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Measuring Thematic Funds Performance via an Approach Based on Observable and Latent Factors

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ABSTRACT

This paper investigates whether thematic equity funds deliver abnormal performance relative to conventional global equity funds. Using Fama-French models augmented with latent factors, we estimate fund-level alphas, and apply the false discovery rate methodology to an estimated three-group mixture distribution, separating good, null and bad performing funds, distinguishing abnormal performance from luck. Thematic funds' alpha distribution meaningfully differs from that of non-thematic funds, after adjusting for hidden exposures. We interpret alphas as measures of abnormal performance, rather than managerial skill. The findings suggest that thematic funds offer distinctive performance characteristics relative to traditional ones.

JEL Classification: G1, C5, C6

1 | Introduction

Thematic funds represent the hallmark of a new investment paradigm, known as thematic investing. This approach seeks to identify and capture opportunities arising from structural shifts or transformations, commonly referred to as "megatrends" (or long-term megatrends) that have the potential to significantly influence economies, reshape business models, and transform societies. The objective of thematic investing is to gain exposure to assets that are likely to benefit from these "megatrends", typically by targeting issuers considered best positioned to capitalise on them. Consequently, this strategy introduces an alternative framework for asset classification, moving beyond traditional region- or sector-based portfolio allocations. Thematic investment encompasses a broader scope and is driven by the anticipated responsiveness to trends that should exhibit two essential characteristics: they must be long-term and irreversible rather than temporary, and they should exert significant influence, transcending economic cycles, to some extent. In essence, thematic investing represents a megatrend-oriented strategy designed to identify the leading issuers of the future. Historically, the global universe of thematic funds has expanded significantly, particularly since 2014–2015, in terms of both the number and variety of funds offered, as well as assets under management. This expansion has been largely driven by bullish equity markets.

According to Morningstar (2024), the number of active thematic funds more than doubled over the 5-year period ending in June 2024, reaching 2776. During the same period, global assets in thematic funds nearly doubled to USD 562 billion from USD 269 billion, although this growth was not linear. Assets peaked at USD 892 billion at the end of 2021 before declining in the subsequent year.

Morningstar notes that global investor interest in thematic funds remains strong. Several explanations can be proposed for this phenomenon. Thematic funds benefit from favourable sentiment and compelling narratives. One possible reason is that thematic investing enables individuals to allocate capital toward themes in which they hold the greatest conviction. In other words, it serves as a means of expressing personal values and beliefs, fostering a sense of participation in driving structural change. For instance, consider an individual concerned about the depletion of already scarce natural resources as a result of overexploitation, such as water sustainability and availability. Thematic funds translate such themes into tangible investment opportunities by providing exposure to companies perceived as well positioned in areas such as water supply, water infrastructure development, and water stress management. These portfolios frequently include industrial firms, public utilities, and issuers in the materials sector, thereby ensuring diversification across industries.

In addition, thematic investing offers investors nontraditional medium- to long-term strategies for allocating savings to assets considered resilient from a forward-looking perspective.

Finally, a key rationale underlying thematic investing, frequently emphasised during the marketing and placement of thematic funds, is their purported capacity to generate alpha, that is, returns exceeding those attributable to general market factors. The claim that thematic funds yield positive alphas rests on a key assumption.

While recent contributions have begun to explore this issue, the financial literature still lacks a rigorous evaluation of whether the observed performance of thematic funds reflects genuine abnormal returns or can be explained by exposure to risk factors. Previous contributions have primarily focused on the role of thematic funds within portfolios and on the problem of determining their weights in asset allocation strategies. This is exemplified by Somefun et al. (2023), who address portfolio construction by proposing asset allocation solutions that account not only for traditional risk factors, but also for five thematic exposures within the satellite component of the portfolio, considering different levels of investor risk aversion. Similarly, Methling and von Nitzsch (2019) propose pragmatic naïve diversification rules for private investors seeking to incorporate a thematic satellite portfolio into a core portfolio. Other literature, in particular Ielasi and Rossolini (2019a), Ielasi and Rossolini (2019b), Bai et al. (2022), and Dong and Doukas (2020) furthered the empirical investigation in the thematic universe.

Our paper is closely related to Bai et al. (2025), who identify investment themes using firm-level textual analysis and construct a thematic concentration index linked to fund performance. While their contribution lies in the identification and measurement of thematic exposures, we take a complementary perspective and assess whether the performance of thematic funds reflects genuine alpha or can instead be explained by exposure to observable and latent risk factors and potential false discoveries.

This article addresses the literature gap in the evaluation of thematic funds, investing in equities of issuers located in developed countries by assessing whether they exhibit statistically significant abnormal performance (alpha) and whether they differ significantly from more traditional global equity funds. This study seeks to determine whether analysing potential investment decisions through a "thematic lens" can enable fund managers to generate additional value (alpha) beyond the returns explained by the portfolio's exposure to a common benchmark and its sensitivities to a set of systematic risk factors.

It is important to distinguish between performance and managerial skill. While positive alpha may reflect managerial ability, it may also arise from luck or exposure to omitted risk factors. In this paper, the focus is on the statistical identification of abnormal performance (alpha), rather than on the structural estimation of managerial skill in the sense of Barras et al. (2022).

Once it has been established which fund population exhibits a greater prevalence of statistically significant positive alphas, the subsequent, more ambitious objective will be to extend our understanding of the distribution of zero- and non-zero-alpha fund managers to the population level. This approach acknowledges that the true alpha of any fund within the population is unobservable and accounts for the necessity of distinguishing statistically significant performance from outcomes that can be attributed to luck.

For the first objective, multi-factor asset pricing models that account for both observable and latent systematic effects (risk factors) are employed, in the vein of Giglio et al. (2021). For the second objective, the False Discovery Rate (FDR), originally developed by Storey (2002), adopted in Barras et al. (2010), and more recently modified and strengthened by Ferson and Chen (2021), is utilised.

In the classical FDR approach, mutual funds come from a mixture of three distributions: one for funds with expected zero alphas, one for funds with expected positive alphas and another for funds with expected negative alphas. However, unlike the classical methodology, the Ferson and Chen (2021) method estimates the location and the fraction of funds in each group. There, the authors proved that, if the fund distribution is correctly estimated, their approach leads to more accurate results, because it allows to better separate statistically significant performance from luck-driven cross-sectional variation. In addition, it allows for the estimation of the probability that a given fund exhibits a positive or negative alpha, exploiting information from the entire cross-section of mutual funds.

In this paper, the approach by Ferson and Chen (2021), which relies exclusively on observable factors, is here extended to include also latent factors, as suggested by Giglio et al. (2021), thereby mitigating the risk of conflating exposure to unobserved, omitted information with alpha. These latent factors capture residual common variation not explained by standard risk factors. Following Giglio et al. (2021), latent factors are estimated through the eigen-decomposition of the covariance matrix of the residuals from the Fama-French models. The accuracy of the estimated alpha distribution is assessed using either the false discovery rate (FDR) or the Kullback–Leibler divergence between the estimated density of the t-values (derived from the data for each Fama-French model, with possible additional latent factors) and the t-value density implied by each alpha configuration, obtained via synthetic data generation.

In addition to improving the measurement of abnormal performance, the paper provides a data-driven economic interpretation of the latent factors. Specifically, we relate the extracted latent components to observable economic and financial variables using a machine learning based selection procedure. The results indicate that latent factors are closely associated with exposures to forward-looking investment segments, such as innovation, technology, sustainability, and other structural transformation themes. This interpretation helps to connect the statistical identification of latent factors and their economic meaning, providing insight into the sources of performance in thematic investing. To our knowledge, this is among the first studies that examine the performance of thematic funds using an innovative approach that incorporates both observable and latent factors to evaluate the presence and distribution of abnormal performance across funds. Therefore, this paper addresses the existing gap in the literature by providing a cross-sectional assessment of different groups of funds (thematic and non-thematic) and distinguishing statistically significant alpha from luck-driven performance. From a practical perspective, the findings have implications for the criteria and effort devoted to fund manager selection.

The remainder of this paper is organised as follows. Section 2 describes the mutual fund data sets employed and provides information on the data used. Section 3 briefly explains the multi-factor asset pricing models selected to estimate funds' abnormal performance. Section 4 illustrates how the enhanced FDR methodology, combining the Ferson and Chen (2021) method with the model adopted in Giglio et al. (2021), is applied to estimate the alpha distribution. Section 5 details the empirical application, with results that allow a comparison of thematic versus non-thematic funds. Further, it offers insights on the latent factors interpretation implementing a novel variable selection technique (Riso et al. 2023). In Section 6, the concluding comments are presented. Appendix A and B illustrate further methodological aspects of the approach here employed, and includes further details on the empirical analysis.

2 | The Data

The data collection process of mutual funds for this study relies on the LSEG Refinitiv database. In line with the introduction, two mutual funds datasets are created to conduct the empirical analysis: a thematic funds dataset and a dataset of conventional (non-thematic) peers, all selected with a focus on global equity investing across developed countries.

The main mutual fund selection criteria adopted in the study can be summarised as follows. For both datasets, the target funds are open-end-funds, domiciled in Europe, UCITS compliant and, therefore, available for sale/placement throughout Europe (so called European passport) and accessible to retail investors. In relation to this, it is worth pointing out that Europe holds the largest proportion of world's thematic fund assets (Morningstar 2024). Then, funds of funds, ETFs index funds and mutual funds using investment strategies other than traditional long-only (usually called Alternative UCITS) are filtered out to have two narrower mutual funds samples composed of investment vehicles that use direct/pure investment strategies and are actively managed. Since a fund can have multiple share classes, an additional screening is applied to include in the datasets only the share class associated with the highest level of annual ongoing charges.

They are inevitably cross-sectional rather than time series data, but significant changes in level of costs of mutual funds are not so frequent. The information is derived from each fund's Key Investor Information Document (KIID) and is stored by Morningstar Direct, unlike the standalone management fee which is not always stored in the database. The decision to select the share class which incorporates the highest costs of running a mutual fund is motivated by the intention to carry out the empirical analysis (both for thematic and conventional funds) referring to the most difficult and challenging situation for the emergence of a pure extra-performance from the fund manager skills. The logical, as well as extremely concrete, assumption is that an investor perceives real skills from the fund manager only in the case extra-performance is achieved net of the costs that weigh on the fund overall NAV. Typically, thematic funds have higher costs. Clearly, this is not the first work that uses net-of-fee returns (see, e.g., Bauer et al. 2005; Elton et al. 2004).

The sample period considered for the empirical analysis covers the time interval from January 2018 to December 2025. It is shorter than in traditional mutual fund studies, reflecting the recent development of thematic funds. As a result, the analysis should be interpreted as evidence on the performance of thematic funds in the current market environment, rather than as long-run stylised facts. The sample period is characterised by a strong expansion of investment themes related to technology, sustainability, and disruptive innovation. Accordingly, the empirical results should be interpreted in the context of this thematic concentration, which reflects the environment in which thematic funds have become most economically relevant, rather than as evidence that necessarily generalises to periods with different thematic dynamics.

For this time span, monthly returns, inclusive of any distributions (therefore in the total return version) and net of the ongoing charges are considered. All returns are denoted in US dollars. Based on the aforementioned selection criteria, the resultant two samples are composed of 149 thematic funds and 1149 non-thematic funds, all with the developed world as investment area.

The identification of thematic funds is based on the classification provided by Morningstar Direct, which assigns funds to thematic categories organised into four broad groups: Technology, Physical World, Social, and Broad Thematic. Each category can be further decomposed into more granular investment themes. In our sample, the largest shares of funds are classified within the Technology and Broad Thematic categories, reflecting the prominence of innovation-driven and cross-sectoral investment strategies in the thematic fund universe.

Furthermore, to obtain reliable alpha estimates when applying multifactor asset pricing models to interpret the performance of funds, a "minimum requirement" in terms of available monthly returns has to be met. It is reasonable to assume that having at least 60/72 observations satisfies the requirement and in the case of this contribution the data availability reaches beyond this threshold, with 96 months.

To mitigate a sort of "incubation bias" (Evans 2010), the inclusion of a fund is conditioned on it beginning operations at least 1 year prior the time span considered in the analysis. In addition, to address potential survivorship bias, the sample includes both surviving and non-surviving funds over the entire observation period (12 non-surviving funds in total). This ensures that the analysis is not restricted to funds that remain active at the end of the sample. Ultimately, it is worth noting that all the funds in the two data sets are accumulating funds and that frequently the same mutual fund management company manages both conventional funds and thematic funds included in the datasets. In Appendix, some summary statistics on both thematic and non-thematic mutual funds are provided by Tables B1 and B2, respectively.

3 | Multi-Factor Models for Funds Performance Evaluation

To evaluate the abnormal performance of mutual funds in the two datasets, an alpha estimate is calculated at the fund level ($\hat{\alpha}_i$). To this end, asset pricing models that capture the cross-section of mutual fund returns must be selected. Considering that the study deals with equity funds, the choice is in favour of using the Fama and French 3-factor model (Fama and French 1992, 1993) and the Fama and French 5-factor model (Fama and French 2015). Employing these multi-factor models complies with the suggestion expressed in Elton and Gruber (2020), underlining that these two models will remain the most popular and standard models for mutual fund studies. Even without taking into account this indication, it is preferable in the specific case not to apply the Carhart 4-factor model (Carhart 1997) due to the limited sample period available.

In these models, the alpha is the intercept of a time series regression of each fund returns (minus the risk-free return) against the excess return from the market portfolio and the returns form further common (risk) factors, all acting together as explanatory variables. Therefore, in terms of interpretation, the fund-level alpha estimate incorporates the 'abnormal' return (good or bad) relative to the return of a passive (or mimicking) portfolio with the same factor exposures (thus, also with the same risk characteristics). The returns for the factor-mimicking portfolios are obtained online from Kenneth French's Library. When the Fama and French 3-factor model is used, the underlying idea is that mutual fund returns are explained by the following economic influences: market factor, size factor (market capitalisation) and value factor

$$\begin{aligned} R_{i,t} - R_{f,t} = {} & \alpha_i + \beta_{\mathrm{M},i}(R_{\mathrm{M},t} - R_{f,t}) \\ & + \beta_{\mathrm{SMB},i}\mathrm{SMB}_t + \beta_{\mathrm{HML},i}\mathrm{HML}_t + \boldsymbol{\phi}_i'\mathbf{w}_{i,t-1} \\ & + \epsilon_{i,t} \quad i = 1, \ldots, N \quad t = 1, \ldots, T \end{aligned} \tag{1}$$

In this time-series regression, $R_{i,t}$ is the log-return for fund i, $R_{f,t}$ is the risk-free return (approximated by the US 1-month T-bill rate in the Kenneth-French's Library), α_i is the alpha of the fund, $R_{\mathrm{M},t}$ is the market return (approximated by the return of a global developed market equity portfolio), SMB_t is the return on a diversified portfolio of small-cap stocks minus the return on a diversified portfolio of large-cap stocks, HML_t is the difference in return between a diversified portfolio of high book-to-market ratio (and high earnings-to-price ratio) stocks, usually called value stocks, and a portfolio consisting of stocks with low book-to-market ratio (and low earnings-to-price ratio) generally referred to as growth stocks. Some additional lagged fund characteristics have been included in the regression to better capture abnormal performance carried by the alphas: age (in log-months), size (measured by total net assets), flows, and a dummy variable signalling the Covid-19 pandemic from March to December of 2020:

$$\mathbf{w}_{i,t-1} = (\mathrm{Age}_{i,t-1}, \log(\mathrm{TNA}_{i,t-1}), \mathrm{Flow}_{i,t-1}, D_{C,t})$$
$$\mathrm{Flow}_{i,t-1} = \frac{\mathrm{TNA}_{i,t-1} - \mathrm{TNA}_{i,t-2}(1 + R_{i,t-1})}{\mathrm{TNA}_{i,t-2}}$$

Lastly, $\epsilon_{i,t}$ is the residual return.

Fama and French innovate the reference model, with respect to Sharpe's CAPM (Sharpe 1964), using the small-minus-big (SMB) factor and the high-minus-low (HML) factor that show a small cap premium and a value premium in terms of returns. When the attention is shifted from the Fama and French 3-factor model (3F) to the 5-factor model (5F), two additional factors contribute to explain the variation in returns across equity mutual funds. They are called the profitability factor robust minus weak (RMW) and the investment factor conservative minus aggressive (CMA). The regression used for measuring skills then becomes

$$\begin{aligned} R_{i,t} - R_{f,t} = {} & \alpha_i + \beta_{\mathrm{M},i}(R_{\mathrm{M},t} - R_{f,t}) \\ & + \beta_{\mathrm{SMB},i}\mathrm{SMB}_t + \beta_{\mathrm{HML},i}\mathrm{HML}_t \\ & + \beta_{\mathrm{RMW},i}\mathrm{RMW}_t + \beta_{\mathrm{CMA},i}\mathrm{CMA}_t + \boldsymbol{\phi}_i'\mathbf{w}_{i,t-1} \\ & + \epsilon_{i,t} \quad i = 1, \ldots, N \quad t = 1, \ldots, T \end{aligned} . \tag{2}$$

Substantially, the profitability factor in an explanatory variable represent the difference between the returns on portfolios of stocks (firms) with robust (high) and weak (low) operating profitability while the investment factor expresses the difference between the returns of stocks (firms) with conservative/aggressive investment strategies. Tables 1 and 2 provide descriptive statistics on the estimated betas of the thematic/non-thematic funds data, for the 3-Factor and 5-Factor models, respectively. Before considering the results of the estimated models, the choice of a global developed market equity portfolio as common reference index deserves attention. Although the funds in the sample are domiciled in Europe, the use of global benchmarks and multi-factor models ensures that potential regional or home biases are captured through factor exposures rather than being attributed to abnormal performance. As a result, the estimated alphas reflect deviations from systematic risk factors rather than geographic allocation effects. By doing this, a "traditional" approach is used to investigate alpha instead of the alternative one proposed by Angelidis et al. (2013) of using the benchmark stated in each fund's prospectus (the so called self-designated benchmark). This is not a random choice: first of all, this article is interested in the evaluation of the alpha that can be extrapolated from an initially broad investment opportunity set that, on the one hand, can be traded by investors and, on the other hand, can be filtered according to different lenses/criteria to achieve original geographical, sectoral or thematic exposures. Secondly, as observed in Braga et al. (2025) and in Kumar et al. (2019), the process of building thematic indices is not comparable with that on which the

TABLE 1 | Descriptive statistics for thematic funds (mean, standard deviation (SD), quartiles (q_j) of the cross-section of coefficients of the Fama-French 3-Factor and 5-Factor models. Neg $_s$ displays the relative frequency of significantly negative estimates, and "Joint F" column displays the frequency of rejections of the joint F-test for $\beta_{\text{RMW},i} = \beta_{\text{CMA},i} = 0$, (at 5% significance level).

Model	Coefficient	Mean	SD	$q_{0.25}$	Median	$q_{0.75}$	Neg $_s$	Joint F
3F	RMRF	0.787	0.189	0.688	0.800	0.917	—	—
	SMB	0.189	0.243	0.044	0.134	0.254	0.154	—
	HML	−0.079	0.199	−0.21	−0.059	0.043	0.705	—
5F	RMRF	0.787	0.202	0.689	0.805	0.915	—	—
	SMB	0.165	0.222	0.019	0.111	0.258	0.194	—
	HML	−0.055	0.166	−0.174	−0.06	0.048	0.671	—
	RMW	−0.045	0.212	−0.153	−0.02	0.087	0.537	0.139
	CMA	−0.033	0.147	−0.115	−0.033	0.067	0.577	

Abbreviations: CMA, conservative minus aggressive; HML, high-minus-low; RMW, robust minus weak; SMB, small-minus-big.

TABLE 2 | Descriptive statistics for non-thematic mutual funds (mean, standard deviation (SD), quartiles (q_j) of the cross-section of coefficients of the Fama-French 3-Factor and 5-Factor models. Neg $_s$ displays the relative frequency of significantly negative estimates, and "Joint F" column displays the frequency of rejections of the joint F-test for $\beta_{\text{RMW},i} = \beta_{\text{CMA},i} = 0$, (at 5% significance level).

Model	Coefficient	Mean	SD	$q_{0.25}$	Median	$q_{0.75}$	Neg $_s$	Joint F
3F	RMRF	0.845	0.138	0.771	0.861	0.926	—	—
	SMB	−0.021	0.181	−0.122	−0.058	0.048	0.658	—
	HML	0.127	0.238	0.013	0.114	0.238	0.227	—
5F	RMRF	0.857	0.138	0.782	0.87	0.935	—	—
	SMB	0.011	0.158	−0.08	−0.024	0.07	0.599	—
	HML	0.054	0.212	−0.061	0.029	0.141	0.397	—
	RMW	0.01	0.173	−0.061	0.025	0.111	0.423	0.257
	CMA	0.15	0.165	0.138	0.236	0.447	0.151	

Abbreviations: CMA, conservative minus aggressive; HML, high-minus-low; RMW, robust minus weak; SMB, small-minus-big.

construction of traditional indices is based, the former being less rule-based and objective. Third, the starting point for the construction of thematic benchmarks is actually a parent index consisting of a global equity index.

The use of the extended 5-factor model provides interesting results. First, it offers a better modelling strategy with respect to the 3-factor model, with non-negligible (joint) contribution from the two added factors, RMW and CMA - by looking at the "Joint F" column in Tables 1 and 2. In addition, the coefficients $\beta_{\cdot,i}$ for the two added factors, RMW and CMA, reveal a notable differentiation between thematic and non-thematic funds. The former exhibits a negative coefficient (−0.045 on average with negatively skewed distribution, according to Table 1, vs. 0.01 on average and a more positively skewed distribution, as it results from Table 2) in non-thematic funds, thus denoting a greater orientation towards issuers with weak operating profitability. This evidence seems coherent for mutual funds investing with a forward-looking approach driven by megatrends since, understandably, they cannot rely on substantial current and especially historical revenues compared to the costs of goods and expenses. With reference to the CMA investment factor, a regression slope with opposite sign is observed for the two datasets: on average it is slightly negative for the thematic funds and positive for the non-thematic funds. Without denying the existence of dispersion around these results, this circumstance seems to reveal a stronger orientation towards including thematic funds in managed portfolio issuers/firms that invest aggressively. Considering the SMB and HML factors, it can be inferred that there is a greater tendency for thematic funds compared to non-thematic funds, towards small capitalisation stocks that are also more growth-based. This seems aligned with the intention of targeting specific investment themes linked to structural transformation that are gaining a foothold in the economic-social context. In fact, this could plausibly lead to the identification of issuers, as well as insights, into their size at an early stage.

4 | A Combined Approach Based on Mixed Models to Assess Funds Skills

From a methodological point of view, the econometric approach followed in the paper is a combination of the ones proposed in Ferson and Chen (2021) and Giglio et al. (2021). The former allows the evaluation of mutual fund performance by testing their alphas and by estimating the locations and proportions of fund groups with negative, zero and positive alphas. The mixture model provides a flexible framework to characterise the cross-sectional distribution of fund performance and to

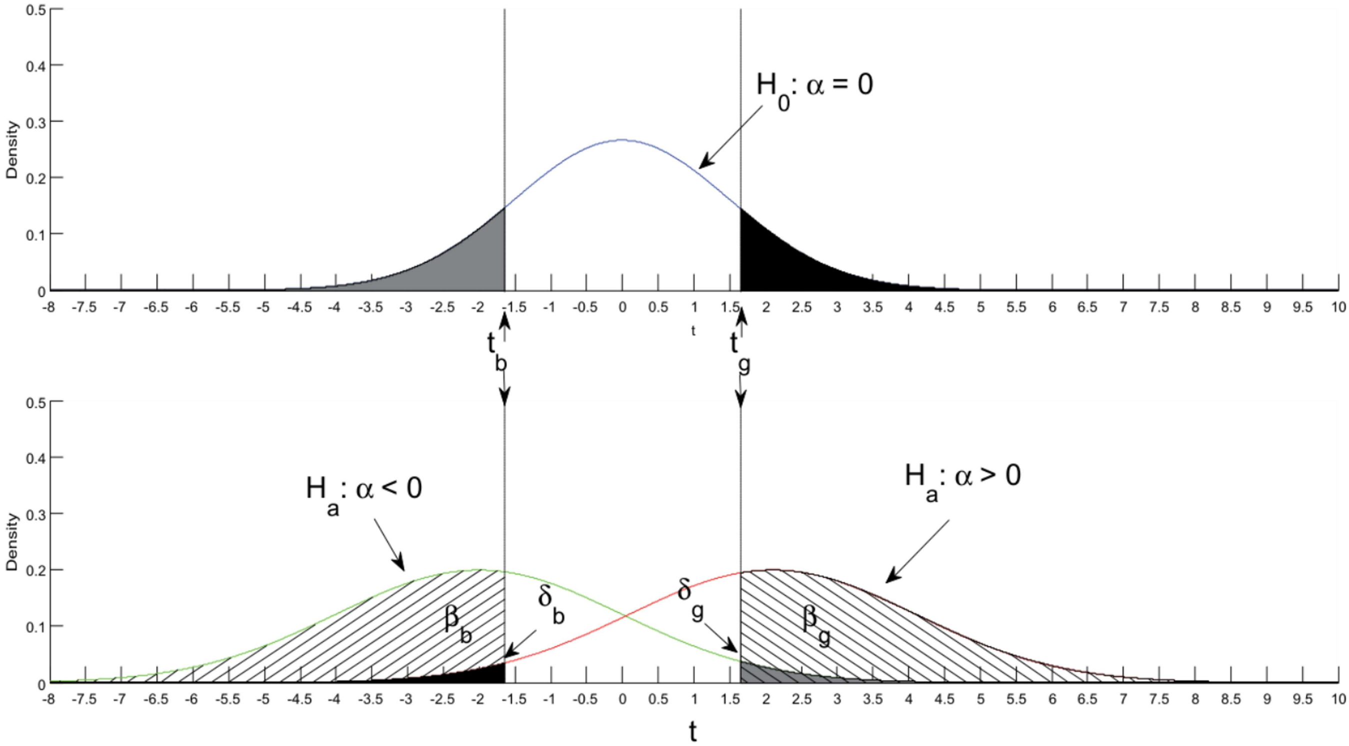

FIGURE 1 | Hypothetical representation of the alpha t-ratio distribution for three subpopulations of funds (*source:* Ferson and Chen 2021). [Color figure can be viewed at wileyonlinelibrary.com]

assign posterior probabilities of belonging to different alpha groups. This approach allows for a data-driven classification of funds into positive-, zero-, and negative-alpha categories, without imposing too restrictive assumptions on the underlying structure of abnormal performance. This approach was originally developed for models using exclusively observable factors and is here extended to latent factors, as the latter can contribute to explaining fund returns and to disentangling exposure to omitted risks from measured alpha (see Giglio et al. (2021)). It is important to note that this framework identifies the fraction of funds with true non-zero alphas and separates these from lucky or unlucky realisations of zero-alpha funds. Thus, it refines the measurement of abnormal performance. However, it does not by itself yield a structural estimate of managerial skill in the sense of Barras et al. (2022), who distinguish skill from scale and value creation.

Following Ferson and Chen (2021), the unconditional distribution of fund's alphas is assumed to be a mixture of three distributions: one of funds with zero alphas, one of funds with alphas greater than zero (good funds) and one of funds with alphas lower than zero (bad funds). These three sub-populations are centred on zero, α_g and α_b, respectively, and each of them include fractions of funds of the entire population equal to π_0, π_g and π_b, respectively, with $\pi_0 + \pi_g + \pi_b = 1$. The distributions of the alpha t-statistics to test the null $H_0 : \alpha = 0$ against the alternative $H_1 : \alpha \neq 0$ can be represented as in Figure 1.

Here, t_b and t_g are the critical values corresponding to a test t of the null with size $\gamma/2$, which is set equal to either 5% or 10% in each tail. Accordingly, the relative frequency of rejections of the null on each tail is given by

$$N_g = \frac{\sum_{i=1}^{N} \mathbf{1}(t_i > t_g)}{N}, N_b = \frac{\sum_{i=1}^{N} \mathbf{1}(t_i < t_b)}{N} \tag{3}$$

where $\mathbf{1}(.)$ is the indicator function. Then, β_g and β_b denote the power of the t-test for zero alpha against the alternative of either good funds ($H_{1,g} : \alpha > 0$) or bad funds ($H_{1,b} : \alpha < 0$) respectively, while δ_g and δ_b are the so-called confusion parameters. The former represents the probability that a bad fund is mistaken for good, while the latter the probability that a good fund is mistaken for bad. According to this model, the set of parameters that need to be estimated is

$$\theta = (\pi_g, \pi_b, \alpha_g, \alpha_b, \beta_g, \beta_b, \delta_g, \delta_b). \tag{4}$$

The estimation process consists of three stages. The first stage involves three simulations. The first simulation determines the critical values t_b and t_g under the null hypothesis of zero alpha funds, along with N_g and N_b. This involves generating data from a model in which $\alpha_i = 0$ for every individual, keeping all other quantities at their estimated value, drawing from the model residuals (see Appendix A). The second simulation is carried out with the same principle, albeit assuming the alternative hypothesis that funds are good. To this end, a grid of possible values for $\alpha_g > 0$ is considered. This allows to determine both β_g and δ_b. Likewise, the third simulation is performed under the alternative that funds are bad and centred on $\alpha_b < 0$. In this case, a set of possible values for α_b is considered. This allows to determine β_b and δ_g. In the second stage, the previous results are gathered to numerically estimate the proportions π_b, π_0 and π_g (see the Appendix for the methodological details). Clearly, the estimates of π_g, π_b and $\pi_0 = 1 - \pi_g - \pi_b$ that result from

TABLE 3 | Alphas cross-sectional distribution obtained from the 3F and 5F models on monthly thematic funds returns.

	3F Alphas		5F Alphas	
	Est. (%)	***t*-ratio**	**Est. (%)**	***t*-ratio**
Fractile				
0.01	−0.547	−3.004	−0.49	−2.869
0.05	−0.399	−1.759	−0.376	−1.779
0.1	−0.355	−1.499	−0.331	−1.358
0.25	−0.236	−0.932	−0.246	−0.911
Median	−0.07	−0.286	−0.06	−0.279
0.75	0.078	0.269	0.09	0.389
0.9	0.207	0.808	0.206	0.808
0.95	0.35	1.163	0.339	1.264
0.99	0.429	1.913	0.419	1.838

TABLE 4 | Expected fractions of lucky (FDR_g) and unlucky (FDR_b) for thematic funds in the Fama-French 3F and 5F models, following either the BSW or the FC methodology.

		BSW		FC	
Model	$\gamma/2$	FDR_g	FDR_b	FDR_g	FDR_b
3F	0.05	0.142	0.052	0.051	0.031
	0.1	0.17	0.071	0.073	0.072
5F	0.05	0.128	0.058	0.049	0.032
	0.1	0.151	0.078	0.089	0.054

Abbreviations: BSW, Barras, Scaillet, and Wermers; FDR, false discovery rate.

this second stage depend on α_g and α_b considered at the first stage. The last stage of the analysis involves the search for the best values α_g and α_b in the grid of alpha parameters (along with related π_b, π_0, and π_g), representing the means of the good and bad funds assumed in the first two simulations. The best alpha distribution can be determined via a goodness-of-fit test, such as the Pearson χ^2 test, the Kolmogorov-Smirnov test or the Dowd test (DTS, Dowd 2020). The latter test is used in the following empirical application, as it is more robust than others, especially in small samples. Its statistic is defined as

$$\text{DTS} = \int_R \frac{|F_i(t_{s,\alpha}) - F_i(t_{r,\alpha})|}{G_i(t_\alpha)(1 - G_i(t_\alpha))}, \tag{5}$$

where $F_i(t_{s,\alpha})$ and $F_i(t_{r,\alpha})$ are the empirical cumulative distributions of the t-statistics obtained from simulated and real data respectively, and $G_i(t_\alpha)$ is the empirical cumulative distribution of a combined sample obtained by merging observed and simulated t-statistics.

Once the best configuration is chosen (i.e., the $\widehat{\theta}$ corresponding to the lowest DTS statistic), the probability model for the alphas can be used to draw further inference. In particular, the Bayes' rule can be employed to assign a specific fund to one of the three sub-populations. The false discovery rates or expected fractions of lucky and unlucky funds computed following Ferson and Chen (2021) are

$$\text{FDR}_g = [(\gamma/2)\pi_0 + \delta_g \pi_g]/N_g \tag{6}$$

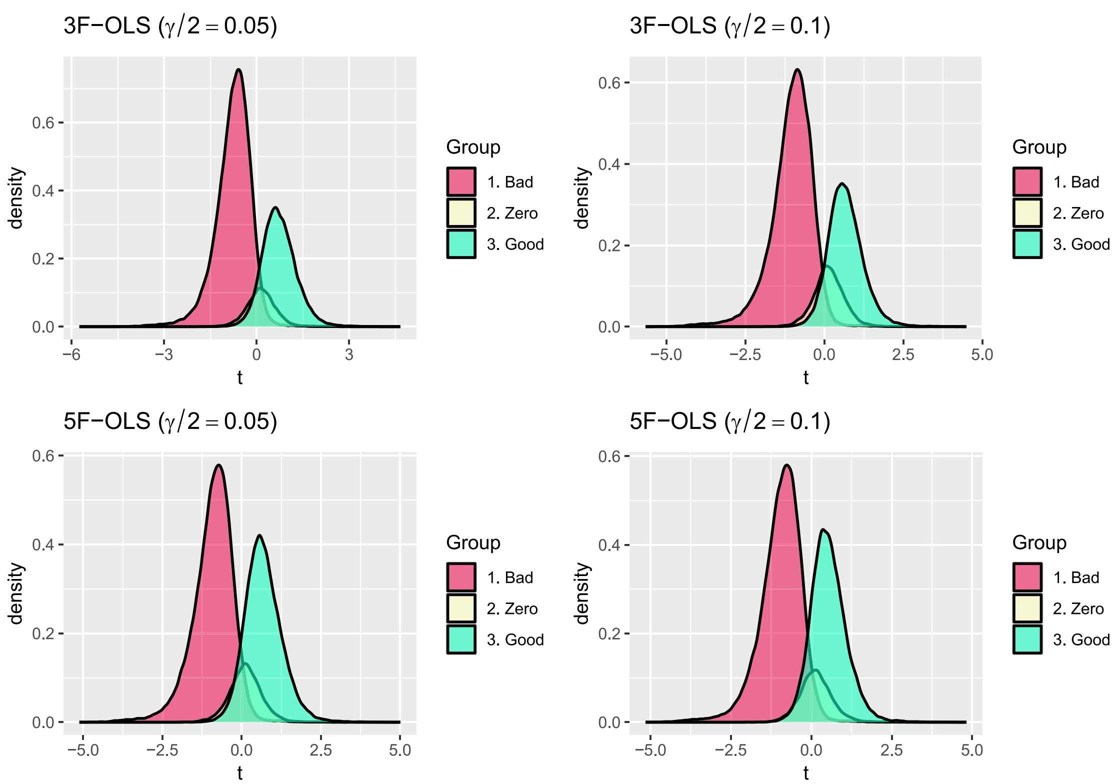

FIGURE 2 | Best simulated alpha mixtures of thematic funds, for the 3F and 5F models. $\gamma/2 = \{0.05, 0.1\}$. [Color figure can be viewed at wileyonlinelibrary.com]

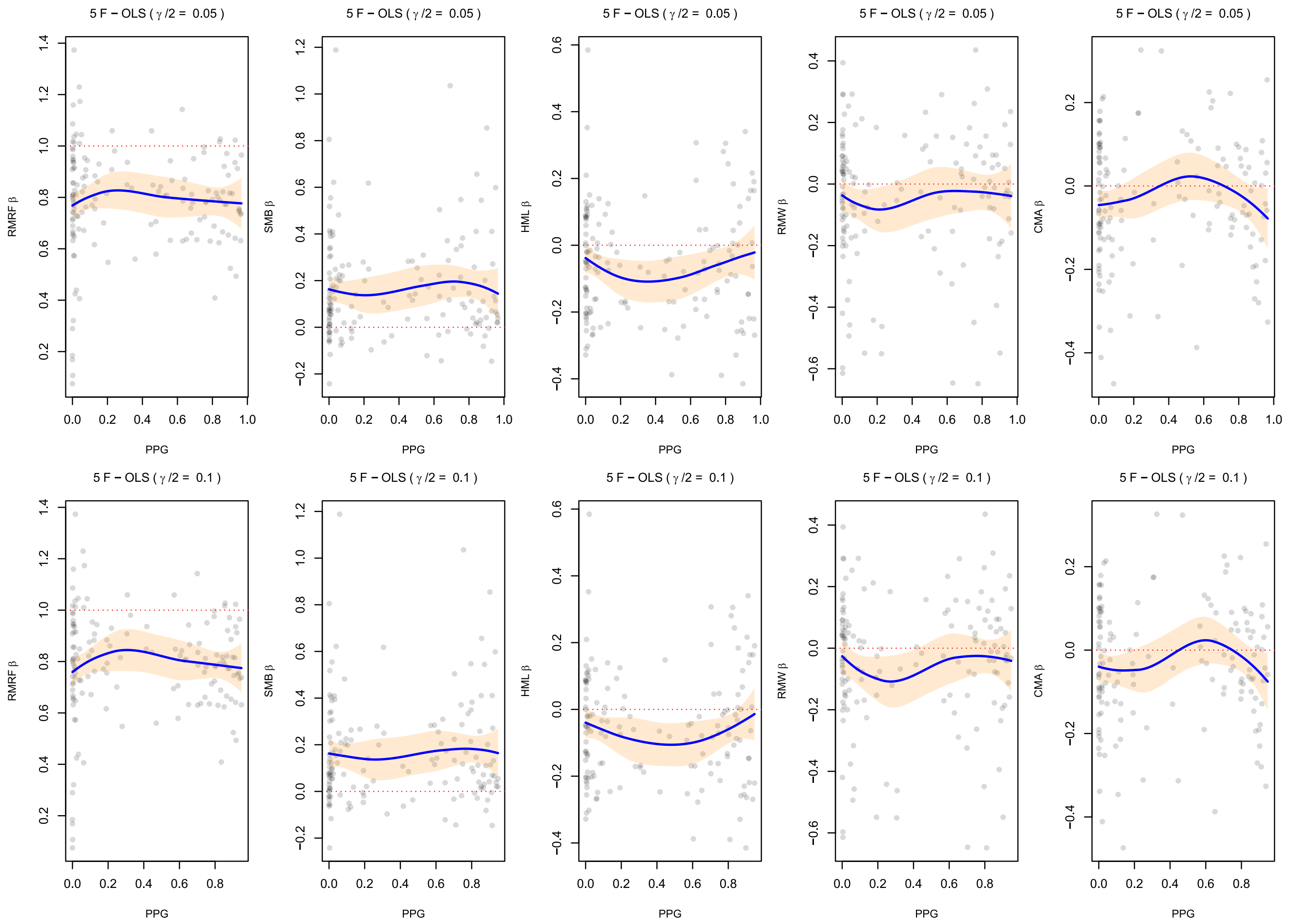

FIGURE 3 | Relationship between the posterior probabilities of of each thematic fund to be good ($\alpha > 0$) and the corresponding estimated coefficients in the 5F model - LOESS line and 95% confidence interval. $\gamma/2 = 0.05$ in the top row, $\gamma/2 = 0.1$ in the bottom row. [Color figure can be viewed at wileyonlinelibrary.com]

$$\text{FDR}_b = [(\gamma/2)\pi_0 + \delta_b\pi_b]/N_b \tag{7}$$

while those computed following the standard approach, that is the one proposed by Barras et al. (2010), are obtained from Equations (6) and (7) by setting to zero the confusion parameters δ_g and δ_g. According to Equation (6), the computation of FDR_g accounts for the very lucky funds, that are funds with negative alphas that are confused with good funds by the test, while in light of Equation (7), the computation of FDR_b accounts for the very unlucky funds, that is funds with positive alphas confused with bad funds by the test. The evaluation of all the presented models is also complemented with the average Kullback–Leibler divergence (KL), across replications of a given estimated mixture:

$$\text{KL}\left(\hat{f}\left(t_m^{(b)}\right), \hat{f}(t_o)\right) = \sum_{i=1}^{N}\left[\frac{1}{2}\hat{f}\left(t_m^{(b)}\right)\log\frac{\hat{f}\left(t_m^{(b)}\right)}{\hat{f}(t_o)} + \frac{1}{2}\hat{f}(t_o)\log\frac{\hat{f}(t_o)}{\hat{f}\left(t_m^{(b)}\right)}\right]$$

$$b = 1, \ldots, B$$

$$\overline{\text{KL}}(\hat{f}(t_m), \hat{f}(t_o)) = \frac{1}{B}\sum_{b=1}^{B}\text{KL}\left(\hat{f}\left(t_m^{(b)}\right), \hat{f}(t_o)\right)$$

Low values of $\overline{\text{KL}}$ indicate that a given mixture approximates well the original data (and model) from which it has been derived.

The analysis developed for the multi-factor models (1) and (2) is then naturally extended to include latent factors. As pointed out in Giglio et al. (2021), the choice of the benchmark model to estimate alphas is crucial. Indeed, there is a possibility that the omission of some important factors leads to attribute to alphas what is merely exposure to some omitted factors. Giglio et al. (2021) showed that alphas computed with a benchmark model that omits some factors can include the risk premia associated with missing factors. This implies a bias in the estimated alphas that affects the result of the t-test for their significance. This effect can be highlighted by considering the following specification of a model that includes both observable and latent factors.[1]

$$r_{i,t} = \alpha_i + \left[\boldsymbol{\beta}'_{o,i}, \boldsymbol{\beta}'_{l,i}\right]\begin{bmatrix}\mathbf{f}_{o,t}\\ \mathbf{f}_{l,t}\end{bmatrix} + \epsilon_{i,t} = \alpha_i + \boldsymbol{\beta}'_{l,i}\boldsymbol{\lambda} + \boldsymbol{\beta}'_{o,i}\mathbf{f}_{o,t} + \boldsymbol{\beta}'_{l,i}(\mathbf{f}_{l,t} - \mathbb{E}(\mathbf{f}_{l,t})) + \epsilon_{i,t} \tag{8}$$

where $\mathbf{f}_{o,t}$ and $\mathbf{f}_{l,t}$ are the (demeaned) observable and latent factors, $\boldsymbol{\beta}_o$ and $\boldsymbol{\beta}_l$ the associated coefficients or exposures and $\boldsymbol{\lambda}_l$ is a vector of risk premia, which tallies with the expected return of $\mathbf{f}_{l,t}$, when the latter are tradable. Equation (8) highlights the potential bias that would affect alphas, in case of omission of latent factors in the specification of the model. In addition, since missing factors could

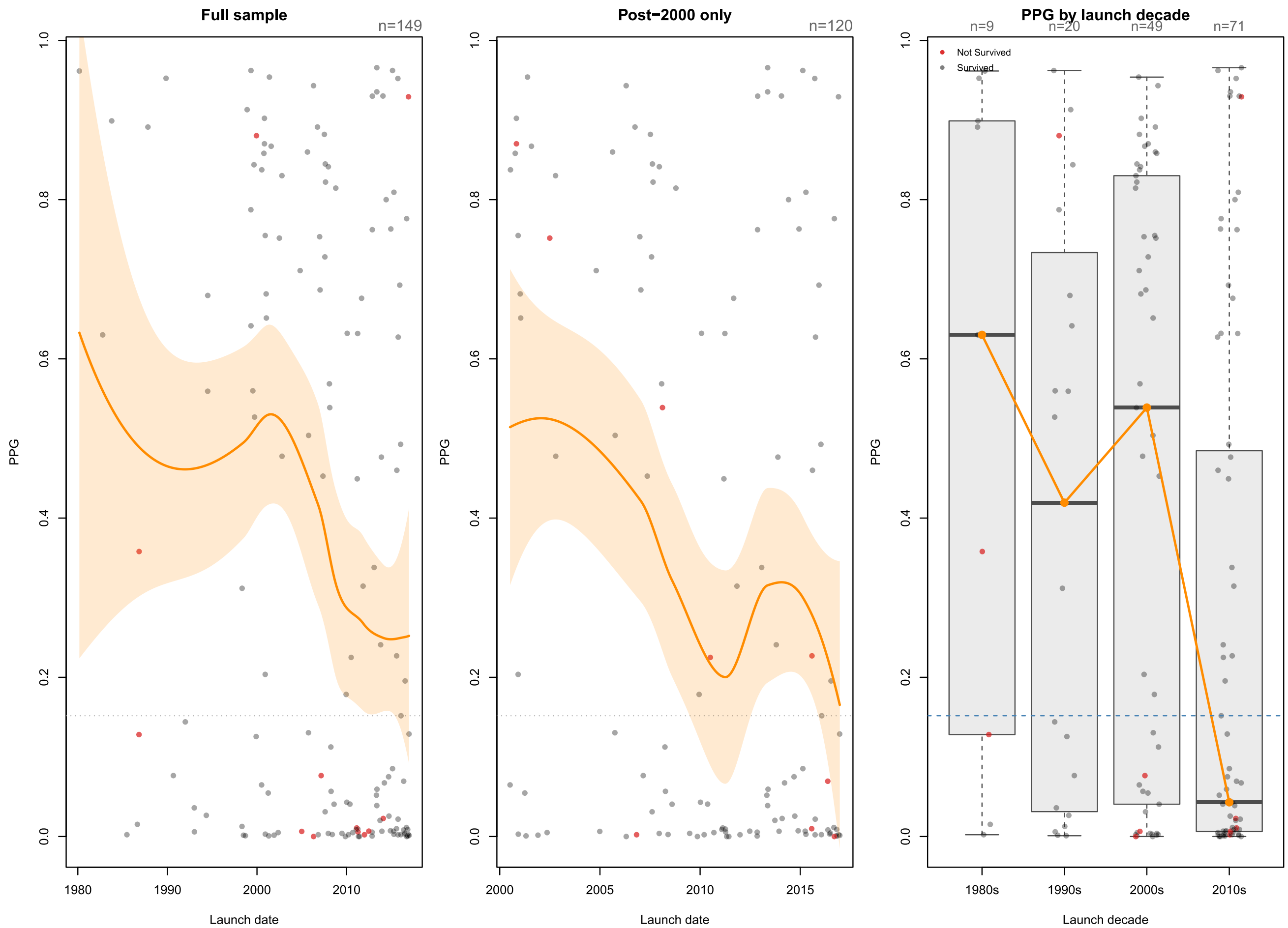

FIGURE 4 | Relationship between the posterior probabilities of of each thematic fund to be good ($\alpha > 0$) in the 5F model and launch date. Red dots denote non-surviving funds. [Color figure can be viewed at wileyonlinelibrary.com]

play the role of idiosyncratic errors, they could affect the t tests for alphas' significance in two different ways. First, by influencing the asymptotic covariance matrix of the alpha estimates and, accordingly, the standard errors of the same. Second, by inducing a strong correlation between the alpha-t-tests that, in turn, would influence the outcomes of the tests. To overcome these negative effects, we propose a novel procedure that integrates the Ferson and Chen (2021) and Giglio et al. (2021) approaches. As in Giglio et al. (2021), the observable component is estimated first

$$\widehat{\beta}_{o,i} = \left(\mathbf{F}_o'\mathbf{M}_T\mathbf{F}_o\right)^{-1}\mathbf{F}_o'\mathbf{M}_T\mathbf{r}_i. \tag{9}$$

Here, $\mathbf{F}_o' = (\mathbf{f}_{o,1}, \ldots, \mathbf{f}_{o,T})$ and $\mathbf{M}_T$ is the de-meaning matrix, $\mathbf{M}_T = \mathbf{I}_T - \frac{1}{T}\mathbf{1}_T\mathbf{1}_T'$. Then, latent factors are extracted from the variance/covariance matrix of the 3F or 5F Fama-French de-meaned residuals $\mathbf{Z} = \mathbf{M}_T(\mathbf{Y} - \mathbf{F}_o\widehat{\beta}_o)$, where $\widehat{\beta}_o = (\widehat{\beta}_{o,1}, \ldots, \widehat{\beta}_{o,N})$. This entails extracting the first K_l eigenvectors $(\mathbf{b}_1, \ldots, \mathbf{b}_{K_l})$ from $\frac{1}{T}\mathbf{ZZ}'$, finally obtaining

$$\widehat{\beta}_l = \sqrt{N}(\mathbf{b}_1, \ldots, \mathbf{b}_{K_l})' \tag{10}$$

and, as a by-product, $\widehat{\beta} = \left(\widehat{\beta}_o', \widehat{\beta}_l'\right)$.

The number of latent factors K_l is determined through the scree plot of the residuals. Then, the slope $\boldsymbol{\lambda}$ is estimated estimating a cross-sectional regression of $\bar{\mathbf{r}}$, the $N \times 1$ vector of return time-averages, on $\widehat{\beta}$

$$\widehat{\lambda} = (\widehat{\beta}'\mathbf{M}_N\widehat{\beta})^{-1}\widehat{\beta}'\mathbf{M}_N\bar{\mathbf{r}} \tag{11}$$

Here, $\mathbf{M}_N$ is the cross-sectional de-meaning matrix, $\mathbf{M}_N = \mathbf{I}_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N'$.

Lastly, the vector of $\boldsymbol{\alpha}$ is estimated by subtracting the risk premia from the average returns

$$\widehat{\alpha} = \bar{\mathbf{r}} - \widehat{\beta}'\widehat{\lambda} \tag{12}$$

The t statistics for each $\widehat{\alpha}_i$ are computed (see Appendix A) and the Ferson and Chen (2021) method is thus adapted to obtain the final alpha distribution.

The treatment of lagged external covariates in $\mathbf{W}$ in the framework of Giglio et al. (2021) is detailed in Appendix. In addition, to deal with the unbalanced nature of the panel, missing observations due to non-surviving funds are handled

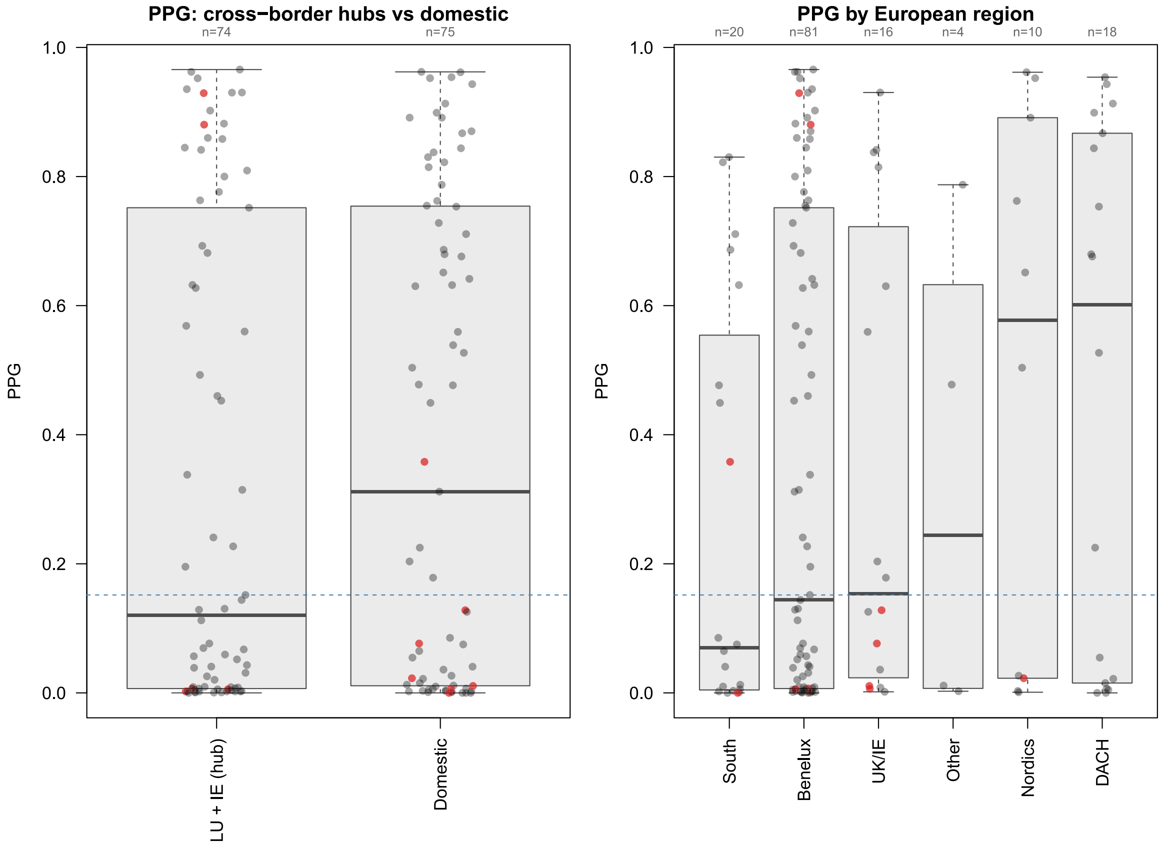

FIGURE 5 | Relationship between the posterior probabilities of of each thematic fund to be good ($\alpha > 0$) in the 5F model and country group. Red dots denote non-surviving funds. South: IT, ES, FR, PT; Benelux: BE, NL, LU; Nordics: SE, NO, FI, DK; DACH: DE, AT, CH; Other: Liechtenstein, San Marino. [Color figure can be viewed at wileyonlinelibrary.com]

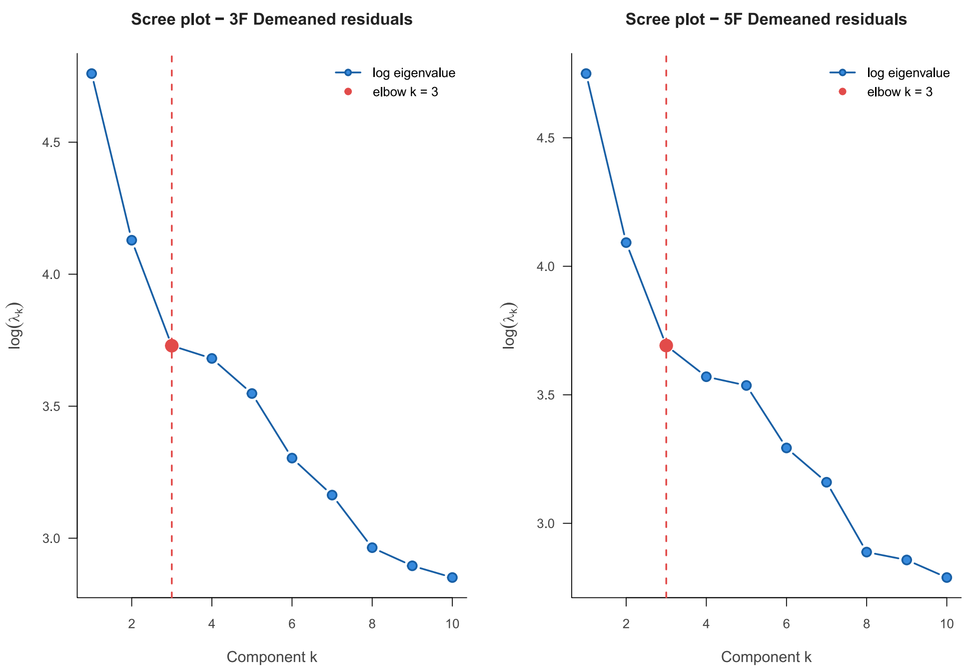

FIGURE 6 | Scree plots of the demeaned residuals of the 3F and 5F models. Thematic funds. [Color figure can be viewed at wileyonlinelibrary.com]

using the matrix completion approach proposed by Giglio et al. (2021), which provides a consistent reconstruction of the return matrix and allows for reliable estimation of fund alphas.

In the next sections, the combined methodology is applied to both thematic and mutual funds to determine their alpha distribution.

5 | The Empirical Application

This section presents the results obtained by applying the combined approach outlined in the previous section on both thematic (ThF) and non-thematic mutual funds (MF). As a particular case, also the Barras et al. (2010) (BSW) method is considered. Regarding the analysis without the inclusion of latent factors, all the t-values across the simulations for the 3F-5F models are computed by using the White heteroskedastic-consistent standard errors (White 1980). Concerning instead the simulations settings under the alternative, the grid of values for α_b and α_g ranges from the 1% quantile to the 99% quantile of the estimated alpha distribution, with a step of 2×10^{-5}. Lastly, the simulation schemes and the generation of the synthetic, model-based t-values are each repeated $B = 1000$ times, and the average DTS statistic is computed as a measure of (mis)match between the simulated t-statistics distribution and the one estimated with the original data. Among all simulated fund alpha distributions, the best is the one with the smallest DTS statistic.

Tables 11 and 12 collects all the relevant results that will be presented in the rest of this section, for each dataset (ThF or MF), and combination of model setup (FC or BSW), number of observed factors (3F or 5F), number of latent factor (0 to 2-3 LF) and size $\gamma/2 = \{0.05, 0.1\}$.

5.1 | Analysis of Thematic Funds

Table 3 displays the distribution of the estimated alphas, and the related t-ratios resulting from the 3F and 5F Fama-French models by quantiles[2], applied to the $N = 149$ selected thematic funds. It can be observed that, while the distribution of the alphas is only mildly negatively skewed in the 3F model, the one of the t-ratios exhibits a significant negative skewness. As for the 5F model, the distribution of the alphas is slightly more positively skewed, while that of the t-ratios has a less severe negative skewness with respect to the 3F model.

Application of FC determines the better alpha distribution. Table B3 in Appendix B reports some configurations of the estimated proportions of negative, positive and zero-alpha funds ($\widehat{\pi}_b$, $\widehat{\pi}_g$ and $\widehat{\pi}_0$), associated with each possible different couple (α_b, α_g) for both the 3F model or with the 5F model at size $\gamma/2 = \{0.05, 0.1\}$. The DTS test results in this table indicate that the best-fitting configurations for the 3F and 5F models are the following

- In the 3F model, with $\gamma/2 = 0.05$, the best fund mixture distribution includes a fraction $\widehat{\pi}_b = 0.62$ of bad thematic funds centred on $\alpha_b = -0.205\%$, a fraction $\widehat{\pi}_b = 0.3$ of good thematic funds centred on $\alpha_g = 0.135\%$, while a portion $\widehat{\pi}_0 = 0.08$ of funds are centred on zero.
- In the 3F model, with $\gamma/2 = 0.1$, the best fund distribution is composed of a fraction $\widehat{\pi}_b = 0.59$ of bad thematic funds centred on $\alpha_b = -0.27\%$, a fraction $\widehat{\pi}_b = 0.3$ of good thematic funds centred on $\alpha_g = 0.135\%$, while a portion $\widehat{\pi}_0 = 0.11$ of funds are centred on zero.
- In the 5F model, with $\gamma/2 = 0.05$, the best fund distribution embodies a fraction $\widehat{\pi}_b = 0.53$ of bad thematic funds centred on $\alpha_b = -0.235\%$, a fraction $\widehat{\pi}_g = 0.37$ of good funds centred on $\alpha_g = 0.125\%$, and a fraction $\widehat{\pi}_0 = 0.1$ of funds centred on zero.
- In the 5F model, with $\gamma/2 = 0.1$, the best fund distribution embodies a fraction $\widehat{\pi}_b = 0.54$ of bad thematic funds centred on $\alpha_b = -0.245\%$, a fraction $\widehat{\pi}_g = 0.37$ of good funds centred on $\alpha_g = 0.09\%$, and a fraction $\widehat{\pi}_0 = 0.09$ of funds centred on zero.

All configurations are characterised by low levels of confusion parameters δ_b and δ_g, while the parameters β_b and β_g highlight that the left side of the mixture retains a higher power value than the right side.

For the sake of comparison, the outcomes of the standard BSW method, obtained by imposing $\beta_b = 1$ and $\delta_g = 0$ in the FC estimation process of π_i, $i = \{0, b\}$, are also reported

- In the 3F model, at $\gamma/2 = 0.05$, a fraction $\widehat{\pi}_b = 0.411$ consists of bad thematic funds centred on $\alpha_b = -0.237\%$, a fraction

TABLE 5 | Expected fractions of lucky (FDR_g) and unlucky (FDR_b) for thematic funds in mixed models, following either the BSW or the FC methodology.

			1LF		2LF		3LF	
Type	**OF model**	$\gamma/2$	FDR_g	FDR_b	FDR_g	FDR_b	FDR_g	FDR_b
FC	3F	0.05	0.0505	0.0448	0.0211	0.0651	0.0464	0.0374
		0.1	0.0482	0.0679	0.12	0.0901	0.0562	0.0784
	5F	0.05	0.0249	0.0381	0.0355	0.0507	0.0585	0.0421
		0.1	0.0491	0.0537	0.056	0.038	0.0264	0.0569
BSW	3F	0.05	0.131	0.0481	0.191	0.023	0.141	0.030
		0.1	0.162	0.066	0.229	0.034	0.186	0.043
	5F	0.05	0.123	0.054	0.187	0.028	0.143	0.027
		0.1	0.159	0.074	0.224	0.039	0.177	0.037

Abbreviations: BSW, Barras, Scaillet, and Wermers; FDR, false discovery rate.

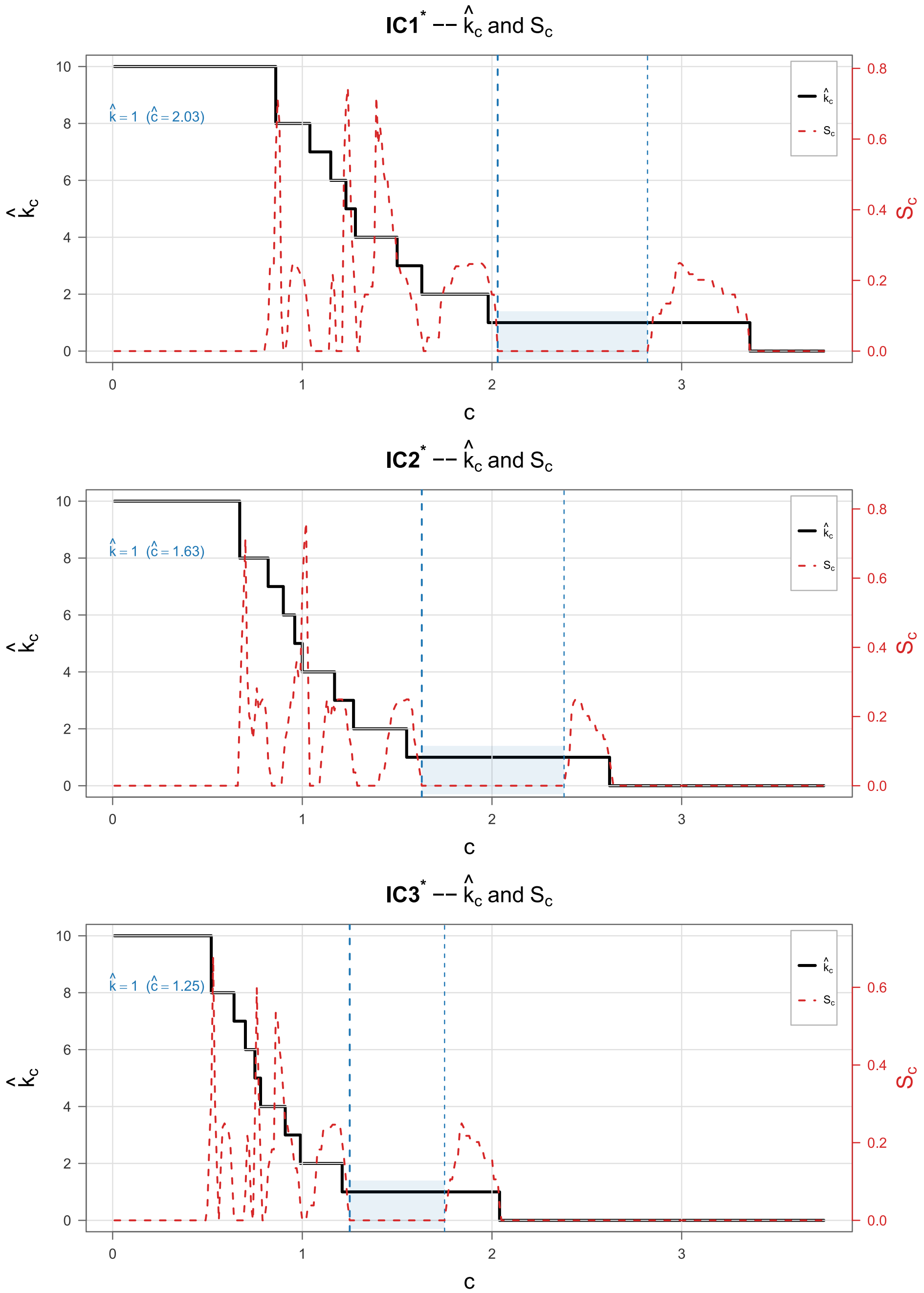

FIGURE 7 | Choice of the best number of latent factor $\hat{k}$ for thematic funds, based on the stability index S_c, as in Alessi et al. (2010). The maximally wide plateau in S_c (secondary y-axis) represents the corresponding number of factors in the primary y-axis. [Color figure can be viewed at wileyonlinelibrary.com]

$\hat{\pi}_b = 0.145$ consists of bad thematic funds centred on $\alpha_b = 0.352\%$, and a fraction $\hat{\pi}_0 = 0.444$ is of funds centred on zero.

- In the 3F model, at $\gamma/2 = 0.1$, a fraction $\hat{\pi}_b = 0.468$ consists of bad thematic funds centred on $\alpha_b = -0.212\%$, a fraction $\hat{\pi}_b = 0.191$ consists of bad thematic funds centred on $\alpha_b = 0.314\%$, and a fraction $\hat{\pi}_0 = 0.341$ is of funds centred on zero.
- In the 5F model, at $\gamma/2 = 0.05$, a fraction $\hat{\pi}_b = 0.378$ consists of bad thematic funds centred on $\alpha_b = -0.232\%$, a fraction $\hat{\pi}_g = 0.167$ is of good funds centred on $\alpha_g = 0.338\%$, and a fraction $\hat{\pi}_0 = 0.455$ is of funds centred on zero.
- In the 5F model, at $\gamma/2 = 0.1$, a fraction $\hat{\pi}_b = 0.434$ consists of bad thematic funds centred on $\alpha_b = -0.208\%$, a fraction $\hat{\pi}_g = 0.22$ of good funds centred on $\alpha_g = 0.303\%$, and a fraction $\hat{\pi}_0 = 0.346$ of funds centred on zero.

It emerges that, especially in the 5F models, BSW engenders higher proportions of zero funds and lower proportions of bad and good funds. In fact the estimation of bad and good funds in

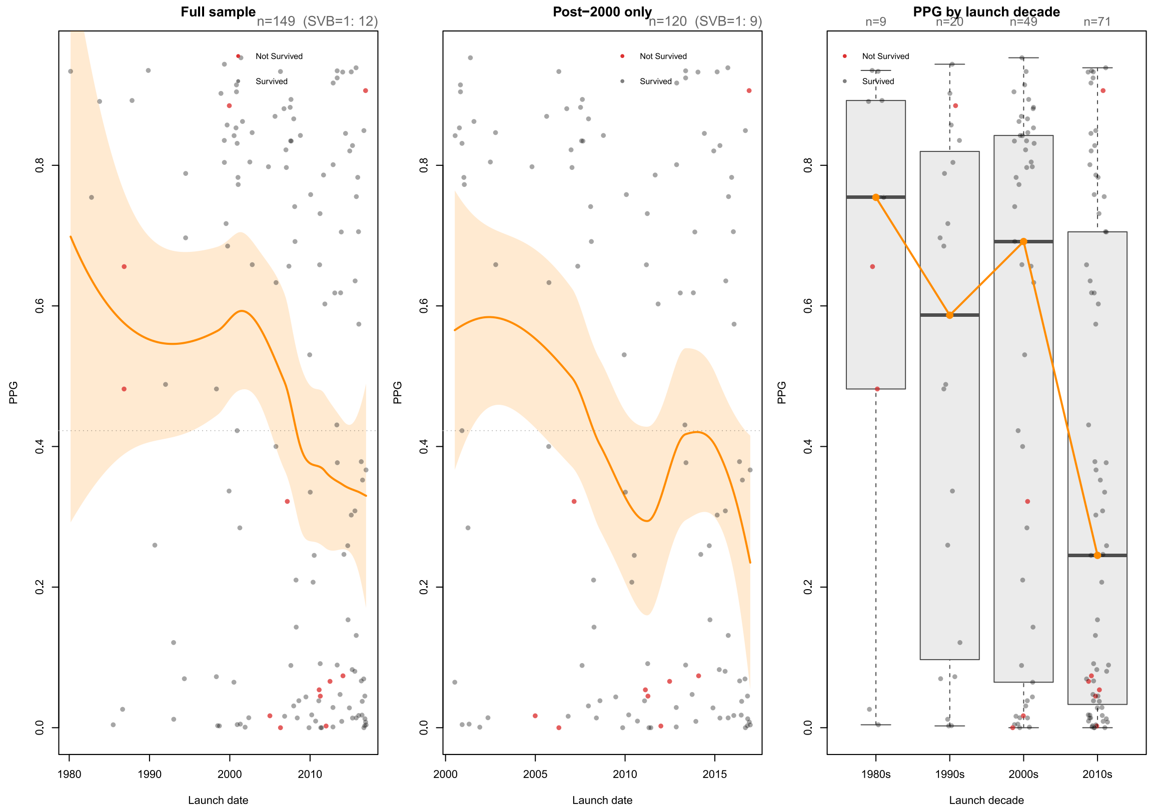

FIGURE 8 | Relationship between the posterior probabilities of of each thematic fund to be good ($\alpha > 0$) in the 5F-1LF model and launch date. Red dots denote non-surviving funds. [Color figure can be viewed at wileyonlinelibrary.com]

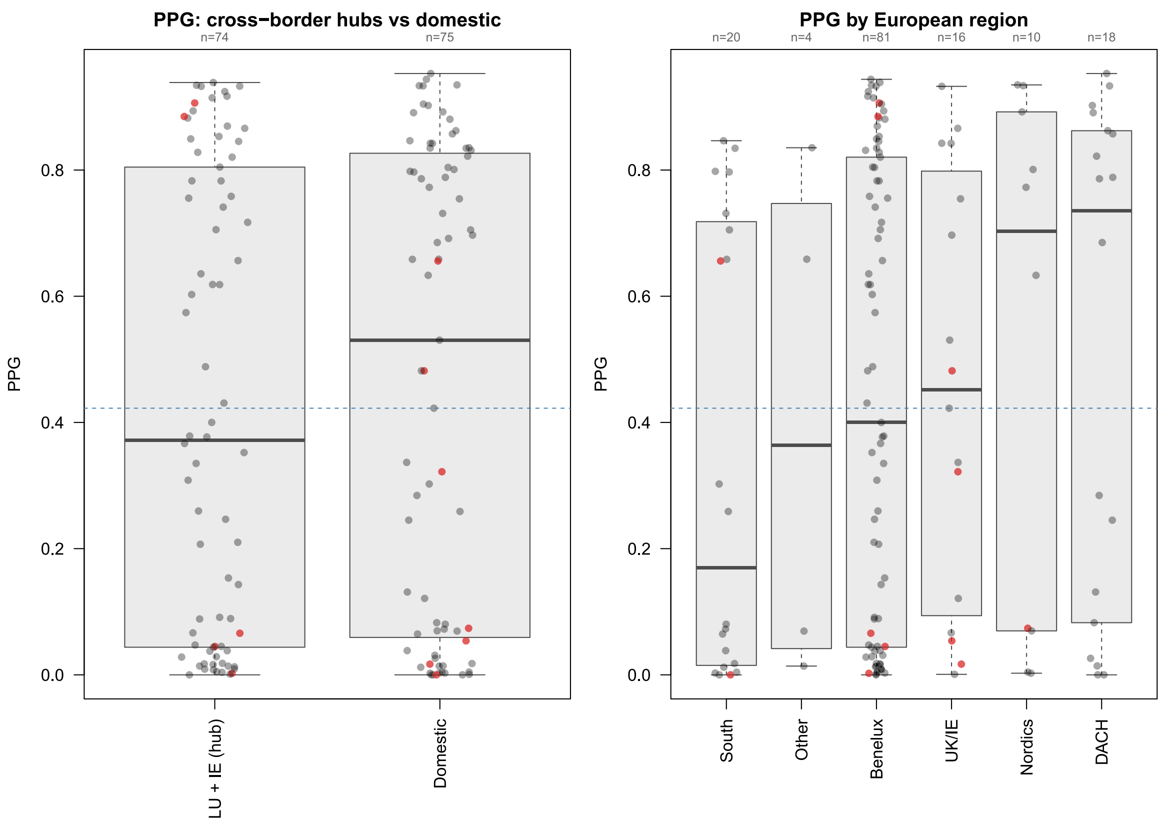

FIGURE 9 | Relationship between the posterior probabilities of of each thematic fund to be good ($\alpha > 0$) in the 5F-1LF model and country group. Red dots denote non-surviving funds. South: IT, ES, FR, PT; Benelux: BE, NL, LU; Nordics: SE, NO, FI, DK; DACH: DE, AT, CH; Other: Liechtenstein, San Marino. [Color figure can be viewed at wileyonlinelibrary.com]

FC also accounts for the confusion parameters.[3] The Ferson and Chen (2021) approach becomes equivalent to the Barras et al. (2010) one, only when it sets the beta parameters, β_g and β_b, equal to one and the confusion parameters equal to zero. Both of these choices introduce bias in the estimators of the fractions of good, bad and zero alpha funds. The FDR, that represents the expected fractions of lucky and unlucky funds, computed as in Equations (7)–(6) and the standard ones (BSW), computed by setting to zero the confusion and the power parameters in these formulas, are displayed in Table 4, for each level $\gamma/2$ and for both the 3F and the 5F models.

It results that BSW provides estimates of the FDR that are generally higher than the ones calculated following FC, especially for the lucky funds. Figure 2 shows the best simulated mixtures of funds' alphas for the 3F and 5F models, at levels $\gamma/2 = 0, 05, 0, 1$. For a given size $\gamma = 0.05, 0.1$ and for each 3F or 5F Fama-French model, the figures depicts the simulated alpha kernel densities of bad funds (in red), good funds (in green) and funds with zero alpha (in yellow). For each configuration, the best final alpha mixture would result from the composition of these three densities with weights $\hat{\pi}_b$, $\hat{\pi}_g$ and $\hat{\pi}_0$ for the density of bad, good and zero alpha, respectively. The figure shows that a discernible three-group mixture is detected in the thematic funds distribution, at both $\gamma/2$ levels, with a polarising tendency on either the bad funds group or the good funds group.

Finally, the posterior probabilities of funds having $\alpha > 0$ (PPG) are calculated via the Bayes' theorem and the employment of kernel density estimators for the alpha densities, using the best-fitting configurations. Analysing them with Fama-French factor coefficients, these probabilities show only a slight increase in PPG with lower coefficient magnitude of CMA, and a higher coefficient magnitude of HML, as displayed in Figure 3. This is a potential indication that once all exogenous drivers and confounders are accounted for, posterior probabilities are a intrinsic measure of performance. In this regard, it is worth noting from Table 1 that both the factors RMW and CMA, which are not included in 3F, are deemed statistically significant (according to the usual F test for linear restrictions) for 14% of the 149 thematic funds. Their inclusion in the model helps identify three distinct groups of funds.

Lastly, looking at Figures 4 and 5, which show the relationship between PPG and launch date/geographical location for the 5F model and $\gamma/2 = 0.05$ (results are nearly identical in the $\gamma/2 = 0.1$ case), it can be observed a general decrease in PPG in the last years, due to the increasing competition in the thematic sector, with a resurgence around 2015—especially looking at

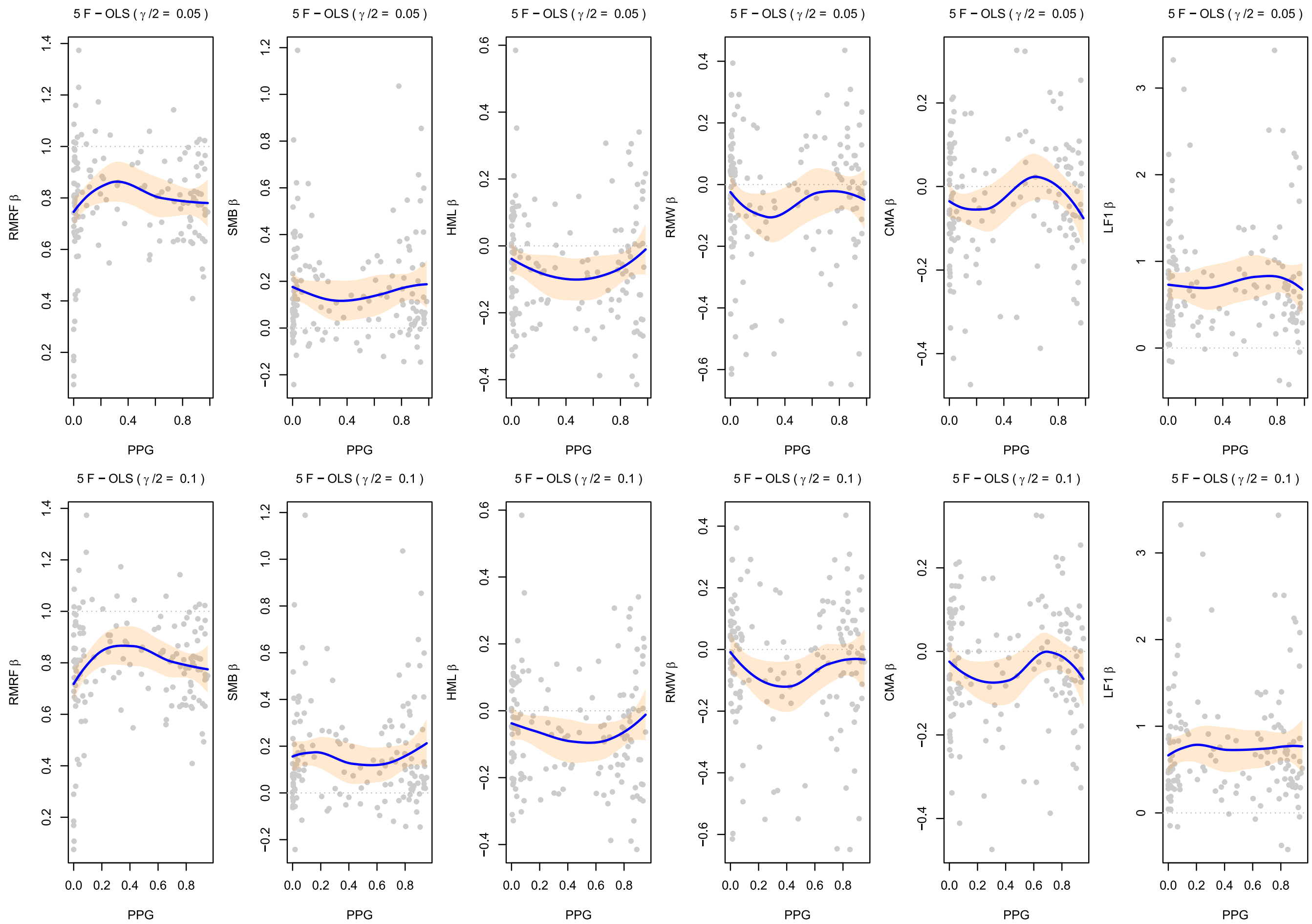

FIGURE 10 | Relationship between the posterior probabilities of $\alpha > 0$ (PPG) for each thematic fund and the coefficients in the 5F model, one latent factor - LOESS line and 95% confidence interval. $\gamma/2 = 0.05$ in the top row, $\gamma/2 = 0.1$ in the bottom row. [Color figure can be viewed at wileyonlinelibrary.com]

the only post-2000 panel. Geographically, funds domiciled in cross-border hubs such as Luxembourg and Ireland have lower PPG, while the best-performing are in the Nordic Europe countries or in the DACH countries (Germany, Austria, Switzerland). This is a purely descriptive consideration, as the boxplot do not show statistically meaningful geographical separation. Generally, funds that have not survived in the entire observation period (highlighted in red) are concentrated in the low PPG area of the graph.

Now, the analysis is also extended to include latent factors in the models. Looking at Figure 6, it seems reasonable to consider, in the mixed model, up to three latent factors. Table 5 provides the false discovery rates for these models. According to the results, the 5F model provides better estimates than the 3F model for $\gamma/2 = 0.1$, while the 3F model is only slightly superior for $\gamma/2 = 0.05$. As observed in the analysis with no latent factors, BSW is almost always outperformed by FC, especially, on the portion related to the lucky funds.

The main difference between mixed models with latent factors and with only observables is the share of bad funds. With the exception of the 1LF model, the 0LF models absorb a sizable amount of either good or zero funds into the bad funds group. As previously noted, FC applied to thematic funds offers a polarised configuration, with a smaller, albeit not negligible,

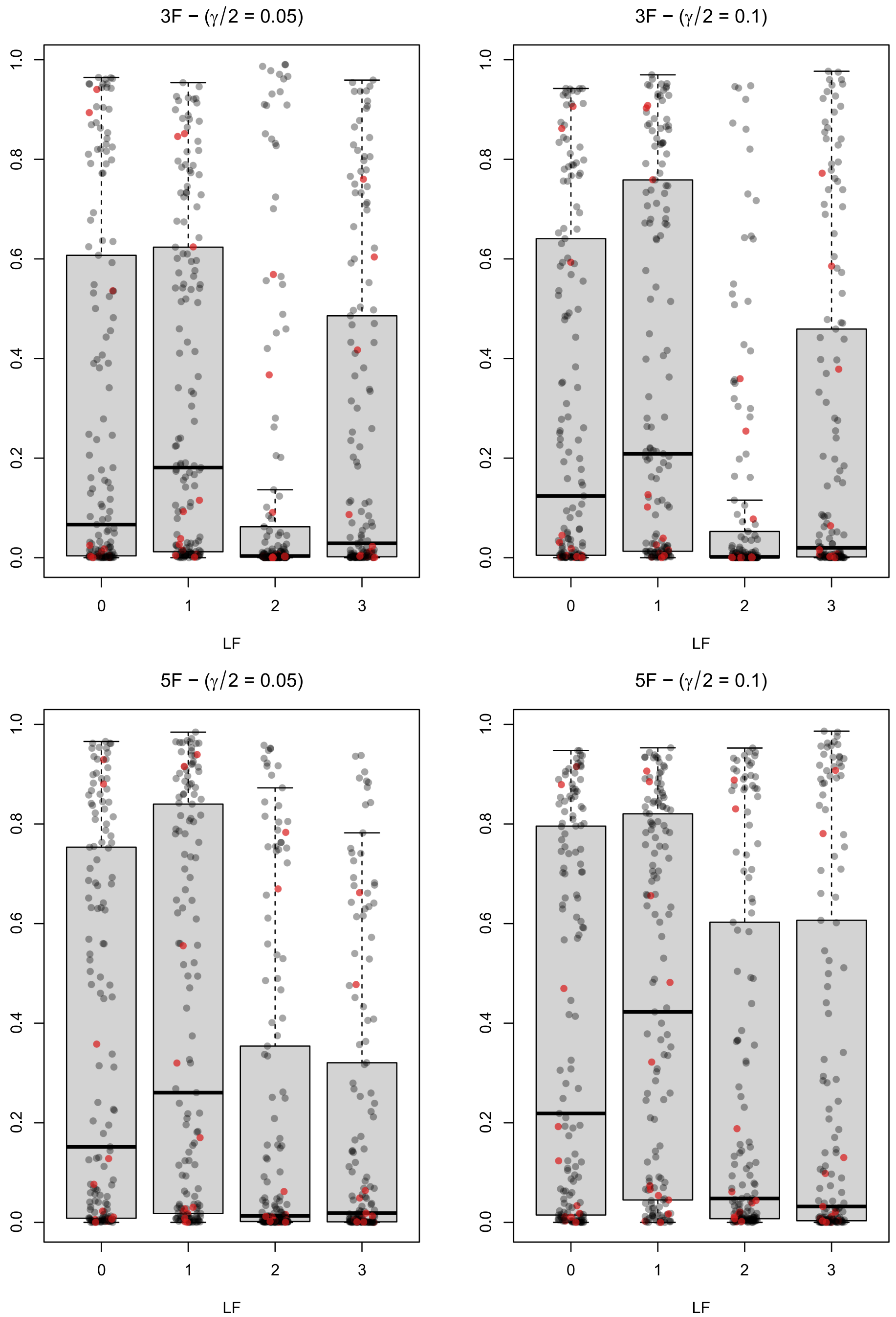

FIGURE 11 | Relationship between the posterior probabilities of $\alpha > 0$ (PPG) for each thematic fund and number of latent factors. Red points denote non-surviving funds. [Color figure can be viewed at wileyonlinelibrary.com]

portion of funds with null alpha. Estimates of the alpha parameters are robust to the composition of the mixture and the number of latent factors, since $\hat{\alpha}_b$ gravitates around −0.25 and $\hat{\alpha}_g$ stays around 0.1 in the majority of FC configurations. BSW, on the contrary, potentially overestimates the centre of the good funds distribution.

In choosing the best suited number of latent factors, the simple graphical analysis of the scree plot has been complemented with three modified information criteria. Alessi et al. (2010) and Gagliardini et al. (2019), make use of the following penalised functions[4]

TABLE 6 | Alphas cross-sectional distribution obtained from the 3F and 5F models on monthly non-thematic funds returns.

	3F Alphas		5F Alphas	
	Est. (%)	***t*-ratio**	**Est. (%)**	***t*-ratio**
Fractile				
0.01	−0.555	−2.227	−0.583	−2.333
0.05	−0.276	−1.559	−0.301	−1.765
0.1	−0.181	−1.097	−0.203	−1.295
0.25	−0.070	−0.401	−0.089	−0.576
Median	0.059	0.395	−0.030	0.206
0.75	0.175	1.223	0.149	1.045
0.9	0.288	1.979	0.261	1.799
0.95	0.382	2.433	0.363	2.204
0.99	0.618	3.008	0.586	2.919

$$\mathrm{IC}_1^c(k) = \log(V(k)) + ck\left(\frac{N+T}{NT}\right)\log\left(\frac{NT}{N+T}\right)$$
$$\mathrm{IC}_2^c(k) = \log(V(k)) + ck\left(\frac{N+T}{NT}\right)\log(\min\{N,T\})$$
$$\mathrm{IC}_3^c(k) = \log(V(k)) + ck\left(\frac{(\sqrt{N}+\sqrt{T})^2}{NT}\right)\log\left(\frac{NT}{(\sqrt{N}+\sqrt{T})^2}\right)$$

In which $V(k)$ is the sum of the remaining eigenvalues of $\frac{\mathbf{Z}_s\mathbf{Z}_s'}{NT}$ (where $\mathbf{Z}_s$ is the fund-standardised version of $\mathbf{Z}$), after removing its largest k eigenvalues. Thus, the optimal number of factors is chosen as

$$k_j^* = \mathrm{argmin}_{k \le k_{\max}} \ \mathrm{IC}_j^c(k) \quad j = 1, 2, 3$$

The constant c has been derived using the stability criterion in Alessi et al. (2010).

Figure 7 shows that the stability index S_c has its longest plateau with $k = 1$ factor. Thus, a mixed model with five observable factors and one latent factors can be deemed to perform the best, by looking at the above criteria, the average KL divergence in Tables 11 and 12, or the FDR estimates given in Table 5. The main results emerging from this model are

- When $\gamma/2 = 0.05$, the optimal fund distribution includes a fraction $\hat{\pi}_b = 0.51$ of bad thematic funds centred on $\alpha_b = -0.235\%$, a fraction $\hat{\pi}_g = 0.42$ of good funds centred

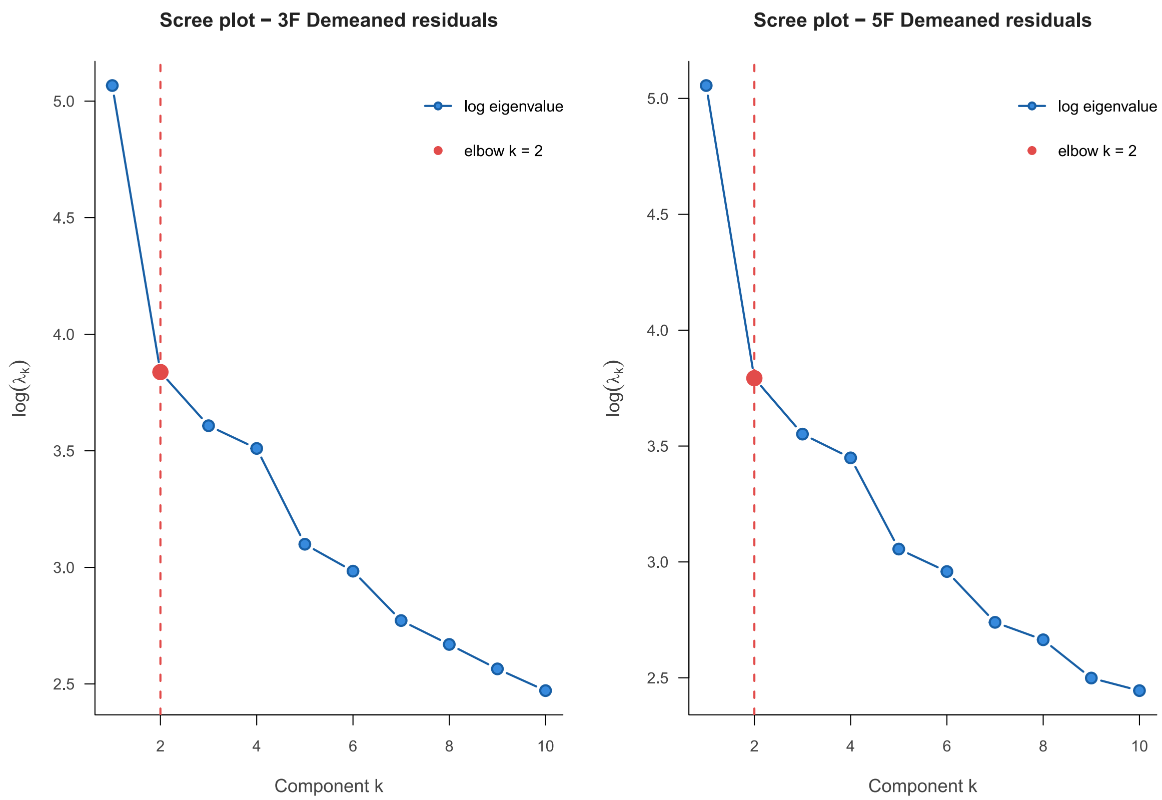

FIGURE 12 | Scree plots of the demeaned residuals of the 3F and 5F models. Non-thematic mutual funds. [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 7 | Expected fractions of lucky (FDR_g) and unlucky (FDR_b) for non-thematic mutual funds, following either the BSW or the FC methodology.

			0LF		1LF		2LF	
Type	**OF Model**	$\gamma/2$	FDR_g	FDR_b	FDR_g	FDR_b	FDR_g	FDR_b
FC	3F	0.05	0.025	0.181	0.05	0.012	0.045	0.048
		0.1	0.055	0.033	0.035	0.079	0.049	0.052
	5F	0.05	0.05	0.359	0.021	0.014	0.004	0.012
		0.1	0.006	0.033	0.013	0.033	0.052	0.033
BSW	3F	0.05	0.228	0.006	0.2	0.007	0.112	0.076
		0.1	0.265	0.009	0.234	0.009	0.152	0.105
	5F	0.05	0.282	0.01	0.265	0.013	0.109	0.081
		0.1	0.317	0.013	0.314	0.017	0.147	0.111

Abbreviations: BSW, Barras, Scaillet, and Wermers; FDR, false discovery rate.

on $\alpha_g = 0.095\%$, and a fraction $\widehat{\pi}_0 = 0.07$ of funds centred on zero. This configuration is characterised by low levels of confusion parameters δ_b and δ_g, while power parameters β_b and β_g favour the left tail of the mixture.

- When $\gamma/2 = 0.1$, the best fund distribution consists of a fraction $\widehat{\pi}_b = 0.44$ of bad thematic funds centred on $\alpha_b = -0.29\%$, a fraction $\widehat{\pi}_g = 0.45$ of good funds centred on $\alpha_g = 0.075\%$, and a fraction $\widehat{\pi}_0 = 0.11$ of funds centred on zero. This configuration shows slightly higher levels of confusion parameter δ_b, with respect to the model having $\gamma/2 = 0.05$, while the left side of the mixture retains a higher power β_b.

Then, Figure 10 shows the relationships between the PPG of each fund and the coefficients of the observed Fama-French factors and the first latent factor in the 5F-1LF model. The relationships found using the 5F models based only on observable factors are generally confirmed. The latent factor loading has a positive mean and exhibits a slight negative curvature corresponding to the right tail of PPG. An ex-post descriptive comparison of PPG and funds' launch dates and country group generally confirms the base OLS model picture, except showing a different geographical ranking: the UK/IE group shows a higher PPG, as seen in Figures 8 and 9.

In summary, complementing the Fama-French model with unobserved factors improves the picture in terms of bad/good fund detection. In particular, both FDR_g and FDR_b improves when considering latent factors, as shown in Table 5, while keeping lower level of confounding and satisfactory power. Interestingly, the distribution of the PPG is altered only when the number of factors exceeds one, as can be seen in Figure 11, Lastly, the empirical analysis does not rely on a purely survivorship-biased sample. Non-surviving funds are explicitly included in the dataset. Exiting funds are more likely to have experienced poor performance, and the PPG is generally lower in the non-surviving subgroup (the red dots in all the Figures presented in the paper confirm this). Therefore, it can be argued that excluding them would lead to overconfident PPG estimation and allocation of funds in the mixture groups.

5.2 | Analysis of Non-Thematic Mutual Funds

The same modelling strategy is applied to $N = 1149$ non-thematic mutual funds. Table 6 shows the alpha and t-ratio cross-sectional distributions in correspondence to relevant fractiles. Looking at the 3F model columns, it emerges that the distribution of the alphas is only slightly negatively skewed, while the t-ratios distribution exhibits a more pronounced negative skewness, especially with respect to thematic funds. In the 5F model, the negative skewness of both the alphas and t-ratios distributions is even more pronounced than those of the 3F model. Moreover, in the 5F model the t-ratios distribution is generally more concentrated on the negative semi-axis, compared to the thematic funds shown in Table 3. From a direct comparison between these results and those of Table 3, it also emerges that the median alpha is negative for both thematic and non-thematic funds. A further argument is that the range of alpha estimates of non-thematic funds, regardless of the asset pricing model used, is narrower than that of thematic funds.

As in the previous section, a mixed model with latent and observable factors has been considered. Both a 3F and a 5F Fama-French models are estimated. Figure 12 displays the scree plots of the 3F and 5F models, suggesting that that at most two latent factors should be extracted from the demeaned residual matrix of the Fama-French model, with either three or five observable factors.

The FC methodology leads mixed conclusions: the optimal alpha distribution might include a negligible amount of either zero alpha funds or good alpha funds, due to the α_g estimate being in several scenarios close to zero, or having the estimated ratio of zero funds negligible. This result follows the lines of the findings of Fama and French (2010). This is corroborated by the search path surface in Figure B2 in Appendix, that hardly finds a unique optimal configuration.

Overall, by looking at Table 7 that provides the FDR for the tails of the distribution, for all models and number of latent factors, and Figure 13, employing information criteria, it emerges that the mixed model with one latent factors can be deemed as generally the optimal choice:

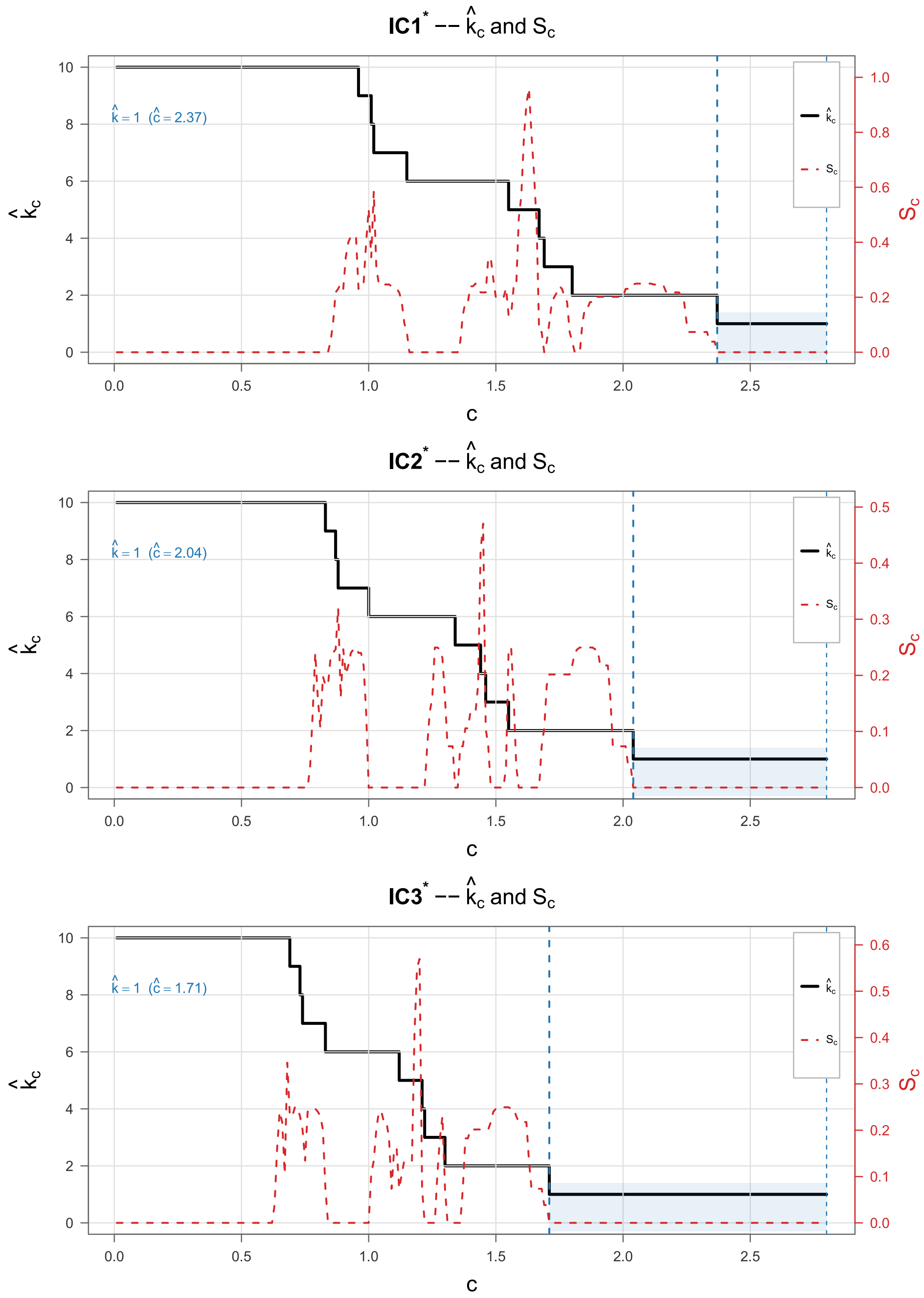

FIGURE 13 | Choice of the best number of latent factor $\widehat{k}$ for non-thematic funds, based on the stability index S_c, as in Alessi et al. (2010). The maximally wide plateau in S_c (secondary y-axis) represents the corresponding number of factors in the primary y-axis. [Color figure can be viewed at wileyonlinelibrary.com]

- Considering the 3F-1LF model with $\gamma/2 = 0.05$, the best alpha fund distribution includes a fraction $\widehat{\pi}_b = 0.71$ of bad non-thematic mutual funds centred on $\alpha_b = -0.42\%$, a fraction $\widehat{\pi}_g = 0.26$ of good funds are centred on $\alpha_g = 0.01\%$, and a fraction $\widehat{\pi}_0 = 0.03$ of funds centred on zero. The power parameter greatly favours the left tail, with $\widehat{\beta}_b = 0.86$.
- Considering the 3F-1LF model with $\gamma/2 = 0.1$, the best alpha fund distribution includes a fraction $\widehat{\pi}_b = 0.98$ of bad non-thematic mutual funds centred on $\alpha_b = -0.245\%$, a fraction $\widehat{\pi}_g = 0.01$ of good funds centred on $\alpha_g = 0.81\%$, and a fraction $\widehat{\pi}_0 = 0.01$ of funds centred on zero. The power parameter greatly favours the left tail, with $\widehat{\beta}_b = 0.88$.
- Considering the 5F-1LF model with $\gamma/2 = 0.05$, the best alpha fund distribution includes a fraction $\widehat{\pi}_b = 0.76$ of bad non-thematic mutual funds centred on $\alpha_b = -0.41\%$, a fraction $\widehat{\pi}_g = 0.21$ of good funds centred on $\alpha_g = 0.01\%$, and a negligible fraction $\widehat{\pi}_0 = 0.03$ of funds centred on zero. The power parameter greatly favours the left tail, with $\widehat{\beta}_b = 0.935$.
- Considering the 5F-1LF model with $\gamma/2 = 0.1$, the best alpha fund distribution includes a fraction $\widehat{\pi}_b = 0.8$ of bad

non-thematic mutual funds centred on $\alpha_b = -0.41\%$, a fraction $\widehat{\pi}_g = 0.18$ of good funds centred on $\alpha_g = 0.01\%$, and a negligible fraction $\widehat{\pi}_0 = 0.02$ of funds centred on zero.

The results of BSW shift the mixture composition toward one with greater percentage of funds with zero alpha and a smaller percentage of good funds, except for the 2LF case. Furthermore, looking at Table 7, it can be seen that BSW produces estimates of the FDR_g that are generally higher than the ones calculated using FC for the lucky funds.

Tables 11 and 12 show the parameters characterising the best configurations for each category of funds, Fama-French model, size and number of latent factors. In addition, standard errors of the estimated proportions $\widehat{\pi_b}$, $\widehat{\pi_0}$ and $\widehat{\pi_g}$ are reported. These quantities have been calculated via simulation for the best-fitting configuration in every scenario, across $B = 1000$ data replications generated from the corresponding alpha distribution. The same table also provides the average Kullback–Leibler Divergence measure between the estimated density, $\widehat{f}(t_o)$, of the t-values obtained from the data (for each Fama-French model and number of latent factors), and the estimated t density, $\widehat{f}(t_m)$, implied by each alpha configuration (i.e., obtained via synthetic data generation).

In short, the results emerging from the analysis on both thematic and non-thematic mutual funds lead to the following conclusions.

Direct comparisons among the Fama-French models show that using a mixed strategy based on five observable factors engenders an alpha distribution whose KL divergence from the empirical counterpart is smaller than others.

In the thematic funds universe, employing a Fama-French model with five factors helps detect an alpha distribution composed of three groups, while the alpha distribution of non-thematic mutual funds turns out to include essentially only two sub-populations. Identifying three groups that characterise the optimal alpha distribution is a considerably easier task in the thematic funds data, when Fama-French models are extended to include latent factors. Second, the Ferson and Chen (2021) methodology, although less extreme in the magnitude of the estimated α_b and α_g, provides a substantial good thematic funds subgroups with a larger π_g estimate with respect to Barras et al. (2010).

5.3 | Latent Factor Interpretation

The latent factors extracted in our framework can be interpreted as capturing common exposures to broad economic drivers associated with structural transformations. In particular, they reflect combinations of thematic, sectoral, and style-related components that are not fully spanned by standard asset pricing factors.

TABLE 8 | List of covariates for latent factor interpretation.

Group	Variable	Description
Extra-Fama	AQRMOML	AQR momentum factor (long-term, 12–1 months)
	AQRMOMS	AQR momentum factor (short-term, 1 month)
	WML	Momentum factor (Fama-French)
	BAB	Betting-against-beta: long low-beta, short high-beta assets
	HMLD	Devil-in-HML: value adjusted for investment and profitability
MSCI GICS	MSCIFN	MSCI Financials
	MSCIIT	MSCI Information Technology
	MSCIEN	MSCI Energy
	MSCIHC	MSCI Health Care
	MSCIIN	MSCI Industrials
	MSCICD	MSCI Consumer Discretionary
	MSCICS	MSCI Consumer Staples
	MSCMT	MSCI Materials
	MSCIUT	MSCI Utilities
MSCI Thematic	ROBO_ETF	Robotics and automation
	ENERGY_ETF	Clean/alternative energy
	HEAL_ETF	Healthcare innovation
	DNA_ETF	Genomics and biotechnology
	DGTL_ETF	Digital economy and fintech
	ARKK_ETF	ARK Innovation (disruptive technology)
	H2O_ETF	Water resources and infrastructure
Macro	GEPU	Global Economic Policy Uncertainty index
	VIX	CBOE Volatility Index
	T10Y2YM	10-year minus 2-year Treasury yield spread

To provide an economic characterisation of these latent components, we adopt a data-driven procedure based on the Best Path Algorithm (BPA) (Riso et al. 2023). The BPA is a variable selection method that identifies, for each latent factor, the subset of observable economic and financial variables that best explains its variation. In our empirical application, the set of candidate variables includes additional conventional factors (e.g., momentum - WML, betting-against-beta - BAB), sectoral indices (MSCI GICS sectors), thematic indices proxied by ETFs (e.g., alternative energy, robotics, fintech and disruptive technology), and macro-financial indicators (e.g., VIX and Treasury yields). All the variables used in the analysis are shown in Table 8.

The results in Table 9 show that, for thematic funds, the latent factors are strongly associated with a subset of thematic investment segments, together with related sectoral and style exposures. In particular, the latent components capture common exposures to forward-looking economic forces linked to technological innovation, sustainability, and structural transformation. These findings suggest that thematic fund managers are primarily exposed to clusters of emerging investment themes rather than to narrowly defined individual sectors.

By contrast Table 10 shows that, for non-thematic funds, the latent factors are primarily explained by traditional sources of systematic risk, such as sectoral allocation and style factors (e.g., momentum), with only limited contribution from

TABLE 9 | BPA - Regression results. Thematic funds.

	3F Model			5F Model		
	LF1	**LF2**	**LF3**	**LF1**	**LF2**	**LF3**
(Intercept)	−0.055	−0.037	−0.070	−0.057	0.006	−0.073
	[0.093]	[0.067]	[0.046]	[0.092]	[0.072]	[0.046]
AQRMOMS	−8.367***		4.613***	−8.127***		4.955***
	[2.714]		[1.413]	[2.689]		[1.611]
HMLD		7.923***				5.500**
		[2.858]				[2.268]
MSCIMT		−0.108***			−0.104***	
		[0.024]			[0.026]	
MSCIUT		−0.064***			−0.066**	
		[0.024]			[0.028]	
MSCIHC			−0.080***			−0.077***
			[0.025]			[0.022]
MSCIIN			−0.080***			
			[0.024]			
MSCIIT					−0.065**	
					[0.032]	
MSCIFN						−0.079***
						[0.022]
ENERGY_ETF	0.153***	−0.046***		0.150***	−0.046***	
	[0.016]	[0.012]		[0.016]	[0.012]	
ARKK_ETF	−0.062***	−0.054***	0.034***	−0.067***	−0.044***	0.028***
	[0.014]	[0.012]	[0.008]	[0.014]	[0.013]	[0.009]
ROBO_ETF	0.102***	0.129***		0.102***	0.155***	
	[0.026]	[0.022]		[0.026]	[0.027]	
DGTL_ETF		0.081***	−0.106***		0.074**	−0.122***
		[0.030]	[0.025]		[0.034]	[0.025]
HEAL_ETF			−0.049***			−0.054***
			[0.018]			[0.018]
DNA_ETF			0.250***			0.228***
			[0.033]			[0.037]
H2O_ETF		0.096***	−0.070***		0.074**	−0.072***
		[0.031]	[0.019]		[0.033]	[0.020]
R^2	0.581	0.646	0.699	0.583	0.649	0.691

Abbreviation: BPA, Best Path Algorithm.
** $p < 0.05$; *** $p < 0.01$.

TABLE 10 | BPA - Regression results. Non-thematic mutual funds.

	3F Model		5F Model	
	LF1	LF2	LF1	LF2
(Intercept)	−0.074	0.037	−0.060	−0.081
	[0.121]	[0.096]	[0.131]	[0.085]
AQRMOMS				10.715***
				[2.659]
AQRMOML	16.754***		22.752***	
	[4.712]		[4.967]	
MSCIMT	−0.205***	0.096***	−0.205***	0.096***
	[0.032]	[0.036]	[0.035]	[0.031]
MSCIFN				−0.096***
				[0.036]
MSCIIN				−0.210***
				[0.051]
ROBO_ETF	0.160***			
	[0.036]			
DGTL_ETF	−0.106**			−0.095**
	[0.048]			[0.042]
ENERGY_ETF	−0.058***		−0.048**	
	[0.019]		[0.020]	
HEAL_ETF				−0.073***
				[0.025]
ARKK_ETF		0.046***		
		[0.016]		
DNA_ETF				0.231***
				[0.055]
R^2	0.498	0.184	0.382	0.418

Abbreviation: BPA, Best Path Algorithm.
** $p < 0.05$; *** $p < 0.01$.

thematic indices. This highlights a clear difference in the nature of the underlying risk exposures between thematic and conventional funds.

From an economic perspective, these results indicate that part of the observed differences in fund performance reflects differential exposure to underlying structural trends that are not fully captured by standard factor models. In this sense, the latent factors act as proxies for omitted common components related to evolving investment opportunities.

Lastly, these findings offer suggestive evidence on the sources of observed differences in fund performance. In particular, they indicate that variation in performance is partly associated with differential exposure to both observable and unobserved sources of systematic variation. Standard factor models may fail to capture relevant dimensions of risk related to emerging thematic segments, so that part of the alpha distribution may reflect exposure to omitted common components rather than purely idiosyncratic performance.

These results also provide insight into why some funds outperform. In particular, they suggest that abnormal performance is partly driven by differential exposure to both observable and unobserved sources of systematic variation. Standard factor models may fail to capture relevant dimensions of risk associated with emerging thematic segments, so that part of the alpha distribution reflects exposure to omitted common components (Tables 11 and 12).

6 | Conclusion

Thematic mutual funds are increasingly becoming part of mainstream investment. Consequently, there is a clear need to conduct detailed research into the distinct performance characteristics of thematic and conventional (non-thematic) mutual funds, a topic previously unexplored.

This paper aims to fill the gap. Specifically, it examines whether a sample of 149 actively managed equity thematic funds delivers greater added value than a sample of 1149 actively managed conventional equity mutual funds, starting from a shared broad investment policy characterised by a focus on developed global equity investing. Both are mutual funds accessible to retail investors. The measure used as proxy for a fund manager's

TABLE 11 | Best configurations for thematic funds, by analytic approach, Fama model, number of LF and size. 0LF denotes the results of Fama-French models estimated without latent factors.

Data	Type	OF Model	LF	$\gamma/2$	$\widehat{\pi}_b$	$\widehat{\pi}_0$	$\widehat{\pi}_g$	$\widehat{\alpha}_b$	$\widehat{\alpha}_g$	t_b	F_b	t_g	F_g	$\widehat{\beta}_b$	$\widehat{\delta}_g$	$\widehat{\beta}_g$	$\widehat{\delta}_b$	$\overline{\text{KL}}$
ThF	FC	3F	0LF	0.05	0.62 (0.218)	0.08 (0.107)	0.3 (0.147)	−0.205	0.135	−0.47	0.445	0.715	0.175	0.71	0.005	0.555	0.03	0.267
				0.1	0.59 (0.138)	0.11 (0.071)	0.3 (0.162)	−0.27	0.12	−0.355	0.475	0.54	0.195	0.78	0.01	0.595	0.04	0.235
			1LF	0.05	0.46 (0.131)	0.17 (0.141)	0.37 (0.246)	−0.26	0.095	−0.665	0.395	0.505	0.205	0.84	0.005	0.535	0.02	0.276
				0.1	0.49 (0.15)	0.1 (0.126)	0.41 (0.228)	−0.245	0.08	−0.71	0.4	0.345	0.25	0.78	0.005	0.59	0.035	0.252
			2LF	0.05	0.8 (0.224)	0.02 (0.16)	0.18 (0.086)	−0.22	0.105	−0.53	0.63	1.01	0.09	0.775	0.005	0.475	0.05	0.498
				0.1	0.79 (0.121)	0.09 (0.078)	0.12 (0.071)	−0.305	0.16	−0.415	0.67	0.845	0.08	0.84	0.005	0.615	0.065	0.34
			3LF	0.05	0.7 (0.132)	0.08 (0.072)	0.22 (0.117)	−0.275	0.095	−0.475	0.575	1.115	0.11	0.81	0.005	0.48	0.025	0.257
				0.1	0.7 (0.15)	0.07 (0.078)	0.23 (0.117)	−0.265	0.11	−0.47	0.58	0.71	0.145	0.805	0.005	0.59	0.055	0.254
		5F	0LF	0.05	0.53 (0.202)	0.1 (0.079)	0.37 (0.197)	−0.235	0.125	−0.57	0.37	0.74	0.215	0.68	0.005	0.55	0.025	0.231
				0.1	0.54 (0.165)	0.09 (0.092)	0.37 (0.192)	−0.245	0.09	−0.475	0.435	0.4	0.235	0.755	0.01	0.605	0.055	0.228
			1LF	0.05	0.51 (0.184)	0.07 (0.095)	0.42 (0.231)	−0.235	0.095	−0.72	0.36	0.565	0.225	0.695	0.005	0.54	0.02	0.211
				0.1	0.44 (0.114)	0.11 (0.151)	0.45 (0.233)	−0.29	0.075	−0.645	0.41	0.285	0.27	0.885	0.005	0.575	0.025	0.239
			2LF	0.05	0.74 (0.17)	0.05 (0.079)	0.21 (0.119)	−0.29	0.095	−0.67	0.56	0.895	0.1	0.75	0.005	0.49	0.035	0.271
				0.1	0.64 (0.129)	0.04 (0.127)	0.32 (0.219)	−0.295	0.045	−0.685	0.61	0.485	0.1	0.935	0.005	0.3	0.03	0.246
			3LF	0.05	0.65 (0.173)	0.13 (0.085)	0.22 (0.145)	−0.255	0.095	−0.66	0.54	0.82	0.13	0.82	0.005	0.555	0.025	0.245
				0.1	0.7 (0.145)	0.03 (0.049)	0.27 (0.127)	−0.285	0.08	−0.52	0.545	0.62	0.165	0.77	0.005	0.58	0.04	0.261
	BSW	3F	0LF	0.05	0.411 (0.14)	0.444 (0.083)	0.145 (0.09)	−0.237	0.352	−0.491	0.423	0.634	0.156	1*	0*	1*	0*	0.486
				0.1	0.468 (0.155)	0.341 (0.084)	0.191 (0.096)	−0.212	0.314	−0.353	0.477	0.463	0.201	1*	0*	1*	0*	0.521
			1LF	0.05	0.425 (0.147)	0.423 (0.083)	0.151 (0.094)	−0.233	0.344	−0.566	0.436	0.641	0.162	1*	0*	1*	0*	0.496
				0.1	0.484 (0.153)	0.324 (0.087)	0.192 (0.088)	−0.21	0.307	−0.42	0.492	0.476	0.2	1*	0*	1*	0*	0.483
			2LF	0.05	0.639 (0.149)	0.292 (0.101)	0.069 (0.061)	−0.262	0.326	−0.581	0.646	0.83	0.076	1*	0*	1*	0*	0.943
				0.1	0.674 (0.156)	0.23 (0.107)	0.095 (0.064)	−0.252	0.298	−0.483	0.679	0.576	0.101	1*	0*	1*	0*	1.011
			3LF	0.05	0.549 (0.157)	0.339 (0.094)	0.112 (0.079)	−0.228	0.317	−0.537	0.558	0.778	0.12	1*	0*	1*	0*	0.723
				0.1	0.603 (0.165)	0.262 (0.108)	0.135 (0.073)	−0.21	0.297	−0.368	0.61	0.623	0.141	1*	0*	1*	0*	0.794
		5F	0LF	0.05	0.378 (0.141)	0.455 (0.079)	0.167 (0.107)	−0.232	0.338	−0.535	0.39	0.618	0.178	1*	0*	1*	0*	0.465
				0.1	0.434 (0.154)	0.346 (0.08)	0.22 (0.102)	−0.208	0.303	−0.396	0.443	0.46	0.229	1*	0*	1*	0*	0.478
			1LF	0.05	0.393 (0.14)	0.44 (0.08)	0.168 (0.097)	−0.227	0.336	−0.607	0.404	0.662	0.179	1*	0*	1*	0*	0.459
				0.1	0.452 (0.159)	0.343 (0.089)	0.206 (0.095)	−0.205	0.307	−0.441	0.461	0.519	0.215	1*	0*	1*	0*	0.462
			2LF	0.05	0.585 (0.155)	0.334 (0.094)	0.081 (0.075)	−0.253	0.324	−0.644	0.593	0.774	0.089	1*	0*	1*	0*	0.896
				0.1	0.643 (0.167)	0.251 (0.111)	0.106 (0.071)	−0.234	0.289	−0.48	0.649	0.565	0.112	1*	0*	1*	0*	0.868
			3LF	0.05	0.578 (0.154)	0.319 (0.093)	0.103 (0.076)	−0.239	0.304	−0.543	0.586	0.783	0.111	1*	0*	1*	0*	0.78
				0.1	0.632 (0.166)	0.239 (0.107)	0.129 (0.073)	−0.222	0.281	−0.387	0.638	0.605	0.135	1*	0*	1*	0*	0.818

*Imposed by the Barras, Scaillet, and Wermers (BSW) methodology.

TABLE 12 | Best configurations for non-thematic mutual funds, by analytic approach, Fama model, number of LF and size. 0LF denotes the results of Fama-French models estimated without latent factors.

Data	Type	OF Model	LF	$\gamma/2$	$\widehat{\pi}_b$	$\widehat{\pi}_0$	$\widehat{\pi}_g$	$\widehat{\alpha}_b$	$\widehat{\alpha}_g$	t_b	F_b	t_g	F_g	$\widehat{\beta}_b$	$\widehat{\delta}_g$	$\widehat{\beta}_g$	$\widehat{\delta}_b$	$\overline{\text{KL}}$
MF	FC	3F	0LF	0.05	0.94 (0.04)	0.01 (0.024)	0.05 (0.026)	−0.5	0.095	−0.36	0.885	1.005	0.02	0.95	0	0.45	0.17	2.134
				0.1	0.97 (0.135)	0.01 (0.129)	0.02 (0.023)	−0.37	0.78	−0.23	0.905	0.845	0.02	0.93	0.005	0.59	0.03	2.212
			1LF	0.05	0.71 (0.038)	0.03 (0.031)	0.26 (0.057)	−0.42	0.01	−1.41	0.62	0.01	0.07	0.86	0	0.27	0.01	2.179
				0.1	0.98 (0.295)	0.01 (0.294)	0.01 (0.02)	−0.245	0.81	−0.48	0.88	0.705	0.03	0.88	0.005	0.55	0.07	2.097
			2LF	0.05	0.46 (0.267)	0.09 (0.073)	0.45 (0.295)	−0.19	0.13	−0.855	0.285	0.795	0.25	0.595	0.015	0.535	0.02	1.063
				0.1	0.49 (0.158)	0.04 (0.088)	0.47 (0.212)	−0.335	0.1	−0.565	0.36	0.605	0.275	0.705	0.02	0.565	0.03	0.491
		5F	0LF	0.05	0.8 (0.18)	0.02 (0.058)	0.18 (0.213)	−0.5	0.03	−0.525	0.85	0.965	0.02	0.985	0.001	0.105	0.38	1.483
				0.1	0.84 (0.217)	0.05 (0.161)	0.11 (0.079)	−0.49	0.02	−0.8	0.8	0.18	0.05	0.95	0.01	0.39	0.001	1.539
			1LF	0.05	0.76 (0.103)	0.03 (0.08)	0.21 (0.172)	−0.41	0.01	−0.68	0.785	0.915	0.03	0.935	0.001	0.13	0.35	1.151
				0.1	0.8 (0.08)	0.02 (0.041)	0.18 (0.079)	−0.41	0.01	−0.81	0.78	0.27	0.06	0.97	0.001	0.3	0.01	1.242
			2LF	0.05	0.78 (0.128)	0.01 (0.039)	0.21 (0.147)	−0.185	0.81	−0.425	0.375	1.375	0.13	0.48	0.001	0.585	0.005	0.682
				0.1	0.57 (0.066)	0.03 (0.063)	0.4 (0.108)	−0.17	0.15	−0.555	0.345	0.82	0.25	0.595	0.025	0.6	0.015	0.657
	BSW	3F	0LF	0.05	0.866 (0.284)	0.112 (0.162)	0.022 (0.151)	−0.08	0.498	−0.516	0.869	0.617	0.025	1*	0*	1*	0*	1.604
				0.1	0.888 (0.262)	0.084 (0.098)	0.029 (0.193)	−0.073	0.498	−0.374	0.891	0.488	0.032	1*	0*	1*	0*	1.706
			1LF	0.05	0.85 (0.147)	0.123 (0.139)	0.026 (0.012)	−0.092	0.513	−0.765	0.854	0.45	0.031	1*	0*	1*	0*	1.3
				0.1	0.886 (0.192)	0.082 (0.146)	0.032 (0.064)	−0.079	0.497	−0.55	0.888	0.363	0.035	1*	0*	1*	0*	1.317
			2LF	0.05	0.308 (0.095)	0.487 (0.251)	0.205 (0.265)	−0.376	0.19	−0.685	0.32	0.808	0.217	1*	0*	1*	0*	0.748
				0.1	0.362 (0.113)	0.391 (0.278)	0.247 (0.3)	−0.353	0.158	−0.502	0.371	0.617	0.257	1*	0*	1*	0*	0.772
		5F	0LF	0.05	0.811 (0.129)	0.164 (0.237)	0.025 (0.252)	−0.076	0.486	−0.715	0.816	0.548	0.029	1*	0*	1*	0*	1.454
				0.1	0.854 (0.156)	0.113 (0.267)	0.033 (0.298)	−0.061	0.474	−0.532	0.857	0.424	0.036	1*	0*	1*	0*	1.528
			1LF	0.05	0.761 (0.127)	0.206 (0.117)	0.033 (0.164)	−0.098	0.484	−0.838	0.766	0.482	0.039	1*	0*	1*	0*	0.979
				0.1	0.824 (0.127)	0.137 (0.113)	0.039 (0.176)	−0.077	0.476	−0.59	0.828	0.404	0.044	1*	0*	1*	0*	1.014
			2LF	0.05	0.293 (0.091)	0.494 (0.107)	0.213 (0.149)	−0.355	0.198	−0.64	0.305	0.863	0.226	1*	0*	1*	0*	0.708
				0.1	0.346 (0.077)	0.395 (0.126)	0.259 (0.174)	−0.333	0.17	−0.467	0.356	0.662	0.269	1*	0*	1*	0*	0.705

*Imposed by the Barras, Scaillet, and Wermers (BSW) methodology.

performance is the alpha obtained by employing the Fama and French 3-factor and 5-factor models, with the addition of unobserved factors.

In the empirical investigation carried out, it is fundamental to disentangle lucky and unlucky funds from funds exhibiting statistically significant abnormal performance. Therefore, the FDR approach in Ferson and Chen (2021) is used and extended, motivated by the objective of identifying statistically significant positive and negative alpha funds. This task is performed to better support mutual fund selection and, more importantly, to have a reliable comparison between thematic versus non-thematic funds.

The results obtained from the empirical investigation reveal interesting differences between thematic funds and their conventional "counterparts". The most noteworthy outcome is the identification of three sub-groups within the former population and essentially two sub-groups within the latter fund category (notably, the zero-alpha group was almost always not detected). This finding is particularly significant as it indicates that thematic funds include a non-negligible proportion of high-performing funds. In other words, the expectation of good performance relative to a multi-factor portfolio is well-founded and should be strongly considered when evaluating thematic funds investing in stocks across developed markets. In fact, it can be argued that there exists a subset of thematic funds exhibiting statistically significant positive alpha, which may be consistent with superior performance after accounting for risk factors. For this subpopulation of thematic funds, the existence of an abnormally positive return, (i.e., independent of the reward for common risk factors), can reasonably lead the reader to interpret such performance as potentially reflecting managerial ability, although it may also be influenced by luck or model specification. Future research in this direction could prove useful and is recommended.

Given these premises, thematic investing appears to be a natural space to implement non-standard issuers analysis approaches and find unknown mispricing. Despite their comparative advantage, it must not be ignored that thematic funds most frequently exhibit negative alpha and that, according to Morningstar (2024), their non-survival rate is crucial, especially in recent years.

The results obtained in this paper also help recognise the connection between alpha generation and the exposure of returns to common factors such as profitability and size. Lastly, the inclusion of unobservable factors improves the parameter estimates characterising the alpha distribution, leading to an estimate of the latter that reduces false rejections. An additional contribution of the paper lies in the economic interpretation of the latent factors. By linking the estimated latent components to observable economic and financial variables, the analysis shows that these factors capture common exposures to forward-looking investment segments associated with structural transformations, such as innovation, technology, and sustainability. This result provides a clearer understanding of the sources of performance in thematic investing. In particular, it suggests that part of the observed differences in fund performance reflects differential exposure to these underlying thematic and structural drivers, rather than purely idiosyncratic or manager-specific effects. This interpretation helps connect statistical factor models and economically meaningful narratives, thus enhancing the interpretability of the presented empirical findings.

In essence, this paper offers several novel insights for selecting fund managers (or investment strategies) for global equity investment across developed countries, and enhances the understanding of thematic investing. The comprehensive and detailed examination carried out with the FDR methodology to detect the presence/absence of statistically significant abnormal performance may open avenues for future research, along with the employment of more flexible distributional assumptions in the underlying model. Lastly, it has to be accounted for that positive alpha does not necessarily entail generating excess returns with respect to the reference market, which investors may prioritise based on an investment policy or the fund manager's stated benchmark.

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The authors declare no conflicts of interest.

Data Availability Statement

The data is available upon request. The data that support the findings of this study are available from the corresponding author upon reasonable request.

Endnotes

[1] In the application, $r_{i,t} = R_{i,t} - R_{f,t}$ is adjusted for the risk-free return, and the first observed factor is $R_{M,t} - R_{f,t}$.

[2] The alphas' of the Fama-French models are estimated via time series regressions because all the factors, being excess returns, are tradable. Otherwise, in the presence of non-tradable factors, the two-pass Fama-MacBeth regressions is recommended for the alphas estimation. In the first stage of this procedure, the coefficient or exposure of the risk-factors are estimated via time series regression. The second stage involves a cross-sectional regression of the average returns onto the estimated coefficients. The residuals of this regression yield estimates of the alphas.

[3] Ferson and Chen (2021) explained that the discrepancy between the results obtained from the two approaches is due to the different computing of fractions of funds with zero, positive and negative alpha. Setting $F = F_g + F_b$, the authors proved that in the BSW case, the fraction of zero funds, π_0 is estimated as follows

$$\widehat{\pi}_{0,BSW} = \frac{(1 - \mathbb{E}(F))}{1 - \gamma}$$

where γ denotes the size of the test, while in FC

$$\widehat{\pi}_{0,FC} = \frac{(\beta - \mathbb{E}(F))}{(\beta - \gamma)}$$

where β denotes the power of a two tailed test. As proved in Ferson and Chen (2021), $\mathbb{E}(\widehat{\pi}_{0,BSW}) > \mathbb{E}(\widehat{\pi}_{0,FS})$ as $\widehat{\pi}_{0,BSW}$ is biased with the bias that depends on β and on confusion parameters δ_b, δ_g.

[4]The IC_3 criterion is the one proposed in Gagliardini et al. (2019), GOS, here renamed for ease of notation.

[5]Generally, the system Equations (A4)–(A5) which is solved numerically with respect to π_g and π_b, taking into account the positivity constraint and the constraint that the sum of π_0, π_g and π_b must be 1.

[6]In particular, the authors found that variations in the values of the parameters $\beta_g, \beta_b, \delta_g, \delta_b$, calibrated via simulations, have a trivial effect on the final results.

[7]The above conditional densities are proportional to the posterior probabilities

$$f\{\widehat{\alpha}_i|\alpha = 0)\} = \frac{Pr\{\alpha = 0|\widehat{\alpha}_i\}f(\widehat{\alpha_i})}{\widehat{\pi}_0}$$
$$f(\widehat{\alpha}_i|\alpha > 0) = \frac{Pr\{\alpha > 0|\widehat{\alpha}_i\}f(\widehat{\alpha_i})}{\widehat{\pi}_g}$$
$$f(\widehat{\alpha}_i|\alpha < 0) = \frac{Pr\{\alpha < 0|\widehat{\alpha}_i\}f(\widehat{\alpha_i})}{\widehat{\pi}_b}$$

[8]The authors suggested the use of the Epanechikov optimal kernel function

$$K(u) = \frac{3}{4}(1 - u^2)I(A)$$

where $I(A)$ denotes the indicator function of the set A.

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Appendix A

Methodology: The Combined Approach

The Ferson and Chen (2021) procedure, combined with the model structure stated in Giglio et al. (2021), is here detailed. It is to be noted that each of the three steps of the first stage of the procedure is repeated for $R = 1000$ replications.

The first simulation aims to determine t_b and t_g. Hence, data is generated under the null model

$$y_{it}^* = \mathbf{f}_t'\widehat{\boldsymbol{\beta}}_i + e_{it}^*, i = 1, \dots, N, t = 1, 2, \dots, T \quad \text{(A1)}$$

Here, $\mathbf{f}_t'$ is the t-th row of the factor matrix $\mathbf{F}$ of Fama-French model (either 3F or 5F, possibly with latent factors), $\widehat{\boldsymbol{\beta}}_i$ are the associated estimated loadings and e_{it}^* are weighted residuals, obtained by weighting the original demeaned residual $\hat{e}_{it}$ of the Fama-French model as

$$e_{it}^* = w_{it}\hat{e}_{it}$$

where $w_{it} = \frac{1}{\sqrt{2}}\eta_{it} + \frac{1}{2}\left(\psi_{it}^2 - 1\right)$, with η_{it} and ψ_{it} denoting independent Standard Normal variables. A bootstrap sample y_{it}^* is generated, as per Equation (A13), by resampling weighted residuals using the wild-bootstrap algorithm of Davidson and Flachaire (2008), Mammen (1993). This synthetic data, generated under the null of zero alpha, has been used to re-estimate the alphas and determine the critical values t_b and t_g, by assuming a size of either the 5% or the 10% in each tail of the t-distribution. The other two simulations are carried out to estimate the powers β_g and β_b and the confusion parameters δ_b and δ_g, for given α_b and α_g. By denoting with $T_{H_{1,g}}$ and $T_{H_{1,b}}$ the distributions of the t-statistic under the alternatives $H_{1,g}$ and $H_{1,b}$, the parameters β_g, β_b, δ_g and δ_b are formally defined as

$$\beta_g = Pr\{T_{H_{1,g}} > t_g\};\ \delta_b = Pr\{T_{H_{1,g}} < t_b\} \quad \text{(A2)}$$

$$\beta_b = Pr\{T_{H_{1,b}} < t_b\};\ \delta_g = Pr\{T_{H_{1,b}} > t_g\} \quad \text{(A3)}$$

These simulations follow a scheme similar to the first one. Specifically, in the second simulation the additional α_g from the grid is added to the simulated data replication. This allows to determine both β_g and δ_b according to Equation (A2). The former parameter is estimated as the fraction of the simulated t-statistics under the "good" alternative that exceeds t_g, while the latter as the fraction of the same statistics that lies below t_b.

The third simulation provides the estimates of β_b and δ_g. Specifically, in the third simulation the negative quantity α_b from the grid is added to the simulated data replication. This allows to determine both β_b and δ_g, according to Equation (A3) The latter parameter is estimated as the fraction of the simulated t-statistics that exceed t_g, while the former as the fraction of the same statistics that lies below t_b, calculated in the first simulation.

In the second stage, the previous results are combined to estimate the proportions π_b, π_0 and π_g. This requires solving the following system

$$\begin{aligned}\mathbb{E}(N_g) &= Pr(\text{reject at } t_g|H_0)\pi_0 + Pr(\text{reject at } t_g|H_{1,b})\pi_b \\ &+ Pr(\text{reject at } t_g|H_{1,g})\pi_g = \frac{\gamma}{2}\pi_0 + \delta_g\pi_b + \beta_g\pi_g\end{aligned} \quad \text{(A4)}$$

$$\begin{aligned}\mathbb{E}(N_b) &= Pr(\text{reject at } t_b|H_0)\pi_0 + Pr(\text{reject at } t_b|H_{1,b})\pi_b \\ &+ Pr(\text{reject at } t_b|H_{1,g})\pi_g = \frac{\gamma}{2}\pi_0 + \beta_b\pi_g + \delta_b\pi_g\end{aligned} \quad \text{(A5)}$$

The solutions are[5]

$$\pi_b = B\left(N_g - \frac{\gamma}{2}\right) + C\left(N_b - \frac{\gamma}{2}\right) \quad \text{(A6)}$$

$$\pi_g = D\left(N_g - \frac{\gamma}{2}\right) + E\left(N_b - \frac{\gamma}{2}\right) \quad \text{(A7)}$$

where N_g and N_b are

$$N_g = \frac{\sum_{i=1}^N \mathbf{1}(t_i > t_g)}{N}, N_b = \frac{\sum_{i=1}^N \mathbf{1}(t_i < t_b)}{N} \quad \text{(A8)}$$

with $\mathbf{1}(.)$ representing the indicator function; B, C, D and E are quantities depending only on γ and the constants $\beta_g, \beta_b, \delta_g, \delta_b$, determined in the first stage of the analysis

$$\begin{aligned}B &= \frac{\beta_b - \gamma/2}{G}, C = \frac{\gamma/2 - \delta_g}{G}, D = \frac{\gamma/2 - \delta_b}{G}, \\ E &= \frac{\beta_g - \gamma/2}{G}\end{aligned} \quad \text{(A9)}$$

and $G = (\frac{\gamma}{2} - \delta_g)(\delta_b - \frac{\gamma}{2}) + (\beta_b - \frac{\gamma}{2})(\beta_g - \frac{\gamma}{2})$.[6] It is worth noting that the estimates of the fractions of funds obtainable by using the standard FDR method proposed in Storey (2002) and Barras et al. (2010) can be derived from Equations (A6) and (A7) by setting $\beta_g = \beta_b = 1$ and $\delta_g = \delta_b = 0$. These estimates, say $\pi_j^s, j = \{0, b, g\}$, turn out to be

$$\pi_0^s = (1 - (N_b + N_g))/(1 - \gamma) \quad \text{(A10)}$$

$$\pi_g^s = N_g - (\gamma/2)\pi_0 \quad \text{(A11)}$$

$$\pi_b^s = N_b - (\gamma/2)\pi_0 \quad \text{(A12)}$$

where $\gamma\pi_0$ is the fraction of zero funds that are erroneously classified. In particular, among these funds, $(\gamma/2)\pi_0$ are erroneously classified as good funds and $(\gamma/2)\pi_0$ as bad funds. The last stage of the analysis consists in scanning the grid of α_g and α_b in search of the values that best represent the original model estimates. This requires a final simulation from the model

$$y_{it}^* = \alpha_i + \mathbf{f}_t'\,\widehat{\boldsymbol{\beta}}_i + e_{it}^*, i = 1, \dots, N, t = 1, \dots, T \quad \text{(A13)}$$

with an additional randomisation layer. For every i, a random number u_i is drawn from the Uniform distribution and

$$\alpha_i = \begin{cases}\alpha_b & \text{if } u_i < \pi_b \\ 0 & \text{if } \pi_b \le u_i < \pi_g \\ \alpha_g & \text{otherwise}\end{cases} \quad \text{(A14)}$$

The resulting synthetic t-values are compared to the one obtained with the original data via the DTS statistic.

Once the best mixture of the three subpopulations of parameters has been determined, inference on α can be carried out. In particular, the Bayes' rule can be employed to assign a specific fund to one of the three sub-populations: the one centred on zero with estimated probability $\widehat{\pi}_0$, the one centred on the best $\widehat{\alpha}_g$ configuration with probability $\widehat{\pi}_g$, or the one centred on the best $\widehat{\alpha}_b$ configuration with probability $\widehat{\pi}_b$, following the steps above. Referring to the i-th fund, the density $f\{\widehat{\alpha}_i\}$ can be expressed in terms of $f\{\widehat{\alpha}_i|\alpha = 0)\}, f(\widehat{\alpha}_i|\alpha > 0)$ and $f(\widehat{\alpha}_i|\alpha < 0)$, as follows

$$f(\widehat{\alpha}_i) = f(\widehat{\alpha}_i|\alpha < 0)\widehat{\pi}_b + f(\widehat{\alpha}_i|\alpha = 0)\widehat{\pi}_0 + f(\widehat{\alpha}_i|\alpha > 0)\widehat{\pi}_g \quad \text{(A15)}$$

Then, $\widehat{\alpha_i}$ is assigned to the set of funds with positive alpha if the following holds

$$f\{\widehat{\alpha}_i|\alpha > 0\}\widehat{\pi}_g > f\{\widehat{\alpha}_i|\alpha < 0\}\widehat{\pi}_b \ \wedge \ f\{\widehat{\alpha}_i|\alpha > 0\}\widehat{\pi}_g > f\{\widehat{\alpha}_i|\alpha = 0\}\widehat{\pi}_0 \tag{A16}$$

while, it is assigned to the set of funds with negative alpha if

$$f\{\widehat{\alpha}_i|\alpha < 0\}\widehat{\pi}_b > f\{\widehat{\alpha}_i|\alpha > 0\}\widehat{\pi}_g \ \wedge \ f\{\widehat{\alpha}_i|\alpha < 0\}\widehat{\pi}_b > f\{\widehat{\alpha}_i|\alpha = 0\}\widehat{\pi}_0 \tag{A17}$$

or to set of funds with zero alpha if[7]

$$f\{\widehat{\alpha}_i|\alpha = 0\}\widehat{\pi}_0 > f\{\widehat{\alpha}_i|\alpha > 0\}\widehat{\pi}_g \ \wedge \ f\{\widehat{\alpha}_i|\alpha = 0\}\widehat{\pi}_0 > f\{\widehat{\alpha}_i|\alpha < 0\}\widehat{\pi}_b \tag{A18}$$

These conditional densities are approximated with the empirical densities of the alphas in each fund sub-population using the Kernel density $K(\widehat{\alpha}_i)$[8]

$$f(\widehat{\alpha}_i|\alpha_j) = K(\widehat{\alpha}_i) = \frac{1}{Nh}\sum_j \frac{K(\widehat{\alpha}_i - \alpha_j)}{h} \tag{A19}$$

where h denotes the bandwidth, $\alpha_j = \{\widehat{\alpha}_b, 0, \widehat{\alpha}_g\}$ and N the sample size. The extension of this methodology to embody latent factors develops as explained in Section 5. In this regard, it is worth noting that Giglio et al. (2021) has proved that the application of this extended approach guarantees that

$$(\widehat{\alpha}_i - \alpha_i)/\sigma_{\widehat{\alpha}_i} \xrightarrow{d} N(0, 1) \quad \text{for any } i \tag{A20}$$

where $\sigma^2_{\widehat{\alpha}_i}$ can be estimated as

$$\widehat{\sigma}^2_{\widehat{\alpha}_i} = \frac{1}{T}\sum_{t=1}^{T} \widehat{e}^2_{i,t}\left(1 - \widehat{\mathbf{f}}_t'\widehat{\boldsymbol{\Sigma}}_f^{-1}\widehat{\boldsymbol{\lambda}}\right)^2 \tag{A21}$$

where $\widehat{e}_{i,t} = y_{i,t} - \bar{y}_i - \widehat{\boldsymbol{\beta}}_i'\widehat{\mathbf{f}}_t$, $\widehat{\mathbf{f}}_t = [\widetilde{\mathbf{f}}_{0,t}, \widehat{\mathbf{f}}_{l,t}]$, with $\widetilde{\mathbf{f}}_{0,t}$ denoting the demeaned observable factors, $\widehat{\mathbf{f}}_{l,t}$ the estimates of the latent factors and $\widehat{\boldsymbol{\Sigma}}_f = \frac{1}{T}\sum_{t=1}^{T}\widehat{\mathbf{f}}_t\widehat{\mathbf{f}}_t'$. Giglio et al. (2021) provides also a more sophisticated algorithm for the alpha's estimation for the case when data panels are unbalanced due to missing data.

In presence of confounders (say in $\mathbf{w}_{i,t-1}$, as in Equation 1 or 2), the estimation undergoes minor adjustments:

Step 1 (time-series regressions).

The three model types are nested in a single specification.

- *Type 1 (observable factors only)*. For each $i = 1, \ldots, N$,

$$\tilde{R}_{i,t} = \boldsymbol{\beta}_i'\tilde{\mathbf{f}}_{o,t} + \boldsymbol{\phi}_i'\widetilde{\mathbf{w}}_{i,t-1} + \epsilon_{i,t},$$

  where tilde notation denotes time-demeaned variables.
- *Type 2 (latent factors only)*. The observable factors $\mathbf{f}_{o,t}$ are omitted; the first-stage regression retains only the fund characteristics and the COVID dummy. Latent factor loadings $\hat{\boldsymbol{\beta}}_{l,i}$ and factors $\hat{\mathbf{f}}_{l,t}$ are then extracted from the $N \times T$ demeaned residual matrix $\hat{\mathbf{Z}}$ via its singular value decomposition.
- *Type 3 (mixed model)*. Both $\mathbf{f}_{o,t}$ and the funds' characteristics enter the first stage; the residuals are then subjected to the same PCA step as in Type 2, yielding additional latent loadings $\hat{\boldsymbol{\beta}}_{l,i}$.

Step 2 (cross-sectional pricing and alpha estimate).

Define the combined loading matrix as

$$\boldsymbol{\beta} = [\hat{\boldsymbol{\beta}}_o \ \hat{\boldsymbol{\phi}}] \quad (\textit{Type } 1)$$
$$\boldsymbol{\beta} = [\hat{\boldsymbol{\phi}} \ \hat{\boldsymbol{\beta}}_l] \quad (\textit{Type } 2)$$
$$\boldsymbol{\beta} = [\hat{\boldsymbol{\beta}}_o \ \hat{\boldsymbol{\phi}} \ \hat{\boldsymbol{\beta}}_l] \ (\textit{Type } 3)$$

Then, the subsequent steps are identical to the original model, following Equations (11)–(12).

Appendix B

Empirical Analysis

Summary Statistics on Thematic and Non-Thematic Mutual Funds

The following tables show some summary statistics of the thematic funds data (Table B1) and non-thematic funds (Table B2).

Both tables have been constructed, as in Ferson and Chen (2021), by considering the cross-sectional distributions of funds for each time period. For each point in time, several fractiles of the cross-section of stock returns are considered, and descriptives are calculated. The tables

TABLE B1 | Descriptive statistics for monthly thematic funds returns for various fractiles cut-points in the cross-section of returns.

	Monthly returns (%)				
Fractile	T_{min}	T_{max}	**Mean**	**SD**	**Rho1**
Thematic Fund Returns: Jan 2018 - Dec 2025 (96 months)					
0.01	0	27	−6.476	5.087	0.006
0.05	0	34	−4.094	4.528	−0.023
0.1	3	38	−2.962	4.367	−0.034
0.25	8	42	−1.284	4.289	−0.072
Median	28	76	0.344	4.199	−0.095
0.75	40	95	2.050	4.219	−0.091
0.9	48	96	3.804	4.431	−0.075
0.95	45	96	5.111	4.765	−0.07
0.99	61	96	7.737	5.759	0.044

show the temporal means of returns not exceeding given quantiles $\tau = 0.01, \ldots, 0.99$ given by

$$\bar{y}_{q_\tau} = \frac{1}{T}\sum_{t=1}^{T} y_{q_\tau,t}$$
$$y_{q_\tau,t} = \inf\{y_{it} : \widehat{F}(y_{it}) \geq \tau\} \quad t = 1, \ldots, T$$

TABLE B2 | Descriptive statistics for monthly non-thematic mutual funds returns for various fractiles cut-points in the cross-section of returns.

	Monthly returns (%)				
Fractile	T_{min}	T_{max}	**Mean**	**SD**	**Rho1**
Non-thematic Fund Returns: Jan 2018–Dec 2025 (96 months)					
0.01	0	30	−6.438	5.44	−0.011
0.05	3	41	−3.494	4.757	−0.031
0.1	17	43	−2.329	4.589	−0.052
0.25	25	50	−0.821	4.437	−0.078
Median	32	65	0.485	4.385	−0.089
0.75	43	79	1.724	4.385	−0.094
0.9	52	90	3.196	4.489	−0.079
0.95	58	94	4.569	4.687	−0.067
0.99	70	96	8.878	6.755	0.006

as well as the standard deviations (SD) and first-order autocorrelations (Rhol). They also report the minimum and maximum number of returns that are equal or lower than the fund's cross-sectional average, for a given quantile τ (the symbol "# (X)" denotes the cardinality of the set X):

$$A = \{y_{it} : y_{it} < \bar{y}_{q_\tau}\}$$
$$T_{\min} = \min_i \{\#(y_{it} \in A), i = 1, \ldots, N\}$$
$$T_{\max} = \max_i \{\#(y_{it} \in A), i = 1, \ldots, N\}$$

Looking at both tables, we see that lower fractiles are associated to fewer funds with negative means. In particular, the standard deviation appears to be less susceptible to the percentile choice, while the first order autocorrelation is almost always around zero. The results of Table B1 reveal that the highest number of returns (34) of thematic mutual funds that are below the corresponding cross-sectional average (−4.096) associated with the 5% quantile is far below the highest number of returns (41) of non-thematic mutual funds below the corresponding average value (−3.494) for the same quantile. A similar discrepancy, between thematic and non-thematic mutual funds, can be observed until the median is reached, after which the highest number of thematic mutual funds below the corresponding median is greater with respect to the non-thematic counterpart. Overall, the range of mean values across funds, for given quantiles, is greater in the non-thematic funds both in the left and right tails.

Simulated Alpha Distributions

Table B3 reports some configurations of the estimated proportions of negative, positive and zero-alpha funds ($\widehat{\pi}_b$, $\widehat{\pi}_g$ and $\widehat{\pi}_0$) associated with each possible different couple (α_b, α_g), resulting from the search of the

TABLE B3 | Estimated proportions $\widehat{\pi}_b$, $\widehat{\pi}_0$, $\widehat{\pi}_g$ associated to selected pairs of values α_b and α_g estimated with the 3F and 5F models for the thematic funds, at $\gamma/2 = \{0.05, 0.1\}$ (no latent factors). The table shows also the corresponding powers $\widehat{\beta}_b$, $\widehat{\beta}_g$, confusion parameters, $\widehat{\delta}_b$, $\widehat{\delta}_g$ and values of the DTS statistic.

Model	$\gamma/2$	$\widehat{\pi}_b$	$\widehat{\pi}_g$	$\widehat{\pi}_0$	α_b	α_g	$\widehat{\beta}_b$	$\widehat{\delta}_g$	$\widehat{\beta}_g$	$\widehat{\delta}_b$	DTS
3F	0.05	0.43	0.48	0.09	−0.455	0.365	0.88	0.005	0.695	0.005	23.3
		0.47	0.09	0.44	−0.365	0.1	0.79	0.005	0.43	0.025	17.1
		0.56	0.04	0.4	−0.315	0.13	0.705	0.005	0.42	0.03	15.9
		0.72	0.21	0.07	−0.12	0.35	0.725	0.005	0.655	0.01	18.7
		0.45	0.21	0.34	−0.42	0.115	0.78	0.005	0.535	0.025	20.9
	0.1	0.43	0.23	0.34	−0.425	0.24	0.78	0.005	0.655	0.03	22.7
		0.58	0.38	0.04	−0.335	0.43	0.995	0.005	0.91	0.005	17.8
		0.82	0.07	0.11	−0.09	0.345	0.695	0.005	0.81	0.015	20.5
		0.28	0.1	0.62	−0.365	0.04	0.865	0.01	0.56	0.01	15.7
		0.37	0.14	0.49	−0.45	0.035	0.915	0.005	0.575	0.025	19.7
5F	0.05	0.39	0.6	0.01	−0.33	0.54	0.9	0.01	0.95	0.005	15.3
		0.22	0.07	0.71	−0.405	0.045	0.9	0.005	0.57	0.01	16.2
		0.42	0.47	0.11	−0.3	0.315	0.825	0.005	0.755	0.005	14.3
		0.47	0.2	0.33	−0.36	0.21	0.78	0.005	0.565	0.015	18.0
		0.94	0.01	0.05	−0.09	0.455	0.57	0.005	0.73	0.005	21.4
	0.1	0.31	0.47	0.22	−0.505	0.285	0.96	0.005	0.875	0.005	25.7
		0.67	0.23	0.1	−0.13	0.33	0.77	0.005	0.75	0.015	17.9
		0.42	0.03	0.55	−0.475	0.04	0.89	0.005	0.51	0.05	24.5
		0.38	0.05	0.57	−0.39	0.025	0.835	0.005	0.485	0.04	16.5
		0.38	0.18	0.44	−0.465	0.09	0.895	0.005	0.615	0.03	21.0

TABLE B4 | Estimated proportions $\widehat{\pi}_b, \widehat{\pi}_0, \widehat{\pi}_g$ associated to selected pairs of values α_b and α_g estimated with the 3F and 5F models for the thematic funds, at $\gamma/2 = \{0.05, 0.1\}$ (1 to 3 latent factors). The table shows also the corresponding powers $\widehat{\beta}_b, \widehat{\beta}_g$, confusion parameters, $\widehat{\delta}_b, \widehat{\delta}_g$ and values of the DTS statistic.

3F- LF	$\gamma/2$	$\widehat{\pi}_b$	$\widehat{\pi}_0$	$\widehat{\pi}_g$	α_b	α_g	$\widehat{\beta}_b$	$\widehat{\delta}_g$	$\widehat{\beta}_g$	$\widehat{\delta}_b$	DTS	5F- LF	$\gamma/2$	$\widehat{\pi}_b$	$\widehat{\pi}_0$	$\widehat{\pi}_g$	α_b	α_g	$\widehat{\beta}_b$	$\widehat{\delta}_g$	$\widehat{\beta}_g$	$\widehat{\delta}_b$	DTS
1	0.05	0.71	0.08	0.21	−0.15	0.285	0.665	0.005	0.67	0.015	20.3	1	0.05	0.47	0.18	0.35	−0.325	0.115	0.775	0.005	0.58	0.03	16.4
		0.67	0.06	0.27	−0.175	0.135	0.665	0.005	0.615	0.025	14.5			0.6	0.31	0.09	−0.14	0.325	0.805	0.005	0.6	0.015	17.6
		0.33	0.28	0.39	−0.47	0.065	0.875	0.005	0.56	0.015	23.6			0.25	0.16	0.59	−0.49	0.055	0.955	0	0.505	0.01	23.3
		0.37	0.53	0.1	−0.345	0.325	0.96	0	0.79	0	16.9			0.41	0.19	0.4	−0.38	0.09	0.73	0.005	0.555	0.02	19.1
		0.44	0.06	0.5	−0.41	0.085	0.74	0.005	0.405	0.035	22.6			0.23	0.17	0.6	−0.53	0.065	0.955	0	0.565	0.005	24.5
	0.1	0.5	0.41	0.09	−0.345	0.335	0.895	0.005	0.82	0.005	18.6		0.1	0.47	0.39	0.14	−0.26	0.32	0.865	0.005	0.77	0.005	16.0
		0.43	0.31	0.26	−0.4	0.245	0.89	0.005	0.66	0.025	22.6			0.35	0.05	0.6	−0.49	0.03	0.975	0	0.535	0.035	27.1
		0.38	0.05	0.57	−0.435	0.04	0.935	0	0.52	0.03	22.2			0.47	0.33	0.2	−0.365	0.28	0.87	0.005	0.7	0.01	22.3
		0.8	0.1	0.1	−0.1	0.35	0.72	0.005	0.81	0.005	20.6			0.33	0.26	0.41	−0.54	0.06	0.905	0	0.725	0.015	29.0
		0.41	0.17	0.42	−0.425	0.075	0.88	0.005	0.57	0.03	21.4			0.4	0.18	0.42	−0.405	0.08	0.845	0.005	0.595	0.03	20.1
2	0.05	0.61	0.35	0.04	−0.5	0.31	0.975	0	0.77	0.005	34.0	2	0.05	0.97	0.01	0.02	−0.085	0.39	0.765	0.005	0.625	0.005	31.4
		0.57	0.1	0.33	−0.49	0.035	0.965	0	0.33	0.05	32.6			0.41	0.3	0.29	−0.415	0.02	0.94	0	0.53	0.01	22.4
		0.53	0.12	0.35	−0.62	0.06	1	0	0.22	0.01	47.0			0.6	0.32	0.08	−0.425	0.245	0.9	0.005	0.73	0.005	26.9
		0.46	0.41	0.13	−0.435	0.28	0.925	0	0.82	0.005	28.4			0.5	0.15	0.35	−0.535	0.03	0.965	0	0.45	0.015	36.6
		0.42	0.47	0.11	−0.595	0.11	0.98	0	0.76	0	40.0			0.58	0.11	0.31	−0.45	0.045	0.87	0	0.35	0.02	28.0
	0.1	0.56	0.25	0.19	−0.37	0.05	0.94	0	0.645	0.01	17.8		0.1	0.58	0.23	0.19	−0.46	0.045	0.935	0	0.655	0.015	28.8
		0.43	0.39	0.18	−0.575	0.075	0.985	0	0.865	0.01	36.5			0.67	0.03	0.3	−0.43	0.05	0.915	0.005	0.42	0.06	27.3
		0.65	0.05	0.3	−0.475	0.035	0.925	0	0.47	0.06	31.6			0.81	0.03	0.16	−0.205	0.105	0.84	0.005	0.565	0.055	17.3
		0.43	0.31	0.26	−0.66	0.01	0.99	0	0.62	0.01	42.8			0.46	0.38	0.16	−0.4	0.3	0.92	0	0.985	0	29.3
		0.75	0.19	0.06	−0.385	0.295	0.935	0	0.67	0.025	23.4			0.58	0.31	0.11	−0.48	0.2	0.91	0	0.84	0.01	31.6
3	0.05	0.55	0.19	0.26	−0.42	0.075	0.83	0	0.505	0.02	28.3	3	0.05	0.75	0.07	0.18	−0.265	0.205	0.805	0.005	0.585	0.025	18.5
		0.73	0.04	0.23	−0.205	0.1	0.695	0.005	0.57	0.025	18.6			0.46	0.04	0.5	−0.34	0.01	0.91	0	0.39	0	19.3
		0.78	0.02	0.2	−0.155	0.15	0.735	0.005	0.61	0.025	21.9			0.75	0.11	0.14	−0.265	0.255	0.805	0.005	0.575	0.025	19.9
		0.49	0.26	0.25	−0.465	0.08	0.795	0	0.62	0.01	31.4			0.59	0.1	0.31	−0.41	0.075	0.915	0	0.38	0.02	27.1
		0.34	0.45	0.21	−0.49	0.12	0.9	0	0.94	0	29.9			0.72	0.05	0.23	−0.32	0.095	0.8	0.005	0.525	0.03	20.5
	0.1	0.84	0.02	0.14	−0.125	0.29	0.78	0.005	0.7	0.015	24.9		0.1	0.36	0.41	0.23	−0.42	0.25	0.935	0	0.955	0	32.1
		0.52	0.04	0.44	−0.44	0.015	0.97	0	0.435	0.025	29.7			0.63	0.05	0.32	−0.36	0.06	0.875	0.005	0.455	0.04	23.7
		0.35	0.43	0.22	−0.4	0.19	0.95	0	0.97	0	25.2			0.53	0.06	0.41	−0.42	0.035	0.97	0	0.43	0.02	27.1
		0.5	0.01	0.49	−0.305	0.06	0.895	0	0.41	0.01	18.8			0.64	0.03	0.33	−0.365	0.065	0.795	0.005	0.495	0.05	24.8
		0.63	0.34	0.03	−0.36	0.435	0.99	0	0.885	0	24.1			0.76	0.12	0.12	−0.14	0.295	0.835	0.005	0.695	0.02	27.2

grid, when the alphas are estimated with the 3F model or with the 5F model at size $\gamma/2 = \{0.05, 0.1\}$. The same table also reports estimates of the power and the confusion parameter for bad and good funds, $\widehat{\beta}_b$, $\widehat{\delta}_g$, and $\widehat{\beta}_g$, $\widehat{\delta}_b$, respectively.

Table B4 provides the parameters that characterise a selection of the alpha distributions obtained using a 3F or a 5F model that includes up to three latent factors.

The landscape of the grid search in the thematic (non-thematic) funds data is reported in Figure B1 (Figure B2). The panels show the values of the DTS test for different (α_b, α_g) pairs. It should be noted that, since simulations are run for different values of t_b and t_g associated to the same pair (α_b, α_g), the DTS values shown in the figures are an average across all instances that share the same alpha pair. Therefore, the minima in the graphs can be different from those reported in Tables 11 and 12.

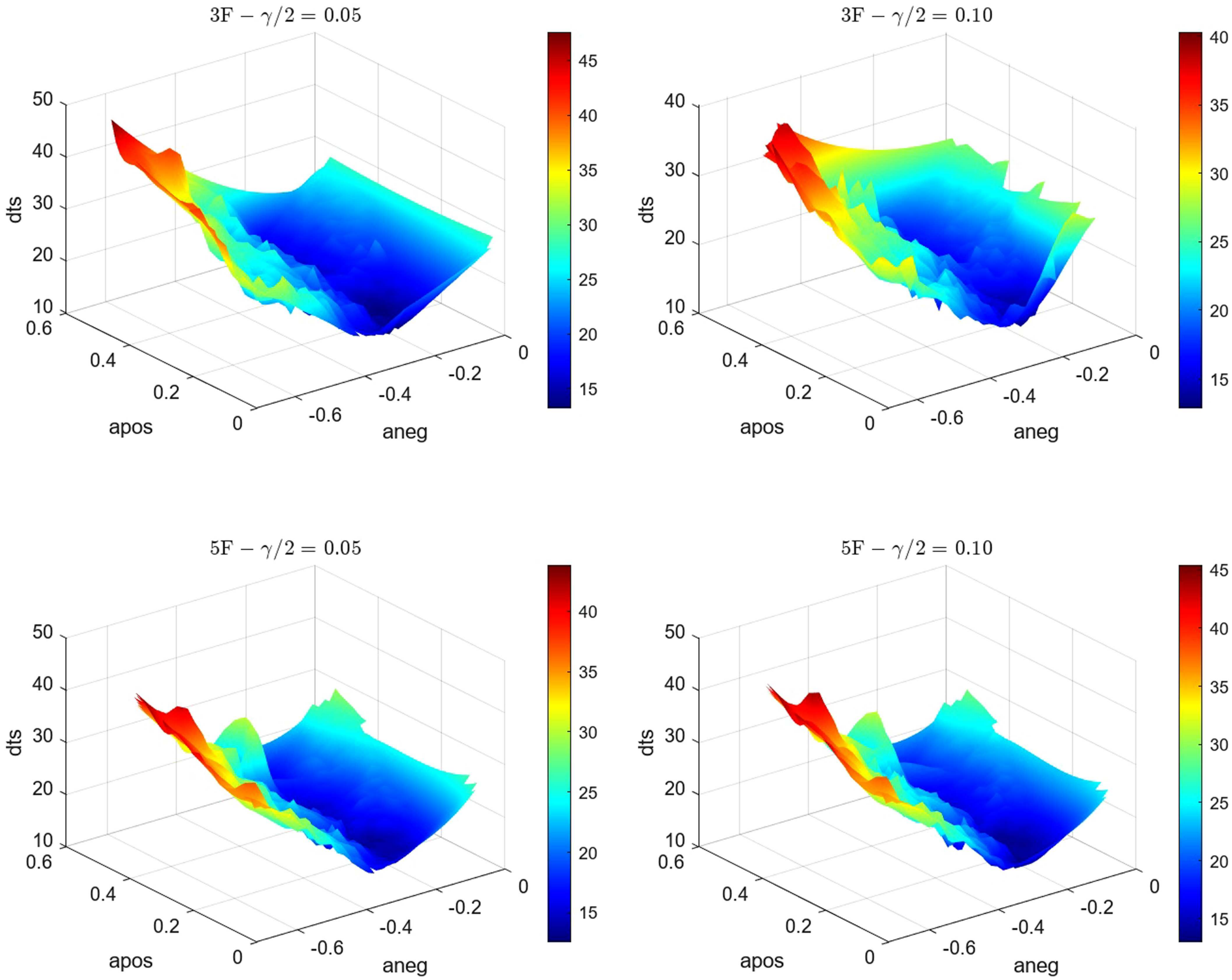

FIGURE B1 | Grid search for the α_b and α_g parameters estimated with the 3F and 5F models (one latent factor), and size $\gamma/2 = \{0.05, 0.1\}$ for the thematic funds, via the DTS test. The DTS values displayed are averages across every duplicate value (α_b, α_g) of the grid. [Color figure can be viewed at wileyonlinelibrary.com]

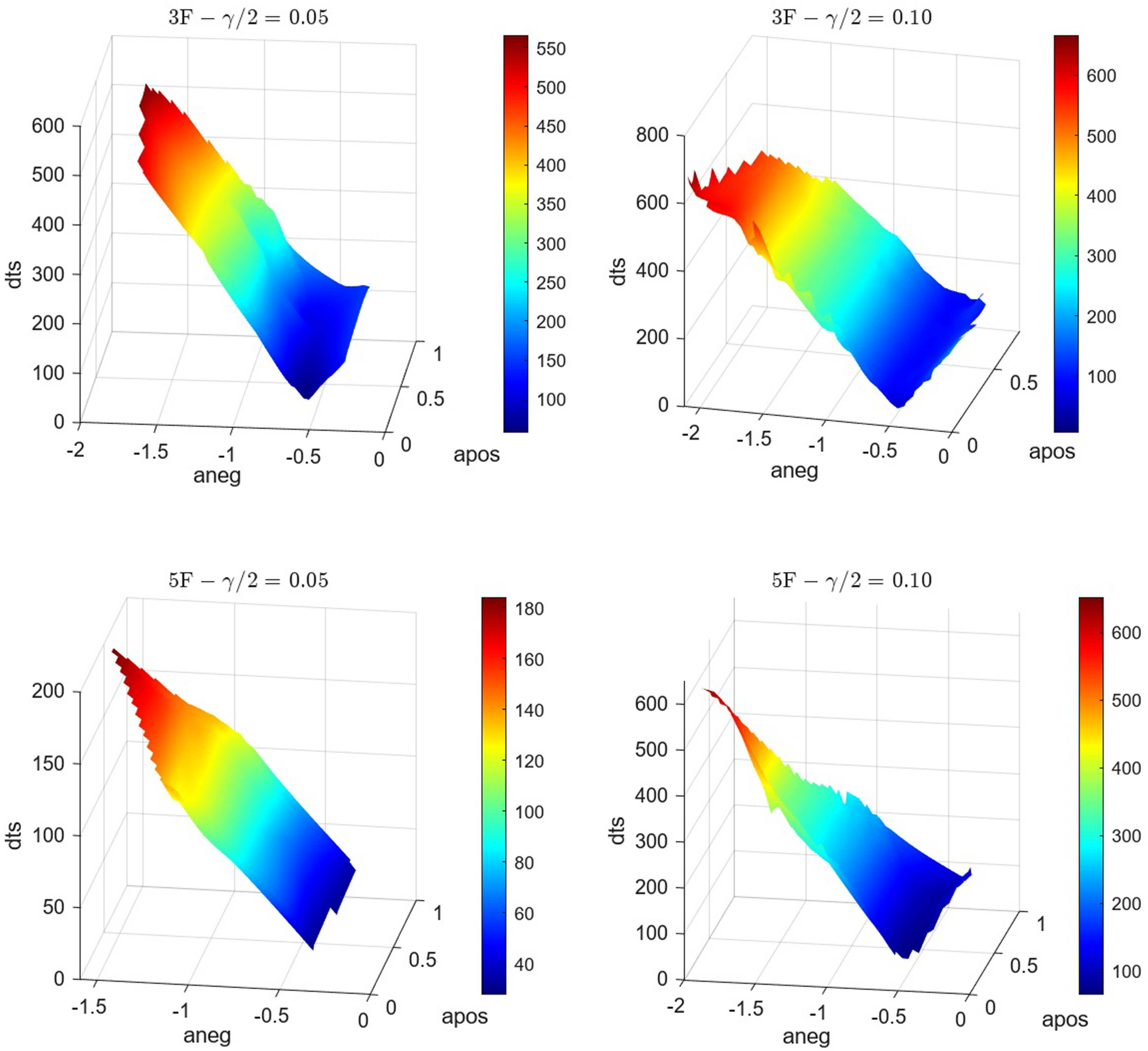

FIGURE B2 | Grid search for the α_b and α_g parameters estimated with the 3F and 5F models (one latent factor), and size $\gamma/2 = \{0.05, 0.1\}$ for the non-thematic funds, via the DTS test. The DTS values displayed are averages across every duplicate value (α_b, α_g) of the grid. [Color figure can be viewed at wileyonlinelibrary.com]