Annals of Operations Research
https://doi.org/10.1007/s10479-023-05687-7

ORIGINAL RESEARCH

Optimal cashback in a cooperative framework for peer-to-peer insurance coverages

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Received: 15 September 2022 / Accepted: 26 October 2023

Abstract

The challenges of using technology in the insurance field are opening new horizons for developing and distributing innovative products. Among these, peer-to-peer insurance schemes attract the interest of policyholders and insurance companies. Different types of peer-to-peer insurance have been introduced, from pure models to hybrid ones, as in the case of the broker model. In this paper, we focus on the broker model, where the groups of peers are formed by an insurance broker according to similar risk characteristics. The participants in the network pay an initial contribution defined by a cooperative rule that must be transparent and shared. A part or the whole of the collected contributions is set aside in a common fund. At the end of the year, if the common fund is sufficient to pay for the claims, the members obtain the excess over-retained premiums that is shared according to a capital allocation rule. We propose a cashback distribution mechanism based on the participant's marginal contribution to the risk, framing the issue in a cooperative game and applying the concept of Shapley value to define an optimal allocation rule of the remaining capital. A numerical application based on a portfolio of motor third-party liability policies is developed to show how the model works.

Keywords Peer-to-peer insurance · Cashback · Capital allocation · Shapley value

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Published online: 29 November 2023

1 Introduction

A variety of breakthrough technologies are providing a fundamental transformation of the insurance industry. Internet of Things, advanced analytics, telematics and the global positioning system, digital platforms, blockchain are important examples of new ways to measure, control, and price risk, to engage with customers, to reduce costs and improve efficiency, and to expand insurability. Emerging technologies provide opportunities for traditional insurers to reinvent themselves but also force them to respond to the competitive challenge posed by new digital players through low-cost technology platforms and different business models.

In this framework, several insurance start-ups are styling themselves as “peer-to-peer”(P2P) insurance being based on a model that differs from traditional insurance. Although there is no common terminological understanding or clarity of the meaning of P2P insurance, the main idea is to offer new forms of insurance coverages based on the P2P insurance mechanism that recovers the ancestral compensation mechanism consisting in using the contributions of the many to balance the misfortunes of the few. In the European Union, P2P has been defined in EIOPA (2019) as a risk-sharing digital network where a group of individuals with mutual interests pool their “premiums”together to insure against risk and to share the risk among them, and where profits are commonly redistributed in case of good claims experience. In the United States, the National Association of Insurance Commissioners (NAIC) has described P2P as an innovation that allows policyholders to pool their capital, self-organize, and self-administer their own insurance. In addition, the core idea has been described in NAIC (2017) as a set of like-minded people with mutual interests group their insurance policies together introducing a sense of control, trust, and transparency while at the same time reducing costs.

Several firms have tried to implement this approach in the insurance sector, with different degrees of success. Indeed, according to a recent survey among the national supervisory authorities of the European Union, the estimated size of the P2P business was very limited and three different models have been developed in practice (see EIOPA, 2019). Following the classification proposed by Rego and Carvalho (2019), the dominant type is the P2P broker model. In this case, an insurance broker creates and manages the coverage. Participants form online groups and a part of the insurance premiums goes into a common fund, while the remaining part is paid to an insurance (or reinsurance) company. Participants share the first layer of cumulative loss, while the higher layers are transferred to an insurance company. If the pool is insufficient to pay for the claims of its members, the insurer pays the excess. Conversely, if the pool is profitable, the excess is distributed to the participants.

A second possibility is represented by the so-called carrier model, which embeds the broker model into a regular insurance policy including participation in the surplus and offering a digital customer relationship. In this case, the mechanism takes advantage of the insurance sector’s risk-bearing capacity and expertise in claim settlement procedures.

The third type of P2P insurance has been developed entirely outside the traditional insurance sector utilizing a self-governing model. Such organizations are made up of self-governing user communities consisting of peers who collectively manage all insurance functions. Communities can be created based on peers’ likeness or insured object (for further details, see, e.g., Denuit & Robert, 2021).

Despite the growth of these innovative solutions, there is still little clarity from a regulatory point of view and a lack of legal definition of these schemes. Indeed, several critical points have been stressed in the literature. In particular, Oryza and Verna (2015) argued that the P2P definition is questionable in both carrier and broker models: passive pools of customers are somewhat affected by the claims experience of other customers in the pool, which is managed by someone else. A recent essay showed that the P2P broker model could result detrimental to customers, in contrast with competitiveness and avoided in the case of compulsory insurance coverage (Marano, 2019). Also, the legal definition of the contract that enables participants to reciprocally insure is still a point unsolved.

Focusing on the technical aspects of the P2P model, several papers have studied alternative allocation rules to split the total loss of the pool among the P2P participants. Transparent and intuitive risk allocation rules have been provided in the literature. In particular, Denuit and Dhaene (2012) propose the conditional mean risk allocation rule, according to which each participant contributes to the conditional expectation of the individual loss brought to the pool, given the total loss of the pool. The advantageous features of the conditional mean risk rule have been investigated in Denuit (2019) and Denuit and Robert (2020). Denuit and Dhaene (2012) show that it is Pareto-optimal for all risk-averse utility maximizers under the hypothesis of co-monotonic structural losses. Abdikerimova and Feng (2020) propose an ex-post contribution rule based on a multivariate extension of the equivalence insurance principle. Indeed, participants are very often asked to pay an initial contribution to enter the scheme. About this, Denuit and Robert (2021a) consider an entry price to participate in a P2P scheme computed according to the classical insurance premium principle, so that it depends on the expected distribution of the single loss and not on that of the total loss. A systematic treatment of different risk-sharing rules for insurance losses is provided in Denuit et al. (2022), including also the conditional mean risk allocation rule.

Feng et al. (2023) derive the optimal risk sharing pooling mechanism by minimizing the sum of the post-pooling variances of the individual agents. Feng et al. (2022) deepen a wide range of risk-sharing schemes within decentralized insurance. Using a general framework, they show how stochastic ordering can be employed to compare existing models and to construct new decentralized insurance schemes. Charpentier et al. (2021) develop a framework for P2P insurance products explicitly based on networks rather than individual characteristics. The optimal designs for non-linear contracts is also studied.

Other scholars outlined that empirical researches should investigate if the lower premiums depend on a lower risk due to this mutual control of the insured persons (see, e.g., Gomber et al., 2017). Clemente and Marano (2020) showed with a simple actuarial model that the convenience of the P2P broker model to the insured is doubtful and pointed out a lack of transparency in the definition of the cashback allocation rule. The definition of the ex-post residual capital allocation policy is indeed a fundamental aspect that needs to be investigated.

This paper tries to solve this issue starting from a classical ex-ante contribution where an initial contribution is paid by the members, from which a final surplus or deficit is generated, and providing an ex-post cashback distribution mechanism based on the participant's marginal contribution to the risk of the P2P scheme. The main advantage of our proposal is to relate the amount of the cashback to the risk borne by each participant overcoming the assumption of an equal cashback for all the members of the pool. Additionally, focusing on a broker scheme, the distribution mechanisms apply in the case of either positive or negative residual amounts. If the residual amount in the pool is positive, it will be distributed as a customized cashback according to a specific rule, while if negative, it represents an additional sum to be paid by participants to share unexpected losses.

The optimal value of the residual capital to be distributed is here obtained by imposing that all participants have an equal marginal contribution to the risk. In the actuarial literature, the first application of the marginal contribution in risk load problems has been proposed by Mango (1998), which used the Shapley value and the game theory for the case of property catastrophic risk. Following this approach, we assess the marginal contribution of each participant to the P2P scheme through the Shapley value and we obtain the optimal value of cashback by setting constant the Shapley value. It is noteworthy that the relation between peer-to-peer and game theory has been explored in Chen et al. (2017). However, our proposal, focusing on the Shapley value for the identification of the cashback rules, provides a different allocation rule with respect to Chen et al. (2017).

A numerical application is further developed considering a portfolio of motor third-party liability policies. The distribution of the optimal shares has been obtained under different scenarios in terms of frequency and average costs. The effect of correlation and diversification between policyholders has been also tested.

The remainder of the paper proceeds as follows. In Sect. 2 we describe the model. In particular, we provide the general framework, we recall the Shapley value, and define the risk allocation criterion in the P2P scheme. Then, we obtain the residual capital distribution rule. In Sect. 3, we illustrate the numerical application based on a portfolio of motor third-party liability coverages. Conclusions follow.

2 The model

Let n be the number of participants to the P2P scheme. Each participant i faces a loss represented by a non negative random variable L_i, such that: $L_i \sim F_{L_i}(l)$, with $F_{L_i}(l)$ cumulative distribution function (c.d.f.) with finite mean and variance.
Let X be the random variable that represents the total loss of the network of participants, defined as follows:

$$X = \sum_{i=1}^{n} L_i$$

It is distributed as $X \sim F_X(x)$. Notice that we consider a general definition of the random variables X and L_i. These definitions are easily applicable to well-known models used in non-life insurance (see, e.g., Daykin et al., 1994 and Kaas et al., 2008)

In the pure self-organizing P2P scheme, members enter without paying any initial commission and contribute ex-post to the total losses realized by the group according to a cooperative risk-sharing mechanism defined a priori. This is the case of the Conditional Mean Rule (CMR) proposed by Denuit (2019), according to which each participant i must contribute to the expected value of the risk brought to the pool, given the fact that the total loss of the pool X is equal to x:

$$h_i^{CMR}(x) = E(L_i|X). \tag{1}$$

where $h_i^{CMR}(x) > 0$ is the monetary contribution of the participant i according to an ex-post CMR. We highlight that a risk-sharing rule has to be defined and acknowledged by all the members at the issue of the contract. However, some of them require an ex-post contribution, as in the case of CMR, and others an initial contribution. Notice that, from 1, we have:

$$\sum_i h_i^{CMR}(x) = \sum_i E[L_i|X] = x \tag{2}$$

Then the sum of contribution exactly matches the realized loss x. Since $E[E(L_i|X)] = E(L_i)$, the sum of contributions is fair.

While the literature has traditionally focused on an ex-post contribution scheme and, in particular, on the CMR as presented in Eqs. 1 and 2, in this work we introduce a different approach, typical of non-pure P2P and broker schemes, where contributions are paid ex-ante. Specifically, in the so-called "broker model", the groups of peers are formed mainly by an insurance broker according to similar risk characteristics and insurance needs. The participants in the network pay an initial contribution defined on a cooperative rule that must be transparent and shared. A part or the whole of the collected contributions is set aside into a common fund, and eventually the other is transferred to a reinsurance company. If the common fund is sufficient to pay for the claims, then the members obtain the excess over-retained premiums, which is shared according to a capital allocation rule. In this paper, we focus on this model based on an ex-ante mechanism where a premium, or entry price, is paid by the policyholder at the issue of the contract, and the whole of the contributions are collected in the common fund.

Let $h_i^{EA}(X) > 0$ be the monetary contribution of the participant i according to an ex-ante risk-sharing rule. To enter the scheme, a generic participant i pays a sum equal to

$$h_i^{EA}(X) = E[E(L_i|X)] = E(L_i), \tag{3}$$

and consequently the sum of the ex-ante total contributions is equal to $E(X)$. However, ex-post, the realized loss may be different from its expected value: for this reason it makes sense to consider an ex-ante safety loading and/or a possible ex-post cashback. We highlight that Eqs. 1 and 2, referring to an ex-post conditional mean rule, always imply that there will be no distribution of surplus from the P2P scheme. The total contributions will be exactly sufficient to cover the realized total loss x. While according to our ex-ante contribution rule (expressed in Eq. 3), it is uncertain whether initial contributions will be exactly sufficient to cover the realized loss, and, therefore, it might produce a final surplus or deficit.

The collected contributions are used to face the total loss experienced by the group. If any amount remains, it is distributed to the members as a cashback according to a specific rule; if not, the members are required to pay an additional sum according to the same distribution mechanism. In this setting, let K be the residual amount after the payment for loss, which can take both positive or negative values. If positive, it represents a cashback, while if negative, it is an additional sum to be paid to cover losses above the initial contributions. Note that K is a random variable depending on the realizations of total loss X and is not fixed ex-ante as in the classical insurance framework dealing with capital distribution. We define the amount K as:

$$K = \sum_{i=1}^{n} h_i^{EA}(X) - X \tag{4}$$

Alternatively, it can be expressed as $K = \sum_{i=1}^{n} [E\,[E(L_i|X)] - L_i]$.

We remind that the ex-post conditional mean rule in Eq. 1 always produces $K = 0$, while the ex-ante contribution rule in Eq. 3 can generate surplus or deficit that is described by the r.v. K.

Notice that, given the definition of the ex-ante contribution expressed in Eq. 3, the average value of K is equal to zero. However, the realized value of K could be different from its expected value involving that a positive or negative amount will be distributed across the P2P participants according to a pre-specified rule.

The definition of K can be set in a classical two steps allocation/distribution scheme. The first step consists of allocating the risk among the participants requiring the definition of an appropriate risk measure, which is necessary to understand how much of the risk is attributable to each participant. After the risk has been allocated, the second step consists into distributing the residual sum K to the members of the P2P insurance scheme.

The computation of the ex-post residual capital to be allocated to the peers is an important issue that needs to be further investigated, even considering that there is no obligation for the P2P insurance scheme to set the allocation rule.

We propose a residual capital distribution mechanism based on the participant's marginal contribution to the risk of the P2P scheme. We find the optimal value of the residual capital to be distributed to the P2P participants by imposing that all participants have an equal marginal contribution to the risk.

In the actuarial literature, the first application of the marginal contribution in risk load problems has been proposed by Mango (1998), which used the Shapley value and the game theory for the allocation of property catastrophic risk. Following this idea, we calculate the marginal contribution of each P2P participant through the Shapley value. Finally, we obtain the optimal value of the residual capital by setting constant the Shapley value. Our model allows the residual amount after the payment for loss to be distributed to the participants according to a transparent risk allocation rule.

2.1 The Shapley value

The Shapley value (Shapley, 1953) is a technique to allocate the economic output of a group to its members. In a cooperative game of a set N with n participants, let S with $S \subseteq N$ be a coalition of players and s with $s \leq n$ the total number of players in the coalition. Let $\mu(S)$ be the characteristic function of the game, the Shapley value represents the amount of risk player i gets to the set N, and is given by:

$$\phi_i(\mu) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} [\mu(S \cup \{i\}) - \mu(S)] \tag{5}$$

where $N \setminus \{i\}$ is the set of all possible coalitions (subsets S) of members when excluding player i. The quantity $[\mu(S \cup \{i\}) - \mu(S)]$ represents the average marginal contribution of player i, averaging over the possible different permutations in which the coalition can be formed. It can be interpreted as the contribution the player demands as fair compensation to enter the game. Hence, the Shapley value can alternatively be written as:

$$\phi_i(\mu) = \frac{1}{n!} \sum_{\psi \in \mathbf{P}(N)} \left[\mu(P^{\psi}(i) \cup \{i\}) - \mu(P^{\psi}(i))\right] \tag{6}$$

where $\mathbf{P}(N)$ is the set of all permutations of N, ψ is a permutation of players in N, $P^{\psi}(i)$ is the coalition made of all predecessors of player i in the order determined by permutation ψ. More formally, $P^{\psi}(i) = \{i \in \psi : \psi(j) < \psi(i)\}$. The sum in Eq. 6 is then divided by $n!$, which is the number of possible orderings of all the agents.

The Shapley value permits to distribute the total gain of a cooperative game to the players, assuming that they all collaborate. It applies in games where the contributions of each actor are unequal, but each player works in cooperation with each other to obtain the gain or pay-off much or more as they would have from acting independently. For further details, the reader may refer to Roth (1988), which provides a thorough survey on the Shapley value.

Shapley (1953) demonstrated that the rule in Eq. 5 (or Eq. 6) is optimal in the sense that it is the unique risk assignment satisfying the following axiomatic properties:

Efficiency The sum of the Shapley values of all members equals the value of the whole coalition:

$$\sum_{i \in N} \phi_i(\mu) = \mu(N)$$

In other words, all the gain is distributed among the members. Each player receives at least as much as he/she would have received if he/she had not participated in the coalition.
Symmetry If there exist two members i and j, where $\mu(S \cup \{i\}) = \mu(S \cup \{j\})$ with $S \subseteq N \setminus \{i, j\}$, then:

$$\phi_i(\mu) = \phi_j(\mu).$$

The symmetry realizes an equity condition, whereby participants who have the same risk are equally treated, or in other words, members that contribute equally to the risk receive the same gain distribution.
Additivity If we combine two games (with the same agent i) described by two functions μ_1 and μ_2 having Shapley values $\phi_i(\mu_1)$ and $\phi_i(\mu_2)$ respectively, then the game $\mu_1 + \mu_2$ has Shapley value $\phi_i(\mu_1 + \mu_2)$. Hence, if we consider a single game in which the coalition S produces the risk $\mu_1(S) + \mu_2(S)$, the agents' payment are the same of the sum of payments in the two separate games.
Axiom of dummy players If agent i is a dummy player (i.e. contributes nothing):

$$\mu(S \cup \{i\}) = \mu(S),$$

then its Shapley value is zero $\phi_i(\mu) = 0$. If a given players has a payoff independent from the others, then its contribution to the risk of the games is only equal to the own risk measure.

2.2 Risk allocation in the P2P scheme and residual capital distribution

Let $\boldsymbol{\kappa} = [K_1, K_2, ..., K_n]$ be a vector whose generic element K_i is the amount of capital distributed to the i^{th} participant, such that $\sum_{i=1}^{n} K_i = K$. We denote as $\hat{\boldsymbol{\kappa}} = [\hat{K}_1, \hat{K}_2, ..., \hat{K}_n]$ the realized value of $\boldsymbol{\kappa}$. To allocate the risk among the P2P insurance members and then distribute the residual capital, we must define an appropriate risk measure, $\mu(X)$, which depends on the total loss and on $\boldsymbol{\kappa}$.

We propose the following risk measure as the difference between the variance of the total loss, $\text{Var}(X)$, and the sum of the variances of the residual amount distributed to each policyholder:

$$\mu(X) = \text{Var}\,(X) - \sum_{i=1}^{n} \text{Var}(K_i) \tag{7}$$

where $\text{Var}(K_i)$ is the variance of the cashback received by the policyholder i.

Several contributions in the literature use variance as a measure of risk, although in different ways. Feng et al. (2023) introduce in the P2P framework the quota risk sharing mechanism, according to which each agent agrees to cover a portion of each other's losses so that each one retains a fixed size of variance. In our proposal, there is a further advancement of this setting by considering not only the total losses and its variability, but also the possible surplus or deficit distributed through the cashback mechanism. In other words, everyone participates in the risk not only by assuming a part of the variance of the total losses but also by absorbing it through the cashback mechanism. The fact that part of the surplus (or deficit) is distributed among the members actually reduces (or increases) the risk due to the variance of total losses.

Therefore, in Eq. 7 the total risk, in which each member must marginally participate, is given by the difference between the variance of total losses and how much of that risk is absorbed by the participant at the time of surplus distribution. This risk, that everyone takes on, is given by the variance of how much of the final surplus (or deficit) is attributed to him/her, and the sum of these variances is the total share of risk absorbed by all members. This is why in the risk measure that we propose, the variance of the total loss is decreased by the sum of the variances of the K_i. Finally, according to Eq. 7, the characteristic function of the game $\mu(X)$ expresses the overall risk of the scheme, which can be interpreted as the difference between the volatility of total losses and what the scheme absorbs in terms of capital distribution. Indeed, while ex ante the total contributions are aggregated into a mutual fund and $Var(X)$ expresses the volatility of aggregate losses, ex post each participant contributes individually and absorbs part of the risk in terms of possible fluctuations of K_i. Hence, the term $\sum_{i=1}^{n} \mathrm{Var}(K_i)$ represents how much of the risk is absorbed with the capital allocation. Once the risk function is defined, the Shapley Value represents the amount of risk that the player i gets to the scheme. According to a fair optimization rule, we impose the constraint according which each participant has the same marginal contribution to whole risk of the scheme, and search for the optimal value $\boldsymbol{\kappa}^* = \left[K_1^*, ..., K_n^*\right]$ by solving the following equation:

$$\phi_i(\mu(X)) = c \quad \forall i \tag{8}$$

where c is a constant value and $\phi_i(\mu(X))$ is the Shapley value.

According to Eq. 8, the optimal allocation rule $\boldsymbol{\kappa}^*$ is defined so that each participant must have the same marginal contribution to the risk for a given risk measure. In particular, the participants who contribute most to the risk of the pool will have to take on the realized losses more or, on the other hand, will be entitled to a greater distribution of the cashback. This ensures that everyone's marginal contribution to risk is identical whatever the realization of the random variable K. In other words, given the realization of the variable K, it will be distributed according to the optimal allocation vector $\boldsymbol{\kappa}^*$ in such a way that all participants have the same marginal contribution to the risk.

To calculate the Shapley value, we start from the expression of the average marginal contribution of participant i in Eq. 5. From Eq. 7, and reminding that $X = \sum_{i=1}^{n} L_i$, we obtain:

$$\begin{aligned}
\mu(S \cup \{i\}) - \mu(S) &= \mathrm{Var}\left(\sum_{j \in S \cup \{i\}} L_j\right) - \mathrm{Var}\left(\sum_{j \in S} L_j\right) - \sum_{j \in S \cup \{i\}} \mathrm{Var}(K_i) + \sum_{j \in S} \mathrm{Var}(K_i) \\
&= \sum_{j,m \in S \cup \{i\}} \mathrm{Cov}\left(L_j, L_m\right) - \sum_{j,m \in S} \mathrm{Cov}\left(L_j, L_m\right) - \mathrm{Var}(K_i) \\
&= \mathrm{Var}\left(L_i\right) + 2 \sum_{j \in S} \mathrm{Cov}\left(L_i, L_j\right) - \mathrm{Var}(K_i).
\end{aligned} \tag{9}$$

Considering the previous expression and applying Eq. 6, we have:

$$\begin{aligned}
\phi_i(\mu(X)) &= \frac{1}{n!} \sum_{\psi \in \mathbf{P}(N)} \left(\mathrm{Var}\left(L_i\right) + 2 \sum_{j \in P^{\psi}(i)} \mathrm{Cov}\left(L_i, L_j\right) - \mathrm{Var}(K_i)\right) \\
&= \mathrm{Var}\left(L_i\right) + \frac{2}{n!} \sum_{j \in N \setminus \{i\}} \sum_{\psi : j \in P^{\psi}(i)} \mathrm{Cov}\left(L_i, L_j\right) - \mathrm{Var}(K_i)
\end{aligned}$$

$$= \text{Var}\left(L_i\right) + \sum_{j \in N \setminus \{i\}} \text{Cov}\left(L_i, L_j\right) = \sum_{j=1}^{n} \text{Cov}\left(L_i, L_j\right) - \text{Var}(K_i)$$
$$= \text{Cov}\left(L_i, X\right) - \text{Var}(K_i) \tag{10}$$

By setting $K_i = k_i \cdot K$ where $K = \sum_{i=1}^{n} h_i^{EA}(X) - X$, we can rewrite Eq. 10 as:

$$\phi_i(\mu(X)) = \text{Cov}\left(L_i, X\right) - k_i^2 \text{Var}(X) \tag{11}$$

Hence, the optimal distribution problem can be formalized as follows:

$$\begin{cases} \phi_i(\mu(X)) = \text{Cov}\left(L_i, X\right) - k_i^2 \text{Var}(X) = c \qquad \forall i \\ \sum_i k_i = 1 \end{cases} \tag{12}$$

The optimal solution is the following:

$$\begin{cases} k_i^* = \frac{\text{Cov}(L_i, X)}{\text{Var}(X)} \\ c = (1 - k_i^*)\text{Cov}(L_i, X) \end{cases} \tag{13}$$

The solution given from Eq. 13 provides the optimal allocation of the residual amount (or ex-post losses) such that for each participant the marginal contribution to the total loss equals those to the final result. In other words, the risk contributed by the individual net of the risk absorbed (in terms of participation to the final result) must be the same for everyone. It is interesting to note that the optimal share k_i^* is equal to the ratio between the additional risk related to the policyholder i and the variance of the total losses. As it could be expected, the additional risk related to the policyholder i takes into account both the variance of the losses L_i and the covariances with the losses of the other policyholders in the portfolio.

3 Numerical example

We apply the proposed approach to a portfolio of motor third-party liability policies. We consider a real portfolio of 1,000 policyholders (i.e. $n = 1,000$) and we assume that the portfolio is managed via a scheme based on both ex-ante contribution and ex-post cashback. The individual losses are separately modelled according to the following frequency-severity model:

$$L_i = \sum_{h=1}^{NC_i} Z_{i,h} \tag{14}$$

with $L_i = 0$ if $NC_i = 0$. Where NC_i is the random variable number of claims of policyholder i and $Z_{i,h}$ is the average cost of claim of the policyholder. We consider classical assumptions of independence between NC_i and $Z_{i,h}$ and that the random variables $Z_{i,h}$ are independent and identical distributed. Alternative scenarios of dependence between policyholders are tested.

In order to model the random variable L_i for each policyholder, we make the following assumptions:

- As usually provided in literature (see, e.g., Daykin et al., 1994), the number of claims distribution is described by the Poisson law ($NC_i \sim Poi(f_i \cdot Q)$), with an expected frequency f_i disturbed by a structure variable Q.

Table 1 Minimum, maximum and average values of the parameters of the model used for each policyholder i

Parameter	Min	Max	Average
f_i	0.02	0.1	0.06
σ_q	0.05	0.05	0.05
m_i	4800	4800	4800
CV_{Z_i}	4	4	4

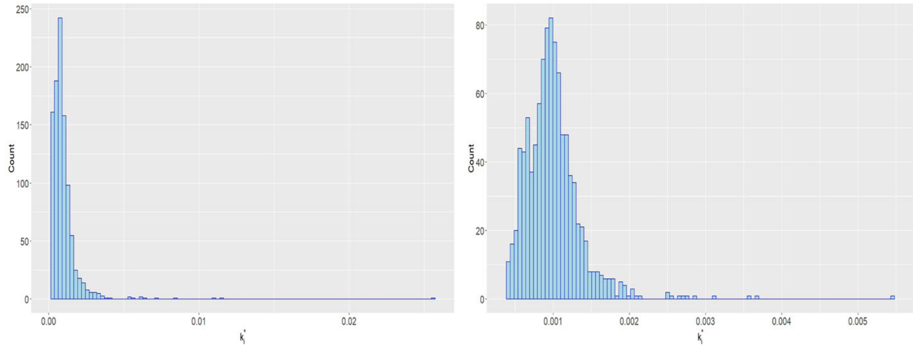

Fig. 1 Distributions of optimal shares k_i^* in case of independence (left hand-side) and correlation (right hand-side) equal to 0.5 between policyholders

- Q is a mixing variable (or contagion parameter) and it describes the parameter uncertainty on the number of claims. It is assumed that $E(Q) = 1$ and that the random variable is defined only for positive values. Following a common approach, we assume that Q is distributed as a Gamma, obtaining NC_i distributed as a Negative Binomial.
- Random variables $Z_{i,h}$ are modelled via LogNormal distributions with mean m_i and coefficient of variation CV_{Z_i}.

We consider a initial scenario characterized by the following parameters (see, e.g., Savelli & Clemente, 2011). To this end, Table 1 displays the minimum, maximum and average values observed between the policyholders in the portfolio:

It is noticeable that we assume that the only difference between policyholders is represented by the frequency f_i. In Fig. 2, left-hand-side, we report the distribution of k_i^* obtained by means of Eq. 13, assuming independence between the policyholders. The optimal shares k_i^* are on average equal to $\frac{1}{n}$, but the distribution shows a positive skewness with a larger number of policyholders with a lower contribution to the risk and hence a lower cashback. This behavior is justified by the features of the portfolio where a low portion of the portfolio is characterized by a high frequency (close to 10%) and hence to a higher risk contribution.

Assuming positive correlations between policyholders, here modeled via a gaussian copula with a correlation coefficient equal to 0.5, we observe that the shares k_i^* are again centered on $\frac{1}{n}$ but a lower variability between policyholders is observed (see Fig. 2, right-hand-side). For instance, k_i^* ranges in the interval $[0.01\%, 2.54\%]$ in case of independence and in the interval $[0.04\%, 0.54\%]$ in case of correlation. We have indeed that the differences in risk contributions are partially compensated by the lower degree of diversification present in the portfolio.

We test then the effect on the results of a portfolio characterized by lower differences in terms of frequencies between policyholders. In particular, results in Fig. 2 are obtained

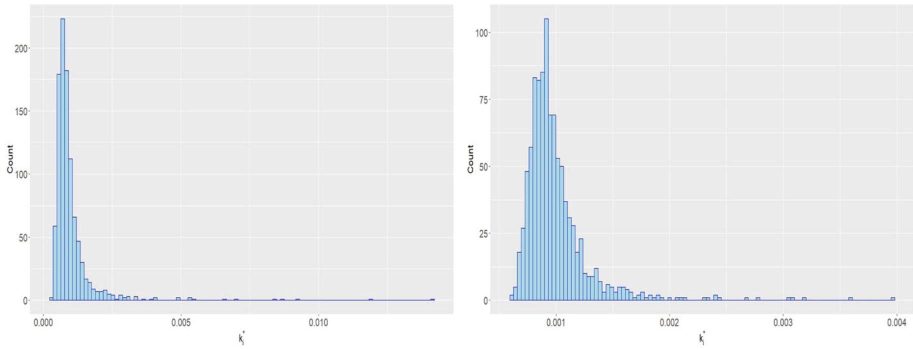

Fig. 2 Distributions of optimal shares k_i^* in case of independence (left hand-side) and correlation (right hand-side) equal to 0.5 between policyholders. With respect to the portfolio represented in Figure, we have a lower variability of frequencies between policyholders

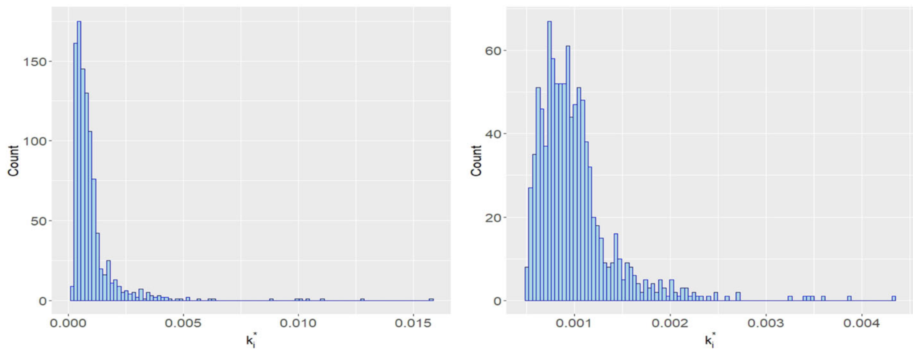

Fig. 3 Distributions of optimal shares k_i^* in case of independence (left hand-side) and correlation (right hand-side) equal to 0.5 between policyholders. In this case, we assume a portfolio where all the policyholders have the same frequency, but with a CV_{Z_i} that varies between 2 and 6

assuming the same parameters in Table 1, but a frequency that moves in the interval [5%, 7%]. As expected, the average values of the distributions in Fig. 2 are equal again to $\frac{1}{n}$ since the portfolios represented in Figs. 1 and 2 are characterized by the same average risk. On the other hand, the lower differences in terms of frequency lead to a lower variability of k_i^* (k_i^* varies now in the intervals [0.03%, 1.4%] and [0.05%, 0.4%], in case of independence and correlation, respectively). Indeed, in the case of an extreme portfolio where all policyholders have the same characteristics, all the shares k_i^* would be equal to $\frac{1}{n}$.

Finally, we consider a third portfolio composed by policyholders with the same frequency (equal to 6%), but with a different variability of the severity. In particular we assume that CV_{Z_i} moves between 2 and 6. The other parameters are equal to the previous portfolios (see Table 1). Although the average k_i^* is always the same, we notice in Fig. 3, a lower variability of k_i^* with respect to the original portfolio (Fig. 1). k_i^* varies indeed in the intervals [0.02%, 1.6%] and [0.04%, 0.5%], in case of independence and correlation, respectively.

4 Conclusions

We are in the early stages of P2P insurance and alternative models have been proposed in the market. Therefore, it is too early to say whether these new schemes can produce disruptive effects in the insurance market. Despite several regulatory issues need to be solved, it is evident that the risks related to the P2P models should be properly evaluated.

On the one hand, great attention has been paid in the literature to the identification of a transparent allocation rule to share the risks between members of the pool. On the other hand, less transparency regards the quantification of the cashback paid to the members in case of a good claim experience. Typically, this amount is shared between the participants without any consideration of the risk borne by each of them.

Therefore, in this paper, we have proposed a model where both profits and unexpected losses are shared among the participants, starting from an ex-ante initial contribution rule, which differs from the CMR under which the initial contribution is based on the expected value of the risk brought to the group of peers. We obtain a new cashback rule by assuming to distribute profits and losses according to the relative contribution of each member to the total risk in a cooperative game through the Shapley value. We have provided an analytical solution based on the assumption that all participants have an equal marginal contribution to the risk. To this aim, we have introduced an ad hoc risk measure, which not only takes into account the variability of total losses but also how much of the risk is absorbed by each member through the cashback mechanism. Our choice starts from the use of variance as a risk measure, proposed in the financial literature and in the context of P2P insurance, and then extends it to include the effect of the cashback mechanism on the risk of the scheme. Further work could extend the proposed model to consider different risk measures.

Finally, a numerical analysis has been developed to analyze the effects of different key drivers, as frequency, claim-size variability and dependence, on the distribution of optimal shares.

Moving from the consideration that the realized loss may not coincide with its expected value, a P2P insurance model should contemplate an ex-ante safety loading to add to the fair premium and/or a cashback to allocate to its members. For instance, in a traditional insurance contract, the premium paid by the policyholder generally includes a safety loading introduced to partially cover unexpected losses and to provide an expected profit. Our approach could be easily extended to consider this additional amount into the ex-ante initial contribution to the P2P scheme.

Funding Open access funding provided by Università degli Studi di Napoli Federico II within the CRUI-CARE Agreement.

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