}} = \frac{1}{\sigma_2 - 1} \frac{\omega_A^*}{M^*} \tag{20}$$

where K_ω represents the overall equilibrium entry cost of each seller (K plus quality investment cost $C(\omega^*)$). The expressions for K_ω and Δ are (25) and (26) in the Appendix. The relative marginal benefit between number of varieties and their quality, (20), depends on the elasticity of substitution between private labels: the more they are substitutes, the more convenient it is to invest in the quality rather than the quantity of varieties.

The equilibrium demand-enhancing investments, quality ω_A^* and number of varieties M^*, are

$$\omega_A^* = \left(\frac{\Delta K_\omega}{\omega^{*\sigma_1-1}} \frac{c^{\sigma_1-1}}{c_A^{\sigma_2-1}} \right)^{\frac{1}{2-\sigma_2}} \left(\frac{\sigma_2 - 1}{b_A f} \right)^{\frac{1}{2(2-\sigma_2)}} \tag{21}$$

$$M^* = \omega_A^* \left(\frac{b_A}{f\left(\sigma_2 - 1\right)} \right)^{\frac{1}{2}} \tag{22}$$

Note that, as the equilibrium values suggest and the first order conditions in the Appendix confirm, quality and number of varieties are strategic complements and co-move in equilibrium.

Introducing endogenous quality-enhancing investments allows us to make an interesting comparison with Etro (2023b). He points out that, when platform varieties and sellers' products are equally differentiated, a platform finds it optimal to go hybrid only if it enjoys a competitive advantage (lower marginal cost and/or higher quality) relative to third-party sellers.[4] That needs not be the case in our setting, where the platform may well enter (i.e., $\omega_A^* > 0$, $M^* > 0$) even in case third-party sellers enjoy a clear competitive advantage.[5]

[4] While in Etro (2023b) quality ω_A is exogenous and an optimal number of varieties M is not derived, we characterize optimal demand-enhancing investments using a quadratic separable cost function. Our main comparison though, holds even if we consider a fixed cost per variety and an exogenous upper bound on the number of platform varieties.

[5] Indeed, since, as shown in the Appendix, $\Delta > 0$ when $\sigma_1 = \sigma_2$, the platform has an incentive to go hybrid no matter its competitive stance vis-à-vis the sellers, provided its costs are not too high in absolute terms.

We now turn to the hybrid platform choice of the optimal fee. Recalling that $A_h(t) = A(t)$ and applying the envelope theorem, the optimal t solves the following first order condition

$$\frac{\theta}{A(t)}\left(1+\frac{t}{1-t}\sigma_1\right)=1+\left(1+\sigma_2\left(1-\left(\sigma_2-\sigma_1\right)\right)\frac{t}{1-t}\right)\frac{p_A(t)V_A(t)}{A(t)}\frac{\sigma_2-\sigma_1}{\sigma_2\left(\sigma_1-1\right)} \tag{23}$$

where from now on we drop the dependency of V_A on ω_A and M for readability.[6] Note how (23) compares to (12): depending on the relative size of the substitution elasticities, the optimal fee may be larger (if $\sigma_1 < \sigma_2$) or smaller (if $\sigma_1 > \sigma_2$) in the hybrid vs. the pure marketplace (a finding already in Etro (2023b)).

5 Welfare and policy implications

The introduction of endogenous demand-enhancing investments allows us to assess how entry and quality-investment decisions of a platform affect consumer welfare while considering its effect on sellers' incentives to invest in quality.

Suppose that the platform has decided to enter its own marketplace and the policymaker is concerned about the potentially negative effects on consumer welfare. Recalling that the aggregator is a sufficient statistic for consumer welfare, we take b, b_A, and f as policy instruments and study how welfare changes as they vary. Note that from (10) the aggregator depends directly only on b. We thus write, for the purpose of this section, $A(t, b)$. We notice further that b, b_A and f influence $A(t, b)$ through changes in the fee, t, by affecting platform investment decisions, ω_A^* and M^*.

We start studying how the demand-enhancing costs of the platform affect welfare. Since they only impact $A(t, b)$ indirectly, the tariff is a sufficient statistic for consumer surplus and we can focus on comparative statics of t with respect to b_A and f. Notice first that, for given fee, an increase in either b_A or f reduces both ω_A^* and M^*. However, via the first order condition (23), this affects the optimal fee. Now, since changes in demand-enhancing investments go through changes in V_A, it is clear from (23) that they will affect the fee in opposite directions depending on which among σ_1 and σ_2 is larger. Consider $\sigma_1 > \sigma_2$, then t increases and welfare $A(t, b)$ decreases. Intuitively, when sellers' products are less differentiated, the platform sets higher markups on its own products. However, to recover third-party sellers' entry and stimulate buyers' expenditure on the marketplace, the platform decreases the commission upon entry. The reduction in demand-enhancing investments (due to higher investment costs) reduces the value of the platform labels, V_A, and their demand, leaving room for the platform to raise the fee. This, in turn, reinforces the negative effect of an increase of investment costs on ω_A^* and M^*. A symmetric analysis holds when $\sigma_1 < \sigma_2$. The next proposition summarizes our discussion.

[6] In the Appendix we provide sufficient conditions for the second order condition to be satisfied.

Proposition 2 *An increase in the hybrid platform demand-enhancing investment costs* (b_A *and or* f) *unambiguously decreases demand-enhancing investments* ω_A^* *and* M^*. *Moreover, it leads to a higher* (*resp. lower*) *ad-valorem fee in equilibrium and to a lower* (*resp. higher*) *consumer welfare* $A(t, b)$ *whenever sellers' products are perceived as more* (*resp. less*) *substitutes than platform varieties, that is* $\sigma_1 > \sigma_2$ (*resp.* $\sigma_1 < \sigma_2$).

Proposition 2 tells us that, whenever sellers' products are less differentiated than platform varieties ($\sigma_1 > \sigma_2$, so that from (23) the fee under platform entry is lower), increasing platform investment costs would lead to a lower welfare. Hence, public authorities concerned with consumer welfare shall not oppose platform entry or investments. To the contrary, whenever sellers' products are more differentiated ($\sigma_1 < \sigma_2$, higher fee under platform entry), platform entry and demand-enhancing investments shall be hindered since curbing them increases consumer welfare. The finding that platform entry is beneficial whenever $\sigma_1 > \sigma_2$ is confirmed by our analysis in a context in which investments are strategic decisions. It relies on the result that, because of free entry, sellers' investment is not crowded-out by platform entry (Proposition 1). Absent this potentially negative effect, it turns out that the platform demand-enhancing investments reinforce the beneficial effect on welfare previously found in the literature.

An implication of Proposition 2 is that, if policymakers were willing to avert platform entry, but unwilling to ban it outright, they could nevertheless raise the costs for the platform (b_A and or f) to the point that $M^* < 1$, thereby effectively making the hybrid business mode unprofitable. Such a policy, which would be welfare enhancing if $\sigma_1 < \sigma_2$ (see (23)), could be implemented via an additional taxation on sales made by a platform on its own marketplace, justifiable on the ground that the platform enjoys a number of competitive advantages because of its role in the marketplace.

We now turn to the effect of sellers' investment cost, b, on welfare. Indeed, a policymaker may be tempted to facilitate sellers' investments by reducing the cost b, both to increase quality to the benefit of consumers and to foster competition with the platform products. We now study under which conditions this is a policy which increases consumer welfare, as measured by the aggregator, $A(t, b)$.

A reduction in b has a direct positive effect on the aggregator, $A(t, b)$, since it increases sellers' product quality and value to consumers. However, a reduction of b may increase the fee t charged by the platform in equilibrium and thereby reduce welfare. If the latter effect is sufficiently strong, reducing b to foster investments leads to a lower welfare.

We star considering a hybrid platform whose impact on welfare is small ($\sigma_1 \approx \sigma_2$). In this case, promoting investments (reducing b) increases welfare. The intuition is as follows: a reduction in b leads to higher quality products, a positive direct effect which induces the platform to increase the fee on sellers to take advantage of their stronger demand. Better sellers' products also push the platform to reduce its demand-enhancing investments and change the fee. However, since the markup difference between sellers and platform products is small, this latter effect is weaker than the former. As a consequence, the fee ends up increasing in response to a decrease of b. However, the change in the fee turns out to be weak enough that its second order negative effect on welfare is more than compensated by the first order effect of a larger quality, leading to a higher welfare overall.

Proposition 3 *Promoting sellers' investments (lowering their cost b) increases the equilibrium fee and the consumer welfare in the hybrid platform mode whenever the degree of product differentiation between sellers' goods is sufficiently close to that of the platform varieties ($\sigma_1 \approx \sigma_2$).*

We now consider a hybrid platform which is detrimental to consumer welfare ($\sigma_1 < \sigma_2$). In this case, promoting sellers' investments (reducing b) increases consumer welfare provided that the surplus derived by the consumption outside of the platform, θ, is large enough relative to that enjoyed by consuming on the marketplace. The intuition relies on the fact that, in this case, the fee decreases: hence, not only sellers offer more valuable products, but also the platform reduces the fee which is passed on to consumers. Why does the platform reduce the fee? On the one hand, the larger sellers' markup ($\sigma_1 < \sigma_2$) and lower platform investments push the platform owner to reduce the fee. On the other hand, the increase in sellers' demand invites the platform to raise t, but tough external competition (high θ) induces it to raise it moderately, since a large increase would cause a strong damage to sellers' demand and, indirectly, to the third-party sales revenue channel. When the external competition is strong enough, the former effect dominates, leading to a reduction of the fee and an overall increase in welfare. The next proposition summarizes.

Proposition 4 *Suppose a hybrid platform is welfare decreasing ($\sigma_1 < \sigma_2$). Then, if the external surplus is large enough compared to the surplus generated by the marketplace, promoting sellers' investments (lowering b) increases consumer welfare and reduces the optimal fee charged by the platform in equilibrium.*

Finally, we consider a hybrid platform which is beneficial to consumer surplus ($\sigma_1 > \sigma_2$). In this case, it can be easily shown that promoting sellers' investments through a decrease of b prompts an increase of the fee as an equilibrium response by the platform. Hence, while the direct effect of investment promotion on consumer surplus is positive since more value is created, the overall effect remains ambiguous and depends on the highlighted trade-off.

Proposition 5 *Suppose a hybrid platform is welfare increasing ($\sigma_1 > \sigma_2$). Then, promoting sellers' investments (lowering b) leads to an increase in the equilibrium value of the fee. The effect on consumer welfare is ambiguous.*

Comparing Propositions 3 and 5, one can see that, if σ_1 is larger than σ_2, then promoting sellers' investments increases the equilibrium fee, but the effect on welfare, which is positive when σ_1 is close to σ_2, may be negative when the elasticities of substitution are significantly different. While analytic conditions for this to happen cannot be derived, numeric examples in which a marginal reduction of b leads to a drop of consumer welfare can be found.[7] In such cases policymakers aiming to foster the competitive stance of sellers vis-à-vis the hybrid platform, should cautiously assess whether or not this is a wise policy, which is an ultimately empirical matter.

A note on total welfare is needed since we introduce quality-enhancing costs related to platform entry. We start noticing that while sellers' profits are nil in either scenario, platform profits are larger when it goes hybrid whenever ω_A^* and M^* are positive. That implies that total surplus, while it clearly increases when the fee decreases in case the platform goes hybrid, may also increase if the fee increases. This is the case when, even though more substitute, platform varieties are almost as differentiated as sellers' products. As equation (23) suggests, when σ_1 is lower but approximates σ_2, the fee is only mildly increased by the hybrid platform. As a result, the subsequent (moderate) reduction in consumer welfare is more than offset by the increase in platform profits when turning hybrid. Conversely, the overall welfare effect of platform entry when platform products are much more differentiated than sellers' cannot be discussed in such a setting since an explicit solution for the equilibrium fee is not available, making the theme fundamentally an empirical matter.

6 Conclusion

We have introduced a simple quality-augmented demand system featuring monopolistic competition in quality and prices of third-party sellers with a sizable platform, which can choose, on top of their quality, the number of its own products to supply. Such demand-enhancing activities constitute an additional instrument for the platform to steer competition in the marketplace by (possibly) influencing sellers' entry, investment, and pricing decisions. They shall be taken into account by policymakers when addressing the welfare implications of hybrid platforms. While, under free entry, sellers' quality investments are not affected by platform entry, the platform owner might increase or decrease the fee upon entry depending on the relative degree of differentiation between its own varieties and those of the sellers. When sellers' products are more differentiated so that the fee increases and platform entry is detrimental to consumers, the policymaker could raise platform costs associated with investments, thereby hindering hybrid mode occurrence and unambiguously healing consumer welfare. In such a scenario, promoting sellers' investments leads to a lower fee, increasing consumer welfare, provided that the external surplus is large enough compared to the surplus by

[7] For instance, consider $\sigma_1 = 1.95$, $\sigma_2 = 1.6$, $\theta = 10$, $l = 950$, $K = 0.001$, $c = c_A = 20$, $b = b_A = 60$ and $f = 0.002$. Then, in equilibrium, $t \approx 0.3$, $n = 12'855$, $\omega^* = 0.005$, $M = 209$, $\omega_A^*/M^* = 0.004$ and $A \approx 25$. While these numbers have no empirical validity outside the model, they illustrate the potential ambiguity of the investment promotion policy.

the marketplace. Nevertheless, when platform products are perceived as more differentiated than sellers' and the hybrid mode is welfare increasing, promoting sellers' investments could damage consumer welfare by involving higher fees. Finally, whenever platform and sellers' varieties are approximately equally differentiated, supporting sellers' investments is beneficial to consumers despite a raise in the fee.

Appendix

Derivation of (21) *and* (22).

We consider the difference between profit functions in the pure (Π_h) and hybrid (Π) settings. Notice that Π does not incorporate either M or ω_A. Denote the profit difference between the hybrid and the pure mode as $\Pi^{diff} \equiv \Pi_h - \Pi$, then

$$\begin{aligned}\Pi^{diff} &= \left(t\left(\sigma_2-\sigma_1\right)+(1-t)\frac{\sigma_2-1}{\sigma_2}\right)\frac{p_A(t)V_A(t,\omega_A,M)}{A_h(t)}l-b\frac{\omega_A^2}{2}-f\frac{M^2}{2}\\ &= MK_\omega\Delta\frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}}\frac{\omega_A^{\sigma_2-1}}{\omega^{\sigma_1-1}}-C\left(\omega_A,M\right)\end{aligned} \tag{24}$$

$$\text{with } K_\omega \equiv K+b\frac{\omega^{*2}}{2}=\frac{2K}{3-\sigma_1} \tag{25}$$

$$\Delta \equiv \frac{\sigma_1^{\sigma_1}}{\sigma_2^{\sigma_2}}\frac{\left(\sigma_2-1\right)^{\sigma_2}}{\left(\sigma_1-1\right)^{\sigma_1}}\left(1+\left(\sigma_2-\sigma_1\right)\frac{\sigma_2}{\sigma_2-1}\frac{t}{1-t}\right)(1-t)^{\sigma_2-\sigma_1} \tag{26}$$

Clearly, in equilibrium it must be $\Delta > 0$, which is trivially true whenever $\sigma_2 > \sigma_1$, while it requires that $t < \frac{1}{1-\frac{\sigma_2(\sigma_2-\sigma_1)}{\sigma_2-1}}$ when $\sigma_2 < \sigma_1$.

Differentiating with respect to M and ω_A we obtain system of first order conditions whose solution is (21) and (22)

$$\begin{aligned}FOC_M &: \quad K_\omega\Delta\frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}}\frac{\omega_A^{\sigma_2-1}}{\omega^{\sigma_1-1}}=C_M\\ FOC_{\omega_A} &: \quad \frac{\left(\sigma_2-1\right)M}{\omega_A}K_\omega\Delta\frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}}\frac{\omega_A^{\sigma_2-1}}{\omega^{\sigma_1-1}}=C_{\omega_A}\end{aligned}$$

from which, dividing side by side, in equilibrium it must hold

$$\frac{C_M}{C_{\omega_A}}=\frac{1}{\sigma_2-1}\frac{\omega_A}{M}.$$

The second derivatives can be written exploiting the FOCs as

$$\begin{aligned}
\frac{\partial^2 \Pi^{diff}}{\partial M^2} &= -\, C_{MM} \\
\frac{\partial^2 \Pi^{diff}}{\partial \omega_A^2} &= \frac{\left(\sigma_2 - 2\right)}{\omega_A} \frac{\left(\sigma_2 - 1\right) M}{\omega_A} K_\omega \Delta \frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}} \frac{\omega_A^{\sigma_2 - 1}}{\omega^{\sigma_1 - 1}} - C_{\omega_A \omega_A} \\
&= \frac{\left(\sigma_2 - 2\right)}{\omega_A} C_{\omega_A} - C_{\omega_A \omega_A}
\end{aligned}$$

while the mixed derivative reads

$$\frac{\sigma_2 - 1}{\omega_A} K_\omega \Delta \frac{c_A^{1-\sigma_2}}{c^{1-\sigma_1}} \frac{\omega_A^{\sigma_2 - 1}}{\omega^{\sigma_1 - 1}} - C_{M\omega_A} \rightarrow \frac{\sigma_2 - 1}{\omega_A} C_M - C_{M\omega_A}$$

For the solution to be a maximum it must be that $\frac{\partial^2 \Pi^{diff}}{\partial M^2} = -C_{MM} < 0$ (which is true under the convex separable cost function) and that the determinant of the Hessian matrix of the maximand Π^{diff} is positive, that is

$$\begin{aligned}
&\det \begin{bmatrix} -C_{MM} & \frac{(\sigma_2 - 1)}{\omega_A} C_M - C_{M\omega_A} \\ \frac{(\sigma_2 - 1)}{\omega_A} C_M - C_{M\omega_A} & \frac{(\sigma_2 - 2)}{\omega_A} C_{\omega_A} - C_{\omega_A \omega_A} \end{bmatrix} = \\
&\qquad = C_{MM} \left(C_{\omega_A \omega_A} - \frac{(\sigma_2 - 2) C_{\omega_A}}{\omega_A} \right) - \left(\frac{C_M \left(\sigma_2 - 1\right)}{\omega_A} - C_{M\omega_A} \right)^2 > 0
\end{aligned}$$

Since $C_{MM} > 0$, in general it must be that $C_{\omega_A \omega_A}$ is sufficiently large for the system to identify a maximum. Under the convex separable cost function the determinant becomes

$$f \left(b_A \left(3 - \sigma_2\right) - f \left(\frac{M\left(\sigma_2 - 1\right)}{\omega_A} \right)^2 \right)$$

which, substituting the equilibrium ratio from (22), becomes $-2fb_A\left(\sigma_2 - 2\right) > 0 \iff \sigma_2 < 2$.

Analysis of second order conditions in the optimal fee problem

Pure setting. Focusing on the pure setting, note that (12) is a rearrangement of the full FOC, $\frac{d\Pi}{dt} = \left(1 - \frac{\theta}{A(t)}\left(1 + \frac{t}{1-t}\sigma_1\right)\right)\left(\sigma_1 - 1\right)l = 0$. The SOC is $\frac{d^2\Pi}{dt^2} = -\frac{\theta}{A(t)} \frac{\sigma_1\left(2 + t\left(\sigma_1 - 1\right)\right)}{(1-t)^2}\left(\sigma_1 - 1\right)l < 0$.

Hybrid setting. Consider now the problem of the hybrid platform. In what follows, using (22) we write V_A as a function of $p_A(t)$ and $\omega_A(t)$ only. The FOC is

$$\begin{aligned}\frac{d\Pi_h}{dt} &= \frac{d\Pi}{dt} + \frac{d\Pi^{diff}}{dt} = 0\\ &= \left(1 - \frac{\theta}{A(t)}\left(1 + \frac{t}{1-t}\sigma_1\right)\right)(\sigma_1 - 1)l\\ &+ \left(1 + \frac{t}{1-t}\sigma_2\left(\sigma_1 - \sigma_2 + 1\right)\right)\frac{p_A(t)V_A\left(p_A(t), \omega_A(t)\right)}{A(t)}\frac{\sigma_2 - \sigma_1}{\sigma_2}l = 0\end{aligned}$$

where we have used the envelope theorem to get rid of the effect of the fee on profits via the price and quality enhancing investments.

To ease readability we study the problem of maximizing $\pi_h \equiv \frac{\Pi_h}{(\sigma_1-1)l}$, which leads to the FOC (23) and simplifies the study of SOCs. We thus have

$$\frac{d\pi_h}{dt} = \frac{d\pi}{dt} + \frac{d\pi^{diff}}{dt} = 0 \tag{27}$$

with

$$\frac{d\pi}{dt} = 1 - \frac{\theta}{A(t)}\left(1 + \frac{t}{1-t}\sigma_1\right) \tag{28}$$

$$\begin{aligned}\frac{d\pi^{diff}}{dt} =& \left(1 + \frac{t}{1-t}\sigma_2\left(\sigma_1 - \sigma_2 + 1\right)\right)\frac{p_A(t)V_A\left(p_A(t), \omega_A(t)\right)}{A(t)}\frac{\sigma_2 - \sigma_1}{\sigma_2\left(\sigma_1 - 1\right)}\\ &> 0 \Leftrightarrow \sigma_2 > \sigma_1\end{aligned} \tag{29}$$

We note here that $\pi^{diff} = 0$ if $\sigma_1 = \sigma_2$, so in this case the SOC under the hybrid mode is the same as that under the pure mode, which, as seen above, is satisfied. We further notice that, in the hybrid mode equilibrium,

$$\frac{d\pi}{dt} < 0 \iff \sigma_2 > \sigma_1 \tag{30}$$

The SOC (for generic $\sigma_1 \neq \sigma_2$) is

$$\frac{d^2\pi_h}{dt^2} = \frac{d^2\pi}{dt^2} + \frac{d^2\pi^{diff}}{dt^2} \tag{31}$$

where (specifying the arguments only when necessary)

$$\frac{d^2\pi(t)}{dt^2} = -\frac{\theta}{A(t)}\frac{\sigma_1\left(2 + t\left(\sigma_1 - 1\right)\right)}{(1-t)^2} \tag{32}$$

and

$$\begin{aligned}\frac{d^2\pi^{diff}(t,p_A(t),\omega_A(t))}{dt^2} =& \frac{d}{dt}\left(\frac{\partial\pi^{diff}}{\partial t}+\frac{\partial\pi^{diff}}{\partial p_A}\frac{dp_A}{dt}+\frac{\partial\pi^{diff}}{\partial\omega_A}\frac{d\omega_A}{dt}\right)\\ =&\frac{\partial\pi^{diff}}{\partial t\partial t}+\left(\frac{\partial\pi^{diff}}{\partial p_A\partial t}\frac{dp_A}{dt}+\frac{\partial\pi^{diff}}{\partial p_A}\frac{dp_A}{dtdt}\right)\\ &+\left(\frac{\partial\pi^{diff}}{\partial\omega_A\partial t}\frac{d\omega_A}{dt}+\frac{\partial\pi^{diff}}{\partial\omega_A}\frac{d\omega_A}{dtdt}\right)\\ =&\frac{\partial\pi^{diff}}{\partial t\partial t}+\frac{\partial\pi^{diff}}{\partial p_A\partial t}\frac{dp_A}{dt}+\frac{\partial\pi^{diff}}{\partial\omega_A\partial t}\frac{d\omega_A}{dt}\end{aligned}$$

since $\frac{\partial\pi^{diff}}{\partial p_A}=\frac{\partial\pi^{diff}}{\partial\omega_A}=0$ by optimality.

We now study the terms of $\frac{d^2\pi^{diff}}{dt^2}$ in turn. Tedious algebra allows to show that

$$\begin{aligned}\frac{\partial\pi^{diff}}{\partial t\partial t} =& \frac{d\pi^{diff}}{dt}\frac{\sigma_2}{(1-t)^2}\frac{1-\left(\sigma_2-\sigma_1\right)}{1+\frac{t}{1-t}\sigma_2\left(\sigma_1-\sigma_2+1\right)}\\ \frac{\partial\pi^{diff}}{\partial t\partial p_A} =& -\frac{d\pi^{diff}}{dt}\frac{\sigma_2-1}{p_A}\\ \frac{\partial\pi^{diff}}{\partial t\partial\omega_A} =& \frac{d\pi^{diff}}{dt}\frac{\sigma_2-1}{\omega_A}\end{aligned}$$

and

$$\frac{dp_A}{dt}=\frac{p_A}{1-t}\qquad\text{and}\qquad\frac{d\omega_A}{dt}=\frac{\omega_A}{1-t}\frac{\sigma_2-\sigma_1}{2-\sigma_2}\hat{\Delta}(t)$$

where

$$\hat{\Delta}(t)\equiv\frac{\sigma_2}{\left(\sigma_2-1\right)(1-t)+\left(\sigma_2-\sigma_1\right)\sigma_2 t}-1>0 \text{ whenever } \Delta>0$$

We then have

$$\begin{aligned}\frac{d^2\pi^{diff}}{dt^2} =& \frac{\partial\pi^{diff}}{\partial t\partial t}+\frac{d\pi^{diff}}{dt}\frac{\sigma_2-1}{1-t}\left(\frac{\sigma_2-1}{1-t}\frac{\sigma_2-\sigma_1}{2-\sigma_2}\hat{\Delta}(t)-1\right)\\ =&\frac{d\pi^{diff}}{dt}\frac{\frac{\sigma_2}{1-t}\frac{1-\left(\sigma_2-\sigma_1\right)}{\left(1+\frac{t}{1-t}\sigma_2\left(\sigma_1-\sigma_2+1\right)\right)}+\left(\sigma_2-1\right)\left(\frac{\sigma_2-1}{1-t}\frac{\sigma_2-\sigma_1}{2-\sigma_2}\hat{\Delta}(t)-1\right)}{1-t}\end{aligned}$$

so that (31) becomes

$$\frac{d^2\pi_h}{dt^2}=\frac{d^2\pi}{dt^2}+\frac{d\pi^{diff}}{dt}\frac{\frac{\sigma_2}{1-t}\frac{1-\left(\sigma_2-\sigma_1\right)}{1+\frac{t}{1-t}\sigma_2\left(\sigma_1-\sigma_2+1\right)}+\left(\sigma_2-1\right)\left(\frac{\sigma_2-1}{1-t}\frac{\sigma_2-\sigma_1}{2-\sigma_2}\hat{\Delta}(t)-1\right)}{1-t}\tag{33}$$

which, since $\frac{d^2\pi}{dt^2}<0$ from (32), is certainly negative for σ_1 and σ_2 close enough (the rightmost ratio takes definite values and $\frac{d\pi^{diff}}{dt}\propto\sigma_2-\sigma_1$).

Comparative statics on welfare (Propositions 2–5)

Preliminaries. Let $x \in \{f, b_A, b\}$. The impact on welfare of a change in x is

$$\frac{dA}{dx} = \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t}\frac{dt}{dx} \tag{34}$$

where A is (10). To do comparative statics we need to assess how the fee varies with x, $\frac{dt}{dx}$.

Denote by F the FOC (27). By the implicit function theorem, in equilibrium it holds

$$\frac{dt}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial t}}$$

We are interested in the sign of $\frac{dt}{dx}$, which, whenever the SOC holds, is the same as that of $\frac{\partial F}{\partial x}$ (since $\frac{\partial F}{\partial t} < 0$).

To study how ω_A^* and M^* vary with f, b_A and b we use (22) to simplify the analysis of $\frac{\partial F}{\partial f}$ and $\frac{\partial F}{\partial b_A}$. Recalling that $\pi_h\left(A(b), f, b_A, \omega_A\left(f, b_A, b\right)\right) = \pi(A(b)) + \pi^{diff}\left(A(b), f, b_A, \omega_A\left(f, b_A, b\right)\right)$, writing $V_A\left(p_A, \omega_A\left(f, b_A, b\right)\right) = p_A^{-\sigma_2}\left(\omega_A^*\left(f, b_A, b\right)\right)^{\sigma_2}\left(\frac{b_A}{f(\sigma_2-1)}\right)^{\frac{1}{2}}$ and omitting the arguments, in what follows we have

$$\begin{aligned}
\frac{\partial F}{\partial f} &= \frac{d\pi_h}{dtdf} = \frac{\partial}{\partial f}\frac{d\pi_h}{dt} + \frac{\partial}{\partial \omega_A}\frac{d\pi_h}{dt}\frac{d\omega_A^*}{df}\\
\frac{\partial F}{\partial b_A} &= \frac{d\pi_h}{dtdb_A} = \frac{\partial}{\partial b_A}\frac{d\pi_h}{dt} + \frac{\partial}{\partial \omega_A}\frac{d\pi_h}{dt}\frac{d\omega_A^*}{db_A}\\
\frac{\partial F}{\partial b} &= \frac{d\pi_h}{dtdb} = \frac{\partial}{\partial A}\frac{d\pi_h}{dt}\frac{dA}{db} + \frac{\partial}{\partial \omega_A}\frac{d\pi_h}{dt}\frac{d\omega_A^*}{db}
\end{aligned}$$

where, evaluating derivatives in equilibrium (using the FOC (27)), we can write

$$\begin{aligned}
\frac{\partial}{\partial f}\frac{d\pi_h}{dt} &= -\frac{1}{2f}\frac{d\pi^{diff}}{dt} & \frac{d\omega_A^*}{df} &= -\frac{\omega_A^*}{2f(2-\sigma_2)} < 0\\
\frac{\partial}{\partial b_A}\frac{d\pi_h}{dt} &= \frac{1}{2b_A}\frac{d\pi^{diff}}{dt} & \frac{d\omega_A^*}{db_A} &= -\frac{\omega_A^*}{2b_A(2-\sigma_2)} < 0\\
\frac{\partial}{\partial \omega_A}\frac{d\pi_h}{dt} &= \frac{\sigma_2}{\omega_A}\frac{d\pi^{diff}}{dt} & \frac{d\omega_A^*}{db_A} &= \frac{\omega_A^*\left(\sigma_1 - 1\right)}{2b\left(2-\sigma_2\right)} < 0\\
\frac{\partial}{\partial A}\frac{d\pi_h}{dt} &= \frac{1}{A} & \frac{dA}{db} &= -\frac{\sigma_1 - 1}{2}\frac{A}{b} < 0
\end{aligned}$$

from which

$$\begin{aligned}
\frac{\partial F}{\partial f} &= -\frac{d\pi^{diff}}{dt}\frac{1}{f}\frac{1}{2-\sigma_2} > 0 \iff \sigma_2 < \sigma_1 \\
\frac{\partial F}{\partial b_A} &= -\frac{d\pi^{diff}}{dt}\frac{1}{b_A}\frac{\sigma_2 - 1}{2-\sigma_2} > 0 \iff \sigma_2 < \sigma_1 \\
\frac{\partial F}{\partial b} &= \frac{\sigma_1 - 1}{2}\frac{1}{b}\left(-1 + \frac{\sigma_2}{2-\sigma_2}\frac{d\pi^{diff}}{dt}\right)
\end{aligned} \tag{35}$$

We now have all the building blocks to prove the propositions of Sect. 5.

Proof of Proposition 2 Note that

$$\frac{d\omega_A^*}{dx} = \frac{\partial \omega_A^*}{\partial x} + \frac{d\omega_A^*}{dt}\frac{dt}{dx} \text{ with } x \in \{f, b_A\}.$$

The result that ω_A^* decreases in either f or b_A follows from the results illustrated above and summarized below

f	$\frac{\partial \omega_A^*}{\partial f}$	$\frac{d\omega_A^*}{dt}$	$\frac{dt}{df}$	$\frac{\partial \omega_A^*}{\partial f} + \frac{d\omega_A^*}{dt}\frac{dt}{df}$
$\sigma_1 > \sigma_2$	$-$	$-$	$+$	$-$
$\sigma_1 < \sigma_2$	$-$	$+$	$-$	$-$

b_A	$\frac{\partial \omega_A^*}{\partial b_A}$	$\frac{d\omega_A^*}{dt}$	$\frac{dt}{db_A}$	$\frac{\partial \omega_A^*}{\partial b_A} + \frac{d\omega_A^*}{dt}\frac{dt}{db_A}$
$\sigma_1 > \sigma_2$	$-$	$-$	$+$	$-$
$\sigma_1 < \sigma_2$	$-$	$+$	$-$	$-$

As to M^*, note that

$$\frac{dM^*}{dx} = \frac{\partial M^*}{\partial x} + \frac{dM^*}{dt}\frac{dt}{dx} \text{ with } x \in \{f, b_A\}$$

where we already know the sign of $\frac{dt}{dx}$. Using (22) one can show that

$$\begin{aligned}
\frac{dM^*}{dt} &= \frac{d\omega_A^*}{dt}\sqrt{\frac{b_A}{f(\sigma_2 - 1)}} \text{ with } \frac{d\omega_A^*}{dt} = \frac{\omega_A}{1-t}\frac{\sigma_2 - \sigma_1}{2-\sigma_2}\hat{\Delta}(t) \\
\frac{\partial M^*}{\partial f} &= -\frac{\omega_A^*}{2f}\sqrt{\frac{b_A}{f(\sigma_2 - 1)}}\frac{3-\sigma_2}{2-\sigma_2} < 0 \\
\frac{\partial M^*}{\partial b_A} &= -\omega_A^*\frac{\sigma_2 - 1}{2-\sigma_2}\sqrt{\frac{b_A}{f(\sigma_2 - 1)}} < 0
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{dM^*}{df} &= \frac{\omega_A^*}{2-\sigma_2}\sqrt{\frac{b_A}{f(\sigma_2 - 1)}}\left(\frac{\sigma_2 - \sigma_1}{1-t}\hat{\Delta}(t)\frac{dt}{df} - \frac{3-\sigma_2}{2f}\right) < 0 \\
\frac{dM^*}{db_A} &= \frac{\omega_A^*}{2-\sigma_2}\sqrt{\frac{b_A}{f(\sigma_2 - 1)}}\left(\frac{\sigma_2 - \sigma_1}{1-t}\hat{\Delta}(t)\frac{dt}{db_A} - (\sigma_2 - 1)\right) < 0
\end{aligned}$$

where the inequalities follow from the observation that the sign of $\frac{dt}{df}$ and $\frac{dt}{db_A}$ is the opposite of that of $\sigma_2 - \sigma_1$.

We next study the impact on welfare of changes in f and b_A. From (34) and (10), noting that $\frac{\partial A}{\partial f} = \frac{\partial A}{\partial b_A} = 0$ and $\frac{\partial A}{\partial t} = -\sigma_1 \frac{A}{1-t} < 0$,

$$\frac{dA}{df} = -\sigma_1 \frac{A}{1-t}\frac{dt}{df} < 0 \iff \sigma_2 < \sigma_1$$

$$\frac{dA}{db_A} = -\sigma_1 \frac{A}{1-t}\frac{dt}{db_A} < 0 \iff \sigma_2 < \sigma_1$$

which completes the proof of Proposition 2.

Proof of Proposition 3 Notice that (34) specializes to

$$\frac{dA}{db} = -\frac{\sigma_1 - 1}{2}\frac{A}{b} - \sigma_1 \frac{A}{1-t}\frac{dt}{db} \tag{36}$$

and that, from (35) and (29), $\frac{dt}{db} < 0$ when σ_1 and σ_2 are close enough and equals zero when they are equal, in which case $\frac{dA}{db} < 0$. By continuity, when the elasticities are different, but close enough, the result stated in the proposition holds.

Proof of Proposition 4 In this case ($\sigma_2 > \sigma_1$) we notice that it follows from (35) and the FOC that

$$\frac{dt}{db} > 0 \iff \frac{\theta}{A(t)}\left(1 + \frac{t}{1-t}\sigma_1\right) > \frac{2}{\sigma_2}$$

Since the LHS of the above inequality is increasing in t, a sufficient condition for $\frac{dt}{db} > 0$ is that $\frac{\theta}{A(0)} > \frac{2}{\sigma_2}$. Hence, from (36) it is clear that lowering b reduces t and increases A, as claimed.

Proof of Proposition 5 In this case ($\sigma_2 < \sigma_1$) it clearly follows from (35) and the observation that $\frac{d\pi^{diff}}{dt} < 0$ that $\frac{dt}{db} < 0$ so that a reduction in b raises the fee, while from (36) its effect on the aggregate is ambiguous.

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Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no Conflict of interest.

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