Structural Change and Economic Dynamics 77 (2026) 274–284

Contents lists available at ScienceDirect

Structural Change and Economic Dynamics

journal homepage: www.elsevier.com/locate/strueco

Global value chains under full employment: An input–output linear optimization

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ARTICLE INFO

JEL classification:
C61
C67
D57
F62
O14

Keywords:
Rectangular choice of techniques
Linear programming
Multi-regional input–output systems
International trade
Industrial planning

ABSTRACT

Building upon Pasinetti's insights, the essay examines the restructuring of global value chains as they transition from being determined by industry level cost-minimization processes to being shaped by supranational coordinated planning aimed at maintaining full employment. By addressing alternative linear programming problems through the integration of the rectangular choice of technology framework within a multi-regional input–output system, we adopt the perspective of a hypothetical supranational social planner tasked with optimizing the level of global income while ensuring full employment within each individual country, thereby preventing forced cross-border migration and regional imbalances in employment. The investigation provides insights for shaping industrial and trade policies, specifically: i. the optimal structure of trade flows to maximize global income and achieve multi-regional full employment; ii. the level and composition of additional final demand required to attain such macroeconomic position; iii. the distribution of the supply of intermediate inputs between multiple providers. Furthermore, at the methodological level, an alternative procedure is proposed to derive effective quantities and prices through input–output linear programming when the number of constraints exceeds the number of variables leading to multiple choices.

> Linear programming is a mathematical technique for the solution of maximum (or minimum) problems when the functions concerned are linear. As a mathematical technique it is neutral with respect to any particular theory.
>
> Pasinetti (1977, p. 180)

1. Introduction

After an extended period of economic and political efforts aimed at mitigating the effects of the financial crisis, the European Union now faces multiple structural changes triggered by geopolitical, climatic, and health-related disruptions. These combined factors suggest that international trade will undergo a significant reconfiguration, both in terms of supply sources and of main destination markets.

The Draghi Report addresses the issue of the EU's role in future world scenarios (Draghi, 2024). The Report highlights that the most pressing challenge for the area lies in its dependence on other superpowers for raw materials and advanced technologies and identifies one of Europe's primary objectives as strengthening its competitive positioning by creating a truly integrated single market, within which firms will need to operate in key strategic sectors. What Draghi proposes is productive specialization in those strategic sectors, whereby European firms should position themselves within the global value chains (GVCs) based on narrowly defined areas of expertise, enhancing competitiveness within the Western trade bloc.

Inside an economic framework characterized by free capital mobility, production specialization based on comparative advantage loses its logical and theoretical primacy in favor of trade patterns driven by absolute advantages. The latter, in fact, have underpinned – over the past decades – the offshoring of segments of production processes from Western to Eastern European countries. Labor market arbitrage, based on wage differentials within the area, has been facilitated by the free movement of capital. Thus, technical specialization became the consequence of the formation of trade networks, rather than its main cause.[1]

In the context of ongoing structural transformation, major firms are, indeed, reformulating their strategies exploring options such as diversifying export markets and supply sources to establish regional production networks (Borin et al., 2023). These changes, however, do not

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[1] On this point, see Bellino and Fratini (2021), Parrinello (2010), and Cucignatto et al. (2025).

https://doi.org/10.1016/j.strueco.2026.01.013
Received 30 November 2024; Received in revised form 5 November 2025; Accepted 25 January 2026
Available online 26 January 2026
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indicate the emergence of a trend towards deglobalization. On the contrary, global trade continues to expand, driven by new digital technologies (Di Sano et al., 2023) and the persistent wage differentials that still make offshoring more advantageous than reshoring, as evidenced by companies relocating parts of their production from China to Vietnam or Mexico (Alfaro and Chor, 2023).

The aim of this research is to explore an alternative strategy that redefines the relationship between trade policy and industrial planning at the supranational level through coordinated action among countries. This involves a shift in perspective from the canonical concept of competitiveness based on pure efficiency – where the optimal structure of trade is conceived as minimizing production costs for each industry – towards the macroeconomic concept of equilibrium – theorized by Leontief, Keynes and, among others, by Pasinetti – which emphasizes the sustained full employment of the labor force. The rationale for this shift stems from the notion that market mechanisms alone are insufficient to ensure equitable distribution of resources among countries. Left unchecked, these mechanisms risk leaving entire sectors or economies behind in terms of production and employment. In other words, supranational public intervention is essential to drive the transition towards a paradigm that prioritizes societal well-being over international competitiveness.

The essay is organized as follows. The second section provides a review of the literature consistent with the theoretical and methodological principles underlying this study. The third section develops a rectangular choice of technology model based on a multi-regional input–output framework (MRIO–RCOT). The fourth section examines the theoretical scenarios to which this framework is applied by solving numerical problems through input–output linear programming (IO–LP). At the methodological level, an alternative procedure is proposed for solving linear programming problems in cases where the number of constraints exceeds the number of variables, often leading to multiple choices. The fifth section discusses the research findings and their implications within the context of a potential redefinition of supranational economic planning. Concluding remarks follow.

2. From linear programming to optimal supply chains

Linear programming is a mathematical technique used to determine optimal solutions to linear functions. Its primary application lies in the efficient allocation of given resources. In this context, traditional input–output analysis represents a special case of linear programming, specifically that in which only one single technique is available for the production of each good and resources are assumed to be indefinitely available (Leontief, 1941).

Kantorovich (1939; 1959) was a pioneer of this approach. His framework enabled the maximization of industrial output while adhering to specific constraints that ensured the rational use of given resources and machinery within the context of socialist planning. Koopmans (1951) further advanced linear programming by examining the relationships between the utilization of scarce resources in production and their implications for prices and economic efficiency.[2]

Pasinetti (1977, Appendix to Chapter VI) discussed the application of linear programming problems in a context different from the traditional one, specifically within the framework of the choice of production techniques as defined by Sraffa (1960). When multiple production methods are available, these can be represented within a rectangular matrix of technical coefficients, with n rows corresponding to the productive sectors and t columns representing the optional techniques that the sectors can choose from, based on a linear objective function.

This approach to input–output analysis has found wide application with Duchin and Levine (2011; 2017), who developed the RCOT model to simultaneously determine the relative quantities and their respective prices (dual solutions) of an economic system, given constraints on final demand and resources. The model can be extended to a MRIO framework (Leontief, 1953; Leontief and Strout, 1963) to determine the optimal structure of international trade based on the simultaneous objective functions and respective constraints of each country (Hammer Strømman and Duchin, 2006; Duchin and Levine, 2012).

This latter extension is used by Garbellini (2021) and Oro (2025) to study how the restructuring of GVCs can be represented through a simultaneous cost-minimization problem addressed at the level of individual industries. The result is that industries optimize the structure of trade exchanges based on absolute advantages rather than comparative advantages, contrary to what is suggested by traditional theory. If capital mobility is allowed, technical specialization becomes a consequence, rather than a cause, of the cost-optimization process undertaken by importing companies.

The linear programming approach is widely employed in environmental and energy optimization analysis to determine technical configurations that minimize CO2 emissions (Vogstad, 2009) and energy consumption levels (He et al., 2017; Kang et al., 2020), or to simulate the dynamic processes of the energy transition (Rocco, 2019).

The following investigation involves the use of IO–LP scenarios adopting a shift in reasoning compared to traditional trade strategies. This approach directly draws from the perspective outlined in Pasinetti's theory.

Pasinetti (1981; 1988) proposes a significant advancement in development planning techniques along Leontevian and Keynesian lines of macroeconomic equilibrium for full employment. Through the formulation of his "natural system" – built upon a closed Leontief system – Pasinetti establishes all macroeconomic conditions that must be met by a system to maintain balanced growth over time in a state of full employment. The perspective of the analysis is thus normative rather than institutional. The growth of consumption, associated with the growth of the labor force, is the exogenous variable that drives the production of goods and capital accumulation. In this sense, vertically integrated subsystems (Pasinetti, 1973; 1977) are reformulated as hyper-integrated sectors, which include in the necessities both the means of production and the additional productive capacity required by the expansion of demand driven by the growing labor force.

Balanced growth requires adherence to the distributive constraint, derived from the Cambridge equation (Kaldor, 1955; Pasinetti, 1962): the "natural" profit level corresponds to the value of the required investments – holding for each single growing subsystem, hence we have as many natural profit rates as there are sectors in the economy – while the "natural" wage remains a constant proportion of labor productivity. In other words, workers are allowed (and required by the balanced growth condition) to purchase all the consumable product, after the gross production has been allocated to the means of production and the new productive capacity to be installed. Consequently, the existence of profit is socially justified by the need to finance only investments, not the consumption of the ownership class of the means of production (Garbellini and Wirkierman, 2014; Bellino, 2021; Wirkierman, 2023).

Pasinetti's framework delineates a fundamental dichotomy between disaggregated (i.e., sectoral) efficiency equilibrium, which is derived from cost-minimization processes at the industry level, and macroeconomic full employment equilibrium. In contrast to the notion that full employment naturally results from real market dynamics, Pasinetti asserts that it must be actively pursued as the primary objective of economic regulation.

[2] As is well known, Kantorovich and Koopmans were awarded the Nobel Prize in Economic Sciences in 1975 "for their contributions to the theory of optimum allocation of resources", several years after Kantorovich had been awarded the Stalin Prize for Mathematics in 1949 in the Soviet Union. This fact granted Kantorovich's contribution a universal recognition that went beyond the ideological differences that characterized the division of the world during the Cold War era.

Table 1
Mathematical notation.

Symbol	Dimension	Description
t	scalar	Number of economic areas
$k = t^m$	scalar	Number of available alternatives for each sector
m	scalar	Number of produced commodities
$n = t \times m$	scalar	Number of sectors = Number of factors
$\mathbf{Q_N}$	$t \times 1$	MRIO vector of total labor availability
$\mathbf{E}$	$t \times n$	Region-wise aggregation matrix
$\overline{\mathbf{A}}$	$m \times m$	Matrix of technical coefficients
$\mathbf{A_{rb}}$	$m \times m$	Matrix of inter-industry transaction coefficients from country r to b
$\mathbf{A}$	$n \times n$	MRIO matrix of technical coefficients
$\mathbf{L}$	$n \times n$	MRIO Leontief inverse matrix
$\mathbf{y}$	$n \times 1$	MRIO vector of final demand (optimal)
$\overline{\mathbf{y}}$	$n \times 1$	MRIO vector of final demand (LP threshold)
$\mathbf{w}_r^{\mathbf{T}}$	$1 \times m$	Vector of nominal hourly wages, country r
$\mathbf{w}^{\mathbf{T}}$	$1 \times n$	MRIO vector of nominal hourly wages (factor prices)
$\boldsymbol{\eta}_r^{\mathbf{T}}$	$1 \times m$	Vector of direct labor coefficients, country r
$\boldsymbol{\eta}^{\mathbf{T}}$	$1 \times n$	MRIO vector of direct labor coefficients (factor requirements)
$\mathbf{F} = \widehat{\boldsymbol{\eta}}$	$n \times n$	MRIO factor requirement matrix
$\mathbf{A}^*$	$n \times (k \times n)$	MRIO augmented matrix
$\mathbf{q}^*$	$(k \times n) \times 1$	MRIO augmented gross quantity vector
$\mathbf{I}^*$	$n \times (k \times n)$	MRIO augmented identity matrix
$\mathbf{F}^* = \widehat{\boldsymbol{\eta}}^*$	$n \times (k \times n)$	MRIO augmented factor requirement matrix
$\mathbf{M}$	$(k \times n) \times n$	MRIO Moore–Penrose pseudo-inverse matrix
$\mathbf{p}^{\mathbf{T}}$	$1 \times n$	MRIO vector of production prices

Conversely, the economic policies of the European Union have centered, in recent decades, on two main principles: labor market flexibility and unrestricted capital mobility. Extensive scholarly literature suggests that this approach to economic integration has driven to unemployment and wage deflation. This outcome is connected to the increasing cross-border labor market arbitrage facilitated by the deregulation of capital flows, which disproportionately benefits multinational companies (Harvey, 2005; Guscina, 2006; Dunn, 2009; Bellofiore and Halevi, 2012; Selwyn, 2015).

In this paper we apply the MRIO-based RCOT framework to investigate the design of supranational planning aimed at defining a structure of trade flows that ensures full employment of the labor force in each country. Our investigation focuses on the formation of GVCs conceptualized as vertically integrated transnational subsystems.

The RCOT framework allows us to determine the relative quantities of produced goods. Given the cost structure and the production technology in use, the choice of techniques essentially represents the selection of the geographical distribution of suppliers. We assume that the technique available for producing goods is the same wherever production plants are located. Therefore, each industry chooses among different suppliers based on the multi-regional price structure, i.e. absolute advantages. By applying the MRIO–RCOT framework we put ourselves from the perspective of a hypothetical supranational social planner tasked with optimizing global income while ensuring full employment within each country, thereby preventing forced cross-border migration and regional imbalances in employment.

3. Methodology: MRIO-based RCOT framework

Let us consider a multi-regional economic system with $t = 3$ economic areas: center (c), semi-periphery (s), and periphery (p) – and $m = 2$ commodities – product 1 and product 2 – which are both used as means of production and are also allocated to final consumption. The sectors operating within the system are, therefore, $n = t \times m = 3 \times 2 = 6$. We assume that the final demand for these goods is satisfied internally by each economy, focusing on the international trade of intermediate inputs.

The production technology implemented in terms of the quantity of inputs required to produce each unit of output, wherever the plants are located, can be expressed through the following matrix:[3]

$$\overline{\mathbf{A}}_{(m\times m)} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

In this context, a_{ij} represents the quantity of input i required to produce one quantity of commodity j. The objective of the analysis is to determine the optimal structure of trade in intermediate inputs within the MRIO matrix, which we define as follows:

$$\mathbf{A}_{(n\times n)} = \begin{bmatrix} \mathbf{A_{cc}} & \mathbf{A_{cs}} & \mathbf{A_{cp}} \\ \mathbf{A_{sc}} & \mathbf{A_{ss}} & \mathbf{A_{sp}} \\ \mathbf{A_{pc}} & \mathbf{A_{ps}} & \mathbf{A_{pp}} \end{bmatrix}, \text{ where } \mathbf{A_{rb}} = \overline{\mathbf{A}}$$

where $\mathbf{A_{rb}}$ represents the matrix of inter-sectoral deliveries coefficients from country r to country b. This matrix is composed by elements of $\overline{\mathbf{A}}$, while the MRIO matrix $\mathbf{A}$ shows the trade patterns, i.e., the geographical position, of the technical coefficients. Hence, the structures of matrices $\mathbf{A_{rb}}$ and, subsequently, of matrix $\mathbf{A}$ have to be determined.[4]

Given the available techniques to produce the two commodities, we perform alternative IO–LP problems to obtain the network of trade flows along with the quantities that optimize the objective functions while respecting the constraints of final demand and available resources. Through each problem explored in the investigation, we can obtain the trade structures specified within the MRIO matrix, where each column represents the single GVC associated with the respective product. Inside the MRIO-based RCOT framework, the choice of techniques becomes the choice of alternative suppliers including domestic producers.

The MRIO vector of nominal hourly wages is:

$$\mathbf{w}^{\mathbf{T}}_{(1\times n)} = \begin{bmatrix} \mathbf{w}^{\mathbf{T}}_{\boldsymbol{c}(1\times m)} & \mathbf{w}^{\mathbf{T}}_{\boldsymbol{s}(1\times m)} & \mathbf{w}^{\mathbf{T}}_{\boldsymbol{p}(1\times m)} \end{bmatrix}$$

The MRIO vector of direct labor coefficients is:

$$\boldsymbol{\eta}^{\mathbf{T}}_{(1\times n)} = \begin{bmatrix} \boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{c}(1\times m)} & \boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{s}(1\times m)} & \boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{p}(1\times m)} \end{bmatrix}$$

We consider labor employed in different sectors as labor of different kinds. In terms of Duchin and Levine's papers, we have six sector-specific production factors (i.e. the number of factors equals the number of sectors n); the corresponding (diagonal) factor requirement matrix is therefore:

$$\mathbf{F}_{(n\times n)} = \widehat{\boldsymbol{\eta}}_{(1\times n)}$$

Each country can decide the geographical origin of the intermediate inputs used in final production; hence, each sector (where, by sector, we mean a country-commodity pair) has $k = t^m = 3^2 = 9$ available alternatives.

The computation requires that all the available alternatives are displayed in the initial data. Hence, the MRIO matrix and the factor requirement matrix have to be augmented in order to include all the options. The augmented matrices are obtained by adding columns/elements for each available alternative for each sector. Let us take one sector as a way of example: the production of commodity i in country r. The augmented MRIO "sub-matrix" is:

[3] Matrices are represented with boldface upper-case letters (e.g., $\mathbf{A}$), all vectors are column vectors represented with boldface lower-case letters, and their transposition is explicitly indicated (e.g., $\mathbf{p}^{\mathbf{T}}$). Vectors with a hat indicate a diagonal matrix with the vector elements on the main diagonal (e.g., $\widehat{\eta}$). Table 1 shows all the mathematical symbols with their respective dimensions and descriptions.

[4] Certainly, the notation $\mathbf{A_{rb}} = \overline{\mathbf{A}}$ is not the most appropriate, but its intended meaning is that $\mathbf{A_{rb}}$ is a matrix composed of the same elements as $\overline{\mathbf{A}}$, though located in positions to be determined through the IO–LP.

$$\mathbf{A}^{*}_{\boldsymbol{ir}(n\times k)} = \begin{bmatrix} a_{11} & a_{11} & a_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 \\ 0 & 0 & 0 & a_{11} & a_{11} & a_{11} & 0 & 0 & 0 \\ 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{11} & a_{11} & a_{11} \\ 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} \end{bmatrix}$$

which describes the features of each alternative option (in column) that the sector producing commodity i in country r can choose from, consisting exclusively of the combination of geographical locations of the suppliers (in row) of the inputs required.

As for the $\mathbf{F}^{*}$ matrix, it can be obtained by repeating each column k times:

$$\mathbf{F}^{*}_{[n\times(k\times n)]} = \begin{bmatrix} \eta_1^c & \eta_1^c & \cdots & \eta_1^c & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \eta_2^c & \eta_2^c & \cdots & \eta_2^c & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & \eta_2^p & \eta_2^p & \cdots & \eta_2^p \end{bmatrix}$$

The same holds for the $\mathbf{I}^{*}$ matrix:

$$\mathbf{I}^{*}_{[n\times(k\times n)]} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & 1 & \cdots & 1 \end{bmatrix}$$

The augmented vector of gross quantities resulting from the IO–LP is:

$$\mathbf{q}^{*}_{[(k\times n)\ \times\ 1]} = \begin{bmatrix} q_1^{c(1)} \\ \vdots \\ q_1^{c(9)} \\ q_2^{c(1)} \\ \vdots \\ q_2^{c(9)} \\ \vdots \\ q_2^{p(1)} \\ \vdots \\ q_2^{p(9)} \end{bmatrix}$$

where $q_i^{r(z)}$ is the gross quantity of commodity i produced in country r under the zth alternative (out of k). The matrices and vector are augmented such that each nth sector has k alternatives to choose from, hence $k \times n = 9 \times 6 = 54$.

The optimal relative quantities depend on the optimal trade flows, hence the MRIO matrix, where the "active" options – those chosen from the various available alternatives – are displayed by the computation. Linear programming, on the other hand, assigns a zero to the options of the augmented MRIO matrix (in column) and the augmented quantity vector (in row) that do not meet the optimal value of the objective function. These correspond to the "inactive" options. By eliminating the inactive alternatives we can transform the rectangular MRIO matrix ($\mathbf{A}^{*}$) into the optimal square MRIO matrix ($\mathbf{A}$) and the augmented quantity vector ($\mathbf{q}^{*}$) into the reduced quantity vector ($\mathbf{q}$).

We explore three alternative linear programming problems. The first aims to describe a situation where autonomous market forces are allowed to freely decide how to configure the trade structure. The second, through a change in the objective function, represents a potential solution in a scenario where a supranational social planner intervenes. The third problem, which is of particular interest for this investigation, involves integrating additional constraints into the previous problem that ensure multi-regional full employment, that is, full employment within each individual country.

Once the square MRIO matrix is obtained, we can derive the corresponding MRIO Leontief inverse matrix $\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1}$, with which we can compute the global production prices of commodities as follows:[5]

$$\begin{aligned} \mathbf{p}^{\mathbf{T}}_{(1\ \times n)} &= \mathbf{w}^{\mathbf{T}}_{(1\ \times n)}\mathbf{F}_{(n\times n)} + \mathbf{p}^{\mathbf{T}}_{(1\ \times n)}\mathbf{A}_{(n\times n)} = \\ &= \mathbf{w}^{\mathbf{T}}_{(1\ \times n)}\mathbf{F}_{(n\times n)}[\mathbf{I}_{(n\times n)} - \mathbf{A}_{(n\times n)}]^{-1} = \\ &= \mathbf{w}^{\mathbf{T}}_{(1\ \times n)}\mathbf{F}_{(n\times n)}\mathbf{L}_{(n\times n)} \end{aligned} \tag{1}$$

This step completes the analysis of the optimal transnational value chains through an MRIO-based RCOT model.

3.1. Cost-minimization without policy constraints

The first problem we consider is the canonical one of seeking efficiency at the industry level, that is, the minimization of production costs (labor costs). Let $\overline{\mathbf{y}}$ denote the threshold of the final demand vector, the IO–LP problem can be expressed as follows:

$$\underset{\mathbf{q}^{*}}{\text{minimize}} W = \mathbf{w}^{\mathbf{T}}_{(1\ \times n)}\mathbf{F}^{*}_{[n\times(k\times n)]}\mathbf{q}^{*}_{[(k\times n)\ \times\ 1]} \tag{2}$$

$$\text{subject to } \left[\mathbf{I}^{*}_{[n\times(k\times n)]} - \mathbf{A}^{*}_{[n\times(k\times n)]}\right]\mathbf{q}^{*}_{[(k\times n)\ \times\ 1]} \geq \overline{\mathbf{y}}_{(n\times\ 1)}$$

where $\mathbf{A}^{*}$, $\mathbf{I}^{*}$, $\mathbf{q}^{*}$ and $\mathbf{w}^{\mathbf{T}}\mathbf{F}^{*}$ represent, respectively, the MRIO matrix, the identity matrix, the vector of quantities produced, and the MRIO labor cost vector augmented according to all the available options for the different sectors. We aim to determine the quantities that minimize production costs, subject to the constraint of reaching the production level that satisfies final demand.

3.2. Income-maximization by the social planner

Here we analyze a different optimization problem. Suppose we take the role of a supranational social planner who intends to modify the trade structure between the different economies involved in order to maximize global income:

$$\begin{aligned} &\underset{\mathbf{q}^{*}}{\text{maximize}}\ V = \mathbf{w}^{\mathbf{T}}_{(1\ \times n)}\mathbf{F}^{*}_{[n\times(k\times n)]}\mathbf{M}_{[(k\times n)\ \times n]}\left[\mathbf{I}^{*}_{[n\times(k\times n)]} - \mathbf{A}^{*}_{[n\times(k\times n)]}\right]\mathbf{q}^{*}_{[(k\times n)\ \times 1]} \\ &\text{subject to } \left[\mathbf{I}^{*}_{[n\times(k\times n)]} - \mathbf{A}^{*}_{[n\times(k\times n)]}\right]\mathbf{q}^{*}_{[(k\times n)\ \times 1]} \leq \overline{\mathbf{y}}_{(n\times 1)} \end{aligned} \tag{3}$$

The objective function consists of the vector of value added coefficients per output unit. In this case, we are looking for the maximum value achievable by the objective function, thus the final demand constraint represents the upper bound of the area within which the optimal quantities must be researched.

Since we are dealing with physical rather than monetary quantities, formulating the objective function – namely, the maximization of global value added – would require the use of production prices which, as discussed above, are endogenous variables within this system. Indeed, in order to compute prices, we must first determine the optimal (square) MRIO matrix and thus its corresponding Leontief inverse. However, the matrix $[\mathbf{I}^{*} - \mathbf{A}^{*}]$ is not invertible, therefore, we can employ the Moore–Penrose pseudo-inverse matrix:

$$\mathbf{M} = [\mathbf{I}^{*} - \mathbf{A}^{*}]^{\mathbf{T}}\left([\mathbf{I}^{*} - \mathbf{A}^{*}][\mathbf{I}^{*} - \mathbf{A}^{*}]^{\mathbf{T}}\right)^{-1}$$

[5] The price results can be obtained, regardless of the derivation of the MRIO Leontief inverse, through the computation of the dual linear program associated with the primal problem under analysis.

which allows for the treatment of underdetermined or overdetermined systems while preserving the linear structure of the model.[6] We use $\mathbf{w}^{\mathrm{T}}\mathbf{F}^{*}\mathbf{M}$ as a generalized estimation of prices, where the pseudo-inverse matrix approximates the economic meaning of the Leontief inverse, i.e., the relationships between prices and labor costs, in a system of potential technologies.

3.3. Income-maximization with multi-regional full employment constraints

Finally, we explore the model of interest for this investigation. The optimization problem faced by the supranational social planner consists of maximizing global income considering that the planner intends not only to meet final demand but also to achieve full employment in each economy. Let $\mathbf{Q}_{\boldsymbol{N}}$ represent the vector of available labor in each country, we write:

$$\begin{aligned} \underset{\mathbf{q}^{*}}{\text{maximize}}\; V = \mathbf{w}^{\mathrm{T}}_{(1\times n)}\mathbf{F}^{*}_{[n\times(k\times n)]}\mathbf{M}_{[(k\times n)\times n]}\left[\mathbf{I}^{*}_{[n\times(k\times n)]} - \mathbf{A}^{*}_{[n\times(k\times n)]}\right]\mathbf{q}^{*}_{[(k\times n)\times 1]} \\ \text{subject to}\left[\mathbf{I}^{*}_{[n\times(k\times n)]} - \mathbf{A}^{*}_{[n\times(k\times n)]}\right]\mathbf{q}^{*}_{[(k\times n)\times 1]} \geq \overline{\mathbf{y}}_{[n\times 1]} \\ \mathbf{E}_{(t\times n)}\mathbf{F}^{*}_{[n\times(k\times n)]}\mathbf{q}^{*}_{[(k\times n)\times 1]} = \mathbf{Q}_{\boldsymbol{N}(t\times 1)} \end{aligned} \tag{4}$$

where $\mathbf{E}$ is an aggregation matrix to sum labor content region-wise:

$$\mathbf{E}_{(t\times n)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Since we aim to obtain the quantities that ensure employment exactly matches the available labor force in each country, the direction of the demand constraint is, in this case, set upward. In other words, we expect the model to reveal the volume and composition of final demand that needs to be added to the existing demand in order to achieve full employment. If final demand constraint were to represent the upper bound of the quantities, we would almost certainly have to relax the employment constraint. Part of the labor force would remain unemployed and the planner would have to abandon the multiple objectives. Therefore, the employment constraints are placed as strict equalities, meaning we expect the entire available labor force in each country to be employed. The reason is that the planner does not only want to achieve full employment globally but also seeks a certain distribution of supply that prevents forced migration of workers from one country to another.

4. Numerical applications

Let us assume that, regardless of the geographical location of the intermediate input suppliers, to produce product 1, 0.4 units of product 1 and 0.2 units of product 2 are required, while to produce product 2, 0.3 units of product 1 and 0.1 units of product 2 are required. Therefore, the matrix of technical coefficients can be written as follows:

$$\overline{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

We introduce the essential assumptions of a center-periphery scheme, i.e., that moving from the center to the periphery, both the hourly wage and labor productivity decrease. Additionally, we assume that product 2 is more labor-intensive than product 1, thus the hourly wage paid for the production of product 2 is lower than that paid for product 1 in any country where they are produced:

$$\mathbf{w}_1^{\mathrm{T}} = \begin{bmatrix} w_1^c & w_1^s & w_1^p \end{bmatrix} = \begin{bmatrix} 40 & 30 & 20 \end{bmatrix}$$

$$\boldsymbol{\eta}_1^{\mathrm{T}} = \begin{bmatrix} \eta_1^c & \eta_1^s & \eta_1^p \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$\mathbf{w}_2^{\mathrm{T}} = \begin{bmatrix} w_2^c & w_2^s & w_2^p \end{bmatrix} = \begin{bmatrix} 35 & 27 & 18 \end{bmatrix}$$

$$\boldsymbol{\eta}_2^{\mathrm{T}} = \begin{bmatrix} \eta_2^c & \eta_2^s & \eta_2^p \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 \end{bmatrix}$$

Therefore, we can write:

$$\mathbf{w}_c^{\mathrm{T}} = \begin{bmatrix} w_1^c & w_2^c \end{bmatrix} = \begin{bmatrix} 40 & 35 \end{bmatrix}$$

$$\boldsymbol{\eta}_c^{\mathrm{T}} = \begin{bmatrix} \eta_1^c & \eta_2^c \end{bmatrix} = \begin{bmatrix} 4 & 5 \end{bmatrix}$$

$$\mathbf{w}_s^{\mathrm{T}} = \begin{bmatrix} w_1^s & w_2^s \end{bmatrix} = \begin{bmatrix} 30 & 27 \end{bmatrix}$$

$$\boldsymbol{\eta}_s^{\mathrm{T}} = \begin{bmatrix} \eta_1^s & \eta_2^s \end{bmatrix} = \begin{bmatrix} 5 & 6 \end{bmatrix}$$

$$\mathbf{w}_p^{\mathrm{T}} = \begin{bmatrix} w_1^p & w_2^p \end{bmatrix} = \begin{bmatrix} 20 & 18 \end{bmatrix}$$

$$\boldsymbol{\eta}_p^{\mathrm{T}} = \begin{bmatrix} \eta_1^p & \eta_2^p \end{bmatrix} = \begin{bmatrix} 6 & 7 \end{bmatrix}$$

Let us define a vector of final demand, assuming that each sector sells an equal quantity of final goods within its own country. Moreover, we assume that moving from the center to the periphery, the available labor force increases:

$$\overline{\mathbf{y}} = \begin{bmatrix} y_1^c \\ y_2^c \\ y_1^s \\ y_2^s \\ y_1^p \\ y_2^p \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}, \quad \mathbf{Q}_{\boldsymbol{N}} = \begin{bmatrix} Q_N^c \\ Q_N^s \\ Q_N^p \end{bmatrix} = \begin{bmatrix} 3500 \\ 4500 \\ 5500 \end{bmatrix}$$

Thus, the total labor force in the system counts 13500 working hours.

The multi-regional structure of labor cost per unit of output can be defined as:

$$\begin{aligned} \mathbf{w}^{\mathrm{T}}\mathbf{F} &= \begin{bmatrix} \mathbf{w}_c^{\mathrm{T}} & \mathbf{w}_s^{\mathrm{T}} & \mathbf{w}_p^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \widehat{\boldsymbol{\eta}_c} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \widehat{\boldsymbol{\eta}_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \widehat{\boldsymbol{\eta}_p} \end{bmatrix} = \\ &= \begin{bmatrix} 160 & 175 & 150 & 162 & 120 & 126 \end{bmatrix} \end{aligned} \tag{5}$$

which represents the coefficients of wages per unit of output associated with the production cost of each supplier. To make the augmented vector of multi-regional labor cost we simply need to compute $\mathbf{w}^{\mathrm{T}}\mathbf{F}^{*}$ to repeat each column of the vector in Eq. (5) $k = 9$ times.

Let us define the augmented sub-matrix (product 1 in the center) in order to include the nine alternative options this single sector can choose from:

$$\mathbf{A}^{*}_{1c} = \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0.4 & 0.4 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 & 0.2 \end{bmatrix}$$

The augmented factor (labor) requirement matrix is:

$$\mathbf{F}^{*} = \begin{bmatrix} 4 & 4 & \cdots & 4 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 5 & 5 & \cdots & 5 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 7 & 7 & \cdots & 7 \end{bmatrix}$$

[6] When a matrix is underdetermined, that is, it has more columns than rows (as in the case of the augmented matrix $[\mathbf{I}^{*} - \mathbf{A}^{*}]$), the Moore–Penrose pseudo-inverse matrix results to be a new rectangular matrix of coefficients with transposed rows and columns. In our case, therefore, the pseudo-inverse $\mathbf{M}$ has $k \times n = 54$ rows and $n = 6$ columns.

4.1. Cost-minimization without policy constraints

Executing program (2) yields the following results, namely the trade structure, total quantities, and final demand that allow for the realization of the minimum production cost value:

$$\mathbf{Aq} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.3 & 0.4 & 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 550 \\ 300 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 450 \\ 200 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} \tag{6}$$

The choice of suppliers falls entirely on the producers from the periphery who, despite the low labor productivity in the economy, benefit from more convenient production processes in terms of labor cost for both products. In this case, producers from the center and semi-periphery delocalize parts of the chains to the periphery and import the intermediate inputs from there. We can assume that the purchasing power required to make these payments comes from the returns realized through their respective FDIs in the periphery.

Regarding the level and distribution of multi-regional employment, we obtain:

$$\mathbf{EFq} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 550 \\ 300 \end{bmatrix} = \begin{bmatrix} 900 \\ 1100 \\ 5400 \end{bmatrix} \tag{7}$$

The number of worked hours does not correspond to the overall labor force, neither at the national nor at the regional level. In fact, the overall unemployment rate remains at 45.18 %.

The Leontief inverse matrix associated with the MRIO matrix is:

$$\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.87 & 0.62 & 0.87 & 0.62 & 1.87 & 0.62 \\ 0.41 & 0.25 & 0.41 & 0.25 & 0.41 & 1.25 \end{bmatrix} \tag{8}$$

The production prices become:

$$\mathbf{p^T} = \mathbf{w^T FL} = [317.5 \quad 281.5 \quad 307.5 \quad 268.5 \quad 277.5 \quad 232.5] \tag{9}$$

Finally, we derive the total production cost:

$$\mathbf{p^T y} = \mathbf{w^T Fq} = 168500 \tag{10}$$

The central and semi-peripheral economies experience unemployment, while the periphery reaches a level close to full employment, but obtains a production structure dependent on foreign investment decisions made by companies from other countries, which are constrained by the results of cost-minimization.

Within this framework, the trade structure can only change in response to a shift in the labor cost structure, implying that, on the one hand, workers from the center and semi-periphery must either endure unemployment or forced migration to the periphery unless they accept a significant wage reduction that makes the substitution of imports – and the consequent reshoring of plans – economically viable. On the other hand, for workers in the periphery it will be more difficult to achieve wage increases, as peripheral firms will need to maintain their competitive position within GVCs as a necessary condition for the inflow of profits through net exports. In summary, the optimal structure of international trade based on autonomous industry decisions proves to be suboptimal when considering the global conditions of the labor market.

4.2. Income-maximization by the social planner

We now arrive at the problem faced by the supranational social planner who seeks to coordinate the integration of various economies within supply chains. The results obtained by computing LP problem (3) are as follows:

$$\mathbf{Aq} = \begin{bmatrix} 0.4 & 0.3 & 0 & 0.3 & 0 & 0 \\ 0.2 & 0.1 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 366.66 \\ 229.62 \\ 166.66 \\ 170.37 \\ 216.66 \\ 100 \end{bmatrix} = \begin{bmatrix} 266.66 \\ 129.62 \\ 66.66 \\ 70.37 \\ 116.66 \\ 0 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} \tag{11}$$

The level and distribution of employment obtained are:

$$\mathbf{EFq} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 366.66 \\ 229.62 \\ 166.66 \\ 170.37 \\ 216.66 \\ 100 \end{bmatrix} = \begin{bmatrix} 2614.81 \\ 1855.55 \\ 2000 \end{bmatrix} \tag{12}$$

The various components of the GVCs are distributed across multiple countries. However, the employment situation worsens. In fact, the globally employed working hours total 6470.37, which implies that the global unemployment rate increases to 52 %.

The Leontief inverse matrix becomes:

$$\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 1.87 & 0.62 & 0.2 & 0.62 & 0.2 & 0.12 \\ 0.41 & 1.25 & 0.41 & 0.13 & 0.04 & 0.02 \\ 0 & 0 & 1.66 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.11 & 0.37 & 0.22 \\ 0 & 0 & 0 & 0 & 1.66 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The production prices are:

$$\mathbf{p^T} = \mathbf{w^T FL} = [372.9 \quad 318.7 \quad 356.2 \quad 304.3 \quad 301.4 \quad 246.8] \tag{14}$$

while the total production cost is:

$$\mathbf{p}^{\mathrm{T}}\mathbf{y} = \mathbf{w}^{\mathrm{T}}\mathbf{F}\mathbf{q} = 190051.9 \tag{15}$$

which becomes logically higher than the cost realized in the previous program.

The maximum level of global income requires that the center becomes the only country to enjoy complete self-sufficiency in both inputs. The semi-periphery, instead, diversifies its production between the two products and remains open to foreign trade. In turn, the periphery becomes self-sufficient in the production of input 1, while it is in a position of dependence on the semi-periphery regarding the supply of input 2.

4.3. Income-maximization with multi-regional full employment constraints

In this IO–LP problem, the social planner aims to coordinate the supply of goods by determining the relative quantities to produce in order to maximize global income while ensuring full employment in each of the three economies considered. The results of solving program (4) can be summarized as follows:

$$\mathbf{A}^{\otimes}\mathbf{q}^{\otimes} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0.3 & 0.4 & 0.3 & 0.4 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 276.47 \\ 473.52 \\ 100 \\ 780 \\ 100 \\ 289.41 \\ 265.49 \\ 272.15 \end{bmatrix} = \begin{bmatrix} 79.64 \\ 0 \\ 680 \\ 0 \\ 189.41 \\ 437.64 \end{bmatrix}, \tag{16}$$

$$\mathbf{y} = \begin{bmatrix} 670.35 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

When the number of constraints exceeds the number of variables, as in this case, some sectors may choose multiple options to respect the additional constraints. Therefore, the quantity vector now contains eight elements corresponding to an eight-column reduced MRIO matrix. This discrepancy indicates that the active suppliers generate a rectangular matrix, which led us to introduce new superscripts ($\mathbf{A}^{\otimes}$ and $\mathbf{q}^{\otimes}$) to reflect the differing number of elements involved in the choice of suppliers. Additionally, the demand constraint is exceeded, with the first sector in the center needing to supply 570.35 additional units of final commodities compared to the initial lower bound constraint.

Theoretically, these results are meaningful because one sector can purchase the same intermediate input from multiple industries (e.g., the first sector in the center and the second sector in the periphery) and, moreover, additional final demand might be required to maximize the level of employment. The model formulation accounts for the structure of trade and the composition of additional final demand needed to resolve the problem.

The only obstacle we encounter is of a technical nature, consisting of the fact that the resulting MRIO matrix remains rectangular and, therefore, non-invertible, which prevents direct computation of production prices and, in general, obtaining a well-defined solution. This obstacle is typically overcome by relaxing the constraints, such as the final demand: if we sufficiently reduce some values in the demand vector, optimization may no longer require the selection of multiple techniques (Duchin and Levine, 2011). This procedure essentially involves going back, modifying the initial setup of the problem and repeating the computation. However, since we are not investigating the choice of available techniques per se but rather analyzing the choice of suppliers with given techniques, it becomes relevant to explore the possibility that the technical coefficients may be distributed along the supply chain between the sectors that produce the same intermediate input.

Therefore, we propose a method to transform the rectangular matrix of active suppliers into a square matrix without implying any discretionary modification of the constraints, thus avoiding the need for recalculation.

We post-multiply the rectangular matrix of active suppliers by the diagonal matrix of the extended quantity vector to obtain the rectangular matrix of intermediate consumption flows $\mathbf{Z}^{\otimes} = \mathbf{A}^{\otimes}\widehat{\mathbf{q}}^{\otimes}$. Subsequently, within this matrix, we can aggregate columns 1 and 2 – which correspond to suppliers of the two inputs needed to produce one unit of product 1 in the center – and columns 7 and 8 – which correspond to suppliers of the two inputs needed to produce one unit of product 2 in the periphery – in order to reduce the rectangular matrix and transform it into a square matrix of multi-regional intermediate flows:

$$\mathbf{Z} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 79.64 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 110.58 & 30 & 312 & 30 & 115.76 & 81.64 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 189.41 & 0 & 0 & 0 & 0 & 0 \\ 150 & 10 & 156 & 10 & 57.88 & 53.76 \end{bmatrix} \tag{17}$$

We can easily derive the reduced vector of quantities produced by aggregating the intermediate demand vector and the final demand vector, both of which are found in the results (16):

$$\mathbf{q} = \mathbf{Z}\mathbf{e} + \mathbf{y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 79.64 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 110.58 & 30 & 312 & 30 & 115.76 & 81.64 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 189.41 & 0 & 0 & 0 & 0 & 0 \\ 150 & 10 & 156 & 10 & 57.88 & 53.76 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 670.35 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 750 \\ 100 \\ 780 \\ 100 \\ 289.41 \\ 537.64 \end{bmatrix} \tag{18}$$

where $\mathbf{e}$ is the summation vector. Finally, we derive the square MRIO matrix of technical coefficients by post-dividing the square matrix of intermediate flows by the diagonal matrix of the reduced vector of total quantities:

$$\mathbf{A} = \mathbf{Z}(\widehat{\mathbf{q}})^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.15 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.15 & 0.3 & 0.4 & 0.3 & 0.4 & 0.15 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix} \tag{19}$$

We now have a new square MRIO matrix of technical coefficients with 6 rows and 6 columns. It may seem that the production technology has changed, but in fact, some technical coefficients have simply been distributed along the value chain between multiple suppliers. To produce product 1 in the center, 0.4 units of product 1 and 0.2 units of product 2 are still required. However, the first sector in the center now purchases 0.15 units of input 1 from the semi-periphery and 0.25 units of the same input from the periphery. Similarly, the second sector in the periphery still requires 0.3 units of product 1 and 0.1 units of product 2, but it purchases 0.15 units of product 1 from the center and 0.15 units of the same input from the semi-periphery.

The employment levels are:

$$\mathbf{EFq} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 750 \\ 100 \\ 780 \\ 100 \\ 289.41 \\ 537.64 \end{bmatrix} = \begin{bmatrix} 3500 \\ 4500 \\ 5500 \end{bmatrix} \quad (20)$$

where we can verify that the unemployment rate turns to 0 % both globally and within each economy.

The Leontief inverse matrix becomes:

$$\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 1.06 & 0.03 & 0.06 & 0.03 & 0.06 & 0.18 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.54 & 0.57 & 1.79 & 0.57 & 0.79 & 0.39 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.26 & 0.01 & 0.01 & 0.01 & 1.01 & 0.04 \\ 0.41 & 0.25 & 0.41 & 0.25 & 0.41 & 1.25 \end{bmatrix} \quad (21)$$

This matrix is less concentrated compared to the inverse matrix obtained from the same objective function without the policy constraints, indicating that sectoral and regional (direct and indirect) interdependencies increase as a result of full employment constraints.

Finally, the MRIO production price vector associated with the optimal solution is:

$$\mathbf{p^T} = \mathbf{w^T FL} = [\,336.3 \quad 300.3 \quad 333.8 \quad 287.3 \quad 303.8 \quad 251.6\,] \quad (22)$$

The production cost becomes higher due to the greater number of quantities that had to be produced in order to respect the additional constraints:

$$\mathbf{p^T y} = \mathbf{w^T Fq} = 373172.9 \quad (23)$$

The situation changes significantly. The center is no longer in a state of complete self-sufficiency, but imports from the other countries. The semi-periphery specializes its supply in product 1. The periphery tends to specialize in product 2 (labor-intensive) because of the larger employable workforce and the lower labor productivity of domestic firms. Moreover, it is relevant to note that since the production stages are delocalized in the peripheral areas, the additional final demand required to achieve full employment in the center is significantly higher than the threshold. The result is that full employment is achieved both globally and within each individual country: no worker remains unemployed or is forced to migrate somewhere else. The Appendix presents the sensitivity analyses for each IO–LP problem considered in this section, along with a brief commentary.

5. Findings and discussion

The results of our programs can be interpreted through the lens of top-down causation, as developed in complex systems theory (Ellis, 2009). In this perspective, macro-level structures – like optimization objectives or physical and institutional constraints – exert causal influence on lower-level configurations, shaping the emergent properties of the system. This mechanism is particularly evident in the model variants explored here, where the structure of trade adjusts endogenously in response to changes in the objective function or in the constraints.

Notably, in program (2), which describes a situation where market mechanisms operate freely and production costs are minimized, we obtain a suboptimal situation in terms of global employment and its distribution. The offshoring in the periphery lead to unemployment in both the center and the semi-periphery while forcing industries in the periphery to continuously exert pressure on labor costs in order to avoid being displaced by the supply chains.

In program (3), the situation changes due to the fact that the structure of trade flows maximizes global income. However, although we observe a broader distribution of production and employment, the overall employment level worsens in absolute terms.

In this sense, the trade patterns emerging from different linear formulations may be understood as outcomes of top-down causation within constrained computational frameworks.

The two preliminary problems help to establish a comparison with a scenario where exchanges are governed by a supranational social planner who wants to maximize global income while imposing multiregional full employment constraints, as explored in program (4). The key results derived from this formulation can be succinctly discussed as follows.

i. The final MRIO-based RCOT model allows for determining the optimal trade flow structure to achieve the maximum global income level while simultaneously ensuring full employment in each country. Unemployment is not only addressed at the global level – an approach that would still force workers to migrate from one country to another, as observed in programs (2) and (3) – but it is minimized in each singular country. This is the key aspect of supranational industrial planning, which aims to ensure that no country is left behind in terms of workers' positions, protecting society both from unemployment and from forced migration.
ii. To reach such equilibrium position, an increase in final demand beyond the initially expected level might be necessary. The model provides the volume and composition of the additional final demand required to meet all policy constraints. The result can also be interpreted as a broad fiscal expansion within a selected country rather than a final demand increase in terms of a precise basket of goods. Therefore, this formulation allows to understand not only the volume of expansionary fiscal policy to be implemented, but also the country that is optimal to stimulate to achieve the specific objectives.
iii. The framework allows for determining the distribution of supply along GVCs. The result obtained in this regard is consistent with a vision based on coordination between countries, preferred over a typical competitive behavior based on the absolute advantages of companies. Indeed, industries are not completely left to operate within a specific area of expertise where they are encouraged to specialize, but the supply of intermediate goods can be distributed among multiple industries located in different countries. This way, potential supply bottlenecks can be prevented, as a particular input is provided by multiple producers.
iv. On a purely methodological level, we have developed an alternative procedure to solve IO–LP problems in those cases where the number of constraints exceeds the number of variables, leading to the choice of multiple techniques/suppliers for the production of the same final good. Typically, one would go back, modifying some values in the constraints of final demand or resources, and recalculating the program so that each sector selects a single technique. This procedure is unsatisfactory in our context, where the choice of suppliers, for given and uniform techniques in the various countries, takes priority over the selection of production methods in the strict sense. Indeed, within our theoretical framework, it is entirely plausible that the supply of the same product is carried out by multiple industries located in different countries. Our method allows for determining the optimal proportions of supply by "unpacking" the single technical coefficient of the intermediate input across the sectors integrated into the GVCs.

These results may prompt reflection on the nature of the EU's supranational policies. Until now, we have witnessed an economic integration model based on capital mobility and labor market flexibility. The combination of these two elements has led to the establishment of a relationship between center and periphery, specifically between the Northern countries on one hand, and the Mediterranean and Eastern countries on the other. In this context, we have observed the multiple offshoring of certain stages of GVCs to peripheral areas, which allowed for minimizing labor costs and, in some cases, benefitting from more flexible labor safety regulations.

This production and trade model has resulted in a process of wage stagnation across the region. This is due both to rising unemployment caused by offshoring (and the subsequent deindustrialization) and to the supply of products that, over time, ended up being sold at lower prices, precisely because they could be imported from abroad.

The situation of imbalance was further exacerbated by the austerity policies implemented to absorb the effects of the 2007–2008 financial crisis. The current accounts of the entire Eurozone were protected. However, the recovery did not result from growing foreign trade and exports; rather, it was the outcome of pressure on wage growth and public spending, which together significantly reduced the imports and increased the price competitiveness of exports in extra-European markets. The impact on income distribution was relevant, not only between workers and producers within each country, but also between the center and the periphery of Europe.

A decisive reorientation of European economic policies appears necessary, one capable of steering the region toward genuine integration founded on coordination among countries in the regulation of labor markets and international capital flows, with the achievement of full employment as the central objective. The goal of increasing competitiveness through wage deflation must, naturally, be redefined. It would involve supporting a "higher cost" with the long-term objective of economic recovery and integration. Industrial policies should be able to restructure GVCs in such a way that final goods are primarily directed towards European households rather than being produced mainly for export outside the region. Similarly, capital mobility should be regulated in a manner that does not undermine the level and distribution of employment within the area.

This shift in objectives – from competitiveness to full employment – that we propose here does not contradict the context of global polarization that the EU is currently facing. A framework of supranational policies oriented towards protecting employment and wage growth is functional to the revitalization of the European domestic market, which has thus far been excluded from export-driven growth models.

6. Concluding remarks

We have proposed an analysis of supranational economic planning starting from a shift in perspective, from the objective of competitiveness towards foreign markets to the objective of full employment toward coordinated multi-regional market. Our investigation thus contrasts with the canonical concept of efficiency, based on minimizing production costs for all industries. However, it is not inconsistent with the primary goals of the EU in the context of the necessary energy transition in times of international uncertainty.

An economic policy approach aimed at making international trade relations more robust (and less conflictual), even at the expense of higher production costs and loss of competitiveness, aligns with the long-term priorities that are now widely accepted. Full employment can therefore return to being a primary objective of European economic regulation, as has been suggested for long by Keynes, Leontief, and Pasinetti.

Our investigation undoubtedly has limitations related to aspects that have been kept "in the background" here, even though they are still highly relevant. In particular, the framework does not include the analysis of trade imbalances or external constraints. An extension of this scheme with trade balances can provide an additional tool to limit the risks of financial crises within the EU caused by sudden stops and capital flights.

Further extensions of this framework could incorporate a perspective of joint production, achievable by integrating the formulation within supply and use tables. The integration of the model within an energy flow table would be essential for designing industrial policies aimed at the energy transition, as decarbonization, alongside employment and distributive policies, is one of the key collective issues of nowadays. By incorporating these elements, the model could provide more detailed insights into how energy raw material flows and production processes across borders interact with labor markets within national economies, ultimately facilitating the development of policies that balance environmental constraints with economic and social development.

CRediT authorship contribution statement

Gianmarco Oro: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization.

Acknowledgements

I wish to thank the two anonymous reviewers for helping to prevent errors and inaccuracies. Full responsibility for any mistakes remains with me.

Appendix. Post-optimal report

In this Appendix we briefly discuss the results obtained from sensitivity analyses conducted for each IO–LP problem considered. We limit this report to the sensitivity analysis of the constraints, which are the quantities that compose the final demand and the available labor forces available in each country.

The shadow price indicates how much the objective function optimal value would increase if the constraint limit were increased by one unit. A value equal to zero indicates that the constraint is non-binding, meaning it does not determine the optimal solution for the objective function. A negative value suggests that the constraint is inversely related to the optimization, i.e., the function's optimal value decreases when the constraint is increased by one unit. The lower and upper limits show the range within which the constraints can vary without changing the shadow prices, highlighting the range of significance for the optimal value. The surplus values are simply the differences between the optimal final demand and resource quantities and those thresholds set as constraints.

Table A.1
Sensitivity analysis for program (2).

Constraint	Shadow price	Lower bound	Upper bound	Surplus
q_1^c	317.5	0	1×10^{30}	0
q_2^c	281.5	1.42×10^{-14}	1×10^{30}	0
q_1^s	307.5	0	1×10^{30}	0
q_2^s	268.5	1.42×10^{-14}	1×10^{30}	0
q_1^p	277.5	− 193.33	1×10^{30}	0
q_2^p	232.5	− 140	1×10^{30}	0

The results of the sensitivity analysis for program (2) are shown in Table A.1. All shadow prices are positive, which is consistent with economic theory in cost-minimization problems, where active (binding) constraints indicate that the productive capacity is fully utilized. In fact, surplus values are equal to zero for each constraint. The lower bounds for some constraints are reported as negative. This does not imply that negative demand is feasible, but rather that the shadow price remains constant even if the constraint were relaxed significantly. The upper bounds are uniformly high, indicating that no explicit bounds were set for these constraints. Therefore, the shadow prices are valid as long as the final demand constraint stays within a practically unconstrained interval.

Table A.2
Sensitivity analysis for program (3).

Constraint	Shadow price	Lower bound	Upper bound	Surplus
q_1^c	346.3	− 95.55	1×10^{30}	0
q_2^c	302.1	− 83.7	1×10^{30}	0
q_1^s	338.3	0	1×10^{30}	0
q_2^s	289.8	− 53.33	1×10^{30}	0
q_1^p	314.3	− 30	1×10^{30}	0
q_2^p	255.5	1.42×10^{-14}	1×10^{30}	0

The post-optimal report for program (3) is shown in Table A.2. All shadow prices are strictly positive, which implies that any increase in final demand leads to a proportional increase in value added. This suggests that all constraints are binding and the current production configuration fully exploits the available capacity under the given final demand. In other terms, the system is operating at a frontier: there is no slack, and expanding any final demand yields clear economic gains. The lower bounds show how much each demand constraint can be relaxed before the shadow price would change. The upper bounds are extremely large, indicating no practical upper constraint was imposed. Thus, the system could absorb increases in final demand while maintaining optimality, at least until demand constraints are triggered elsewhere.

Table A.3
Sensitivity analysis for program (4).

Constraint	Shadow price	Lower bound	Upper bound	Surplus
q_1^c	0	-1×10^{30}	1×10^{30}	570.3
q_2^c	− 15.6	-4.26×10^{-14}	268.9	0
q_1^s	− 8	20.3	181.6	0
q_2^s	− 20.72	11.1	187.4	0
q_1^p	− 32	20.3	181.6	0
q_2^p	− 50.1	9.73	232.1	0
Q_N^c	36.6	2947.3	4123.8	0
Q_N^s	29.3	3643.2	5440.2	0
Q_N^p	24.4	4471.8	6335.5	0

The post-optimal report for program (4) is shown in Table A.3. The results show that the shadow prices associated with some final demand constraints are negative, indicating that increasing demand in these sectors would lead to a reduction in the overall value added. This suggests that such sectors are less efficient under the constraint of full employment. In contrast, the shadow price of zero for some demand constraints implies that their marginal variation has no effect on the objective function, which is further confirmed by the presence of a surplus. More critically, the constraints related to the availability of labor in each country are all binding, as shown by their zero surplus.

Data availability

No data was used for the research described in the article.

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