Structural Change and Economic Growth: Production in the Long Run — A generalisation in terms of vertically hyper-integrated sectors

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Abstract Pasinetti's (1981) *Structural Change and Economic Growth* provides a complete and far reaching theoretical framework for the study of structural change, and therefore of economic development, rooted in in the Classical-Sraffian tradition.

Some attempts have been made, both in the '80s — for instance Siniscalco (1982) and Momigliano & Siniscalco (1986) — and more recently — e.g. Montresor & Vittucci Marzetti (2007a) and Montresor & Vittucci Marzetti (2008) — to use this framework for empirical purposes. However, all these attempts are based on Pasinetti's (1973) paper, i.e. on vertically integrated analysis. It is my contention that, as a consequence, they failed to recognise, and therefore to take advantage of, the main analytical feature of the 1981 book, namely vertical *hyper-integration*.

Actually, when trying to overcome the simplifying assumptions made by Pasinetti (1981) as regards the description of the technique, the starting point should be Pasinetti (1988), and not Pasinetti (1973), the latter being an intermediate step leading to the former.

After having highlighted the key differences between Pasinetti (1973) and Pasinetti (1988) — in order to show Pasinetti's (1981) vertically *hyper*-integrated character — and having generalised — by reintroducing inter-industry relations and allowing for more complex dynamics of economic magnitudes — the analytical framework provided by Pasinetti (1981) itself as to *production in the short run* (see Garbellini 2010a), the aim of the present paper is that of facing the issue of *production in the long run*, i.e. of extending the above mentioned generalisation to the 'general multisector dynamic model' (Pasinetti 1981, chapter V) presented by Pasinetti in his 1981 book.

This conceptual clarification and analytical generalisation is intended to be the first step of a line of research aiming at using, and extending, the present frame-

work to perform empirical analyses and study the behaviour of actual economic systems.

Keywords Natural system, vertically integrated sectors, vertically hyper-integrated sectors, functional income distribution, natural rates of profit, natural prices.

JEL classification B51,L16,O41

1 Introduction

In his 1981 book, Pasinetti goes into many topics concerning economic *theory* — e.g. the accumulation of capital — and *reality* — e.g. international relations. Anyway, the most complete and general formulation of the quantity and price systems used as a starting point for the development of the whole framework is given in Pasinetti (1988), where the notion of vertically hyper-integrated sector — or growing subsystem — is rigorously introduced.

After having clarified the vertically hyper-integrated character of Pasinetti's (1981) framework, and having restated and generalised the quantity and price systems, their solutions, and the equilibrium conditions characterising production in the *short run* (see Garbellini 2010a), this paper aims at doing the same with the topics touched upon in the second part of the book, i.e. that devoted to production in the *long run*.

More specifically — after providing some basic notation in section 2 and reassessing production in the short run in section 3 — section 4 sets up the general multi-sector dynamic model: the initial conditions and the laws of motion are stated, and therefore the 'dynamic' equilibrium conditions are derived.

Then, section 5 touches upon the topic of *changes in labour productivity*, singling out how the change in (total) labour productivity in each vertically hyper-integrated sector i ($i = 1, 2, \dots, m$) is the (weighted) average of the rate of change of *direct*, *indirect* and *hyper-indirect* labour productivity — or, alternatively, of the rate of change of direct and indirect labour productivity for consumption commodity i and direct and indirect labour productivity for the corresponding *additional* productive capacity. Some reflections are made on the usefulness of this analytical decomposition for empirical purposes.

Section 6 is a note on the degrees of freedom left open when the price system is considered *through time*, and the implications that they have on the choice of the *numéraire*.

Section 7 then goes through the structural dynamics of physical quantities and commodity prices, stressing how the whole structure of the economic system — looking both at the physical and at the value side — is continuously changing through time, due to the presence of non-uniform (among vertically hyper-integrated sectors) and of non-steady rates of growth of sectoral demand for consumption commodities and labour productivity — and to their intermingled dynamics.

Section 8 first recalls the difference between capital intensity — as expressed by the capital/output ratios(s) — and degree of mechanisation — as expressed by the capital/labour ratio(s) — and, moreover, between the sectoral and the aggregate expressions for such ratios. Then, the dynamics of both the sectoral and the aggregate ratios is analysed, in order to single out the corresponding determinants.

Section 9 introduces the 'natural' economic system; first, the particular theory of income distribution leading to it is briefly exposed, the 'natural' rate of profits are defined, and the 'natural' price system(s) — together with their properties and features — are stated (section 9.1). Then, the particular configuration of *sectoral* capital/output and capital/labour ratios *within the 'natural' economic system* is analysed, in order to single out the determinants of their dynamics through time as opposed to those characterising them when prices are not the 'natural' ones (section 9.2). Third, the concepts of 'standard rate of growth of productivity' — and hence of 'dynamic standard commodity' — are introduced (section 9.3).

Then, section 10 deals with the issue of the choice of the *numéraire* — with special reference to a conventional unit of account, thereby reaching a definition, to be used in the last section, of the general rate of price inflation — for the price system, again looking at the implications of such a choice on the closure of the two degrees of freedom left open.

Finally, section 11 closes the essay, by introducing the concept of 'natural' rate of interest, and hence extending the principle of labour income and value distribution also to those exchanges that shift purchasing power through time.

Some final remarks are provided in section 12.

2 Basic notation

Consider an economic system in which m commodities, denoted by subscript i ($i = 1, 2, \dots, m$) are produced. Such commodities can be used *either* as (pure) consumption goods *or* as intermediate commodities, *or* both.

Moreover, make the simplifying assumption that those commodities used as means of production are completely used up in each period, and therefore have to be replaced entirely.[1]

[1] No treatment of fixed capital is made here. This simplification is intended to be a first step to be followed by a complete treatment of this issue too. However, since extending

The economic system can be described by:

$\mathbf{q}$	$=$	$[q_i]$:	vector of total quantities;
$\mathbf{x}$	$=$	$[x_i]$:	vector of final demand for consumption goods;
$\mathbf{j}$	$=$	$[j_i]$:	vector of final demand for investment goods;
$\mathbf{y}$	$=$	$[y_i]$:	vector of final demand, with $y_i = x_i + j_i$, $i = 1, 2, \ldots, m$;
$\mathbf{A}$	$=$	$[a_{ij}]$:	matrix of inter-industry coefficients;
$\mathbf{a}_{ni}$	$=$	$[a_{ni}]$:	vector of direct labour requirements;
$\mathbf{a}_{in}$	$=$	$[a_{in}]$:	vector of demand coefficients for consumption goods: $x_i = a_{in} x_n$;
$\mathbf{a}_{k_i n}$	$=$	$[a_{k_i n}]$:	vector of demand coefficients for new investment: $j_i = a_{k_i n} x_n$;
$\mathbf{s}$	$=$	$[s_i]$:	vector of intermediate commodities necessary for the production of quantities q_i;
$\mathbf{p}$	$=$	$[p_i]$:	vector of commodity prices;
		x_n:	total labour.
		g:	rate of growth of population;
		r_i:	rate of growth of per-capita (average) demand of commodity i as a final good; $(i = 1, \ldots, m)$

the description of the technology in use introduces many complications, I have decided to limit myself, for the time being, to consider circulating capital only.

and the derived magnitudes obtained in Garbellini (2010a) are:

$$\mathbf{H} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{A}(\mathbf{I} + \mathbf{H})$$
$$\mathbf{v}^T = \mathbf{a}_{ni}^T(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{a}_{ni}^T(\mathbf{I} + \mathbf{H})$$
$$\mathbf{M} = \mathbf{H}(\mathbf{I} - \mathbf{H}\widehat{\mathbf{c}})^{-1}$$
$$\mathbf{z}^T = \mathbf{v}^T(\mathbf{I} - \mathbf{H}\widehat{\mathbf{c}})^{-1}$$
$$\mathbf{M}^{(i)} = \mathbf{H}(\mathbf{I} - \mathbf{H}c_i)^{-1} = [\mathbf{m}_1^{(i)} \ldots, \mathbf{m}_i^*, \ldots, \mathbf{m}_m^{(i)}], \qquad i = 1, 2, \ldots, m$$
$$\mathbf{z}^{(i)T} = \mathbf{v}(\mathbf{I} - \mathbf{H}c_i)^{-1} = [z_1^{(i)}, \ldots, z_i^*, \ldots, z_m^{(i)}], \qquad i = 1, 2, \ldots, m$$
$$\mathbf{z}_k^{(i)T} = \mathbf{z}^{(i)T}\mathbf{M}^{(i)} = [z_{k_1}^{(i)}, \ldots, z_{k_i}^*, \ldots, z_{k_m}^{(i)}]$$
$$\overline{\mathbf{M}} = [\mathbf{m}_1^*, \ldots, \mathbf{m}_i^*, \ldots, \mathbf{m}_m^*]$$
$$\overline{\mathbf{z}}^T = [z_1^*, \ldots, z_i^*, \ldots, z_m^*]$$
$$\overline{\mathbf{z}}_k^T = \overline{\mathbf{z}}^T\overline{\mathbf{M}} = [z_{k_1}, \ldots, z_{k_i}, \ldots, z_{k_m}]$$
$$\mathbf{\Phi}(\pi) = [\boldsymbol{\phi}_i(\pi)] = \big(\mathbf{I} - \overline{\mathbf{M}}(\pi\mathbf{I} - \widehat{\mathbf{c}})\big)^{-1}$$
$$\mathbf{\Phi}^{(i)}(\pi) = [\boldsymbol{\phi}_i^{(i)}(\pi)] = \Big(\mathbf{I} - \mathbf{M}^{(i)}(\pi - c_i)\Big)^{-1}$$
$$\mathbf{\Phi}_k(\pi) = [\boldsymbol{\phi}_{k_i}(\pi)] = \big(\mathbf{I} - (\pi\mathbf{I} - \widehat{\mathbf{c}})\overline{\mathbf{M}}\big)^{-1}$$
$$\mathbf{D}^{(i)} = [\mathbf{d}_i^{(i)}] = \frac{\mathrm{d}}{\mathrm{dc_i}}(\mathbf{I} - \mathbf{H}c_i)^{-1} = \big(\mathbf{I} - (\mathbf{H}c_i)^2\big)^{-1}$$

All throughout the paper, the following conventions will be observed:

- All vectors and matrices will be denoted by boldface symbols, while all scalar quantities by normal type ones;
- all matrices will be denoted by upper case letters, while all vectors by lower case ones;
- all vectors will be intended as column vectors; row vectors will be denoted by transposed vectors;
- a vector with a hat will denote a diagonal matrix with the element of the corresponding vector on the main diagonal.

3 Production in the short run: a reassessment

In order to be able to face the dynamic part of the framework, it is worth briefly recalling the quantity and price systems, their solutions, and the equilibrium conditions guaranteeing full employment — of the labour force and of productive capacity — and full expenditure of income.[2]

The quantity and price systems can be written, respectively, as:

$$\begin{bmatrix} \mathbf{I} & \mathbf{O} & -\mathbf{a}_{in} \\ -\mathbf{I} & \mathbf{I} & -\widehat{\mathbf{c}}\mathbf{a}_{in} \\ -\mathbf{a}_{ni}^T & -\mathbf{a}_{ni}^T\overline{\mathbf{M}} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x_k} \\ x_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ 0 \end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} \mathbf{p}^T & \mathbf{p}_k^T & w \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{O} & -\mathbf{a}_{in} \\ -(\pi\mathbf{I}-\widehat{\mathbf{c}}) & \mathbf{I}-(\pi\mathbf{I}-\widehat{\mathbf{c}})\overline{\mathbf{M}} & (\pi\mathbf{I}-\widehat{\mathbf{c}})\mathbf{a}_{in} \\ -\overline{\mathbf{z}}^T & -\overline{\mathbf{z}}^T\overline{\mathbf{M}} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0}^T & \mathbf{0}^T & 0 \end{bmatrix} \tag{3.2}$$

their solutions being, respectively:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{x}_k \\ x_n \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{in}\overline{x}_n \\ (\mathbf{I}+\widehat{\mathbf{c}})\mathbf{a}_{in}\overline{x}_n \\ \overline{x}_n \end{bmatrix} \tag{3.3}$$

$$\begin{bmatrix} \mathbf{p}^T & \mathbf{p}_k^T & w \end{bmatrix} = \begin{bmatrix} \overline{w}\left[\overline{\mathbf{z}}^T\left(\mathbf{I}-\overline{\mathbf{M}}(\pi\mathbf{I}-\widehat{\mathbf{c}})\right)^{-1}\right] \\ \overline{w}\left[\overline{\mathbf{z}}^T\overline{\mathbf{M}}\left(\mathbf{I}-(\pi\mathbf{I}-\widehat{\mathbf{c}})\overline{\mathbf{M}}\right)^{-1}\right] \\ \overline{w} \end{bmatrix}^T = \begin{bmatrix} \overline{w}\overline{\mathbf{z}}^T\boldsymbol{\Phi}(\pi) \\ \overline{w}\overline{\mathbf{z}}_k^T\boldsymbol{\Phi}_k(\pi) \\ \overline{w} \end{bmatrix}^T \tag{3.4}$$

Moreover, the intermediate commodities price vectors for vertically hyper-integrated sector i is given by:

$$\mathbf{p}_k^{(i)T} = \overline{w}\mathbf{z}_k^{(i)T} + \mathbf{p}_k^{(i)T}\mathbf{M}^{(i)}(\pi - c_i) \tag{3.5}$$

and therefore

$$\mathbf{p}_k^{(i)T} = \overline{w}\mathbf{z}_k^{(i)T}(\mathbf{I}-\mathbf{M}(\pi-c_i))^{-1} = \overline{w}\mathbf{z}_k^{(i)T}\boldsymbol{\Phi}^{(i)}(\pi) \tag{3.6}$$

Full employment of the labour force and full expenditure of income — the *flows* of the economic system — are guaranteed by a *macroeconomic condition*, analytically emerging as a condition for systems (3.1) and (3.2) to have non trivial solutions. Such a condition, though derived within a multi-sectoral framework, is independent of the number of sectors conforming the economic system as a whole, and therefore emerges as being a truly *macroeconomic* condition. Pasinetti (1981) calls it *effective demand condition*:

$$\mathbf{a}_{ni,t}^T\mathbf{a}_{in} + \mathbf{a}_{ni}^T\overline{\mathbf{M}}\mathbf{a}_{in} + \mathbf{a}_{ni}^T\overline{\mathbf{M}}\widehat{\mathbf{c}}\mathbf{a}_{in} \equiv \overline{\mathbf{z}}^T\mathbf{a}_{in} = 1 \tag{3.7}$$

[2]For details, see Garbellini (2010a).

On the contrary, full utilisation of the productive capacity — the *stocks* of the economic system — is guaranteed by a whole *series of sectoral conditions*, ensuring that the number of units of productive capacity available *at the beginning* of the time period is exactly that necessary for the satisfaction of final demand:

$$\mathbf{x} = \mathbf{k} \tag{3.8}$$

Moreover, it is worth recalling the definitions of vertically hyper-integrated productive capacity and labour.

A unit of vertically hyper-integrated productive capacity for sector i is the set of all intermediate commodities directly, indirectly, and *hyper-indirectly* needed for the production of one unit of commodity i as a final consumption good.

In the same way, the vertically hyper-integrated labour coefficient for sector i is the amount of labour directly, indirectly, and *hyper-indirectly* needed for the production of one unit of commodity i as a consumption good.

We can now go on and set up the general multi-sector dynamic model.

4 Setting up a general multi-sector dynamic model

Now, following Pasinetti (1981, section 1, chapter V) we shall define the *initial conditions* of the economic system and the *laws of motion* of the main economic variables.

At time 0, the economic system is characterised by:

(i) A series of m *stocks* of intermediate commodities expressed in units of vertically hyper-integrated productive capacity:

$$k_{i,0}, \qquad (i = 1, 2, \ldots, m); \tag{4.1}$$

(ii) an *exogenous* population $\overline{x}_{n,0}$;

(iii) a series of m technical coefficients, representing the quantity of labour *directly* necessary for the production of one unit of each of the m commodities produced in the economic system as a whole:

$$a_{ni,0}, \qquad (i = 1, 2, \ldots, m); \tag{4.2}$$

(iv) a series of m *per capita* (average) consumption coefficients:

$$a_{in,0}, \qquad (i = 1, 2, \ldots, m); \tag{4.3}$$

(v) a series of m investment coefficients, i.e. the average per capita demand for the m commodities produced in the economic system as new investment goods:

$$a_{k_i n,0}, \qquad (i = 1, 2, \ldots, m); \tag{4.4}$$

At time zero, all these coefficients are such as to satisfy the relations defining equilibrium in the economic system as a whole. This means that all the technical, consumption and investment coefficients under (iii), (iv) and (v) are such as to satisfy *macroeconomic condition* (3.7) at time zero:[3]

$$\mathbf{a}_{ni,t}^{T}\mathbf{a}_{in,t} + \mathbf{a}_{ni,t}^{T}\overline{\mathbf{M}}\mathbf{a}_{in,t} + \mathbf{a}_{ni,t}^{T}\overline{\mathbf{M}}\widehat{\mathbf{c}}_{t+1}\mathbf{a}_{in,t} = 1, \qquad t = 0 \tag{4.6}$$

Moreover, this means that — in each vertically hyper-integrated sector i, ($i = 1, 2, \ldots, m$) — the stocks of capital goods expressed in units of vertical hyper-integrated productive capacity under (i) satisfy the series of sectoral conditions (3.8) for full utilisation of productive capacity, i.e.:

$$k_{i,0} = x_{i,0} \qquad \forall i = 1, 2, \ldots, m \tag{4.7}$$

Macroeconomic condition (4.6), referring to the *flows* of the economic system, and the series of *sectoral* conditions (4.7), referring to the *stocks*, are the equilibrium condition within a single period of time. Of course, the fact that they might be satisfied within a single time period t does not imply that any automatism will enable the economic system to do the same in the following time periods as well.

In order to define the concept of equilibrium in a dynamic framework,[4] we now have to describe the way in which the relevant variables move through time.

Differently from what Pasinetti (1981) did, we will describe such movements using *discrete*, rather than continuous, time. This will introduce some additional

[3]Notice that, in principle, matrix $\overline{\mathbf{M}}$ should be dated too — even if we are making the assumption that inter-industry coefficients in matrix $\mathbf{A}$ are not changing — since it depends on the rates of growth of sectoral per capita demand, which — as we are going to see in a moment — are themselves changing through time.

Anyway, from one period to the following one, we will consider matrix $\overline{\mathbf{M}}$ as constant too, since the change is very small. Specifically:

$$\mathbf{M}_{t}^{(i)} - \mathbf{M}_{t-1}^{(i)} = \mathbf{H}^2 r_{i,t}\sigma_{r_{i,t+1}}\left(\mathbf{I} + \mathbf{H}r_{i,t}(2+\sigma_{r_{i,t+1}}) + \mathbf{H}^2 r_{i,t+1}^2(3 + 3\sigma_{r_{i,t+1}} + \sigma_{r_{i,t+1}}^2) + \ldots\right)$$

where

$$\sigma_{r_{i,t}} = (r_{i,t} - r_{i,t-1})/r_{i,t-1} \tag{4.5}$$

i.e. $\sigma_{r_{i,t}}$ is the *speed* with which per capita demand for consumption commodity i ($i = 1, 2, \ldots, m$) changes through time.

The order of magnitude of this difference is clearly very small, though not necessarily irrelevant. For the time being, however, in order not to complicate too much notation and derivation, we will assume the $\mathbf{M}^{(i)}$, and thus also $\overline{\mathbf{M}}$, to be constant through time.

[4]As already explained elsewhere (see Garbellini & Wirkierman 2010b), here we do not have an equilibrium position which is automatically maintained through time; rather, we have *a series* of *situations* of equilibrium, which have to be actively pursued through the choice of an appropriate amount of new investments.

analytical complications, but — together with the re-introduction of the whole set of inter-industry relations — will also allow us to analyse more in detail the dynamics of the main economic variables.

The dynamics of population and of labour, demand, and investment coefficients are the following:

(i) Population increases over time at a *steady* rate g:

$$\overline{x}_{n,t} = \overline{x}_{n,0}(1+g)^t \tag{4.8}$$

(ii) Direct labour coefficients change through time at the *non-steady* rates $\varrho_{i,t}$, which are different from sector to sector:

$$a_{ni,t} = a_{ni,t-1}(1-\varrho_{i,t}), \qquad i = 1, 2, \ldots, m \tag{4.9}$$

(iii) Demand coefficients change through time at the *non-steady* rates $r_{i,t}$, different from sector to sector:

$$a_{in,t} = a_{in,t-1}(1+r_{i,t}) \tag{4.10}$$

Of course, demand and labour coefficients cannot be negative, since this would have no economic meaning.

As stated above, no automatism guarantees that, once satisfied at time zero, conditions (3.7) and (3.8) continue to hold also for $t = 1, 2, \ldots$. The dynamics under (i), (ii) and (iii) are such as to continually change the structure of the net output and of relative labour productivities; therefore, full employment of the labour force and of productive capacity, together with full expenditure of income, are tasks to be actively pursued through *institutional mechanisms*.

What this framework can tell us is which conditions, *if satisfied* — and *given* each time period's coefficients — allow us to move the economic system from the equilibrium position entailed by the structure of the economic system in one time period to that entailed by the structure of the following one.

By dating all magnitudes whose movements through time have just been introduced, effective demand condition 3.7 can be written as:

$$\mathbf{a}_{ni,t}^T \mathbf{a}_{in,t} + \mathbf{a}_{ni,t}^T \overline{\mathbf{M}} \mathbf{a}_{in,t} + \mathbf{a}_{ni,t}^T \overline{\mathbf{M}} \widehat{\mathbf{c}}_{t+1} \mathbf{a}_{in,t} = 1 \tag{4.11}$$

At this stage of the analysis, Pasinetti (1981) derived what he then called the *capital accumulation conditions*, a series of *sectoral* conditions concerning equilibrium new investments, i.e. guaranteeing the evolution through time of the number of units of vertically hyper-integrated productive capacity available at the beginning of each time period in line with the evolution of final demand (for consumption commodities):

$$a_{k_i n,t} = c_{i,t+1} a_{in,t}, \qquad i = 1, 2, \ldots, m$$

Since we have started developing the reformulation of the whole framework from the more general analytical formulation presented in Pasinetti (1988), where such conditions — and their derivation — were already taken for granted, they have been already introduced in the price and quantity systems, and therefore in the macroeconomic condition (4.11). Anyway, it is worth doing a step backwards in order to explicitly derive them as conditions for 'equilibrium' capital accumulation.

As we hinted at before, these conditions, *if* satisfied, allow to keep productive capacity fully utilised period after period, i.e. drives *capital accumulation* in line with the evolution of final demand for consumption commodities — and with *technical progress*.

The variation of the stock of capital available in each vertically hyper-integrated sector i at the beginning time period t is given by the amount of intermediate commodities bought for the sake of new investment in the previous one. If stock equilibrium is to be maintained from time period t to $t+1$, the number of units of vertically hyper-integrated productive capacity to be devoted to the expansion of productive capacity itself must be the same as the variation of demand for the corresponding consumption good. I.e.:

$$k_{i,t+1} - k_{i,t} = x_{i,t+1} - x_{i,t} = a_{k_i n,t}\overline{x}_{n,t} \tag{4.12}$$

and hence:

$$a_{in,t+1}\overline{x}_{n,t+1} - a_{in,t}\overline{x}_{n,t} = a_{k_i n,t}\overline{x}_{n,t} \tag{4.13}$$

By using expressions (4.8) and (4.10) and rearranging, what we get is:

$$a_{k_i n,t} = (g + r_{i,t+1})a_{in,t} = c_{i,t+1}a_{in,t} \tag{4.14}$$

i.e. Pasinetti's (1981) capital accumulation conditions, though formulated in *discrete time*.

Equilibrium investments in period t are therefore determined by the expansion of demand from t to $t+1$, and are those investments which ensure the expansion of productive capacity to be exactly in line with the movements of sectoral total demand for consumption commodities.

Expression (4.14) shows the advantage of using discrete, rather than continuous, time. The capital accumulation conditions originally derived by Pasinetti (1981, Chapter V, p. 86) are slightly different from (4.14), i.e.:

$$a_{k_i n}(t) = (g + r_i)a_{in}(t)$$

Using continuous rather than discrete time is a matter of analytical simplicity. When such a choice is made, assuming non-steady rates of change of variables does not make sense, since it would introduce exactly the same type of analytical complications that carrying out a continuous time analysis is intended to avoid.[5] As a consequence, steady rates of growth are thus assumed, making impossible to distinguish between current, past and future rates of change of per capita demand. On the contrary, by using discrete time and hence allowing for non-steady rates of change, it is possible to make it clear that new investment do not depend on the change of demand from period $t-1$ to t, but from t to $t+1$, i.e. on the change of demand from the time period in which the investment decision has to be taken, to the following one.

5 Direct, indirect and hyper-indirect labour productivity

Item (ii) in the list of the dynamics depicted in the previous section concerns the movements of *direct labour* requirements. However, in the present framework, we are not only interested in these coefficient, but also in the vertically hyper-integrated ones. The latter can be written as:[6]

$$z^*_{i,t} = a_{ni,t} + \mathbf{a}^T_{ni,t}\mathbf{m}^*_i + c_{i,t+1}\mathbf{a}^T_{ni,t}\mathbf{m}^*_i, \qquad i = 1, 2, \ldots, m \tag{5.1}$$

or

$$\overline{\mathbf{z}}^T = \mathbf{a}^T_{ni,t} + \mathbf{a}^T_{ni,t}\overline{\mathbf{M}} + \mathbf{a}^T_{ni,t}\overline{\mathbf{M}}_t\widehat{\mathbf{c}}_{t+1} \tag{5.2}$$

For each vertically hyper-integrated sector i ($i = 1, 2, \ldots, m$) expression (4.9) describes the movement of the *first* addendum of this sum, whose rate of change from time period $t-1$ to t is given by $\varrho_{i,t}$. The second addendum is *indirect labour*, i.e. labour indirectly required to replace *all* those intermediate commodities used up during the production process for producing *both* final consumption commodity i *and* the whole *set* of intermediate commodities to be devoted to new investment. The third one is *hyper-indirect labour*, i.e. the quantity of labour necessary for the production of additional productive capacity. By defining $\varrho_{k_i,t}$ and $\varrho_{k_{hi},t}$, respectively, the rates of change of the second and third addenda, the rate of change of the whole vertically hyper-integrated labour coefficient can be written as:

$$\varrho'_{z_i,t} = \varrho_{i,t}\frac{a_{ni,t-1}}{z_{i,t-1}} + \varrho_{k_i,t}\frac{\mathbf{a}^T_{ni,t-1}\mathbf{m}^*_i}{z_{i,t-1}} + \varrho_{k^{c_i}_i,t}\frac{c_{i,t+1}\mathbf{a}^T_{ni,t-1}\mathbf{m}^*_i}{z_{i,t-1}} \tag{5.3}$$

[5] For a discussion on this point, see Garbellini (2010b, section 3.4).

[6] To see how this decomposition can be derived, see Garbellini (2010a, section 4).

i.e. as the *weighted average* of the rates of change of the three addenda, the weights being the proportion of each of them to the total.

Clearly, both $\varrho_{k_i,t}$ and $\varrho_{k_{hi},t}$ are themselves weighted averages of the direct labour coefficients. More specifically, the rate of change of indirect labour productivity is given by:

$$\varrho_{k_i,t} = \frac{\mathbf{a}_{ni,t-1}^T \widehat{\boldsymbol{\varrho}}_t \mathbf{m}_i^*}{\mathbf{a}_{ni,t-1}^T \mathbf{m}_i^*}, \qquad i = 1, 2, \ldots, m \tag{5.4}$$

i.e. by the weighted average of the ϱ_i's, the weights being the ratios, at time $t-1$, of the direct labour necessary to produce each commodity i $(i = 1, 2, \ldots, m)$ entering $\mathbf{m}_i^*$ to the *total* direct labour necessary to produce the whole unit of vertically hyper-integrated productive capacity.

Finally, the rate of change of hyper-indirect labour productivity is given by:

$$\varrho_{k_i^{c_i},t} = \frac{\mathbf{a}_{ni,t}^T \mathbf{m}_i^* c_{i,t+1} - \mathbf{a}_{n,t-1}^T \mathbf{m}^* c_{i,t}}{\mathbf{a}_{n,t-1}^T \mathbf{m}_i^* c_{i,t}} = \frac{\mathbf{a}_{ni,t}^T (\widehat{\boldsymbol{\varrho}} - \sigma_{r_i,t} \mathbf{I}) \mathbf{m}_i^*}{\mathbf{a}_{ni,t}^T \mathbf{m}_i^*} \tag{5.5}$$

where $\sigma_{r_i,t}$ is given by expression (4.5). Rearranging and substituting expression (5.4) into equation (5.5), the latter can be rewritten as:

$$\varrho_{k_i^{c_i},t} = \varrho_{k_i,t} - \sigma_{r_i,t+1} \tag{5.6}$$

i.e., hyper-indirect labour increases or decreases with respect to indirect one in proportion to the *speed* of change of per-capita demand for the corresponding final consumption commodity. This means that overall labour productivity — intended as the amount of working hours necessary to produce one unit of the corresponding consumption commodity and to make it possible to keep demand satisfied in the following period too — increases or decreases, *ceteris paribus*, when the growth of demand *decelerates* or *accelerates*, respectively.

This opens up the question of whether or not the vertically hyper-integrated labour coefficients are a good measure for labour productivity, since they are influenced not only by technical coefficients, but also by the movements through time of demand for consumption goods. The order of magnitude of this last component is likely to be very small, but when dealing with sectors experiencing great expansion (or contraction) this might not necessarily be so.[7]

Further decompositions of the vertically hyper-indirect labour coefficients can be obtained, when useful for specific tasks. For example, we could write $z_{i,t}$ as:

$$z_{i,t} = a_{ni,t} + \mathbf{a}_{ni,t}^T \mathbf{h}_i + \mathbf{a}_{ni,t}^T c_{i,t+1} \mathbf{m}_i^* + \mathbf{a}_{ni,t}^T \mathbf{H} c_{i,t+1} \mathbf{m}_i^* \tag{5.7}$$

[7] The theoretical and empirical problems connected to the measurement of productivity changes in vertically integrated and vertically *hyper*-integrated terms are treated more in depth in Garbellini & Wirkierman (2010a).

where the first and second addenda are, respectively, direct and indirect labour for the production of consumption commodity i — i.e. vertically integrated labour — while the third and fourth are direct and indirect labour for the production of additional productive capacity — i.e. vertically integrated labour for the production of additional productive capacity:

$$z_{i,t} = \mathbf{a}_{ni,t}^T(\mathbf{I}+\mathbf{H})\mathbf{e}^{(i)} + \mathbf{a}_{ni,t}^T(\mathbf{I}+\mathbf{H})c_{i,t+1}\overline{\mathbf{M}}\mathbf{e}^{(i)} \tag{5.8}$$

This last decomposition gives us a further hint about the differences between the vertically integrated and the vertically hyper-integrated approach. In the former, indirect labour coefficients simply indicate that amount of working time devoted, both directly and indirectly, to replace the intermediate commodities used up for the production of one unit of the final consumption commodity produced in the (vertically integrated) sector: $\mathbf{a}_{ni.t}^T\mathbf{H}$. On the contrary, what Pasinetti (1988) calls indirect labour is something more: it is the amount of working hours directly and indirectly necessary for the replacement of intermediate commodities used up for the production of one unit of final consumption commodity i $(i = 1, 2, \ldots, m)$ — $\mathbf{a}_{ni,t}^T\mathbf{He}^{(i)}$ — *and* for the production of the whole set of intermediate commodities composing one unit of the corresponding (additional) vertically hyper-integrated productive capacity — $\mathbf{a}_{ni,t}^T\mathbf{Hm}_i^*$.

Moreover, Pasinetti's (1973, section 9) paper explicitly treats higher order vertical integration, where by vertically integrated sectors of second order are the vertically integrated sectors producing the units of productive capacity. Expression (5.8) therefore shows that vertically hyper-integrated labour is the sum of vertically integrated labour of first and second order, when by productive capacity we mean vertically hyper-integrated one.

Expression (5.8) can also help us in dealing with the problem of labour productivity: each vertically hyper-integrated labour coefficient is decomposed in two parts. While the second is influenced by the movements of demand, the first one reflects purely technological factors. The relationships between the two gives us an idea of the weight of new investments, i.e. of *capital accumulation*, on the production effort to be put forward by each vertically hyper-integrated sector to keep equilibrium through time.

Finally, we can compute the rate of change of the *labour equivalents*[8] — $\varrho_{i,t}^{(e)}$ and $\varrho_{k_i,t}^{(e)}$ $(i = 1, 2, \ldots, m)$, for consumption commodities and units of productive capacity, respectively — such that

$$z_{i,t}^{(e)} = (1 - \varrho_{i,t}^{(e)})z_{i,t-1}^{(e)} \tag{5.9}$$

[8]For a definition of labour equivalents and of labour transformation matrix see Garbellini (2010a, section 6).

and

$$z_{k_i,t}^{(e)} = (1 - \varrho_{k_i,t}^{(e)}) z_{k_i,t-1}^{(e)}, \qquad i = 1, 2, \ldots, m \tag{5.10}$$

By recalling that $z_{i,t}^{(e)} = \overline{\mathbf{z}}_t^T \boldsymbol{\phi}_{i,t}(\pi)$ and that $z_{k_i,t}^{(e)} = z_{i,t}^{(e)} \overline{\mathbf{M}} = \overline{\mathbf{z}}_t^T \overline{\mathbf{M}} \boldsymbol{\phi}_{k_i,t}(\pi)$, the expressions for $\varrho_{i,t}^{(e)}$ and $\varrho_{k_i,t}^{(e)}$ can be written as:

$$\varrho_{i,t}^{(e)} = \frac{\overline{\mathbf{z}}_{t-1}^T (\widehat{\boldsymbol{\varrho}}_t' - \widehat{\boldsymbol{\sigma}}_{\phi_{i,t}}) \boldsymbol{\phi}_{i,t-1}(\pi)}{\overline{\mathbf{z}}_{t-1}^T \boldsymbol{\phi}_{i,t-1}(\pi)} = \frac{\overline{\mathbf{z}}_{t-1}^T (\widehat{\boldsymbol{\varrho}}_t' - \widehat{\boldsymbol{\sigma}}_{\phi_{i,t}}) \boldsymbol{\phi}_{i,t-1}(\pi)}{z_{i,t}^{(e)}} \tag{5.11}$$

and

$$\varrho_{k_i,t}^{(e)} = \frac{\overline{\mathbf{z}}_{t-1}^T (\widehat{\boldsymbol{\varrho}}' - \widehat{\boldsymbol{\sigma}}_{\phi_{i,t}}) \boldsymbol{\phi}_{i,t-1}(\pi) \overline{\mathbf{M}}}{\overline{\mathbf{z}}_{t-1}^T \boldsymbol{\phi}_{i,t-1}(\pi) \overline{\mathbf{M}}} = \frac{\overline{\mathbf{z}}_{t-1}^T (\widehat{\boldsymbol{\varrho}}' - \widehat{\boldsymbol{\sigma}}_{\phi_{i,t}}) \overline{\mathbf{M}} \boldsymbol{\phi}_{k_i,t-1}(\pi)}{z_{k_i,t}^{(e)}} \tag{5.12}$$

where $\widehat{\boldsymbol{\sigma}}_{\phi_{i,t}}$ is a diagonal matrices whose elements are the rates of change from time period $t-1$ to t of the corresponding elements of vector $\boldsymbol{\phi}_{i,t}(\pi)$.

Both $\varrho_{i,t}^{(e)}$ and $\varrho_{k_i,t}^{(e)}$ are weighted averages of the difference between the rate of change of each element of column i of the labour transformation matrix and the corresponding rate of growth of vertically hyper-integrated labour.

Finally, it is worth specifying a series of 'hypothetical' magnitudes, to use Pasinetti's (1988) terminology, which are associated to those elements of vectors $\mathbf{z}^{(i)T}$ $(i = 1, 2, \ldots, m)$ different from the i-th one.

We have defined ϱ_i' as the rate of change of labour productivity in vertically hyper-integrated sector i. I.e., ϱ_i' is the opposite of the rate of change of the corresponding vertically hyper-integrated labour coefficient z_i^*, which is the i-th element of vector $\mathbf{z}^{(i)T}$; we should also define the rate of change of the remaining $m-1$ elements. Let therefore $-\varrho_j^{(i)\prime}$ be the rate of change through time of the j-th element of vector $\mathbf{z}^{(i)T}$, with $i = 1, 2, \ldots, m$ and $j \neq i$.

6 The price system: choice of the *numéraire* and degrees of freedom

Before analysing the dynamics of relative physical quantities and relative prices, it is worth spending a few words on the price system, and on the meaning, when time is inserted into the picture, of choosing a *numéraire*.

As explained by Pasinetti (1981, Chapter V, section 12), the price system is characterised by *two* degrees of freedom, one concerning the *initial price* of the *numéraire* commodity, and one concerning the *rate of change* of such a price through time.

In particular, choosing labour as the *numéraire* commodity — and therefore keeping the wage rate fixed — actually means closing these two degrees of freedom as follows:

$$\begin{cases} w_0 = \overline{w} = 1 \\ \sigma_{w,t} = 0, \quad \forall t = 1, 2, \dots \end{cases} \tag{6.1}$$

But we can also choose the price of any commodity, or composite commodity, as the *numéraire* of the price system. If, for example, we chose commodity h as a basis, this would amount at setting:

$$\begin{cases} p_{h,0} = 1 \\ \sigma^{(h)}_{p_{h,t}} = 0, \quad \forall t = 1, 2, \dots \end{cases} \tag{6.2}$$

In order to express all prices in terms of commodity h, what we are left to do is expressing both the (real) wage rate at time zero and its rate of change through time *in terms of commodity* h itself, i.e.:

$$\begin{cases} w^{(h)}_0 = \dfrac{1}{\overline{\mathbf{z}}^T_0 \left(\mathbf{I} - \overline{\mathbf{M}}(\pi \mathbf{I} - \widehat{\mathbf{c}}_1)\right)^{-1} \mathbf{e}^{(h)}} = \dfrac{1}{z^e_{h,0}(\pi)} \\ \\ \sigma^{(h)}_{w,t} = -\varrho^{(e)\prime}_{h,t} \end{cases} \tag{6.3}$$

The real wage rate increases/decreases in the same proportion as the labour *equivalent* content of the *numéraire* commodity decreases/increases.

By inserting expression (6.3) into the price system, all prices will automatically be expressed in terms of commodity h; in the same way, by inserting expression (6.1), all prices will automatically be expressed in terms of labour.

7 Structural dynamics of physical quantities and commodity prices

We can now explicitly state the dynamics of both *relative* physical quantities and prices, respectively:

$$\begin{cases} \mathbf{x}_t = \mathbf{a}_{in,t} \overline{x}_{n,t} = (\mathbf{I} + \widehat{\mathbf{c}}_t) \mathbf{a}_{in,t-1} \overline{x}_{n,t-1} \\ \mathbf{x}_{k,t} = (\mathbf{I} + \widehat{\mathbf{c}}_{t+1}) \mathbf{a}_{in,t} \overline{x}_{n,t} = (\mathbf{I} + \widehat{\mathbf{c}}_t + \widehat{\mathbf{c}}_{t+1}) \mathbf{a}_{in,t-1} \overline{x}_{n,t-1} \end{cases} \tag{7.1}$$

$$\begin{cases} \mathbf{p}^T_t = \overline{w} \mathbf{z}^{eT}_t = \overline{w} \mathbf{z}^{eT}_{t-1} (\mathbf{I} - \widehat{\boldsymbol{\varrho}}^{(e)}_{i,t}) \\ \mathbf{p}^T_{k,t} = \overline{w} \mathbf{z}^{eT}_{k,t} = \overline{w} \mathbf{z}^{eT}_{k,t-1} (\mathbf{I} - \widehat{\boldsymbol{\varrho}}^{(e)}_{k_i,t}) \varrho^{(e)}_{k_i,t} \end{cases} \tag{7.2}$$

the corresponding rates of change through time therefore being:

$$\begin{cases} \sigma_{x_i,t} = c_{i,t} \\ \sigma_{x_{k_i},t} = c_{i,t} + c_{i,t+1} \end{cases} \qquad i = 1, 2, \ldots, m \tag{7.3}$$

$$\begin{cases} \sigma_{p_i,t} = -\varrho^{(e)}_{i,t} \\ \sigma_{p_{k_i},t} = -\varrho^{(e)}_{k_i,t} \end{cases} \qquad i = 1, 2, \ldots, m \tag{7.4}$$

As it is apparent from expressions (7.1) and (7.2), the whole structure of physical quantities and relative prices continuously changes through time, since all the rates of change — of per capita demand and of labour productivity — are different from sector to sector, and from time period to time period.

The very fact of having introduced the more complete description of technology, i.e. the complete matrix of inter-industry relations, and of having chosen to use discrete, rather than continuous, time, make such dynamics much more complex than in Pasinetti's (1981, Chapter V, p. 92) original formulation.

As to the quantity system, the analytical formulation of the dynamics of physical quantities is not affected by the introduction of the complete matrix **A**: we are working with units of vertically hyper-integrated productive capacity, which still are such whatever their physical content. Clearly, changes in the technique affect this physical content. But this problem, thanks to the adoption of these particular units of measurement for intermediate commodities, can be kept separate from that of capital accumulation. As Pasinetti himself states:

> [T]he notion of a physical unit of productive capacity, by being defined with reference to the commodity that is produced, continues to make sense, as a physical unit, whatever complications technical change may cause to its composition in terms of ordinary commodities.
>
> (Pasinetti 1973, p. 24).

On the contrary, the fact of introducing discrete, rather than continuous, time — and therefore of having the possibility of taking into account changes, from time period to time period, of the various rates of growth — makes it clear that the rate of change of the *number* of units of productive capacity to be produced in each vertically hyper-integrated sector in time period t crucially depends on the rate of change of demand for the corresponding consumption commodity *both* from time period $t-1$ to t *and* from time period t to $t+1$.

As to the price system, things are much more complicated than for physical quantities, since, *with the complete description the technique in use*, the price of each consumption commodity, and of the corresponding unit of vertically hyper-integrated productive capacity, comes to depend on the prices of *all* the others, which are directly or indirectly used in the corresponding vertically hyper-integrated sector. The dynamics of prices, as clearly emerges from expression

(5.4), depend on the change in productivity in *all* industries entering the sector, as well as on the difference between the *future* rate of change of demand for consumption commodities — that is to say, the rate of new investment required for the system to keep productive capacity fully utilised, and therefore to be in stock equilibrium, period after period — and the rate of profit, which determines the amount of resources that are available, at the end of each time period, for new investments themselves.

In particular, changes in the r_i's, $(i = 1, 2, \dots, m)$ affect both the vertically hyper-integrated labour coefficients — which increase as future demand for the corresponding consumption commodity increases with respect to the current one — and matrices $\mathbf{\Phi}(\pi)$ and $\mathbf{\Phi}_k(\pi)$.

The change through time of the elements of matrix $\mathbf{\Phi}(\pi)$ (and $\mathbf{\Phi}_k(\pi)$) itself are analytically quite complex, and therefore cannot be explicitly computed and singled out. Anyway, such elements do not change directly together with the rates of change of per capita demand, but rather through the difference between the rate(s) of profit and the rate of change of total demand for each consumption commodity, i.e. $(g + r_{i,t+1})$, $(i = 1, 2, \dots, m)$.

Hence, the price of any consumption commodity — and of the corresponding units of vertically hyper-integrated productive capacity — depend on the rates of change of demand for the corresponding consumption commodity and of labour productivity in *all* sectors. The value, at current prices, created in the economic system as a whole as a consequence of production activity is distributed among the different sectors according to whether their own rate of growth, *and* those of the others, are greater than, smaller than, or equal to the rate of profit.[9]

8 Sectoral and aggregate magnitudes through time

After discussing the determinants of the movements through time of relative quantities and prices, let us now analyse the structural dynamics of some other relevant economic magnitudes, namely the *capital/output* ratio(s), the *capital/labour* ratio(s), and the product per worker.

As stated elsewhere,[10] there is a deep difference between the concept of capital intensity, as summarised by sectoral capital/output ratios, and degree of mechanisation, as expressed by sectoral capital/labour ratios.

The capital/output ratio for vertically hyper-integrated sector i is the ratio of

[9] For details on the shifts of (vertically hyper-integrated) labour value from sector to sector as a consequence of the rate of profit being different from (sectoral) rates of change o demand for consumption commodities, see Garbellini (2010a, section 6).

[10] See Pasinetti (1981, Chapter IX, sections 4-7) and Garbellini (2010a, section 7).

two quantities of *labour equivalents*: the labour equivalent associated to the production of the units of vertically hyper-integrated productive capacity available at the beginning of the time period — i.e. $k_{i,t}$ units of productive capacity, evaluated at their current price $\overline{w}z^{e}_{k_i,t}(\pi)$ — and the labour equivalent associated to the production of the final (consumption) commodity — i.e. $x_{i,t}$ units od commodity i evaluated at its current price $\overline{w}z^{e}_{i,t}(\pi)$.

The wage rate — appearing both at the numerator and at the denominator — cancels out, and therefore its dynamics does not affect the movements of the capital/output ratios. Anyway the rate of profit, or better, the difference between the rate of profit and the rate of growth of each vertically hyper-integrated sector, does. In fact, matrices $\mathbf{\Phi}_t(\pi)$ and $\mathbf{\Phi}_{k,t}(\pi)$ depend on the actual rate of profit, and therefore labour equivalents $z^{e}_{i,t}(\pi)$ and $z^{e}_{k_i,t}(\pi)$ do depend on it as well.

The information provided by the degree of capital intensity, as Pasinetti (1981) explains in detail, are relevant for two kinds of problems. The first concerns the effect of sectoral investments on the flow of net production. The second concerns the process of price formation, since the higher the capital/output ratio in sector i, the higher the incidence of capital — i.e. of profit mark-up on the stock of existing capital — in the price of the corresponding final commodity.

In particular, the sectoral capital/output ratio for vertically hyper-integrated sector i $(i = 1, 2, \ldots, m)$ can be written as:[11]

$$\gamma_{i,t} = \frac{z^{e}_{k_i,t}(\pi)k_{i,t}}{z^{e}_{i,t}(\pi)x_{i,t}} = \frac{z^{e}_{k_i,t}(\pi)}{z^{e}_{i,t}(\pi)} = \frac{z^{e}_{k_i,t-1}(\pi)(1-\varrho^{(e)}_{k_i,t})}{z^{e}_{i,t-1}(\pi)(1-\varrho^{(e)}_{i,t})} \tag{8.1}$$

its rate of change through time therefore being:

$$\sigma_{\gamma_i,t} = \frac{\varrho^{(e)}_{i,t} - \varrho^{(e)}_{k_i,t}}{1-\varrho^{(e)}_{i,t}} \tag{8.2}$$

By looking at expression (8.2), we see that the dynamics of the degree of capital intensity in each vertically hyper-integrated sector i $(i = 1, 2, \ldots, m)$ crucially depend on the rate of change of the labour equivalent for both the final consumption commodity and the corresponding units of vertically hyper-integrated productive

[11]The second equality in (8.1) reflects the fact that, since we are looking for those conditions that allow to keep the economic system in equilibrium, we start from a situation of stock-equilibrium, in which $k_{i,t} = x_{i,t}$. Anyway, if we wanted to analyse a specific, concrete situation, using actual data, it would be quite likely that $k_{i,t} \neq x_{i,t}$. See Garbellini & Wirkierman (2010b) for details.

capacity. In particular, $\sigma_{\gamma_i,t} \lesseqgtr 0$ according to whether

$$\varrho_{i,t}^{(e)} \lesseqgtr \frac{1+\varrho_{k_i,t}^{(e)}}{2}$$

Recalling expressions (5.11) and (5.12), we see that the rates of change through time of the labour equivalents $\varrho_{i,t}^{(e)}$ and $\varrho_{k_i,t}^{(e)}$ depend not only on labour productivity in the corresponding vertically hyper-integrated sector, but also on the changes in the elements of the corresponding columns of matrices $\mathbf{\Phi}(\pi)$ and $\mathbf{\Phi}_k(\pi)$, i.e. on how demand for consumption commodity i changes, from t to $t+1$, with respect to the rate of profit. This in its turn depends not only on whether demand decreases or increases, but also on the speed with which such a decrease or increase takes place.

At the aggregate level, the capital/output ratio can be written as:

$$\Gamma_t = \frac{\mathbf{z}_{k_i,t}^{eT}\mathbf{a}_{in,t}}{\mathbf{z}_{i,t}^{eT}\mathbf{a}_{in,t}} \simeq \frac{\mathbf{z}_{k_i,t-1}^{eT}\left(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{k_i,t}^{(e)}+\widehat{\mathbf{r}}_{i,t}\right)\mathbf{a}_{in,t-1}}{\mathbf{z}_{i,t-1}^{eT}\left(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{i,t}^{(e)}+\widehat{\mathbf{r}}_{i,t}\right)\mathbf{a}_{in,t-1}} \tag{8.3}$$

its rate of change through time therefore being a quite complicated expression:

$$\sigma_{\Gamma_t} = \frac{\mathbf{z}_{k_i,t-1}^{eT}\left((\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{k_i,t}^{(e)}+\widehat{\mathbf{r}}_{i,t})\mathbf{a}_{in,t-1}\mathbf{z}_{t-1}^{eT}-\mathbf{a}_{in,t-1}\mathbf{z}_{t-1}^{eT}(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{i,t}^{(e)}+\widehat{\mathbf{r}}_{i,t})\right)\mathbf{a}_{in,t-1}}{\mathbf{z}_{t-1}^{eT}(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{i,t}^{(e)}+\widehat{\mathbf{r}}_{i,t})\mathbf{a}_{in,t-1}\mathbf{z}_{k_i,t-1}^{eT}\mathbf{a}_{in,t-1}} \tag{8.4}$$

Clearly, the structural dynamics of the aggregate ratio is much more complicated than that of the sectoral ones, since it depends not only on technology — and income distribution — but also on the whole *structure* of final demand for consumption commodities. Actually, starting from a multi-sectoral framework, it is possible to see how macroeconomic magnitudes and their dynamics crucially depend on an extraordinarily complicated interaction at the level of the sectoral ones; their determination depending not only on technology and income distribution, but also on the very sectoral structure of the economic system as a whole.

Moreover, expression (8.4) shows that the aggregate capital intensity depends not only on the determinants of changes in the labour equivalent associated to the production of *all* consumption commodities and units of vertically hyper-integrated productive capacity, but also on the rate of change of demand for the final consumption commodities produced by *all* sectors.

As to the capital/labour ratios, the information they provide is useful in facing problems concerning labour employment; more precisely, those problems relating technical progress and employment:

> Changes in the degree of mechanisation, as expressed by the capital labour ratio, mean changes in the size of employment associated with any given amount of capital goods, expressed at (average) constant prices.
>
> (Pasinetti 1981, p. 183)

The wage rate, differently from the case of the capital/output ratios, cannot be factored out here, since it only appears at the numerator of the sectoral ratios; therefore its dynamics — in addition to those of vertically hyper-integrated labour productivity and of the labour equivalent for the production of the sector-specific unit of vertically hyper-integrated productive capacity — do affect the variations of the sectoral degrees of mechanisation.

Sector i's degree of mechanisation is not the ratio of two quantities of labour equivalent, but the ratio of a quantity of labour equivalent — for the production of the stock of units of vertically hyper-integrated productive capacity available at the beginning of the time period — and a quantity of labour — the vertically hyper-integrated labour necessary for the production of $x_{i,t}$ units of commodity i, the final output of the production process carried out in vertically hyper-integrated sector i.

In particular, the capital/labour ratio for vertically hyper-integrated sector i $(i = 1, 2, \ldots, m)$ is given by:

$$\theta_{i,t} = \frac{\overline{w} z^e_{k_i,t}(\pi) k_{i,t}}{z^*_{i,t} x_{i,t}} = \frac{\overline{w} z^e_{k_i,t}(\pi)}{z^*_{i,t}} = \frac{\overline{w} z^e_{k_i,t-1}(\pi)\left(1 - \varrho^{(e)}_{k_i,t}\right)}{z^*_{i,t-1}\left(1 - \varrho'_{i,t}\right)} \tag{8.5}$$

its rates of change through time therefore being:

$$\sigma_{\theta_{i,t}} = \frac{\varrho'_{i,t} - \varrho^{(e)}_{k_i,t}}{1 - \varrho'_{i,t}} \tag{8.6}$$

The capital/labour ratio for sector i $(i = 1, 2, \ldots, m)$ depends negatively on vertically hyper-integrated labour productivity; moreover, it increases together with the corresponding labour equivalent for the production of one unit of productive capacity. As it can be seen, the difference with respect to the rate of change of the capital/output ratios is that the latter involve the rate of change through time of two quantities of labour equivalent — the numerator and the denominator of the ratios themselves — while the former involve the rate of change of a quantity of labour equivalent — for producing the corresponding vertically hyper-integrated productive capacity, i.e. the numerator of the ratios — and of a physical quantity of labour — for producing the final consumption commodity, i.e. the denominator of the ratios.

At the aggregate level, the capital/labour ratio for the economic system as a whole is given by:

$$\Theta_t = \frac{\overline{w}\mathbf{z}^{e^T}_{k,t}\mathbf{a}_{in,t}}{\overline{\mathbf{z}}^T_t\mathbf{a}_{in,t}} = \frac{\overline{w}\mathbf{z}^{e^T}_{k,t-1}\left(\mathbf{I} - \widehat{\boldsymbol{\varrho}}^{(e)}_{k_i,t} + \widehat{\mathbf{r}}_t\right)\mathbf{a}_{in,t-1}}{\overline{\mathbf{z}}^T_{t-1}\left(\mathbf{I} - \widehat{\boldsymbol{\varrho}}'_t + \widehat{\mathbf{r}}_t\right)\mathbf{a}_{in,t-1}} \tag{8.7}$$

its rate of change through time again being quite a complex expression:

$$\sigma_{\Theta_{i,t}} = \frac{\mathbf{z}^{e^T}_{k,t-1}\left((\mathbf{I} - \widehat{\boldsymbol{\varrho}}^{(e)}_{k_i,t} + \widehat{\mathbf{r}}_t)\mathbf{a}_{in,t-1}\overline{\mathbf{z}}^T_{t-1} - \mathbf{a}_{in,t-1}\overline{\mathbf{z}}^T_{t-1}(\mathbf{I} - \widehat{\boldsymbol{\varrho}}'_t + \widehat{\mathbf{r}}_t)\right)\mathbf{a}_{in,t-1}}{\overline{\mathbf{z}}^T_{t-1}(\mathbf{I} - \widehat{\boldsymbol{\varrho}}'_t + \widehat{\mathbf{r}}_t)\mathbf{a}_{in,t-1}\mathbf{z}^{e^T}_{k,t-1}\mathbf{a}_{in,t-1}} \tag{8.8}$$

Also in this case, we can see that the aggregate dynamics is further complicated by the effect of changes in the *composition* of final demand for consumption commodities, interacting with the effect of technological progress and of changes in the rate of profit.

Finally, we can consider the product per worker. The sectoral products are given by:

$$y_{i,t} = \frac{\overline{w}z^e_{i,t}(\pi)}{z^*_{i,t}} = \frac{\overline{w}z^e_{i,t-1}(\pi)(1 - \varrho^{(e)}_{i,t})}{z^*_{i,t-1}(1 - \varrho'_{i,t})} \tag{8.9}$$

their rates of change through time being:

$$\sigma_{y_{i,t}} = \frac{\varrho'_{i,t} - \varrho^{(e)}_{i,t}}{1 - \varrho'_{i,t}} \tag{8.10}$$

Clearly, the product per worker decreases either when there is an increase in the quantity of labour equivalent necessary for the production of consumption commodity i — which, with the corresponding vertically hyper-integrated labour remaining constant, is implied by an increase in the rate of profit with respect to the rate of growth of the sector — or when there is an increase in vertically hyper-integrated labour productivity — which, if the corresponding labour equivalent remains the same, implies exactly the opposite.

The aggregate product per worker is finally given by:

$$Y_t = \frac{\overline{w}\mathbf{z}^{e^T}_t\mathbf{a}_{in,t}}{\overline{\mathbf{z}}^T_t\mathbf{a}_{in,t}} \tag{8.11}$$

its rate of change through time being:

$$\sigma_{Y_{i,t}} = \frac{\mathbf{z}_{t-1}^{eT}\left((\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{i,t}^{(e)}+\widehat{\mathbf{r}}_t)\mathbf{a}_{in,t-1}\mathbf{z}_{t-1}^T-\mathbf{a}_{in,t-1}\overline{\mathbf{z}}_{t-1}^T(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_t'+\widehat{\mathbf{r}}_t)\right)\mathbf{a}_{in,t-1}}{\overline{\mathbf{z}}_{t-1}^T(\mathbf{I}-\widehat{\boldsymbol{\varrho}}_{i,t}^{(e)}+\widehat{\mathbf{r}}_t)\mathbf{a}_{in,t-1}\mathbf{z}_{t-1}^{eT}\mathbf{a}_{in,t-1}} \tag{8.12}$$

To conclude, we might say that there is a deep difference between sectoral and aggregate magnitudes in general. The former only depend on technology and on the specific configuration of income distribution, as well as on the particular movements of the rate of growth of the corresponding vertically hyper-integrated sector with respect to the (uniform or sectoral) rate(s) of profit. The latter crucially depend also on the very structure of the economic system as a whole, and on the way in which such a structure changes through time due to the change in the structure of final demand for consumption goods, and therefore to the related dynamics of capital accumulation — and thus, of technical progress, which clearly affects all the sectoral and aggregate movements taking place in the economic system.

As to aggregate magnitudes, we may add that singling out and isolating the determinants of their movements through time is a very complicated task; a deep understanding of the changes taking place in the economic system can only rely on the joint analysis of the dynamics of sectoral magnitudes. Looking at the aggregates only gives us a very superficial and primitive idea of what is going on at the fundamental level.

9 The 'natural' economic system

Expressions (7.1) and (7.2) are such as to satisfy the macroeconomic condition for *flow* equilibrium (4.6), the set of sectoral conditions for *stock* equilibrium (4.7), and *capital accumulation conditions* (4.14).

Quantities (7.1) are precisely those quantities that allow to satisfy, *period after period*, final demand for consumption goods, while keeping labour force and productive capacity full employed, and therefore providing for those new investments, according to conditions (4.14), that maintain capital accumulation in line with the evolution of effective demand.

Prices (7.2) are the other side of the coin: they are precisely those *exchange ratios* which, given the distributive variables, are *necessary* for the economic system to produce, period after period, exactly equilibrium quantities (7.1).

To be more precise, what we have is a whole *set* of equilibrium configurations of relative prices and relative quantities, one for each possible combination of the distributive variables. The next step of the analysis, to catch up Pasinetti (1981), consists in choosing one of these configurations, and specifically the one defining the 'natural' economic system. That is to say, the 'natural' economic system is given by expressions (7.1) and (7.2) closed by means of a particular *theory of distribution*, giving a particular combination of the rate(s) of profit and therefore, given the *numéraire*, of the wage rate.

The aim of this section is that of sketching such a theory of income distribution and the main consequences and implications of its adoption as the closure of the price system.

9.1 'Natural' rates of profit and 'natural' prices

The theory of income distribution underlying Pasinetti's (1981) approach is discussed in detail in Garbellini & Wirkierman (2010b, section 4.1). Suffice here to recall that Pasinetti's purpose is "to develop first of all a theory which remains neutral with respect to the institutional organisation of society" (Pasinetti 1981, p. 25). In order to close the price system according to such a purpose, he had to find out a way of treating income distribution independently of institutional considerations.

How is it possible to do so, when "the way in which income is distributed crucially depends on the character of the *social relations of production*, no less than on cultural, ethic, legal considerations, that is to say, precisely on the institutional set-up of society" (Garbellini & Wirkierman 2010b, section 4.1, p. 21)?

The main idea is that of attributing to wages and profits two different *functions*: while the former provide for the purchasing power which must absorb the production of *consumption commodities*, i.e. those commodities *not re-entering the circular flow*, the latter must provide for the purchasing power necessary for ensuring equilibrium *capital accumulation*, i.e. for absorbing *new investment* commodities which, on the contrary, *do re-enter the circular flow*.

In few words, Pasinetti states a theory of *functional income distribution*, according to which each vertically hyper-integrated sector i ($i = 1, 2, \ldots, m$) has its own 'natural' rate of profit, exactly equal to its own specific rate of growth: $g + r_i$.

Therefore, the 'natural' rates of profit to be used to close the price system are given by:

$$\pi^n_{i,t} = g + r_{i,t+1} = c_{i,t+1}, \quad i = 1, 2, \ldots, m \tag{9.1}$$

The main difference with respect to Pasinetti (1981) already emerged in Pasinetti (1988): when we close the price system with the natural rates of profit, we clearly do not have a uniform rate of profit anymore, but a whole series of m *sectoral* rates of profit $\pi^n_{i,t}$, $i = 1, 2, \ldots, m$. Therefore, we also have m 'natural' price systems, one for each vertically hyper-integrated sector:

$$\begin{cases} \mathbf{p}^{nT}_{i,t} = \overline{w}\mathbf{z}^{(i)T}_t \\ \mathbf{p}^{nT}_{k_i,t} = \overline{w}\mathbf{z}^{(i)T}_t\mathbf{M}^{(i)} \end{cases} \qquad i = 1, 2, \ldots, m \tag{9.2}$$

or, when embodied labour is adopted as the *numéraire* commodity for the price system, and therefore we set $\overline{w} = 1$:

$$\begin{cases} \mathbf{p}^{(i)nT}_{i(w),t} = \mathbf{z}^{(i)T}_t \\ \mathbf{p}^{nT}_{k_i(w),t} = \mathbf{z}^{(i)T}_t\mathbf{M}^{(i)} \end{cases} \qquad i = 1, 2, \ldots, m \tag{9.3}$$

Each vertically hyper integrated sector i is therefore characterised by a specific 'natural' price system. The first line of expression (9.2) — or, equivalently, of expression (9.3) — gives the prices of *all* the m commodities produced in the sector *in ordinary units*. The i-th element of vector $\mathbf{p}^{nT}_{i,t}$ $(i = 1, 2, \ldots, m)$ — or $\mathbf{p}^{nT}_{i(w),t}$ — is the 'natural' price of *consumption commodity* i, i.e. the final consumption commodity defining the vertically hyper-integrated sector. The other $m-1$ elements are defined by Pasinetti (1988) 'hypothetical' prices: *if* all the m commodities produced in the economic system as a whole were produced *as consumption commodities* in vertically hyper-integrated sector i, these would be the corresponding prices. In fact however (as explained in detail in Garbellini 2010a, sections 5.3 and 6) they can also be seen as the prices, in ordinary units, of the single commodities entering vertically hyper-integrated productive capacity, and therefore used to compute the corresponding prices $p^{(i)}_{k_j,t}$ $(i, j = 1, 2, \ldots, m)$.

The whole vector is thus used in the second line of expression (9.2) — or, equivalently, of expression (9.3) — to compute the price of the units of vertically hyper-integrated productive capacity. Also here, the relevant element of vector $\mathbf{p}^{nT}_{k_i,t}$ — or $\mathbf{p}^{nT}_{k_i(w),t}$ — is the i-th one. The other elements would be the prices of the units of productive capacity for consumption commodities $j \neq i$, were they produced in vertically hyper-integrated sector i as final consumption commodities. However, exactly as for the elements of vector $\mathbf{p}^{nT}_{i,t}$ other than the i-th one, they can also be seen as the prices of each single component of vertically hyper-integrated productive capacity for producing one unit of productive capacity itself, and are therefore those prices that we would use in case we wanted to compute the price of such 'higher order' productive capacity: $\mathbf{p}^{nT}_{k_i,t}\mathbf{M}^{(i)}$.

The just given definition of the 'natural prices' immediately implies a reflection on the relation between the fundamental and the institutional stages of the analysis:

> after developing our analysis independently of institutions, it may well emerge that some of the 'natural' features of an economic system may be

> impossible to achieve within a particular institutional set-up. In fact, the foregoing analysis precisely points at the 'natural rates of profit' as a most clear example of this type of impossibility.
>
> (Pasinetti 1981, p. 151)

That is to say, Pasinetti states, the 'natural' configuration of income distribution, and therefore of prices, is impossible to be achieved within a capitalist economic system. This impossibility is even more clear here — as well as in Pasinetti (1988) — than in Pasinetti (1981), due to the re-introduction of the more general description of the technique in use.

In the simplified formulation adopted by Pasinetti (1981), in fact, any commodity is produced *either* as a consumption good, *or* as an intermediate commodity; moreover, each intermediate commodity is utilised as an input by only one specific sector, and does not have any role in the others. The price of a consumption commodity simply depends on the amount of labour necessary for its production and on the price of its own 'capital' good; thus, the introduction of the (non-uniform) 'natural' rates of profit, as long as we keep institutional considerations outside the picture, does not create too many complications, since interactions between different industries are ruled out. The only incompatibility with a capitalist social mode of production is that in such an institutional framework the rate of profit is generally thought of as being uniform; in the 'natural' economic system this could only happen in presence of uniform rates of change of demand for final consumption commodities, which clearly is quite an irrealistic option.

In the general case, things are much more complicated. As it appears from expressions (9.2) and (9.3), when we use the complete matrix $\mathbf{A}$, and then close the price system with the 'natural' rates of profit, we get *a whole series* of m 'natural' price systems, one for each vertically hyper-integrated sector i ($i = 1, 2, \ldots, m$). This means that each commodity produced in the economic system as a whole has only one natural price as a *consumption good*, and m different natural prices as an *intermediate commodity*, in ordinary units, according to the specific vertically hyper-integrated sector whose corresponding productive capacity it is part of. Indeed, this is something more than non-uniformity in the rate of profit of different *industries*; since one single industry enters more than one vertically hyper-integrated sector — m, if it produces a basic commodity — and since all the activities participating in the same vertically hyper-integrated sector do charge the same rate of profit, we should have different rates of profit *within the very same industry*. Clearly, this is at odds with what we can actually observe in any capitalist economic system.

More specifically, if we consider vertically hyper-integrated sector i with non-uniform rates of profit $\pi_{i,t}$, the corresponding prices are given by

$$\begin{cases} \mathbf{p}_t^{(i)T} = \overline{w}\mathbf{z}_t^{(i)T} \left(\mathbf{I} - \mathbf{M}^{(i)}(\pi_{i,t} - c_{i,t+1})\right)^{-1} \\ \mathbf{p}_{k,t}^{(i)T} = \overline{w}\mathbf{z}_t^{(i)T}\mathbf{M}^{(i)} \left(\mathbf{I} - \mathbf{M}^{(i)}(\pi_{i,t} - c_{i,t+1})\right)^{-1} \end{cases} \tag{9.4}$$

which, when $\pi_{i,t} = \pi_{i,t}^n$ reduces to:

$$\begin{cases} \mathbf{p}_t^{(i)nT} = \overline{w}\mathbf{z}_t^{(i)} \\ \mathbf{p}_{k,t}^{(i)nT} = \overline{w}\mathbf{z}_t^{(i)T}\mathbf{M}^{(i)} = \overline{w}\mathbf{z}_{k,t}^{(i)T} \end{cases} \tag{9.5}$$

Since there are m expression like (9.4), or (9.5) — one for each vertically hyper-integrated sector — each commodity j ($j = 1, 2, \ldots, m$) has one price when considered as the consumption good produced by vertically hyper-integrated sector j— specifically, the j-th component of the corresponding price vector $\mathbf{p}_t^{(j)T}$, or $\mathbf{p}_t^{(j)nT}$ — and one price when considered as an intermediate commodity for each vertically hyper-integrated sector i, that is to say when it is part of its vertically hyper-integrated productive capacity — specifically, the j-th component of the corresponding price vector $\mathbf{p}_t^{(i)T}$ or $\mathbf{p}_t^{(i)nT}$, with $i = 1, 2, \ldots, m$.

Expression (9.5) also makes another characteristic of the 'natural' economic system come to the fore. We can in fact see that the 'natural' price of each consumption commodity, and of each unit of vertically hyper-integrated productive capacity, is given by the product of the wage rate and the corresponding vertically hyper-integrated labour coefficient. The implication is straightforward: thanks to the redefinition of the concept of net output, labour embodied is thought of not only as direct and indirect, but also ad *hyper-indirect* labour. Together with the adoption of the particular theory of income distribution sketched at the beginning of the present section, and implying the 'natural' rates of profit, it is then possible to go back to a *pure labour theory of value*, even when a rate of profit does exist and when intermediate commodities are considered. All 'natural' profits immediately translate into new investments, and thus into wages for those labourers producing new investment commodities; the 'natural' price of each commodity is exactly equal to its 'labour value', labour embodied and labour commanded thus coming to coincide.

To conclude this section, we may now go back to the analysis of section 6, to look more in detail at the dynamics of the real wage rate within the 'natural' economic system, i.e. when 'natural' rates of profit are adopted as the closure of the price system.

In this case, if *consumption* commodity h is chosen as the *numéraire* of the price system(s), its price will be kept fixed to 1 in all time periods; therefore, its rate of change through time will be $\sigma_{p_h,t} = \sigma_{p_h} = 0$. Hence, the real wage rate — and its rate of change through time — in terms of commodity h will be:

$$\begin{cases} w_t^{(h)} = \dfrac{1}{z^*_{h,t}}, \quad \forall t \\ \\ \sigma_{w_t}^{(h)} = \varrho'_{h,t} \end{cases} \tag{9.6}$$

Any increase/decrease in the vertically hyper-integrated labour coefficient for the commodity whose price is chosen as the *numéraire* immediately translates into an equal decrease/increase in the real wage rate. Therefore, all increases in labour productivity immediately translate into a corresponding increase in labourers' real purchasing power: all price reductions made possible by an increased labour productivity are gained by labourers themselves, instead of being (partially) absorbed by profits.

9.2 Sectoral capital/output and capital/labour ratios in the 'natural' economic system

It is worth spending a few words on the meaning that capital/output and capital/labour ratios come to acquire within the 'natural' economic system.

First of all, there is a major difference with respect to the sectoral capital/output ratios computed for the general case; when evaluated at current prices different from the 'natural' ones, the stock of capital available at the beginning of the production process is given by the product of the wage rate and the corresponding labour *equivalent*; the same holds for sectoral output. As a consequence, the sectoral capital/output ratios are ratios of two quantities of labour equivalent. On the contrary, within the 'natural' economic system they are ratios of *physical quantities of labour*, i.e.:

$$\gamma^n_{i,t} = \frac{p^n_{k_i,t} k_{i,t}}{p^n_{i,t} x_{i,t}} \equiv \frac{\mathbf{z}_t^{(i)T} \mathbf{m}_i^*}{z^*_{i,t}} = \frac{\mathbf{z}_{t-1}^{(i)T}(\mathbf{I} - \widehat{\boldsymbol{\varrho}}_t^{(i)\prime})\mathbf{m}_i^*}{z^*_{i,t-1}(1 - \varrho'_{i,t})} \tag{9.7}$$

or

$$\gamma^n_{i,t} = \frac{z^*_{k_i,t}}{z^*_{i,t}} = \frac{z^*_{k_i,t-1}(1 - \varrho'_{k_i,t})}{z^*_{i,t-1}(1 - \varrho'_{i,t})} \tag{9.8}$$

where

$$\varrho'_{k_i,t} = -\frac{\mathbf{z}_t^{(i)T}\mathbf{m}_i^* - \mathbf{z}_{t-1}^{(i)T}\mathbf{m}_i^*}{\mathbf{z}_{t-1}^{(i)T}\mathbf{m}_i^*} = -\frac{\mathbf{z}_{t-1}^{(i)T}(\mathbf{I} - \widehat{\boldsymbol{\varrho}}_t^{(i)\prime})\mathbf{m}_i^* - \mathbf{z}_{t-1}^{(i)T}\mathbf{m}_i^*}{\mathbf{z}_{t-1}^{(i)T}\mathbf{m}_i^*} = \frac{\mathbf{z}_{t-1}^{(i)T}\widehat{\boldsymbol{\varrho}}_t^{(i)\prime}\mathbf{m}_i^*}{\mathbf{z}_{t-1}^{(i)T}\mathbf{m}_i^*} \tag{9.9}$$

where $\varrho_{k_i,t}^{(i)}$ is the rate of change from t to $t+1$ of the vertically hyper-integrated labour necessary for the production of one unit of productive capacity for vertically hyper-integrated sector i $(i = 1, 2, \ldots, m)$, $\mathbf{z}^{(i)T}\mathbf{m}_i^*$.

The rate of change from t to $t+1$ of such ratios is given by:

$$\sigma_{\gamma_{i,t}^n} = \frac{\mathbf{z}_{t-1}^{(i)T}(\varrho'_{i,t}\mathbf{I} - \widehat{\boldsymbol{\varrho}}_t^{(i)\prime})\mathbf{m}_i^*}{\mathbf{z}_{t-1}^{(i)T}(1 - \varrho'_{i,t})\mathbf{m}_i^*} \tag{9.10}$$

or:

$$\sigma_{\gamma_{i,t}^n} = \frac{\varrho'_{i,t} - \varrho'_{k_i,t}}{1 - \varrho'_{i,t}} \tag{9.11}$$

which means that the 'natural' capital output ratio — i.e. the ratio of vertically hyper-integrated labour embodied in one unit of vertically hyper-integrated productive capacity to that embodied in one unit of final output — increases when (vertically hyper-integrated) labour increases in the production of consumption commodities more than in the production of the corresponding units of productive capacity.

The degree of mechanisation of vertically hyper-integrated sector i $(i = 1, 2, \ldots, m)$ is the ratio between the *stock* of capital, evaluated at current prices, and the *flow* of labour employed in the production process:

$$\theta_{i,t}^n = \frac{p_{k_i,t}^n k_{i,t}}{z_{i,t}^* x_{i,t}} = \frac{p_{k_i,t}^n x_{i,t}}{z_{i,t}^* x_{i,t}} = \frac{w_t \mathbf{z}_t^{(i)T}\mathbf{m}_i^*}{z_{i,t}^*} = \frac{w_{t-1}\mathbf{z}_{t-1}^{(i)T}\left(\mathbf{I}(1 + \sigma_{w,t}^{(h)}) - \widehat{\boldsymbol{\varrho}}_t^{(i)\prime}\right)\mathbf{m}_i^*}{z_{i,t-1}^*(1 + \varrho'_{i,t})} \tag{9.12}$$

or:

$$\theta_{i,t}^n = \frac{w_t z_{k_i,t}^*}{z_{i,t}^*} = \frac{w_{t-1} z_{k_i,t-1}^*(1 + \sigma_{w,t}^{(h)} - \varrho'_{k_i,t})}{z_{i,t-1}^*(1 - \varrho'_{i,t})} \tag{9.13}$$

its rate of change through time being:

$$\sigma_{\theta_{i,t}^n} = \frac{\mathbf{z}_{t-1}^{(i)T}\left(\mathbf{I}(\sigma_{w,t}^{(h)} - \varrho'_{i,t}) - \widehat{\boldsymbol{\varrho}}_{k_i,t}^{(i)\prime}\right)\mathbf{m}_i^*}{\mathbf{z}_{t-1}^{(i)T}(1 - \varrho'_{i,t})\mathbf{m}_i^*} \tag{9.14}$$

or

$$\sigma_{\theta_{i,t}^n} = \frac{\sigma_{w,t}^{(h)} - \varrho'_{k_i,t} + \varrho'_{i,t}}{1 - \varrho'_{i,t}} \tag{9.15}$$

Expression (9.15) tells us that the 'natural' degree of mechanisation increases when (vertically hyper-integrated) labour productivity in the production of the units of productive capacity is greater than the sum of labour productivity increases in the production of the corresponding consumption commodity and the rate of growth of the wage rate — that of course is different according to which commodity h ($h = 1, 2, \ldots$) is chosen as the *numéraire* of the price system.

With respect to the dynamics of the 'natural' capital intensity, therefore, here the role played by the movements of the wage rate is therefore apparent. Within the 'natural' economic system, such ratios change because of changes in the quantity of labour *embodied in the units of productive capacity* and because of changes in the wage rate (which of course are influenced by the choice of the *numéraire* too). Therefore, capital intensity and degree of mechanisation change in the same direction, and actually in the very same proportion, only when the wage rate is constant, and therefore the second effect — which is specific to capital/labour ratios — disappears.

In all other cases, their movements through time do not go, in principle, in the same direction. Differences in their trajectories are not predictable, and the capital/labour ratios cannot be taken as indicators of capital intensity.[12]

9.3 The 'standard rate of growth of productivity' and the 'dynamic standard commodity'

As a further characteristic of the 'natural' economic system, Pasinetti (1981, Chapter V, sections 13-14) introduces the 'standard rate of growth of productivity', and hence the 'dynamic standard commodity', which then he uses — among the other things — to define the concept of 'natural' rate of interest. In a few words, the basic original idea is that the 'standard rate of growth of productivity' is a weighted average of the rates of growth of labour productivity in the various sectors, which can also be seen as the rate of growth of productivity of a particular composite commodity — the 'dynamic standard commodity' — which, if used as the *numéraire* of the price system, possesses the remarkable property of keeping the average price level constant through time.

Let us first of all briefly summarise the analytical formulation of the 'standard rate of growth of productivity' originally put forward by Pasinetti (1981), though with the modification of considering discrete, rather than continuous, time.

According to Pasinetti (1981), the 'standard rate of growth of productivity' is a *weighted average* of the rates of growth of productivity in the m vertically hyper-integrated sectors composing the economic system as a whole, the weights

[12] For details about the consequences of doing so, see Pasinetti (1981, Chapter IX, section 7).

being $\lambda_{i,t}$ $(i = 1, 2, \ldots, m)$:

$$\varrho_t^* = \sum_{i=1}^{m} \lambda_{i,t} \varrho'_{i,t} \tag{9.16}$$

The problem is now that of finding the proper weights to compute this average; Pasinetti solves the problem by observing that the addenda entering the macroeconomic condition:

$$\sum_{i=1}^{m} a_{in,t} z_{i,t}^* = 1 \tag{9.17}$$

besides adding up to 1 — which is of course a necessary requirement for them to be the weights to be used to compute a weighted average — represent the *proportion* of total labour required by the each vertically hyper-integrated sector. Hence, they are precisely the weights we are looking for:

$$\varrho_t^* = \sum_{i=1}^{m} \lambda_{i,t} \varrho'_{i,t} = \sum_{i=1}^{m} \left(a_{in,t} z_{i,t}^* \right) \varrho'_{i,t} \tag{9.18}$$

By having a closer look at the λ_i's, by direct examination of the macroeconomic condition, written as:

$$\sum_{i=1}^{m} z_{i,t}^* a_{in,t} = \sum_{i=1}^{m} \lambda_{i,t} = 1 \tag{9.19}$$

we can stress first of all that such weights change themselves through time, and are therefore in principle different from period to period. Moreover, each $\lambda_{i,t}$ can be seen in two ways:

(i) as the proportion of total labour employed by vertically hyper-integrated sector i — also when we are not within the 'natural' economic system; and

(ii) *only within* the 'natural' economic system, as the proportion of the total wages spent for buying consumption commodity i.

Therefore, ϱ_t^* can be seen as the rate of change in the vertically hyper-integrated labour coefficient, from time period $t-1$ to t, of a hypothetical (composite) sector producing a particular *composite commodity* — the 'dynamic standard commodity'. Let us call $z_{s,t}$ such a (composite) labour coefficient (where the subscript s stands for 'standard' commodity).

We can now go back to the beginning of the present section, recalling the main property of the 'dynamic standard commodity': when it is used as the *numéraire* of the price system, the (average) price level remains constant through time. Pasinetti (1981) exploits this property, as already hinted at above, to arrive to the definition of the 'natural' rate of interest,[13] and specifically to express the 'real' and 'nominal' rate of inflation.

It is therefore clear that when Pasinetti refers to the average price level, to be kept constant in terms of the 'dynamic standard commodity' — whose composition, it is worth stressing, changes through time — he is referring to a magnitudes which is related to the *purchasing power* of the average consumer, since its dynamics is at the basis of the idea of *price inflation*.

The purchasing power of the average consumer — or better, of his/her income — depends not only on (relative) prices, but also by the basket of goods he/she wants, or needs, to consume in every specific point in time. That is to say, when we try to compute changes in the individuals' purchasing power, we do not care about the absolute changes of prices, but about the interaction of such changes with the shifts of the composition of final demand. If the price of a certain consumption commodity undergoes a great increase relatively to those of the others, but it is a very small fraction, in the final period, of the actually consumed basket of goods, the effect on the change in households real purchasing power will be very small indeed.

With this idea in mind, we clearly cannot but define the purchasing power of the average consumer — or the average wage earner, which within the 'natural' economic system is precisely the same — at time t as the real value of the wage rate in terms of the specific basket of goods actually consumed, i.e. the composite commodity $[a_{1n,t} \; a_{2n,t} \; \ldots \; a_{mn,t}]$. Saying that the purchasing power of the wage rate is constant through time thus amount at saying that *the real wage rate*, expressed in terms of such a composite commodity, is constant through time.

In terms of whatever *numéraire* we may arbitrarily choose for the price system, in analytical terms such a condition can be written as:

$$\frac{w_t}{\sum_{i=1}^{m} p^*_{i,t} a_{in,t}} = \frac{w_t}{w_t \sum_{i=1}^{m} z^*_{i,t} a_{in,t}} = \frac{1}{\sum_{i=1}^{m} z^*_{i,t} a_{in,t}} = 1, \quad \forall t \tag{9.20}$$

Hence, within the 'natural' economic system, the real purchasing power of the wage rate, defined as we did define it above, is *always* constant, whatever the *numéraire* we choose for the price system.

This is an interesting conclusion, revealing another feature of the 'natural' economic system: in real terms, with respect to the basket of goods actually composing final demand for consumption commodities, the real purchasing power of the wage rate is constant through time.

But yet, up to now we have been talking of the purchasing power of the wage rate, not of the average price level. Consistently with the definition of purchasing

[13] See section 11.

power itself, the average price level for time period t can be defined as the price of the basket of goods actually consumed by the average consumer:

$$\overline{p}_t^* = \sum_{i=1}^{m} p_{i,t}^* a_{in,t} = w_t \sum_{i=1}^{m} z_{i,t}^* a_{in,t} \tag{9.21}$$

We now want to see whether the 'dynamic standard commodity', as defined by Pasinetti (1981), keeps the average price level constant through time when used as the *numéraire* commodity.

As we have seen at the end of the previous section, if we want to use such a commodity as the *numéraire* of the price system, we do not need to know its composition; we simply have to express the wage rate, both in a specific point in time and through time, in terms of it, and inserting the resulting expression into the price system. This would amount at setting $p_{s,t}^* = w_t z_{s,t} = 1$, and therefore:

$$\begin{cases} w_t^{(s)} = \dfrac{1}{z_{s,t}^*} \\ \\ \sigma_{w_t}^{(s)} = \varrho_t^* \end{cases} \tag{9.22}$$

When the 'dynamic standard commodity' is used as the *numéraire* of the price system, any decrease in total (vertically hyper-integrated) labour necessary for its production translates into a proportional increase in the real wage rate. It is important to stress that we can use such a commodity as the *numéraire* even *without knowing its composition.*

Moreover, when the 'dynamic standard commodity' is used as the *numéraire* of the price system, the rate of change of the price of any commodity i ($i = 1, 2, \ldots, m$) is given by:

$$\sigma_{p_{i,t}}^{(s)} = \varrho_t^* - \varrho_{i,t}' \tag{9.23}$$

i.e. by the difference between the 'standard rate of growth of productivity' and the rate of change of the corresponding vertically hyper integrated labour coefficient. This means that the 'natural' price of a commodity increases when the vertically hyper-integrated labour productivity is smaller than the (weighted) average, and decreases when it is higher.

Let us now go back to expression (9.21), and compute its changes through time when the price of the 'dynamic standard commodity' is the *numéraire* of the price system:

$$\overline{p}_t^* - \overline{p}_{t-1}^* = w_{t-1}(1+\varrho_t^*)\sum_{i=1}^{m} z_{i,t-1}^* a_{in,t-1}(1-\varrho_{i,t}'+r_{i,t}) - w_{t-1}\sum_{i=1}^{m} z_{i,t-1}^* a_{in,t-1} =$$

$$= w_{t-1}\sum_{i=1}^{m} z_{i,t-1}^* a_{in,t-1}(\varrho_t^* - \varrho_{i,t}' + r_{i,t})$$

In order for this change to be equal to zero, i.e. for the average price level to be constant through time, the following condition should be holding:

$$w_{t-1}\sum_{i=1}^{m} z_{i,t-1}^* a_{in,t-1}(\varrho_t^* - \varrho_{i,t}' + r_{i,t}) = 0$$

i.e.:

$$\varrho_t^* = \sum_{i=1}^{m} z_{i,t-1}^* a_{in,t-1}(\varrho_{i,t}' - r_{i,t}) = \sum_{i=1}^{m} \lambda_{i,t}(\varrho_{i,t}' - r_{i,t}) \tag{9.24}$$

We immediately notice the difference with respect to Pasinetti’s (1981) original formulation: the ‘standard rate of growth of productivity’, as it emerges from our formulation, is a weighted average — the weights being the $\lambda_{i,t}$s — not of the rates of growth of labour productivity in the m vertically hyper-integrated sectors, but of the difference between such rates and the rate of growth of final per-capita demand for the corresponding consumption commodity.

This conclusion is the quite obvious consequence of what stated above: changes in the purchasing power, and thus in the average price level, do depend not only on the variations of (relative) prices, but also, and in a very relevant way, for the variations in the composition of final demand. Therefore, a ‘standard rate of growth of productivity’ has to take into account both determinants.

I may add that this reformulation allows us to overcome a further difficulty connected with Pasinetti’s (1981) original definition of the ‘standard rate of growth of productivity’ and thus of the ‘dynamic standard commodity’. The latter do make sense *only* within the natural economic system, where prices and labour values do coincide, the weights $\lambda_{i,t}$s do have the double meaning mentioned above, and therefore the rate of change of the real purchasing power do coincide with the rate of change of labour productivity in the vertically hyper-integrated sector producing the *numéraire* (composite) commodity. More specifically, Pasinetti follows the following reasoning: prices change because of changes, and exactly in the same proportion as, labour requirements change. Therefore, if we choose the weighted average of the rates of change of labour productivity in the different vertically hyper-integrated sectors obtained by using the $\lambda_{i,t}$s as the weights — which are both the proportion of sectoral to total labour, but also of the average per-capita income spent for buying consumption commodity i with respect to the total — as the *numéraire* of the price system, one half of the prices will increase, and the other half decrease, on *weighted* average, the positive and the negative changes therefore canceling out. Clearly, this reasoning only holds *within* the 'natural' economic system. But as soon as we consider prices different from the natural ones, their changes are not caused by and exactly in the same proportion as changes in vertically hyper-integrated labour requirements, but are caused by and exactly in the same proportion as changes in the corresponding labour equivalents. Changes in distributive variables, and not only in labour productivity, come to affect the average price level.

On the contrary, the reformulation given in this section do make sense both within and outside the 'natural' economic system, since it implies the definition of the 'dynamic standard commodity' as a composite commodity that, whatever the rates of profit, if used as the *numéraire* of the price system keeps, *by definition*, the average price level constant through time. The 'standard rate of growth of productivity', outside the natural economic system, is not to be seen as the rate of change of the real purchasing power of the wage rate when the 'dynamic standard commodity' is adopted as the *numéraire* of the price system: such a rate of change is given by the associated reduction in the corresponding *labour equivalent*;[14] anyway, it is precisely the rate of change of labour productivity in the associated, hypothetical (composite) vertically hyper-integrated sector producing the 'dynamic standard commodity' itself.

10 The 'natural' price system through time: choice of the *numéraire* and rate of inflation

All the magnitudes considered, and analysed, so far are *real* magnitudes, i.e. magnitudes whose value is expressed in terms of some physical commodity, or composite commodity, or of labour. A *conventional unit of account*, such as paper money, needs to be introduced in order to be able of treating *nominal* magnitudes; at the

[14]To be more precise, this rate of change, call it $\sigma_{w,t}^{(s)}$, where the superscript s stands for *standard* commodity, is given by:

$$\sigma_{w,t}^{(s)} = \frac{\mathbf{z}_{t-1}^{(e)T}\left(\widehat{\boldsymbol{\varrho}}_{t-1}^{(e)\prime} - \widehat{\mathbf{r}}_{t-1}\right)\mathbf{a}_{in,t-1}}{\mathbf{z}_{t-1}^{(e)T}\mathbf{a}_{in,t-1}}$$

same time, a way of establishing a relation between nominal and real magnitudes is necessary in order to attach a concrete meaning to the former.

When paper money is used as the basis for the price system, expressing commodity prices in terms of it requires to close the same two degrees of freedom left open in the (relative) commodity price system, exactly in the same way as we did when choosing any physical commodity as the *numéraire*. that is to say, we have to arbitrarily fix the initial value, and the rate of change through time, of the wage rate in terms of money, by setting:

$$\begin{cases} w_0 = w_0^{(M)} \\ \sigma_{w_t} = \sigma_{w_t}^{(M)} \end{cases} \tag{10.1}$$

As Pasinetti (1981, p. 162) points out, not only $w_0^{(M)}$, but also $\sigma_{w_t}^{(M)}$ can be arbitrarily fixed at *any* level; nonetheless, such a rate of change would be a purely nominal one, as long as we do not give it a physical content by establishing a relation between it and real magnitudes.

Let us now adopt such a conventional unit of account as the *numéraire* of the price system, and compute the rate of change through time of the *nominal* (average) price level:

$$\begin{aligned} \sigma_{\overline{p}_t^{(M)*}} &= \frac{w_{t-1}^{(M)} \left((1+\sigma_{w_t}^{(M)}) \sum_{i=1}^m \lambda_{i,t-1}(1+r_{i,t}-\varrho'_{i,t}) - \sum_{i=1}^m \lambda_{i,t-1} \right)}{w_{t-1}^{(M)} \sum_{i=1}^m \lambda_{i,t-1}} = \\ &= \sigma_{w_t}^{(M)} - \sum_{i=1}^m \lambda_{i,t-1}(\varrho'_{i,t} - r_{i,t}) = \sigma_{w_t}^{(M)} - \varrho_t^* \end{aligned} \tag{10.2}$$

It immediately appears from expression (10.2) that an *ideal situation* of *nominal* (average) *price stability* — that can be taken as a reference point — would occur in the special case in which $\sigma_{w_t}^{(M)} = \varrho_t^*$.

Any time that $\sigma_{w_t}^{(M)} > \varrho_t^*$, the general nominal price level would be increasing, and therefore we would be in a situation of *price inflation*; by contrast, any time that $\sigma_{w_t}^{(M)} < \varrho_t^*$ the nominal price level is decreasing, and we are in a situation of *price deflation*.

Accordingly, we can therefore define the *general level of price inflation* ($\sigma_{A,t}^{(M)}$) as:

$$\sigma_{A,t}^{(M)} = \sigma_{w_t}^{(M)} - \varrho_t^*$$

and hence write the rate of change through time of the ‘natural’ price of any (consumption) commodity i ($i = 1, 2, \ldots, m$) as:

$$\sigma_{p_{i,t}^*}^{(M)} = \sigma_{w_t}^{(M)} - \varrho'_{i,t} \equiv \left(\sigma_{w_t}^{(M)} - \varrho_t^* \right) + \left(\varrho_t^* - \varrho'_{i,t} \right) \equiv \sigma_{A,t}^{(M)} + \left(\varrho_t^* - \varrho'_{i,t} \right) \tag{10.3}$$

As it is clear from expression (10.3), the rate of change through time of the price of a commodity is made up by two components: the general level of price inflation, affecting *all* prices, over and above the specific changes in labour productivity; and a *sector-specific* component, namely $(\varrho_t^* - \varrho'_{i,t})$, affecting only the price of the commodity produced in the corresponding vertically hyper-integrated sector.

11 The 'natural' rate of interest

We are now in the position to move — step by step, as Pasinetti (1981) does — towards the definition of a 'natural' rate of interest. Doing so implies introducing into the picture a whole set of assets and liabilities, i.e. of debt and credit relations to be stipulated between individuals, or group of individuals, and that cancel out at the aggregate level.

As Pasinetti notices,

> [t]he immediate consequence of the introduction of financial assets and liabilities into our analysis is that it becomes no longer indifferent which commodity is chosen as the *numéraire* of the price system [...]. For, the choice of the *numéraire* ties down all debts and credit to being constant through time in terms of the particular commodity chosen as the *numéraire*; while, at the same time, all 'natural' prices are changing in terms of that *numéraire*. (Pasinetti 1981, p. 158)

To see how this happens, let us first suppose to choose the price of commodity h as the *numéraire* of the price system. Be $i^{(h)}$ the interest rate stipulated between the borrower and the lender on the amount of the loan; consider a loan stipulated at time t — with a *zero rate of interest* — and expiring at time $t+1$.

Since h is the *numéraire* commodity, the loan is stipulated in terms of it, which means that a certain amount of purchasing power, in terms of commodity h, as been lent at time t, and *exactly the same amount* of purchasing power in terms of the *numéraire* must be given back at time $t+1$, at expiration of the loan. Clearly, such a purchasing power is kept constant through time in terms of the *numéraire* commodity, but will not be constant also in terms of all other commodities (or composite commodities).

In particular, let us consider commodity i $(i = 1, 2, \ldots, m)$. As usual, $\varrho'_{i,t}$ is the rate of change of productivity in vertically hyper-integrated sector i, and $\varrho'_{h,t} - \varrho'_{i,t}$ is the rate of change of the ('natural') price of commodity i itself when the *numéraire* is the price of commodity h. Be $q_{i,t}^{(h)}$ the number of units of commodity i that could be bought, with the amount of the loan, at time t. This means that the original value of the loan, at current prices, in terms of commodity i is given by:

$$p_{i,t}^{(h)*}q_{i,t}^{(h)} = p_{i,t}^{(h)*}(1+\varrho'_{h,t}-\varrho'_{i,t})(1-\varrho'_{h,t}+\varrho'_{i,t})q_{i,t}^{(h)} \equiv p_{i,t+1}^{(h)*}q_{i,t+1}^{(h)}$$

and therefore:

$$q_{i,t+1}^{(h)} = (1-\varrho'_{h,t}+\varrho'_{i,t})q_{i,t}^{(h)} \tag{11.1}$$

Whenever $\varrho'_{i,t} > \varrho'_{h,t}$, the purchasing power in terms of commodity i is greater at the expiration of the loan than at time t: the loan has undergone a *revaluation*, in terms of commodity i, at the rate $\varrho'_{i,t}-\varrho'_{h,t}$. On the contrary, whenever $\varrho'_{i,t} < \varrho'_{h,t}$, the purchasing power has decreased from t to $t+1$, and therefore the loan, in terms of commodity j, has undergone a *devaluation* at the rate $\varrho'_{h,t}-\varrho'_{i,t}$.

Similarly, if the amount of the loan at time t allowed to command $x_n^{(h)}$ hours of labour, its real value, in terms of labour, at time t is given by:

$$w_t^{(h)}x_{n,t}^{(h)} = w_t^{(h)}(1+\varrho'_{h,t})(1-\varrho'_{h,t})x_{n,t}^{(h)} \equiv w_{t+1}^{(h)}x_{n,t+1}^{(h)}$$

and therefore

$$x_{n,t+1}^{(h)} = (1-\varrho'_{h,t})x_{n,t}^{(h)}$$

Whenever $\varrho'_{h,t} > 0$ the loan undergoes a *revaluation* in terms of labour at the rate $\varrho'_{h,t}$; whenever $\varrho'_{h,t} < 0$ the loan undergoes a *devaluation* at the rate $-\varrho'_{h,t}$.

Let us now suppose that the wage rate is chosen as the *numéraire* of the price system, and that the conditions of the loan are exactly the same as those of the previous case. Consider again any commodity $(i = 1, 2, \ldots, m)$, recalling that in this case the rate of change of any price with respect to to labour is given by $-\varrho'_{i,t}$. The initial value of the loan is therefore given by:

$$p_{i,t}^{(w)*}q_{i,t}^{(w)} = p_{i,t}^{(w)*}(1-\varrho'_{i,t})(1+\varrho'_{i,t})q_{i,t}^{(w)} \equiv p_{i,t+1}^{(w)*}q_{i,t+1}^{(w)} \tag{11.2}$$

and therefore:

$$q_{i,t+1}^{(w)} = (1+\varrho'_{i,t})q_{i,t}^{(w)} \tag{11.3}$$

Hence, whenever the rate of change of productivity in vertically hyper-integrated sector i is positive, the loan undergoes a *revaluation* at the rate $\varrho'_{i,t}$; while, whenever it is negative, it undergoes a *devaluation* at the rate $-\varrho'_{i,t}$.

In a few words,

> [...] the existence of financial assets and liabilities, when coupled with a structural dynamics of natural prices, implies the existence, not of one rate of interest, but a whole series of rates of interest. More precisely, it implies the existence of a particular own-rate of interest for each commodity.
>
> (Pasinetti 1981, p. 159)

To be more precise, we have *many* series of own-rates of interest, one for each *numéraire* commodity, or composite commodity, we might decide to choose for the price system.

Such own-rates of interest are implied by the very *structural dynamics* of commodity 'natural' prices, and are therefore *always* present, on all assets and liabilities, over and above whatever rate of interest that might be stipulated by the lender and the borrower on the loan itself, which adds up to them. Going back to the example in which h is the *numéraire* commodity, if the rate of interest on the loan has been decided to be $i^{(h)} \neq 0$, then the series of own-rates of interest would be given by:

$$\begin{cases} \left(i^{(h)} + \varrho'_{i,t} - \varrho'_{h,t}\right) & \text{for commodity } i, \quad i = 1, 2, \ldots, m \\ \left(i^{(h)} + \varrho'_{i,t}\right) & \text{for labour} \end{cases} \tag{11.4}$$

Clearly, these are all *real* own-rates of interest; if we want to talk about *nominal* ones, we have to introduce paper money — or any other conventional unit of account — as the basis of the price system.

If all assets and liabilities are all stipulated in terms of paper money, with a nominal interest rate equal to $i^{(M)}$, then the series of own-rates of interest would be:

$$\begin{cases} \left(i^{(M)} - \sigma^{(M)}_{A,t} - (\varrho^*_t - \varrho'_{i,t})\right) & \text{for commodity } i, \quad i = 1, 2, \ldots, m \\ \left(i^{(M)} - \sigma^{(M)}_{A,t} - \varrho^*_t\right) & \text{for labour} \end{cases} \tag{11.5}$$

We can however consider a *special case*, i.e. the nominal own-rate of interest for the 'dynamic standard commodity':

$$\left(i^{(M)} - \sigma^{(M)}_{A,t} - (\varrho^*_t - \varrho^*_t)\right) = \left(i^{(M)} - \sigma^{(M)}_{A,t}\right) \tag{11.6}$$

> which, as we may see, is simply the difference between the money rate of interest and the rate of inflation. [...] It represents a sort of average 'real' rate of interest for the economic system as a whole. We may call it the 'standard' real rate of interest.
>
> (Pasinetti 1981, p. 165)

And what about the 'natural' rate of interest? Pasinetti (1981) states the problem in a very clear and effective way:

> A *whole structure* of rates of interest exists in any case, whatever the actual 'nominal' rate of interest (even if it were fixed at zero) and whatever the *numéraire* chosen as the basis of the price system. In other words, a whole structure of own-rates of interest — all of them 'real' rates of interest — is unavoidably inherent in the structural dynamics of relative prices.
>
> [...]
>
> [T]he problem to be solved — within the present theoretical framework, may be stated in the following manner. From the infinite number of possible levels of the actual rate of interest (and by implication of the structure of own-rates of interest), is there a particular one that may be called the 'natural' level of the rate of interest? (And, by implication is there a 'natural' level of the whole structure of the own-rates of interest?)
>
> (Pasinetti 1981, p. 166)

The answer Pasinetti gives to this question is as straightforward as the question itself:

> In an economic system in which all contributions to, and and benefits from, the production process are regulated on the basis of quantities of labour, the 'natural' rate of interest cannot but be a zero rate of interest in terms of labour.
>
> (Pasinetti 1981, p. 166)

Or, as he concludes at the end of the chapter:

> [w]e may well say that income is distributed according to a 'labour principle of income distribution'.
>
> (Pasinetti 1981, p. 169)

Or again, to put in another way, the main characteristic of the 'natural' economic system is the equivalence of labour embodied and labour commanded. This must hold not only within a single period of time, but also *through time*. When labour productivity increases, the wage rate increases proportionally. This means that — as we have shown above — if a certain amount of purchasing power can command, at time t an amount of labour equal to $x_{n,t}^{(h)}$ (h being whatever *numéraire* we have chosen for the price system), at time $t+1$ it will be able to command only a quantity $x_{n,t+1}^{(h)} = x_{n,t}^{(h)}(1 - \varrho'_{h,t})$. In order to restore the equivalence between labour embodied and commanded, therefore, the lent/borrowed amount of purchasing power must be 'augmented' through an interest rate — the *'natural' interest rate* — equal to ϱ'_h, that is equal to *the rate of growth of the wage rate, in terms of whatever numéraire is actually the basis of the price system.*

In short, in order to preserve *through time* the equivalence between labour embodied and labour commanded — i.e., in order to preserve the main characteristic of the 'natural' price system — the particular level of the rate of interest that may be called the 'natural' rate of interest is given by:

$$\begin{cases} \sigma_{w_t}^{(h)} & \text{if commodity } h \text{ is the } \textit{numéraire} \text{ commodity} \\ \sigma_{w_t}^{(s)} & \text{if the `dynamic standard commodity' is the } \textit{numéraire} \text{ commodity} \\ \sigma_{w_t}^{(M)} & \text{if paper money is the } \textit{numéraire} \text{ commodity} \\ \sigma_{w_t}^{(w)} = 0 & \text{if labour is the } \textit{numéraire} \text{ commodity} \end{cases}$$

12 Conclusions

As stated in the Introduction, the main task of the present paper was that of introducing in Pasinetti's (1981) and Pasinetti's (1988) original formulations — besides the complete description of the technique in use, already introduced in Garbellini (2010a) — discrete, rather than continuous, time, and thus non-steady rates of change of both sectoral (average) per-capita demand for consumption commodities and sectoral labour productivities. As explained in Garbellini (2010b, section 3.4), this aimed at increasing the realism of the whole formulation, at the same time making it more suitable for empirical analyses and allowing to see more in detail the dynamics of the main economic magnitudes of interest.

In particular, the introduction of the most general description of the technique in use in each time period — i.e., of the whole set of inter-industry relations — allowed to deepen, with respect to Pasinetti (1981), the analysis of the dynamics of commodity relative prices and of labour productivities.

As to the relative price system, the simplifying assumptions made in Pasinetti (1981) on the relations between the industries producing inputs and outputs were such as to make the price of each final consumption commodity i ($i = 1, 2, \ldots, m$) depend on the quantity of vertically hyper-integrated labour used up for its production and on the price of the corresponding intermediate commodity k_i only. The price of each unit of vertically hyper-integrated productive capacity k_i, in its turn, depends on the cost of vertically hyper-integrated labour employed for its production — and, in the more complex formulation (see Garbellini 2010a, appendix A.4) also on its own price — only. In fact, inter-industry relations within vertically hyper-integrated sectors are reduced to the exchanges between each industry producing a final consumption commodity and the one producing the corresponding unit of productive capacity.

However, when all inter-industry relations are re-introduced into the picture, the price of each consumption commodity depends on the price of *all* the others, in the case of both final consumption commodities and units of vertically hyper-integrated productive capacity. Value creation thus depends on a very complex network of relations, reproduced to a sector-specific extent within *each* vertically hyper-integrated sector. In fact, all (basic commodities producing) industries enter all vertically hyper-integrated sectors.

This is also reflected in the analysis of labour productivities. In Pasinetti (1981), each rate of change of vertically hyper-integrated labour productivity $\varrho'_{i,t}$ $(i = 1, 2, \ldots, m)$ is the weighted average of changes in direct labour productivity in the industry producing final consumption commodity i, i.e. $\varrho_{i,t}$, and in the industry producing the corresponding intermediate commodity k_i, i.e. $\varrho_{k_i,t}$, the weights being the relative importance of the two industries in constituting the whole sector. But as soon as all inter-industry relations are concerned, and vertically hyper-integrated sectors are considered, besides having the vertically hyper-integrated component too — i.e. besides considering labour productivity in the production of additional productive capacity — the very changes in the (vertically hyper-integrated) labour productivity for the production of intermediate commodities $\varrho_{k_i,t}$ become weighted averages of *all* the $\varrho_{i,t}$s, since productive capacities made up not by one single commodity, but by *all* the (basic) commodities produced in the economic system as a whole. We can therefore think of many decompositions of vertically hyper-integrated labour productivities, to analyse the production of consumption commodities, of the different units of productive capacities, of their own production, and so on, according to the specific 'layer' of the productive structure, and of the structure of the single vertically hyper-integrated sectors, we want to analyse.

Differently from the case of the relative price system, the physical quantity side of the production process, both in a single period of time and through time, is not affected by the introduction of the more complete description of the technique in use in every points in time. The fact of using productive capacities as units of measurement for intermediate commodities prevents it from complicating the analysis, since it involves more complex changes in the *composition* of such productive capacities, which is a problem, as it has been stated in section 7, at page 16, that can be kept separated from that of the analysis of capital accumulation, precisely thanks to the adoption of such a unit of measurement.

Hence, the introduction of the complete inter-industry relations mainly affects the *value creation side* of the production process.

On the contrary, the dynamics of physical quantities — specifically, the dynamics of the 'equilibrium' quantities of the unit of productive capacity to be produced in each vertically hyper-integrated sector, and therefore capital accumulation — become much more complicated with the introduction of discrete, rather than continuous, time and thus of non-steady rates of change of demand

and labour requirements. In particular, the rate of change from time period t to time period $t+1$ of the number of unit of productive capacity in capital stock available at the beginning of the production process is given by the sum of the rate of change of demand from $t-1$ to t — and therefore on the evolution of such a stock through all the past time periods — and from t to $t+1$.[15] This is a consequence of the fact that the very dynamics of capital accumulation given by *capital accumulation conditions* (4.14) involve the variation of demand for consumption commodities from the currently considered to the following time period, i.e. c_{t+1}. Though being quite an intuitive result, since capital accumulation's task is that of providing the economic system, in the *future* time periods, with an adequate stock of productive capacity, it is not possible to singling it out using continuous time — a choice that, as explained in Garbellini (2010b, section 3.4), is made by Pasinetti for a matter of analytical convenience; introducing non-steady rates of change of the main observable, exogenous variables would make the convenience of making such an assumption disappear, therefore making it more reasonable to use discrete time.

The introduction of more complex movements through time of demand and labour requirements, thus, mainly affects the *physical quantity side* of the production process.

A further final remark that I would like to make here concerns the reformulation, with respect to Pasinetti (1981), of what he calls the 'standard rate of growth of productivity', and therefore of the 'dynamic standard commodity'. As stated in section 9.3, the main property of such a composite commodity is that, when used as the *numéraire* of the price system, it makes the average price level constant through time. Its definition thus is the answer Pasinetti tries to give to Ricardo's problem of finding an 'invariable standard' of value. However, Pasinetti's (1981) does, in my opinion, suffer from two shortcomings.

The first shortcoming is that, in defining the 'standard rate of growth of productivity', Pasinetti (1981) does not take into account the role played by the rates of change of (average) per-capita demand on the composition of the basket of goods defining, in each time period, the real purchasing power of the average consumer, and therefore on the very definition of average price level.

The second one is that such definition is valid only *within the natural economic system*; as explained at the end of section 9.3, when the rate(s) of profit are not the 'natural' ones, and therefore prices are different from (vertically hyper-integrated) labour values, the logic adopted by Pasinetti (1981) in defining the 'standard rate of growth of productivity' does not work anymore. By reformulating the 'standard rate of growth of productivity', and thus the 'dynamic standard commodity', as in

[15]See equations (7.1) and (7.3).

expression (9.24), both this shortcomings, in fact closely connected to each other, have been overcome.

To conclude, what emerges from this attempt at extending to 'production in the long run' the generalisation of Pasinetti's (1981) framework developed in Garbellini (2010a) is that the analysis of capital accumulation, and in general of the structural dynamics of the physical quantity side of the economic system, is not complicated by the introduction of the more complete description of the technique in use; Pasinetti's contention — i.e. that the adoption of vertically hyper-integrated sectors as the object of the analysis, and of the units of vertically hyper-integrated productive capacity as the units of measurement for intermediate commodities, does make it possible to overcome the difficulties in the analysis of capital accumulation entailed by the presence of growth and technical progress — is fully confirmed. The analysis can however be enriched, and its degree of realism improved, by adopting discrete time and allowing for non-steady rates of growth of per-capita (average) demand for consumption goods and of labour productivities.

On the other hand, considering the network of inter-industry relations in all its complexity allows us to make the whole framework suitable for empirical applications and institutional analysis; that is to say, it allows us to take full advantage of all the reflections, suggestions and intuition put forward by Pasinetti's (1981) book, and to catch up in the most fruitful way the intellectual heritage of Piero Sraffa, that today, fifty years after the publication of his masterpiece, is still awaiting to exploit all its potentialities.

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