Received: 18 January 2022 | Revised: 25 May 2022 | Accepted: 3 October 2022

DOI: 10.1111/theo.12441

ORIGINAL ARTICLE

A critique of Malpass's argument against Supervaluationism

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Abstract

Supervaluationism is one of the most discussed approaches to the semantics of future tense sentences in a branching time. In this paper, we consider the criticism advanced by Malpass against Supervaluationism. This criticism relies on the fact that supervaluationists must accept as supertrue disjunctions whose disjuncts are not only supertrue—which supervaluationists are ready to acknowledge—but also not satisfiable. In order to show this, Malpass proposes a formula, FF_1, which shows the existence of a satisfiable disjunction with unsatisfiable disjuncts in supervaluationist models. In reply, we show that formula FF_1 cannot be expressed *within* a model (whether Ockhamist or supervaluationist) because it quantifies on models. It can be correctly characterised only within a meta-model that has the resources to quantify on various models. However, once that is done, FF_1 is, for the advocates of Supervaluationism, no more demanding than other disjunctions because it just generalises at the meta-theoretical level what supervaluationists already acknowledge at the theoretical level.

KEYWORDS

branching future, future tense semantics, Supervaluationism

As is well known, the semantics of future tense sentences in a branching time (BT) model is controversial. One of the most discussed approaches is Supervaluationism (Thomason, 1970). Supervaluationism is a kind of para-complete semantics; however, compared to other semantic frameworks that allow truth value gaps, Supervaluationism has the advantage of preserving the validity of formulas such as $\mathbf{F}\varphi \vee \mathbf{F}\neg\varphi$ and the invalidity of formulas such as $\mathbf{Fn}\varphi \wedge \mathbf{Fn}\neg\varphi$,[1] which seem very intuitive.

[1]The meaning of **Fn** is "in *n* time units it will be the case that …".

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Supervaluationism has received much criticism. Here we will consider the challenge advanced by Malpass (2013). Malpass refers to an argument put forward by Graff Fara (2010) against "canonical Supervaluationism" (for instance, that of Fine, 1975) and readapts it to Thomason's Supervaluationism (TS). Malpass's aim is to show the superiority of the Supervaluational Thin Red Line (STRL)—which he and Wawer defended in Malpass and Wawer (2012)—with respect to TS; the STRL model is in fact immune to Fara's objection.

In this paper, we will show that Malpass's criticism of TS fails and that, at least from this point of view, STRL is not superior to TS. Obviously, it might be superior for other reasons. The structure of this paper is as follows: in section 1, the semantics of TS is reviewed; in section 2, Malpass's criticism of TS is presented; in section 3, we show why this criticism is misplaced; section 4 concludes the paper.

1 | THE SEMANTICS OF TS

Following MacFarlane (2008) and Wawer (2014), we can consider TS as a post-semantic model based on an Ockhamist semantic model. In turn, Ockhamist semantics is based on BT structures. Therefore, we will first define BT structures, then the Ockhamist semantics and finally the TS post-semantics.

1.1 | BT structure

A BT structure (BT) is a couple $\langle T, < \rangle$, where T is a non-empty set of times, $\{t_1,\ldots,t_n\}$, and $<$ is a binary relation defined on T. $<$ is asymmetric and transitive and satisfies (at least) the conditions of Backwards Linearity (BL) and of Historical Connectness (HL)

(BL) $\forall t, t_1, t_2 (t_1 < t \wedge t_2 < t) \Rightarrow (t_1 = t_2 \vee t_1 < t_2 \vee t_2 < t_1)$
(HC) $\forall t_1 \forall t_2 \exists t (t \leq t_1 \wedge t \leq t_2)$.

(BL) states that two instants of the past of t are either identical or ordered by $<$; this implies that, for every instant t, there is one and only one past history. (HC) asserts that all the instants are connected in the past. The maximal subsets of instants linearly ordered in T are referred to as *histories* (h)—the possible courses of events in the world. The conditions on $<$ guarantee that the branching is always directed towards the future and never towards the past: two histories that are separated cannot rejoin.

1.2 | Ockhamist semantics

We can define an evaluation function $V: Var \mapsto \wp(T)$ that maps every propositional letter p onto a set of instants at which p is true. A model $\boldsymbol{M}$ is, then, a couple $\langle \mathsf{BT}, V \rangle$.

Ockhamist semantics have the following clauses:

$$\begin{aligned}
M,t/h \vDash_{Ock} p &\Leftrightarrow t \in V(p) \\
M,t/h \vDash_{Ock} \neg\varphi &\Leftrightarrow M,t/h \nvDash_{Ock} \varphi \\
M,t/h \vDash_{Ock} \varphi \wedge \psi &\Leftrightarrow M,t/h \vDash_{Ock} \varphi \text{ and } M,t/h \vDash_{Ock} \psi \\
M,t/h \vDash_{Ock} \mathbf{P}\varphi &\Leftrightarrow \exists t'(t' < t \wedge M,t'/h \vDash_{Ock} \varphi) \\
M,t/h \vDash_{Ock} \mathbf{F}\varphi &\Leftrightarrow \exists t'(t' > t \wedge t' \in h \wedge M,t'/h \vDash_{Ock} \varphi) \\
M,t/h \vDash_{Ock} \Diamond\varphi &\Leftrightarrow \exists h'(t \in h' \wedge M,t/h' \vDash_{Ock} \varphi)
\end{aligned}$$

In Ockhamist semantics, formulas are always evaluated with respect to an instant and a history passing through that instant. t/h means that the formula is evaluated with respect to instant t and history h, where $t \in h$. Since branching is possible only towards the future, the specification of the history h is irrelevant for the clause for the past operator **P**: all the histories passing through instant t coincide for all moments previous to t. The historical parameter h becomes relevant for the future operator **F** because the histories passing through t can diverge for the instants following t. In this case, a formula can be true at an instant subsequent to t in a history h' and false at an instant subsequent to t in another history h''.

As usual, we can define the necessity operator: $\Box =_{def} \neg \Diamond \neg$. Notice that because all histories coincide at the instant of evaluation and at the previous instants, $\varphi \rightarrow \Box\varphi$ is valid, with φ being atomic or containing the operator **P** only. This accounts for the "closure" of the present and the past compared to the openness of the future. The past is determined, whereas the future is still (partly) indeterminate. Therefore, $\Diamond$ and $\Box$ are to be intended as historical possibility and necessity respectively.

Based on the possibility operator $\Diamond$, we can also define the operator $\mathfrak{C}$, whose intuitive meaning is "It is contingent that …":

Contingency

$$\mathfrak{C}\varphi =_{def} \Diamond\varphi \wedge \Diamond\neg\varphi$$

It is easy to show that only future events can be contingent: if φ is atomic or past-tensed, $\mathfrak{C}\varphi$ is false with respect to any evaluation instant.

1.3 | Supervaluationism

Within Ockhamist semantics, every formula is evaluable only if the historical parameter is specified. However, this can be unsatisfactory. If one is asked whether a sentence like "It will rain tomorrow" is true or false, it would be odd to answer, "It is true in the histories in which it is true that it will rain tomorrow, and it is false in the histories in which it is false that it will rain tomorrow".[2] We would need to evaluate this sentence at the time of evaluation *tout court* without the history parameter.

Several post-semantics that solve this problem have been proposed. TS is one of them. It quantifies on the histories $\mathcal{H}_t$ to which the time of evaluation t belongs. Specifically, a formula is (super)true if it is true in each of these histories, (super)false if it is false in each of these histories, and indeterminate otherwise. In symbols:

$$M,t \vDash_{sup} \varphi \;\Leftrightarrow\; \forall h \in \mathcal{H}_t (M,t/h \vDash_{Ock} \varphi)$$
$$M,t \,\Dashv_{sup} \varphi \;\Leftrightarrow\; \forall h \in \mathcal{H}_t (M,t/h \,\Dashv_{Ock} \neg\varphi)$$

As for the clauses for atomic and past tense sentences, TS gives the same results as Ockhamist semantics because every history in $\mathcal{H}_t$ coincides at time t and at every time preceding t. However for the sentences containing the future operator **F**, the situation is radically different. A formula such as $\mathbf{F}\varphi$ is sup-true only if in every history passing through t there is a time subsequent to t at which φ is true, sup-false if in every history passing through t there is no time subsequent to t at which φ is true, indeterminate otherwise.

An interesting result is that if a proposition such as $\mathbf{F}\varphi$ is contingent—that is, if in at least one history there is a time subsequent to t at which φ is true and in at least one history there is

[2]In other words, our interlocutor might reply: "Okay, but what will actually happen tomorrow?"

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not such a time—then the truth values of $\mathbf{F}\varphi$ and $\mathbf{F}\neg\varphi$ are indeterminate at t, but $\mathbf{F}\varphi \vee \mathbf{F}\neg\varphi$ is sup-true at t because in every history passing through t, it is true that $\mathbf{F}\varphi \vee \mathbf{F}\neg\varphi$. Analogously, $\mathbf{Fn}\varphi \wedge \mathbf{Fn}\neg\varphi$ is sup-false at t because it is false in every history departing from t.

TS has been blamed for advancing a non-completely compositional semantics of connectives: classically, a disjunction is true if at least one of the disjuncts is true; however, in TS, a disjunction can be true even if none of its disjuncts is true. Supervaluationists reply that, although this is odd in some contexts, it is in agreement with our intuitions regarding the openness of the future. If the future is really open and indeterminate, we are prepared to judge neither a future contingent nor its negation as true, but we are ready to accept their disjunction. For instance, supposing that (1) is a future contingent:

(1) It will rain tomorrow

we tend to deny that (1) and its negation (2) are true:

(2) It will not rain tomorrow

However, we have no difficulty accepting their disjunction:

(3) It will rain or it will not rain tomorrow

Supervaluationists hold that we evaluate (1)–(3) in this way because we believe that things *could* go in one way or another, and it is indeterminate in which way they will go. We are, however, sure that they will go in one way or another.

Finally, note that if $M,t/h \vDash_{Ock} \Diamond\varphi$, then $M,t \vDash_{sup} \Diamond\varphi$. Indeed, $M,t/h \vDash_{Ock} \Diamond\varphi$ if φ is true in at least one history passing through t. Therefore, it is true in every history passing through t that there is at least one history passing through t in which φ is true. It also follows that if $M,t/h \vDash_{Ock} \mathfrak{C}\varphi$, then $M,t \vDash_{sup} \mathfrak{C}\varphi$.

2 | MALPASS'S CRITICISM

Malpass's argument against TS is based on the criticism of Graff Fara (2010) against canonical Supervaluationism. The basic idea of Fara and Malpass is that supervaluationists not only must consider disjunctions whose disjuncts are not true as true, but they must also consider as true disjunctions whose disjuncts *cannot be satisfiable*, that is untrue in every model. According to Malpass, in such cases, the classical supervaluationist rejoinder—according to which neither of the disjuncts is (super)true or (super)false because things could go in one way or another, and it is indeterminate in which way they will go—is undermined. In Malpass' counterexample, disjuncts are unsatisfiable and thus *cannot* be true.

To formulate this criticism, Malpass defines the satisfiability operator $\Diamond_s$:

Definition of the satisfiablility operator $\Diamond_s$ *(cf. Malpass, 2013, p. 273)*
$M,t/h \vDash_{Ock} \Diamond_s\varphi \iff \exists M' \exists t' \in M' (M',t' \vDash_{sup} \varphi)$.

In other words: a formula is satisfiable with respect to a model M and a couple time/history t/h iff there is a model M' and a time t' belonging to M' in which this formula is true in M' at t'. Hence, $\Diamond_s\varphi$ is true iff φ is true in some supervaluationist model. As we will see, this definition is puzzling, but for the sake of argument, we will assume it in this section.

Then, Malpass introduces a formula (FF_1), which states that a disjunction is sup-true even though each disjunct is not satisfiable:

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(FF_1) $(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p) \wedge \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \wedge \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

This formula is satisfiable in TS if there is at least one model M and one time t belonging to M with respect to which it is sup-true. According to Malpass, this is the case. Let us see why.

FF_1 is true if its three conjuncts are true. Let us begin with the first conjunct. This is equivalent to $\mathfrak{C}\mathbf{F}p \wedge (\mathbf{F}p \vee \neg\mathbf{F}p)$. Suppose that $\mathbf{F}p$ is a future contingent at t. In this case, $\mathfrak{C}\mathbf{F}p$ is sup-true at t. In fact, this formula is true if both $\Diamond\mathbf{F}p$ and $\Diamond\neg\mathbf{F}p$ are Ock-true. This is the case if there is at least one history passing through t in which $\mathbf{F}p$ is true and at least one history in which $\mathbf{F}p$ is false. This is ensured by the fact that $\mathbf{F}p$ is a future contingent. Moreover, $\mathbf{F}p \vee \neg\mathbf{F}p$ is also true in this situation. Indeed, for every history passing through t, $\mathbf{F}p$ is Ock-true or Ock-false. Therefore, $\mathbf{F}p \vee \neg\mathbf{F}p$ is Ock-true for every couple t/h. Then, for the supervaluationist clauses, this formula will be sup-true with respect to t. As a result, the first conjunct of FF_1 is true in TS if $\mathbf{F}p$ is a contingent future and, thus, is satisfiable in TS.

Let us turn to the second conjunct. It says that $\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$ is not satisfiable in TS. This is so if there is no M and no t belonging to M such that the formula is sup-true at M,t. Now, in TS, $\mathbf{F}p$ is sup-true at t if $\mathbf{F}p$ is true in every history stemming from t. Instead, $\mathfrak{C}\mathbf{F}p$ is sup-true at t only if $\mathbf{F}p$ is a future contingent and thus only if there is at least one history passing through t in which $\neg\mathbf{F}p$ is true. Therefore, these two formulas cannot both be sup-true with respect to t. Therefore, for no M,t, $\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$ is sup-true. This makes the second conjunct of FF_1 true.

The same reasoning applies to the third conjunct. $\neg\mathbf{F}p$ is sup-true iff it is Ock-true in every history. This formula is Ock-true with respect to a history iff there is no future instant at which p is true. Instead, $\mathfrak{C}\mathbf{F}p$ is sup-true only if $\mathbf{F}p$ is a future contingent and thus only if there is at least one history passing through t in which $\mathbf{F}p$ is true. Thus, these two formulas cannot both be sup-true with respect to t. Therefore, for no M,t, $\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p$ is sup-true. This makes the third conjunct of FF_1 true. Since there is at least one supervaluationist model in which the three conjuncts are true, FF_1 is satisfiable in TS.

In STRL, that is, in the alternative semantics proposed by Malpass, there is one history that is privileged over the others. This is the actual history of the world, which, starting from Belnap and Green (1994), is often called the *Thin Red Line* (TRL). Now, a formula such as $\mathbf{F}p$ has different truth conditions in STRL depending on whether the instant of evaluation belongs to the TRL. If it does not belong to the TRL, this formula has the usual supervaluationist truth conditions, but if it does belong to the TRL, the formula is true iff there is an instant t' *belonging to the TRL* and subsequent to that of evaluation at which p is true.

In STRL, FF_1 is not satisfiable. In fact, if the instant of evaluation is on the TRL, $\mathbf{F}p$ can be true (or false) even though it is a future contingent. Its truth or falsity depends on what happens in the TRL, whereas its contingency also depends on the other histories passing through the instant of evaluation. For instance, it is possible that $\mathbf{F}p$ is true at t, because there is an instant subsequent to t on the TRL at which p is true, but it is also contingent because in a history different from the TRL, p is true at no instant subsequent to t. Then, there is at least one STRL model and one time belonging to this model at which $\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$ is true. Therefore, the second conjunct of FF_1 is not true in STRL. It follows that STRL is immune from Fara's criticism.

3 | PROBLEMS WITH MALPASS'S CRITICISM

In this section, we argue that Malpass's argument fails. In particular, we show that formula FF_1 is no more demanding for the supervaluationist than the other disjunctions.

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Let us start by noticing that if in a supervaluationist model M, $\mathbf{F}p$ is a future contingent with respect to an instant t, then the supervaluationist is committed to the three following assumptions:

(4) $M,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$
(5) $M,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p$
(6) $M,t \vDash_{sup} (\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

This is one of the cases in which the supervaluationist admits that a disjunction can be sup-true even though the disjuncts are not. The point at issue is whether FF_1 is more committing than (4)–(6).

FF_1 contains the satisfiability operator $\Diamond_s$. Unfortunately, Malpass's definition of this operator is flawed. Let us return to the original definition:

$$M,t/h \vDash_{Ock} \Diamond_s \varphi \;\Leftrightarrow\; \exists M' \exists t' \in M' (M',t' \vDash_{sup} \varphi)$$

This definition introduces an existential quantification over models. This, *per se*, is not a problem, as Malpass correctly points out:

> This operator worked by quantifying existentially over "supervaluational models". In this BT context, this means to quantify existentially over different BT-models. This makes the analysis slightly less than standard, as quantifying over models is to quantify into a somewhat unusual domain of reference; the domain is a domain whose elements are domains. This is a consequence of the fact that the satisfiability is usually a meta-linguistic notion, employed in the proof theory for a logic, in which it makes more sense to quantify over models. Therefore, the introduction of satisfiability as an object-language operator is itself somewhat non-standard. The procedure is intuitive enough however, and poses no outright logical difficulties of which I am aware. (Malpass, 2013, p. 273)

The problem with Malpass's argument is here. It is a subtle point and it is worth pausing on it.[3] The supervaluationist satisfiability operator $\Diamond_s$ is an existential quantifier whose intended domain is constituted by a class of models, that is, (set theoretic) structures on the elements of which the formulas of a certain language are interpreted. Now, the introduction of this quantifier is clearly licit, as is well known from the standard model theory. However, to introduce it, the passage to a meta-theory with the resources to refer to the models of the object-theory is necessary. On the contrary, Malpass assumes that an Ockhamist structure can *represent* supervaluationist satisfiability. Nevertheless, this is impossible: the quantification over models cannot be embedded into an Ockhamist model because the quantification domain of such model is constituted just by a class of times T and by a class of maximal subsets of instants (that is, histories). Thus, an Ockhamist model does not have the resources to quantify on models. It is important to notice that, here, the problem does not concern the specific temporal semantics one is choosing. In other words, the issue is not about Ockhamist or supervaluationist structures. The quantification on meta-theoretical structures, such as models (whether or not supervaluationist), is not available *within the model of the object theory*.

In order to correctly formulate Malpass's criticism, we have to introduce a meta-structure in which we are able to "talk" about the various models of BT.

[3]We want to thank an anonymous referee for encouraging us to deepen this point.

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3.1 | Construction of the meta-model

Even if we offer here just a sketch of the definition of the meta-structure, we provide a framework able to adequately characterise Malpass's argument.

Let MM be our meta-model. It is constituted by a pair $<\mathbb{M}, I>$, where $\mathbb{M}$ is a set of models and I is an interpretation function that maps any formula of the language to a subclass of models in which the formula is satisfiable. Let us notice that, while in the construction of a standard semantics, the interpretation function pairs elements of the language with elements of the structure (whether times, worlds, and so on), here the interpretation function acts at a meta-theoretical level. Let us assume, for instance, that φ is satisfiable by two models m_1 and m_2. Thus, we have that $m_1 \in I(\varphi)$ and $m_2 \in I(\varphi)$. m_1 and m_2 can differ in the ontology of their domains. It is important to notice that, in this sketch of the meta-theoretical semantics, we do not mention possible accessibility relations among models.

For our purposes, we consider the subclass of $\mathbb{M}$ constituted by supervaluationist models. Thus, we have that:

$$MM, m \vDash \varphi \Leftrightarrow m, t \vDash_{sup} \varphi$$

That is, within the galaxy of models, φ is satisfied in m iff m is a model in which φ is supertrue. It should be noted that the time parameter occurs in the right part of the bi-conditional (after all, the supervaluations are temporally indexed). In the left part, however, there is no reference to times; this is intuitive, since from a meta-meta-theoretical point of view, we are interested in the satisfiability relationships, and these are, in a sense, timeless. We keep, therefore, the parameter t in the biconditional, considering it as arbitrary.[4]

Let us now introduce a clause for modality, which is crucial in Malpass's argument:

$$MM, m \vDash \Diamond_s \varphi \Leftrightarrow \exists m, m, t \vDash_{sup} \varphi$$

Let us notice that negation behaves as follows:

$$\begin{aligned} MM, m \vDash \neg \Diamond_s \varphi \;&\Leftrightarrow\; \neg \exists m, m, t \vDash_{sup} \varphi, \text{ that is} \\ &\Leftrightarrow\; \forall m, m, t \nvDash_{sup} \varphi \end{aligned}$$

What does it mean that φ is not satisfiable (within the class of supervaluationist models)? It means that there is no model in which φ is sup-true; that is, in every model, φ is not supertrue. Let us notice that this clearly does not mean that φ is superfalse in every model, since $m, t \nvDash_{sup} \varphi$ and $m, t \vDash_{sup} \neg \varphi$ are not equivalent. Accordingly, it is not possible to infer from

$$MM, m \vDash \neg \Diamond_s \varphi$$

the following:

$$MM, m \vDash \Box_s \neg \varphi$$

[4] As an anonymous referee pointed out, one could consider the parameter t as bound by an existential quantifier. This is plausible: φ is satisfiable if and only if there is a model m in which φ is supertrue. But according to the supervaluationism, supertruth is defined at an instant of time. So, t could be considered as implicitly existentially quantified.

In fact, the former formula says that, in every model, φ is not supertrue, whereas the latter says that, in every model, it is superfalse. This shows, among other things, how $\Diamond_s$ differs from a standard possibility operator.[5]

Now we have the logical resources to represent Malpass's argument. As we have seen, (FF_1) consists of three parts:

(A) $\neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p)$
(B) $\neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$
(C) $(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

Embedding these three claims in our framework, we get

(A') $MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p)$
(B') $MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$
(C') $MM,m \vDash (\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

Based on the above, we have that

$$MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \Leftrightarrow \forall m,m,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$$

This means that in any supervaluationist model, the conjunction claiming the existence of a true contingent future is not sup-true. Thus, (A') is true. Once again, (A') states that, in every model, the formula $\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$ is not supertrue; (A') does not claim that, in every model, this formula is superfalse. Analogously,

$$MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p) \Leftrightarrow \forall m,m,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p$$

Namely, there is no supervaluationist model in which it is supertrue that p is a false contingent future. Thus, (B') too is true. However, also in this case, we have that, in every model, $\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p$ is not supertrue; we do not have that, in every model, the formula is superfalse.

Let us move to the third conjunct, which is a disjunction:

$$MM,m \vDash (\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$$

As noted above, this is equivalent to

$$MM,m \vDash \mathfrak{C}\mathbf{F}p \wedge (\mathbf{F}p \vee \neg\mathbf{F}p)$$

Consider the first conjunct; it is true in the meta-metamodel MM if $\mathfrak{C}\mathbf{F}p$ is satisfiable in m. Let us take an arbitrary supervaluationist model m where $\mathfrak{C}\mathbf{F}p$ is satisfiable. This means that $m,t \vDash_{sup} \mathfrak{C}\mathbf{F}p$; that is, $\forall h \in \mathcal{H}_t, m,t/h \vDash_{Ock} \mathfrak{C}\mathbf{F}p$. According to the definition of contingency, this means in turn that $\exists h', \exists t' > t, t'/h' \vDash_{Ock} p$ and $\exists h'', \neg\exists t'' > t, t''/h'' \vDash_{Ock} p$. Therefore, this model features (at least) one branch in which p holds in the future and (at least) another branch in which $\neg p$ holds in the future.

In this supervaluationist model, it is not sup-true that $\mathbf{F}p$, and it is not sup-true that $\neg\mathbf{F}p$, since neither $\mathbf{F}p$ nor $\neg\mathbf{F}p$ are true in all branches; however, it is sup-true that $\mathbf{F}p \vee \neg\mathbf{F}p$. It follows that, in this model, the conjunction $\mathfrak{C}\mathbf{F}p \wedge (\mathbf{F}p \vee \neg\mathbf{F}p)$ is sup-true. Therefore, (C') is true.

The argument shows that there is no supervaluationist model in which there are (super)true future contingents, and analogously, there is no supervaluationist model in which there are

[5]Let us recall that the passage from $\neg\Diamond$ to $\Box\neg$ is allowed *within* supervaluationist models.

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(super)false future contingents. The argument, however, does *not* show that in all models, p is a contingent future. Notably, it does *not* show that the conjunction at issue is necessary; it is true within the models in which p is a future contingent.

3.2 | An evaluation of FF_1

Is Malpass's argument still valid when formulated in this more accurate fashion? The aim of his criticism is to show that, in the case at play, the supervaluationist cannot exploit the classical rejoinder according to which none of the disjuncts are supertrue, since both *could* take place, and it is indeterminate which one will take place. As far as FF_1 is concerned—Malpass claims—none of the disjuncts *can* take place. Indeed, FF_1 states the existence of a true disjunction whose disjuncts cannot be satisfiable.

In this section, we show that, if we consider Malpass's formula from a meta-theoretic point of view, it simply generalises at the level of all models what the supervaluationist claims at the level of one particular model. However, if the supervaluationist conception of future contingents is not problematic with respect to a particular model, it should not be problematic even when generalised to every model in general. Therefore, Malpass's criticism fails.

Let us see the argument in more detail. As we have seen, the supervaluationist is committed to (4)–(6). The supervaluationist maintains that if $\mathbf{F}p$ is a future contingent with respect to M,t, then things can go in one way or another, and it is now indeterminate how things will go; therefore, both $\mathbf{F}p$ and its negation are neither supertrue nor superfalse with respect to M,t. However, the disjunction of $\mathbf{F}p$ and its negation is supertrue.

When correctly evaluated at the meta-theoretical level, FF_1 says that this is true not only with respect to a particular M,t, but with respect to *any* model and time. Let us see why.

(A') simply states that what (4) claims with regard to a single model and instant of time holds for *every* model. Let us repeat (4) and (A') for convenience:

(4) $M,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p$
(A') $MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p)$

(4) states that it cannot be supertrue at M,t that $\mathbf{F}p$ is a future contingent and that $\mathbf{F}p$. In other words, if $\mathbf{F}p$ is a future contingent at at M,t, it cannot be supertrue at M,t. (A') says that there is *no* supervaluationist model in which $\mathbf{F}p$ is both a future contingent and supertrue. In fact, recall that $\neg\Diamond_s(\varphi)$ means that φ cannot be supertrue in any model (not that it is superfalse in every model). Therefore, (A') is the meta-theoretical version of (4).

Likewise, (B') states that what (5) claims with regard to a single model and instant of time holds for every model. Again, let us repeat (5) and (B') for convenience:

(5) $M,t \nvDash_{sup} \mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p$
(B') $MM,m \vDash \neg\Diamond_s(\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

(5) states that if $\mathbf{F}p$ is a future contingent at M,t, $\neg\mathbf{F}p$ cannot be supertrue at M,t. (B') states that in every supervaluationist model in which $\mathbf{F}p$ is a future contingent $\neg\mathbf{F}p$ is not supertrue in that model. Again, (B') is the metatheoretical version of (5).

(C') is the meta-theoretical analogue of (6):

(6) $M,t \vDash_{sup} (\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$
(C') $MM,m \vDash (\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$

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(6) says that when $\mathbf{F}p$ is a future contingent at M,t, then the disjunction $\mathbf{F}p \vee \neg\mathbf{F}p$ is supertrue at M,t.[6](C') generalises, at a meta-level, what the supervaluationist claims at the single model level: in every model, when $\mathbf{F}p$ is a future contingent, the disjunction $\mathbf{F}p \vee \neg\mathbf{F}p$ is supertrue.

Therefore, FF_1 translates at the meta-theoretical level what the supervaluationist already claims at the theoretical level. Future contingents cannot be sup-true or sup-false, not only with respect to particular M,t but also with respect to *any* model, since in *every* situation, it will always be possible that a future contingent will or will not take place. Moreover, even if it is indeterminate if a future contingent will take place or not, it is (always) certain that either it will take place or it will not take place. Therefore, the disjunction $\mathbf{F}p \vee \neg\mathbf{F}p$ is true not only with respect to a particular M,t, but it will *always* be true. Once again, this holds for *every* model.

We have seen the supervaluationist classical rejoinder with respect to (4)–(6): both a future contingent and its negation *could* be true with respect to M,t. Accordingly, it is indeterminate which of them is true. However, the world will certainly unfold in one way or other and, therefore, the disjunction of the future contingent and its negation is true with respect to M,t. However, since there are no specific assumptions on a particular model and time, it is possible to generalise this rejoinder to the whole class of models in which there are future contingent situations. Thus, the supervaluationist can say that in every model and at every time at which it is open whether p will be true in the future, both $\mathbf{F}p$ and $\neg\mathbf{F}p$ are not supertrue because both could be true. Nevertheless, the disjunction $\mathbf{F}p \vee \neg\mathbf{F}p$ is supertrue in every model at every time in which $\mathbf{F}p$ is a future contingent because the world will surely unfold in one way or other. FF_1, when considered from the meta-theoretical point of view, says exactly that: with respect to any model, future contingents are untrue but their disjunction is supertrue. It is not clear why this extension should be troublesome for the supervaluationist; she can happily claim that her rejoinder does not hold for a model only but that it can be applied to all models.

To appreciate how innocent is the passage from the theoretical level to the meta-theoretical level, consider the subset of $\mathbb{M}$ constituted by the models in which $\mathbf{F}p$ is a future contingent in each point of the structure—that is, from any point of the structure, there is at least one history in which p is true at every instant in the future and another history in which $\neg p$ is true at every instant in the future. Let M_1 be this particular subset of models. Thus, for the supervaluationist, given an arbitrary $m \in M_1$, FF_2 is true:

$$(FF_2)\ M_1,m \vDash (\mathbf{F}p \vee \neg\mathbf{F}p) \wedge \neg\Diamond_s\mathbf{F}p \wedge \neg\Diamond_s\neg\mathbf{F}p$$

In other words, in every model in which $\mathbf{F}p$ is a future contingent, $\mathbf{F}p$ is never supertrue nor superfalse; but, on the other hand, the disjunction $\mathbf{F}p \vee \neg\mathbf{F}p$ is supertrue. Now, it is not clear why FF_2 should be troublesome for the supervaluationist: this formula states the supervaluationist view of future contingents at the meta-theoretical level. If FF_2 is not problematic for the advocate of supervaluationism, neither should FF_1 be.

4 | CONCLUSION

Malpass's criticism of TS relies on the fact that supervaluationists must accept as supertrue disjunctions whose disjuncts are not only non-supertrue—which the advocates of Supervaluationism are ready to acknowledge—but also not satisfiable. In reply, we showed that formula FF_1 cannot be expressed *within* a model (whether Ockhamist or supervaluationist) because it quantifies on models. It can be correctly characterised only within a meta-model that has the resources to quantify on various models. Once that is done, FF_1 is, for the advocate of

[6]Recall that $(\mathfrak{C}\mathbf{F}p \wedge \mathbf{F}p) \vee (\mathfrak{C}\mathbf{F}p \wedge \neg\mathbf{F}p)$ is equivalent to $\mathfrak{C}\mathbf{F}p \wedge (\mathbf{F}p \vee \neg\mathbf{F}p)$.

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Supervaluationism, no more demanding than her classical rejoinder because it just generalises at the meta-theoretical level what supervaluationists already acknowledge at the theoretical level.

We think, therefore, that Malpass's criticism misses the mark and, at least in this respect, that STRL is no better than TS. Obviously, there can be other reasons for criticising Supervaluationism. However, their analysis is beyond the aim of this work.[7]

ACKNOWLEDGEMENT

Open Access Funding provided by Università Cattolica del Sacro Cuore within the CRUI-CARE Agreement.

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How to cite this article: De Florio, C. & Frigerio, A. (2023) A critique of Malpass's argument against Supervaluationism. *Theoria*, 89(1), 31–41. Available from: https://doi.org/10.1111/theo.12441

[7]We would like to thank an anonymous referee for some helpful comments on an earlier version of our paper, which helped to improve and clarify it.

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