Foundations of Science
https://doi.org/10.1007/s10699-023-09912-1

Bayesian Practical Inference

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Accepted: 12 March 2023

Abstract
In this essay, we will try to provide a formal analysis of practical inference, attentive to the various phases in which it is articulated, and being so capable of explaining both the logical conclusiveness of the inference and the probabilistic nature of its conclusion. An innovative purpose of this article is to show how the final deliberation leading to action—the ultimate practical judgment—takes place according to a logic consistent with the use of Bayes' theorem. This is why we refer to Bayesian practical inference in the title of this essay.

Keywords Bayesian practical inference · Bayes' theorem · Practical inference · Nomological vs. probabilistic explanation · Natural vs. human sciences

1 Introduction

In this introductory part, it is our intention to present some fundamental aspects of practical inference, highlighting the most debated and controversial topics that have received scholarly attention over the years. This overview will be useful to contextualize our research and its aims.

(A) What kind of practical inference?

In the current literature, many forms of practical inference are discussed and applied. It is therefore appropriated to clarify which type of practical inference is being analyzed in this paper. Clarke (1985) distinguishes two classes of practical inference:

"There is a second class of inferences also similar to practical inferences (…). These are inferences used to explain a past action or predict a future action. In contrast to normative 'ought' and 'rational' conclusions their conclusions are not used to evaluate actions. The

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Published online: 26 April 2023

premises from which they are inferred instead provide reasons why an action was or will be performed, not reasons for its performance" (p. 37).

The inferences of the first class end with a proposition in a deontic sense, for example:

I ought to do M

My doing L is necessary to do M if C

C obtains

I ought to do L

while those of the second class end with a description of the action, for example:

X wants E

X believes that X's doing M is necessary to attain E if C

X believes that C obtains

X does M

The former justifies the assertion of an obligation, the latter explains an action.

The focus of our study is the rigorous analysis and reconstruction of the practical inferences of the second class.[1] This does not exclude the consideration of inferences of the first type as well, in view of the fact that the structure of practical inference arises from the systematic combination of practical inferences of various kinds.

(B) Is practical inference logically conclusive?

A main reason why practical inference is rejected as a logical inference is that it is normally judged to be logically inconclusive.[2] One proof of this is normally considered to be the phenomenon of *akrasia*—the so-called phenomenon of weakness of will—according to which a subject often acts contrary to her interests and against her knowledge. This problem will be discussed in depth later. Here we merely show that, as far as logical conclusiveness is concerned, there is a deep symmetry between the practical inference model and C.G. Hempel's DN model. Both are logical models of explanation. The practical inference

[1] The author who made this notion of practical inference central was von Wright (1971) who, in turn, drew inspiration from Anscombe's (1957) book on practical syllogism. At the origin of these ideas is Aristotle, The Nicomachean Ethics, 1147 a 25–30. Practical inference has also been taken up and developed by other authors. Besides Clarke (1985), Audi (1989), McCann (1998), Castellani (2000) and Mele (2017).

[2] The strongest criticism comes from von Wright himself with the example of the tyrant (1971, pp. 116–117): "If this sort of case can be imagined, it shows that the conclusion of a practical inference does not follow with logical necessity from the premises". In "On so called Practical Inference"(1972) von Wright claims: "This objection (that is, something happens which prevents X from doing A) may appear so grave that, in fact, it shows that action cannot possibly follow logically from premises about intentions and epistemic attitudes" (p. 48); and, further on, he states "So in any use of the argument there is a time gap involved between premises and conclusion and with this gap there is also a rift in the logical connection between the intention and epistemic attitude on the one hand and the action on the other hand" (p. 50). This fact also leads many authors to maintain that practical inference is a defeasible non-monotonic form of inference. See for example Spohn (2019). The theme is also present in authors sensitive to the treatment of practical inference in medieval practical philosophy. See, for example, Hoffman and Michon (2017, p. 29): "What makes human choices contingent is that, while the premises of a practical inference might remain unchanged and its soundness undisputed, the practical inference can be defeated. In contrast to sound theoretical inferences, sound practical inferences are nonmonotonic (which means that a sound inference can be undone by adding new information)". More on this topic later.

model is intended to be a logical model of explanation of rational action, just as Hempel's DN model is a logical model of explanation of physical events. Is it perhaps because both DN and practical inference are logical models that they do not admit of empirical falsification? DN falsification concerns the empirical laws of the model but not its logical part. A physical object may deviate from the assumed nomological regularity—and this will be proof that the law is false—but it cannot deviate from the logical laws implicit in the model. The same happens in the explanation of action through the model of practical inference. If the subject acts rationally and her action is in blatant departure from the model's prescription, the logical nature of the inferential model cannot be said to be falsified, but some of the premises assumed are. In other words, the failure to adhere to the conclusion of the practical inference, in the execution of the action by an agent, is not the falsification of the inferential model, but of some of the premises that, in the agent's view, should guarantee the rationality of that particular action.

(C) Is practical inference incompatible with freedom of action?

A further objection is often made to practical inference. Its logical character is seen as incompatible with the fact that a rational action is normally understood as a free action.[3] If the nexus between the end intended by a rational agent and the action is seen as a relation of logical consequence, how is it possible for the action to be a free action? Is it free because the aspiration to achieve that end is itself free or because the choice of means to achieve the end is free? Or for both reasons? And how are opposing aspirations composed? The model of practical inference we propose intends to address this issue as well. Of course, it is not the primary intention of this paper to address the issue of free will, but we are involved in it, as the model of action explanation we propose denies that the subject is necessitated—i.e. is not free—in the choice and execution of its action plans. The idea of composing practical inference with the use of Bayes' theorem depends precisely on the intention to give space, in addition to the vertical line of the premise-consequence relationship, to the horizontal line of probabilistic assumption of premises. The grafting of probability into the premises assumes that these latter are contingent, which provides space for the subject to freely influence the process of preference construction.

The essay is organized as follows. In Sect. 2, the structure of practical inference is presented; in Sect. 3 the fundamental objection of logical inconclusiveness of inference is discussed; in Sect. 4 the inferential process of choice and its dynamics are analyzed; in Sect. 5 the transformation of practical inference into Bayesian practical inference is proposed.

[3] See Michon (2011, p. 2) where he states: "Causal determinism or necessitation is not the only threat to freedom: since necessitation is the core idea of determinism, all kinds of necessitation would have the same consequence. Aristotle's sea battle argument and the discussion of future contingents point towards a form of logical necessitation, that is reinforced by the idea of an infallible and omniscient knower of the future. But there is another classical discussion that bears on a possible necessitation of action by the reasons considered by the agent". In Hoffman-Michon (2017, p. 28) the authors ask themselves: "In the course of deliberation, several practical inferences may have been tested, but only one has been deemed successful. The successful one is the reasoning that explains why this particular action is to be done (or why it was done)". But does the agent truly have alternative possibilities? The solution the authors propose is: "What saves the contingency of a choice is the fact that the conclusion of a practical inference does not necessarily follow upon its premises". It is also worth mentioning Van Inwagen's Mind Argument, which in Van Inwagen (1983), referring to Leibniz, touches on the problematic of not only causal, but also logical determinism of free will.

2 Practical Inference's Structure

2.1 *Intentional Practical Inference* (IPI)

Practical inference refers to a particular kind of inference whereby an agent's actions can be explained on the basis of her intending certain ends that the agent recognizes them as worthy of being pursued. The connection between these ends and the action takes place through a series of inferential chains, each of which leads to the choice, depending on the particular conditions in which the agent finds herself operating, of the most appropriate means to achieve those ends. An example of a practical inference—consisting of one single inference element—is the following:

Intentional premise	Subject x wants to bring about a state of affairs p	W(x,p)
Epistemic premise	Subject x believes that in order to bring about p it is necessary to bring about q	B(x, ❑(p → q))
Conclusion	Subject x brings about q	q

The inference consists of an intentional premise—characterized by the will operator W—an epistemic premise—characterized by the belief operator B—, and a propositional conclusion.[4] The presence of the will and belief operators in the two premises is decisive because it highlights the gap between the practical-inferential explanatory model of action and the deductive-nomological model of the natural sciences. Whereas in DN explanation arises from the presence of universal laws, in which no epistemic or practical operators of any kind appear, in the scheme just presented epistemic and intentional operators perform the explanatory function without the need for universal premises. This is possible because it is precisely the logical-semantic principles that govern the use of these operators that, so to speak, bear the burden of explaining the connection between premises and conclusion. To fully understand how this happens, it is appropriate to distinguish two parts of practical inference, each of which is a particular form of it: the first is intentional practical inference (IPI), while the second is resolution practical inference (RPI). The intentional practical inference underlying the previous example can be schematized as follows:

Intentional premise	Subject x wants to bring about a state of affairs p	W(x,p)
Epistemic premise	Subject x believes that in order to bring about p it is necessary to bring about q	B(x, ❑(p → q))
Intentional conclusion	Subject x wants q	W(x,q)

As one can easily realize, the IPI consists of the two premises of the initial PI and a new conclusion: an intentional conclusion, characterized, like the premise of the same name, by the intentional operator of the will. This circumstance is significant because it allows us to

[4] The conclusion is formulated through the propositional variable q. A more logically refined analysis would be to replace this variable with a proposition characterized modally through some sort of actualization operator. We do not think it appropriate to make this complication because it is unnecessary for our purposes. It would still have to be a *factive* operator, that is, such as to imply the propositional variable we are interested in. This is also why we will say, indifferently, that the conclusion of a practical inference is the description of an *action* or the *result* of it.

understand the two main differences existing between IPI and RPI. First, in IPI, the inference conclusion refers not to a fact, the action's result, but to the intentional operator of will. As a consequence, the conclusion of IPI appears intuitively as a logical consequence of the relative premises, whereas it is not so for RPI and the whole PI (see below). For if x wants p and is aware of the fact that obtaining p in turn demands that the intermediate goal q be pursued, then it is entirely rational for x to want q as well. Not wanting q in the knowledge that p implies q is a sign that the actual will to achieve p is also missing, since wanting the necessary means— known *qua* necessary—is a necessary condition of will itself.

2.2 *Resolution Practical Inference* (RPI)

The second component of practical inference is resolution practical inference RPI. It takes the following form:

Intentional premise	Subject x wants q	W(x, q)
Non-impediment condition	Subject x is not prevented from bringing about q	c_{ni}
Conclusion	Subject x brings about q	q

Note the difference with intentional inference. The premise of RPI is intentional—in fact, it coincides with the conclusion of IPI—but the conclusion of RPI is the description of an action's result. Consequently, there is no immediate intuition of the fact that the conclusion follows logically from the premises, as is the case with intentional inference. The difference can also be grasped from an ontological point of view. While in IPI a relation between only mental states is at stake, in RPI a relation between a mental state, the descriptive condition of non-impediment and an empirical behavioral fact is at stake.[5] However, the semantics underlying the will operator, which will be discussed shortly in Sect. 4, is the key to showing that even in RPI between premises and conclusion there is a relation of logical consequence.

3 Objection of Inconclusiveness to Practical Inference

The most important objection to the scheme of practical inference concerns its logical conclusiveness. As we have already said, von Wright himself presents serious doubts about the logical conclusiveness of the inference by citing the famous example of the person who planned to shoot the tyrant ((1971), pp. 116–117). The killer is categorically determined to kill the tyrant. She enters his room and draws her loaded pistol to carry out the deliberate

[5] The condition of non-impediment can be attributed the defect of being a way of immunizing the explanatory derivation of the action from any possible falsification. It is as if through it one exorcised the non-occurrence of the action, while maintaining the truth of the premises, by claiming as a reason the fact that there was something that prevented it. But those who make this objection disregard the nature of the condition of non-impediment. It does not express a condition internal to the subjectivity of the agent, which brings her decision into play before it takes place. The impediment is logically subsequent to the decision and only after the realization of this it is reasonable for the subject to move on to another inference. In other words, for the action to take place there must be no physical impediment to its execution and this cannot affect the independent process of decision.

enterprise. At the last moment, however, she is seized with doubt as to what to do and refrains from committing the crime. According to von Wright, this is proof that it is possible for the premises of the inference to be true while the conclusion is false, which means that the conclusion is not a logical consequence of the premises.

However, if we reflect on the above analysis of the inference's structure, we can see that the description of the action follows logically from the premises. All the cases that can be devised as possible falsifiers of the inference are cases in which some premise of the inference is false. Instead, the principles governing the inference's structure are not falsifiable. It is clear that our volitions are subject to change and that a change in them involves a change in the course of action. Similarly, if we change a means will, this depends either on change of end will or belief in the means-end connection. In any case, if the state of truth of one of the premises changes, this is not the case for the logical relation between premises and conclusion.

This way of looking at things seems to us to be an answer to the objection through which the accusation of inconclusiveness often passes. Some authors (see Clarke, footnote 1, and Hoffmann-Michon, and Spohn, footnote 2) hold that all forms of practical inference are defeasible forms of argumentation and that therefore the logic underlying them is a non-monotonic logic. A non-monotonic argumentation is characterized by the fact that it is sensitive to the addition of further premises. If any new premises are added to these, then the conclusion of the argument can be falsified. The difference from a classical—monotonic—argumentation is that the latter is correct for any strengthening of the premises. However, in our case the conclusiveness of the inference does not only depend on the correctness of the logic involved in it, but on the fact that the set of premises—in the concrete situation of practical reasoning—must fulfil the *requirement of overall relevance*. This means that the premises from which the inference is drawn must cover the totality of the conditions that are relevant to the explanation of the concluding action. It is therefore not possible to add a new premise that does not contradict the truth of the others, but in this case, since the truth of the premises is lacking, the essential condition of conclusiveness is missing.

Given the logical conclusiveness of practical inference, i.e. that there are no cases imaginable of falsification without a variation in the truth state of some premises, how, then, is it possible to verify that in a given situation, the use of practical inference is appropriate? Such appropriateness is only given in two situations: 1. The action is performed and there is evidence —or at least clues—attesting the intentional framework of the subject. 2. The action is not yet performed, but the set of conditions that would lead the subject to perform it is reasonably determinable. In the first scenario, practical inference shows its explanatory role, as is the case, for example, in historical explanation: one explains the events that happened, but one cannot make predictions about the future. In the second scenario, practical inference also shows its predictive capacity. In the first case, falsification consists of showing that the premises constituting the intentional framework of the inference are disjunctively false. In the second scenario, falsification consists of showing that the expected action does not take place and that, consequently, some of the expected premises have not taken place. In both scenarios, the outcome of the inference is probabilistic.

That the outcome of the inference is probabilistic, however, is not inconsistent with the fact that the practical inference is logically conclusive. It is not true that human action, having to follow with necessity from the premises, cannot be simply probable—and therefore contingent. Nor is it true that to guarantee freedom of action, which presupposes that action is merely probable, this cannot necessarily follow from premises. This is not true because the inference can be both logically conclusive and involve a probabilistic consequence.

When one speaks of probabilistic explanations, one normally alludes to explanations in which the explanatory laws have a probabilistic structure. However, in the case of practical inference, there are no laws of any kind, not even probabilistic ones. There are only logical-analytic principles that do not deviate from necessity. How then is it possible for the consequence to be probabilistic? It is possible because it is sufficient that *some* of the inference's premises be contingent to guarantee the space of probability. If this condition is verified, it is easy to understand that the contingency of the premise is followed by the contingency of the conclusion, even if, once the truth of the premise is established, the conclusion necessarily follows.

In short, the logical conclusiveness of practical inference is not an obstacle to the probability of action and, thus, to freedom of choice. Of course, freedom of choice is not only contingency. Contingency can also belong to random events, which happened but might not have happened, whereas freedom belongs to actions that depend on the control of the agent's free will. The contingency we are interested in is therefore the contingency of the act of will that initiates the conclusive inference. For the subject to act freely, it is necessary that this will be free. It is therefore important to get an idea of the inferential process that normally leads to an act of free will or choice.

4 Inferential Process of Choice

4.1 Notion of Will

Let us therefore begin with a closer examination of the notion of will conveyed by the will operator W introduced in Sect. 2.

The will operator W(x, p), i.e., the will for p by subject x, is understood as equivalent to "x's belief in the optimality of p".[6] At the basis of this definition is the idea that subject x wants to carry out—i.e., decides to carry out among a set of alternative courses of action—that action, which at that moment and in that given situation, appears to her as the best possible —i.e., optimal—with respect to the ordering of her concrete preferences *hic et nunc*. A state of affairs is optimal for a subject if it represents what is most desirable for her in a given situation and, precisely for this reason, impels her to action. What is important is not its value in the abstract—i.e., considered independent of the singular characteristics of the moment—but the value that the state of affairs manifests to the subject at that precise moment, with all the characteristics of desirability *hic et nunc*. This is the reason why when the optimality of p enters the range of subject x's belief, at that very moment, this belief becomes the will for x to realize p. In conclusion, the following equivalence applies:[7]

$$\mathrm{W}(\mathrm{x}, \mathrm{p}) \text{ iff } \mathrm{B}\left(\mathrm{x}, \mathrm{O}_t(\mathrm{x}, \ \mathrm{p})\right)$$

[6] An exhaustive treatment of the will operator would require the construction of its precise semantics, capable of defining the deontic concept of optimality. Here we refer back to technical treatises such Kutschera (1982), pp. 1–28, Corradini (1989), pp. 91–106, Galvan (1991), pp. 127–205 and Castellani (2000), pp. 29–50. In any case, it is sufficient to work in mixed deontic systems, such as in Åquist's (1987) KT5Q system, in which the constant Q defines the set of optimal worlds.

[7] Note that this formula is an equivalence and not a real definition. In the latter case we would have written $\mathrm{W(x,p)} =_{\mathrm{def}} \mathrm{B(x,O_t(x,\ p))}$. We are, however, granted the liberty of sometimes saying that the will is optimality belief for pure reasons of linguistic expressiveness.

Note the peculiarity of the relationship that this equivalence establishes between volition and the conviction of optimality. Volition is equivalent to a conviction of optimality: it is not possible for x to want something that does not appear to x, in the concrete situation she finds herself in, to be maximally preferable—that is, all things considered, preferable to any concrete alternative. On the other hand, it is not possible to judge something that one does not want to be concretely optimal.

It is important to be aware of the theory underlying this analysis of the will operator. It rests on the distinction between *axiological beliefs*—in values held to be objective by the subject—and *preferential beliefs*. This distinction is fundamental insofar as, on the one hand, it takes account of the influence that the objectivity of values exerts on the subject's intentions—and hence her action. On the other hand, it prevents one from falling into the meshes of an intellectualist conception of the good when conceiving the idea of willing. With regard to the first aspect, this theory presents an approach that allows it to overcome the fracture, present in many objectivist conceptions of value—as in the Kantian conception—, between duty and interest. With regard to the other aspect, it seems to us that our theory is able to avoid the removal of the autonomous function of the will in practical decision-making processes, as seems to take place in conceptions of Socratic-Platonic origin, in which the will necessarily follows from the recognition of the good. There is actually a particular meaning of the good according to which the will necessarily follows from the knowledge of it. It is the good with respect to the subject's *hic et nunc* expectations (preferential beliefs), whereby subject x's belief that p is in that sense optimal means the same as x's willing p. Actually, between the intentional plane of agent x and the epistemic plane of her beliefs there is a correspondence principle according to which that x wants p *hic et nunc* means that x believes that p is *hic et nunc* the optimal thing for her to do or pursue. It is not asserted that x wants p because p is held by x to be optimal in itself—these are axiological beliefs—but because p is held by x to be optimal with respect to her concrete expectations and preferences of the moment—these are preferential beliefs. As we shall see in a moment, this solves the problem of akrasia at its roots.

4.2 From Intentional to Resolution Inference

In Sect. 2, we focused on inferences in which the intentional premises are volitional but never on inferences in which intentional non-volitional premises are involved. Now, we want to consider inference forms in which the latter occur. What is an intentional non-volitional premise? It is a proposition declaring the subject's belief that something is good for her, but without this being the object of an actual volition. In other words, it is a *prima facie* belief in positivity, from which it is rational to derive further beliefs in positivity. It is then entirely plausible that practical inference begins with one or more branches having the form of the following intentional branch:

Intentional non-volitional premise	Subject x believes in p's positivity	B(x, Pos(p))
Epistemic premise	Subject x believes that to bring about p it is necessary to bring about q	$B(x, \Box(p \rightarrow q))$
Intentional non-volitional conclusion	Subject x believes in q's positivity	B(x, Pos(q))

What is the difference with the scheme of volitional inference? The difference is that in the latter to want q it is not enough to believe that q is abstractly positive. It is necessary that it retains its positivity given all the intentional beliefs that come into the subject's mind at a certain moment. The object of intention q must appear to the subject as concretely positive and, for this reason, worthy of being desired. Having arrived at B(x, Pos(q)) does not yet mean having arrived at B(x, O_t(x, q)), i.e., —owing to the latter's equivalence with W—willing q. As it were, x has not yet crossed the line between doing q and not doing it, she has not yet arrived at the so-called *ultimate practical judgment*, she is still in suspense. Only the ultimate practical judgment makes it possible to pass from B(x, Pos(q)) to willing q.[8]

Secondly, in the dynamics of the act of willing, there are many non-volitional intentional inferences that compete with each other and contribute to the formation of the act itself. Some of them are rejected at the outset and set aside. Others contribute together to the formation of the ultimate practical judgment. However, they all have a similar structure to that described in the previous diagram and the same vocation to move from the abstract plane of positivity to the concrete plane of optimality.

The existence, in the dynamics of will, of two distinct planes, the plane of positivity beliefs—such as B(x, Pos(q))—and the plane of optimality beliefs—such as B(x, O_t(x, q))—is definitely relevant for at least two basic reasons.

First, the existence of two distinct planes makes it possible to provide a cogent answer to the objection that practical inference cannot be a model for explaining action because it cannot explain the phenomenon of *akrasia*. The phenomenon of *akrasia,* in fact, consists in the contrast between what the agent does and what she thinks she should do. Now, if one theorizes the distinction between the subject's axiological beliefs—i.e., her beliefs about what she herself believes to be objectively positive—and preferential beliefs—i.e., what she believes to be concretely positive for herself at the moment of acting, then it is quite conceivable that the subject decides differently from what she herself believes she ought to do. She decides differently from what the objective principles she herself believes to be valid prescribe, because those principles, however abstractly shared, have not become part of the subjective preferential order determined *hic et nunc* by her concrete will.

Second, it is in the space between these two planes, that the self, aware of the choice to be made, intervenes with all its weight. The "I" has the function of transforming the simple belief in the positivity of p into the belief that p is optimal so that p becomes the object of effective volition. It is in the gaps in this passage that, for libertarians, the exercise of free will takes place. Here, the inference's resolution phase is realized.

The resolution principle—in the first box of the following diagram—expresses precisely x's assumption of the axiological positivity of q as the determinant factor of the preferential ordering that underlies x's choice of q as the optimal action to be desired and consequently carried out.

[8] Here, too, the advocate of the defeasibility of practical arguments might have something to say. It is, in fact, possible that the positivity of q inherited from p comes into conflict with some other belief in positivity—that of r, for instance. Which is in that case the winning positivity: that of q or that of r? It is not possible to answer one way or the other, because it is not a resolution inference. However, this does not mean that the positivity of q is not inherited from the positivity of p. It is only a matter of *prima facie* positivity, as, after all, is that of p.

Resolution principle	$B(x,Pq) \rightarrow B(x, O_t(x,q))$ (x is resolutely convinced of the positivity of q)
Equivalence of $W(x,q)$ to $B(x, O_t(x,q))$	$W(x,q)$ (x wants q)
Non-impediment condition	c_{ni} (There are no impediments to bringing about q)
Conclusion	q (q is brought about)

Clearly, one can see that the resolution principle is *not* a logical principle. The implication it expresses is the result of the *idiographic* intervention of the subject. Thus, its truth goes hand in hand with the aleatory nature of the personal will, and for this reason, the action q that follows is as free as the will to q.

Even though the dynamic leading to the resolution principle takes place under the banner of the freedom of the subject and thus has an undoubtedly random character, it does not escape the laws of Bayesian probability. The next section serves to justify this claim. We will show how Bayesian inference can be combined with practical inference and how, consequently, practical inference can gain an important advantage from becoming a Bayesian practical inference.

5 Bayesian Practical Inference

5.1 Bayesian Inference

The explanatory virtue of practical inference is consistent with the explanatory use of Bayesian inference applied to very diverse phenomena. Bayesian inference is used, for example, to explain the phenomenon of object perception or the mechanism leading to species' evolutionary differentiation and other phenomena.[9] Let us focus briefly on perceptual dynamism (Kersten & Yuilley, 2003).

Let us denote by S the (occurrence of the) object that is at the origin of a given sensory stimulus. Let I be the (occurrence of the) image that the perceiver makes of S. According to Bayesian explanation, the perception phenomenon would be the result of the composition of two elements: the expectation on the part of the subject of I, irrespective of the stimulus—be it indicated in probabilistic terms through P(I)—and the expectation on the part of the subject of I on the basis of the stimulus determined by object S. The composition occurs according to the following formula:

$$P(S|I) = \frac{P(S) \cdot (P(I|S)}{P(I)}$$

It states that the perceiver's degree of belief that I is the image of object S is directly proportional to her degree of belief that object S is there independently of the stimulus that generates image I and to her degree of belief that I is generated by object S, while it is inversely proportional to the degree of expectation of image I.

This means first that the larger the belief that S exists regardless of what is perceived, the larger the belief that I will be the image of S. Second, the larger the belief that I is

[9] Geisler and Diehl (2002), Campbell (2016), Gergely and Csibra (2003), Baker et al. (2014).

generated by object S, the larger the belief that I is the image of S. On the other hand, the smaller the expectation of perceiving I, i.e.,—the greater the surprise of having such a perception—the greater the belief that S is given under condition I.

The consequence of Bayes' formula is, therefore, that perception is a compromise[10] between the reliability of the image features, represented by P(I|S) and the prior knowledge P(S). It is also important to note that some perceptions may be driven more by prior knowledge and others more by data. The less reliable the image features are (e.g. they are fuzzy), the more perception is influenced by prior knowledge. Finally, the dynamism of perception, i.e., the "focusing" of perceptual activity occurs through a recursive iteration of the same inference.

All these features are decisive factors in the structural consistency between practical and Bayesian inference. Practical inference explains the occurrence of an action—such as the killing of a person—or the development of a course of actions leading to a certain final action. In turn, Bayesian inference explains a certain perception or the process leading to the perception of the details of an image. Even though they are different procedures, they can interact. Since practical inference is explanatory if the premises are true and these have a contingent character, accounting for the correctness of practical inference means ascertaining, as far as possible, how the subject arrived at the situation described by the inference's premises. Bayesian inference can be of help here. As we shall see in the next section, the use of Bayes' theorem to make the "branches" of practical inference that contribute to the same final action's explanation interact seems very significant.

5.2 Bayesian Practical Inference

Let us start by studying a concrete case of practical inference. Take the above diagram containing the resolution principle and interpret letter p as indicating a certain political project. Let letter q, on the other hand, stand for the action of voting in support of the party that is considered essential to the realization of that project. It is then possible to construct the following inferential scheme.

Intentional non-volitional premise	Subject x believes in p's positivity	B(x, Pos(p))
Epistemic premise	Subject x believes that to bring about project p, q is necessary	B(x, ❑(p→q))
Intentional non-volitional conclusion	Subject x believes in q's positivity	B(x, Pos(q))

Of course, this scheme only contains the intentional non-volitional part of the practical inference. To carry out voting action q, the resolution principle is required.

Resolution principle	B(x,Pos(q))→B(x,Ot(x,q)) (x is resolutely convinced of q's positivity)
Non-impediment condition	c_{ni} (No impediment for voting action q)
Conclusion	q (x votes q)

[10] See Manjaly and Iglesias (2020), p. 6.

The resolution part of practical inference is not obvious because the content of the resolution principle is not obvious. There may be other political parties that pursue the same political project p and may therefore be able to win the goodwill of x. Why should x not also consider these parties for the vote?

Normally, voters consider the whole range of parties inspired by the same general political project and then decide which party to choose on the basis of more specific factors. But, what does it mean to broaden considerations to more specific factors? From the point of view of practical inference, it means that for each of these factors, the subject constructs a new inferential branch in her mind, places it next to the branch that leads to the possible choice of q and makes it interact with q.

Let us assume, for the sake of argument, that there is only one alternative party to that supporting program p. According to the two intentional branches of the practical tree, the vote for each of the parties is positive. We denote the vote for the first party by a and the vote for the second party by b. At this point in the decision-making process, subject x must find the optimal vote. How will she do this?

To choose, she will have to judge the two parties from different value viewpoints with respect to the general political project p, which is abstractly identical for both. One of these may be the increase in *welfare* in terms of wealth creation, which the two parties' programs envisage achieving in their own way. Often, then, the voter's judgment also depends to a considerable extent on the degree of *social sensitivity*—that is, the level of *equity* in income distribution—that she is willing to attribute to the two parties, on the basis of what they have done in past legislations in view of social justice, of promises kept and those broken, and so on.

How does the comparison take place? Let us denote by W the well-being value and by J the justice value, both values characterizing, according to subject x's judgment, project p. The subject is thus faced with three cases. She may judge the project p to be optimal for the justice aspect or the welfare aspect or for both aspects interacting in a certain way. In all three cases she will have to decide for one of the parties. How she will make the choice? It is at this stage that Bayesian inference can be invoked. It is then a question of using Bayes' theorem to fill in the middle boxes of the following scheme, where the first level represents the optimality belief of W, or J or W∧J, while the third level represents the optimality belief of vote a (vote of majority for the first party) or vote b (vote of majority for the second party).

Volitional premise	$B(x, O_t(W))$	$B(x, O_t(J))$	$B(x, O_t(W \wedge J))$
Bayes' inference Volitional conclusion	$B(x, O_t(a))$ or $B(x, O_t(b))$	$B(x, O_t(a))$ or $B(x, O_t(b))$	$B(x, O_t(a))$ or $B(x, O_t(b))$

It is reasonable to describe the situation through the following initial assumptions:

1. $P(a) = P(b) = 0.5$. Let us assume, in fact, that a priori the two parties are considered by x as equals. There is no prior reason to vote for one over the other.
2. P(W) and P(J) are assumed to be close to 0.5, because the subject has no reason to believe that values W and J are realized regardless of the intervention of the two parties.
3. P(W∧J) is assumed to be very low—0.275—due to the difficulties, which the subject is well aware of, of achieving W and J in harmony.

4. The following conditional probabilities are justified by the case's history:

$$P(W|a) = 0.75,\ P(J|a) = 0.2,\ P(W|b) = 0.45,\ P(J|b) = 0.8$$

Let us try to calculate:P(a|W), P(a|J), P(a|W ∧ J), P(b|W), P(b|J), P(b|W ∧ J)

$$P(a|W) = \frac{P(W|a) \cdot P(a)}{P(W)} = \frac{0.75 \cdot 0.5}{0.6} = 0.625$$

$$P(a|J) = \frac{P(J|a) \cdot P(a)}{P(J)} = \frac{0.2 \cdot 0.5}{0.5} = 0.2$$

$$P(a|W \wedge J) = \frac{P(W \wedge J|a) \cdot P(a)}{P(W \wedge J)} = \frac{0.15 \cdot 0.5}{0.275} = 0.\overline{27}$$

$$P(b|W) = \frac{P(W|b) \cdot P(b)}{P(W)} = \frac{0.45 \cdot 0.5}{0.6} = 0.375$$

$$P(b|J) = \frac{P(J|b) \cdot P(b)}{P(J)} = \frac{0.8 \cdot 0.5}{0.5} = 0.8$$

$$P(b|W \wedge J) = \frac{P(W \wedge J|b) \cdot P(b)}{P(W \wedge J)} = \frac{0.4 \cdot 0.5}{0.275} = 0.\overline{72}$$

This results in:

1. $P(a|W) > P(b|W)$: i.e., expectation of first party's incidence in realizing W is higher than in the case of second party.
2. $P(a|J) < P(b|J)$: i.e., expectation of second party's incidence regarding the realization of J is much higher than in the case of first party.
3. $P(a|W \wedge J) < P(b|W \wedge J)$: i.e., expectation of second party's incidence regarding the realization of $W \wedge J$ is higher than in the case of second party.

Thus, the previous incomplete resolution scheme can be completed as follows:

Volitional premise	$B(x, O_t(W))$	$B(x, O_t(J))$	$B(x, O_t(W \wedge J))$
Bayes' inference	$P(a\|W) > P(b\|W)$	$P(a\|J) < P(b\|J)$	$P(a\|W \wedge J) < P(b\|W \wedge J)$
Volitional conclusion	$B(x, O(a))$	$B(x, O(b))$	$B(x, O_t(b))$

5.3 Predictive Value of Inferential Explanation

The above inference describes the deliberation of the voting action of a single person x. It remains an open problem whether conclusions can be generalized. It only makes sense to

generalize if one deems person x as representative of a similar group of persons. However, it is difficult to establish the conditions for such similarity, and in any case, one cannot go beyond the probability of the premises. The reason lies in the close connection between the intentional and the resolution part of practical inference, which, as we have seen, is linked to the free decision of the individual.

The idiographic character of practical inference makes it difficult to account for generalization, but not for another typical function of science, namely prediction. Even if it is doubtful that one can generalize the inferential explanation of a specific action, it makes perfect sense to think that a practical inference capable of explaining a specific action (or course of action) also has potential predictive value.

Certainly not in the sense that that action would be predictable if the situation described by the premises were to be repeated exactly. This could not take place since the conditions expressed by the premises are historically determined and the past, for a subject, cannot repeat itself. However, the subject has memory, and it is therefore conceivable that, similar to the perception processes mentioned above, she makes use of Bayes' theorem to readjust the pursuit of the same ends to the new situation. Thus, Bayes' procedure also provides us here with a method for seeing how the agent *might* act if the situation were to change in a particular way. Let us return to the case of our voter, assuming that she is in a new voting cycle. How will she decide?

Let us suppose that, in the eyes of our voter, the second party—for whom she voted—was seriously committed to value J but did not achieve much to improve the social situation, having failed to gather the necessary consensus of the other political forces. In contrast, let the credit for the improvement of the social situation be attributed to the first party, which, not caring too much about justice, has managed to reverse the economic situation and start a period of innovation. Let our voter also be aware that if the action taken by the first party were to be blocked after the new elections, the damage would be very serious, as there would be no conditions for continuity. What then will be the consequences of these considerations in restructuring the new practical inference? What can we reasonably *predict* by readjusting the above examined scheme to the new situation? Let us denote with a' the new vote for the first party e b' the new vote for the second party.

It is reasonable to assume that:

1. Probabilities P(a') and P(b') are updated with P(a') = 0.27 and P(b') = 0.72, respectively.
2. Conditional probabilities, reflecting the voter's thoughts on the incidence of the two voting alternatives, are $P(W \wedge J|a') = \mathbf{0.6}$ and $P(W \wedge J|b') = \mathbf{0.20}$. Let the other variables be kept identical.

Then, it is to be expected that this time the voter's decision is, albeit reluctantly—i.e., with a low percentage—in favor of the first party.

$$P(a'|W \wedge J) = \frac{P(W \wedge J|a') \cdot P(a')}{P(W \wedge J)} = \frac{\mathbf{0.6} \cdot [0,27]}{0.275} = 0.58$$

$$P(b'|W \wedge J) = \frac{P(W \wedge J|b') \cdot P(b')}{P(W \wedge J)} = \frac{\mathbf{0.20} \cdot [0,72]}{0.275} = 0.52$$

The corrective application of Bayes' theorem made in this section clearly highlights the importance of experience in the evaluation of the probabilities to be assigned when the Bayesian argument is iterated—as usually said: The posterior probabilities of today are the prior probabilities of tomorrow. This also seems to us a distinct advantage of using Bayes in assessing the optimality of an action over the procedure of decision theory. In decision theory the probabilities are given, in Bayes the probabilities are gradually reshaped in dependence on new knowledge. And this, on the other hand, is more in keeping with the dynamic function of a practical inference for the explanatory purposes of action.

6 Conclusions

In the preceding pages we attempted to answer the main objections to practical inference.

First, practical inference is logically conclusive, in the sense that if the premises of the inference are true, then the conclusion, i.e., the action's description, follows with necessity. Action does not occur only if some premise of the inference is not true. This does not imply that the action cannot be freely chosen by the subject. In fact, the choice takes place through a process of deliberation on the part of the subject. Such a process determines the passage to the resolution phase of practical inference and, for this reason, is decisive in ensuring the truth of the premises, which are sufficient conditions for the action. Precisely for this reason, practical inference, although logically conclusive, has probabilistic value.

Action probability occurs both in the explanation of the action that has already taken place *ex post actu* and in the prediction. The explanation succeeds if all the conditions that constitute the premises of the inference are true, which can only be ascertained after the action has taken place. On the other hand, if the action is not performed but the set of conditions that would lead the subject to perform it is reasonably determinable, the action can be probabilistically predicted. In the first scenario, practical inference shows only its explanatory role, in the second scenario, practical inference also shows its predictive capacity.

The main intention of our proposal, however, was to distinguish the various levels on which the dynamics of action draw. Close to the decision and the execution of the action is the level of the concrete preferential order, which acts as the motor of the will. At a more abstract level there is the domain of value beliefs that influence choice, meaning that they contribute to the formation of the concrete preferential order, but do not coincide with it. Bayes' theorem plays an important role in establishing the connection between the abstract level of value beliefs and the concrete level of preferences.

In fact, we used Bayes' theorem to explain the resolution phase of practical inference, i.e., the phase in which the ultimate practical judgment takes place. We think that the use of Bayesian inference for the purposes of practical inference is only just beginning and that much still needs to be done to make their interaction more rigorous. However, our attempts to show that Bayesian inference can be integrated with practical inference seem to us to be sufficient to be able to speak of "Bayesian Practical Inference".

Funding Open access funding provided by Università Cattolica del Sacro Cuore within the CRUI-CARE Agreement.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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