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How Many Worlds Could There Be? David Lewis and Advanced Modalizing

Lorenzo Azzano | Ciro De Florio | Aldo Frigerio

Università Cattolica del Sacro Cuore, Milan, Italy

Correspondence: Lorenzo Azzano (lorenzo.az@hotmail.it)

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ABSTRACT

Advanced modalizing, namely, possibilities and necessities concerning modal space itself, is problematic for a Lewis-style analysis of modality. A popular solution, proposed by Divers, postulates explicit semantic clauses for a collapse of advanced modalizing, to the conclusion that all such matters are, if true in the first place, both possible and necessary. But this is problematic: Divers' solution not only leaves no room for "advanced" contingency, but also renders alternative approaches to Lewisian metaphysics not merely false or metaphysically impossible but analytically false. In this paper, we propose a new framework for advanced modalizing that introduces two novel modal operators, which evaluate sentences not at worlds but at galaxies. This move avoids the collapse of modality and potentially restores a measure of contingency to advanced modal discourse. A key strength of our approach is that it enables the expression of alternative metaphysical positions within a unified modal framework.

1 | Introduction

A Lewis-style analysis of modality requires some metaphysical baggage: a plurality of worlds, so-and-so constructed. But once those metaphysical resources are deployed, new modal questions immediately arise: could there be more, or fewer, worlds than there currently are? Could they be differently constructed? These are only some of the issues falling under the umbrella term of "advanced modalizing." A somewhat orthodox answer, one embraced by Lewis himself, is that such matters are necessary: the plurality of worlds may provide some measure of contingency but it is, itself, necessary in its nature and composition.

We find this strategy unsatisfactory for several reasons, as we will see in Section 3. First, the strategy works only for quantified sentences. Second, it claims that certain sentences are necessary, although—at least at first glance—they do not appear to be so. Finally, it ends up regarding the alternative options to Lewisian metaphysics as not merely false, but analytically false. This outcome appears implausible, even from a Lewisian standpoint.

In this paper, we sketch a new semantic framework for advanced modalizing (FAM) that is devised to solve these difficulties. It primarily concerns the introduction of two new modal operators, which shift the parameter of evaluation of the relevant sentences not across worlds, as the normal modal operators do, but across what we call *galaxies*, namely, collections of accessible worlds. A recurring theme in the debate is that advanced modalizing revolves around content that is "too big" to be represented in a single world, thus jeopardizing the idea that the relevant sentences can be true at any world and thus possible: we introduce the notion of truth at a galaxy and the corresponding possibility operator to assuage this difficulty. Under the right circumstances, some contingency is provided as well.

Here is a brief summary of contents. In Section 2 we present a Lewis-style analysis of modality and the debate around advanced modalizing. In Section 3 we introduce a somewhat orthodox strategy to deal with it: according to Divers' redundancy intepretation, all (true) advanced modalizing is both possible and necessary; we also discuss the problems of this strategy. The

pars construens of the paper begins in Section 4, by introducing the idea that advanced modalizing revolves around what we call *modally undecidable sentences*: if they are true at all, they are not true at any world (and similarly for falsehood). How are they true or false, then? In Section 5, we introduce and discuss the notion of a galaxy and truth at a galaxy, which will lead, in Section 6, to the introduction of a new possibility operator and the resolution of the initial problem of advanced modalizing. In Section 7 we will discuss how to recover contingency in this new framework: this mostly amounts to a selection of frames, namely, a metaphysical choice. Unlike previous attempts to deal with advanced modalizing, our framework is per se neutral vis-à-vis these choices, which we take as a strength. Finally, Section 8 answers a potential objection concerning absolutely unrestricted quantification, while Section 9 provides some concluding remarks.

2 | LMR, LAM, and Advanced Modalizing

According to Lewis' modal realism (for simplicity, LMR), the actual world does not hold a privileged status compared to other possible worlds: they are all ontologically on a par as maximal sums of spatiotemporally-related objects.[1] The term "actual" only has indexical value: every inhabitant of each possible world will take their own world as actual (Lewis 1970).

LMR functions as the metaphysical underpinning of an analysis of modality, which we will call Lewis' analysis of modality or LAM for brevity. Although Lewis initially used world-talk to build a translation schema for modal talk (Lewis 1968), the mature theory is commonly understood as providing a semantic analysis of modal talk (regimented through the use of possibility and necessity sentential operators) in the form of truth conditions for modal sentences.[2] Such truth-conditions are frugal insofar as they only rely on LMR, first-order quantification over worlds, and counterpart theory (once we venture beyond propositional modal semantics). The ideology of LMR is itself parsimonious, mostly borrowing from Lewis' extensional mereology.

LAM endorses the basic principle of Kripkean modal semantics, according to which "possibly, p" is true if and only if there exists a possible world (accessible from the world of evaluation) at which p is true. Necessity is treated as the dual of possibility. The notion of truth at a world is ultimately reducible to truth within a restricted domain. As noted by (Lewis 1986, 5), "at w_1" operates analogously to locative expressions such as "in Australia" in the sentence "there are n kangaroos in Australia." That is, it functions as a domain restriction. While possible worlds may serve as representations of alternative possibilities, they are, in Lewis's framework, spatiotemporally isolated regions of reality. To say that a sentence is true at a world thus means that it expresses a truth about a specific region of reality. For example, if a possible world w_1 contains unicorns, then the sentence "there are unicorns" is true-at-w_1 just in case it is simpliciter true that unicorns exist in w_1.[3] Modal operators shift the world parameter, causing the sentence to be evaluated at a world different from the original world of utterance. While truth-at is not an inherently modal notion, it is crucial for the purpose of the analysis of modality, precisely because it restricts truth: by tweaking these restrictions, we obtain different truth values for different possibility and necessity sentences. In the form of a slogan: no restrictions, no modality.

Despite its renown, the LMR + LAM package is not without problems. One of them is the following: it is a constitutive claim of LMR that:

1. There are many worlds.[4]

By possibility introduction, from (1) it follows that

2. Possibly, there are many worlds.

But herein lies a problem: according to LAM, (2) is false. Details vary, depending on which stage of Lewis' theory we consider (Divers 1999, 221–227), but what all variants of LAM have in common is that (2) is true iff at some world w (accessible from the evaluating world), it is true that there are many worlds. But this cannot be; given that worlds are maximal sums of spatiotemporally related entities, in no world are there many worlds.

To be fair, Lewis (1986, 6) concedes that sometimes a quantifier is not restricted by a world modifier in whose scope it appears, as, for instance, in "at some small worlds, there is a natural number too big to measure any class of individuals," whose intended meaning is preserved by taking "any," but not "there is" to be bound by the world modifier: the excessively big natural number need not be *in* the world for the sentence to be true. But we believe that sentences like (2) afford no such mercy; only a single instance of quantification over worlds appears in the scope of the modal operator, and there can be no doubt about the formalization required to preserve its meaning: sentence (2) means that it is possible that there are many worlds, which, in LAM, amounts to asking whether at some world it is true that there are many worlds.[5]

What to do, then? The options are limited; (1) is a foundational claim without which LMR and LAM simply cannot get off the ground.[6] Neither does it seem viable to reject the inference from (1) to (2): the rule of possibility-introduction, or equivalently the T axiom, is crucial to a minimal characterization of any alethic modality.[7] The best option would be to accommodate the theory for (2) to be true.

The issue concerning the truth-value of (2) is not an isolated one. Consider e, the class of (unrestrictedly) all elephants in all possible worlds, and suppose that in Lewis' plurality of worlds there are n elephants. Thus, (3) is true:

3. e has n members.

By possibility introduction, it should also hold that

4. Possibly, e has n members.

Unfortunately, by the standard of LAM, (4) is false again: if e is the class of all elephants in all possible worlds, e has n members at no world. What (2) and (4) have in common is that they

concern the modalization of content that is "too big" to be represented or contained within a single world. They are far from the only ones; such claims are part of a bigger family of *advanced modal claims*, in Divers' words:

> [a]dvanced modal claims are modal claims about entities other than spatiotemporally unified individuals (perhaps, then, spatiotemporally disunified individuals, sets, numbers, properties, propositions and events) (Divers (1999, 217)).

Advanced modalizing presents a problem for the supporter of LMR + LAM: how can sentences like (2) and (4) be true? To concede that they are not can be taken as a strike against the theory itself (see Jago (2016)). More generally, advanced modalizing is also an opportunity to reflect on the modal status of possible world metaphysics: is it necessary for the plurality of worlds to be the way it is, or could it have been otherwise?

Moreover, if advanced modalizing presents a challenge for any analysis of modality, it is especially problematic for the proponent of LMR + LAM. The claim that, for example, there could have been a different number of oranges than there actually are is, for the standard Lewisian, a perfectly legitimate modal statement—one that can be paraphrased within the framework of possible worlds. The claim that there could have been a different number of possible worlds than there actually are appears to be structurally analogous. One possible strategy for rejecting its legitimacy is to argue that oranges and worlds are not ontologically on a par. Yet this line of response seems unavailable to the defender of LMR: just as there is no ontological distinction between actual and possible worlds, there is no ontological distinction between worlds and their constituents.

3 | Divers' Redundancy Interpretation

A prominent solution to the problem raised by advanced modal claims is Divers' *Redundancy Interpretation*, or DRI (Divers 1999). The insight behind DRI relies on the typically Lewisian understanding of modal operators as scope-altering devices. As we have seen, "possibly" has a semantic of shifting the domain of evaluation to another world, but crucially, this shift requires the original quantification to be world-restricted. Most of the time, this is not a problem: quantification is often implicitly world-restricted. But the case of (1) is peculiar; the quantification in (1) is *unrestricted* for the sentence to be true, so the modal operator cannot have its intended semantic effect: it is, for all intents and purposes, redundant. This leads Divers (Divers 1999, 230) to an interpretation of advanced modalizing which is effectively a collapse of modality:

> restricting '*P*' to cases expressing transworld states of affairs, we have the possibility principle:
>
> (AP) 'It is possible that *P*' is true if and only if *P*.
>
> from which we derive each of the following:
>
> (AN) 'It is necessary that *P*' is true if and only if *P*
>
> (AN) 'It is impossible that *P*' is true if and only if not *P*
>
> (AC) 'It is contingent that *P*' is true if and only if *P* and not *P*.

This is DRI. According to (AP), (1) is true; so it is possible that there are many worlds. By (AN), it is also necessary. Note that, because of (AC), advanced modal contingent propositions are contradictory; thus, assuming LMR to be internally coherent, nothing about it is contingent: not that there are worlds, nor how many there are, nor how they are.

This conclusion is consistent with what Lewis (1986, 80, 125–126) himself thought of the modal status of the plurality of worlds; contingency is provided at the local level by way of quantifier restrictions: without it, we are left with necessities. Relatedly, Divers (2014) agrees with Williamson (2013, 16–17) that LMR validates necessitism, in the almost trivial sense that, when quantifiers are understood unrestrictedly, thus making modal operators redundant, "necessarily everything is necessarily something" is equivalent to the logical truth that everything is something.

DRI has many virtues. First and foremost, it preserves the truth of both (1) and (2), thus saving the inference from the former to the latter. But it displays at least three flaws. First, there is an issue of *generality*, as noted by Roca-Royes (2020, 4687). While the principles from (AP) to (AC) are meant to function on any arbitrary case "*p*" of advanced modalizing, DRI appears to be only motivated in quantificational contexts, in which modal operators alter the scope of quantifiers. Indeed, Divers (1999, 230) claims that DRI "applies to claims of the type 'It is possible that there is some x such that $A(x)$' when and only when the quantification in the contained non-modal sentence is not world-restricted." However, as previously mentioned, the issue of modal statements concerning entities that are "too big" to be contained within a single world does not appear to pertain solely to quantified sentences. (4) serves as an example of a non-quantified sentence that concerns entities of this nature. The question, therefore, is whether DRI should apply to such sentences. For example, is the semantic effect of "possibly" in (4) redundant? Some effort has been made on this front; for example, Divers and Parry (2018) have generalized DRI to account for non-quantificational sentences. Yet a general treatment of advanced modalizing in the spirit of DRI has not yet been proposed and the domain to which DRI should be applied is not clear.

Second, there is an issue concerning intuitions about the truth conditions of certain advanced modal sentences. To be clear, intuitions about the contingency of advanced modalizing may concern all cases; for example, about how many worlds could there be, or about their nature, for example, Skyrms (1976, 332). But certain cases indeed appear to be more pressing; consider, for instance, (5), an example from Jago (2016, 4):

5. Anna is taller than Bill

Where Anna and Bill are not worldmates. There is a sense in which (5) concerns entities that are not contained within a

single world. Assuming that Anna's height is indeed greater than Bill's, if we apply DRI to statements of this kind, we obtain that it is necessary that Anna is taller than Bill. However, this seems counterintuitive. While it appears acceptable that sentences such as (1), which concern the structure of the worlds, can be regarded as necessary, sentences like (5) do not seem to be necessary. Therefore, DRI does not seem to yield intuitively correct results in this case. There are ways for the supporters of DRI to bite the bullet, as in Divers and Parry (2018, 417); however, it is an open question whether these attempts are successful.

Third, DRI faces another issue related to the source of necessity and the structure of worlds. Consider, for instance, sentences predicating properties often judged as essential, such as "Socrates is a man." According to LMR, the structure of worlds is such that sentences like this are considered non-necessary because the universe of worlds has an unrestricted combinatorial structure, and thus there exists a world where Socrates is not a man.

However, since for many it is intuitively true that essentialism holds, we can imagine a different structure of worlds, distinct from that hypothesized by LMR, in which there is no world where Socrates is not a man. The issue here, however, is not so much the correctness of essentialism but rather that, if the DRI + LMR + LAM package is assumed; essentialism is not merely false but analytically false. Recall that, according to DRI, false advanced modal sentences are considered contradictory. Consequently, we cannot even hypothesize a "situation" (for lack of a better term) in which essentialism is true. We find it problematic; in particular, we find it problematic that the DRI + LMR + LAM package of modal semantics makes other options in modal metaphysics not simply false, but almost inexpressible; we would want our language to be able to coherently describe alternative configurations of the modal space, if only to claim that they do not occur. After all, this is precisely what metaphysicians do when they discuss the various possible configurations that the universe of worlds might assume. To frame such a discussion, we need a language that allows us to speak of these configurations and thus enables the comparison between different metaphysical theories. Unfortunately, DRI + LMR + LAM makes this impossible. In conclusion, DRI cannot tell us anything about how advanced modal sentences could coherently be false, or how the modal space would need to be for it to be contingent. If an extension of LAM could achieve this result, this would vastly improve its expressive power as an analysis of modality. In our minds, this extension of LAM could function as an extended logical framework capable of expressing different and incompatible metaphysical theories (and their modal status as necessary!).

4 | Modally Undecidable Sentences

Perhaps we ought to take a step back. Some of the issues above suggest that there are deeper problems to be discussed here, not concerning the modal status of what Divers called "trans-world" sentences, but their very truth. Consider some such sentences, like (1) and (3) from above:

1. There are many worlds
3. *e* [the class of unrestrictedly all elephants] has *n* members

We have begun this paper by claiming that (1) and (3) are true, but such a claim requires closer consideration. After all, we have established that (1) and (3) are not true *at any world*: at no world there are many worlds, and at no world there are *n* elephants. Once again the problem is generalizable:

5. Anna is taller than Bill

is not true in any world, assuming again that Anna and Bill are not world-mates

Sentences (1), (3), and (5), as different as they are, share a semantic feature: they are not true at any world, even if, in some *other* sense, they are true, and perhaps even necessary, if DRI is to be believed! Some of them, like (1), are constitutive of LMR itself. For now, let us say that they are true *simpliciter*, or true unrestrictedly; in the next section we will see that something more needs to be said on this notion of truth, which bears important consequences on the treatment of advanced modalizing.

Before that, however, there is another peculiar semantic feature that these sentences all share, and which is worth discussing. Let us assume that p is "there are many worlds" and consider an arbitrary possible world, w_1. Now, in w_1, p is false, since, as previously stated, there is nothing in w_1 in virtue of which p is true: $w_1 \not\models p$. However, if one assumes a standard semantics for negation, it follows that $w_1 \models \neg p$, meaning that in w_1 it should be true that there are not many worlds. Moreover, since w_1 is arbitrarily chosen, the argument can be reiterated, leading to the conclusion that it is necessarily true (in w_1) that there are not many worlds. So, the negation of (1) would be necessarily true.

This result is impalatable. As we stated above, (1) not only appears to be true, but also seems to be one of the fundamental assumptions of *R*. If we wish to avoid this result, we must not only claim that (1) is not true at a world, but that the negation of (1) is not true either. Simply put, (1) is not a sentence that is suitable to be evaluated as true or false at a world because (1) concerns entities that are "too big" to be contained within a single world. Therefore, (1) is neither true nor false at a world.

We can draw a comparison between sentences of advanced modalizing evaluated at a world and what Strawson (1950) stated regarding sentences containing improper definite descriptions. Take for instance: "the present king of France is bald." According to Strawson, such a sentence is, strictly speaking, neither true nor false. Indeed, in order to evaluate a statement like "the *N* is *P*," we must assess whether the referent of the definite description "the *N*" is within the extension of *P* or not. This sentence is true if such a referent belongs to the set of *P*s, and false if it belongs to the set of non-*P*s. However, if the referent does not exist, we cannot say that it belongs to either set. Thus, we cannot even begin to evaluate the sentence as either true or false: both "the present king of France is bald" and "the present king of France is not bald" are devoid of truth value. The same seems to be true of "there are many worlds": we cannot even begin to evaluate

it *at one world*. Therefore, it is neither true nor false at a world. Sentences like (1) and its negation are, in this sense, *modally undecidable*, and thus belong to the class

$$\mathrm{MU} = \{p \mid \forall w, w \not\models p \wedge w \not\models \neg p\}$$

These peculiar semantic features of MU may help us in another way, by allowing us to clarify the very notion of *advanced modalizing*. (1), (3), and (5) are very different sentences: one is about worlds; one is about individuals and set-theoretical membership; one is about individuals and the relation of being-taller-than. As different as they are, they all have something in common: they all belong to MU. We argue that the reason why they all belong to MU is that they are the wrong kind of sentences to be true at a world. This semantic feature is the mark of advanced modalizing.[8]

5 | Introducing Galaxies

To repeat, modally undecidable sentences can never be true or false at a world. Yet there is an obvious sense in which many of them are true. If LMR is correct, then it is true that there are many worlds—even if at no worlds there are many worlds. And if Lewis' universalist mereology is correct, for any collection of entities, for example, two non-worldmates such as Gandalf and the Eiffel Tower, it is true that their fusion exists, even if this bizarre entity is not entirely located in any world.

What can we say about this notion of truth that outstrips truth at a world? The answer to this question is particularly pressing if we consider that on any loosely Kripkean modal semantics like LAM, truth-conditions for modal sentences are parasitic on truth-conditions for non-modal sentences; so if we have trouble deciding the truth-conditions for (1), we will have trouble on the truth-conditions for (2); indeed, a main component of the difficulty concerning advanced modalizing and possibility-introduction, as in the (1) and (2) inference, is that the non-modal sentence appears to be true, even though we cannot find any world for it to be true in.

There is an obvious solution to this difficulty: to the extent in which LAM frames truth at a world as a restriction of truth *simpliciter*, then the natural thing to say here is that while (1) is not true in every restricted reading, it is true *simpliciter* in its unrestricted reading. In other words, we could simply say that sentences in MU are true *simpliciter*, unrestricted truths that are not true "of" or "at" any specific region of the modal space.

There is a problem, however, sentences in MU can be modalized. As we have seen, in LAM modals have a semantic effect in the case of sentences that are true at a world, by changing the world of evaluation; but if sentences in MU are understood as truth *simpliciter*, which are not true at any world, then modalization achieves very little (remember our slogan: no restriction, no modality!). This is of course a vindication of Divers' redundancy interpretation, whose problems we have discussed above.

An alternative would be the following: although sentences in MU are not true at worlds, they are not true simpliciter: for some X, which we must now identify, they are true at X, such that a nontrivial notion of possibility and necessity can be defined on the back of that X. So, what is X? We now introduce the concept of galaxy. A galaxy is a collection of accessible worlds, such that propositions in MU are true or false at galaxies.

Let us keep things simple for now, and confine ourselves to propositional logic. Let us introduce an evaluation function that maps propositions in MU onto sets of galaxies, galaxies at which the propositions that constitute the function's argument are true. Thus we have

6. $I_G\colon \mathsf{MU} \to \mathcal{P}(\mathbb{G})$

where $\mathbb{G}$ is the set of galaxies $\{g_1, \ldots, g_n, \ldots\}$ and $\mathcal{P}(\mathbb{G})$ is the powerset of $\mathbb{G}$. For the moment there is very little we must assume on galaxies; let us only assume that $\mathbb{G}$ is non-empty, and that every galaxy g is a collection of related worlds.

In what follows, we shall postulate that the class of all possible worlds (which we call $\mathbb{W}$) is divided into galaxies of worlds. Every world belongs to a galaxy, and each world can belong to at most one galaxy. That is,

$$\forall w \in \mathbb{W}, \exists g \in \mathbb{G}, w \in g$$

$$\forall w \forall g, g' \left(\left(w \in g \wedge w \in g' \right) \rightarrow g = g' \right)$$

Moreover, the worlds of a galaxy are all connected by an accessibility relation, that is to say:

7. $g = \{w_i, w_j \mid R(w_i, w_j)\}$.

On this basis, we can now define truth at a galaxy in the following way. For any proposition p and any galaxy g such that $g \in \mathbb{G}$,

8. if $p \in \mathsf{MU}$, then $g \vDash p$ iff $g \in I_G(p)$

That is to say: a modally undecidable proposition is true at a galaxy iff that galaxy belongs to the set of galaxies in which the proposition is true. In other words, sentences in MU are true or false, not relative to a world, but a galaxy: such sentences do not concern what happens in a specific world but rather what happens in a galaxy of worlds. Notice that if p is true at galaxy g (i.e., $g \vDash p$), p is not true at any possible world w that belongs to g, precisely due to its nature as a modally undecidable sentence.

Before moving to the applications of this framework, especially those concerning advanced modalizing, it is worth saying something about the relation between the two conceptions of truth: truth at a world and truth at a galaxy. Truths at a galaxy are never true at a world, but what about the other way round? May sentences that are true at a world also be true at a galaxy? A naive interpretation might claim that everything that is the case at a world also is the case at the relative galaxy; after all, a galaxy is but a bigger portion of the modal space comprising many worlds, so every sentence true at a world perhaps is also true at the relative galaxy (here is again another "Lewisian" comparison: everything that is the case in London is also the case in the United Kingdom). But this

cannot be right; this account would sanction that some contradictions are true at a galaxy, simply by virtue of different worlds validating incompatible sentences. Example: at some world there are elephants, and at some other world, there are no elephants. But if truth at a world entailed truth at a galaxy, by conjunction introduction at some galaxy it would be the case that there are and there are not elephants.

The account that we prefer is one according to which (8) is not simply a conditional, but a biconditional: it is not simply the case that propositions in MU may be true at galaxies, it is also the case that all propositions that may be true at galaxies belong to MU, and thus are never true at worlds. The set of truths is thus bi-partitioned into truths at worlds and truths at galaxies, that is intra-wordly truths, and extra-worldly truths.

The notion of (truth at) a galaxy paves the way for a resolution to the problem of advanced modalizing, that we will illustrate in the next section. As we will see in Section 7, the insight of a galaxy of worlds has been hinted at in the writings of Lewis himself, and its introduction will raise all sorts of interesting questions (first and foremost: is not restriction to a galaxy, so defined as a collection of worlds, trivial? Is not there only *one* galaxy? To anticipate a bit, our answer is: yes, *for Lewis*).

6 | Advanced Modalizing, Again

Propositions true at a galaxy belong to MU, namely, the class of extra-worldly propositions that are neither true nor false at any world because they concern entities too big to be contained in a world or relations among entities belonging to different worlds. Now that we have this new alethic notion at our disposal, we can define a corresponding notion of possibility. The basic idea is of course an extension of the standard Kripkean modality: given a proposition p that belongs to MU, it is "possible" that p (in this new sense) if there is a galaxy in which p is true. Necessity is, as usual, the dual of possibility. A bit more formally, for any p such that $p \in \mathsf{MU}$,

9. $g \vDash \blacklozenge p$ iff $\exists g_1, R(g, g_1), g_1 \vDash p$

In the semantic clause for our $\blacklozenge$ modal operator, we need an accessibility relation between galaxies that mimics the one between possible worlds. Formally characterizing the nature of the accessibility relation does not serve us here, however, just as we can be general about the accessibility relation between worlds. It only suffices to say that R is at least reflexive, to ensure an alethic interpretation of $\blacklozenge$ which obeys the T axiom.

Let us call "$\blacklozenge$-possibility" the possibility generated by the $\blacklozenge$ sentential operator whose semantic behavior is described in (9), and, contrariwise, let us call "$\Diamond$-possibility" the possibility generated by the standard LAM interpretation of the $\Diamond$ operator. (Also, let us call $\blacksquare$-necessity the sense of necessity regimented by the operator "$\blacksquare$", viz., the dual of "$\blacklozenge$").

Let us call FAM as a system that incorporates both "$\Diamond$" and "$\blacklozenge$," along with their respective interpretations. We will not present all the logical details surrounding FAM. However a couple of things about FAM are worth remarking. First, there are some conditions on *acceptable* sentences: if p is an extra-worldly sentence, "$\Diamond p$" is not acceptable; in the same vien, if p is an intra-worldly sentence, "$\blacklozenge p$" is not acceptable.[9] It is easy to see that nesting of the two operators is not acceptable. Since standard modal semantics assigns truth conditions for "$\Diamond p$" it follows that $\Diamond p$ does not belong to MU; thus, "$\blacklozenge \Diamond p$" is never acceptable. In addition, if p belongs to MU, complex formulas such as "$\blacklozenge p$" or "$\blacksquare p$" are acceptable: their truth-conditions in the right-hand side of (9) involves quantification over galaxies, which is unsuitable to be restricted to worlds. That is, if $\blacklozenge p$ belongs to MU, then $\Diamond \blacklozenge p$ is not acceptable. In this sense, FAM is not properly speaking a bimodal system, as there are no two distinct modalities defined on the same (fragment of) language; FAM only concerns the "$\blacklozenge$" modality and it allows us evaluate advanced modalizing.

We now have all the tools to resolve the original problem of advanced modalizing, namely, the (1) and (2) inference. To recap:

1. There are many worlds,
2. Possibly, there are many worlds

The slightly regimented English of (2) is ambiguous vis-à-vis our two senses of possibility, namely, $\Diamond$-possibility and $\blacklozenge$-possibility. In other words,

10. $\Diamond$ (there are many worlds)
11. $\blacklozenge$ (there are many worlds)

Assuming that (1) belongs to MU, (10) is not acceptable. To the extent in which $\Diamond$ is only defined on intra-worldly sentences, its inferential behavior is similarly restricted, so we should not be expecting to obtain (10) from (1) through possibility-introduction, and have an acceptable sentence in return. $\Diamond$-introduction, and the corresponding T-axiom, hold within the intra-worldly fragment of language, to which (1) does not belong.

(11) however is acceptable, and possibly true, depending on our assumptions on $\mathbb{G}$. Even within the confines of LMR, in which there is only one galaxy, we can find a galaxy in which there are many worlds: ours (assuming again that R is reflexive). So given some reasonable assumption concerning LMR, (11) is true.

Before we move on, an important clarification is required. (10) is not acceptable because (1) does belong to MU, namely, it is a modally undecidable sentence. So to the extent in which we restrict ourselves to world-only models (without galaxies), (1) does not provide a counterexample to the standard T axiom "$p \rightarrow \Diamond p$", for the simple reason that (1) cannot be plugged in as the antecedent: that there are many worlds is neither true nor false at any world. This trumps a potential objection against FAM, namely, that of changing the subject. In the words of Parsons (2012, 148) nomenclature changed to fit our article,

> [a] fan of advanced modalizing could make "is necessarily-true" ambiguous for themselves and their conversational partners by introducing a new and hitherto unknown meaning for that term. But this would not solve any problems connected with the old

> meaning. For we would still use the simple analysis to analyse the ordinary sense of the modal idioms. [O]n the simple analysis "there are at least two possible worlds" comes out necessarily-false. That is, if [FAM] is true, then ordinary modalizing invalidates the T rule—"there are at least two possible worlds" is both true and (ordinarily) necessarily false! To say that there is some other, technical, extraordinary, advanced, sense of possibility that does not have this problem does not remove it from the ordinary sense. Only rejecting [FAM], the ordinary analysis, or the T rule (as applied to ordinary modality) could do that.

We agree that the simple postulation of galaxies and our black modality, per se, does not solve the problem concerning the transition from (1) to (10): what solves it is the well-motivated assumption (or so we argue) that (1) is modally undecidable, and thus provides no counterexample to "$p \rightarrow \Diamond p$" when standard world-only models are employed. Within such models, T says that whatever is true at the world of utterance, is true at some world; or that whatever is true at all worlds, is true at the world of utterance. But (1) is never true (nor false) at any world. So we are not avoiding the original question: we are, rather, dispelling it.

7 | FAM, Uniqueness, and Lewis

It is now time to say a bit more about the interpretative frame of FAM, and most specifically the set of galaxies of $\mathbb{G}$. This set-up may appear baroque and ontologically unseemly at first, at least if we start with Lewis' opinion on the matter: "[t]here is but one totality of worlds" (Lewis 1986, 80).

Let us see how to reconstruct Lewis' position in FAM. If $\mathbb{G} = \{g\}$, that is if there is only one galaxy g, the results of FAM coincide with those of DRI. Assuming that $R(g, g)$, every extra-worldly truth that is true at g is also true at some galaxy in $\mathbb{G}$; it is also true at all galaxies in $\mathbb{G}$. Hence, if $\mathbb{G} = \{g\}$, every proposition that is true at g is also ⧫-possible and ■-necessary. For example, it is both ⧫-possible and ■-necessary that there are many worlds. If Anna is taller than Bill, it is both ⧫-possible and ■-necessary that Anna is taller than Bill.[10]

On the other hand, if a MU-proposition p is false at g, then $\neg \blacklozenge p$ and $\blacksquare \neg p$ are true. So, it is not possible that there is just one world, and it is necessary that there is not just one world.

Since these results coincide with those of DRI, what is the point of FAM, then? The point is that these results are not owed to a collapse of modality *by fiat*, as in DRI; they depend on a metaphysical choice underpinning the selection of frames. More specifically, they depend on the fact that, according to Lewis' *metaphysical* theory, there is only one galaxy. Here is an analogy: in a "Spinozian" set of worlds $\mathbb{W}$, everything is necessary because $\mathbb{W}$ contains just one world. In a "Lewisian" set of galaxies $\mathbb{G}$, every extra-wordly truth is necessary because there is only one galaxy in $\mathbb{G}$. That the unicity of the plurality of worlds is responsible for the necessity of advanced modalizing is something we find hinted in the writings of Lewis themselves; for example, "[t]here is but one totality of worlds [...] it could not have been different" (Lewis 1986, 80). This train of thought is irrecoverable in DRI: no matter the motivation that led Divers to this account, on DRI an extra-worldly truth is necessary *by definition*. FAM is a better competitor in this regard. Case in point: change something in $\mathbb{G}$, and the results of FAM begin to diverge from those of DRI. This is a significant gain in expressive power, in the sense that FAM has the resources to evaluate advanced modalizations even in a context where certain metaphysical features of LMR were to fail.

Let us call *Uniqueness* the claim that $\mathbb{G}$ has only one member, namely, that there is only one galaxy, one plurality of worlds; assuming Uniqueness no contingency can be recovered for advanced modalizing. So the idea is quite natural: reject Uniqueness, and recover some contingency. Let us call g_L a galaxy consisting of the plurality of worlds described in LMR, and g_S a Spinozian galaxy with a single world w_S in it (i.e., $g_S = \{w_S\}$; galaxy g_S is entirely comprised of a lonely world whose only accessible world is itself). Now, let us suppose that these two galaxies, namely, the Lewisian one, consisting of a plurality of possible worlds, and the Spinozian one, consisting of a single possible world, are the only two galaxies in the absolute modal space: $\mathbb{G} = \{g_L, g_S\}$. The advanced modalized sentence "there are many worlds" is true at some galaxy but false in another, and it is therefore contingent (in the sense that it is ⧫-possible but not ■-necessary). That is to say, we are recovering an interpretation of modal discourse that allows us to say: there are many worlds, but there *could* only be one. So supposing that we live in a world w such that $w \in g_L$, we might very transparently say that while there are many worlds, if Spinoza had been correct, there would only be one. What about Anna and Bill? Let us assume that Anna is taller than Bill, a sentence which is true in this galaxy. If the sentence is *false* in another galaxy, then, again, we recover its contingency.[11]

These are only some of the results that can be achieved by rejecting Uniqueness; many others can be formulated using the examples above. Lack of contingency is, as we have seen in the beginning of the paper, a potential issue concerning some of the advanced modal sentences, so supporters of FAM may feel some pressure to reject Uniqueness and embrace a plurality of galaxies. Why not do it?

In Lewis' writings we can find an argumentative strand against this proposal, which may be worth discussing. It can be found in a passage from the *On the Plurality of Worlds* discussing the problem of trans-world causation; the corresponding counterfactual "is supposed to hold—where?" (Lewis 1986, 78), which at least hints at the notion of truth at a galaxy. But Lewis' take on the project is highly critical. To provide a quote:

> [w]e think of the totality of all the possible worlds as if it were one grand world, and that starts us thinking that there are other ways the grand world might have been. [...] But this is thoroughly mistaken. If I am right, the many worlds already provide for contingency, and there is no sense in providing for it all over again. Or else I am wrong, and the many worlds do not provide for genuine

> contingency. [...] But then it makes no sense to repeat the very method you think has failed, only on a grander scale.
>
> (Lewis 1986, 80)

We disagree on this diagnosis. The many worlds *have* in fact provided for contingency in a vast class of cases, what we would call "ordinary modalizing"; but they are insufficient in other cases, that is, the extra-worldly sentences where the notion of truth at a world breaks down. (Many) galaxies can be brought into the theory, in the same spirit worlds were, to provide for contingency in these other cases as well. Unless one can provide an argument against the idea of treating galaxies as, functionally speaking from a semantic point of view, "super-worlds" through biconditional clauses like (9), the notion seems fruitful: it allows one to express within the theory itself how the truth-value of modal sentences varies based on different metaphysical presuppositions.

This last observation is crucial to our argumentative strategy, to the scope and ambitions of FAM. Roughly put, embracing and rejecting Uniqueness is a metaphysical choice, and it is not the main goal here. The goal is rather to sketch a framework of modal semantics which allows us to clearly see the modal consequences of such choices. This leaves many matters open which we do not aim to resolve; case in point, opinions differ amongst the authors of this very paper concerning the philosophical pull towards the contingency of "Anna is taller than Bill"; so the case for Uniqueness may vary from person to person.

We can imagine FAM being employed by metaphysicians of different sympathies, with different features provided to the class of galaxies. Consider a metaphysical disagreement between what we may call a Lewisian (viz., someone who believes in a plurality of possible worlds), and a Spinozian (viz., someone who believes that there is only one possible world); but there are degrees to this disagreement. In its most radical interpretation, the Spinozian might believe not only that there is one world, but that necessarily there is one world, which FAM understands as $\mathbb{G} = \{g_S\}$; and similarly the Lewisian might believe that necessarily there are many worlds, namely, $\mathbb{G} = \{g_L\}$.

But FAM has a conciliatory option for a Spinozian and a Lewisian with a moderate approach; namely, both might claim that $\mathbb{G} = \{g_L, g_S\}$. So the moderate Spinozian might hold that while there is only one world, there might be more than one; and contrariwise the moderate Lewisian might hold that while there are many worlds, there might only be one.

But there is more to the expressive power of FAM. Even a Spinozian who holds that a plurality of worlds is metaphysically impossible, might claim that a plurality of worlds is epistemically possible, or conceivable. We remain nonconfrontational about the formal features of the R relation also to remain neutral on the interpretation of this new modality. Similarly, an orthodox Lewisian who follows Lewis in its stipulation that other galaxies are metaphysically impossible, can accept FAM under a suitably weak interpretation of the modality. They might say that other galaxies are conceivable, but unreal. What these options have in common, and what allows us to be somewhat nonconfrontational on this matter, is that FAM preserves the way in which metaphysicians actually operate. A metaphysician may consider a certain claim X to be necessary, but also wonder, and investigate through nontrivial modal semantics, what would it be like for X to be false.

Unlike DRI, FAM without Uniqueness achieves this result. Far from being a problem to be solved, advanced modalizing becomes a fertile test-bed for metaphysical theories, their modal claims, and their relations, all of which can be discussed within the confines of a single semantic meta-language. This is what FAM ultimately is: the common language amongst competing metaphysical views.

8 | Worlds, Galaxies, and Unrestrictedly Everything

One final issue needs to be addressed, that concerns absolutely unrestricted quantification. Remember one of the examples that started this paper, for example,

3. *e* [the class of unrestrictedly all elephants] has *n* members

The reader may remember the original problem: how can (3) be true at a world, if at no world there are *n* elephants? FAM concedes that it is not. (3) can, at best, be true at galaxies. A critic would argue: the shift from worlds to galaxies accomplishes nothing, for the same problem can pop up again. If *n* is the number of unrestrictedly all elephants, scattered across galaxies, it is perfectly *possible* that (3) is true, while not true at *any* galaxy, assuming that there are elephants in at least two galaxies. It is only within the confines of Uniqueness that quantification in a galaxy and unrestricted quantification converge. Obviously, it would not help to move from galaxies to *clusters of galaxies*: we would face an infinite regress. So, the argument goes, if we are facing an infinite regress, why bother with galaxies in the first place? This might be welcome news for the orthodox Lewisian.[12]

The standard answer is that, although we admit that the regress is infinite, some ground *has* in fact been covered. We can claim that there are *n* elephants in LMR's galaxy, and that perhaps that might not have been the case (if Uniqueness is rejected). We can claim that Anna is taller than Bill, and perhaps that might not have been the case. Another advantage of the account is that we can clearly express the difference between *existence* and *existence at a world* by the semantic features of the relative sentences (whether or not they belong to MU); which is a welcome result for those metaphysicians, like Lewis, who posit trans-world individuals like sums and classes, which is not easily expressible in LAM: in FAM, such entities are not "impossible individuals" (Lewis 1986, 211). More generally, we can discuss potential failures of LMR in less "desertic" galaxies (e.g., an ersatzist galaxy, an essentialist galaxy, etc....) within a shared account of modality. These are nontrivial results.

But we concede that absolutely unrestricted quantification interacts just as problematically with truth at a galaxy as it does with truth at a world. So, while it may be true *simpliciter* that there are

(unrestrictedly) n elephants, it is difficult to attribute that truth to a world or a galaxy; so we are not in a position to claim that sentences like (3) are, strictly speaking, $\Diamond$-possible or $\blacklozenge$-possible, at least not without resorting to primitive modality, a collapse of modality, or some other similar resource. But this is hardly surprising; remember our slogan: no restriction, no modality! The same goes for all matters that concern unrestricted quantification, such as absolute cardinality (if such a thing can be sensibly expressed). We might be in a position to claim that in FAM's non-empty set of galaxies there are n things, but we may have trouble expressing the fact that there could have been more, or less. Similarly, we encounter difficulties in formulating the claim that there could exist more galaxies than there actually are. We concede all of this: but the admission of galaxies is useful. Even if absolutely unrestricted quantifications remain a sore spot, galaxy-restricted quantification achieves more than world-restricted quantification does.

9 | Conclusion

Could there be more, or less, worlds than there are? Could they be differently construed, according perhaps to some restricted recombination principle that leaves some gap in the modal space? By virtue of a standard Kripke-style semantics, these questions are underpinned by a further problem: how can the non-modal portion of such sentences be true at a world at all?

We concede that they cannot. But neither are they true simpliciter, a standard answer amongst those who favor a collapse of modality, like Divers. They are rather true *at a galaxy*. Our framework for advanced modalizing, or FAM, with its new possibility operator can solve the issues of advanced modalizing, so that each verdict of contingency or necessity can be traced back to a particular metaphysical selection of frames. For example, to get some degree of contingency to the questions above we must reject the claim that there is only one plurality of worlds, namely, we must reject Uniqueness. While there is some philosophical pull in this direction, FAM remains per se neutral on such matters—something we take to be a testament to its expressive strength as a framework for articulating the modal implications of philosophical theorizing.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Endnotes

[1] LMR is presented in its mature formulation in (Lewis 1986). For a succinct formulation (see Rosen 1990, 333 and Jago 2016, 1).

[2] For details on Lewis' understanding of "analysis" relative to modality (see Divers and Fletcher 2020).

[3] While we may claim that truth-at is a more fundamental notion than truth simpliciter (Lycan 1994, 33–34), we might also claim that the two are one and the same.

[4] The semantics of "many" is complex. Probably the basic meaning of "many A are B" is that the cardinality of the set $A \cap B$ is greater than a certain contextually established standard ($| A \cap B | > n$, where n is a natural number established by the context). There are other meanings of "many." For example, it can mean that a high number of A are B, that is $| A \cap B | / | A | > r$, where r is a rational number between 0 and 1, again established by the context. Probably there are other further meanings (Lappin 1988; Peters and Westerståhl 2006, 6.3). These questions are largely orthogonal to the topic of this paper. For sake of simplicity, here we will assume that "many" simply means "at least 2."

[5] Relatedly, a relevant distinction here might be the one between what Adams (1981) calls "truth in" versus "truth at" a world, and what Kit Fine calls "inner" versus "outer" sense of truth; in Fine's words: "a proposition (or statement) will be true at a world in the inner sense conditionally upon the individuals it mentions existing at that world, and true at a world in the outer sense regardless of the existence of those individuals in the world" (Fine 2005). So one might try to somehow dissolve the puzzle by exploiting the intuition that (1) can be true in the *outer* sense at a world, but not in the *inner* sense. That said, it is unclear how this distinction would work within the setting of LMR + LAM where, explicitly, world-operators function as quantifier-restrictions of the domain associated with *that* world. In other words, a Lewisian setting affords no truly "outer" sense of truth, insofar as worlds are regions of reality such that truth at worlds is truth *simpliciter* circumscribed to that region.

[6] Positions like fictionalism (Rosen 1990) and agnosticism (Divers 2004) endeavor to preserve LAM without the ontological commitment conveyed by sentences such as (1). We do not consider these questions in this paper.

[7] A supporter of this option would perhaps be Hudson (1999). Hudson's rejection of the T axiom is based on a problematic passage from Lewis (1986, 210–220) concerning "trans-world individuals": given Lewis' universalist mereology, any two non-worldmates compose an entity, say e, composed of non-worldmates e_1 and e_2; by the light of mereology, "e exists" is true; however, e exists at no world, as no world entirely contains it. Hence "e exists" is true simpliciter, but also necessarily false at any world. We will come back to the semantic status of such sentences in Section 4.

[8] To be clear, we have characterized advanced modalizing through the notion of modal undecidability. But the question remains open as to whether there are propositions whose truth value is undecided across worlds and are not, albeit intuitively, advanced modalizing. We leave this matter for another time.

[9] There is of course a familiarity between acceptability and well-formedness. The difference is of course that well-formedness is a syntactic feature of sentences, whereas acceptability in this context revolves around semantic features, namely, whether the sentence belongs to MU.

[10] An anonymous referee suggests to consider the following scenario: a galaxy g_1 has at least four worlds. Anna is in w_1 and is 1.7 m tall; Bill is in w_2 and is 1.5 m tall; in w_3 there is a 1.5 m Anna-counterpart; and in w_4 there is a 1.7 m Bill-counterpart. The referee asks whether in this scenario is $\blacklozenge$-possible both that Anna is shorter than Bill and Bill is shorter than Anna. Within our semantics, we did not explicitly address the issue of the rigidity of proper names with respect to evaluations over galaxies, which is precisely the concern raised here. Two options are available. (i) "Anna" designates a single individual within the galaxy, for example, Anna living in w_1. In that case, the advanced modalized sentence "Anna is taller than Bill" is not merely $\blacklozenge$-possible, but $\blacksquare$-necessary. However, (ii) Anna might instead refer to all of Anna's counterparts within the galaxy. In that case, as the reviewer correctly observes, it would be true in g both that Anna is taller than Bill and that Bill is taller than Anna. These are contradictory sentences, and their contradictoriness depends on the "plural" reference borne by the proper names. For this reason, we believe that

the first solution is arguably preferable. Despite the interest of the question concerning the denotation of proper names in our framework, we will not pursue this issue further here.

[11] We have merely sketched a semantics for truth-in-a-galaxy at the propositional level; going to the first-order will probably require an extension of counterpart theory to function. We leave this for another occasion.

[12] An anonymous reviewer suggests a way to define the set of galaxies as a solution to address the problem of absolute cardinality. The set of galaxies is the powerset of the Lewisian set of worlds. This is an interesting suggestion worth discussing. In particular, we believe that this proposal has certain merits. First of all, it actually provides a criterion for generating the set of galaxies required by our models (and so, a way to generate the modal space) which may be of independent value. That said, such a feature is also the source of some limitation of the purported account: because the starting point is the Lewisian set of worlds, we will never find any world, in any galaxy, that is also not present in the initial pluriverse; so that, in effect, whatever Lewis deems $\Diamond$-impossible, is also $\Diamond$-impossible in any other galaxy. Given that our goal was to set up a shared framework for modalizing across competing metaphysical theories, we believe this to be too restrictive. In general, we acknowledge that our account generates a "duplication" of worlds so to speak. Curiously, many of these will be otherwise indistinguishable worlds belonging to different galaxies; so, given that positioning in a galaxy determines which $\Diamond$-possibilities hold, that entails that there are modal differences between these worlds that are not underpinned by any "intrinsic" differences between them, but only underpinned by their relative positioning within the overall structure of worlds. Although not a knock-down argument per se, this is an observation worth making. However, we do not agree that the proposed strategy has any advantage concerning the modalization concerning absolute cardinalities, such as for example, "there could be more elephants than there are". While the proposed strategy may have some tools to actually calculate the absolute number of elephants across all galaxies, it does not offer any particular insight as to how the relative modalization ought to be evaluated: namely, even if the set of galaxies is the powerset of the Lewisian galaxy, that does not provide any tool to clarify how or why such a set of galaxy could have been different (in this particular instance: how that absolute number of elephants could have been another).

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