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Abstract

The article studies the optimal redistribution system, achieved by direct taxation, indirect taxation and public provision of the *pseudo-necessary* good, when individuals, who differ in productivity, can take hidden actions (tax evasion by moral hazard) and have hidden information (tax evasion by adverse selection). It proves that any Government willing to effectively reallocate resources among individuals has to undertake measures against tax evasion, i.e. to establish tax evasion fines.

Keywords: Redistribution; Tax Evasion; Asymmetric Information
JEL Classification: H23; H42; H26; D82

1. Introduction

Atkinson (1977) argues that the choice between direct and indirect taxes is one of the crucial issues of taxation policy due to the challenging theoretical questions arising and its significant policy relevance (see, among others, Cremer et al., 2001). Since taxation is a key instrument to redistribute among individuals, assessing the re-distributional power of differential commodity taxation versus nonlinear income taxation is of great interest in the literature on optimal taxation (Saez, 2002).

The role of differential commodity taxation has been seriously undermined by the seminal paper of Atkinson and Stiglitz (1976); they study the

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optimal direct and indirect tax mix problem in presence of individuals differing for the sole earning ability and show that nonlinear income taxation does not need to be supplemented by commodity taxation when preferences are weakly separable in labour supply and produced goods.

As Boadway and Pestieau (2002) have pointed out, a number of studies have tried to further investigate the "Atkinson-Stiglitz" theorem. In a framework of strong homogeneity of preferences for consumption goods, while Mirrlees (1976) shows that commodity taxation is desirable on goods that are relatively more preferred by the high skilled individuals, Christiansen (1984) proves that goods that are complementary with leisure should be taxed. Boadway and Pestieau (2002) consider how robust the "Atkinson-Stiglitz" theorem is with respect to differences in needs or endowments of goods, more than one type of labour supply, differences in preference for leisure, and restrictions on policy instruments; they conclude that in developing countries the compliance and administration costs of income taxation are so high that tax authorities have to rely on indirect taxation.

The recent literature focuses on the role of indirect taxes when individuals are heterogeneous in one or more characteristics. Cremer et al. (2001) assume individuals differ in several unobservable characteristics (i.e. productivity and endowment) and prove that differential commodity taxation is a useful instrument of tax policy even if preferences are separable between labour and produced goods. Saez (2002) assumes individuals are heterogeneous in earnings and tastes and shows that a small tax on a given commodity is desirable if high income earners have a relatively higher taste for this commodity or if consumption of this commodity increases with leisure.

There is also a large literature on the economics of tax evasion: while some authors are interested in tax evasion on the indirect taxes (see, among others, Cremer and Gahavari, 1993), others are interested in tax evasion on the direct taxes (see, among others, Cremer and Gahavari, 1996).

Most of these articles follow the standard approach originated in Allingham and Sandmo (1972) and treat the decision of how much tax to evade as one of choosing a consumption stream under uncertainty: taxpayers, faced with a given probability of penalty, will choose the amount of evasion which maximizes their expected utilities (see Boadway and Sato, 2000).

This article investigates the tax evasion problem from an innovative perspective, which is relying on the analytical categories proper of the asymmetric information setting: individuals can take hidden actions to affect their labour supply and commodity demands (tax evasion by moral hazard) and hide information about their productivity (tax evasion by adverse selection).

If individuals only take hidden actions, the Government can implement the redistribution system without auditing them but imposing the incentive constraints; on the contrary, if individuals also hide information, to implement the redistribution system the Government has to audit and to punish them.

The remainder of the article proceeds as follows. Section 2 describes the setup of the model, Section 3 considers a public information environment without and with the redistribution system, in Section 4 the individuals can take hidden actions and in Section 5 the individuals can take hidden actions and also have hidden information. Finally, Section 6 concludes.

2. The Model

In the economy there are two commodities $i = 1, 2$ (the *pseudo-necessity* and the *pseudo-luxury*) and two individual types $h = A, B$ (the more efficient or high income workers and the less efficient or low income workers).

The population n is partitioned into the two groups A, B with $n = n^A + n^B$. Assuming the population size is large, the probabilities $\pi^h = \pi^A, \pi^B$, with $\pi^A + \pi^B = 1$, are also the population share between the two individual types.

Both the individual types A, B are constrained by the same time endowment T and the same unearned lump-sum income $I = 0$[1]. With L^h labour and ℓ^h leisure, the time endowment is $L^h + \ell^h = T$, and hence the full income y^h of the individual type $h = A, B$ is:

$$y^h = \left(w^h - \tau^h\right) \cdot T = m^h \cdot T, h = A, B \qquad (1)$$

(see De Bartolome, 1990; Kaiser, 1993), where w^h is the gross wage rate, τ^h the labour income tax and m^h the net wage rate. The consumer price q_i for the good $i = 1, 2$ is:

$$q_i = p_i + t_i, i = 1, 2, \qquad (2)$$

where p_i is the producer price and t_i the commodity tax.

[1]The assumption of constant returns to scale, together with competitive behaviour, implies that the firms earn zero profits. Therefore, the households receive no profit income and the lump-sum income is zero (Myles, 1995).

The redistribution systems will be illustrated presenting a computational model where the parametrisation is chosen mainly for numerical convenience and considering a Klein-Rubin (Klein and Rubin, 1948) or Stone-Geary (Stone, 1954; Geary, 1950) utility function (*KR-SG*) and Cobb-Douglas production functions (*CD*)[2].

3. Public Information

The household's problem (*Utility Maximization Problem - UMP*) is:

$$\begin{aligned} & U^h = \max_{\mathbf{x}^h, \ell^h} U\left(\mathbf{x}^h, \ell^h\right) \\ & s.t. \sum_{i=1,2} p_i \cdot x_i^h + w^h \cdot \ell^h \leq w^h \cdot T \ \ \left(\lambda^h\right), \end{aligned} \tag{3}$$

with $\mathbf{x}^h = (x_1^h, x_2^h)$ (see, among others, Deaton and Muellbauer, 1981; Kaiser, 1993). The *FOCs* of the *UMP* (3) entail:

$$\begin{aligned} p_i &= \tfrac{1}{\lambda^h} \cdot \tfrac{\partial U^h}{\partial x_i^h} \\ w^h &= \tfrac{1}{\lambda^h} \cdot \tfrac{\partial U^h}{\partial \ell^h}. \end{aligned} \tag{4}$$

Therefore, the marginal rate of substitution is:

$$MRS_{1,2} = \frac{p_1}{p_2} = \frac{\frac{\partial U^h}{x_1^h}}{\frac{\partial U^h}{x_2^h}}, \tag{5}$$

and with $\lambda^h = \frac{\partial U^h}{\partial \ell^h} \cdot \frac{1}{w^h}$ it is also possible to write:

$$\frac{\partial U^h}{\partial x_i^h} = \frac{\partial U^h}{\partial \ell^h} \cdot \frac{p_i}{w^h}. \tag{6}$$

The demand of good i by individual h is $x_i^h = x_i(\mathbf{p}, w^h)$ and the leisure demand by individual h is $\ell^h = \ell(\mathbf{p}, w^h)$, with $\mathbf{p} = (p_1, p_2)$. The properties of the *Walrasian demand functions* are analysed in Appendix A.

[2] The parameter values are $\pi^A = \pi^B = 0.5$ and $T = 100$, $A_1 = 1.1$, $A_2 = 1$ and $\alpha_1^A = 0.55$, $\alpha_2^A = 0.6$ (and then $\alpha_1^B = 0.45$, $\alpha_2^B = 0.4$), $\alpha = 0.33$, $\beta = 0.33$, $\gamma = 0.33$ and $b_1 = 13$, $b_2 = -13$, $b_\ell = 0$.

The dual problem (*Expenditure Minimization Problem - EMP*) is:

$$
\begin{array}{ll}
e^h & = \min_{\mathbf{x}^h,\ell^h} \sum_{i=1,2} p_i \cdot x_i^h + w^h \cdot \ell^h \\
s.t. & U\left(\mathbf{x}^h, \ell^h\right) \geq U \ \left(\nu^h\right).
\end{array} \tag{7}
$$

The *FOCs* of the *EMP* (7) entail:

$$
\begin{array}{ll}
p_i & = \nu^h \cdot \frac{\partial U^h}{\partial x_i^h} \\
w^h & = \nu^h \cdot \frac{\partial U^h}{\partial \ell^h},
\end{array} \tag{8}
$$

and therefore with $\frac{1}{\nu^h} = \frac{\partial U^h}{\partial \ell^h} \cdot \frac{1}{w^h}$ it is possible to obtain equation (6).

The set of optimal commodities and leisure quantities in the *EMP* (7) are denoted by $h_i(\mathbf{p}, w^h, U)$ for $i = 1, 2$ and by $h_L(\mathbf{p}, w^h, U)$.

Considering the Walrasian demand functions $x_i(\mathbf{p}, w^h, I)$ and $\ell(\mathbf{p}, w^h, I)$ allows to relate the Hicksian and Walrasian demands as in (C.1). Since $I = 0$, the equations in (B.6) and in (C.2) can be rewritten as:

$$
\begin{array}{l}
\frac{\partial x_i\left(\mathbf{p},w^h\right)}{\partial p_j} = \frac{\partial h_i\left(\mathbf{p},w^h,U\right)}{\partial p_j} = S_{ij}^h \quad \text{and} \quad \frac{\partial L\left(\mathbf{p},w^h\right)}{\partial p_j} = \frac{\partial h_L\left(\mathbf{p},w^h,U\right)}{\partial p_j} = S_{Lj}^h, \\
\frac{\partial x_i\left(\mathbf{p},w^h\right)}{\partial w^h} = \frac{\partial h_i\left(\mathbf{p},w^h,U\right)}{\partial w^h} = I_{iL}^h \quad \text{and} \quad \frac{\partial L\left(\mathbf{p},w^h\right)}{\partial w^h} = \frac{\partial h_L\left(\mathbf{p},w^h,U\right)}{\partial w^h} = I_{LL}^h,
\end{array} \tag{9}
$$

with $i, j = 1, 2$, where S_{ij}^h and S_{Lj}^h are the substitution effects and I_{iL}^h and I_{LL}^h are the *pseudo-income* effects.

Multiplying the first equation in (9) by $\frac{p_j}{x_i^h}$, the second equation by $\frac{p_j}{L^h}$, the third equation by $\frac{w^h}{x_i^h}$ and the fourth equation by $\frac{w^h}{L^h}$ allows to obtain the direct and cross elasticities:

$$
\begin{array}{l}
\varepsilon_{ij}^h = S_{ij}^h \cdot \frac{p_j}{x_j} \quad \text{and } \varepsilon_{Lj}^h = S_{Lj}^h \cdot \frac{p_j}{L^h}, \\
\varepsilon_{iL}^h = I_{iL}^h \cdot \frac{w^h}{x_i^h} \quad \text{and } \varepsilon_{LL}^h = I_{LL}^h \cdot \frac{w^h}{L^h}
\end{array} \tag{10}
$$

with $i, j = 1, 2$. If the good $i = 1$ is a *pseudo-necessity*, then the *pseudo-income* elasticity is between 0 and 1, that is $0 \leq \varepsilon_{1L}^h < 1 \Rightarrow 0 \leq \frac{\partial x_1^h}{\partial w^h} \cdot \frac{w^h}{x_1^h} < 1 \Rightarrow \frac{\partial x_1^h}{\partial w^h} < \frac{x_1^h}{w^h}$ (the good 1 is substitute to leisure and his demand is inelastic to wage rate) and if the good $i = 2$ is a *pseudo-luxury* then the *pseudo-income* elasticity is greater than 1, that is $\varepsilon_{2L}^h \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial w^h} \cdot \frac{w^h}{x_2^h} \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial w^h} \geq \frac{x_2^h}{w^h}$ (the good 2 is substitute to leisure and his demand is elastic to wage rate).

Considering a log transformation of the *KR-SG* utility function, the household choices are determined by the *UMP*:

$$\begin{aligned} U^h = \max_{\mathbf{x}^h, L^h} \quad & \alpha \cdot ln\left(x_1^h - b_1\right) + \beta \cdot ln\left(x_2^h - b_2\right) + \\ & +\gamma \cdot ln\left(\left(T - L^h\right) - b_\ell\right) \\ s.t. \sum_{i=1,2} & p_i \cdot x_i^h - w^h \cdot L^h \leq 0 \ \left(\lambda^h\right), \end{aligned} \tag{11}$$

with $\alpha + \beta + \gamma = 1$ and $b_1 > 0$ (the good 1 ia a *pseudo-necessity*), $b_2 < 0$ (the good 2 ia a *pseudo-luxury*), and $b_\ell = 0$; the demand system of individual $h = A, B$ for the goods $i = 1, 2$ and leisure ℓ is:

$$\begin{aligned} x_1^h &= b_1 + \alpha \cdot \frac{w^h \cdot (T - b_\ell) \sum\limits_{i=1,2} p_i \cdot b_i}{p_1}, \\ x_2^h &= b_2 + \beta \cdot \frac{w^h \cdot (T - b_\ell) \sum\limits_{i=1,2} p_i \cdot b_i}{p_2}, \\ T - L^h &= b_\ell + \gamma \cdot \frac{w^h \cdot (T - b_\ell) \sum\limits_{i=1,2} p_i \cdot b_i}{w^h}. \end{aligned} \tag{12}$$

whose elasticities ε_{1j}^h, ε_{2j}^h and ε_{Lj}^h (with $j = 1, 2, L$) in (10) are provided in Appendix D (see equations in (D.1), (D.2) and (D.3)).

Considering the production side of the economy and assuming the production function is $f(\mathbf{L}_i; \mathbf{n})$, where $\mathbf{L}_i = (L_i^A, L_i^B)$ and $\mathbf{n} = (n^A, n^B)$, the *Profit Maximization Problem (PMP)* of the good $i = 1, 2$ is:

$$\Pi_i = \max_{\mathbf{L}_i} p_i \cdot f\left(\mathbf{L}_i; \mathbf{n}\right) - \sum_{h=A,B} w^h \cdot n^h \cdot L_i^h. \tag{13}$$

Therefore, the marginal rate of transformation is:

$$MRT_{1,2} = \frac{p_1}{p_2} = \frac{\frac{\partial f_2}{\partial L_2^h}}{\frac{\partial f_1}{\partial L_1^h}}, h = A, B \tag{14}$$

and the marginal rate of technical substitution is:

$$MRTS^{A,B} = \frac{w^A}{w^B} = \frac{\frac{\partial f_i}{\partial L_i^A}}{\frac{\partial f_i}{\partial L_i^B}}, i = 1, 2. \tag{15}$$

Assuming $\frac{\partial f_1}{\partial L_1^h} > \frac{\partial f_2}{\partial L_2^h}$ yields to $p_1 < p_2$ and assuming $\frac{\partial f_i}{\partial L_i^A} > \frac{\partial f_i}{\partial L_i^B}$ yields to $w^A > w^B$.

The *clearing conditions* are ensured since prices p_1 and p_2 guarantee the goods markets are in equilibrium:

$$\begin{aligned} p_1 &: f\left(\mathbf{L}_1;\mathbf{n}\right) = n^A \cdot x_1^A + n^B \cdot x_1^B, \\ p_2 &: f\left(\mathbf{L}_2;\mathbf{n}\right) = n^A \cdot x_2^A + n^B \cdot x_2^B; \end{aligned} \tag{16}$$

and wage rates w^A and w^B guarantee the labour market is in equilibrium:

$$\begin{aligned} w^A &: L_1^A + L_2^A = L^A, \\ w^B &: L_1^B + L_2^B = L^B. \end{aligned} \tag{17}$$

A *CD* production function $f(\mathbf{L}_i; A_i, \boldsymbol{\alpha}_i, \mathbf{n})$ is adopted for both commodities $i = 1, 2$, where $\boldsymbol{\alpha}_i = (\alpha_i^A, \alpha_i^B)$. Therefore, the *PMP* of the good $i = 1, 2$ is:

$$\Pi_i = \max_{\mathbf{L}_i} p_i \cdot \left(A_i \cdot \left(n^A \cdot L_i^A\right)^{\alpha_i^A} \cdot \left(n^B \cdot L_i^B\right)^{\alpha_i^B}\right) - \sum_{h=A,B} w^h \cdot n^h \cdot L_i^h, \tag{18}$$

and the production functions in (18) are characterized by $A_1 > A_2$, which signifies the assumption that the rate of labour-augmenting technological progress A_i is higher in the production of good 1 than in the production of good 2[3], and by constant returns to scale $\alpha_i^A + \alpha_i^B = 1$.

Moreover, it is assumed that the production functions in (18) are characterized not only by $\alpha_i^A > \alpha_i^B$ (type A individuals are more efficient than type B individuals), but also by $\alpha_2^A > \alpha_1^A$ (type A individuals are more efficient in the production of good 2, that is the production of good 2 is A-labour intensive with respect to the production of good 1) and then, with $\alpha_i^B = 1 - \alpha_i^A$, by $\alpha_1^B > \alpha_2^B$ (type B individuals are more efficient in the production of good 1, that is the production of good 1 is B-labour intensive with respect to the production of good 2).

Therefore, the marginal rate of transformation is:

$$MRT_{1,2} = \frac{p_1}{p_2} = \frac{A_2 \cdot \alpha_2^A \cdot \left(\frac{n^B \cdot L_2^B}{n^A \cdot L_2^A}\right)^{\alpha_2^B}}{A_1 \cdot \alpha_1^A \cdot \left(\frac{n^B \cdot L_1^B}{n^A \cdot L_1^A}\right)^{\alpha_1^B}} = \frac{A_2 \cdot \alpha_2^B \cdot \left(\frac{n^A \cdot L_2^A}{n^B \cdot L_2^B}\right)^{\alpha_2^A}}{A_1 \cdot \alpha_1^B \cdot \left(\frac{n^A \cdot L_1^A}{n^B \cdot L_1^B}\right)^{\alpha_1^A}}, \tag{19}$$

[3]In other words, the production of commodity 1 benefits more from technological progress and/or is more capital intensive than the production of commodity 2. In fact, luxuries are often produced in small batches, using labour intensive methods, in contrast to necessities, which tend to be produced on a larger scale, using capital intensive methods (Burkett, 2006).

and the marginal rate of technical substitution is:

$$MRTS^{A,B} = \frac{w^A}{w^B} = \frac{A_1 \cdot \alpha_1^A \cdot \left(\frac{n^B \cdot L_1^B}{n^A \cdot L_1^A}\right)^{\alpha_1^B}}{A_1 \cdot \alpha_1^B \cdot \left(\frac{n^A \cdot L_1^A}{n^B \cdot L_1^B}\right)^{\alpha_1^A}} = \frac{A_2 \cdot \alpha_2^A \cdot \left(\frac{n^B \cdot L_2^B}{n^A \cdot L_2^A}\right)^{\alpha_2^B}}{A_2 \cdot \alpha_2^B \cdot \left(\frac{n^A \cdot L_2^A}{n^B \cdot L_2^B}\right)^{\alpha_2^A}}. \quad (20)$$

Assuming $A_1 > A_2$ entails $\frac{\partial f(\mathbf{L}_1; A_1, \boldsymbol{\alpha}_1, \mathbf{n})}{\partial L_1^h} > \frac{\partial f(\mathbf{L}_2; A_1, \boldsymbol{\alpha}_2, \mathbf{n})}{\partial L_2^h}$ and then $p_1 < p_2$ and assuming $\alpha_i^A > \alpha_i^B$ entails $\frac{\partial f(\mathbf{L}_i; A_i, \boldsymbol{\alpha}_i, \mathbf{n})}{\partial L_i^A} > \frac{\partial f(\mathbf{L}_i; A_i, \boldsymbol{\alpha}_i, \mathbf{n})}{\partial L_i^B}$ and then $w^A > w^B$.

The results of the simulation in case of public information are presented in table 1, which displays (i) quantities and utilities, (ii) prices and expenditures and (iii) elasticities.

Table 1: Public Information

	x_1^h	x_2^h	L_1^h	L_2^h	L^h	U^h
A	33.5711	5.7761	56.8262	8.4821	65.3084	3.1714
B	29.5513	2.1071	58.0731	7.0630	65.1361	3.0296

	p_1	p_2	w^h	$y^h = e^h$
A	1	1.0956	0.6109	39.8994
B			0.4891	31.8598

	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h
A	-0.7406	0.1400	0.6005	-0.6779	-2.5079	3.1858	0.1108	-0.1214	0.0106
B	-0.7053	0.1590	0.5462	-1.8584	-5.1337	6.9921	0.1387	-0.1520	0.0133

From table 1 it is possible to verify that good 1 is a *pseudo-necessity* since $\varepsilon_{1L}^A = 0.6005 < 1$ and $\varepsilon_{1L}^B = 0.5462 < 1$ and the good 2 is a *pseudo-luxury* since $\varepsilon_{2L}^A = 3.1854 \geq 1$ and $\varepsilon_{2L}^B = 6.9921 \geq 1$[4]. From table 1 it is also possible to verify that $w^A > w^B$ and $p_2 > p_1$ (with p_1 as numeraire).

Moreover—given the utility and the production functions—the following results have been obtained: (i) the labour supply of more efficient individuals A is higher than the labour supply of less efficient individuals B ($L^A > L^B$), (ii) both the more efficient individuals A and the less efficient individuals B contribute more to the production of the *pseudo-necessary* good 1 ($L_1^h > L_2^h$), but (iii) the less efficient individuals B contribute more than the more efficient

[4]Both intuitively and empirically (see, among others, Kokoski, 2003) necessities are less price sensitive (i.e. they have an inelastic demand) and luxuries are more price sensitive (i.e. they have an elastic demand); in fact, table 1 shows also that $|\varepsilon_{11}^h| < 1$ and $|\varepsilon_{22}^h| \geq 1$ $(h = A, B)$.

individuals A to the production of the *pseudo-necessary* good 1 ($L_1^B > L_1^A$) and (iv) the more efficient individuals A contribute more than the less efficient individuals B to the production of the *pseudo-luxury* good 2 ($L_2^A > L_2^B$).

3.1. Redistribution System & Public Provision of the Pseudo-necessity

Out of the taxation revenues, the Government publicly provides the good $i = 1$ (the *pseudo-necessity*) and each individual $h = A, B$ receives the quantity x_G. Therefore the *ex-post* household's problem (UMP) is

$$\begin{aligned} U^h &= \max_{\mathbf{x}^h, \ell^h} U\left(x_1^h + x_G, x_2^h, \ell^h\right) \\ s.t. & \sum_{i=1,2} q_i \cdot x_i^h + m^h \cdot \ell^h \leq m^h \cdot T \ \ \left(\lambda^h\right) \end{aligned} \tag{21}$$

(see, among others, Deaton and Muellbauer, 1981; Kaiser, 1993). The *FOCs* of the UMP (21) yield:

$$\begin{aligned} q_i &= \tfrac{1}{\lambda^h} \cdot \tfrac{\partial U^h}{\partial x_i^h} \\ m^h &= \tfrac{1}{\lambda^h} \cdot \tfrac{\partial U^h}{\partial \ell^h}. \end{aligned} \tag{22}$$

Therefore, the marginal rate of substitution is:

$$MRS_{1,2} = \frac{q_1}{q_2} = \frac{\frac{\partial U^h}{x_1^h}}{\frac{\partial U^h}{x_2^h}}, \tag{23}$$

and with $\lambda^h = \frac{\partial U^h}{\partial \ell^h} \cdot \frac{1}{m^h}$ it is also possible to write:

$$\frac{\partial U^h}{\partial x_i^h} = \frac{\partial U^h}{\partial \ell^h} \cdot \frac{q_i}{m^h}. \tag{24}$$

The demand of good i by individual h is $x_i^h = x_i(\mathbf{q}, m^h, x_G)$ and the leisure demand by individual h is $\ell^h = \ell(\mathbf{q}, m^h, x_G)$, with $\mathbf{q} = (q_1, q_2)$. Moreover, the elasticities in (10) can be rewritten as:

$$\begin{aligned} \varepsilon_{ij}^h &= \tfrac{\partial x_i\left(\mathbf{q}, m^h, x_G\right)}{\partial q_j} \cdot \tfrac{q_j}{x_i^h} = S_{ij}^h \cdot \tfrac{q_j}{x_j}, \ \varepsilon_{Lj}^h = \tfrac{\partial L\left(\mathbf{q}, m^h, x_G\right)}{\partial q_j} \cdot \tfrac{q_j}{L^h} = S_{Lj}^h \cdot \tfrac{q_j}{L^h}, \\ \varepsilon_{iL}^h &= \tfrac{\partial x_i\left(\mathbf{q}, m^h, x_G\right)}{\partial m^h} \cdot \tfrac{m^h}{x_i^h} = I_{iL}^h \cdot \tfrac{m^h}{x_i^h}, \ \varepsilon_{LL}^h = \tfrac{\partial L\left(\mathbf{q}, m^h, x_G\right)}{\partial m^h} \cdot \tfrac{m^h}{L^h} = I_{LL}^h \cdot \tfrac{m^h}{L^h} \end{aligned} \tag{25}$$

with $i, j = 1, 2$. If the good $i = 1$ is a *pseudo-necessity* then the *pseudo-income* elasticity is between 0 and 1, that is $0 \leq \varepsilon_{1L}^h < 1 \Rightarrow 0 \leq \frac{\partial x_1^h}{\partial m^h} \cdot \frac{m^h}{x_1^h} <$

$1 \Rightarrow \frac{\partial x_1^h}{\partial m^h} < \frac{x_1^h}{m^h}$, and if the good $i = 2$ is a *pseudo-luxury* then the *pseudo-income* elasticity is greater than 1, that is $\varepsilon_{2L}^h \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial m^h} \cdot \frac{m^h}{x_2^h} \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial m^h} \geq \frac{x_2^h}{m^h}$.

The Government redistribute resources among individuals through indirect taxes $\mathbf{t} = (t_1, t_2)$, direct taxes $\boldsymbol{\tau} = (\tau^A, \tau^B)$ and the publicly provided good G. Denoting $\mathcal{V}^h = \mathcal{V}(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h)$, $x_i^h = x_i(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h)$ and $L^h = L(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h)$, the *Government Problem* (*GP*) is:

$$\begin{aligned} W &= \max_{\mathbf{t},\boldsymbol{\tau},x_G} W\left(\pi^A \cdot \mathcal{V}^A, \pi^B \cdot \mathcal{V}^B\right) \\ s.t. &\quad \sum_{h=A,B} \pi^h \cdot \left(\tau^h \cdot L^h + \sum_{i=1,2} t_i \cdot x_i^h\right) \geq p_1 \cdot x_G \left(\lambda_G\right). \end{aligned} \tag{26}$$

Considering the elasticities in (25) the *FOCs* of the *GP* (26) are:

$$\begin{aligned} \frac{\sum_{h=A,B} \pi^h \cdot (\tau^h \cdot \varepsilon_{L1}^h \cdot L^h + \sum_{i=1,2} t_i \cdot \varepsilon_{i1}^h \cdot x_i^h)}{q_1 \cdot x_1} &= -1 - \frac{1}{\lambda_G} \cdot \frac{\sum_{h=A,B} \pi^h \cdot \beta_1^h}{x_1}, \\ \frac{\sum_{h=A,B} \pi^h \cdot (\tau^h \cdot \varepsilon_{L2}^h \cdot L^h + \sum_{i=1,2} t_i \cdot \varepsilon_{i2}^h \cdot x_i^h)}{q_2 \cdot x_2} &= -1 - \frac{1}{\lambda_G} \cdot \frac{\sum_{h=A,B} \pi^h \cdot \beta_2^h}{x_2}, \\ \frac{\tau^A \cdot \varepsilon_{LL}^A \cdot L^A + \sum_{i=1,2} t_i \cdot \varepsilon_{iL}^A \cdot x_i^A}{m^A \cdot L^A} &= 1 - \frac{1}{\lambda_G} \cdot \frac{\beta_L^A}{L^A}, \\ \frac{\tau^B \cdot \varepsilon_{LL}^B \cdot L^B + \sum_{i=1,2} t_i \cdot \varepsilon_{iL}^B \cdot x_i^B}{m^B \cdot L^B} &= 1 - \frac{1}{\lambda_G} \cdot \frac{\beta_L^B}{L^B}, \\ \sum_{h=A,B} \pi^h \cdot \left(\tau^h \cdot \frac{\partial L^h}{\partial x_G} + \sum_{i=1,2} t_i \cdot \frac{\partial x_i^h}{\partial x_G}\right) &= p_1 - \frac{1}{\lambda_G} \cdot \sum_{h=A,B} \pi^h \cdot \beta_G^h, \end{aligned} \tag{27}$$

where $\beta_i^h = \frac{\partial W}{\partial \mathcal{V}^h} \cdot \frac{\partial \mathcal{V}^h}{\partial q_i}$ is the (gross) marginal social evaluation of individual h's utility with respect to good i, $\beta_L^h = \frac{\partial W}{\partial \mathcal{V}^h} \cdot \frac{\partial \mathcal{V}^h}{\partial m^h}$ is the (gross) marginal social evaluation of the h's utility with respect to labour L and $\beta_G^h = \frac{\partial W}{\partial \mathcal{V}^h} \cdot \frac{\partial \mathcal{V}^h}{\partial x_G}$ is the (gross) marginal social evaluation of the h's utility with respect to the publicly provided good G.

Considering a log transformation of the *KR-SG* utility function, the *UMP* (21) is:

$$\begin{aligned} U^h &= \max_{\mathbf{x}^h, L^h} \alpha \cdot ln\left(x_1^h + x_G - b_1\right) + \beta \cdot ln\left(x_2^h - b_2\right) + \\ &\quad +\gamma \cdot ln\left(\left(T - L^h\right) - b_\ell\right) \\ s.t. &\quad \sum_{i=1,2} q_i \cdot x_i^h \leq m^h \cdot L^h \ \left(\lambda^h\right), \end{aligned} \tag{28}$$

and with $\varphi\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)=m^h\cdot(T-b_\ell)+q_1\cdot x_G-\sum_{i=1,2}q_i\cdot b_i$ the demand system of individual $h=A,B$ for the goods $i=1,2$ and leisure ℓ is:

$$\begin{aligned} x_1^h+x_G &= b_1+\alpha\cdot\frac{\varphi\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{q_1},\\ x_2^h &= b_2+\beta\cdot\frac{\varphi\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{q_2},\\ T-L^h &= b_\ell+\gamma\cdot\frac{\varphi\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{m^h}. \end{aligned} \tag{29}$$

Hence with $\varphi^h=\varphi\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)$ the indirect utility of individual h is:

$$\mathcal{V}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)=\alpha\cdot ln\left(\alpha\cdot\frac{\varphi^h}{q_1}\right)+\beta\cdot ln\left(\beta\cdot\frac{\varphi^h}{q_2}\right)+\gamma\cdot ln\left(\gamma\cdot\frac{\varphi^h}{m^h}\right) \tag{30}$$

and, with $\mathcal{V}^h=\mathcal{V}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)$, $x_i^h=x_i(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h)$ and $L^h=L(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h)$, the *GP* (26) is:

$$\begin{aligned} W &= \max_{\mathbf{t},\boldsymbol{\tau},x_G}\sum_{h=A,B}\beta^h\cdot\pi^h\cdot\mathcal{V}^h\\ s.t. &\quad \sum_{h=A,B}\pi^h\cdot\left(\tau^h\cdot L^h+\sum_{i=1,2}t_i\cdot x_i^h\right)\geq p_1\cdot x_G\left(\lambda_G\right), \end{aligned} \tag{31}$$

where β^h is the weight assigned by the Government to individual h. Therefore, the *FOCs* (27) are obtained, where the elasticities ε_{1j}^h, ε_{2j}^h and ε_{Lj}^h (with $j=1,2,L$) in (25) and the (gross) marginal social evaluations of h's utility of good $i=1,2$, labour L and the publicly provided good G are provided in Appendix D (see equations in (D.4), (D.5), (D.6) and (D.7)).

Moreover, the *FOC* with respect to x_G implies:

$$\begin{aligned} &\sum_{h=A,B}\pi^h\cdot\left(\tau^h\cdot\frac{\partial L^h}{\partial x_G}+\sum_{i=1,2}t_i\cdot\frac{\partial x_i^h}{\partial x_G}\right)=\\ &\sum_{h=A,B}\pi^h\cdot q_1\cdot\left(\alpha\cdot\frac{t_1}{q_1}+\beta\cdot\frac{t_2}{q_2}-\gamma\cdot\frac{\tau^h}{m^h}\right)-t_1. \end{aligned} \tag{32}$$

Therefore, the public provision x_G is function of the ratios between indirect taxes and consumer prices $\frac{t_i}{q_i}$ ($i=1,2$) and between direct taxes and net wage rates $\frac{\tau^h}{m^h}$ ($h=A,B$).

The simulation is implemented considering both the Utilitarian social welfare function $\beta^A=\beta^B=1$ (see tables 2 and 3) and the Rawlsian social welfare function $\beta^A<\beta^B$ (see tables 4 and 5).

There exists a set of possible redistribution systems $(\mathbf{t}, \boldsymbol{\tau}, x_G)$ which maximize the social welfare function W in (31): while tables 2 and 4 show the redistribution system characterized by $t_1, t_2 > 0$ and $\tau^A, \tau^B < 0$ (i.e. while the goods $i = 1, 2$ are taxed, the wage rates of both individuals $h = A, B$ are subsidized), tables 3 and 5 show the redistribution system characterized by $t_1, t_2, \tau^B < 0$ and $\tau^A > 0$ (i.e. while the wage rate of individual $h = A$ is taxed, both the goods $i = 1, 2$ and the wage rate of individual $h = B$ are subsidized).

Tables 2, 3, 4 and 5 display (i) quantities and utilities, (ii) prices, taxation levels and expenditures, (iii) elasticities and (iv) (gross) marginal social evaluations.

Table 2: Redistribution System with Public Information: the Utilitarian case ($\beta^A = \beta^B = 1$) with $t_1, t_2 > 0$ and $\tau^A, \tau^B < 0$

	x_G	x_1^h			x_2^h		L_1^h	L_2^h	L^h	U^h	W
A	0.0056	31.5799			3.9615		56.7597	8.4685	65.2281	3.1052	3.1043
B		31.5312			3.9171		58.1561	7.0700	65.2260	3.1035	
	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$	
A	1	0.2413	1.2413	1.0957	0.2644	1.3601	0.6117	-0.0719	0.6836	44.5877	
B							0.4884	-0.1933	0.6817	44.4668	
	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h		
A	-0.7243	0.1489	0.5755	-0.9879	-3.1986	4.1865	0.1230	-0.1348	0.0118		
B	-0.7239	0.1491	0.5748	-0.9991	-3.2236	4.2227	0.1233	-0.1352	0.0119		
	β_G^h	β_1^h			β_2^h			β_L^h			
A	0.01776	-0.4517			-0.0567			0.9331			
B	0.01780	-0.4522			-0.0562			0.9355			

Tables 2 and 3 allow to appreciate that in the Utilitarian case both the redistribution systems $(t_1, t_2 > 0; \tau^A, \tau^B < 0)$ and $(t_1, t_2, \tau^B < 0; \tau^A > 0)$ lead to the same outcome (i.e., same utilities). Moreover, the public provision x_G is positive, yet very low.

Tables 4 and 5 allow to appreciate that also in the Rawlsian case both the redistribution systems $(t_1, t_2 > 0; \tau^A, \tau^B < 0)$ and $(t_1, t_2, \tau^B < 0; \tau^A > 0)$ lead to the same outcome (i.e., same utilities), while the public provision x_G is zero.

Obviously both in the Utilitarian case and in the Rawlsian case the redistribution systems make type A individuals worse off (tables 2 and 3 show that $3.1052 < 3.1714$ and tables 4 and 5 that $3.1043 < 3.1714$) and type B individuals better off (tables 2 and 3 show that $3.1035 > 3.0296$ and tables

Table 3: Redistribution System with Public Information: the Utilitarian case ($\beta^A = \beta^B = 1$) with $t_1, t_2, \tau^B < 0$ and $\tau^A > 0$

	x_G	x_1^h	x_2^h	L_1^h	L_2^h	L^h	U^h	W
A	0.0056	31.5799	3.9615	56.7597	8.4685	65.2281	3.1052	3.1043
B		31.5312	3.9171	58.1561	7.0700	65.2260	3.1035	

	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$
A	1	-0.1074	0.8926	1.0957	-0.1177	0.9780	0.6116	0.1201	0.4915	32.0615
B							0.4884	-0.0018	0.4902	31.9746

	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h
A	-0.7243	0.1489	0.5755	-0.9879	-3.1986	4.1865	0.1230	-0.1348	0.0118
B	-0.7239	0.1491	0.5748	-0.9991	-3.2236	4.2227	0.1233	-0.1352	0.0119

	β_G^h	β_1^h	β_2^h	β_L^h
A	0.01776	-0.6282	-0.0788	1.2976
B	0.01780	-0.6289	-0.0781	1.3010

4 and 5 that 3.1043 > 3.0296).

Finally, it is appropriate to stress the difference between the Utilitarian case and the Rawlsian case: tables 2 and 3 show that $U^A > U^B$ and tables 4 and 5 that $U^A = U^B = W$. However, the Utilitarian social welfare function and the Rawlsian social welfare function lead to the same welfare level: $W = 3.1043$.

4. Moral Hazard

In what follows it is analysed the redistribution problem when the individual types $h = A, B$ are observable, but the labour choice L^h and the consumption choices x_i^h made by the individuals belonging to both groups $h = A, B$ are not observable. The moral hazard equilibrium is analysed applying the *first-order condition approach* (see Mas-Colell et al., 1995).

Ex-post the individuals maximize their utility given the direct and indirect taxation levels and the public provision of the *pseudo-necessity*:

$$\begin{aligned} \widetilde{U}^h &= \max_{\widetilde{\mathbf{x}}^h, \widetilde{L}^h} U\left(\widetilde{x}_1^h + x_G, \widetilde{x}_2^h, T - \widetilde{L}^h\right) \\ s.t. &\sum_{i=1,2} p_i \cdot \widetilde{x}_i^h + \sum_{i=1,2} t_i \cdot x_i^h \leq w^h \cdot \widetilde{L}^h - \tau^h \cdot L^h \left(\widetilde{\lambda}^h\right) \end{aligned} \tag{33}$$

with $\widetilde{x}_i^h > x_i^h$ and $\widetilde{L}^h > L^h$, where x_i^h and L^h are the solution of the *UMP* (21).

Table 4: Redistribution System with Public Information: the Rawlsian case ($\beta^A = 0.9987$ and $\beta^B = 1.0013$) with $t_1, t_2 > 0$ and $\tau^A, \tau^B < 0$

	x_G	x_1^h	x_2^h	L_1^h	L_2^h	L^h	U^h	W
A	0	31.5621	3.9402	56.7604	8.4702	65.2306	3.1043	3.1043
B		31.5621	3.9402	58.1590	7.0716	65.2306	3.1043	

	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$
A	1	0.8369	1.8369	1.0957	0.9170	2.0127	0.6117	-0.3987	1.0104	65.9076
B							0.4884	-0.5220	1.0104	65.9076

	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h
A	-0.7240	0.1489	0.5751	-0.9936	-3.2106	4.2042	0.1232	-0.1350	0.0118
B	-0.7240	0.1489	0.5751	-0.9936	-3.2106	4.2042	0.1232	-0.1350	0.0118

	β_G^h	β_1^h	β_2^h	β_L^h
A	0.01778	-0.3055	-0.0381	0.6313
B	0.01778	-0.3055	-0.0381	0.6313

The Government implements the redistribution system (taxation levels t_i and τ^h and public provision x_G) given that both type A and type B individuals choose the optimal demand systems for the goods $i = 1, 2$ and leisure ℓ in response to the redistribution system itself, i.e. subject to the incentive constraints (see Platoni, 2010). Therefore, the Government considers the commodity demands $\widetilde{x}_i^h = \widetilde{x}_i^h(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h)$, the labour supply $\widetilde{L}^h = \widetilde{L}^h(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h)$ and hence the indirect utility $\widetilde{\mathcal{V}}^h = \widetilde{\mathcal{V}}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right)$.

Using a log transformation of the *KR-SG* utility function, the *UMP* (33) is:

$$\begin{aligned} \widetilde{U}^h &= \max_{\widetilde{\mathbf{x}}^h, \widetilde{L}^h} \alpha \cdot ln\left(\widetilde{x}_1^h + x_G - b_1\right) + \beta \cdot ln\left(\widetilde{x}_2^h - b_2\right) + \\ &\quad + \gamma \cdot ln\left(\left(T - \widetilde{L}^h\right) - b_\ell\right) \\ s.t. &\quad \sum_{i=1,2} p_i \cdot \widetilde{x}_i^h + \sum_{i=1,2} t_i \cdot x_i^h \leq w^h \cdot \widetilde{L}^h - \tau^h \cdot L^h \left(\widetilde{\lambda}^h\right). \end{aligned} \tag{34}$$

Imposing $x_i^h = \widetilde{x}_i^h$ and $L^h = \widetilde{L}^h$ and considering $\widetilde{\varphi}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right) = \frac{m^h \cdot (T - b_\ell) + q_1 \cdot x_G - \sum_{i=1,2} q_i \cdot b_i}{\alpha \cdot \frac{q_1}{p_1} + \beta \cdot \frac{q_2}{p_2} + \gamma \cdot \frac{m^h}{w^h}}$, the demand system of individual $h = A, B$ for the goods $i = 1, 2$ and leisure ℓ is:

$$\begin{aligned} \widetilde{x}_1^h + x_G &= b_1 + \alpha \cdot \frac{\widetilde{\varphi}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right)}{p_1}, \\ \widetilde{x}_2^h &= b_2 + \beta \cdot \frac{\widetilde{\varphi}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right)}{p_2}, \\ T - \widetilde{L}^h &= b_\ell + \gamma \cdot \frac{\widetilde{\varphi}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right)}{w^h}; \end{aligned} \tag{35}$$

Table 5: Redistribution System with Public Information: the Rawlsian case ($\beta^A = 0.9987$ and $\beta^B = 1.0013$) with $t_1, t_2, \tau^B < 0$ and $\tau^A > 0$

	x_G	x_1^h	x_2^h	L_1^h	L_2^h	L^h	U^h	W
A	0	31.5621	3.9402	56.7604	8.4702	65.2306	3.1043	3.1043
B		31.5621	3.9402	58.1590	7.0716	65.2306	3.1043	

	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$
A	1	-0.1061	0.8939	1.0957	-0.1162	0.9795	0.6117	0.1200	0.4917	32.0729
B							0.4884	-0.0033	0.4917	32.0729

	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h
A	-0.7240	0.1489	0.5751	-0.9936	-3.2106	4.2042	0.1232	-0.1350	0.0118
B	-0.7240	0.1489	0.5751	-0.9936	-3.2106	4.2042	0.1232	-0.1350	0.0118

	β_G^h	β_1^h	β_2^h	β_L^h
A	0.01778	-0.6277	-0.0784	1.2973
B	0.01778	-0.6277	-0.0784	1.2973

then the demand system of individual $h = A, B$ (35) is function of the ratios between consumer and producer prices $\frac{q_i}{p_i}$ $(i = 1, 2)$ and between net and gross wage rates $\frac{m^h}{w^h}$. Hence, with $\widetilde{\varphi}^h = \widetilde{\varphi}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right)$ the indirect utility of type h individuals is:

$$\widetilde{\mathcal{V}}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right) = \alpha \cdot ln\left(\alpha \cdot \frac{\widetilde{\varphi}^h}{p_1}\right) + \beta \cdot ln\left(\beta \cdot \frac{\widetilde{\varphi}^h}{p_2}\right) + \gamma \cdot ln\left(\gamma \cdot \frac{\widetilde{\varphi}^h}{w^h}\right). \tag{36}$$

Therefore, from the *GP* (31) the *FOCs* (27) are obtained, where the elasticities $\widetilde{\varepsilon}_{1j}^h$, $\widetilde{\varepsilon}_{2j}^h$ and $\widetilde{\varepsilon}_{Lj}^h$ (with $j = 1, 2, L$) in (25) are provided in Appendix D (see equations in (D.8), (D.9) and (D.10)).

Moreover, the *FOC* with respect to x_G implies:

$$\begin{array}{c} \sum\limits_{h=A,B} \pi^h \cdot \left(\tau^h \cdot \frac{\partial L^h}{\partial x_G} + \sum\limits_{i=1,2} t_i \cdot \frac{\partial x_i^h}{\partial x_G}\right) = \\ \sum\limits_{h=A,B} \pi^h \cdot q_1 \cdot \frac{\alpha \cdot \frac{t_1}{p_1} + \beta \cdot \frac{t_2}{p_2} - \gamma \cdot \frac{\tau^h}{w^h}}{\alpha \cdot \frac{q_1}{p_1} + \beta \cdot \frac{q_2}{p_2} + \gamma \cdot \frac{m^h}{w^h}} - t_1 \end{array} \tag{37}$$

and then the public provision x_G is function not only of the ratios between indirect taxes and producer prices $\frac{t_i}{p_i}$ $(i = 1, 2)$ and between direct taxes and gross wage rates $\frac{\tau^h}{w^h}$ $(h = A, B)$, but also of the ratios between consumer and producer prices $\frac{q_i}{p_i}$ $(i = 1, 2)$ and between net and gross wage rates $\frac{m^h}{w^h}$ $(h = A, B)$.

Both in the Utilitarian and in the Rawlsian case the only redistribution system which maximizes the social welfare function $\widetilde{W}$ in (31) is the "corner" solution characterized by $t_1 = -p_1$ and then $q_1 = 0$[5] (see tables 6 and 7). This means that the residual demand of the *pseudo-necessity* is completely subsidized[6].

Tables 6 and 7 display (i) quantities and utilities, (ii) prices, taxation levels and expenditures, (iii) elasticities and (iv) (gross) marginal social evaluations.

Table 6: Redistribution System with Moral Hazard: the Utilitarian case ($\beta^A = \beta^B = 1$) with $-t_1 = p_1$

	x_G	x_1^h			x_2^h		L_1^h	L_2^h	L^h	U^h	W
A	26.2676	4.9348			3.6880		59.1196	8.9326	68.0522	3.0642	3.1056
B		5.6288			4.3243		55.2860	6.8065	62.0925	3.1470	
	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$	
A	1	-1	0	1.0907	1.3973	2.4880	0.5870	0.4522	0.1348	9.1759	
B							0.5136	0.3403	0.1733	10.7589	
	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h		
A	0	-0.6771	1.2581	0	-0.9060	1.6835	0	0.0506	-0.0940		
B	0	-0.6667	1.4158	0	-0.8678	1.8429	0	0.0623	-0.1322		
	β_G^h	β_1^h			β_2^h			β_L^h			
A	0	-0.1077			-0.0805			1.4849			
B	0	-0.1133			-0.0871			1.2501			

In presence of moral hazard the maximization of the Utilitarian social welfare function leads to $\widetilde{U}^A < \widetilde{U}^B$, that is the more efficient individuals (type A individuals) are worse off than the less efficient ones (type B individuals), while with public information the more efficient individuals were better off (see tables 2 and 3).

Therefore, the Rawlsian social welfare function does not benefit the less efficient individuals (type B individuals), but the more efficient ones (type A individuals): $\beta^A > 1$ and $\beta^B < 1$.

Indeed with moral hazard while in the Utilitarian case type A individuals are worse off and type B individuals are better off (from table 6 $\widetilde{U}^A < U^A$ with $3.0642 < 3.1052 < 3.1714$ and $\widetilde{U}^B > U^B$ with $3.1470 > 3.1035 >$

[5]The only solutions considered are those characterized by $q_i \geq 0$.

[6]Tables 6 and 7 show that the demand of the *pseudo-necessity* is perfectly inelastic, i.e. ε_{11}^h=0 ($h = A, B$).

Table 7: Redistribution System with Moral Hazard: the Rawlsian case ($\beta^A = 1.0523$ and $\beta^B = 0.9477$) with $-t_1 = p_1$

	x_G	x_1^h			x_2^h		L_1^h	L_2^h	L^h	U^h	W
A	27.7443	4.3297			4.4585		58.2559	8.7557	67.0116	3.1054	3.1054
B		3.2809			3.4985		56.2901	6.8935	63.1837	3.1054	
	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$	
A	1	-1	0	1.0925	1.4082	2.5007	0.5957	0.4293	0.1664	11.1494	
B							0.5044	0.3660	0.1384	8.7488	
	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h		
A	0	-0.9148	1.6776	0	-0.8883	1.6291	0	0.0609	-0.1117		
B	0	-0.9491	2.0556	0	-0.8900	1.9277	0	0.0508	-0.1100		
	β_G^h	β_1^h			β_2^h			β_L^h			
A	0	-0.0927			-0.0955			1.4349			
B	0	-0.0671			-0.0715			1.2917			

3.0296), in the Rawlsian case both type A and type B individuals are better off (from table 7 $\widetilde{U}^A > U^A$ with $3.1054 > 3.1043 < 3.1714$ and $\widetilde{U}^B > U^B$ with $3.1054 > 3.1043 > 3.0296$).

Finally, while in presence of public information the Utilitarian and Rawlsian social welfare functions have the same level ($W = 3.1043$), in case of moral hazard the level of the Utilitarian social welfare function ($\widetilde{W} = 3.1056$ with $\widetilde{U}^A < U^A$ and $\widetilde{U}^B > U^B$) is higher than the one of the Rawlsian ($\widetilde{W} = 3.1054$ with $\widetilde{U}^A > U^A$ and $\widetilde{U}^B > U^B$). However, the most appealing result is that the social welfare function is higher with moral hazard (3.1056 and 3.1054) than with pubic information (3.1043).

5. Moral Hazard and Adverse Selection

In what follows both the labour choice L^h and the consumption choice x_i^h made by the individuals belonging to both groups $h = A, B$ and the individual types $h = A, B$ are not observable.

Therefore, *ex-post* while type B individuals decide to maximize their utility given the direct and indirect taxation levels and the public provision of the *pseudo-necessity* as in (33), type A individuals decide to maximize their utility (i) given the direct and indirect taxation levels and the public provision of the *pseudo-necessity* and (ii) mimicking type B individuals since

$\tau^B < \tau^A$ and then $\hat{U}^A > \widetilde{U}^A$:

$$
\begin{aligned}
\hat{U}^A \quad &= \max_{\hat{\mathbf{x}}^A, \hat{L}^A} U\left(\hat{x}_1^A + x_G, \hat{x}_2^A, T - \hat{L}^A\right) \\
s.t. \quad & \sum_{i=1,2} p_i \cdot \hat{x}_i^A + \sum_{i=1,2} t_i \cdot x_i^B \le w^A \cdot \hat{L}^A - \tau^B \cdot L^B \left(\hat{\lambda}^A\right),
\end{aligned} \tag{38}
$$

where x_i^B and L^B are the solution of the *UMP* (21).

The Government implements the redistribution system (taxation levels t_i and τ^h and public provision x_G) given that type A and B individuals choose the optimal demand systems for the goods $i = 1, 2$ and leisure ℓ in response to the redistribution system itself (incentive constraints) and subject to the constraint (incentive-compatible or self-selection constraint) that type A individuals should not mimic type B individuals (see Platoni, 2010). Therefore the *GP* is:

$$
\begin{aligned}
\check{W} \quad &= \max_{\check{\mathbf{t}}, \check{\boldsymbol{\tau}}, \check{x}_G} W\left(\pi^A \cdot \check{\mathcal{V}}^A, \pi^B \cdot \check{\mathcal{V}}^B\right) \\
s.t. \quad & \sum_{h=A,B} \pi^h \cdot \left(\check{\tau}^h \cdot \check{L}^h + \sum_{i=1,2} \check{t}_i \cdot \check{x}_i^h\right) \ge p_1 \cdot \check{x}_G \left(\check{\lambda}_G\right) \\
& \check{\mathcal{V}}^A \ge \hat{\mathcal{V}}^A \left(\lambda_{AS}\right),
\end{aligned} \tag{39}
$$

where $\check{x}_i^h = \widetilde{x}_i(\check{\mathbf{t}}, \check{\tau}^h, \check{x}_G; \mathbf{p}, w^h)$, $\check{L}^h = \widetilde{L}(\check{\mathbf{t}}, \check{\tau}^h, \check{x}_G; \mathbf{p}, w^h)$, $\check{\mathcal{V}}^h = \widetilde{\mathcal{V}}(\check{\mathbf{t}}, \check{\tau}^h, \check{x}_G; \mathbf{p}, w^h)$ and $\hat{\mathcal{V}}^A = \hat{\mathcal{V}}(\check{\mathbf{t}}, \check{\tau}^B, \check{x}_G; \mathbf{p}, \mathbf{w})$, with $\mathbf{w} = (w^A, w^B)$. From the *GP* (39) it is possible to obtain the *FOCs*:

$$
\begin{aligned}
\frac{\sum_{h=A,B} \pi^h \cdot (\check{\tau}^h \cdot \check{\varepsilon}_{L1}^h \cdot \check{L}^h + \sum_{i=1,2} \check{t}_i \cdot \check{\varepsilon}_{i1}^h \cdot \check{x}_i^h)}{\check{q}_1 \cdot \check{x}_1} &= -1 - \frac{1}{\check{\lambda}_G} \cdot \frac{\sum_{h=A,B} \pi^h \cdot \check{\beta}_1^h + \lambda_{AS} \cdot \sigma_1}{\check{x}_1}, \\
\frac{\sum_{h=A,B} \pi^h \cdot (\check{\tau}^h \cdot \check{\varepsilon}_{L2}^h \cdot \check{L}^h + \sum_{i=1,2} \check{t}_i \cdot \check{\varepsilon}_{i2}^h \cdot \check{x}_i^h)}{\check{q}_2 \cdot \check{x}_2} &= -1 - \frac{1}{\check{\lambda}_G} \cdot \frac{\sum_{h=A,B} \pi^h \cdot \check{\beta}_2^h + \lambda_{AS} \cdot \sigma_2}{\check{x}_2}, \\
\frac{\check{\tau}^A \cdot \check{\varepsilon}_{LL}^A \cdot \check{L}^A + \sum_{i=1,2} \check{t}_i \cdot \check{\varepsilon}_{iL}^A \cdot \check{x}_i^A}{\check{m}^A \cdot \check{L}^A} &= 1 - \frac{1}{\check{\lambda}_G} \cdot \frac{\check{\beta}_L^A + \frac{\lambda_{AS}}{\pi^A} \cdot \sigma^A}{\check{L}^A} = \\
&= 1 - \frac{1}{\check{\lambda}_G} \cdot \frac{\left(1 + \frac{\lambda_{AS}}{\pi^A}\right) \cdot \check{\beta}_L^A}{\check{L}^A}, \\
\frac{\check{\tau}^B \cdot \check{\varepsilon}_{LL}^B \cdot \check{L}^B + \sum_{i=1,2} \check{t}_i \cdot \check{\varepsilon}_{iL}^B \cdot \check{x}_i^B}{\check{m}^B \cdot \check{L}^B} &= 1 - \frac{1}{\check{\lambda}_G} \cdot \frac{\check{\beta}_L^B + \frac{\lambda_{AS}}{\pi^B} \cdot \hat{\sigma}^A}{\check{L}^B}, \\
\sum_{h=A,B} \pi^h \cdot \left(\check{\tau}^h \cdot \frac{\partial \check{L}^h}{\partial \check{x}_G} + \sum_{i=1,2} \check{t}_i \cdot \frac{\partial \check{x}_i^h}{\partial \check{x}_G}\right) &= p_1 - \frac{1}{\check{\lambda}_G} \cdot \sum_{h=A,B} \pi^h \cdot \check{\beta}_G^h + \\
&\quad - \frac{\lambda_{AS}}{\check{\lambda}_G} \cdot \sigma_G,
\end{aligned} \tag{40}
$$

where $\sigma_1 = \frac{\partial \breve{W}}{\partial \breve{\mathcal{V}}^A} \cdot \frac{\partial \breve{\mathcal{V}}^A}{\partial \breve{q}_1} - \frac{\partial \breve{W}}{\partial \hat{\mathcal{V}}^A} \cdot \frac{\partial \hat{\mathcal{V}}^A}{\partial \breve{q}_1}$, $\sigma_2 = \frac{\partial \breve{W}}{\partial \breve{\mathcal{V}}^A} \cdot \frac{\partial \breve{\mathcal{V}}^A}{\partial \breve{q}_2} - \frac{\partial \breve{W}}{\partial \hat{\mathcal{V}}^A} \cdot \frac{\partial \hat{\mathcal{V}}^A}{\partial \breve{q}_2}$, $\sigma^A = \breve{\beta}_L^A = \frac{\partial \breve{W}}{\partial \breve{\mathcal{V}}^A} \cdot \frac{\partial \breve{\mathcal{V}}^A}{\partial \breve{m}^A}$, $\hat{\sigma}^A = -\frac{\partial \breve{W}}{\partial \hat{\mathcal{V}}^A} \cdot \frac{\partial \hat{\mathcal{V}}^A}{\partial \breve{m}^B}$ and $\sigma_G = \frac{\partial \breve{W}}{\partial \breve{\mathcal{V}}^A} \cdot \frac{\partial \breve{\mathcal{V}}^A}{\partial \breve{x}_G} - \frac{\partial \breve{W}}{\partial \hat{\mathcal{V}}^A} \cdot \frac{\partial \hat{\mathcal{V}}^A}{\partial \breve{x}_G}$.

Considering a log transformation of the *KR-SG* utility function, the *UMP* (38) is:

$$\begin{aligned} \hat{U}^A &= \max_{\hat{\mathbf{x}}^A, \hat{L}^A} \ \alpha \cdot ln\left(\hat{x}_1^A + x_G - b_1\right) + \beta \cdot ln\left(\hat{x}_2^A - b_2\right) + \\ &\quad + \gamma \cdot ln\left(\left(T - \hat{L}^A\right) - b_\ell\right) \\ s.t. &\quad \sum_{i=1,2} p_i \cdot \hat{x}_i^A + \sum_{i=1,2} t_i \cdot \widetilde{x}_i^B \leq w^A \cdot \hat{L}^A - \tau^B \cdot \widetilde{L}^B \left(\hat{\lambda}^A\right), \end{aligned} \tag{41}$$

and with $\hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right) = \left(w^A - w^B\right) \cdot (T - b_\ell) + \frac{m^B \cdot (T - b_\ell) + q_1 \cdot x_G - \sum_{i=1,2} q_i \cdot b_i}{\alpha \cdot \frac{q_1}{p_1} + \beta \cdot \frac{q_2}{p_2} + \gamma \cdot \frac{m^B}{w^B}}$ the demand system of the mimicking type A individuals for the goods $i = 1, 2$ and leisure ℓ is:

$$\begin{aligned} \hat{x}_1^A + x_G &= b_1 + \alpha \cdot \frac{\hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right)}{p_1}, \\ \hat{x}_2^A &= b_2 + \beta \cdot \frac{\hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right)}{p_2}, \\ T - \hat{L}^A &= b_\ell + \gamma \cdot \frac{\hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right)}{w^A}. \end{aligned} \tag{42}$$

Therefore, the demand system of the mimicking type A individuals (42) is function of the gap between type A and type B gross wage rates $w^A - w^B$ and of the ratios between consumer and producer prices $\frac{q_i}{p_i}$ $(i = 1, 2)$ and between net and gross wage rates of type B individuals $\frac{m^B}{w^B}$. Hence, with $\hat{\varphi}^A = \hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right)$ the indirect utility of the mimicking type A individuals is:

$$\begin{aligned} \hat{\mathcal{V}}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right) = &\quad \alpha \cdot ln\left(\alpha \cdot \frac{\hat{\varphi}^A}{p_1}\right) + \beta \cdot ln\left(\beta \cdot \frac{\hat{\varphi}^A}{p_2}\right) + \\ &+ \gamma \cdot ln\left(\gamma \cdot \frac{\hat{\varphi}^A}{w^A}\right) \end{aligned} \tag{43}$$

and with $\hat{\mathcal{V}}^A = \hat{\mathcal{V}}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right)$ the *GP* is:

$$\begin{aligned} \breve{W} &= \max_{\breve{\mathbf{t}}, \breve{\boldsymbol{\tau}}, \breve{x}_G} \sum_{h=A,B} \beta^h \cdot \pi^h \cdot \breve{\mathcal{V}}^h \\ s.t. &\quad \sum_{h=A,B} \pi^h \cdot \left(\breve{\tau}^h \cdot \breve{L}^h + \sum_{i=1,2} \breve{t}_i \cdot \breve{x}_i^h\right) \geq p_1 \cdot \breve{x}_G \left(\breve{\lambda}_G\right) \\ &\quad \breve{\mathcal{V}}^A \geq \hat{\mathcal{V}}^A \left(\lambda_{AS}\right) \end{aligned} \tag{44}$$

which yields the *FOCs* in (40), where the elasticities $\breve{\varepsilon}_{1j}^h$, the elasticities $\breve{\varepsilon}_{2j}^h$ and the elasticities $\breve{\varepsilon}_{Lj}^h$, with $j = 1, 2, L$, are the same as in (D.8), (D.9) and

(D.10), where the (gross) marginal social evaluations of the h's utility with respect to good $i = 1, 2$ ($\check{\beta}_i^h$), to labour L ($\check{\beta}_L^h$) and to publicly provided good G ($\check{\beta}_G^h$) are the same as in (D.7), and where σ_1, σ_2, σ^A, $\hat{\sigma}^A$ and σ_G are provided in Appendix D (see equations in (D.11)).

Since the value of λ_{AS} withdraws the effects of different values of the weights β^A and β^B, the results of the Rawlsian case are equal to the results of the Utilitarian case $\forall \beta^h$. Moreover, the only redistribution system which maximizes the social welfare function $\check{W}$ in (44) is the "corner" solution characterized by $t_1 = -p_1$ and then by $q_1 = 0$ (see table 8), i.e. the residual demand of the *pseudo-necessity* is completely subsidized[7].

(i) Quantities and utilities, (ii) prices, taxation levels and expenditures, (iii) elasticities and (iv) (gross) marginal social evaluations are displayed in table 8. In addition the simulation yields $\sigma_1 = -0.0703$, $\sigma_2 = -0.0639$, $\sigma^A = \frac{\check{\beta}_L^A}{\beta^A} = 1.1939$, $\hat{\sigma}^A = 1.2891$ and $\sigma_G = 0$.

Table 8: Redistribution System with Moral Hazard and Adverse Selection with $-t_1 = p_1$

	x_G	x_1^h			x_2^h		L_1^h	L_2^h	L^h	U^h	W
A	27.4092	6.1619			5.7761		56.8262	8.4821	65.3084	3.1714	3.1005
B		2.1421			2.1071		58.0731	7.0630	65.1361	3.0296	
	p_1	t_1	q_1	p_2	t_2	q_2	w^h	τ^h	m^h	$y^h = e^h$	
A	1	-1	0	1.0956	1.4086	2.5042	0.6109	0.3894	0.2215	14.4645	
B							0.4891	0.4081	0.0810	5.2765	
	ε_{11}^h	ε_{12}^h	ε_{1L}^h	ε_{21}^h	ε_{22}^h	ε_{2L}^h	ε_{L1}^h	ε_{L2}^h	ε_{LL}^h		
A	0	-0.8057	1.4449	0	-0.8595	1.5414	0	0.0783	-0.1405		
B	0	-0.9153	2.0503	0	-0.9305	2.0843	0	0.0310	-0.0695		
	β_G^h	β_1^h			β_2^h			β_L^h			
A	0	-0.1126			-0.1056			1.1939			
B	0	-0.0527			-0.0518			1.6022			

In absence of a tax evasion fine the utilities obtained under both moral hazard and adverse selection are the same as those obtained without Government intervention (compare tables 1 and 8): $\check{U}^A = U^A = 3.1714$ and $\check{U}^B = U^B = 3.0296$.

[7]Table 8 shows that the demand of the *pseudo-necessity* is perfectly inelastic, i.e. ε_{11}^h=0 ($h = A, B$).

5.1. Tax Evasion Fine

This section assumes the Government may audit the individuals who have hidden information (tax evasion by adverse selection) such that the Government intervenes to punish only tax evasion by adverse selection.

If type A individuals have hidden information, their expected indirect utility is:

$$E\left(\hat{\mathcal{V}}^A\right) = (1-\pi)\cdot\hat{\mathcal{V}}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right) + \pi\cdot\hat{\mathcal{V}}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right) \quad (45)$$

with π the probability of being audited and $\hat{F}^A$ the fine applied in case of tax evasion by adverse selection.

The Government decides both the optimal redistribution system $\left(\breve{\mathbf{t}},\breve{\boldsymbol{\tau}},\breve{x}_G\right)$ and the amount of the fine $\hat{F}^A$. Therefore, the GP is:

$$\begin{aligned}
\breve{W} &= \max_{\breve{\mathbf{t}},\breve{\boldsymbol{\tau}},\breve{x}_G,\hat{F}^A} W\left(\pi^A\cdot\breve{\mathcal{V}}^A,\pi^B\cdot\breve{\mathcal{V}}^B\right)\\
s.t. \quad & \sum_{h=A,B}\pi^h\cdot\left(\breve{\tau}^h\cdot\breve{L}^h+\sum_{i=1,2}\breve{t}_i\cdot\breve{x}_i^h\right)\geq p_1\cdot\breve{x}_G\left(\breve{\lambda}_G\right)\\
& \breve{\mathcal{V}}^A \geq E\left(\hat{\mathcal{V}}^A\right)\left(\lambda_{AS}\right),
\end{aligned} \quad (46)$$

where $\breve{\mathcal{V}}^h = \widetilde{\mathcal{V}}(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$ and with $\breve{x}_i^h = \widetilde{x}_i(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$ and $\breve{L}^h = \widetilde{L}(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$. Since

$$-\lambda_{AS}\cdot\frac{\partial E\left(\hat{\mathcal{V}}^A\right)}{\partial\hat{F}^A} = 0 \rightarrow \lambda_{AS} = 0 \quad (47)$$

the problem reverts to the *FOCs* in (27), where the elasticities are the ones in (D.8), (D.9) and (D.10) and where the (gross) marginal social evaluations of the h's utility are the ones in (D.7). Therefore $\hat{F}^A$ is determined from $\breve{\mathcal{V}}^A = E\left(\hat{\mathcal{V}}^A\right)$.

As in the previous sections, a log transformation of the *KR-SG* utility function is considered. If type A individuals have hidden information their demand system for the goods $i=1,2$ and leisure ℓ is the one in (42) when not audited, while when audited is:

$$\begin{aligned}
\hat{x}_1^A + x_G &= b_1+\alpha\cdot\frac{\hat{\varphi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{p_1},\\
\hat{x}_2^A &= b_2+\beta\cdot\frac{\hat{\varphi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{p_2},\\
T-\hat{L}^A &= b_\ell+\gamma\cdot\frac{\hat{\varphi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{w^A}
\end{aligned} \quad (48)$$

with $\hat{\varphi}\left(\mathbf{t}, \tau^B, x_G, \hat{F}^A; \mathbf{p}, \mathbf{w}\right) = \hat{\varphi}\left(\mathbf{t}, \tau^B, x_G; \mathbf{p}, \mathbf{w}\right) - \hat{F}^A$.

Considering an audit probability $\pi = 0.5$, in the Utilitarian case the results obtained are equal to those displayed in table 6 with $\hat{F}^A = 17.5105$ and in the Rawlsian case the results are equal to those displayed in table 7 with $\hat{F}^A = 11.3464$ (i.e., reversion to the moral hazard outcomes).

In Appendix E it is assumed the Government may audit both the individuals having hidden information (tax evasion by adverse selection) and the individuals taking hidden actions (tax evasion by moral hazard); obviously there is a reversion to the public information outcomes (see tables 2, 3, 4 and 5).

6. Conclusions

The article considers an economy characterized by two commodities (the *pseudo-necessity* and the *pseudo-luxury*) and by two individual types (the more efficient and the less efficient workers) and analyses a redistribution system implemented through direct and indirect taxation and the public provision of the *pseudo-necessity*.

When the economy is characterized by moral hazard only, (i) the level of social welfare is higher than the level achieved in case of public information and (ii) if the Government maximizes an Utilitarian social welfare function the more efficient individuals are worse off than the less efficient ones.

Hence to face the tax evasion by moral hazard the Government can impose the incentive constraints[8] without limiting, but rather strengthening, the redistributive purposes of the taxation system (with the public provision of the *pseudo-necessity*). On the contrary, if the Government faced the tax evasion both by moral hazard and adverse selection imposing not only the incentive constraints, but also the incentive-compatible (or self-selection) constraints, then the Government would not reach the redistributive aim of the public intervention.

Therefore, if individuals not only take hidden actions (tax evasion by moral hazard), but they also exploit hidden information (tax evasion by adverse selection), the Government succeeds in redistributing resources only if a tax evasion fine is established. Hence, as suggested by Cremer and

[8] In Italy the policy tools representing the incentive constraints are the “studi di settore”and the “redditometro”.

Gahavari (1993), the set of Government policy tools should include the audit strategy as well as the tax rates.

Appendices

A. Properties of the Walrasian Demand Functions

The implications of homogeneity of degree zero come from $x_i(\mathbf{p}, w^h, I) = x_i(\alpha \cdot \mathbf{p}, \alpha \cdot w^h, \alpha \cdot I)$ and $\ell(\mathbf{p}, w^h, I) = \ell(\alpha \cdot \mathbf{p}, \alpha \cdot w^h, \alpha \cdot I)$. Differentiating these expressions with respect to α and evaluating the derivative at $\alpha = 1$ allows to get the results (A.1).

Proposition A.1. *(see Mas-Colell et al., 1995) If the Walrasian demand functions $x_i(\boldsymbol{p}, w^h, I)$ and $\ell(\boldsymbol{p}, w^h, I)$ are homogeneous of degree zero in $\boldsymbol{p}$, w^h and I, then for all $\boldsymbol{p}$, w^h and I:*

$$\begin{aligned} &\frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I} \cdot I + \sum_{j=1,2} \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} \cdot p_j + \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} \cdot w^h = 0, \\ &\frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial I} \cdot I + \sum_{j=1,2} \frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} \cdot p_j + \frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} \cdot w^h = 0. \end{aligned} \tag{A.1}$$

By the *Walras' law*, it is known that:

$$\sum_{i=1,2} p_i \cdot x_i\left(\mathbf{p}, w^h, I\right) + w^h \cdot \ell\left(\mathbf{p}, w^h, I\right) = w^h \cdot T + I \tag{A.2}$$

and then:

$$\begin{aligned} &\sum_{i=1,2} p_i \cdot x_i\left(\mathbf{p}, w^h, I\right) + w^h \cdot \left(\ell\left(\mathbf{p}, w^h, I\right) - T\right) &= \\ = &\sum_{i=1,2} p_i \cdot x_i\left(\mathbf{p}, w^h, I\right) - w^h \cdot L\left(\mathbf{p}, w^h, I\right) &= I \end{aligned} \tag{A.3}$$

for all $\mathbf{p}$, w^h and I.

Proposition A.2. *(see Mas-Colell et al., 1995) If the Walrasian demand functions $x_i(\boldsymbol{p}, w^h, I)$ and $\ell(\boldsymbol{p}, w^h, I)$ satisfy the Walras' law, then for all $\boldsymbol{p}$ and w^h:*

$$\begin{aligned} &\sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I} + w^h \cdot \frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial I} &= \\ = &\sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I} &= 1. \end{aligned} \tag{A.4}$$

Proposition A.3. *(see Mas-Colell et al., 1995) If the Walrasian demand functions $x_i(\boldsymbol{p}, w^h, I)$ and $\ell(\boldsymbol{p}, w^h, I)$ satisfy the Walras' law, then for all $\boldsymbol{p}$ and w^h:*

$$\begin{array}{rll} & \sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} + w^h \cdot \frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} + x_j\left(\boldsymbol{p}, w^h, I\right) & = \\ = & \sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} + x_j\left(\boldsymbol{p}, w^h, I\right) & = 0, \\ & \sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} + w^h \cdot \frac{\partial \ell\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} + \left(\ell\left(\boldsymbol{p}, w^h, I\right) - T\right) & = \\ = & \sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} - L\left(\boldsymbol{p}, w^h, I\right) & = 0. \end{array} \tag{A.5}$$

B. Hicksian Demands, Slutsky Equations and Roy's Identities

Proposition B.1. *(see Mas-Colell et al., 1995) Suppose that $U(\cdot)$ is a continuous utility function representing a locally nonsatiated and strictly convex preference relation $\succcurlyeq$ defined on the consumption set $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{L}) = \mathbb{R}^3_+$. For all $\boldsymbol{p}$, w^h and U, the Hicksian demands $h_i(\boldsymbol{p}, w^h, U)$ for $i = 1, 2$ and $h_L(\boldsymbol{p}, w^h, U)$ are:*

$$\begin{array}{rl} h_i\left(\boldsymbol{p}, w^h, U\right) = & \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_i}, \\ h_L\left(\boldsymbol{p}, w^h, U\right) = & -\frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h}. \end{array} \tag{B.1}$$

Proof B.1. *(see Mas-Colell et al., 1995) Using the chain rule, the change in expenditure:*

$$e\left(\boldsymbol{p}, w^h, U\right) = \sum_{i=1,2} p_i \cdot h_i\left(\boldsymbol{p}, w^h, U\right) - w^h \cdot h_L\left(\boldsymbol{p}, w^h, U\right) \tag{B.2}$$

can be written as:

$$\begin{array}{rl} \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_i} & = h_i\left(\boldsymbol{p}, w^h, U\right) + \\ & + \sum_{j=1,2} p_j \cdot \frac{\partial h_j\left(\boldsymbol{p},w^h,U\right)}{\partial p_i} - w^h \cdot \frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial p_i}, \\ \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} & = -h_L\left(\boldsymbol{p}, w^h, U\right) + \\ & + \sum_{j=1,2} p_i \cdot \frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} - w^h \cdot \frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial w^h}. \end{array} \tag{B.3}$$

From equation (8) of the EMP (7) it is possible to write:

$$\begin{aligned} \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_i} &= h_i\left(\boldsymbol{p},w^h,U\right) + \nu^h \cdot \left(\sum_{j=1,2} \frac{\partial U^h}{\partial x_j^h}\cdot\frac{\partial h_j^h}{\partial p_i} - \frac{\partial U^h}{\partial L^h}\cdot\frac{\partial h_L^h}{\partial p_i}\right), \\ \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= -h_L\left(\boldsymbol{p},w^h,U\right) + \nu^h \cdot \left(\sum_{j=1,2} \frac{\partial U^h}{\partial x_j^h}\cdot\frac{\partial h_i^h}{\partial w^h} - \frac{\partial U^h}{\partial L^h}\cdot\frac{\partial h_L^h}{\partial w^h}\right). \end{aligned} \tag{B.4}$$

Since the constraint $U(\boldsymbol{x}^h, T-L^h) = U(h_1(\boldsymbol{p},w^h,U), h_2(\boldsymbol{p},w^h,U), T - h_L(\boldsymbol{p},w^h,U)) = U$ *holds for all* $\boldsymbol{p}$ *and* w^h*, it is known that:*

$$\begin{aligned} \sum_{j=1,2} \frac{\partial U^h}{\partial x_j^h}\cdot\frac{\partial h_j^h}{\partial p_i} - \frac{\partial U^h}{\partial L^h}\cdot\frac{\partial h_L^h}{\partial p_i} &= 0, \\ \sum_{j=1,2} \frac{\partial U^h}{\partial x_j^h}\cdot\frac{\partial h_i^h}{\partial w^h} - \frac{\partial U^h}{\partial L^h}\cdot\frac{\partial h_L^h}{\partial w^h} &= 0 \end{aligned} \tag{B.5}$$

and then the (B.1) is obtained.

Proposition B.2. ***Slutsky Equations*** *(see Mas-Colell et al., 1995) Suppose that* $U(\cdot)$ *is a continuous utility function representing a locally non-satiated and strictly convex preference relation* $\succcurlyeq$ *defined on the consumption set* $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{L}) = \mathbb{R}^3_+$*. Then, for all* $(\boldsymbol{p}, w^h)$ *and* $U = \mathcal{V}(\boldsymbol{p}, w^h, I)$ *it is possible to write:*

$$\begin{aligned} S_{ij}^h &= \frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &&= \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} + \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot x_j\left(\boldsymbol{p},w^h,I\right), \\ S_{Lj}^h &= \frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &&= \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} + \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot x_j\left(\boldsymbol{p},w^h,I\right); \\ S_{iL}^h &= \frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &&= \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} - \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot L\left(\boldsymbol{p},w^h,I\right), \\ S_{LL}^h &= \frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &&= \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} - \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot L\left(\boldsymbol{p},w^h,I\right). \end{aligned} \tag{B.6}$$

(see Hicks, 1946). Therefore, the overall effect of a price change or of a wage change can be decomposed into a substitution effect and an income effect (see Abbott and Ashenfelter, 1976):

$$\begin{aligned} \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= S_{ij}^h - \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot x_j\left(\boldsymbol{p},w^h,I\right), \\ \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= S_{Lj}^h - \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot x_j\left(\boldsymbol{p},w^h,I\right); \\ \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= S_{iL}^h + \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot L\left(\boldsymbol{p},w^h,I\right), \\ \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= S_{LL}^h + \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I}\cdot L\left(\boldsymbol{p},w^h,I\right). \end{aligned} \tag{B.7}$$

Proof B.2. *(see Mas-Colell et al., 1995) Since* $e(\boldsymbol{p}, w^h, U) = \sum_{i=1,2} p_i \cdot x_i^h - x_i^h - w^h \cdot L^h$*, differentiating* $h_i(\boldsymbol{p}, w^h, U) = x_i(\boldsymbol{p}, w^h, e(\boldsymbol{p}, w^h, U))$ *and* $h_L(\boldsymbol{p}, w^h, U) = L(\boldsymbol{p}, w^h, e(\boldsymbol{p}, w^h, U))$ *with respect to* p_j *and* w^h *and evaluating at* $(\boldsymbol{p}, w^h, U)$*, allows to get:*

$$\begin{aligned}
\frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &= \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_j}, \\
\frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &= \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_j}; \\
\frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h}, \\
\frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h}.
\end{aligned} \tag{B.8}$$

Using equation (B.3) yields

$$\begin{aligned}
\frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &= \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_j\left(\boldsymbol{p},w^h,U\right), \\
\frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &= \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_j\left(\boldsymbol{p},w^h,U\right); \\
\frac{\partial h_i\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial x_i\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_L\left(\boldsymbol{p},w^h,U\right), \\
\frac{\partial h_L\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial L\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_L\left(\boldsymbol{p},w^h,U\right).
\end{aligned} \tag{B.9}$$

Finally since $e(\boldsymbol{p}, w^h, U) = I$*,* $h_i(\boldsymbol{p}, w^h, U) = x_i(\boldsymbol{p}, w^h, I)$ *and* $h_L(\boldsymbol{p}, w^h, U) = L(\boldsymbol{p}, w^h, I)$ *the equations in (B.6) are obtained.*

Proposition B.3. ***Roy's Identities*** *(see Mas-Colell et al., 1995) Suppose that* $U(\cdot)$ *is a continuous utility function representing a locally non-satiated and strictly convex preference relation* $\succcurlyeq$ *defined on the consumption set* $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{L}) = \mathbb{R}^3_+$*. Suppose also that the indirect utility function is differentiable at* $(\bar{p}, \bar{w}) \gg 0$*. Then*

$$\begin{aligned}
\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= -\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial I} \cdot x_j\left(\boldsymbol{p},w^h,I\right), \\
\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} &= \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial I} \cdot L\left(\boldsymbol{p},w^h,I\right).
\end{aligned} \tag{B.10}$$

Proof B.3. ***First Proof*** *(see Mas-Colell et al., 1995) Assume that* $x_i(\boldsymbol{p}, w^h, I)$ *and* $L(\boldsymbol{p}, w^h, I)$ *are differentiable and* $x_i(\bar{p}, \bar{w}^h, \bar{I}) \gg 0$ *and* $L(\bar{p}, \bar{w}^h, \bar{I}) \gg 0$*. By the chain rule it is possible to write:*

$$\begin{aligned}
\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= \sum_{i=1,2} \frac{\partial U^h}{\partial x_i^h} \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} - \frac{\partial U^h}{\partial \ell^h} \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j}, \\
\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} &= \sum_{i=1,2} \frac{\partial U^h}{\partial x_i^h} \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} - \frac{\partial U^h}{\partial \ell^h} \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial w^h}.
\end{aligned} \tag{B.11}$$

Substituting $\frac{\partial U^h}{\partial x_i^h} = \lambda^h \cdot p_i$ *and* $\frac{\partial U^h}{\partial \ell^h} = \lambda^h \cdot w^h$ *and using the FOCs in (4) of the UMP in (3) yields:*

$$\begin{aligned} \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= \lambda^h \cdot \left(\sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} \right), \\ \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} &= \lambda^h \cdot \left(\sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} \right). \end{aligned} \tag{B.12}$$

If the Walrasian demand functions $x_i(\boldsymbol{p}, w^h, I)$ *and* $\ell(\boldsymbol{p}, w^h, I)$ *satisfy the Walras' law, i.e. the equations in (A.5), then*

$$\begin{aligned} \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial p_j} &= -\lambda^h \cdot x_j\left(\boldsymbol{p},w^h,I\right), \\ \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial w^h} &= \lambda^h \cdot L\left(\boldsymbol{p},w^h,I\right). \end{aligned} \tag{B.13}$$

By the chain rule, the change in utility from a marginal increase in I *is given by*

$$\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial I} = \sum_{i=1,2} \frac{\partial U^h}{\partial x_i^h} \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I} - \frac{\partial U^h}{\partial \ell^h} \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I}. \tag{B.14}$$

From the FOCs (4) of the UMP (3) it is possible to write:

$$\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial I} = \lambda^h \cdot \left(\sum_{i=1,2} p_i \cdot \frac{\partial x_i\left(\boldsymbol{p},w^h,I\right)}{\partial I} - w^h \cdot \frac{\partial L\left(\boldsymbol{p},w^h,I\right)}{\partial I} \right) \tag{B.15}$$

and the (A.4) allows to obtain the marginal utility of income:

$$\frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,I\right)}{\partial I} = \lambda^h. \tag{B.16}$$

Therefore the (B.10) is obtained.

Second Proof *(see Mas-Colell et al., 1995) Assume* $U = \mathcal{V}(\boldsymbol{p}, w^h, I)$. *Since* $\mathcal{V}(\boldsymbol{p}, w^h, e(\boldsymbol{p}, w^h, U)) = U$ *holds for all* $\boldsymbol{p}$ *and* w^h, *differentiating with respect to* p_j *and evaluating at* $p_j = \bar{p}_j$ *yields to:*

$$\begin{aligned} \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial p_j} &= 0, \\ \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot \frac{\partial e\left(\boldsymbol{p},w^h,U\right)}{\partial w^h} &= 0. \end{aligned} \tag{B.17}$$

Substituting the (B.1) into the (B.17) allows to get:

$$\begin{array}{ll} \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial p_j} + \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_i\left(\boldsymbol{p},w^h,U\right) & = 0, \\ \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial w^h} + \frac{\partial \mathcal{V}\left(\boldsymbol{p},w^h,e\left(\boldsymbol{p},w^h,U\right)\right)}{\partial I} \cdot h_L\left(\boldsymbol{p},w^h,U\right) & = 0. \end{array} \tag{B.18}$$

Finally, since $e(\boldsymbol{p},w^h,U) = I$, the (B.10) is obtained.

C. Substitution and Income Effects

Consider the Walrasian demand functions $x_i(\mathbf{p},w^h,I)$ and $\ell(\mathbf{p},w^h,I)$ allows to relate the Hicksian and Walrasian demands as follows:

$$\begin{array}{rcl} h_i\left(\mathbf{p},w^h,U\right) & = & x_i\left(\mathbf{p},w^h,e\left(\mathbf{p},w^h,U\right)\right), \\ h_L\left(\mathbf{p},w^h,U\right) & = & L\left(\mathbf{p},w^h,e\left(\mathbf{p},w^h,U\right)\right); \\ x_i\left(\mathbf{p},w^h,I\right) & = & h_i\left(\mathbf{p},w^h,\mathcal{V}\left(\mathbf{p},w^h,I\right)\right), \\ L\left(\mathbf{p},w^h,I\right) & = & h_L\left(\mathbf{p},w^h,\mathcal{V}\left(\mathbf{p},w^h,I\right)\right). \end{array} \tag{C.1}$$

The equations (B.6) can be rewritten as:

$$\begin{array}{ll} \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial p_j} & = \frac{\partial h_i\left(\mathbf{p},w^h,U\right)}{\partial p_j} - \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot x_j\left(\mathbf{p},w^h,I\right) = \\ & = S^h_{ij} - \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot x_j\left(\mathbf{p},w^h,I\right), \\ \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial p_j} & = \frac{\partial h_L\left(\mathbf{p},w^h,U\right)}{\partial p_j} - \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot x_j\left(\mathbf{p},w^h,I\right) = \\ & = S^h_{Lj} - \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot x_j\left(\mathbf{p},w^h,I\right); \\ \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial p_j} & = \frac{\partial h_i\left(\mathbf{p},w^h,U\right)}{\partial w^h} + \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot L\left(\mathbf{p},w^h,I\right) = \\ & = S^h_{iL} + \frac{\partial x_i\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot L\left(\mathbf{p},w^h,I\right), \\ \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial p_j} & = \frac{\partial h_L\left(\mathbf{p},w^h,U\right)}{\partial w^h} + \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot L\left(\mathbf{p},w^h,I\right) = \\ & = S^h_{LL} + \frac{\partial L\left(\mathbf{p},w^h,I\right)}{\partial I} \cdot L\left(\mathbf{p},w^h,I\right). \end{array} \tag{C.2}$$

If the first equation in (C.2) is multiplied by $\frac{p_j}{x_i^h}$, the second by $\frac{p_j}{L^h}$, the third by $\frac{w^h}{x_i^h}$, the fourth by $\frac{w^h}{L^h}$ and the last terms (income effects) by $\frac{I}{I}$, then it is possible to write:

$$\begin{aligned}
\underbrace{\frac{\partial x_i\big(\mathbf{p},w^h,I\big)}{\partial p_j}\cdot\frac{p_j}{x_i^h}}_{\varepsilon_{ij}^h} &= \underbrace{S_{ij}^h\cdot\frac{p_j}{x_i^h}}_{\tilde{\varepsilon}_{ij}^h} - \underbrace{\frac{p_j\cdot x_j\big(\mathbf{p},w^h,I\big)}{I}}_{s_j^h}\cdot\underbrace{\frac{\partial x_i\big(\mathbf{p},w^h,I\big)}{\partial I}\cdot\frac{I}{x_i^h}}_{\eta_i^h},\\
\underbrace{\frac{\partial L\big(\mathbf{p},w^h,I\big)}{\partial p_j}\cdot\frac{p_j}{L^h}}_{\varepsilon_{Lj}^h} &= \underbrace{S_{Lj}^h\cdot\frac{p_j}{L^h}}_{\tilde{\varepsilon}_{Lj}^h} - \underbrace{\frac{p_j\cdot x_j\big(\mathbf{p},w^h,I\big)}{I}}_{s_j^h}\cdot\underbrace{\frac{\partial L\big(\mathbf{p},w^h,I\big)}{\partial I}\cdot\frac{I}{L^h}}_{\eta_L^h};\\
\underbrace{\frac{\partial x_i\big(\mathbf{p},w^h,I\big)}{\partial w^h}\cdot\frac{w^h}{x_i^h}}_{\varepsilon_{iL}^h} &= \underbrace{S_{iL}^h\cdot\frac{w^h}{x_i^h}}_{\tilde{\varepsilon}_{iL}^h} - \underbrace{\frac{w^h\cdot L\big(\mathbf{p},w^h,I\big)}{I}}_{s_L^h}\cdot\underbrace{\frac{\partial x_i\big(\mathbf{p},w^h,I\big)}{\partial I}\cdot\frac{I}{x_i^h}}_{\eta_i^h},\\
\underbrace{\frac{\partial L\big(\mathbf{p},w^h,I\big)}{\partial w^h}\cdot\frac{w^h}{L^h}}_{\varepsilon_{LL}^h} &= \underbrace{S_{LL}^h\cdot\frac{w^h}{L^h}}_{\tilde{\varepsilon}_{LL}^h} - \underbrace{\frac{w^h\cdot L\big(\mathbf{p},w^h,I\big)}{I}}_{s_L^h}\cdot\underbrace{\frac{\partial L\big(\mathbf{p},w^h,I\big)}{\partial I}\cdot\frac{I}{L^h}}_{\eta_L^h}.
\end{aligned}\tag{C.3}$$

If the good $i = 1$ is a necessity then the income elasticity is between 0 and 1, that is $0 \leq \eta_1^h < 1 \Rightarrow 0 \leq \frac{\partial x_1^h}{\partial I}\cdot\frac{I}{x_1^h} < 1 \Rightarrow \frac{\partial x_1^h}{\partial I} < \frac{x_1^h}{I}$, and if the good $i = 2$ is a luxury then the income elasticity is greater than 1, that is $\eta_2^h \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial I}\cdot\frac{I}{x_2^h} \geq 1 \Rightarrow \frac{\partial x_2^h}{\partial I} \geq \frac{x_2^h}{I}$.

D. *KR-SG* Utility Function: Elasticities, (Gross) Marginal Social Evaluations and Adverse Selection Terms

With public information (section 3) and considering a log transformation of the *KR-SG* utility function, the elasticities ε_{1j}^h, with $j = 1, 2, L$, in (10) are:

$$\begin{aligned}
\varepsilon_{11}^h &= -\alpha\cdot\frac{w^h\cdot(T-b_\ell)-p_2\cdot b_2}{(p_1)^2}\cdot\frac{p_1}{x_1^h},\\
\varepsilon_{12}^h &= -\alpha\cdot\frac{b_2}{p_1}\cdot\frac{p_2}{x_1^h},\\
\varepsilon_{1L}^h &= \alpha\cdot\frac{T-b_\ell}{p_1}\cdot\frac{w^h}{x_1^h}
\end{aligned}\tag{D.1}$$

with $\varepsilon_{11}^h < 0$, $\varepsilon_{12}^h > 0$ and $\varepsilon_{1L}^h > 0$; the elasticities ε_{2j}^h are:

$$\begin{aligned}
\varepsilon_{21}^h &= -\beta\cdot\frac{b_1}{p_2}\cdot\frac{p_1}{x_2^h},\\
\varepsilon_{22}^h &= -\beta\cdot\frac{w^h\cdot(T-b_\ell)-p_1\cdot b_1}{(p_2)^2}\cdot\frac{p_2}{x_2^h},\\
\varepsilon_{2L}^h &= \beta\cdot\frac{T-b_\ell}{p_2}\cdot\frac{w^h}{x_2^h}
\end{aligned}\tag{D.2}$$

with $\varepsilon_{21}^h < 0$, $\varepsilon_{22}^h < 0$ and $\varepsilon_{2L}^h > 0$; the elasticities ε_{Lj}^h are:

$$\begin{aligned}
\varepsilon_{L1}^h &= \gamma\cdot\frac{b_1}{w^h}\cdot\frac{p_1}{L^h},\\
\varepsilon_{L2}^h &= \gamma\cdot\frac{b_2}{w^h}\cdot\frac{p_2}{L^h},\\
\varepsilon_{LL}^h &= -\gamma\cdot\frac{\sum_{i=1,2} p_i\cdot b_i}{(w^h)^2}\cdot\frac{w^h}{L^h}
\end{aligned}\tag{D.3}$$

with $\varepsilon_{L1}^h > 0$ and $\varepsilon_{L2}^h < 0$.

If the Government redistribute resources among individuals (subsection 3.1), in case of a log transformation of the *KR-SG* utility function the elasticities ε_{1j}^h, with $j = 1, 2, L$, in (25) and considered in the *FOCs* (27) are:

$$\begin{aligned} \varepsilon_{11}^h &= -\alpha \cdot \tfrac{m^h \cdot (T - b_\ell) - q_2 \cdot b_2}{(q_1)^2} \cdot \tfrac{q_1}{x_1^h}, \\ \varepsilon_{12}^h &= -\alpha \cdot \tfrac{b_2}{q_1} \cdot \tfrac{q_2}{x_1^h}, \\ \varepsilon_{1L}^h &= \alpha \cdot \tfrac{T - b_\ell}{q_1} \cdot \tfrac{m^h}{x_1^h}; \end{aligned} \tag{D.4}$$

the elasticities ε_{2j}^h are:

$$\begin{aligned} \varepsilon_{21}^h &= -\beta \cdot \tfrac{b_1 - x_G}{q_2} \cdot \tfrac{q_1}{x_2^h}, \\ \varepsilon_{22}^h &= -\beta \cdot \tfrac{m^h \cdot (T - b_\ell) + q_1 \cdot x_G - q_1 \cdot b_1}{(q_2)^2} \cdot \tfrac{q_2}{x_2^h}, \\ \varepsilon_{2L}^h &= \beta \cdot \tfrac{T - b_\ell}{q_2} \cdot \tfrac{m^h}{x_2^h}; \end{aligned} \tag{D.5}$$

and the elasticities ε_{Lj}^h are:

$$\begin{aligned} \varepsilon_{L1}^h &= \gamma \cdot \tfrac{b_1 - x_G}{m^h} \cdot \tfrac{q_1}{L^h}, \\ \varepsilon_{L2}^h &= \gamma \cdot \tfrac{b_2}{m^h} \cdot \tfrac{q_2}{L^h}, \\ \varepsilon_{LL}^h &= -\gamma \cdot \tfrac{\sum_{i=1,2} q_i \cdot b_i - q_1 \cdot x_G}{(m^h)^2} \cdot \tfrac{m^h}{L^h}. \end{aligned} \tag{D.6}$$

Moreover, the (gross) marginal social evaluations of h's utility of good $i = 1, 2$, labour L and the publicly provided good G considered in the *FOCs* (27) are:

$$\begin{aligned} \beta_i^h &= -\beta^h \cdot \tfrac{1}{m^h \cdot (T - b_\ell) + q_1 \cdot x_G - \sum_{i=1,2} q_i \cdot b_i} \cdot x_i^h, \\ \beta_L^h &= \beta^h \cdot \tfrac{1}{m^h \cdot (T - b_\ell) + q_1 \cdot x_G - \sum_{i=1,2} q_i \cdot b_i} \cdot L^h, \\ \beta_G^h &= \beta^h \cdot \tfrac{1}{m^h \cdot (T - b_\ell) + q_1 \cdot x_G - \sum_{i=1,2} q_i \cdot b_i} \cdot q_1. \end{aligned} \tag{D.7}$$

With moral hazard (section 4), in case of a log transformation of the *KR-SG* utility function the elasticities $\widetilde{\varepsilon}_{1j}^h$, with $j = 1, 2, L$, in (25) and considered in the *FOCs* (27) are:

$$\begin{aligned} \widetilde{\varepsilon}_{11}^h &= \widetilde{S}_{11}^h \cdot \tfrac{\widetilde{q}_1}{\widetilde{x}_1^h} = -\alpha \cdot \tfrac{\widetilde{x}_1^h}{p_1 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \tfrac{\widetilde{q}_1}{\widetilde{x}_1^h}, \\ \widetilde{\varepsilon}_{12}^h &= \widetilde{S}_{12}^h \cdot \tfrac{\widetilde{q}_2}{\widetilde{x}_1^h} = -\alpha \cdot \tfrac{\widetilde{x}_2^h}{p_2 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \tfrac{\widetilde{q}_2}{\widetilde{x}_1^h}, \\ \widetilde{\varepsilon}_{1L}^h &= \widetilde{I}_{1L}^h \cdot \tfrac{\widetilde{m}^h}{\widetilde{x}_1^h} = \alpha \cdot \tfrac{\widetilde{L}^h}{w^h \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \tfrac{\widetilde{m}^h}{x_1^h}; \end{aligned} \tag{D.8}$$

the elasticities $\widetilde{\varepsilon}^h_{2j}$ are:

$$\begin{aligned}
\widetilde{\varepsilon}^h_{21} &= \widetilde{S}^h_{21} \cdot \frac{\widetilde{q}_1}{\widetilde{x}^h_2} = -\beta \cdot \frac{\widetilde{x}^h_1}{p_1 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{q}_1}{\widetilde{x}^h_2}, \\
\widetilde{\varepsilon}^h_{22} &= \widetilde{S}^h_{22} \cdot \frac{\widetilde{q}_2}{\widetilde{x}^h_2} = -\beta \cdot \frac{\widetilde{x}^h_2}{p_2 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{q}_2}{\widetilde{x}^h_2}, \\
\widetilde{\varepsilon}^h_{2L} &= \widetilde{I}^h_{2L} \cdot \frac{\widetilde{m}^h}{\widetilde{x}^h_2} = \beta \cdot \frac{\widetilde{L}^h}{w^h \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{m}^h}{\widetilde{x}^h_2};
\end{aligned} \tag{D.9}$$

and the elasticities $\widetilde{\varepsilon}^h_{Lj}$ are:

$$\begin{aligned}
\widetilde{\varepsilon}^h_{L1} &= \widetilde{S}^h_{L1} \cdot \frac{\widetilde{q}_1}{\widetilde{L}^h} = \gamma \cdot \frac{\widetilde{x}^h_1}{p_1 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{q}_1}{\widetilde{L}^h}, \\
\widetilde{\varepsilon}^h_{L2} &= \widetilde{S}^h_{L2} \cdot \frac{\widetilde{q}_2}{\widetilde{L}^h} = \gamma \cdot \frac{\widetilde{x}^h_2}{p_2 \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{q}_2}{\widetilde{L}^h}, \\
\widetilde{\varepsilon}^h_{LL} &= \widetilde{I}^h_{LL} \cdot \frac{\widetilde{m}^h}{\widetilde{L}^h} = -\gamma \cdot \frac{\widetilde{L}^h}{w^h \cdot \left(\alpha \cdot \frac{\widetilde{q}_1}{p_1} + \beta \cdot \frac{\widetilde{q}_2}{p_2} + \gamma \cdot \frac{\widetilde{m}^h}{w^h}\right)} \cdot \frac{\widetilde{m}^h}{\widetilde{L}^h}.
\end{aligned} \tag{D.10}$$

With moral hazard and adverse selection (section 5), in case of a log transformation of the *KR-SG* utility function the terms σ_1, σ_2, σ^A, $\hat{\sigma}^A$ and σ_G in the *FOCs* (40) are:

$$\begin{aligned}
\sigma_1 &= -\frac{1}{\check{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^A}{w^A}\right)} \cdot \check{x}^A_1 + \frac{1}{\hat{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^B}{w^B}\right)} \cdot \check{x}^B_1, \\
\sigma_2 &= -\frac{1}{\check{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^A}{w^A}\right)} \cdot \check{x}^A_2 + \frac{1}{\hat{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^B}{w^B}\right)} \cdot \check{x}^B_2, \\
\sigma^A &= \check{\beta}^A_L = \frac{1}{\check{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^A}{w^A}\right)} \cdot \check{L}^A, \\
\hat{\sigma}^A &= -\frac{1}{\hat{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^B}{w^B}\right)} \cdot \check{L}^B, \\
\sigma_G &= \left(\frac{1}{\check{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^A}{w^A}\right)} - \frac{1}{\hat{\varphi}^A \cdot \left(\alpha \cdot \frac{\check{q}_1}{p_1} + \beta \cdot \frac{\check{q}_2}{p_2} + \gamma \cdot \frac{\check{m}^B}{w^B}\right)}\right) \cdot \check{q}_1.
\end{aligned} \tag{D.11}$$

E. Tax Evasion Fines

In what follows it is assumed the Government may audit both the individuals taking hidden actions (tax evasion by moral hazard) and the individuals having hidden information (tax evasion by adverse selection). Therefore, the Government intervenes to punish tax evasion both by moral hazard and by adverse selection.

The expected indirect utility of individuals taking hidden actions is:

$$E\left(\widetilde{\mathcal{V}}^h\right) = (1-\pi) \cdot \widetilde{\mathcal{V}}\left(\mathbf{t}, \tau^h, x_G; \mathbf{p}, w^h\right) + \pi \cdot \widetilde{\mathcal{V}}\left(\mathbf{t}, \tau^h, x_G, \widetilde{F}^h; \mathbf{p}, w^h\right) \tag{E.1}$$

with $\widetilde{F}^h$ the tax evasion by moral hazard fines.

If type A individuals do not take hidden actions, but have hidden information, their expected indirect utility is:

$$E\left(\hat{\mathcal{V}}^A\right) = (1-\pi)\cdot\hat{\mathcal{V}}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right) + \pi\cdot\hat{\mathcal{V}}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right) \quad \text{(E.2)}$$

with $\hat{F}^A$ being the fine for tax evasion by adverse selection.

The Government decides not only the optimal redistribution system, but also the amount of fines $\widetilde{F}^h$ and $\hat{F}^A$. Therefore, the GP is:

$$\begin{aligned}
\breve{W} \quad &= \max_{\breve{\mathbf{t}},\breve{\boldsymbol{\tau}},\breve{x}_G,\widetilde{\mathbf{F}},\hat{F}^A} W\left(\pi^A\cdot\breve{\mathcal{V}}^A,\pi^B\cdot\breve{\mathcal{V}}^B\right) \\
s.t. \quad &\sum_{h=A,B}\pi^h\cdot\left(\breve{\tau}^h\cdot\breve{L}^h+\sum_{i=1,2}\breve{t}_i\cdot\breve{x}_i^h\right)-p_1\cdot\breve{x}_G\geq 0\left(\breve{\lambda}_G\right) \\
&\breve{\mathcal{V}}^h\geq E\left(\widetilde{\mathcal{V}}^h\right)\left(\lambda_{MH}^h\right) \\
&\breve{\mathcal{V}}^A\geq E\left(\hat{\mathcal{V}}^A\right)\left(\lambda_{AS}\right),
\end{aligned} \quad \text{(E.3)}$$

where $\breve{\mathcal{V}}^h = \mathcal{V}(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$ and with $\breve{x}_i^h = x_i(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$ and $\breve{L}^h = L(\breve{\mathbf{t}},\breve{\tau}^h,\breve{x}_G;\mathbf{p},w^h)$. Since

$$\begin{aligned}
-\lambda_{MH}^h\cdot\frac{\partial E\left(\widetilde{\mathcal{V}}^h\right)}{\partial\widetilde{F}^h} &= 0 \quad \rightarrow \lambda_{MH}^h = 0, \\
-\lambda_{AS}\cdot\frac{\partial E\left(\hat{\mathcal{V}}^A\right)}{\partial\hat{F}^A} &= 0 \quad \rightarrow \lambda_{AS} = 0
\end{aligned} \quad \text{(E.4)}$$

the problem entails the *FOCs* (27), where the elasticities are the ones in (D.4), (D.5) and (D.6) and where the (gross) marginal social evaluations of the h's utility are the ones in (D.7); therefore $\widetilde{F}^h$ are determined from $\breve{\mathcal{V}}^h = E\left(\widetilde{\mathcal{V}}^h\right)$ and $\hat{F}^A$ is determined from $\breve{\mathcal{V}}^A = E\left(\hat{\mathcal{V}}^A\right)$.

Considering a log transformation of the *KR-SG* utility function, the demand system for the goods $i=1,2$ and leisure ℓ of individuals taking hidden actions and not audited is:

$$\begin{aligned}
\widetilde{x}_1^h + x_G &= b_1 + \alpha\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{p_1}, \\
\widetilde{x}_2^h &= b_2 + \beta\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{p_2}, \\
T-\widetilde{L}^h &= b_\ell + \gamma\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)}{w^h}
\end{aligned} \quad \text{(E.5)}$$

with $\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right) = \left(m^h\cdot(T-b_\ell)+q_1\cdot x_G-\sum_{i=1,2}q_i\cdot b_i\right)\cdot\left(\alpha\cdot\frac{p_1}{q_1}+\beta\cdot\frac{p_2}{q_2}+\gamma\cdot\frac{w^h}{m^h}\right)$, while when audited it is:

$$\begin{aligned}\widetilde{x}_1^h+x_G &= b_1+\alpha\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G,\widetilde{F}^h;\mathbf{p},w^h\right)}{p_1},\\ \widetilde{x}_2^h &= b_2+\beta\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G,\widetilde{F}^h;\mathbf{p},w^h\right)}{p_2},\\ T-\widetilde{L}^h &= b_\ell+\gamma\cdot\frac{\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G,\widetilde{F}^h;\mathbf{p},w^h\right)}{w^h}\end{aligned} \tag{E.6}$$

with $\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G,\widetilde{F}^h;\mathbf{p},w^h\right)=\widetilde{\psi}\left(\mathbf{t},\tau^h,x_G;\mathbf{p},w^h\right)-\widetilde{F}^h$.

If type A individuals do not take hidden actions, but have hidden information, their demand system for the goods $i=1,2$ and leisure ℓ when not audited is:

$$\begin{aligned}\hat{x}_1^A+x_G &= b_1+\alpha\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right)}{p_1},\\ \hat{x}_2^A &= b_2+\beta\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right)}{p_2},\\ T-\hat{L}^A &= b_\ell+\gamma\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right)}{w^A}\end{aligned} \tag{E.7}$$

with $\hat{\psi}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right) = \left(w^A-w^B\right)\cdot(T-b_\ell)+ \\ +\left(m^B\cdot(T-b_\ell)+q_1\cdot x_G-\sum_{i=1,2}q_i\cdot b_i\right)\cdot\left(\alpha\cdot\frac{p_1}{q_1}+\beta\cdot\frac{p_2}{q_2}+\gamma\cdot\frac{w^B}{m^B}\right)$, while when audited it is:

$$\begin{aligned}\hat{x}_1^A+x_G &= b_1+\alpha\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{p_1},\\ \hat{x}_2^A &= b_2+\beta\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{p_2},\\ T-\hat{L}^A &= b_\ell+\gamma\cdot\frac{\hat{\psi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)}{w^A}\end{aligned} \tag{E.8}$$

with $\hat{\psi}\left(\mathbf{t},\tau^B,x_G,\hat{F}^A;\mathbf{p},\mathbf{w}\right)=\hat{\psi}\left(\mathbf{t},\tau^B,x_G;\mathbf{p},\mathbf{w}\right)-\hat{F}^A$.

Considering an audit probability $\pi=0.5$ allows to obtain in the Utilitarian case the results displayed in tables 2 and 3 with $\widetilde{F}^A=0.1459$, $\widetilde{F}^B=0.1645$ and $\hat{F}^A=15.0613$ and in the Rawlsian case the results displayed in tables 4 and 5 with $\widetilde{F}^A=0.1492$, $\widetilde{F}^B=0.1689$ and $\hat{F}^A=15.2370$.

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